1 Introduction

There are different forms of energy because there are different kinds of forces.

- Gravitational energy
- Elastic energy
- Thermal energy
- Radiant energy
- Electrical energy
- Nuclear energy
- Mass energy

Force is an agent of change, **energy** is often defined as the ability to do work, and **work** is one way of transferring energy from one system to another. The **Law of Conservation of Energy** (aka the **First Law of Thermodynamics**) says that if you account for all its various forms, the total amount of energy in a given process will stay constant (*conserved*).

2 Work

If $\mathbf{F}$ is the force, and $d\boldsymbol{x}$ is an infinitesimal amount of displacement, then the work done is:

$$W = \int \mathbf{F} \cdot d\boldsymbol{x}$$

If $\mathbf{F}$ is constant, then it can be removed from the integral, and since $\mathbf{F} \cdot \mathbf{r} = (F \cos\theta)r$ for any angle θ, then $W = (F \cos\theta)r$.

Unit. The unit of work is the newton-meter ($\mathrm{N} \cdot \mathrm{m}$), which is also called a joule (J).

Note. Though work depends on two vectors ($\mathbf{F}$ and $\mathbf{r}$), work itself is *not* a vector.
This is a result of the dot product, whereby two vectors are multiplied to produce a scalar. Another result of the dot product is that only the component of the force that is parallel (or antiparallel) to the displacement does any work.

Note. Any force or component of a force that is perpendicular to the direction that an object actually moves cannot do work because the displacement in that direction is zero.

Note. If a graph of force as a function of position or displacement is given, the work done by the force is the definite integral whose boundaries correspond to the displacement, Δx.

3 Kinetic Energy

Consider an object at rest ($v_0 = 0$) with mass m and a steady force $\mathbf{F}$ is being exerted on it, pushing it in a straight line. The object's acceleration is $a = \frac{F_{\text{net}}}{m}$, so after travelling distance Δx under the force, its final speed v is:

$$v^2 = v_0^2 + 2a(x - x_0) = 2a\Delta x = 2\frac{F_{\text{net}}}{m}\Delta x \rightarrow F_{\text{net}}\Delta x = \frac{1}{2}mv^2$$

> **Note.** This equation is given by the Big Five #5.

But the quantity $F_{net}\Delta x$ is the **total work** done by the force, so $W_r = \frac{1}{2}mv^2$. The work done on the object has transferred energy to it, in the amount $\frac{1}{2}mv^2$.

The energy an object possesses by virtue of its motion is therefore defined as $\frac{1}{2}mv^2$ and is called **kinetic energy**:

$$K = \frac{1}{2}mv^2$$