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$$A = \begin{bmatrix} 3 & 1 \\ 6 & 2 \end{bmatrix} = U \Sigma V^T$$ 1

$$AA^T = \begin{bmatrix} 3 & 1 \\ 6 & 2 \end{bmatrix} \begin{bmatrix} 3 & 6 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 10 & 20 \\ 20 & 40 \end{bmatrix}$$

$$\det(AA^T \lambda I) = 0$$

$$\det(AA^T \lambda I) = I \begin{bmatrix} 10-\lambda & 20 \\ 20 & 40-\lambda \end{bmatrix} = (10-\lambda)(40-\lambda) - 400 =$$

$$= 400 - 10\lambda - 40\lambda + \lambda^2 - 400 = \lambda^2 - 50\lambda$$

$$\lambda^2 - 50\lambda = 0$$

$$\lambda(\lambda - 50) = 0 \qquad \lambda_1 = 50 \qquad \sigma_1 = \sqrt{50} = 5\sqrt{2} \approx 7$$

$$\lambda_2 = 0 \qquad \sigma_2 = 0$$

$$\cancel{v_1 = \sigma_1^{-1} A^T u_1}$$

$$AA^T u_1 = \lambda_1 u_1$$

$$(AA^T - \lambda_1 I)u_1 = 0$$

$u_1' =$

$$\begin{bmatrix} 10-5\sqrt{10} & 20 \\ 20 & 40-5\sqrt{10} \end{bmatrix} \begin{bmatrix} u_1^1 \\ u_1^2 \end{bmatrix} = 0$$

$$(10 - 2\sqrt{5})\cdot u_1^1 + 20 u_1^2 = 0 \qquad 20 u_1^1 + (40 - 2\sqrt{5}) u_1^2 = 0$$

$$u_1^1 = \frac{-20}{10 - 5\sqrt{10}} u_1^2 \qquad u_1^1 = \frac{5\sqrt{10} - 40}{20} u_1^2$$

$$\cancel{u_1^1 = \frac{-20}{2.92} u_{12}^2 = -6.84 \quad u_1^1 =}$$

$$u_1^1 = \frac{-20}{-40} u_1^2 = \frac{1}{2} u_1^2 \qquad u_1^1 = \frac{10}{20} u_{12} = \frac{1}{2} u_1^2$$

$$u_1 = \alpha [1 \; 2]$$

$\|v_1\|_2 = 1$

$\alpha = \frac{1}{\|v_1\|_2} = \frac{1}{\sqrt{5}}$ $\quad v_1 = \frac{1}{\sqrt{5}}[1 \ 2]^T$

$AA^T v_2 = \lambda_2 v_2$

$$\begin{bmatrix} 10 & 20 \\ 20 & 40 \end{bmatrix} \begin{bmatrix} v_2^1 \\ v_2^2 \end{bmatrix} = 0$$

$10 v_2^1 + 20 v_2^2 = 0 \qquad 20 v_2^1 + 40 v_2^2 = 0$

$v_2^1 = \frac{-20}{10} v_2^2 = -2 v_2^2 \qquad v_2^1 = \frac{-40}{20} v_2^2 = -2 v_2^2$

$v_2 = \alpha [-2 \ {+1}]^T = \frac{1}{\sqrt{5}} [-2 \ {+1}]^T$

$$U = \frac{1}{\sqrt{5}} \begin{bmatrix} 1 & -2 \\ 2 & +1 \end{bmatrix}$$

$$V_1 = \sigma_1^{-1} A^T v_1 = \frac{1}{5\sqrt{2}} \begin{bmatrix} 3 & 6 \\ 1 & 2 \end{bmatrix} \frac{1}{\sqrt{5}} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \frac{1}{5\sqrt{10}} \begin{bmatrix} 15 \\ 5 \end{bmatrix} = \frac{1}{\sqrt{10}} \begin{bmatrix} \frac{3}{\sqrt{10}} \\ \frac{1}{\sqrt{10}} \end{bmatrix} = \frac{1}{\sqrt{10}} \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$\cancel{V_2 = \sigma_2^{-1} A^T v_2 = \frac{1}{5\sqrt{2}} \begin{bmatrix} 3 & 6 \\ 1 & 2 \end{bmatrix} \frac{1}{\sqrt{5}} \begin{bmatrix} 2 \\ -1 \end{bmatrix} =}$$

$$A^T A = \begin{bmatrix} 45 & 15 \\ 15 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 45 & 15 \\ 15 & 5 \end{bmatrix} \begin{bmatrix} v_2^1 \\ v_2^2 \end{bmatrix} = 0$$

$A^T A \, v_2 = 0$

$45 v_2^1 + 15 v_2^2 = 0 \qquad 15 v_2^1 + 5 v_2^2 = 0$

$3 v_2^1 = - v_2^2 \qquad 3 v_2^1 = - v_2^1$

$$V = \frac{1}{\sqrt{10}} \begin{bmatrix} 3 & -1 \\ 1 & 3 \end{bmatrix}$$

$V_2 = \alpha [-1 \ \ 3]^T$

$\alpha = \frac{1}{\|V_2\|_2} = \frac{1}{\sqrt{10}}$

$$A = \frac{1}{\sqrt{5}} \begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 5\sqrt{2} & 0 \\ 0 & 0 \end{bmatrix} \frac{1}{\sqrt{10}} \begin{bmatrix} 3 & -1 \\ 1 & 3 \end{bmatrix}$$