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MDPI

Article

Optimal Selection of Stock Portfolios Using Multi-Criteria Decision-Making Methods

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Abstract: In the past, investors used their own or others' experiences to achieve their goals. With the development of financial management, investors' choices became more scientific. They could select the optimal choice by using different models and combining the results with their experiences. In portfolio optimization, the main issue is the optimal selection of the assets and securities that can be provided with a certain amount of capital. In the present study, the problem of optimization, i.e., maximizing stock portfolio returns and minimizing risk, has been studied. Therefore, this study discussed comprehensive modeling for the optimal selection of stock portfolios using multi-criteria decision-making methods in companies listed on the Tehran Stock Exchange. A sample of 79 companies listed on the Tehran Stock Exchange was used to conduct this research. After simulating the data and programming them with MATLAB software, the cumulative data analysis model was performed, and 24 companies were selected. This research data were collected from the financial statements of companies listed on the Tehran Stock Exchange in 2020. The primary purpose of this study was a comprehensive modeling for the optimal selection of stock portfolios using multi-criteria decision-making methods in companies listed on the Tehran Stock Exchange. The index in the Tehran Stock Exchange can be used to provide a comprehensive and optimal model for the stock portfolio; different multi-index decision-making methods (TOPSIS method), the taxonomy method (Taxonomy), ARAS method, VIKOR method, The COPRAS method and the WASPAS method can all identify the optimal stock portfolio and the best stock portfolio for the highest return.

Keywords: portfolio selection; MADM methods; MCDM

MSC: 90C40

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1. Introduction

The topic of optimal portfolio selection is the cornerstone of the financial industry [1]. The optimal allocation of resources at the market level, the ability to meet the needs of market participants, and the risk management of investments are some of the challenges of today's capital markets. Investors' decisions require methods, tools, and criteria to identify and measure each investment opportunity's potential value and risk, including "stocks" [2]. The optimal allocation of resources at the market level, the ability to meet the needs of market participants and, in an investment, the risk of decisions should be controlled and reduced if possible. One of the ways to manage and mitigate investment risk is to form a stock portfolio and diversify all types of assets [3]. Therefore, one of the most critical concerns for investors in the financial markets is choosing a stock or portfolio that is optimal in terms of profitability. Many methods have been introduced to determine the stock portfolio. Selecting the optimal stock portfolio is one of the purposes of portfolio management [4,5]. An investor is faced with the problem of choosing a portfolio among a large number of assets. The need to consider the number of investments and the share of each asset in that portfolio makes the decision-making process seem complicated [6]. Stock portfolio selection, maintenance, and other related decisions have always been associated with the two approaches of investors' risk aversion and the balance between the risk and expected returns. In the Markowitz model and the expanded models based on it, the main goal is to reduce the amount of risk for a certain level of return or to maximize the return for a certain amount of risk [7]. Markowitz's theory was the introduction of the synergy issue into the field of securities. The use of Markowitz's mean-variance method in the problem of choosing an investment portfolio has been the focus of research activities and served as a basis for the development of modern financial theory [8,9].

Different investors with different investment levels share one goal: to achieve a portfolio of assets that brings the least possible risk while meeting their expected returns [10]. This makes the solution of the stock portfolio optimization problem complicated. It is difficult to solve with the help of conventional mathematical methods and requires the use of innovative (inaccurate) methods. The problem of choosing an optimal set of assets is one of the theories of the capital market, which is also of particular importance in micro- and macroeconomic issues [8]; this issue has always been an attractive and practical issue in the financial markets. Optimizing and diversifying the stock portfolio has become an efficient tool to develop financial markets and help investors make decisions [11]. In the portfolio optimization literature, there is almost a consensus that investors need a model that includes uncertainty [12].

At present, Multiple-Attribute Decision-Making (MADM) models are used as the leading investment tool to cover the different risk and return preferences of investors and the uncertainty of the stock market in financial investment models. Multiple-Attribute Decision-Making is a branch of operations research that provides a solid mathematical foundation for the multiple attributes (intrinsic) nature of the ranking problem. The variety of indicators affecting financial decisions (such as the evaluation environment and goals), the complexity of the economic, commercial and financial environments, and the subjective nature of most financial decisions are only some of the features of the MADM modeling framework. Therefore, MADM methods are suitable for the study of many financial decision-making issues [13].

In portfolio optimization, the main issue is the optimal selection of assets and securities that can be prepared with a certain amount of capital. Although minimizing risk and predicting investment returns seems simple, several methods are used to form an optimal portfolio [8]. This has been proven to be a multidimensional problem in portfolio selection, and many researchers adopted the Multi-Criteria Decision-Making (MCDM) approach to solve it [14]. Although all researchers tried to bring efficiency to portfolio models, it is challenging to develop an effective portfolio, especially in an uncertain, dynamic environment [15]. MADM is a branch of multi-criteria decision-making [16,17].

This type of decision-making includes models and methods, divided into two categories: compensatory and non-compensatory models. In most cases, making decisions based on several criteria or indicators is desirable. Criteria may be quantitative or qualitative. In multi-criteria decision-making methods, several evaluation criteria are used, instead of one optimality measurement criterion. Multi-Criteria Decision-Making (MCDM) models are divided into two main categories: Multi-Objective Decision-Making (MODM) models and MADM models [18]. MODM models are used for design purposes, and MADM models are used to select the best option. The main difference between MODM and MADM models is that the first is defined in the continuous decision-making area and the second is defined in the discrete decision-making area. Compensatory models were used in the current research. Such models consist of indicators that interact with each other, which means that the unfavorable values of one index can be covered by the favorable values of another index. This research was conducted with the aim of completing a comprehensive modeling for the optimal stock portfolio selection using MADM methods in companies admitted to the Tehran Stock Exchange.

The application of theory to the optimal selection of efficient portfolios begins with the study by Markowitz (1952), and benefits from vital contributions, such as the contributions of Sharpe (1964) and Stephen (1976), among other authors who have made significant progress on outstanding problems. In portfolio selection decisions, multi-criteria decision-making (MCDM) considers a wide variety of criteria or objectives that are part of the optimization process that the investor must perform [19]. Researchers [19] investigate a methodology for making decisions in the stock market using the AHP-TOPSIS multi-criteria technique. They used multi-criteria methods to find an appropriate balance between profitability and risk in the stock investment decision process in the Colombian stock market. Wu et al. [20] studied an integrated method to deal with the portfolio allocation problem based on the multi-criteria group decision-making method (MCGDM) considering the consensus process in a type-2 interval fuzzy environment. Mohammed [21] presented and applied the concept and techniques of multi-criteria decision-making in a fuzzy environment to prioritize and select projects in portfolio management. The results showed that it is important to measure the weight of the criteria in the fuzzy TOPSIS technique, adjust the ranking of other projects, and determine the best project to achieve the desired levels.

To achieve a more reliable result, a hybrid approach is used in this research so that the challenges in the investigated problem can be taken into consideration. For this purpose, since the high number of options in a decision-making problem is problematic and poses difficulties when obtaining the opinions of experts, we first reduced the number of options by using the data envelopment analysis model and removing inefficient ones. Furthermore, as different multi-attribute decision-making approaches have unique characteristics, we used a combination of widely used approaches, and finally, combined the obtained results obtained. This research can contribute to the financial and investment knowledge, using the attention and evaluation of multiple indicators and criteria in ranking and evaluating the efficiency of the stock portfolio.

2. Research Literature

2.1. Modern Portfolio Theory

The modern theory of portfolio selection, in which the rate of return and risk are considered simultaneously, was presented by Markowitz in 1952 [8]. In this model, variance is used to measure risk [22]. For the first time, he determined a relationship between portfolio risks and returns. In Modern Portfolio Theory (MPT), Markowitz (1952) discussed portfolio selection and proposed the expected return (mean) and variance of total portfolio returns as measures of portfolio selection, both as a possible hypothesis about actual behavior and as an acknowledgment of how investors operate [23]. Optimizing the portfolio means choosing the best combination of financial assets to maximize the return on the investment portfolio and minimize the portfolio's risk as much as possible [17]. The basic idea of modern portfolio theory is that if one invests in assets that are not fully correlated, the risk of those assets will offset each other, and a fixed return can be obtained with less risk. The main assumptions of the Markowitz model are [24]:

- Investors are risk-averse and have increasing expected utility, and the final utility curve of their wealth is decreasing;
- Investors choose their stock portfolio based on the average and variance of expected returns. Therefore, their indifference curves are a function of the expected rate of return and variance;
- Every investment option is infinitely divisible;
- Investors have a "one-period" "time" horizon, and this is the same for all investors;
- Investors prefer higher returns at a certain level of risk, and vice versa; for a certain level of return, they want the lowest risk (anti-recession assumption).

One of the most critical issues in the previous models is the failure to consider multiple attributes and criteria when evaluating the efficiency of the stock portfolio. Therefore, multi-criteria decision-making approaches should be used to solve this shortcoming. In this situation, the data envelopment analysis method will be one of the multi-criteria decision-making approaches that can be used to realize this [25]. Algorithms that exist to solve optimization problems can be divided into two categories: exact algorithms and approximate algorithms. Exact algorithms are able to accurately find the optimal solution. Still, approximate algorithms can find near-optimal solutions for complex optimization problems and are divided into heuristic, meta-heuristic and ultra-heuristic algorithms. The two main problems of heuristic algorithms are their placement in local optima and their inability to be applied to various problems. The meta-heuristic algorithms that are presented to solve the problems of heuristic algorithms are among the types of approximate optimization algorithms that have solutions that deviate from the local optimum and can be used in a wide range of problems [26]. The research carried out in connection with the problem of stock portfolio optimization, which used meta-heuristic methods for the solution, can be divided into two general groups. The first group is those who considered only risk or return as the objective function, and modeled and solved the problem in the form of a single-objective optimization problem. The second category is those who defined the problem as a dual- or multi-objective problem and aimed to find solutions that simultaneously optimized various objective functions [10].

2.2. Portfolio and MCDM Approaches

Failure to consider multiple attributes and criteria for portfolio evaluation is one of the most significant drawbacks of previous models. Therefore, MCDM approaches should be used to solve this shortcoming. Data envelopment analysis (DEA) is one of the MCDM approaches that can be used to achieve this [27]. Notably, no existing studies have paid enough attention to the economic conditions governing financial markets, especially the dependence of domestic industries on exchange rate fluctuations, and the lack of identification and use of variables affecting capital absorption and forecasting the developments of financial markets, especially TSE. DEA generally acts as a primary filter and separates efficient shares. Suppose that the number of efficient shares exceeds a certain threshold, or real-world assumptions, such as cardinality restriction, are used in the optimization model. In that case, the problem-solving process will have computational complexities that cannot be solved through traditional mathematical approaches on time [27].

MPT, in which the rate of return and risk are simultaneously considered, was proposed by Markowitz in 1952. MPT uses variance to measure risk. Markowitz was the first to establish a relationship between portfolio risk and expected return. He added that every investor could build an optimal portfolio using the two determinants of risk and return. MPT considers an efficient portfolio. An efficient portfolio is a portfolio with minimum risk for return or maximum return for a given level of risk. Markowitz's model offers portfolios that maximize investors' returns while minimizing risk, but the selection is not limited to just two criteria. Despite the popularity of MPT, this model has been criticized over the years, including for its non-normalized return distribution. MPT has two drawbacks: computations and failure to consider investors' interests [27].

2.3. Optimization Algorithms

In recent years, numerous research has been conducted to incorporate uncertainty into mathematical models. One way to account for uncertainty is to consider non-deterministic parameters and numbers as intervals. An interval can determine the uncertainty in the upper and lower bounds. The main advantage of the interval methods is that they only calculate the interval bounds determining these uncertainty bounds [28]. Optimization problem-solving algorithms can be classified into two categories: exact and approximate. Exact algorithms can find the precise optimal solution, while approximate algorithms can find near-optimal solutions for NP-hard optimization problems. Approximate algorithms are further divided into three categories: heuristic, meta-heuristic, and hyper-heuristic. The two main disadvantages of heuristic algorithms are: (1) they trap in the local optimal solutions and (2) their inability to be applied to various problems. Meta-heuristic algorithms, proposed for solving heuristic problems, are approximate optimization algorithms with local optimal solutions and can be applied to a wide range of problems [29].

Studies on portfolio optimization using meta-heuristic approaches can be divided into two general categories The first category considers risk or return as an objective function and models and solves the problem as a single-objective optimization problem. The second category defines the problem as a two- or multi-objective problem, aiming to find solutions that simultaneously optimize various objective functions [10].

2.4. MADM Approaches

2.4.1. TOPSIS

The technique for Order of Preference by Similarity to the Ideal Solution (TOPSIS) [30] is one of the most popular MADM approaches, based on a simple and convenient logic proposed in 1981 [31]. This method involves ranking alternatives by comparing multiple criteria based on the distance between the ideal solution and the negative ideal solution [32]. To express this, TOPSIS forms a (positive) ideal solution (PIS) and a negative ideal (anti-ideal) solution (NIS), and prioritizes alternatives based on the shortest distance from the PIS and the farthest distance from the NIS. PIS is usually the best alternative solution. NIS is generally worse than other solutions. In other words, the PIS maximizes the benefit criteria and minimizes the cost criteria, whereas the NIS maximizes the cost criteria and minimizes the benefit criteria.

The following steps should be taken to solve the decision-making problem using TOPSIS:

Step 1: Construct a decision matrix and quantify the criteria.

Step 2: Create a descaled (normalized) decision matrix.

First, create the decision matrix. Since the criteria have different scales, the decision matrix is descaled using a Euclidean norm algorithm, and the matrix N_1 is constructed.

Step 3: Calculate the weighted descaled decision matrix (V).

This step first obtains the weight of each criterion and determines a square matrix containing criteria weights ($w_{n\times n}$). The main diameter entries are the weight of each criterion, and the other entries are zero. Then, obtain the descaled weighted decision matrix by multiplying the weighted matrix by the descaled decision matrix.

$$V = N_D \times W_{n\times n}$$
$$V = \begin{bmatrix} v_{12} \cdots & \cdots v1_n \\ \vdots & \vdots \\ & \ddots & \\ \vdots & \vdots \\ v_{m1} \cdots & \cdots v_{mn} \end{bmatrix} \tag{1}$$

Step 4: Determine PIS and NIS.

A_j^+ = [The best value vector of each alternative in $v_j^+ \,\Big|\, V$]

$A_j^- =$ [The worst value vector of each alternative in $v_j^- \,\Big|\, V$]

For positive alternatives, the best value is the maximum, and the worst is the minimum. For negative alternatives, the best value is the minimum, and the worst is the maximum.

Step 5: Obtain the distance of each alternative from PIS, i.e., A_j^+ and NIS, i.e., A_j^-.

Use the following equations to find the distance of each alternative from A_j^+ and A_j^-.

$$S_i^+ = \sqrt{\sum_{j=1}^{n}\left(V_{i\,j-}V_j^+\right)^2}\ i = 1,2,\ldots,m$$
$$S_i^- = \sqrt{\sum_{j=1}^{n}\left(V_{i\,j-}V_j^-\right)^2}\ i = 1,2,\ldots,m \tag{2}$$

S_i^+ denotes the distance between each A_j^+, and S_i^- indicates the distance of each alternative from A_j^-.

Step 6: Calculate the relative closeness coefficient of each alternative to the ideal solution.

Using the following equation, determine the relative closeness coefficient of each alternative (Cl^*i) and rank the alternatives based on Cl^*i. After determining Cl^*i for each alternative, the alternative with the maximum Cl^*i is ranked first. It can be argued that this alternative has the farthest distance from A_j^- and is the closest alternative to A_j^+.

$$Cl^*i = \frac{S_i^-}{S_i^- + S_i^+} \tag{3}$$

2.4.2. ARAS

The Additive Ratio Assessment (ARAS) method was proposed in 2010 [33]. The ARAS method has significant advantages over other MADM methods. In this method, the degree of utility for each alternative is calculated based on comparison with the Pareto optimal solution, the alternative described by optimal index values. Additionally, techniques such as TOPSIS, VIKOR, SAW, AHP, COPRAS, etc., are exposed to the rank inversion phenomenon [34].

ARAS aims to select the best alternative based on several criteria, and the other options are ranked by determining the utility degree of each choice. The model's features include the following: (1) it is one of the compensatory methods, and (2) in this method, qualitative attributes must be converted into quantitative ones.

In this matrix, alternatives and criteria are described based on the information received from the DM, as follows:

$$X = \begin{bmatrix} r_{01} & \cdots & r_{oj} & \cdots & r_{on} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ r_{i1} & \cdots & r_{ij} & \cdots & r_{in} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ r_{m1} & \cdots & r_{mj} & \cdots & r_{mn} \end{bmatrix};\ i = 0,\ldots,m,\ j = 1,\ldots,n \tag{4}$$

As can be seen, r_{oj} indicates the decision matrix entry for alternative i in criterion j, and r_{oj} is the optimal value of criterion *j*. If r_{oj} is indeterminate, you can use Equation (5) when the criterion is positive and Equation (6) when the criterion is negative.

$$R_{oj} = \max r_{ij};\ i = 0,\ \ldots m,\ j = 1,\ \ldots,n \tag{5}$$

$$r_{oj} = \min r_{ij};\ i = 0,\ \ldots m,\ j = 1,\ \ldots,n \tag{6}$$

In this method, the DM determines the criteria weights, $[w_1, w_2, \ldots, w_n]$, considering the descaling property ($\sum_{j=1}^{n} w_j = 1$). First, the decision matrix should be descaled to utilize this method. The following equation is used to descale (normalize) the decision matrix:

$$r_{ij}^* = \frac{r_{ij}}{\sum_{i=0}^{m} r_{ij}};\ i = 0,\ \ldots,m,\ j = 1,\ \ldots,n \tag{7}$$

Then, the values for the weighted descaled decision matrix are determined. Considering the criteria weight, $[w_1, w_2, \ldots, w_n]$, the weighted descaled values of each criterion are determined based on the following equation:

$$\hat{r}_{ij} = r^*_{ij};\ i = 0, \ldots, m,\ j = 1, \ldots, n \tag{8}$$

The optimality function is obtained in the subsequent step. It is better to have a more significant value for the optimality function (S_i). S_i is determined for alternative i based on the following equation:

$$S_i = \sum_{j=1}^{n} \hat{x}_{ij};\ i = 0,\ \ldots,\ m,\ j = 1, \ldots, n \tag{9}$$

The utility degree (k_i) is used for the final ranking of alternatives. k_i is in the range [0, 1]. k_i for alternative i is calculated using the following equation:

$$k_i = \frac{S_i}{S_o};\ i = 0, \ldots, m \tag{10}$$

where S_o denotes the best value of S_i for alternative i.

In the final ranking, k_i values are sorted in descending order, and an alternative with the maximum k_i is selected as the best alternative.

2.4.3. Taxonomy Method

Taxonomy was initially proposed in 1763 [35], and was developed in 1950 by a group of Polish mathematicians [36]. In 1968, Hellwig [37] proposed taxonomy as a tool to classify and determine the degree of development. Taxonomy is an excellent method for grading, classifying, and comparing different activities according to the utility degree of that activity from attributes. The model's features include the following: (1) it is one of the compensatory methods, (2) in this method, qualitative attributes must be converted into quantitative ones, and (3) the attributes are independent. The classical taxonomy method is presented as follows [36]:

Step 1. Calculation of the average and standard deviation of the features:

$$\overline{\alpha}_j = \frac{1}{m}\sum_{i=1}^{m} \alpha_{ij};\ j = 1, 2, \ldots, n. \tag{11}$$

$$S_{\hat{j}} = \sqrt{\frac{1}{m}\sum_{i=1}^{m} \left(\alpha_{ij} - \overline{\alpha}_j\right)^2};\ j = 1, 2, \ldots, n. \tag{12}$$

Step 2. Since, in matrix decision-making, alternative solutions have different measurement scales in terms of attributes, this step aims to balance the different units; therefore, the following formula is used to achieve this goal:

$$L_{i\hat{j}} = \frac{\alpha_{ij} - \overline{\alpha}_j}{S_{\hat{r}}};\ i = 1,\ \ldots,\ m,\ j = 1, \ldots, n. \tag{13}$$

Step 3. Calculate the distance from each alternative to other alternatives using the following formula:

$$P_{ab} = \sqrt{\sum_{j=1}^{m} \left(l_{aj} - l_{bj}\right)^2}; \tag{14}$$

where a and b represent the evaluated options to facilitate a comparison of the two options, and the following combined distance matrix can be obtained:

$$P = \begin{bmatrix} p11 & \ldots & p1j & \ldots & p1n \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ pi1 & \ldots & pij & \ldots & pin \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ pm1 & \ldots & pmj & \ldots & pmn \end{bmatrix}_{m\times n} ;\ i = 1, \ldots, m,\ j = 1, \ldots, n. \quad (15)$$

Step 4. Calculate the mean and standard deviation of the minimum distance in each row according to the calculation formula:

$$\acute{O} = \frac{1}{m}\sum_{i=1}^{m} O_{\hat{i}}; \quad (16)$$

$$S_{\hat{o}} = \sqrt{\frac{1}{m}\sum_{i=1}^{m} \left(O_{\hat{i}} - \acute{O}\right)^{2}}. \quad (17)$$

In this calculation, $O_{\hat{i}}$ shows the optimal distance of each row. Then, Formula (17) is used to determine the range that the composite distance matrix should have.

$$O = \acute{O} \pm S_{\hat{0}}. \quad (18)$$

If any row has a value outside this range, it will not work, and the mean and standard deviation of each row must be recalculated.

Step 5. The standardized matrix calculation development pattern is used:

$$L_{io} = \sqrt{\sum_{j=1}^{n} \left(L_{ij} - L_{0j}\right)^{2}};\ i = 1, \ldots, m. \quad (19)$$

where L_{io} represents the ideal value of feature *j*th, depending on whether this feature is of benefit type or negative type. L_{ij} represents the standard value of feature j in the *i*th selection.

Step 6. Calculation of development height:

$$L_{O} = \overline{L}_{io} + 2S_{Lio}. \quad (20)$$

Then, calculate the final order of progress using the following formula:

$$F_{i} = \frac{L_{io}}{L_{o}};\ i = 1, \ldots, m. \quad (21)$$

2.4.4. VIKOR

Opricovic originally developed the VIKOR method in 1998 [38]. VIKOR is one of the compromise methods in compensatory models, i.e., an alternative related to this subgroup will be preferred if it is the closest alternative to the ideal solution. The TOPSIS method allocates a solution based on its distance from the negative and positive ideal solutions, but the relativity of these distances is not considered. The VIKOR method allows for DMs to make their own assessments at the initial level [39]. This method is an effective MCDM tool, especially when the decision-maker cannot express his preferences at the beginning of the system design [40]. The VIKOR method generally focuses on ranking alternatives, selecting an alternative with a set of conflicting criteria, and providing a compromise solution that can help the DM reach the final solution. The model's features include the following: (1) it is one of the compensatory methods, (2) in this method, criteria must be independent, and (3) qualitative criteria must be converted into quantitative ones. This method is a type of compromise programming that uses the LP-metric method to find the closest alternative to the optimal solution:

$$L_{pi} = \left\{ \sum_{j=1}^{n} \left[\frac{w_j \left(f_j^0 - f_{ij} \right)}{\left(f_j^0 - f_j^- \right)} \right] P \right\}^{1/P} \quad 1 \leq P \leq \infty;\ j = 1, 2, \ldots, m. \tag{22}$$

where w_j indicates the criteria weight announced by the decision-maker; P represents the determinant parameter of the LP family; f_{ij} denotes the value of alternative i in criterion j. f_j^* is the best (optimal) f_{ij}, and f_j^- is the worst f_{ij}.

L_{ij} is represented by S_i, which is equal to:

$$S_i = \sum\nolimits_{j=1}^{n} w_j \frac{f_j^* - f_{ij}}{f_j^* - f_j^-} \quad j = 1, \ldots, n;\ i = 1, \ldots, m. \tag{23}$$

L_{pi} is represented by R_i, which is equal to:

$$R_i = \mathrm{Max} \left[w_j \frac{f_j^* - f_{ij}}{f_j^* - f_j^-} \right] \quad j = 1, \ldots, n;\ i = 1, \ldots, m. \tag{24}$$

2.4.5. COPRAS

The "Complex Proportional Assessment", or COPRAS method, was introduced by Zavadskas and Kaklauskas [41]. This method calculates the answer by considering the best solution ratio [42]. The superiority of the mentioned method compared to other MCDM methods is that it is dependent on the importance and degree of directness, and the applicability of the considered versions, based on the criteria that explain the alternatives, weights and values [43]. Therefore, this method can be used to maximize or minimize the criteria to evaluate more than one criterion [40]. COPRAS separately assesses the effect of both minimum and maximum criteria on the result evaluation. The method's features include the following: (1) it is one of the compensatory methods and (2) the method contains qualitative criteria that must be converted into quantitative ones.

2.4.6. WASPAS

The Weighted Aggregates Sum Product Assessment (WASPAS) method was first proposed in 2012 [44]. WASPAS combines the weighted sum model (WSM) and the weighted product model (WPM) [45,46]. Zavadskas et al. (2012) [44] studied the WASPAS method, and finally concluded that it is more powerful than WSM and WPM. They also showed which WASPAS method was more accurate than the others. This method provides the performance values of the alternatives according to the criteria by using the criteria weights to solve MCDM problems. As a result of this solution, the alternatives are ranked from best to worst. In addition, the method tries to achieve a high level of estimation stability by optimizing the weighted integral function [46]. This method determines the relative importance of each criterion and then evaluates and prioritizes the alternative. The method's features include the following: (1) it is one of the compensatory methods, (2) in this method, criteria are independent, and (3) qualitative criteria must be converted into quantitative ones.

2.4.7. DEA Method

DEA is a widely used non-parametric and linear programming technique for evaluating the relative efficiency of DMUs. Since the DEA can handle multiple inputs and outputs, it is a suitable technique for discovering relationships that remain unclear in other methods [47]. DEA is an effective method to evaluate the relative efficiency of the same type of decision-making units (DMU) using multi-indicator input and output data based on mathematical programming tools and optimization methods [48]. Data Envelopment Analysis (DEA) can directly evaluate the performance of each decision-making unit (DMU) by combining multiple inputs and outputs, without the need for information on the relationship between inputs and outputs [49].

2.4.8. Mean Rank Method

The mean rank method determines the average rank obtained by each alternative, and any alternative with a lower average rank will be selected as the best (optimal) alternative.

Borda Count Method

The Borda count method constructs an $m \times m$ matrix, where m is the number of alternatives. The entries of the square matrix are determined based on the number of wins. Therefore, if the number of alternative wins in the row is higher than that of alternative wins in the column, then M is placed in that entry, as the number of higher alternative ranks in the row is greater than that of higher alternative ranks in the column in different decision-making approaches. Suppose the number of alternative wins in the row is equal to or is less than the number of alternative wins in the column. In that case, X is placed in that entry, as the number of higher alternative ranks in the row is equal to or less than that of higher alternative ranks in the column in different decision-making approaches. Then, the number of wins of each alternative is determined, which is equal to the sum of the number of M in each row. Finally, the number of wins is the basis for ranking, i.e., any alternative with more wins will be ranked higher.

Copeland Method

Copeland, as in the Borda count method, creates an $m \times m$ matrix, where m is the number of alternatives. The entries of the square matrix are determined based on the number of wins. Suppose the number of alternative wins in the row is higher than that of alternative wins in the column. In that case, M is placed in that entry, as the number of higher alternative ranks in the row is greater than that of higher alternative ranks in the column in different decision-making approaches. Suppose the number of alternative wins in the row is equal to or less than the number of alternative wins in the column. In that case, X is placed in that entry, as the number of higher alternative ranks in the row is equal to or less than that of higher alternative ranks in the column in different decision-making approaches. Then, the number of wins of each alternative is determined, which is equal to the sum of the number of M in each row. The number of losses of each option is also specified, equal to the sum of the M in each column. Finally, the difference between wins and losses is also determined, based on the ranking, i.e., each alternative with a more significant difference between its wins and losses is ranked higher.

3. Research Method

As stated in the previous sections, the methods used in this research were used to select the optimal stock portfolio. Figure 1 shows the research flow.

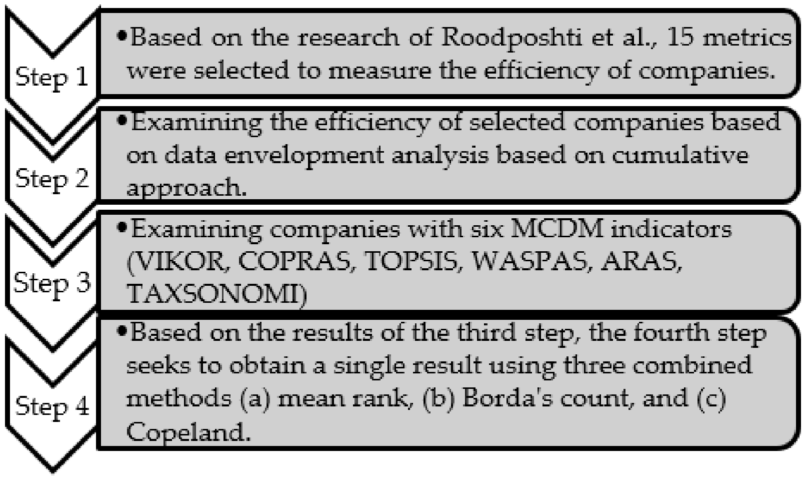

Figure 1. The research flow [50].

3.1. Population

The research population comprises all the companies listed on the TSE during 2020. The data relating to 149 companies were analyzed according to the collected data. Based on the research of Roodposhti et al. [50], 15 metrics were selected to measure the efficiency of the companies. According to the opinion of experts, including 4 university professors and 4 experts in the capital market, and 2 individual investors, three indicators were selected as input (turnover of total assets, account receivable turnover, inventory turnover) and two indicators as output (sales growth and ROA). A total of 70 companies were excluded as they had incomplete information and seasonal operations. Therefore, we used data from 79 companies for computations. It is noteworthy that data from the previous year, 2019, were needed to calculate some variables. After simulating and programming the data using MATLAB software, a cumulative DEA model was developed, and 24 companies were selected.

3.2. Evaluation Metrics

This study of Roodposhti et al. [50] used fifteen metrics from five financial ratio classes, each of which will be explained in detail below. Among the growth criteria, sales growth rate, net income growth rate, and earnings per share (EPS) growth rate are selected, which are calculated as follows.

$$\text{Sales growth rate} = \frac{(\text{Current period sales} - \text{Prior period sales})}{\text{Prior period sales}}$$

The sales growth rate is a metric to measure the efficiency of a company's sales team, which is determined to increase sales revenue over a fixed period. Understanding the sales growth rate is a crucial metric by which companies can make decisions based on input data.

$$\text{Net income growth rate} = \frac{(\text{Current period net income} - \text{Prior period net income})}{\text{Prior period net income}}$$

Net income growth indicates how much the company's net profit has increased compared to the prior period.

$$\text{EPS growth rate} = \frac{(\text{Current period EPS} - \text{Prior period EPS})}{\text{Prior period EPS}}$$

EPS growth implies the EPS growth rate over the years (periods).

Net EPS, net profit margin (net profits to net sales), operating margin, return on assets (ROA), and return on equity (ROE) were selected as profitability ratios. These metrics are calculated as follows:

$$\text{EPS} = \frac{(\text{Net income})}{\text{Total number of shares}}$$

EPS is one of the essential financial statistics considered by investors and financial analysts. EPS is calculated by dividing earnings after tax (EAT) by the total number of shares, indicating the profit the company has earned per common share during a specific period.

$$\text{Net profit margin} = \frac{(\text{Net profits})}{\text{Net sales}}$$

Net profit margin reflects an effective pricing policy, i.e., optimally controlling the sales costs.

$$\text{Operating margin} = \frac{(\text{Operating income})}{\text{Revenue (sales)}}$$

The operating margin considers all operating costs, including production and selling, and general and administrative expenses (SG&A). That is why calculating the operating margin is essential to assess the company's profitability from its business (operating activities).

$$\text{ROA} = \frac{\text{Net income}}{\text{Total assets}}$$

ROA is one of the primary metrics used to evaluate corporate performance. This metric indicates a company's profitability for its total assets.

$$\text{ROE} = \frac{\text{Net income}}{\text{Total equity}}$$

For business owners, ROE is more important than other ratios because it shows the return on their investment.

Total asset turnover, account receivable turnover, and inventory turnover are selected from the profitability ratios. These metrics are calculated as follows:

$$\text{Total asset turnover} = \frac{\text{Net sales}}{\text{Total assets}}$$

The total assets turnover ratio shows how the assets' turnover affects corporate earnings. This ratio indicates how well the company uses its assets to generate income. By comparing this ratio to previous periods, whether the increase in assets affects the company's earnings more can be determined.

$$\text{Accounts receivable turnover} = \frac{\text{Net sales}}{\text{Average account receivables}}$$

The accounts' receivable turnover ratio indicates how often the company's receivables are collected during a year. For instance, a ratio of 2 indicates that the company collects and cashes its receivables twice a year on average. This ratio is used to calculate the average period during which a company collects its receivables.

$$\text{Inventory turnover} = \frac{\text{Cost of goods sold}}{\text{Average inventories}}$$

The inventory turnover ratio shows how often the company has sold and replenished its inventory of goods and materials during a specific period (e.g., a fiscal year).

Financial risk and beta coefficient were selected as risk ratios. These metrics are computed as follows:

$$\beta = \frac{\mathrm{Cov}(r_a, r_b)}{\mathrm{Var}(r_b)}$$

In financial science, the beta (β) coefficient measures the systematic risk of an investment or a portfolio of financial assets relative to the market portfolio risk. β coefficient refers to the slope in a linear regression equation and a correlation coefficient, indicating stock price volatility compared to the overall market in which it operates.

$$\text{Financial risk} = \frac{\text{Total debts}}{\text{Total assets}}$$

Financial risk is the possibility that shareholders will lose capital when investing in a company with debt. Financial risk occurs when the company's cash flow is inadequate to meet its financial obligations. When a company uses debt financing, its creditors will collect their debts before the shareholders if it goes bankrupt.

Price to dividend and price to book value per share (BVPS) were selected from the market ratios. These metrics are calculated as follows:

$$\text{Price to EPS} = \frac{\text{Price per share}}{\text{EPS}}$$

Capital market investors usually refer to the price-to-earnings (P/E) ratio as a tool for valuing a company. P/E is the most common ratio, which is vital for investors, analysts, portfolio managers, consultants, etc., in the capital market. P/E's merit is its ability to represent the relationship between market value and EPS using a mathematical number. P/E indicates the relationship between the price an investor pays for a share, the company's prospects, and expected profits.

$$\text{Price to BVPS} = \frac{\text{Price per share}}{\text{Total number of shares/equities}}$$

Companies use the price-to-book (P/B) ratio to compare a firm's market value with its book value. Book value is a firm's net asset value, computed as total assets minus intangible assets (patents, goodwill) and liabilities. All metrics are presented in Table 1.

Table 1. All metrics [50].

	Metric	Formula
1	Sales growth rate =	(Current period sales − prior period sales)/prior period sales
2	Net income growth rate =	(Current period net income − prior period net income)/prior period net income
3	EPS growth rate =	(Current period EPS − prior period EPS)/prior period EPS
4	EPS =	Net income/total number of shares
5	Net profit margin =	Net profits/net Sales
6	Operating margin =	Operating income/revenue (or sales)
7	ROA =	Net income/total assets
8	ROE =	Net income/total equity
9	Total asset turnover =	Net sales/total assets
10	Accounts receivable turnover =	Net sales/average account receivables
11	Inventory turnover =	Cost of goods sold/average inventories
12	Beta =	$\beta = \frac{\mathrm{Cov}(r_a, r_b)}{\mathrm{Var}(r_b)}$
13	Financial risk =	Total debts/total assets
14	Price to EPS =	Price per share EPS
15	Price to BVPS =	Price per share/total number of shares/equities

4. Findings

Initially, the companies listed on TSE during 2020 were selected from six industries, i.e., automotive, pharmaceutical, food, cement, chemical, and machinery. After evaluating

149 companies, 70 were excluded from other computational steps as they had incomplete information and seasonal operations. Finally, we used data from 79 companies for computations. Then, preliminary computations for 2020 were completed based on fifteen criteria selected from five different financial ratio categories. After extracting fifteen criteria for DEA analysis, three criteria were considered as inputs and four as outputs based on the experts' viewpoints. It is noteworthy that the DEA technique was used to prioritize efficient companies.

This study evaluated 24 companies ranked based on fifteen criteria using six MCDM approaches (VIKOR-COPRAS-TOPSIS-WASPAS-ARAS-Taxonomy). MCDM seeks to select the best (optimal) alternative over other alternatives. Implementing these six approaches requires the weighting of the criteria mentioned above. The sum of the weights for all criteria must be equal to 1. The expert determined the positive and negative criteria. The weights of the indicators were 0.04, 0.05, 0.05, 0.02, 0.04, 0.05, 0.09, 0.08, 0.09, 0.10, 0.12, 0.08, 0.05, 0.07, and 0.07. After implementing each method, each company's rank was obtained according to the fifteen criteria reported in Table 2.

As shown in Table 2, each method's ranking result was different, and it is impossible to select a particular investment company based on a single method. For instance, Company 1 ranked first based on the taxonomic technique, and Company 21 ranked first based on the ARAS. Therefore, we integrated these six methods to obtain a single result using three combined methods: (a) mean rank, (b) Borda count, and (c) Copeland. The results obtained from each method are presented separately in Tables 3 and 4.

Table 2. Ranking of each company according to fifteen metrics.

Companies Numbers	VIKOR	COPRAS	TOPSIS	WASPAS	Taxonomy
1	2	24	6	23	1
2	11	19	13	22	12
3	16	14	19	9	11
4	5	7	4	3	3
5	10	5	5	6	4
6	24	16	12	20	24
7	4	10	22	5	16
8	20	15	16	14	19
9	22	22	24	24	22
10	18	21	17	17	14
11	15	11	11	11	6
12	7	1	1	1	21
13	21	9	9	19	20
14	8	20	18	7	9
15	12	4	3	10	7
16	3	23	20	15	15
17	13	8	8	13	13
18	14	12	21	18	17
19	19	3	7	12	18
20	1	6	10	2	2
21	23	2	2	16	23
22	6	18	23	8	5
23	9	13	15	4	8
24	17	17	14	21	10

After combining six methods according to the mean rank method, Company 20 took the first rank and was recognized as a suitable alternative for investment. Table 4 shows Company 12, which ranked 1st in the Borda count and Copeland methods. Table 5 summarizes the results of the mean rank, Borda count, and Copeland methods.

Table 3. Combined results according to mean rank method.

Companies Numbers	Taxonomy	ARAS	WASPAS	TOPSIS	COPRAS	VIKOR	Mean Rank	Final Rank
1	1	3	23	6	24	2	9.8	6
2	12	9	22	13	19	11	14.3	17
3	11	14	9	19	14	16	13.8	14
4	3	6	3	4	7	5	4.6	2
5	4	15	6	5	5	10	7.5	4
6	24	21	20	12	16	24	19.5	23
7	16	22	5	22	10	4	13.1	13
8	19	4	14	16	15	20	14.6	19
9	22	12	24	24	22	22	21	24
10	14	18	17	17	21	18	17.5	21
11	6	13	11	11	11	15	11.1	8
12	21	2	1	1	1	7	5.5	3
13	20	17	19	9	9	21	15.8	20
14	9	11	7	18	20	8	12.1	11
15	7	19	10	3	4	12	9.1	5
16	15	8	15	20	23	3	14	16
17	13	10	13	8	8	13	10.8	7
18	17	23	18	21	12	14	17.5	22
19	18	24	12	7	3	19	13.8	15
20	2	5	2	10	6	1	4.3	1
21	23	1	16	2	2	23	11.1	9
22	5	16	8	23	18	6	12.6	12
23	8	20	4	15	13	9	11.5	10
24	10	7	21	14	17	17	14.3	18

Figure 2 compares the mean rank, Borda count, and Copeland methods for the 24 studied companies.

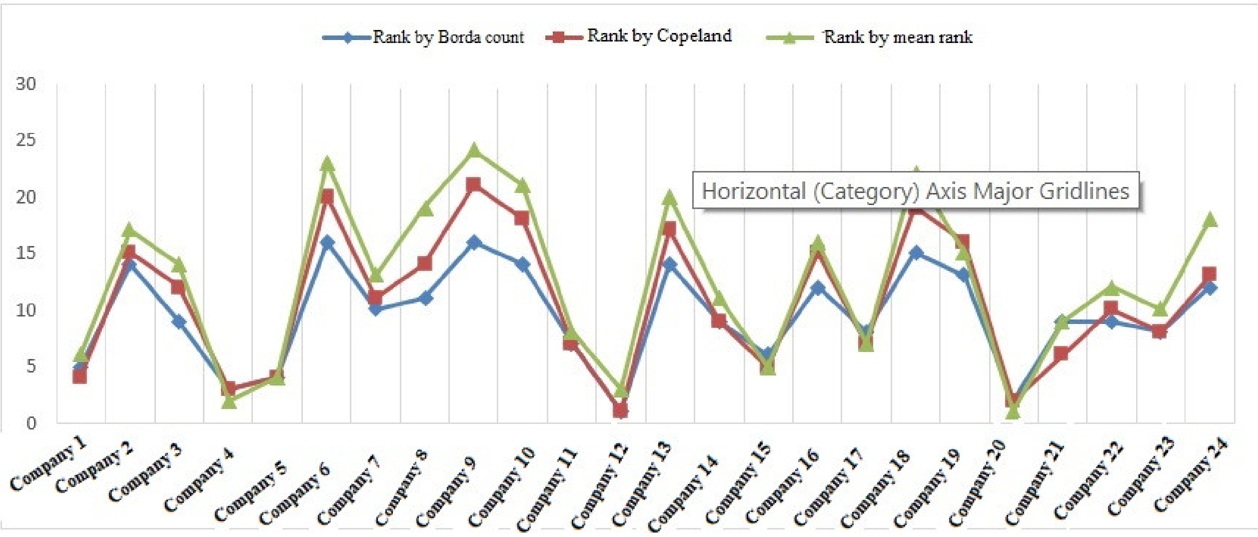

Figure 2. Comparison of three mean ranks, Borda count, and Copeland methods for 24 companies.

According to Table 5 and experts' opinion, five companies that ranked first or had a better rank were selected (for example, Company 12 ranked first Borda count and Copeland method and third in the mean ranks, and Compant 20 ranked second in the Borda count and Copeland method and first in the mean ranks). Then, 10 criteria were analyzed for data homogeneity to confirm the results shown in Table 6 and Figure 3.

Table 4. Combined results according to Borda count and Copeland methods.

	Company 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	Total Wins	Borda Count	Difference between Wins and Losses	Copeland
Company 1		1	1	0	0	1	1	1	1	1	1	0	1	1	0	1	1	1	1	0	0	1	1	1	17	5	15	4
2	0		0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	3	14	−8	15
3	0	0		0	0	1	0	1	1	1	0	0	1	0	0	1	0	1	1	0	0	0	0	0	8	9	−2	12
4	0	1	1		1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	0	0	1	1	1	19	3	17	3
5	0	1	1	0		1	1	1	1	1	1	0	1	1	1	1	1	1	1	0	0	1	1	1	18	4	15	4
6	0	0	0	0	0		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	16	−21	20
7	0	0	0	0	0	1		1	1	0	0	0	0	0	0	0	0	1	1	0	0	1	1	0	7	10	−1	11
8	0	0	0	0	0	1	0		1	1	0	0	1	0	0	1	0	0	1	0	0	0	0	0	6	11	−6	14
9	0	0	0	0	0	0	0	0		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	16	−22	21
10	0	0	0	0	0	1	0	0	1		0	0	0	0	0	0	0	1	0	0	0	0	0	0	3	14	−13	18
11	0	1	1	0	0	1	0	1	1	1		0	1	0	0	1	0	1	1	0	0	0	1	1	12	7	5	7
12	1	1	1	1	1	1	1	1	1	1	1		1	1	1	1	1	1	1	1	1	1	1	1	23	1	23	1
13	0	0	0	0	0	1	0	0	1	0	0	0		0	0	0	0	0	0	0	0	1	0	0	3	14	−12	17
14	0	0	1	0	0	1	0	0	1	1	0	0	1		0	1	0	1	1	0	0	0	0	0	8	9	2	9
15	0	1	1	0	0	1	1	1	1	1	1	0	1	0		1	1	1	1	0	0	0	1	1	15	6	10	5
16	0	0	0	0	0	1	1	0	1	0	0	0	1	0	0		0	1	0	0	0	0	0	0	5	12	−8	15
17	0	0	1	0	0	1	1	1	1	1	1	0	1	0	0	1		1	0	0	0	0	0	1	11	8	5	7
18	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0		0	0	0	0	0	0	2	15	−15	19
19	0	0	0	0	0	1	0	1	1	0	0	0	1	0	0	0	0	0		0	0	0	0	0	4	13	−9	16
20	0	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1		0	1	1	1	20	2	19	2
21	1	1	0	0	0	1	0	0	1	1	0	0	1	0	0	0	0	1	0	0		0	0	1	8	9	7	6
22	0	1	0	0	0	1	0	0	1	1	0	0	1	0	1	0	0	1	1	0	0		0	0	8	9	1	10
23	0	1	1	0	0	1	0	1	1	1	0	0	0	1	0	1	0	1	1	0	0	0		1	11	8	3	8
24	0	1	0	0	0	1	0	0	1	1	0	0	0	0	0	1	0	0	0	0	0	0	0		5	12	−5	13
Total losses	2	11	10	2	3	21	8	12	22	16	7	0	15	6	5	13	6	17	13	1	1	7	8	10	216			

Table 5. Summarized results of mean rank, Borda count, and Copeland methods.

Companies Number	Rank by Borda Count	Rank by Copeland	Rank by Mean Rank
1	5	4	6
2	14	15	17
3	9	12	14
4	3	3	2
5	4	4	4
6	16	20	23
7	10	11	13
8	11	14	19
9	16	21	24
10	14	18	21
11	7	7	8
12	1	1	3
13	14	17	20
14	9	9	11
15	6	5	5
16	12	15	16
17	8	7	7
18	15	19	22
19	13	16	15
20	2	2	1
21	9	6	9
22	9	10	12
23	8	8	10
24	12	13	18

Table 6. Ten criteria for five top-ranked companies.

Companies	Sales Growth (1)	Net Income Growth (2)	EPS Growth (3)	Net Profit Margin (4)	Operating Margin (5)	ROA (6)	ROE (7)	Total Assets Turnover (8)	Financial Risk (9)	β Coefficient (10)
1	1/9404	1/9611	−1/9611	0/0074	0/0076	0/0430	0/2129	3/7761	0/7931	0/0399
4	0/5572	0/8219	−0/0890	0/6213	0/6399	0/5423	0/8020	0/8728	0/3035	0/6841
5	0/9013	0/9935	0/9935	0/3286	0/3873	0/2339	0/7790	0/7118	0/6460	0/3720
12	0/1774	1/2144	1/2144	−0/1549	−0/1348	−0/2298	0/0987	1/4829	0/9867	1/2314
20	0/8756	3/3155	3/3155	0/3543	0/4779	0/2550	0/5979	0/7197	0/4246	0/3366

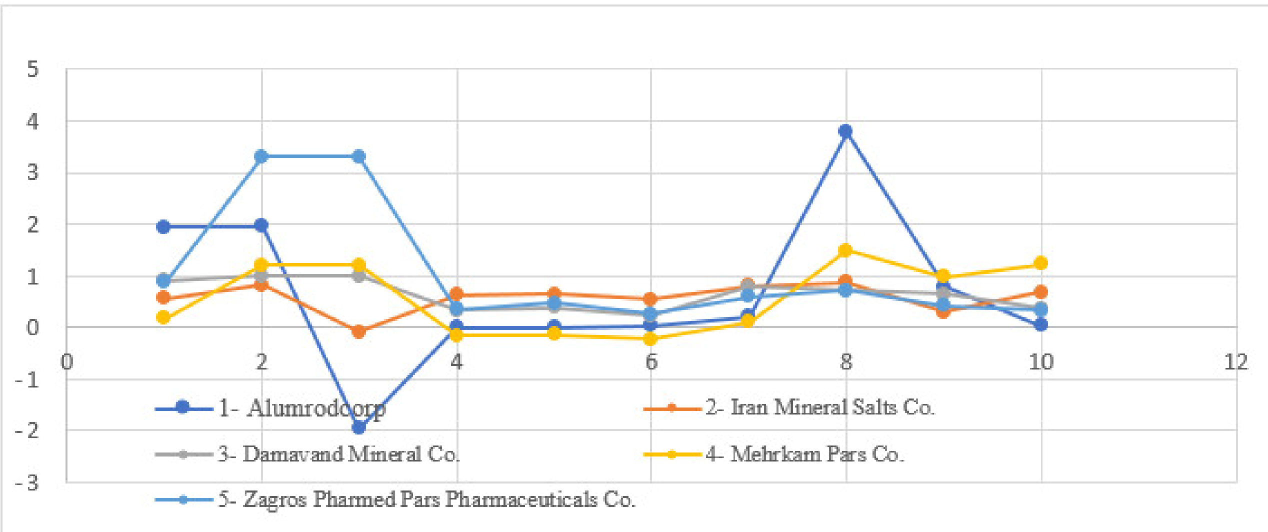

Figure 3. Comparison of 10 criteria for five top-ranked companies.

5. Discussion and Conclusions

Financial markets have recently experienced a large amount of volatility. The globalization of the economy has increased the contagion of financial crises, leading to increased volatility in financial markets. The sharp fall in the stock prices of the capital markets has caused significant losses to their activists. Therefore, the activity in financial markets will be accompanied by uncertainty and risk, which is essential for investors to measure. Creating an asset portfolio is one of the ways to control investment risk.

An efficient portfolio is the optimal combination of assets to minimize portfolio risk for a given return. Risk and return on capital (ROC) are key components of investment decisions. Rational investors consider the optimal returns and avoid risk. They also make rational decisions that maximize their expected returns. Therefore, investors' utility is a function of expected return and risk, the two fundamental parameters of investment decisions. In other words, the asset portfolio optimization problem seeks a portfolio that produces less standard deviation (risk) and more expected value. This study aimed to conduct a comprehensive modeling for the optimal stock portfolio selection using MADM approaches for companies listed on the TSE. The results of the research hypothesis test demonstrated that achieving a comprehensive and optimal portfolio model in TSE is possible using MADM approaches. Different MADM approaches (TOPSIS, TAXONOMY, ARAS, VIKOR, COPRAS, and WASPAS) can all be used to identify the optimal and best portfolios to obtain the maximum return.

Mendonça et al. (2020) [51] applied MADM to the financial portfolio optimization problem. According to the findings, the current cumulative return methods represent safe investments for the analyzed period, and the aggressive profile obtains more profit with more risk. Researchers evaluated Athens Stock Exchange (ASE) stocks using an MADM approach according to a decision-support system [52]. This method is based on fundamental analysis ratios and uses the UTA method to classify stocks from the best to the worst and consider the investor's risk-taking ability. This system, which was designed for natural and legal investors, utilizes the relevant data and applies it to real-world conditions to update the data. This finding aligns with this study.

Chen explored the artificial bee colony (ABC) algorithm in the portfolio optimization problem [53]. In this study, returns on risky assets were fuzzy numbers. The studied model was based on the cardinality-constrained mean-variance (CCMV), upper and lower limits of stock weights, and transaction cost. Then, the study algorithm was modified due to the improvements in its performance. Finally, the performance of the proposed ABC algorithm was tested using a series of numerical examples. The results reveal that this algorithm has an outstanding performance, which is identical to the results of this study. Scholars investigated and compared two modified particle swarm optimization (PSO) and modified harmony search algorithm (HSA) algorithms in portfolio optimization using the CCMV model [54]. PSO modifications were made to the learning coefficients and inertia weights, and HSA modifications were made to the memory consideration rate. The results of these algorithms when optimizing 31 assets up to 225 assets indicated that adjusted HSA outperforms adjusted the PSO regarding speed. This finding aligns with the present study.

Research Limitations

From a theoretical perspective, the issue of stock portfolio selection in the case of risk minimization can be solved using mathematical formulas and through a quadratic equation. However, in practice and the real world, considering the number of choices that exist in capital markets, the mathematical approach used to solve this model is limited and requires extensive calculations and planning. As stock market behavior does not follow a linear pattern, common linear methods cannot be used to describe this behavior and be useful. Considering the conditions of uncertainty when the investor determines the effective factors in the investment process, including the exact amount of return and stock risk, an attempt was made in this article to provide a model using non-linear programming, as well as its solution method for optimal portfolio selection. Since the goal of an investment is to obtain the minimum risk in return for an acceptable amount of return, an optimization model was used to minimize the adverse risk based on a certain amount of return.

Author Contributions: D.J. planned the scheme, initiated the project, and suggested the simulation; M.I. conducted the numerical simulation and analyzed the results; S.A.E. and A.A. developed the simulation result and modeling. H.A.E.-W.K. examined the theory validation. The manuscript was written with the contribution of all authors. All authors have read and agreed to the published version of the manuscript.

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