

$$A = \begin{vmatrix} 4 & 9 & 2 \\ 3 & 5 & 7 \\ 8 & 1 & 6 \end{vmatrix}$$

①

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$$z^{(1)} = \begin{vmatrix} 1 \\ 1 \\ 1 \end{vmatrix}$$

$$w^{(1)} = A z^{(1)} = \begin{vmatrix} 4 & 9 & 2 \\ 3 & 5 & 7 \\ 8 & 1 & 6 \end{vmatrix} \begin{vmatrix} 1 \\ 1 \\ 1 \end{vmatrix} = \begin{vmatrix} 15 \\ 15 \\ 15 \end{vmatrix} \quad \lambda^{(1)} = 15$$

$$e^{(1)} = \left\| \begin{vmatrix} 15 \\ 15 \\ 15 \end{vmatrix} - 15 \begin{vmatrix} 1 \\ 1 \\ 1 \end{vmatrix} \right\| = \left\| \begin{vmatrix} 0 \\ 0 \\ 0 \end{vmatrix} \right\| = 0 < \epsilon$$

$$\lambda = 15$$

② $\|A\|_2 = |\lambda| = 15$

③ $A^{-1} = \frac{1}{\det(A)} \cdot (A^D)^T$

$$\begin{aligned} \det(A) &= \sum_{i=1,2,3} A_{1i} (-1)^{1+i} \det(M_{1i}) = (-1)^2 \cdot 4 \cdot \det \begin{vmatrix} 5 & 7 \\ 1 & 6 \end{vmatrix} \\ &+ (-1)^3 \cdot 9 \cdot \det \begin{vmatrix} 3 & 7 \\ 8 & 6 \end{vmatrix} + (-1)^4 \cdot 2 \cdot \det \begin{vmatrix} 3 & 5 \\ 8 & 1 \end{vmatrix} = 4(30-7) - \\ &9(18-56) + 2(3-40) = 82 + 342 - 74 = 360 \end{aligned}$$

$$A^D = \begin{vmatrix} 57 & -37 & 35 \\ 16 & 86 & 81 \\ 92 & 42 & -49 \\ -16 & 86 & 81 \\ 92 & -42 & 49 \\ 57 & 37 & 35 \end{vmatrix} = \begin{vmatrix} 30-7 & -(18-56) & 3-40 \\ -(54-3) & 24-16 & -(4-12) \\ 63-10 & -(28-6) & 20-21 \end{vmatrix} =$$

$$= \begin{vmatrix} 23 & 38 & -37 \\ -51 & 8 & 68 \\ 53 & -22 & -7 \end{vmatrix}$$

$$(A^D)^T = \begin{vmatrix} 23 & -51 & 53 \\ 38 & 8 & -22 \\ -37 & 68 & -7 \end{vmatrix}$$

$$A^{-1} = \begin{vmatrix} \frac{23}{360} & \frac{-51}{360} & \frac{53}{360} \\ \frac{38}{360} & \frac{8}{360} & \frac{-22}{360} \\ \frac{-37}{360} & \frac{68}{360} & \frac{-7}{360} \end{vmatrix} = \begin{vmatrix} 0.06 & -0.14 & 0.14 \\ 0.1 & 0.02 & -0.06 \\ -0.1 & 0.18 & -0.02 \end{vmatrix}$$

$$z^{(1)} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad w^{(1)} = A z^{(1)} = \begin{bmatrix} 0.066 \\ 0.066 \\ 0.066 \end{bmatrix} \quad \lambda^{(1)} = 0.066$$

$$e^{(1)} = \left\| \begin{bmatrix} 0.066 \\ 0.066 \\ 0.066 \end{bmatrix} - 0.066 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\| = 0 < \epsilon$$

$$\lambda = 0.066$$

$$\|A^{-1}\|_2 = 0.066$$

$$\textcircled{4} \text{ Cond}_2 = \|A\|_2 \|A^{-1}\|_2 = 15 \cdot 0.066 = 0.99 \approx 1$$