# 1 Introduction

There are different forms of energy because there are different kinds of forces.

- Gravitational energy
- Elastic energy
- Thermal energy
- Radiant energy
- Electrical energy
- Nuclear energy
- Mass energy

**Definition.** **Force** is an agent of change, **energy** is often defined as the ability to do work, and **work** is one way of transferring energy from one system to another.

The **Law of Conservation of Energy** (aka the **First Law of Thermodynamics**) says that if you account for all its various forms, the total amount of energy in a given process will stay constant (*conserved*).

# 2 Work

**Definition.** Work is the energy transferred to or from an object via the application of force along a displacement.

If $\mathbf{F}$ is the force, and $d\boldsymbol{x}$ is an infinitesimal amount of displacement, then the work done is:

$$W = \int \mathbf{F} \cdot d\boldsymbol{x}$$

If $\mathbf{F}$ is constant, then it can be removed from the integral, and since $\mathbf{F} \cdot \mathbf{r} = (F \cos\theta)r$ for any angle $\theta$, then $W = (F \cos\theta)r$.

**Unit.** The unit of work is the newton-meter ($\mathrm{N} \cdot \mathrm{m}$), which is also called a joule (J).

**Note.** Though work depends on two vectors ($\mathbf{F}$ and $\mathbf{r}$), work itself is *not* a vector.
This is a result of the dot product, whereby two vectors are multiplied to produce a scalar. Another result of the dot product is that only the component of the force that is parallel (or antiparallel) to the displacement does any work.

**Note.** Any force or component of a force that is perpendicular to the direction that an object actually moves cannot do work because the displacement in that direction is zero.

**Note.** If a graph of force as a function of position or displacement is given, the work done by the force is the definite integral whose boundaries correspond to the displacement, $\Delta x$.

## 3 Kinetic Energy

**Definition.** Kinetic energy, denoted by $K$, is the energy an object has by virtue of its motion.

Consider an object at rest ($v_0 = 0$) with mass $m$ and a steady force $\mathbf{F}$ is being exerted on it, pushing it in a straight line. The object's acceleration is $a = \dfrac{F_{\text{net}}}{m}$, so after travelling distance $\Delta x$ under the force, its final speed $v$ is:

$$v^2 = v_0^2 + 2a(x - x_0) = 2a\Delta x = 2\frac{F_{\text{net}}}{m}\Delta x \Rightarrow F_{\text{net}}\Delta x = \frac{1}{2}mv^2$$

**Note.** This equation is given by the Big Five #5.

But the quantity $F_{net}\Delta x$ is the **total work** done by the force, so $W_r = \dfrac{1}{2}mv^2$. The work done on the object has transferred energy to it, in the amount $\dfrac{1}{2}mv^2$.

The energy an object possesses by virtue of its motion is therefore defined as $\dfrac{1}{2}mv^2$ and is called **kinetic energy**:

$$K = \frac{1}{2}mv^2$$

**Unit.** Kinetic energy is expressed in joules.

**Note.** Kinetic energy is a scalar quantity.

# 4 The Work-Energy Theorem

**Theorem.** The total work done on an object — equivalently, the work done by the net force — will equal its change in kinetic energy:

$$W_{\text{total}} = \Delta K$$

**Note.** This is the extension of the derivation in Section 3 (Kinetic Energy) to an object with a non-zero initial speed.

# 5 Potential Energy

**Definition.** Potential energy, denoted by $U$, is the energy an object has by virtue of its position.

**Note.** Potential energy is independent of motion.

Because there are different types of forces, there are different types of potential energy.

## 5.1 Gravitational Potential Energy

**Definition.** Gravitational potential energy, denoted by $U_{\text{grav}}$, is the energy stored by virtue of an object's position in a gravitational field.

**Example.** Consider a ball with mass $m$ being lifted from the floor to a tabletop of height $h$. The work done by gravity during the ball's ascent was:

$$W_{\text{by gravity}} = -F_w h = -mgh$$

If an object of mass $m$ is raised a height $h$ (which is small enough that $g$ stays essentially constant over this height change), then the increase in the object's $U_{\text{grav}}$ is:

$$\Delta U_{\text{grav}} = mgh$$

**Note.** This equation does not depend on the path taken by the object. Thus, gravity is said to be a **conservative** force.

# 6 Conservation of Mechanical Energy

**Definition.** Mechanical energy, denoted by $E$, is the sum of an object's kinetic and potential energies:

$$E = K + U$$

If the net work done by nonconservative forces is zero, the total mechanical energy of an object is conserved:

$$K_\mathrm{i} + U_\mathrm{i} = K_\mathrm{f} + U_\mathrm{f}$$

# 7 Potential Energy Curves

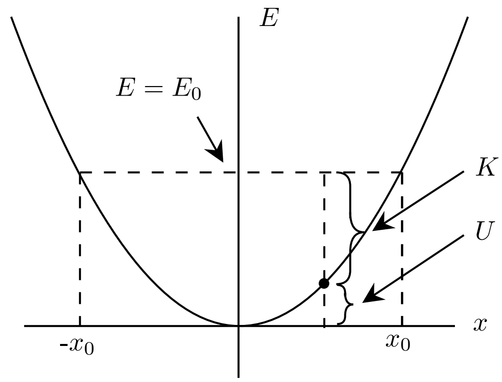
The behavior of a system can be analyzed if we are given a graph of its potential energy, $U(x)$, and its mechanical energy, $E$:

$$K + U = E \Rightarrow \frac{1}{2}mv^2 + U(x) = E$$

which can be solved for $v$, the velocity at position $x$:

$$v = \pm\sqrt{\frac{2}{m}[E - U(x)]}$$

Consider the following potential energy curve:




The graph shows how $U$ varies with $x$. A particular value of the total energy, $E = E_0$, is also shown. Motion of an object whose potential energy is given by $U(x)$ and which has a mechanical energy of $E_0$ is confined to the region $-x_0 \leq x \leq x_0$, because only in this range is $E_0 \geq U(x)$. At each $x$ in this range, the kinetic energy $K = E_0 - U(x)$ is positive. however, if $x > x_0$ (or if $x < -x_0$), then $U(x) > E_0$, which is physically impossible because the difference, $E_0 - U(x)$, which should give $K$, is negative.