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CSCI 3104

CPU: 2.8 GHz Intel Core i7

Ram: 16 GB 1600 MHz DDR3

OSX Yosemite

Homework #3

On my honor, as a University of Colorado at Boulder student, I have neither given nor received any unauthorized help.

1.

1. Algorithm A: $a=4, b=2, d=1$
 $T(n) = 4 \cdot T(n/2) + O(n)$

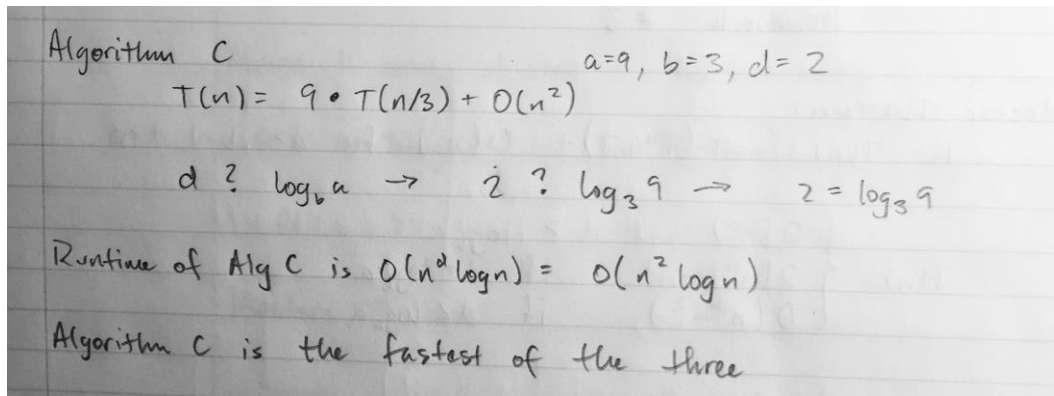
$d \geq \log_b a \rightarrow 1 \geq \log_2 4 \rightarrow 1 < \log_2 4$

Therefore runtime of Alg A is $O(n^{\log_b a}) = O(n^{\log_2 4}) = O(n^2)$

Algorithm B:
 $T(n) = 2 \cdot T(n-1) + O(1)$

$T(1) =$
 $T(2) = 2 \cdot T(1)$
 $T(3) = 2 \cdot T(2) = 2 \cdot 2 \cdot T(1)$
 $T(4) = 2 \cdot T(3) = 2 \cdot 2 \cdot 2 \cdot T(1)$

Runtime of Alg B is $O(2^n)$



2.

- a. Find the minimum and maximum numbers in the array.
- b. Create a new Array A of size M both set at 0.
- c. Scan through the array, and for each element $X[i]$, increment $A[\min\{X[i]\} + X[i]]$.
- d. Create a new array $Y[1 \dots n]$.
- e. Scan through $A[]$ again, for each element $A[i]$, put $A[i]$ values of I into the next empty slots of $Y[]$.

The output of the sorted array is found by scanning through $A[i]$ and outputting the values in order. It takes a constant amount of work to scan through arrays of size n and M so it takes $O(n + M)$ time.

3.

3. k-way merge sort

- k sorted arrays, each with n elements
- combine into a single sorted array of kn elements

a

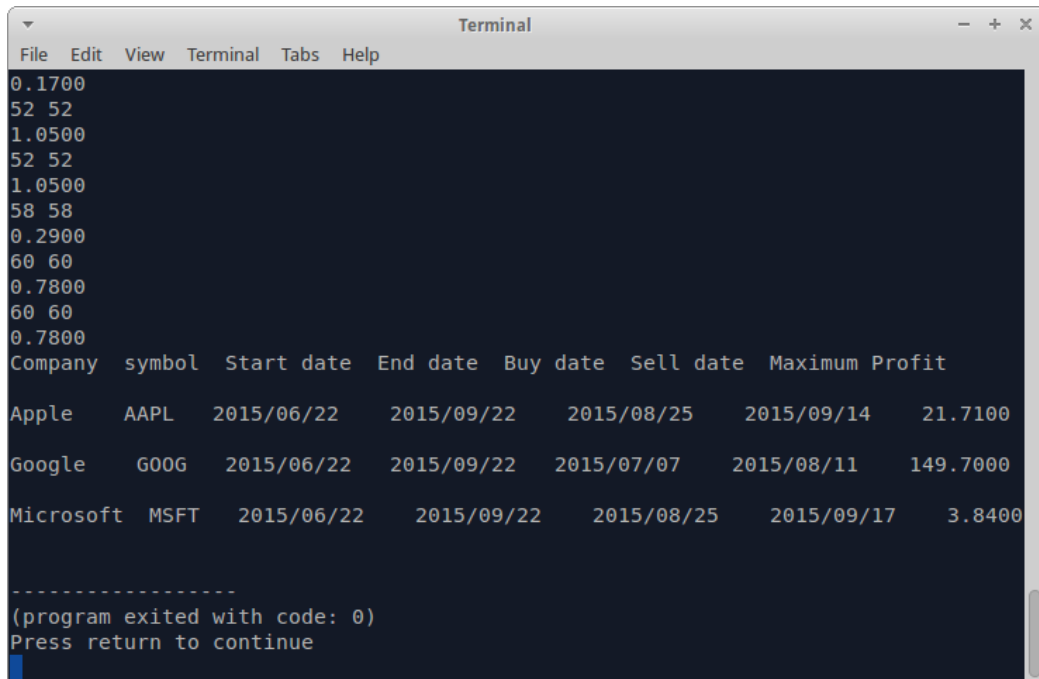
Merge of A_1 and A_2 takes $O(n_1 + n_2) \rightarrow (n+n) \rightarrow 2n$
 Merge of A_1, A_2 and A_3 : $2n+n$
 Merge of A_1, A_2, A_3 and A_4 : $3n+n$
 Merge of k arrays: $(n+n) + (2n+n) + (3n+n) + \dots + ((k-1)n+n)$
 $= 2n + 3n + 4n + \dots + kn - n + n$
 $= 2n + 3n + 4n + \dots + kn$
 $= O(k^2 n)$

3b Instead of doing it the first way, merge the first array w/ the second, the third array with the fourth, so on for k arrays.

- To merge k arrays of size n into $k/2$ arrays of size $2n$ takes $O(kn)$

- This occurs a number of $O(\log k)$ times
- Total run time is $O(\log k) \cdot O(kn) = O(kn \log k)$

4.



```
0.1700
52 52
1.0500
52 52
1.0500
58 58
0.2900
60 60
0.7800
60 60
0.7800
Company  symbol  Start date  End date  Buy date  Sell date  Maximum Profit
Apple    AAPL    2015/06/22   2015/09/22   2015/08/25   2015/09/14   21.7100
Google   GOOG    2015/06/22   2015/09/22   2015/07/07   2015/08/11  149.7000
Microsoft MSFT    2015/06/22   2015/09/22   2015/08/25   2015/09/17    3.8400

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