

Metallization of molecular and atomic hydrogen in 2D under high pressure

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Outline

① Motivation

Hydrogen systems - metallization

Hydrogen planes

② Method and Model

Method: Exact Diagonalization Ab Initio (EDABI)

③ Benchmark: H_2 molecule

④ Two-dimensional molecular crystal

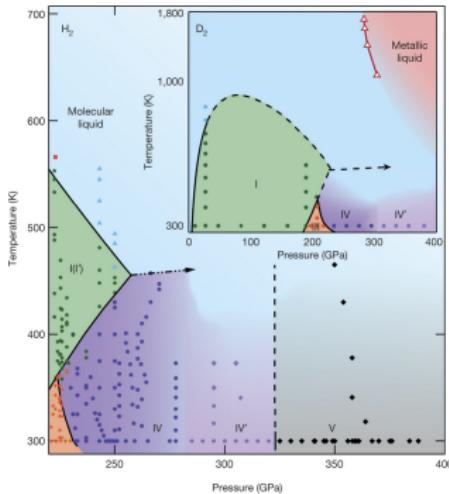
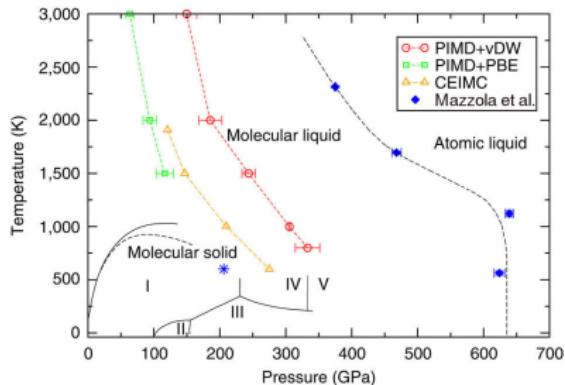
Model

Results

⑤ Atomic triangular plane

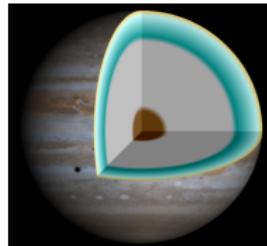
⑥ Summary

Hydrogen metallization



G. Mazzola, S. Yunoki and S. Sorella, Nature Communications 5, 3487, (2014) ↑

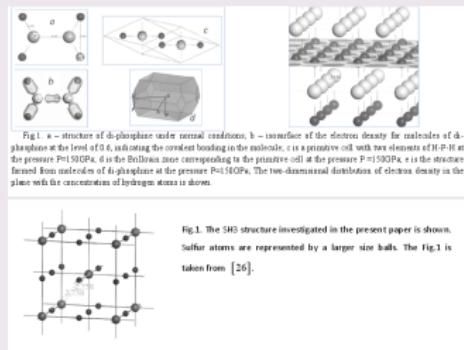
P. Dalladay-Simpson, R.T. Howie and E. Gregoryanz, Nature, 529, (2016) - phase V ↗



Jupiter interior
- potentially
metallic
hydrogen.
[en.wikipedia.org/wiki/
Metallic_hydrogen](https://en.wikipedia.org/wiki/Metallic_hydrogen)

Hydrogen planes

DFT calculations



PH₄ - N. Degtyarenko, E. A. Mazur, J. Exp. Theor. Phys. **123**, 2 (2016),
H, H₂S, H₃S - N. Degtyarenko, E. A. Mazur, unpublished work

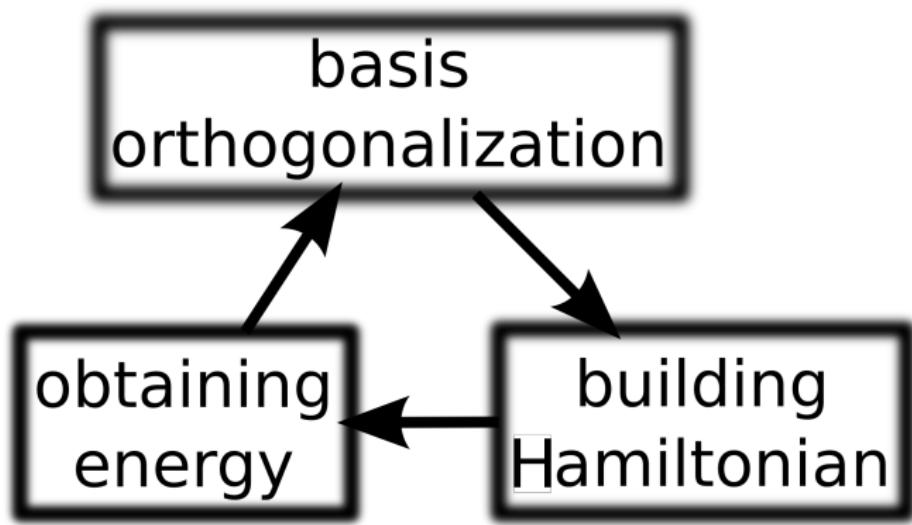
Comment

Two-dimensional planes contribute to the metallization and superconductivity with strong hydrogen-hydrogen correlations and electron-phonon coupling.

Method: Exact Diagonalization Ab Initio (EDABI)

Exact Diagonalization **ab Initio** approach

Quantum mechanical method to describe light-element structures with proper correlations picture, combining the first and second quantization.



Hamiltonian

$$\hat{\mathcal{H}} = \sum_{\sigma, i, j} t_{ij} \hat{c}_{i\sigma}^\dagger \hat{c}_{j\sigma} + \sum_{\substack{i, j, k, l \\ \sigma, \sigma'}} V_{ijkl} \hat{c}_{i\sigma}^\dagger \hat{c}_{j\sigma}^\dagger \hat{c}_{l\sigma'} \hat{c}_{k\sigma} + \mathcal{V}_{c-c}$$

$$\{\hat{c}_{i\sigma}^\dagger, \hat{c}_{j\bar{\sigma}}^\dagger\} \equiv \{\hat{c}_{i\sigma}, \hat{c}_{j\bar{\sigma}}\} \equiv 0 \quad \text{and} \quad \{\hat{c}_{i\sigma}^\dagger, \hat{c}_{j\bar{\sigma}}\} \equiv \delta_{ij} \delta_{\sigma\bar{\sigma}}$$

Microscopic parameters

$$t_{ij} \stackrel{\text{a.u.}}{=} \left\langle w(\mathbf{r})_i \right| - \nabla^2 - \sum_{k=1}^n \frac{2}{|\mathbf{r} - \mathbf{R}_k|} \left| w(\mathbf{r})_j \right\rangle$$

$$V_{ijkl} \stackrel{\text{a.u.}}{=} \left\langle w(\mathbf{r})_i w(\mathbf{r}')_j \right| \frac{2}{|\mathbf{r} - \mathbf{r}'|} \left| w(\mathbf{r})_k w(\mathbf{r}')_l \right\rangle$$

Benchmark: H_2 molecule

Extended basis

Orbitals: $1s, 2s, 2p_x, 2p_y, 2p_z$

Results

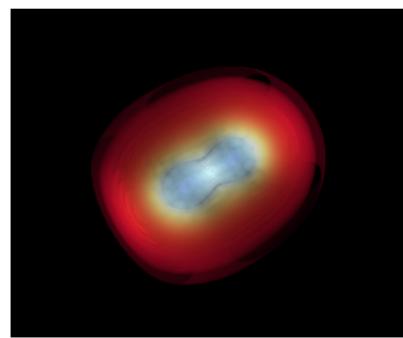
$$R_B = 1.395(1.4010) \text{ } a_0,$$

$$E_B = -2.33766(-2.3291) \text{ } Ry.$$

Comparable to the best with only 4 variational parameters!

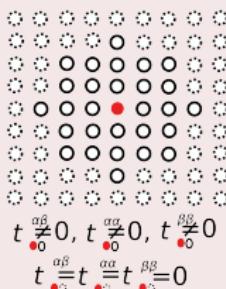
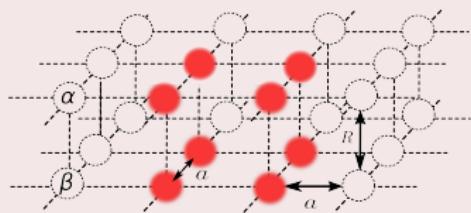
(Kołos, Wolniewicz, Piszczałkowski)

<http://cccbdb.nist.gov/energy2.asp>

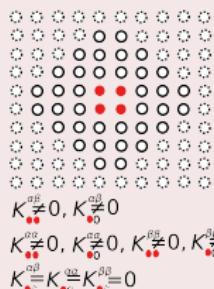


Two-dimensional molecular crystal - model

Two-dimensional molecular lattice



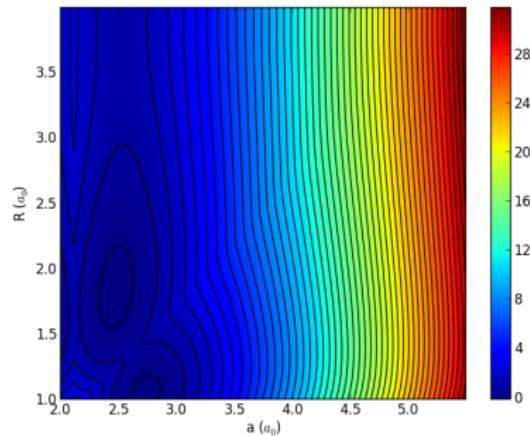
$$t_{\bullet}^{a\beta} \neq 0, t_{\bullet}^{aa} \neq 0, t_{\bullet}^{\beta\beta} \neq 0$$
$$t_{\bullet}^{a\beta} = t_{\bullet}^{aa} = t_{\bullet}^{\beta\beta} = 0$$



$$K_{\bullet}^{a\beta} \neq 0, K_{\bullet}^{aa} \neq 0$$
$$K_{\bullet}^{a\beta} \neq 0, K_{\bullet}^{aa} \neq 0, K_{\bullet}^{bb} \neq 0, K_{\bullet}^{\beta\beta} \neq 0$$
$$K_{\bullet}^{a\beta} = K_{\bullet}^{aa} = K_{\bullet}^{\beta\beta} = 0$$

- periodic boundary conditions in xy plane
- 8 atoms in supercell
- hoppings up to 13th neighbor
- Coulomb repulsion K_{ij} up to 13th neighbor

Function of state

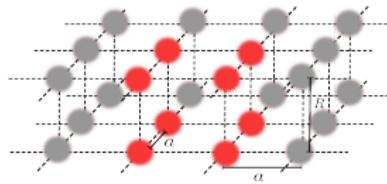


Generalized enthalpy near the metal–insulator transition

Generalized enthalpy
Generalized pressure

$$p \Leftrightarrow p_{2D} (Rya_0^{-2})$$

$$\begin{aligned} H/\text{atom} \equiv \\ E_G/\text{atom} + p_{2D}a^2/2 \end{aligned}$$



Software: QMT



QMT

(Quantum Metallization Tools)

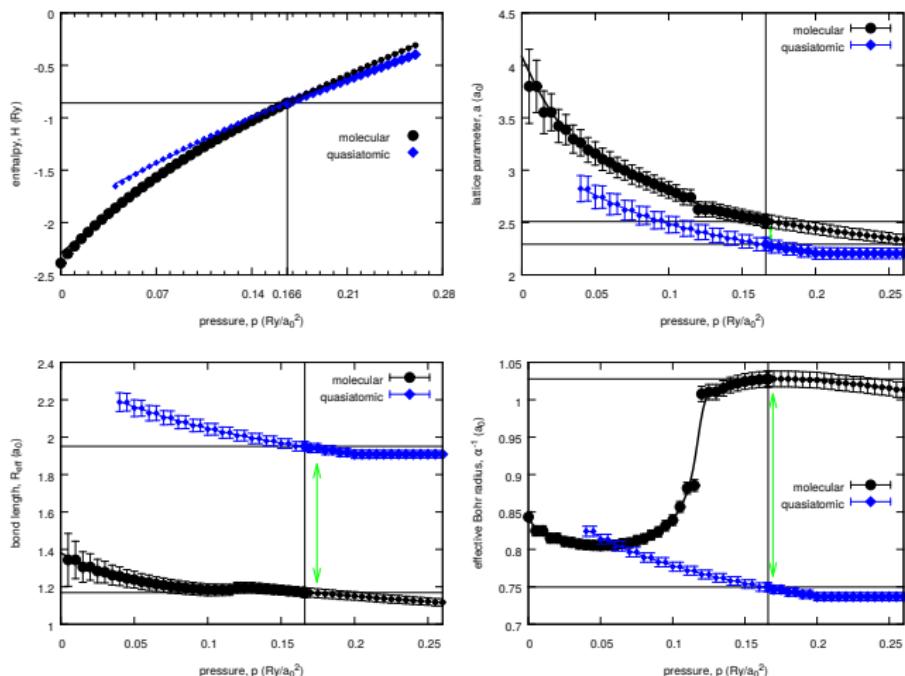
bitbucket.org/azja/qmt

Process-pool solution for integral calculation

AB, APK, JS, Comput. Phys. Commun. **197**, 7 (2015)



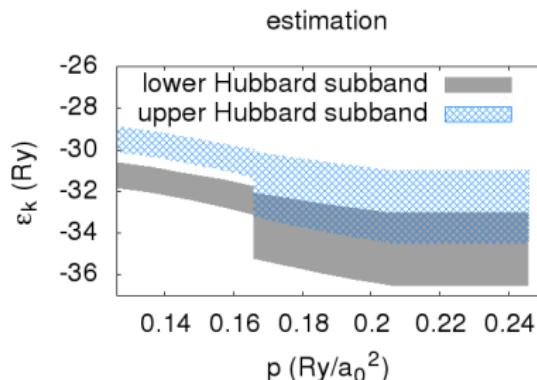
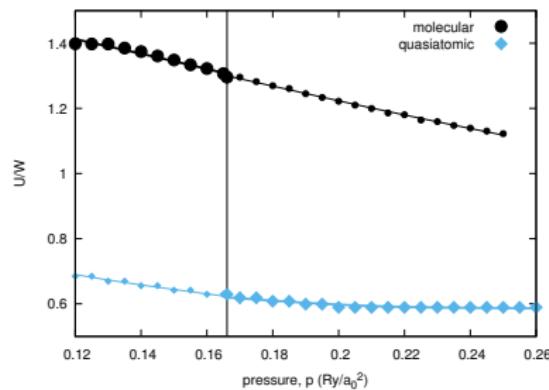
Enthalpy and structural parameters



Transition

Metal-insulator

Nature of transition

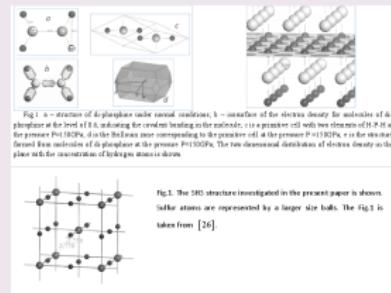


Metallization of solid hydrogen

We observe concomitant atomization and metallization of molecular plane, as shown on on-site Coulomb repulsion U to bandwidth W ratio, as well as the overlap of Hubbard subbands after transition.

Atomic triangular plane

DFT calculations



PH_4 - N. Degtyarenko, E. A. Mazur, J. Exp. Theor. Phys. **123**, 2 (2016),
 H, H_2S, H_3S - N. Degtyarenko, E. A. Mazur, unpublished work

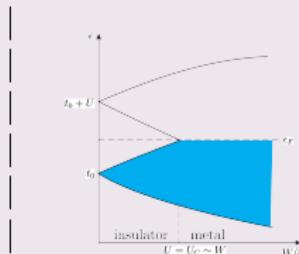
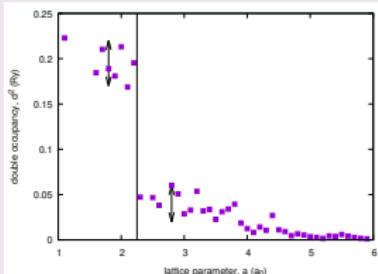
Model



Constant interaction radius
 $R_{cutoff} = 50a_0$,
 No. of interaction parameters:
 $M_{a=6.0a_0} = 36$
 $M_{a=1.0a_0} = 645$
 Hoppings up to 17th neighbor.

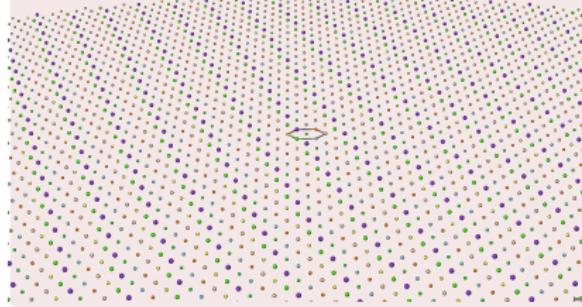
Atomic triangular plane

Results - Metallization under pressure



Wstęp do fizyki materii skondensowanej, Józef Spałek, PWN, Warszawa 2015

Model



Constant interaction radius

$$R_{\text{cutoff}} = 50a_0,$$

No. of interaction parameters:

$$M_{a=6.0a_0} = 36$$

$$M_{a=1.0a_0} = 645$$

Hoppings up to 17th neighbor.

Summary

- successful *ab-initio* description of infinite systems with proper correlation picture
- atomization and metallization pressure estimated for molecular plane $p_C = 0.166 \text{ Ry}/a_0^2$
- metallization of atomic planes in di-phosphine under pressure confirmed

To do

- charge and spin gap (more computational resources)
- larger supercell size (VMC - already in the testing phase)
- phonon spectra

Thank you!



EDABI:

Computational procedure:

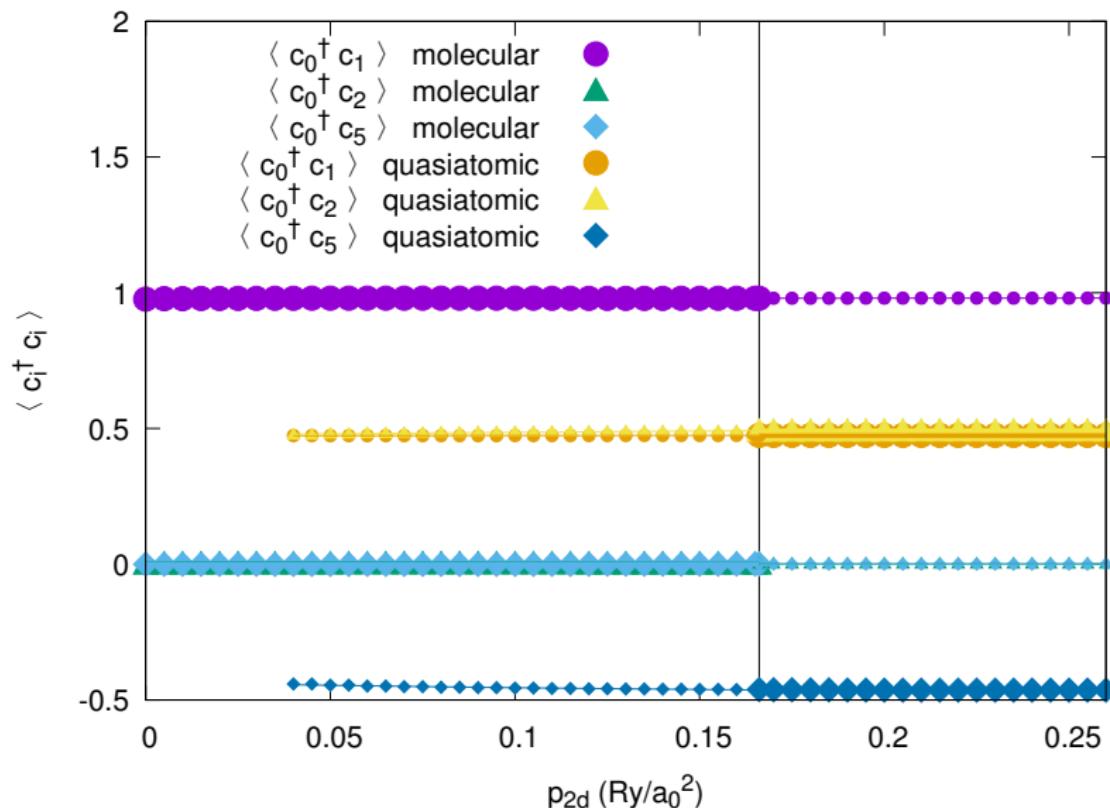
- starting with the single-particle basis $\{w_i^{\{\alpha(i)\}}(r)\}$ (eg. LCAO)
- calculating the microscopic parameters $t_{ij}(w_i, w_j)$, $V_{ijkl}(w_i, w_j, w_k, w_l)$
- diagonalizing the Hamiltonian $\hat{H}_N(\{t_{ij}\}, \{V_{ijkl}\})$ in the second quantization picture

$$\hat{H}|\Psi_N\rangle = E_G|\Psi_N\rangle$$

t_{ij} V_{ijkl} \rightarrow $O(z^2 N_G^2)$ \rightarrow E_G \rightarrow \hat{H}

$$\begin{cases} \langle w_i | w_j \rangle = \delta_{ij} \\ \hat{H} |\Psi_N\rangle = E_G \left(\{w_i^{\{\alpha(i)\}}\} \right) |\Psi_N\rangle \\ \delta_{w_i} E_G \left(\{w_i^{\{\alpha(i)\}}\} \right) = 0 \end{cases}$$

Correlation functions



Dispersion relation

