1 Introduction

Originally, Newton expressed his Second Law in words:

> The alteration of motion is...proportional to the...force impressed...

By "motion", he meant the product of mass and velocity, a vector quantity known as **linear momentum**, which is denoted by $\mathbf{p}$:

$$\mathbf{p} = mv$$

So the original Second Law read $\Delta\mathbf{p} \propto \mathbf{F}$, or, equivalently, $\mathbf{F} \propto \Delta\mathbf{p}$. But a large force that acts for a short period of time can produce the same change in linear momentum as a small force acting for a greater period of time. Knowing this, if we take the average force, $\overline{\mathbf{F}}$, that acts over the time interval Δt, the proportion can be expressed as:

$$\overline{\mathbf{F}} = \frac{\Delta\mathbf{p}}{\Delta t}$$

This equation becomes $\mathbf{F} = m\mathbf{a}$, since $\dfrac{\Delta\mathbf{p}}{\Delta t} = \dfrac{\Delta(m\mathbf{v})}{\Delta t} = m(\dfrac{\Delta\mathbf{v}}{\Delta t} = m\mathbf{a})$ (assuming that m remains constant). If we take the limit as $\Delta t \to 0$, then the equation above takes the form:

$$\mathbf{F} = \frac{d\mathbf{p}}{dt}$$

2 Impulse

Definition. Impulse, denoted by $\mathbf{J}$, is the product of force and the time during which it acts.

$$\mathbf{J} = \overline{\mathbf{F}}\Delta t$$

Note. Impulse is a vector quantity.

Theorem. In terms of impulse, Newton's Second Law can be written in yet another form:

$$\mathbf{J} = \Delta\mathbf{p}$$

Sometimes this is referred to as the **impulse-momentum theorem**.

If $\mathbf{F}$ varies with time over the interval during which it accts, then the impulse delivered by the force $\mathbf{F} = \mathbf{F}(\text{t})$ from time $t = t_1$ to $t = t_2$ is given by the following integral:

$$\mathbf{J} = \int_{t_1}^{t_2} F(t)dt$$

Note. On the equation sheet for the free-response section, this information will be represented as follows:
$$\mathbf{J} = \int \mathbf{F}dt = \Delta\mathbf{p}$$

If a graph of force-versus-time is given, then the impulse of force $\mathbf{F}$ as it acts from t_1 to t_2 is equal to the area bounded by the graph of $\mathbf{F}$, the t-axis, and the vertical lines associated with t_1 and t_2 as shown in the following graph.
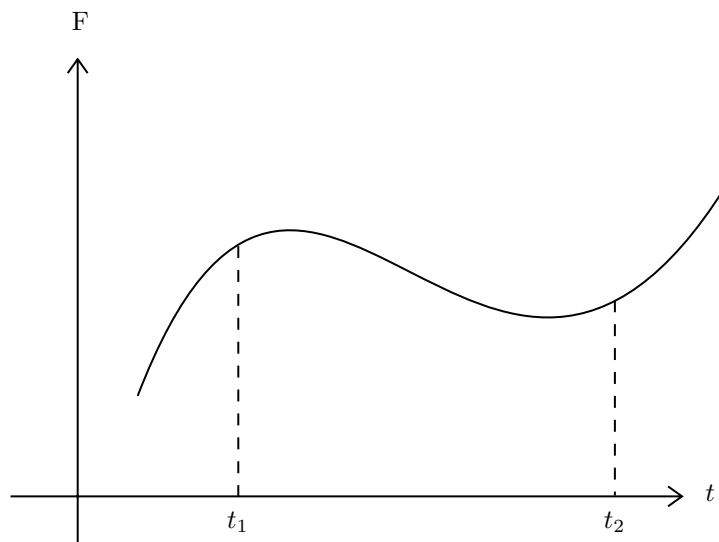
3   Conservation of Linear Momentum

According to Newton's Second and Third Law, the impulse delivered to an object is equal to the resulting change in its linear momentum, $\mathbf{J} = \Delta\mathbf{p}$, the two interacting objects experience equal but opposite momentum changes (assuming that there are no external forces), which implies that the total linear momentum of the system remains constant.

> **Law.** The **Law of Conservation of Linear Momentum** states that, in an isolated system, the total linear momentum will remain constant.
>
> $$m_1 \overrightarrow{v}_{1i} + m_2 \overrightarrow{v}_{2i} = m_1 \overrightarrow{v}_{1f} + m_2 \overrightarrow{v}_{2f}$$

4   Collisions

CoLM is routinely used to analyze **collisions**. The objects whose collision we will analyze form the *system*, and although the objects exert forces on each other during the impact, these forces are only *internal*. The system's total linear momentum is conserved if there is no net external force on the system.

Collisions are classified into two major categories:

- A collision is said to be **elastic** if kinetic energy is conserved.

    > **Example.** Most macroscopic collisions are never truly elastic, because there is always a change in energy due to energy transferred as heat, deformation of the objects, and the sound of the impact. However, if the loss of initial kinetic energy is small enough to be ignored, then the collision can be treated as virtually elastic.

- A collision is said to be **inelastic** if the total kinetic energy ids different after the collision.

    > **Example.** An extreme example is complete inelasticity after the collision. In this case, the objects stick together after the collision and move as one afterward.

In all cases of isolated collisions, Conservation of Linear Momentum states that

$$\text{total } \mathbf{p}_{\text{before collision}} = \text{total } \mathbf{p}_{\text{after collision}}$$

5 Center of Mass

> **Definition.** The **center of mass** is the point where all of the mass of an object can be considered to be concentrated; it's the dot that represents the object of interest in a free-body diagram.

For a homogeneous body (that is, one for which the density is uniform throughout), the center of mass is at the geometric center.

If there's a collection of discrete particles, the center of mass of the system can be determined mathematically as follows.

1. Consider the case where the particles all lie on a straight line (x-axis).

2. Select some point to be the origin ($x = 0$) and determine the positions of each particle on the axis.

3. Multiply each position value by the mass of the particle at that location, and get the sum for all the particles.

4. Divide this sum by the total mass, and the resulting x-value is the center of mass.

$$x_{\text{cm}} = \frac{m_1 x_1 + m_2 x_2 + \cdots + m_n x_n}{m_1 + m_2 + \cdots + m_n}$$

The system of particles behaves in many respects as if all its mass, $M = m_1 + m_2 + \cdots + m_n$, were concentrated at a single location, x_{cm}.

If the system consists of objects that are not confined to the same straight line, use the x_{cm} equation to find their center of mass, and this equation

$$y_{\text{cm}} = \frac{m_1 y_1 + m_2 y_2 + \cdots + m_n y_n}{m_1 + m_2 + \cdots + m_n}$$

to find the y-coordinate of their center of mass.

From the equation

$$x_{\text{cm}} = \frac{m_1 x_1 + m_2 x_2 + \cdots + m_n x_n}{M}$$

we can derive

$$M v_{\text{cm}} = m_1 v_1 + m_2 v_2 + \cdots + m_n v_n$$

So, the total linear momentum of all the particles in the system is the same as $M v_{\text{cm}}$, the linear momentum of a single particle (whose mass is equal to the system's total mass) moving with the velocity of the center of mass.

Differentiating again establishes the following:

$$F_{\text{net}} = M \mathbf{a}_{\text{cm}}$$

This states that if the net external force on the system is zero, then the center of mass will not accelerate.

Note. On the Table of Information, this information will be represented as follows:

$$\mathbf{r}_{\mathrm{cm}} = \sum m\mathbf{r} / \sum m$$

x_1 x_2

x_3

x

m_1 0 m_2 m_3

is equivalent to:

x_{cm}

x

M 0