1. Problem

What is the derivative of $f(x) = x^4 e^{3x}$, evaluated at x = 0.69?

Solution

Using the product rule for $f(x) = g(x) \cdot h(x)$, where $g(x) := x^4$ and $h(x) := e^{3x}$, we obtain

$$f'(x) = [g(x) \cdot h(x)]' = g'(x) \cdot h(x) + g(x) \cdot h'(x)$$

$$= 4x^{4-1} \cdot e^{3x} + x^4 \cdot e^{3x} \cdot 3$$

$$= e^{3x} \cdot (4x^3 + 3x^4)$$

$$= e^{3x} \cdot x^3 \cdot (4 + 3x).$$

Evaluated at x = 0.69, the answer is

$$e^{3 \cdot 0.69} \cdot 0.69^3 \cdot (4 + 3 \cdot 0.69) = 15.802491.$$

Thus, rounded to two digits we have f'(0.69) = 15.80.