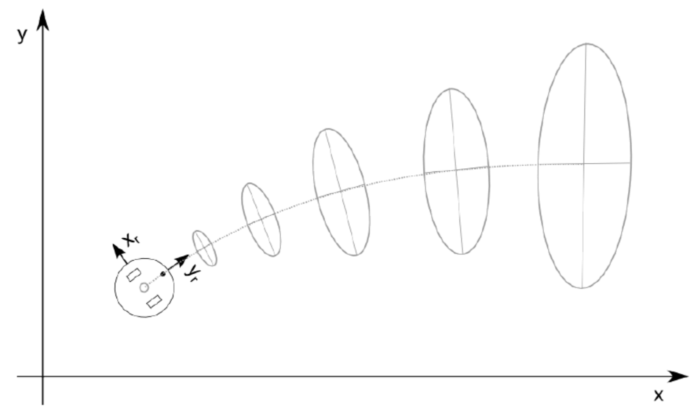
Localization

Chapter 9

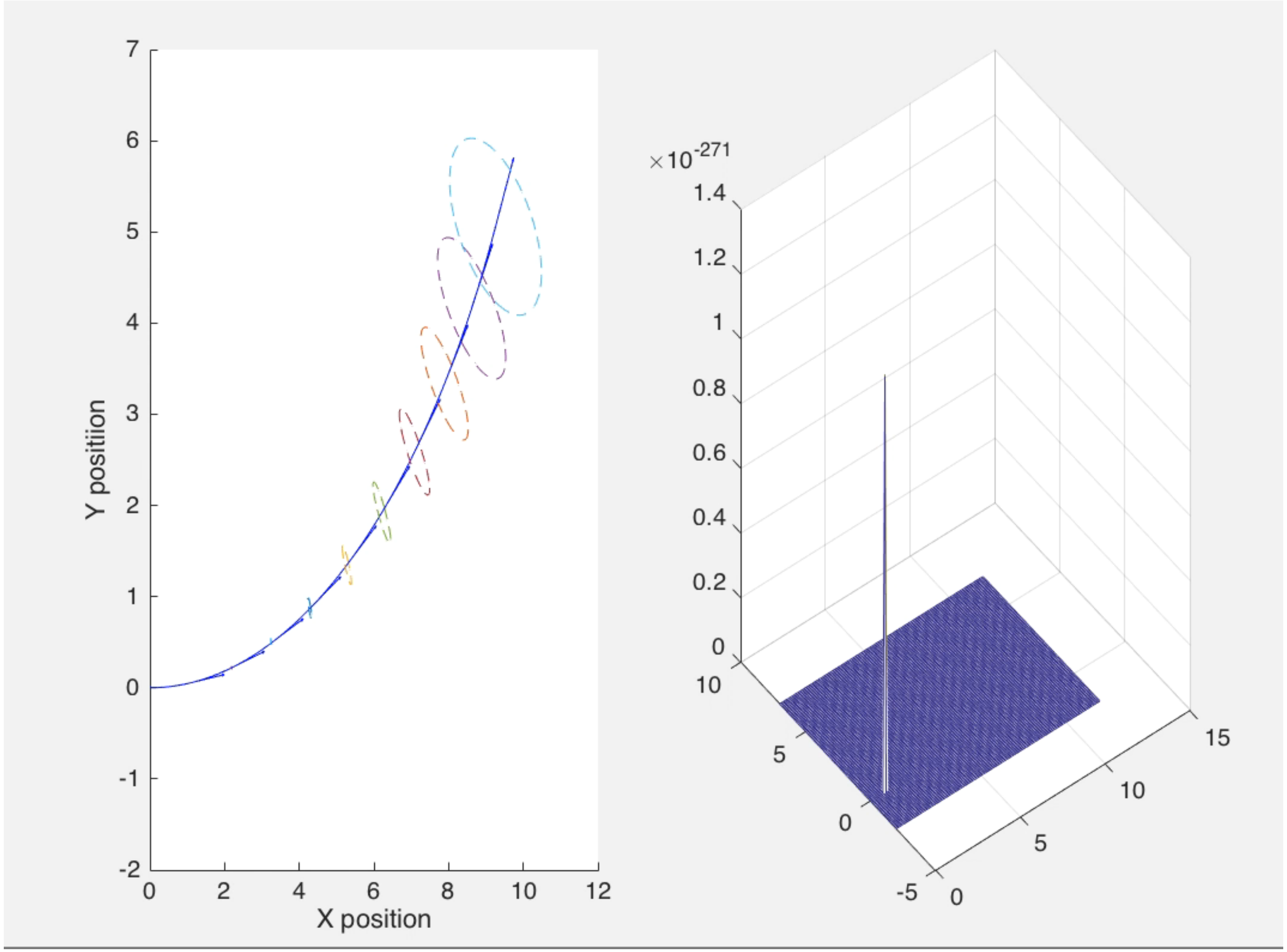
Last Week

- Random variables
- Probability distributions
  - The Uniform distribution
  - The Gaussian distribution
- Summing two random variables: convolution
- Error propagation
- Example: odometry



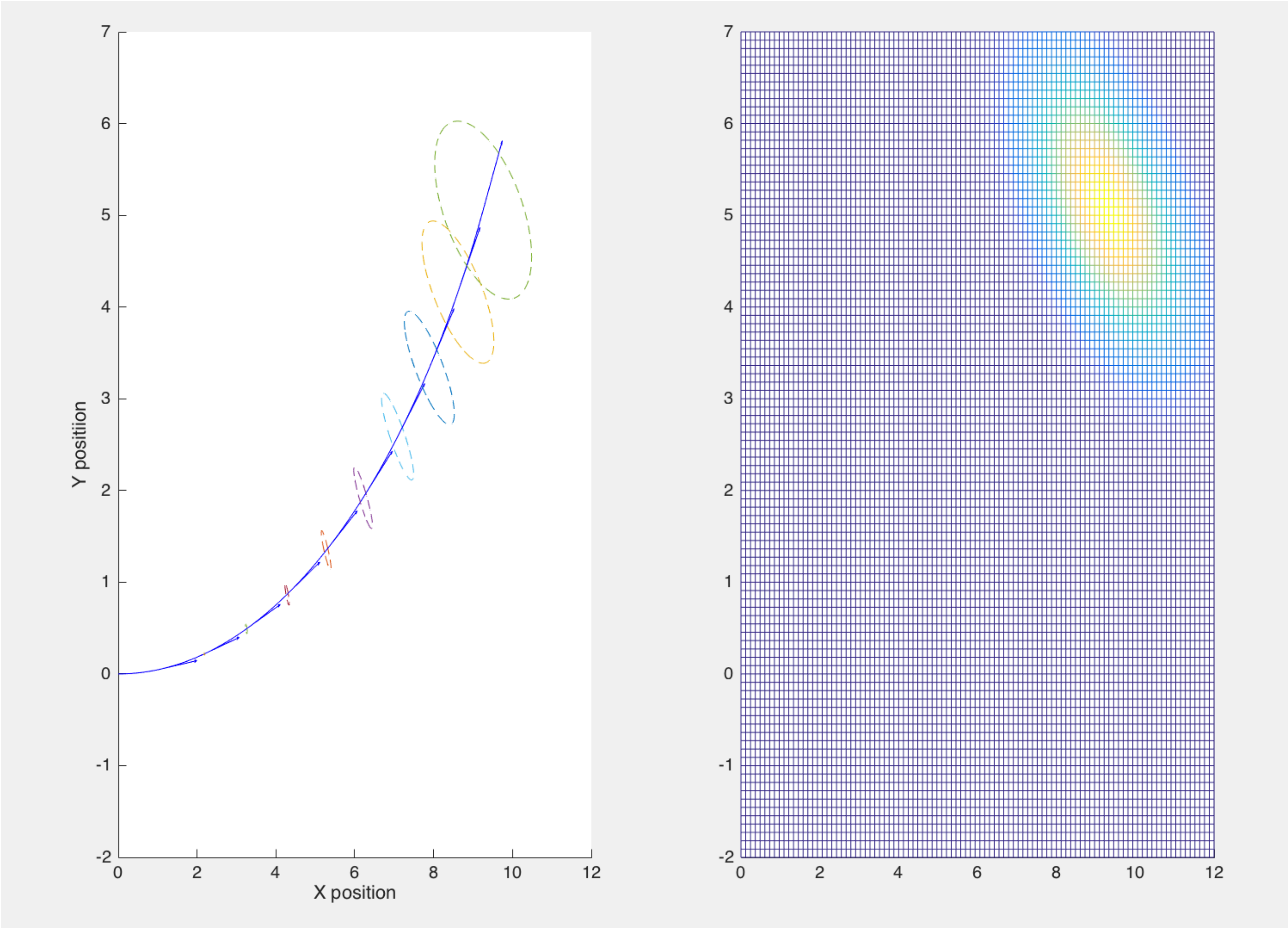
$$\sigma_y^2 = \frac{\partial df}{\partial x}^2 \sigma_x^2$$

Odometry



Y position
X position
7
6
5
4
3
2
1
0
-1
-2
0
2
4
6
8
10
12
×10^-271
1.4
1.2
1
0.8
0.6
0.4
0.2
0
10
5
0
-5
0
5
10
15

How can we use this information?



Bayes' rule

A and B happen together

$$P(A \cap B) = P(A)P(B|A) = P(B)P(A|B)$$

$$\downarrow$$

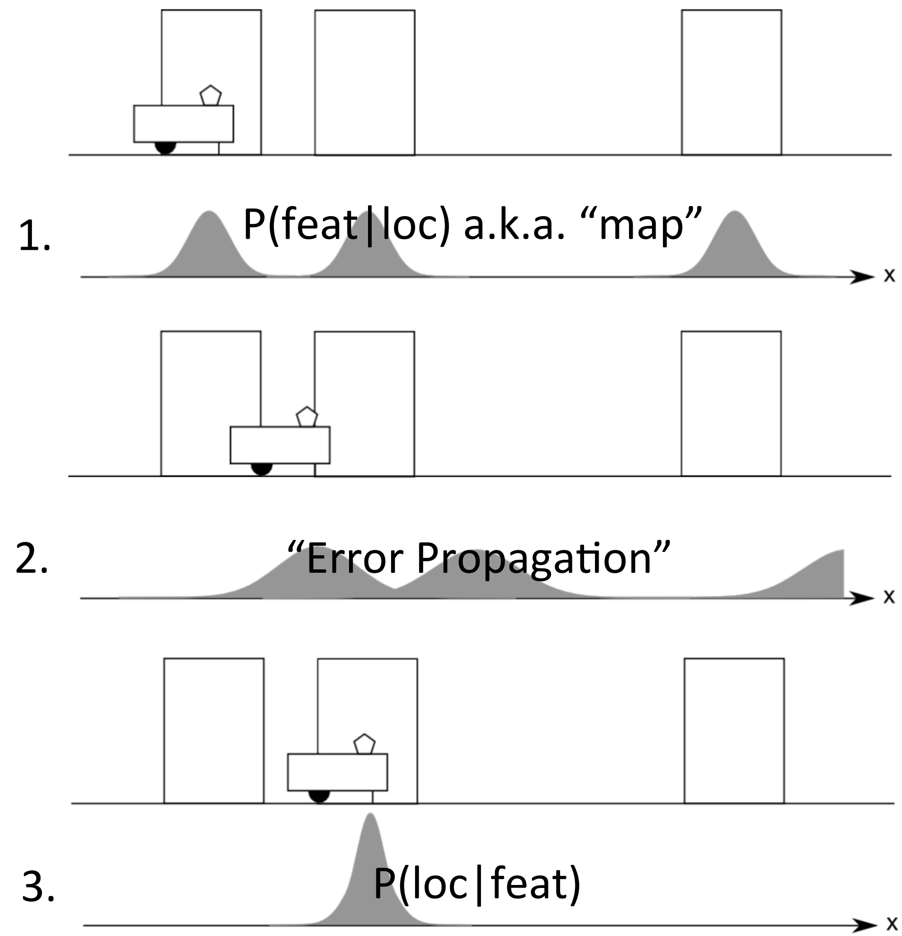
$$P(A|B) = \frac{P(A)P(B|A)}{P(B)}$$

Example: Localization

$$P(loc|feat) = \frac{P(loc)P(feat|loc)}{P(feat)}$$

Need:

1. P(loc) : Probability to be at a location *loc*
2. P(feat|loc): Probability to see a feature at *loc*
3. P(feat): Normalization factor



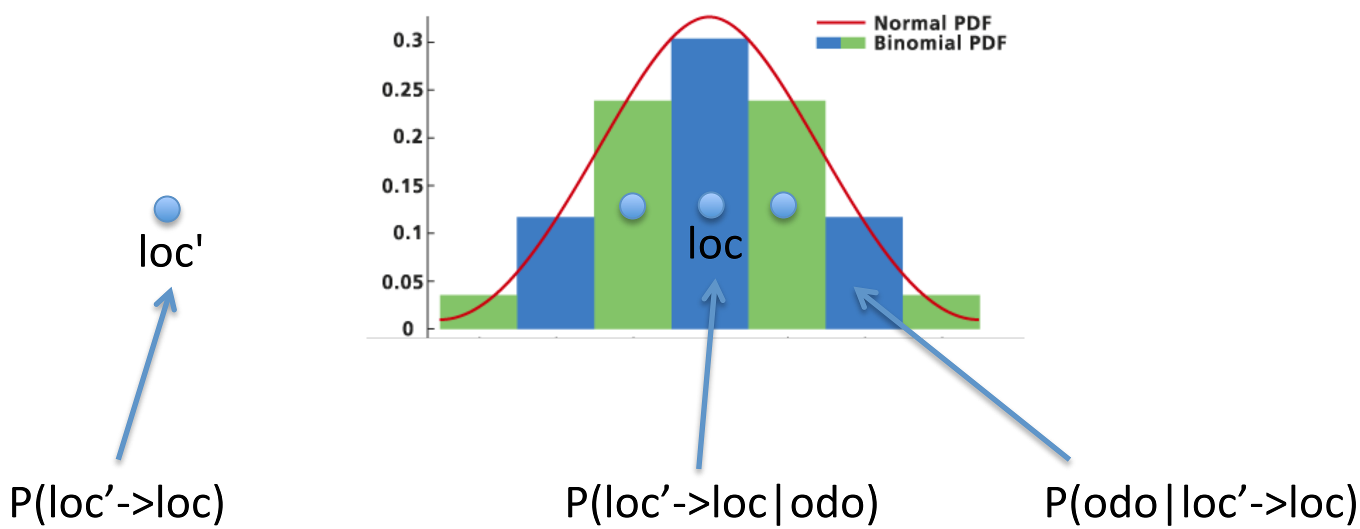
Bayes' rule allows us to exploit perception events given a map of the environment.

Discrete Error Propagation

$$P(A|B) = \frac{P(A)P(B|A)}{P(B)}$$

What is the probability that I drove 1m from *loc'* to *loc*, given that my odometer told me that I drove 1m?

$$P(loc' -> loc|odo) = P(loc' -> loc)P(odo|loc' -> loc)/P(odo)$$



Problem 1: I could end up anywhere, not only at loc.
Problem 2: I could have started out anywhere.

Solution

- 1. Calculate the probability to have arrived from anvwhere

$$P(loc|odo) = \sum_{loc'} P(loc' -> loc)P(odo|loc' -> loc)$$

- 2. Calculate this probability for all locations

P(loc1|odo), P(loc2|odo), ….

- => Sum of two probability distributions

Robot location * Robot Motion Model

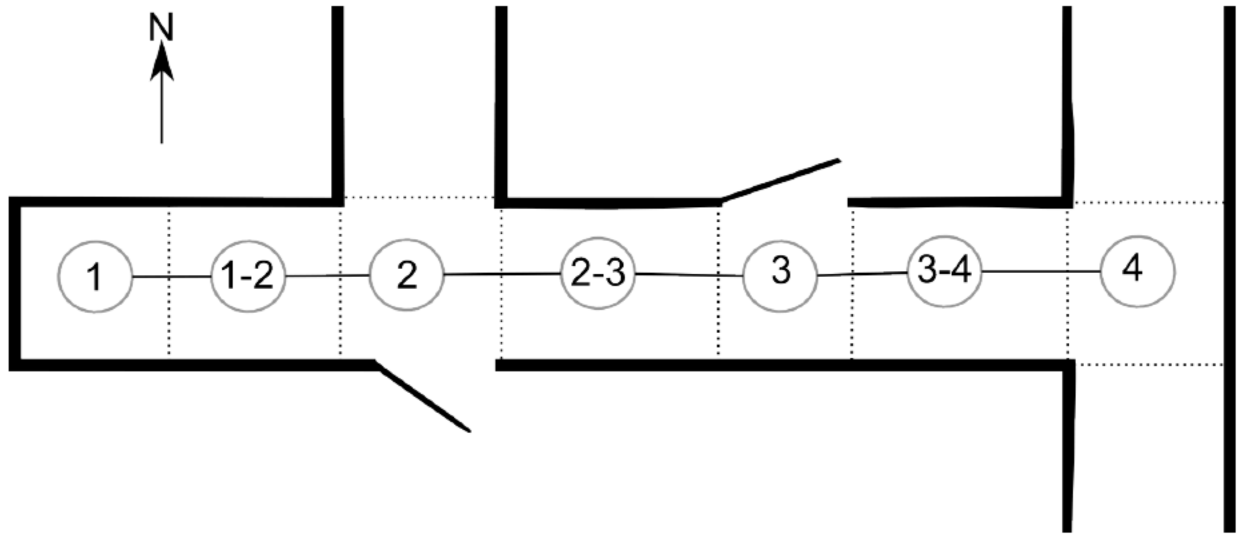
$$P(Z=z) = \sum_{k=-\infty}^{\infty} P(X=k)P(Y=z-k)$$

So far:

1. Action update: propagate the likelihood of the robot's location ("Error propagation")
2. Perception update: use Bayes' rule to update posterior probability *P(loc|feat)* using knowledge about where features are

This is known as *Markov Localization*. Problem: How to deal with sensor uncertainty?

1D Example (Nourbaksh, 1994)

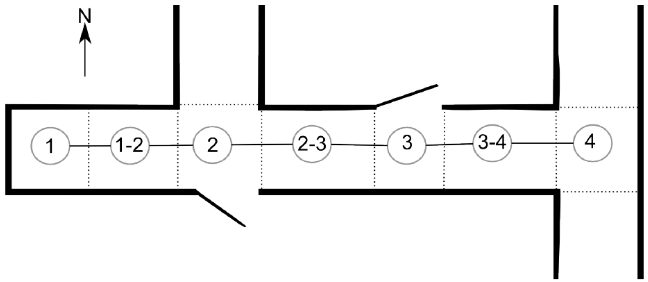


7 locations

Features and their likelihood P(feat|loc)

	Wall	Closed dr	Open dr	Open hwy	Foyer
Nothing detected	70%	40%	5%	0.1%	30%
Closed door detected	30%	60%	0%	0%	5%
Open door detected	0%	0%	90%	10%	15%
Open hallway detected	0%	0%	0.1%	90%	50%

Example



1. Initialization P(1-2)=0.8 and P(2-3)=0.2
2. Robot drives east until it reports
   "open hallway left and open door to its right"

	Wall	Closed dr	Open dr	Open hwy	Foyer
Nothing detected	70%	40%	5%	0.1%	30%
Closed door detected	30%	60%	0%	0%	5%
Open door detected	0%	0%	90%	10%	15%
Open hallway detected	0%	0%	0.1%	90%	50%

1. Perception update:

P(feat|1) =0; P(feat|1-2)=0; **P(feat|2)=0.9*0.9**; P(feat|2-3)=0; P(feat|3)=0; P(feat|3-4)=0; **P(feat|4)=0.9*0.1**

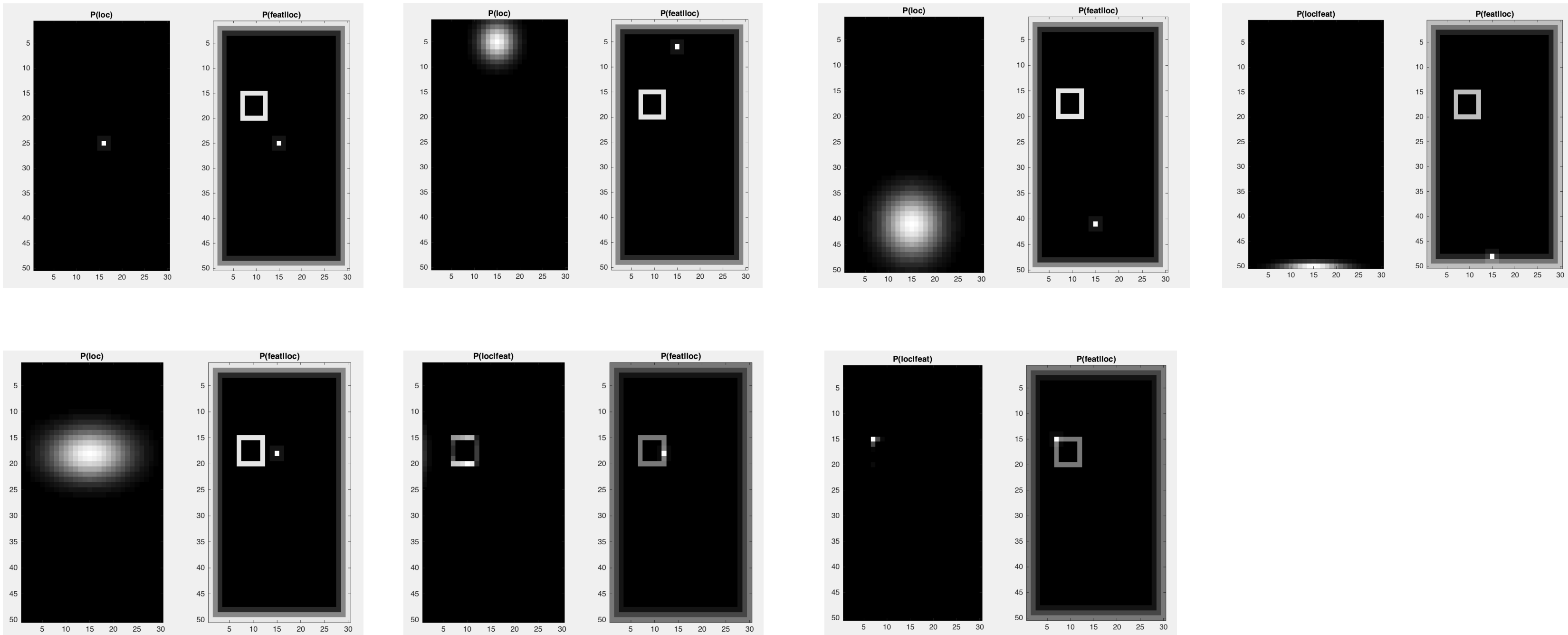
2. Action update:

P(2|feat)=P(2)P(feat|2) = **0.8***0.81=64.8%
P(4|feat)=P(4)*P(feat|4)=(**0.2***0.05*0.7*0.7*0.7+**0.8***0.001*0.05*0.7*0.7*0.5*0.7*0.7*0.7)*P(feat|4)=0.03%

3. Normalize

P(2) = 99.95%
P(4) = 0.005%

2D Example



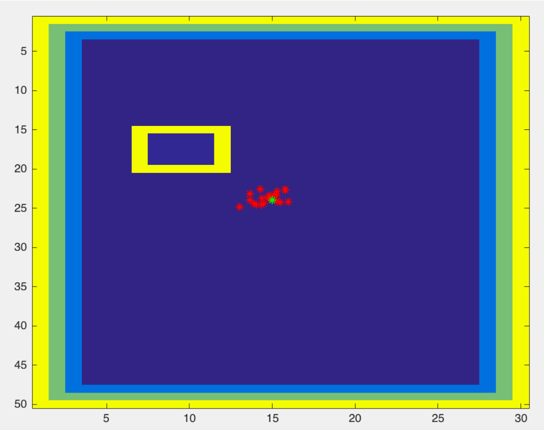
Problem: Need to update the probability for every cell on the map, which is very inefficient and does not scale.

(Matlab demo)

Solution: Sampling-based approach (Particle Filter)

1. Sample random “particles” around possible belief candidates
2. Action update: move particles like the robot, add noise using its motion model
3. Perception update: evaluate the likelihood of each particle to generate this perception event
4. Bookkeeping: remove unlikely particles and replicate likely ones

Demo



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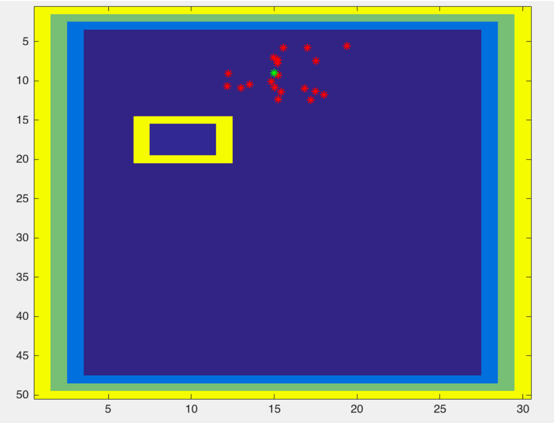

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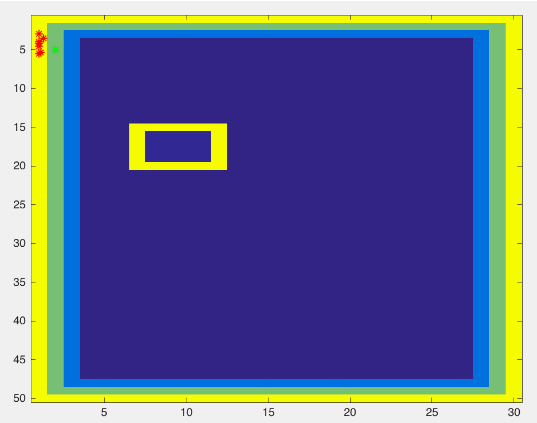

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Summary

- Bayes' rule provides a formal framework to use information about known features in a map
- Together with "error propagation" this is known as "Markov Localization"
- The problem can be computationally simplified using a "Particle Filter"
- Both are very general methods for state estimation, not limited to localization
- Both approaches are able to solve the "dropped robot" problem