Brownian Motion

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Abstract

In this paper Boltzmann's constant was measured by observing Brownian motion of latex particles in water and in an attempt to replicate the result published by Nakroshis [1]. Averaging separate measurements along the x,y axis results in obtaining the value $k_B = (1.32 \pm 0.15) \times 10^{-23} m^2 kg/(s^2 K)$.

1 Introduction

Brownian motion is a phenomenon first described in 1827 by the biologist Robert Brown [2], who observed that small parts of pollen grains move randomly in water. In 1905 this phenomenon was explained by Albert Einstein [3] and in 1908 it was experimentally verified by Jean Perrin [4]. The significance of Brownian motion is that its explanation requires the existence atoms and molecules.

This paper will describe a rendition of an experiment performed by Nakroshis [1], which is based on Perrin's original experiment. The motion of latex particles in water is recorded using a set-up consisting of microscope and digital camera, from which a track of a single particle is extracted. This data is then used to obtain the variance of the displacements, which in turn allows to get a value for the Boltzmann constant.

2 Theory

Brownian motion is essentially a magnification of the thermal motion. Thermal motion refers to the random movement of particles, such as molecules, proportional to their temperature. These particles are, however, too small to observe directly, but if there are larger particles present, the randomly moving molecules will exert forces on the large particles, which will in turn also exhibit random motion. These large particles, such as latex spheres are observable with an optical microscope and their movement can be recorded and quantified.

In order to quantify the motion, a formula for the average squared displacements needs to be obtained [5] [6] from Stoke's law and molecular kinetic theory, which is also known as the Stokes-Einstein equation.

$$D = \frac{k_b T}{6\eta\pi R} \tag{1}$$

Here D is the diffusion coefficient, η is the viscosity of the medium, T is the temperature and R is the radius of the particle. The diffusion coefficient for 1-D can be rewritten as $D =< (\Delta x)^2 > /(2t)$

$$< (\Delta x)^2 >= \frac{2k_bT}{6\eta\pi R} \tag{2}$$

$$k_b = \frac{6\sigma_x^2\pi\eta R}{2tT} \tag{3}$$

From equation (2) the Boltzmann constant can be expressed in order to obtain experimental result from the data. The mean squared displacement is replaced with the variance, as it is necessary to account for any flow of the medium (non-Brownian motion). The mean squared displacement equals to the variance if the mean is 0.

3 Experiment Apparatus

The experiment apparatus consists of an optical microscope and a camera, connected to computer for data collection. The set-up is calibrated using a calibration slide and it was determined that the collected images have a ratio of $7.7549\mu m$/pixel. The sample studied is a suspension of $0.5\mu m$ latex spheres in water and is imaged every 2 seconds.

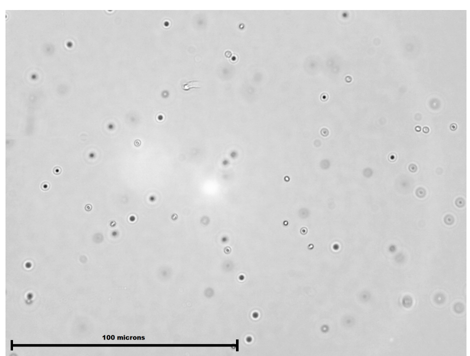

Figure 1: Latex particles. A series of images like this one, taken at uniform time intervals allows to record the motion of the particle.

4 Data

The ImageJ analysis software is used to track the position of a particle across the collected images and it's positions are recorded in a .csv file. The motion that the particle traces out is recorded in Figure 2.

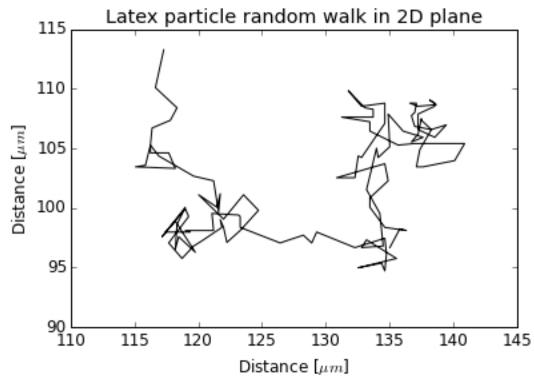

Figure 2: Random walk. A plot of the recoded motion, which can be separated into individual displacements along the x and y axes.

Individual displacements are determined along the x and y axes and plotted in histograms. It can be observed that the distributions of step sizes are Gaussian, which is to be expected since it's a measurement of random motion.

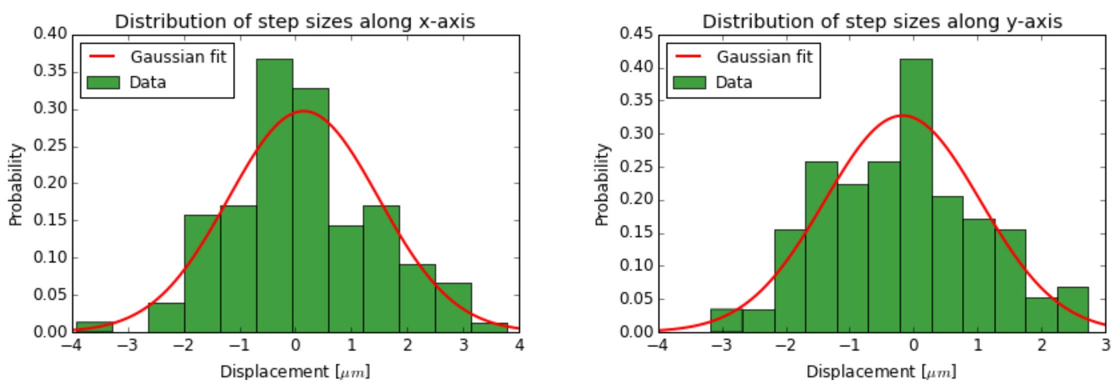

Figure 3: Step size distribution.

Chauvenet's criterion is applied and one y-displacement is discarded as an outlier.

Variable	Value	Uncertainty
$\eta\ [kg/(sm)]$	1×10^{-3}	5×10^{-5}
$R[m]$	0.5×10^{-6}	0.25×10^{-7}
$T[K]$	293	2
$t[s]$	2	0

Table 1: Variables. Input parameters and associated uncertainties necessary in order to obtain the k_B value.

5 Results

The variance of the two x,y datasets is obtained and preliminary values k_B can be calculated by plugging into equation (3). To obtain uncertainty on k_B, the uncertainties on the variables need to be propagated using equation (4).

$$\sigma^2_{k_B} = \sqrt{\left(\frac{dk_B}{d\eta}\right)^2 \sigma^2_\eta + \left(\frac{dk_B}{dT}\right)^2 \sigma^2_T + \left(\frac{dk_B}{dR}\right)^2 \sigma^2_R + \left(\frac{dk_B}{d\sigma^2_x}\right)^2 \sigma^2_{\sigma^2_x}} \tag{4}$$

In order to obtain the uncertainty on the sample variance for equation (4), a Monte Carlo simulation of the measurement is performed, where for the obtained mean and variance, a new set of displacements is randomly generated. This is repeated 10 000 times. Each time, the new value of variance is recorded. For this meta-sample of variances, the variance of the variances is computed, which gives the desired uncertainty on original variance of the sample needed for the error propagation.

The values in table 2 are obtained from the x and y datasets.

Dataset	Value of k_B $[m^2kg/(s^2K)]$	Uncertainty $[m^2kg/(s^2K)]$
x-axis	1.45×10^{-23}	0.22×10^{-23}
y-axis	1.20×10^{-23}	0.21×10^{-23}

Table 2: Experimental k_B values.

A consistency check is conducted by comparing the absolute difference of the measurements to zero, within the combined variance of the measurements.

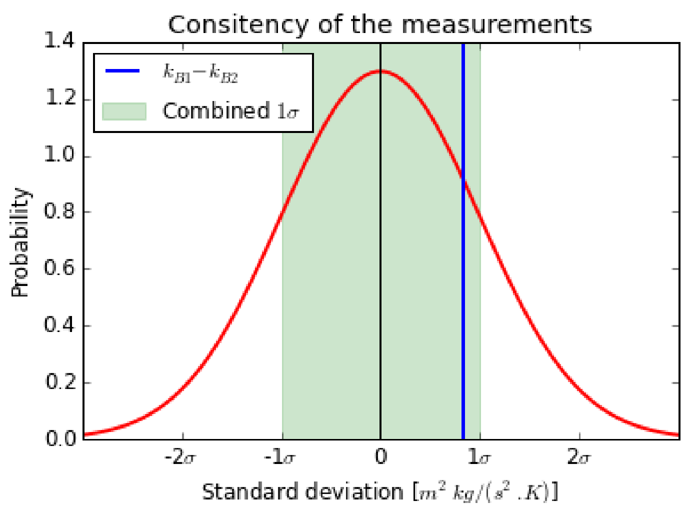

Figure 4: Consistency. The measurement absolute difference is within one combined standard deviation from 0 and therefore it can be said that the measurements are consistent.

A final value is obtained through a weighted mean of the measurements along x and y axes by equation (5).

$$k_B = \frac{\sum k_{Bi}\sigma_i^{-2}}{\sum \sigma_i^{-2}} \qquad \sigma^2_{k_B} = \frac{1}{\sum \sigma_i^{-2}} \tag{5}$$

This yields the final value of $k_B = 1.32 \times 10^{-23} \pm 0.15 \times 10^{-23} m^2kg/(s^2K)$.

6 Conclusion

The obtained value is within the experimental uncertainty of the real value and it can be concluded that the experimental approach was valid.

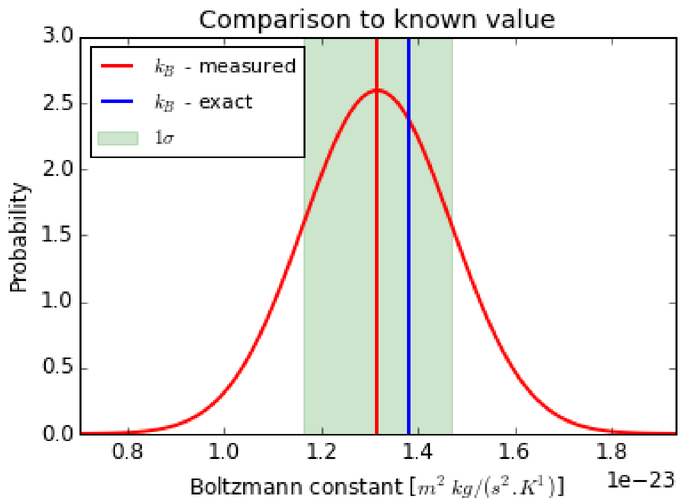

Figure 5: Comparison to exact value

The used set-up suffers several drawbacks. The largest uncertainty contribution comes from the variance (mean squared displacement) and could be reduced by replacing manual tracking by automated computer recognition. This would enable collecting significantly more data, which in turn would reduce error. Furthermore the temperature of the environment could be more controled, as any error on it propagates into the error on viscosity. Lastly the measurement could be improved by perfecting the sample preparation and analysing and reducing the edge effects and flows.

Nevertheless, despite the drawbacks, the obtained k_B value is within 10% of the accepted value.

References

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