

**Status.** This is done up until "Rotational Dynamics".

**Note.** These were taken from "The Princeton Review AP Physics C Prep 2021".

## 1 Introduction

All motion is some combination of **translation** and **rotation**.

## 2 Rotational Kinematic

Mark several dots along a radius on a disk, and call this radius the *reference line*. If the disk rotates about its center, the movement of these dots can represent angular displacement, angular velocity, and angular acceleration.

If the disk rotates as a rigid body, then all dots have the same **angular displacement**,  $\Delta\theta$ .

**Definition.** A body is considered a **rigid body** when all points along a radial line always have the same angular displacement.

**Definition.** The time rate-of-change of angular displacement gives **angular velocity**, denoted by  $\omega$ .

Average angular acceleration:

$$\bar{\omega} = \frac{\Delta\theta}{\Delta t}$$

Instantaneous angular acceleration ( $\Delta t \rightarrow 0$ ):

$$\omega = \frac{d\theta}{dt}$$

**Definition.** The time rate-of-change of angular velocity gives **angular acceleration**, denoted by  $\alpha$ .

Average angular acceleration:

$$\bar{\alpha} = \frac{\Delta\omega}{\Delta t}$$

Instantaneous angular acceleration:

$$\alpha = \frac{d\omega}{dt}$$

In the above scenario, all points undergo the same angular displacement in any given time interval; this means that all points on the disk have the same angular velocity,  $\omega$ , but not all points have the same linear velocity,  $v$ . This follows the definition of the **radian** measure. In radians, the angular displacement,  $\Delta\theta$ , is related to the arc length,  $\Delta s$ , by the equation

$$\Delta\theta = \frac{\Delta s}{r}$$

Rearranging this equation and dividing by  $\Delta t$

$$\begin{aligned}\Delta s = r\Delta\theta &\Rightarrow \frac{\Delta s}{\Delta t} = r \frac{\Delta\theta}{\Delta t} \Rightarrow \bar{v} = r\bar{\omega} \\ ds = r d\theta &\Rightarrow \frac{ds}{dt} = r \frac{d\theta}{dt} \Rightarrow v = r\omega\end{aligned}$$

Therefore, the greater the value of  $r$ , the greater the value of  $v$ . Points on the rotating body farther from the rotation axis move more quickly than those closer to the rotation axis.

From the equation  $v = r\omega$ , one can derive the relationship that connects angular acceleration and linear acceleration. Differentiating both sides with respect to  $t$  (holding  $r$  constant), gives

$$\frac{dv}{dt} = r \frac{d\omega}{dt} \Rightarrow a = r\alpha$$

**Note.** The acceleration  $a$  in this equation is *not* centripetal acceleration; it's tangential acceleration, which arises from a change in speed caused by an angular acceleration. By contrast, centripetal acceleration does not produce a change in speed.

Often, tangential acceleration is denoted as  $a_t$  and centripetal acceleration as  $a_c$ .

**Note.** One can derive an expression for centripetal acceleration in terms of angular speed.

$$a_c = \frac{v^2}{r} = \frac{(r\omega)^2}{r} = \omega^2 r$$

This will be expressed on the equation sheet for the free-response section as

$$a_c = \frac{v^2}{r} = \omega^2 r$$

### 3 The Big Five For Rotational Motion

	<i>Missing variable</i>	
Big Five #1:	$\Delta\theta = \bar{\omega}t$	$\alpha$
Big Five #2:	$\omega = \omega_0 + \alpha t$	$\Delta\theta$
Big Five #3:	$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$	$\omega$
Big Five #4:	$\Delta\theta = \omega t - \frac{1}{2}\alpha(t)^2$	$\omega_0$
Big Five #5:	$\omega^2 = \omega_0^2 + 2\alpha\Delta\theta$	$t$

**Note.** In Big Five #1, because angular acceleration is constant, the average angular velocity is the average of the initial and the final angular velocity:  $\bar{\omega} = \frac{1}{2}(\omega_0 + \omega)$ . Also, if  $t_i = 0$ , then  $\Delta t = t_f - t_i = t - 0 = t$ , so  $t$  can be written instead of  $\Delta t$  in the first four equations.

	<i>Translational</i>	<i>Rotational</i>	<i>Connection</i>
displacement:	$\Delta x$	$\Delta\theta$	$\Delta x = r\Delta\theta$
velocity:	$v$	$\omega$	$v = r\omega$
acceleration:	$a$	$\alpha$	$a = r\alpha$
Big Five #1:	$\Delta x = x - x_0 = \bar{v}t$	$\Delta\theta = \bar{\omega}t$	
Big Five #2:	$v = v_0 + at$	$\omega = \omega_0 + \alpha t$	
Big Five #3:	$x = x_0 + v_0 t + \frac{1}{2}at^2$	$\Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2$	
Big Five #4:	$x = x_0 + vt - \frac{1}{2}at^2$	$\Delta\theta = \omega t - \frac{1}{2}\alpha t^2$	
Big Five #5:	$v^2 = v_0^2 + 2a(x - x_0)$	$\omega^2 = \omega_0^2 + 2\alpha\Delta\theta$	