MADR-2025-Project-Tasks-04

Diffusion maps - toy examples

Implement the diffusion maps algorithm yourself. Test your implementation on the following graphs (here we assume that the starting point is the similarity graph, not a data set):

- a ring graph,
- a complete graph,
- a ring graph with a link (here assume that the number of edges n is even and add one edge to the graph between nodes 1 and $n/2$),
- two disjoint ring graphs.

Assume first that the weights are equal for all the edges, then experiment with different weights. Experiment with different numbers of vertices (besides the case of the complete graph, where essentially it only makes sense to use the algorith for $n = 3$ and $n = 4$). See what effect using different values of the diffusion parameter q in $\Phi^{(q)}$ brings.

Use the diffusion mappings implementation from the pyDiffMap library (or any other trustworthy library or package) for the above graphs - compare the results with the ones you've got using your implementation.

Diffusion maps - two famous nonlinear shapes

Test your implementation of the diffusion maps algorithm (and the one from the pyDiffMap library) on the Swiss roll and S-curve datasets (generators for both datasets are available in the scikit-learn library). Use the Gaussian kernel to generate the similarity graphs (try different values of the bandwidth parameter to see the effect that this has). Assume that the input data $x_1, \dots, x_n \in \mathbb{R}^p$ have an additional color attribute that should be preserved when generating the diffusion coordinates $\Phi^{(q)}(x_1), \dots, \Phi^{(q)}(x_n)$.

Diffusion maps - MiNI and Politechnika Warszawska logos

Consider the image of the MiNI PW logo such as the one from file **MiNI.jpg**.

- Rotate the logo $20, 40, \ldots, 180$ degrees. Represent each image by a $N \times N$ matrix, with each entry of the matrix representing the color value (in gray scale).
- Flatten each matrix to a point in N^2-dimensional space.
- Find the one dimensional diffusion embedding of the points you obtained, using the Gaussian similarity kernel. Plot the diffusion coordinate the angle the original image was rotated by. What kind of information is carried by the diffusion coordinate?
- Find and visualize the two dimensional diffusion embedding of the points, using the Gaussian similarity kernel.
- Repeat the above steps with Politechnika Warszawska logo such as the one from file **PW.jpg**.