Speed of Light

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Abstract

In this paper the speed of light is measured using Foucault method and PASCO Speed of Light Apparatus. Obtained result is compared to the defined value of 299792458 m/s. The speed of light is measured in two settings and the final value is found to be $c = 2.98 \pm 0.08 \times 10^8$ m/s.

1 Introduction

Speed of light is an important universal physical constant for many reasons, the most significant being that it is the maximum speed at which matter and information can travel [1]. As such it figures in various theories such as relativity and electromagnetism. It is denoted c.

Speed of light, being a relatively large quantity was for a significant part of history believed unmeasurable or infinite. The first attempt to measure a finite value is usually attributed to Galileo, but he was unable to obtain any meaningful result. The first actual measurement was done by Rømer [2] in 1675 by observing Jupiter's moon Io, but was quite inaccurate at only 70% of the actual value. The first laboratory experiment was designed much later in 1849 by Fizeau [3], who obtained a value about 5% larger than the currently accepted number. Improving upon this method Foucault [4] obtained a value within 0.6% in 1862. Finally this method was perfected by Michelson [5] in 1926, who obtained a value accurate to several km/s.

Other methods were used since, but as it was observed that the speed of light could be measured more accurately than the length of a meter, it was decided to specify a fixed value for speed of light and redefine meter.

> The metre is the length of the path travelled by light in vacuum during a time interval of 1/299792458 of a second.[7].

Therefore the speed of light is since 1983 defined exactly as 299792458 m/s. This paper will describe rendition of Foucault's experiment, albeit using modern equipment.

2 Experimental Apparatus

The basis of Foucault's experimental apparatus is a light source (laser), a rotating mirror and a fixed reflecting mirror. Light travels to the rotating mirror, which reflects it towards the fixed mirror, which reflects it back. Now if the rotating mirror is stationary the beam is reflected right back towards the source. However, if the rotating mirror moves slightly, the beam will be reflected back at an angle. The speed of light is then determined from measuring the angle, given the variables.

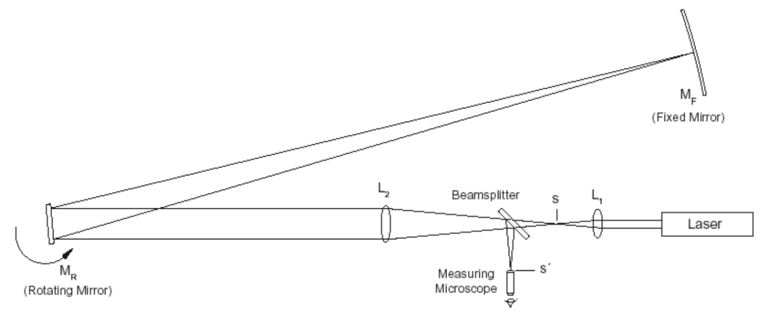

Figure 1: Foucault method. A rotating mirror reflects the original pulse at one angle and when the pulse returns, it reflects the pulse back towards the source at a slightly different angle, which can be measured. [6]

More precisely, the laser beam passes through the lens L_1 and is focused to a point image at the point S. The image then passes through the lens L_2, which is set up to focus the image onto the fixed mirror M_F, after it is reflected from the rotating mirror M_R. The returning beam is reflected by the rotating mirror M_R at a different angle than before, and is refocused by the lens L_2 into the point S'. In order to measure the displacement $\Delta S = S - S'$, a beam splitter intercepts the path of the returning beam and allows the displacement to be measured through microscope as shown in Figure 1.

The analysis can be simplified by considering the virtual images instead of the reflected ones, which have to be equivalent as shown in Figure 2. The length of arc is $r\theta$, which will approximate the displacement at M_F due to change in angle of M_R. But any change $\Delta\theta$ has to be factored in twice since by the law of reflection it affects both incident and reflected beam. From this, Equation (1) is obtained.

$$\Delta S = S_1 - S = 2D\Delta\theta \tag{1}$$

Furthermore the magnification due to thin lens is $m = \frac{h_i}{h_o} = \frac{-d_i}{d_o}$ so the measured displacement in Equation (2) is obtained by combining this with Equation (1) and using the absolute value of the magnification.

$$\Delta s = \Delta s' = \frac{d_i}{d_0}\Delta S = \frac{A}{B+D}\Delta S = \frac{2DA\Delta\theta}{B+D} \tag{2}$$

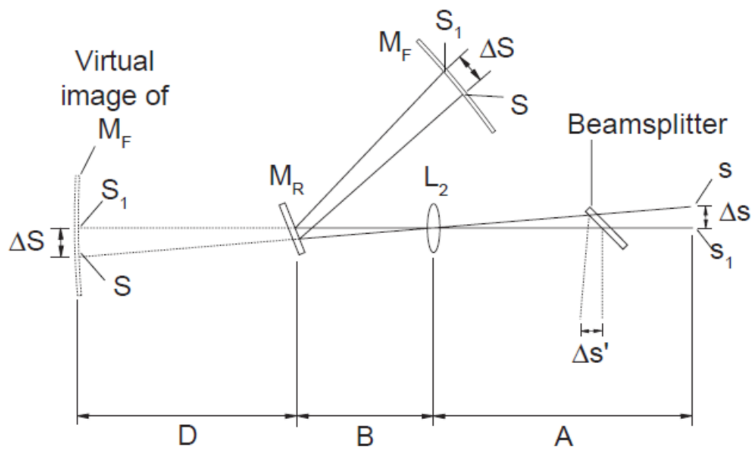

Figure 2: Foucault method. Virtual images are used to simplify calculations. [6]

$\Delta\theta$ is expressed in terms of angular velocity time in Equation (3).

$$\Delta\theta = \omega t = \omega \frac{s}{v} = \omega \frac{2D}{c} \tag{3}$$

Equations (2) and (3) are combined to give an expression relating the speed of light to the displacement of the beam in Equation (4).

$$\Delta s' = \frac{4AD^2\omega}{(B+D)c} \tag{4}$$

By rearranging Equation (4), a more useful relation is obtained, which gives c in terms of the slope $\frac{\Delta s'}{\omega}$, so that fitting can be performed on the experimental data to obtain a best-fit result.

$$c = \frac{4AD^2}{(B+D)} \frac{\omega}{\Delta s'} = \frac{4AD^2}{(B+D)} \left(\frac{\Delta s'}{\omega}\right)^{-1} = \frac{4AD^2}{(B+D)m} \tag{5}$$

In Equation (5), c is the speed of light, A, B, D are distances and m is the slope of a line fitted through the measured displacement values at various angular velocities.

For more details, in-depth description of the experimental set-up and instruction on how to conduct the measurement consult the PASCO Speed of Light Apparatus manual [6]. A diagram of the apparatus is included in the Appendix (Figure 6).

3 Data

Two sets of data are collected with the fixed mirror at distances $D_1 = 5$ m, $D_2 = 3$ m and with angular velocities $\omega = -1500, -750, 750, 1500$ rps.

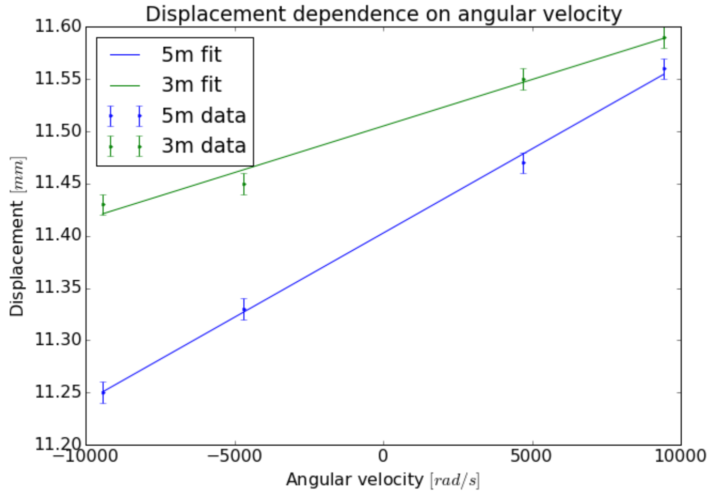

Figure 3: Fitting experimental data. Linear fit of the form $y = mx + k$ is employed and goodness of fit is assessed based on χ^2.

A linear model is fitted to the data in Figure 3, such that $\chi^2 = \sum_{i=1}^{N} \frac{[y_i-(a+bx_i)]^2}{\sigma_i^2}$ of the fit is minimized. For a linear model the values of a, b, that minimize χ^2 can be obtained analytically and are found through Equations (7) and (8), where Δ is the determinant of matrix M.

$$M = \begin{bmatrix} \sum \frac{1}{\sigma_i^2} & \sum \frac{x_i}{\sigma_i^2} \\ \sum \frac{x_i}{\sigma_i^2} & \sum \frac{x_i^2}{\sigma_i^2} \end{bmatrix} \tag{6}$$

$$a = \frac{1}{\Delta}\left(\sum \frac{x_i^2}{\sigma_i^2} \sum \frac{y_i^2}{\sigma_i^2} - \sum \frac{x_i^2}{\sigma_i^2} \sum \frac{x_i y_i}{\sigma_i^2}\right) \tag{7}$$

$$b = \frac{1}{\Delta}\left(\sum \frac{1}{\sigma_i^2} \sum \frac{y_i x_i}{\sigma_i^2} - \sum \frac{x_i}{\sigma_i^2} \sum \frac{y_i}{\sigma_i^2}\right) \tag{8}$$

The associated uncertainties are obtained through Equations (9) and (10).

$$\sigma_a^2 = \frac{1}{\Delta} \sum \frac{x_i^2}{\sigma_i^2} \tag{9}$$

$$\sigma_b^2 = \frac{1}{\Delta} \sum \frac{1}{\sigma_i^2} \tag{10}$$

Dataset	m $[ms/rad]$	k $[m]$	σ_m $[ms/rad]$	χ^2	χ^2 probability
$D_1 = 5\ m$	1.61×10^{-8}	0.011403	4.5×10^{-19}	1.15	0.56
$D_2 = 3\ m$	0.89×10^{-8}	0.011505	4.5×10^{-19}	2.6	0.27

Table 1: Linear fits and goodness of fits. χ^2 probability is based on 2 degrees of freedom of the linear model and is well within the expected probability range, confirming that the fit is valid.

Variable	Value	Uncertainty
A $[m]$	0.26	0.01
B $[m]$	0.472	0.01
D_1 $[m]$	5	0.02
D_2 $[m]$	3	0.02

Table 2: Distances. Input parameters and associated uncertainties necessary in order to obtain the value of c .

4 Results

A value for c is obtained from each dataset with the fit parameters in Table 1 and measurements in Table 2. The uncertainties on c are obtained through propagation of error in Equation (11).

$$\sigma_c^2 = \sqrt{\left(\frac{dc}{dA}\right)^2 \sigma_A^2 + \left(\frac{dc}{dB}\right)^2 \sigma_B^2 + \left(\frac{dc}{dD}\right)^2 \sigma_D^2 + \left(\frac{dc}{dm}\right)^2 \sigma_m^2} \tag{11}$$

The values obtained are listed in Table 3.

Dataset	Value of c $[m/s]$	Uncertainty $[m/s]$
$D_1 = 5\ m$	2.94×10^8	0.11×10^8
$D_2 = 3\ m$	3.02×10^8	0.11×10^8

Table 3: Experimental c values.

A consistency check is conducted by comparing the absolute difference of obtained values to zero, within their combined standard deviation $\sigma^2 = \sigma_1^2 + \sigma_2^2$.

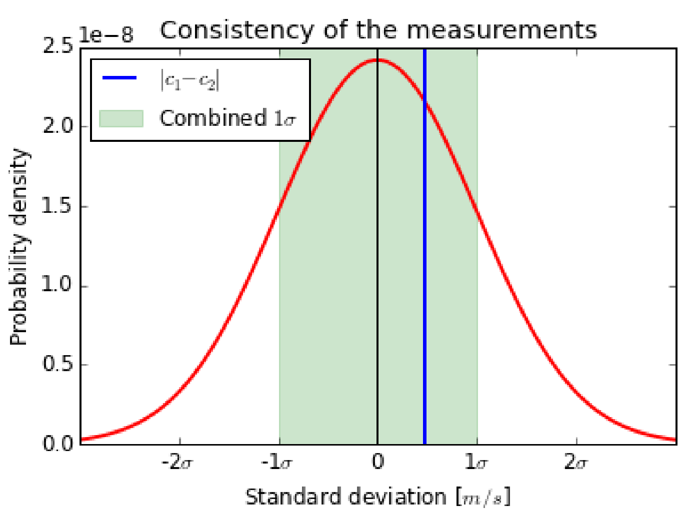

Figure 4: Consistency. The absolute difference of measurements obtained from the two datasets c_1, c_2 is within one combined standard deviation from 0 and therefore it can be said that the measurements are consistent.

A final result is obtained through a weighted mean of the two values by Equation (12).

$$c = \frac{\sum c_i \sigma_i^{-2}}{\sum \sigma_i^{-2}} \qquad \sigma_c^2 = \frac{1}{\sum \sigma_i^{-2}} \tag{12}$$

This yields the final value of $c = 2.98 \pm 0.08 \times 10^8$ m/s.

5 Conclusion

The obtained value is within the experimental uncertainty of the accepted value and it can be concluded that the experimental approach was valid.

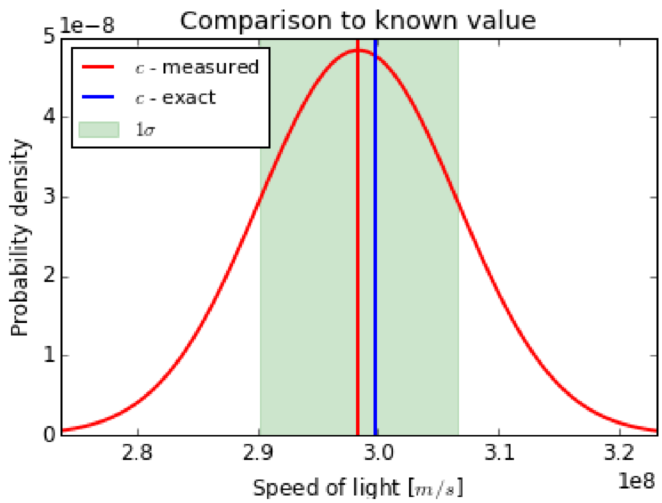

Figure 5: Comparison to exact value

The used set-up suffers several drawbacks. The largest uncertainties come from the distance measurements of the apparatus and they could be reduced significantly by a set-up with better control over the distances, such as one where the elements would be moving on fixed tracks and their position adjusted through gears, instead of sliding. This experiment also per instruction neglects the error on the angular velocity of the rotating mirror, as it would likely be negligible compared to other sources of error.

Nevertheless, despite the drawbacks, the obtained c value is within 0.5% of the exact, defined value.

References

[1] J-P. Uzan, B. Leclercq *The Natural Laws of the Universe: Understanding Fundamental Constants.*, Springer. pp. 434. (2008).

[2] O. Rømer, *Dmonstration touchant le mouvement de la lumire trouv par M. Rmer de l'Academie Royale des Sciences*, Journal des savans, pp. 22336. (1676).

[3] T. Hockey, *The Biographical Encyclopedia of Astronomers*, Springer Publishing, pp. 725-726. (2014).

[4] J.L. Foucault, *Dtermination exprimentale de la vitesse de la lumire: parallaxe du Soleil"*, Comptes rendus de l'Acadmie des sciences 55: pp. 501503, 792796. (1862).

[5] A.A. Michelson, *Measurement of the Velocity of Light in a Partial Vacuum*, Astrophysical Journal 82, pp. 26-61. (1935).

[6] PASCO, *Speed of Light Experiment*, https://www.pasco.com/prodCatalog/EX/EX-9932_speed-of-light-experiment. (Retrieved 4/8/2106).

[7] *Resolution 1 of the 17th CGPM*, (1983).

Appendix

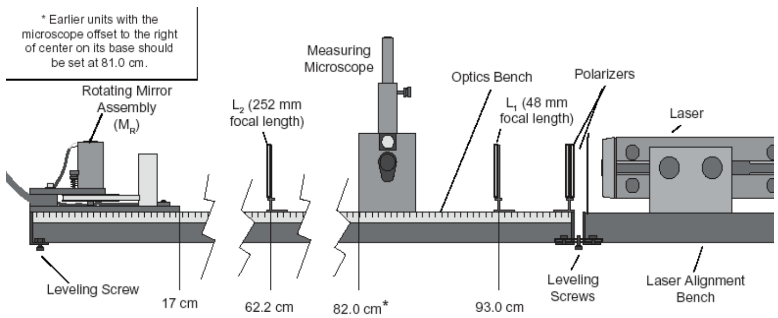

Figure 6: Experimental apparatus.