Andrew Rutherford

CSCI 3302

Homework 1

1. A standard lawnmower has two degrees of freedom: it can move forward/backward, and can pivot side to side if rotated about the rear axle. You are still able to mow your entire lawn because doing so requires balancing the lawnmower on the rear two wheels, allowing for sideward movement.
2. Objects driving on a plane can have a maximum of four degrees of freedom

3 a

$$\vec{a} \bullet \vec{b} = \cos\theta$$

$$\begin{pmatrix} \cos 45 \\ -\sin 45 \\ 0 \end{pmatrix} \bullet \begin{pmatrix} \sin 45 \\ \cos 45 \\ 0 \end{pmatrix} = \cos\theta$$

$$\left(\left(\frac{\sqrt{2}}{2}\right) \cdot \left(\frac{\sqrt{2}}{2}\right)\right) + \left(\left(-\frac{\sqrt{2}}{2}\right) \cdot \left(\frac{\sqrt{2}}{2}\right)\right) + (0) = \cos\theta$$

$$2/4 \bullet -2/4 = \cos\theta \rightarrow 0 = \cos\theta$$

$$\arccos(\cos\theta) = 0 \rightarrow \theta = \arccos(0) \rightarrow \theta = \pi/2$$

b $(0,0,1)$

4 a

$$ {}^{A}X_B \qquad {}^{A}Y_B \qquad {}^{A}Z_B $$

$$\begin{pmatrix} X_B \cdot X_A \\ X_B \cdot Y_A \\ X_B \cdot Z_A \end{pmatrix} \quad \begin{pmatrix} Y_B \cdot X_A \\ Y_B \cdot Y_A \\ Y_B \cdot Z_A \end{pmatrix} \quad \begin{pmatrix} Z_B \cdot X_A \\ Z_B \cdot Y_A \\ Z_B \cdot Z_A \end{pmatrix}$$

$$\begin{pmatrix} X_B \cdot X_A & Y_B \cdot X_A & Z_B \cdot X_A \\ X_B \cdot Y_A & Y_B \cdot Y_A & Z_B \cdot Y_A \\ X_B \cdot Z_A & Y_B \cdot Z_A & Z_B \cdot Z_A \end{pmatrix} = {}^{A}_{B}R$$

b

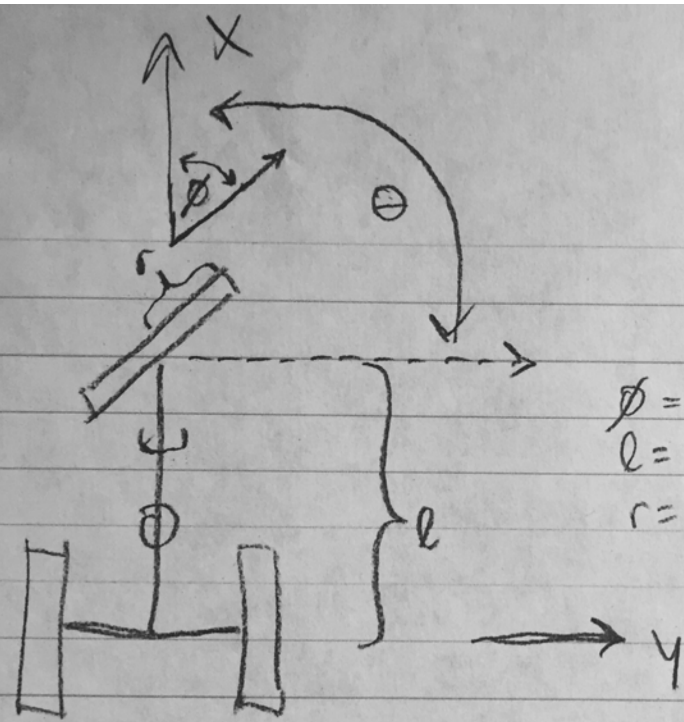
$$\left({}^{A}_{B}R\right)(0,1,0)^T = \begin{pmatrix} Y_B \cdot X_A \\ Y_B \cdot Y_A \\ Y_B \cdot Z_A \end{pmatrix}$$

c

$$ {}^{B}X_A \qquad {}^{B}Y_A \qquad {}^{B}Z_A $$

$$\begin{pmatrix} X_A \cdot X_B \\ X_A \cdot Y_B \\ X_A \cdot Z_B \end{pmatrix} \quad \begin{pmatrix} Y_A \cdot X_B \\ Y_A \cdot Y_B \\ Y_A \cdot Z_B \end{pmatrix} \quad \begin{pmatrix} Z_A \cdot X_B \\ Z_A \cdot Y_B \\ Z_A \cdot Z_B \end{pmatrix}$$

$$\begin{pmatrix} X_A \cdot X_B & Y_A \cdot X_B & Z_A \cdot X_B \\ X_A \cdot Y_B & Y_A \cdot Y_B & Z_A \cdot Y_B \\ X_A \cdot Z_B & Y_A \cdot Z_B & Z_A \cdot Z_B \end{pmatrix} = {}^{B}_{A}R$$



ϕ = front wheel angle
l = distance between axles
r = radius of front wheel

Forward velocity:

$$v_f = r\dot{\omega}\cos\phi$$

Angular Velocity

$$\dot{\theta} = \frac{r}{l}\dot{\omega}\sin\phi$$