1 Introduction

There are different forms of energy because there are different kinds of forces.

- Gravitational energy
- Elastic energy
- Thermal energy
- Radiant energy
- Electrical energy
- Nuclear energy
- Mass energy

Definition. **Force** is an agent of change, **energy** is often defined as the ability to do work, and **work** is one way of transferring energy from one system to another.

The **Law of Conservation of Energy** (aka the **First Law of Thermodynamics**) says that if you account for all its various forms, the total amount of energy in a given process will stay constant (*conserved*).

2 Work

Definition. Work is the energy transferred to or from an object via the application of force along a displacement.

If $\mathbf{F}$ is the force, and $d\boldsymbol{x}$ is an infinitesimal amount of displacement, then the work done is:

$$W = \int \mathbf{F} \cdot d\boldsymbol{x}$$

If $\mathbf{F}$ is constant, then it can be removed from the integral, and since $\mathbf{F} \cdot \mathbf{r} = (F \cos\theta) r$ for any angle θ, then $W = (F \cos\theta) r$.

Unit. The unit of work is the newton-meter $(\mathrm{N} \cdot \mathrm{m})$, which is also called a joule (J).

Note. Though work depends on two vectors ($\mathbf{F}$ and $\mathbf{r}$), work itself is *not* a vector.
This is a result of the dot product, whereby two vectors are multiplied to produce a scalar. Another result of the dot product is that only the component of the force that is parallel (or antiparallel) to the displacement does any work.

Note. Any force or component of a force that is perpendicular to the direction that an object actually moves cannot do work because the displacement in that direction is zero.

Note. If a graph of force as a function of position or displacement is given, the work done by the force is the definite integral whose boundaries correspond to the displacement, Δx.

3 Kinetic Energy

Definition. Kinetic energy, denoted by K, is the energy an object has by virtue of its motion.

Consider an object at rest ($v_0 = 0$) with mass m and a steady force $\mathbf{F}$ is being exerted on it, pushing it in a straight line. The object's acceleration is $a = \frac{F_{\text{net}}}{m}$, so after travelling distance Δx under the force, its final speed v is:

$$v^2 = v_0^2 + 2a(x - x_0) = 2a\Delta x = 2\frac{F_{\text{net}}}{m}\Delta x \Rightarrow F_{\text{net}}\Delta x = \frac{1}{2}mv^2$$

Note. This equation is given by the Big Five #5.

But the quantity $F_{net}\Delta x$ is the **total work** done by the force, so $W_r = \frac{1}{2}mv^2$. The work done on the object has transferred energy to it, in the amount $\frac{1}{2}mv^2$.

The energy an object possesses by virtue of its motion is therefore defined as $\frac{1}{2}mv^2$ and is called **kinetic energy**:

$$K = \frac{1}{2}mv^2$$

Unit. Kinetic energy is expressed in joules.

Note. Kinetic energy is a scalar quantity.

4 The Work-Energy Theorem

Theorem. The total work done on an object — equivalently, the work done by the net force — will equal its change in kinetic energy:

$$W_{\text{total}} = \Delta K$$

Note. This is the extension of the derivation in Section 3 (Kinetic Energy) to an object with a non-zero initial speed.

5 Potential Energy

Definition. Potential energy, denoted by U, is the energy an object has by virtue of its position.

Note. Potential energy is independent of motion.

Because there are different types of forces, there are different types of potential energy.

5.1 Gravitational Potential Energy

Definition. Gravitational potential energy, denoted by U_{grav}, is the energy stored by virtue of an object's position in a gravitational field.

Example. Consider a ball with mass m being lifted from the floor to a tabletop of height h. The work done by gravity during the ball's ascent was:

$$W_{\text{by gravity}} = -F_w h = -mgh$$

If an object of mass m is raised a height h (which is small enough that g stays essentially constant over this height change), then the increase in the object's U_{grav} is:

$$\Delta U_{\text{grav}} = mgh$$

Note. This equation does not depend on the path taken by the object. Thus, gravity is said to be a **conservative** force.

6 Conservation of Mechanical Energy

Definition. Mechanical energy, denoted by E, is the sum of an object's kinetic and potential energies:

$$E = K + U$$

If the net work done by nonconservative forces is zero, the total mechanical energy of an object is conserved:

$$K_{\mathrm{i}} + U_{\mathrm{i}} = K_{\mathrm{f}} + U_{\mathrm{f}}$$

7 Potential Energy Curves

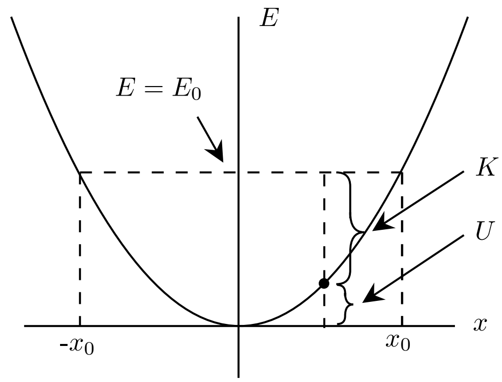
The behavior of a system can be analyzed if we are given a graph of its potential energy, $U(x)$, and its mechanical energy, E:

$$K + U = E \Rightarrow \frac{1}{2}mv^2 + U(x) = E$$

which can be solved for v, the velocity at position x:

$$v = \pm\sqrt{\frac{2}{m}[E - U(x)]}$$

Consider the following potential energy curve:



The graph shows how U varies with x. A particular value of the total energy, $E = E_0$, is also shown. Motion of an object whose potential energy is given by $U(x)$ and which has a mechanical energy of E_0 is confined to the region $-x_0 \leq x \leq x_0$, because only in this range is $E_0 \geq U(x)$. At each x in this range, the kinetic energy $K = E_0 - U(x)$ is positive. however, if $x > x_0$ (or if $x < -x_0$), then $U(x) > E_0$, which is physically impossible because the difference, $E_0 - U(x)$, which should give K, is negative.