Note. These were taken from "The Princeton Review AP Physics C Prep 2021".

1 Introduction

All motion is some combination of **translation** and **rotation**.

2 Rotational Kinematic

Mark several dots along a radius on a disk, and call this radius the *reference line*. If the disk rotates about its center, the movement of these dots can represent angular displacement, angular velocity, and angular acceleration.

If the disk rotates as a rigid body, then all dots have the same **angular displacement**, $\Delta\theta$.

Definition. A body is considered a **rigid body** when all points along a radial line always have the same angular displacement.

Definition. The time rate-of-change of angular displacement gives **angular velocity**, denoted by ω.

Average angular acceleration:

$$\overline{\omega} = \frac{\Delta\theta}{\Delta t}$$

Instantaneous angular acceleration ($\Delta t \to 0$):

$$\omega = \frac{d\theta}{dt}$$

Definition. The time rate-of-change of angular velocity gives **angular acceleration**, denoted by α.

Average angular acceleration:

$$\overline{\alpha} = \frac{\Delta\omega}{\Delta t}$$

Instantaneous angular acceleration:

$$\alpha = \frac{d\omega}{dt}$$

In the above scenario, all points undergo the same angular displacement in any given time interval; this means that all points on the disk have the same angular velocity, ω, but not all points have the same linear velocity, v. This follows the definition of the **radian** measure. In radians, the angular displacement, $\Delta\theta$, is related to the arc length, Δs, by the equation

$$\Delta\theta = \frac{\Delta s}{r}$$

Rearranging this equation and dividing by Δt

$$\begin{aligned} \Delta s = r\Delta\theta \quad &\Rightarrow \quad \frac{\Delta s}{\Delta t} = r\frac{\Delta\theta}{\Delta t} \quad \Rightarrow \quad \overline{v} = r\overline{\omega} \\ ds = rd\theta \quad &\Rightarrow \quad \frac{ds}{dt} = r\frac{d\theta}{dt} \quad \Rightarrow \quad v = r\omega \end{aligned}$$

Therefore, the greater the value of r, the greater the value of v. Points on the rotating body farther from the rotation axis move more quickly than those closer to the rotation axis.

From the equation $v = r\omega$, one can derive the relationship that connects angular acceleration and linear acceleration. Differentiating both sides with respect to t (holding r constant), gives

$$\frac{dv}{dt} = r\frac{d\omega}{dt} \quad \Rightarrow \quad a = r\alpha$$

Note. The acceleration a in this equation is *not* centripetal acceleration; it's tangential acceleration, which arises from a change in sped caused by an angular acceleration. By contrast, centripetal acceleration does not produce a change in speed.

Often, tangential acceleration is denoted as a_t and centripetal acceleration as a_c.

Note. One can derive an expression for centripetal acceleration in terms of angular speed.

$$a_c = \frac{v^2}{r} = \frac{(r\omega)^2}{r} = \omega^2 r$$

This will be expressed on the equation sheet for the free-response section as

$$a_c = \frac{v^2}{r} = \omega^2 r$$

3 The Big Five For Rotational Motion

		Missing variable
Big Five #1:	$\Delta\theta = \overline{\omega}t$	α
Big Five #2:	$\omega = \omega_0 + \alpha t$	$\Delta\theta$
Big Five #3:	$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$	ω
Big Five #4:	$\Delta\theta = \omega t - \frac{1}{2}\alpha(t)^2$	ω_0
Big Five #5:	$\omega^2 = \omega_0^2 + 2\alpha\Delta\theta$	t

Note. In Big Five #1, because angular acceleration is constant, the average angular velocity is the average of the initial and the final angular velocity: $\overline{\omega} = \frac{1}{2}(\omega_0 + \omega)$. Also, if $t_i = 0$, then $\Delta t = t_f - t_i = t - 0 = t$, so t can be written instead of Δt in the first four equations.

	Translational	*Rotational*	*Connection*
displacement:	Δx	$\Delta\theta$	$\Delta x = r\Delta\theta$
velocity:	v	ω	$v = r\omega$
acceleration:	a	α	$a = r\alpha$
Big Five #1:	$\Delta x = x - x_0 = \overline{v}t$	$\Delta\theta = \overline{\omega}t$	
Big Five #2:	$v = v_0 + at$	$\omega = \omega_0 + \alpha t$	
Big Five #3:	$x = x_0 + v_0 t + \frac{1}{2}at^2$	$\Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2$	
Big Five #4:	$x = x_0 + vt - \frac{1}{2}at^2$	$\Delta\theta = \omega t - \frac{1}{2}\alpha t^2$	
Big Five #5:	$v^2 = v_0^2 + 2a(x - x_0)$	$\omega^2 = \omega_0^2 + 2\alpha\Delta\theta$	