

Andrew Rutherford

CSCI 3104

CPU: 2.8 GHz Intel Core i7

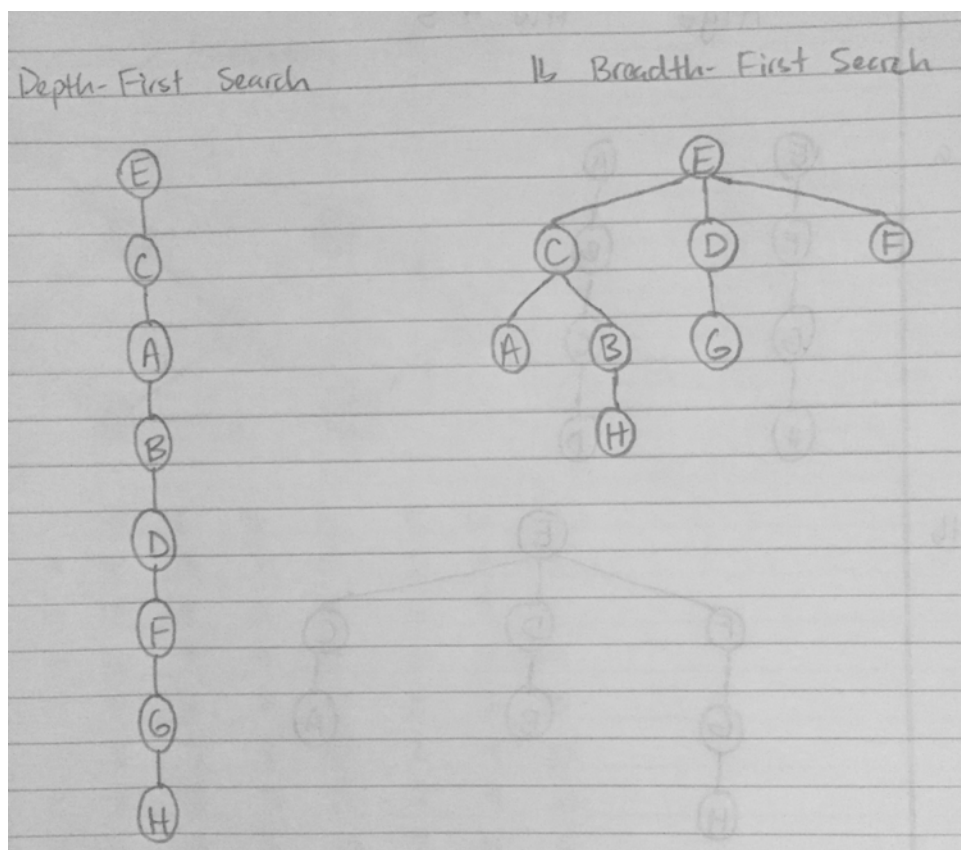
Ram: 16 GB 1600 MHz DDR3

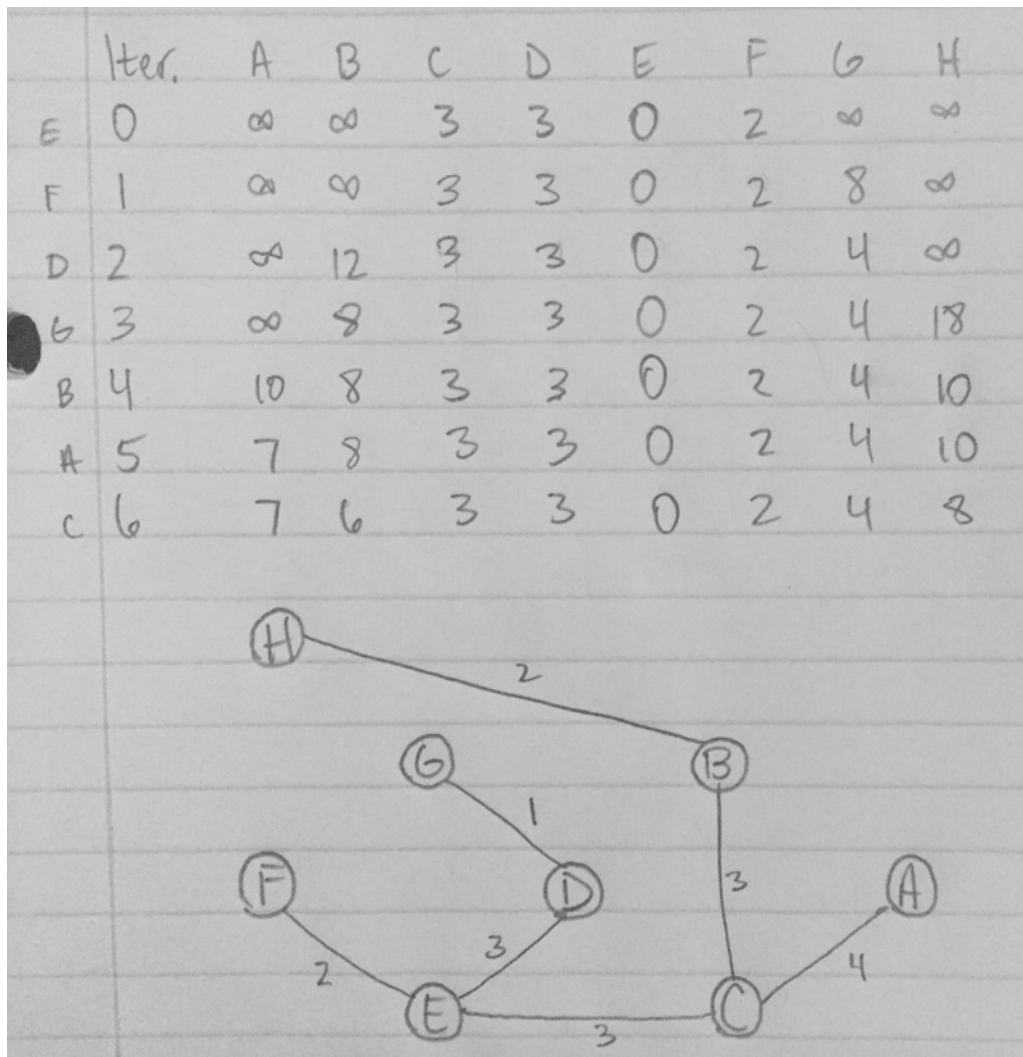
OSX Yosemite

#### Homework #4

On my honor, as a University of Colorado at Boulder student, I have neither given nor received any unauthorized help.

1.





- Perform a Breadth-First Search starting from  $u$ , and create variable  $\text{num\_paths}(x)$  for the number of paths from  $u$  to  $x$ , for all vertices  $x$ . If  $x_1, x_2, x_3, \dots, x_k$  are vertices of depth  $l$  in the BFS tree, and  $x$  is a vertex of depth  $l + 1$  such that  $(x_1, x), \dots, (x_k, x)$  are contained in  $E$ . So,

$$\text{num\_paths}(x) = \text{num\_paths}(x_1) + \dots + \text{num\_paths}(x_k).$$

$\text{num\_paths}(x) = 0$  for all vertices  $x \neq u$  and  $\text{num\_paths}(u) = 1$ .

$\text{num\_paths}(y) = \text{num\_paths}(y) + \text{num\_paths}(x)$  for each edge  $(x, y)$  that goes down a level in the tree. BFS is only changed to do one extra operation per edge, so it takes linear time.
- If  $P$  is the shortest path from vertex  $u$  to  $v$  passing through  $v_0$ , then between  $v_0$  and  $v$ ,  $P$  must follow the shortest path from  $v_0$  to  $v$ . Also, between  $u$  and  $v_0$ ,  $P$  must also follow the shortest

path from  $v_0$  to  $u$  in the reverse graph, which must exist because the graph is strongly connected.

The shortest path from a node  $u$  to a node  $v$  through node  $v_0$  can be found by running Dijkstra's algorithm twice, once on graph  $G$ , and once on the reverse graph of  $G$ . The time complexity is dominated by looking up all the  $O(|V|^2)$  pairs of distances.