Measurement of the acceleration of gravity by locating the focal point of a rotating water surface

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Abstract

This article decribes an inexpensive, yet experimentally challengenging way to determine the acceleration of gravity g, using a rotating liquid surface and a laserpointer. The main components of this metod is that a rotating liquid surface will form a parabola and that this parabola will focus the light to a known focal point.

1 Introduction

The acceleration of gravity, g, has been measured countless times in the classroom. Usually this is done with some sort of pendulum or free fall. In a case of experimental simplicity most of theese methods are hard to beat. However, they do suffer from a sort of “dullness” – the physics behind them are quite basic and only involves Newtonian mechanics. From the student’s point of view, a pendulum experiment for instance won’t excite much experimental creativity.

A more thought-provoking way to measure g is to study the surface of a liquid in a rotating bowl. By studying the centrifugala force in the rotating bowl alongside gravity it can readily be shown that the gradient of the liquid surface in the bowl will be proportional to the distance from the axis of rotation. This corresponds to a parabola. More precisley the surface will follow the equation [1, 2]

$$z(r) = \frac{r^2\omega^2}{2g} \tag{1}$$

with the notation as in Figure 2. In 2010 Šabatka and Dvǒrák [1] confirmed this experimentally using a set of metal rods.

The fact that the surface is parabolic can then be used optically. A parabolic mirror will reflect any vertically incoming light to the focal point of that parabola. This property was used by Berg[2] in 1990 to focus light into an optical image.

The focus of the parabola in (1) is located at the height [3]

$$h = \frac{g}{2\omega^2}$$

above the vertex of the parabola. So by locating the focal point of the parabola in (1) the value of g can then be caluclated as

$$g = 2h\omega^2.$$

As we can see here, only the focal height h and the angular frequency ω influence the value of g making this method experimentally quite clean with only two parameters to measure.

Neither of these steps require very complicated theoretical caluculations, but the steps are not all trivial. All in all the main principle here will give the students an exercise in combinding knowledge from two parts of physics. At the same time the student's experimental creativity can be used as there are many ways of locating the focal point.

A similar method have even been used as part of an experimental problem in the International Physics Olympiad (IPhO) 2001[4]. IPhO is an international physics competition for talented high school students from about 80 countries. So this method already have quite good gounds as a challenging lab exercise.

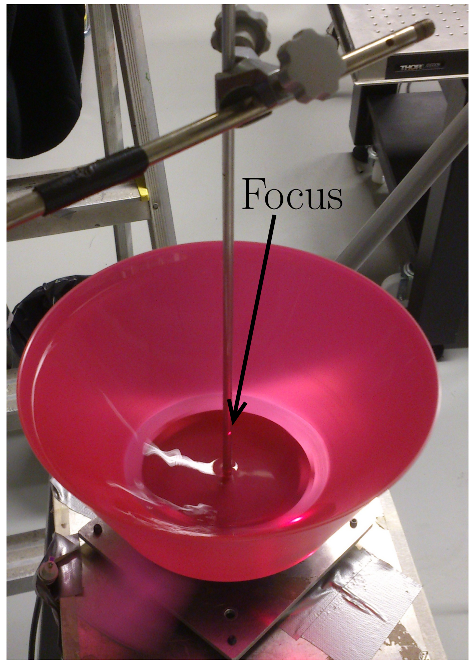

Figure 1: A photograph of the setup running. The reflected beam from a laser pointer (outside frame) can be seen on the central rod.

2 The apparatus and measuring method

The apparatus shown in Figure 1 consists of a bowl of water on a spinning disk, a laser-pointer, a central rod marking out the axis of rotation, and a photodiode for measureing the speed of rotation. The spinning disk was driven by a DC motor so the rotational speed could be adjusted by varying the voltage applied to the motor.

The focal point will be located somewhere along the axis of rotation. So when shining a laser vertically down on the parabolic surface the refection will hit the central rod and the height h over the vertex can be measured using a narrow ruler.

Then by varying the rotational speed and measureing the focal height for different speeds a linnear regression could be made for g as the slope of the cuve of h plotted against $1/(2\omega^2)$. Measurements were taken at two different radial distances from the center to verify that the radial distance has no impact on the measured value of g.

The rotational speed ω was measured with a photodiode connected to a digital oscilloscope wich could record several periods of rotation. Though a more readily availible option is to connect the photodiode to a computers microphone input and record the signal as a "sound" file on the computer, as [1] did. Since ω is one of the two parameters directly determining the value of g, it's recomendet to measure the periods with some kind of digital logging device such as an oscilloscope or computer, to minimize uncertainty in this parameter.

2.1 Calibration

Firstly the laser pointer has to be perfecly vertical. This can neatly be calibrated by shin-

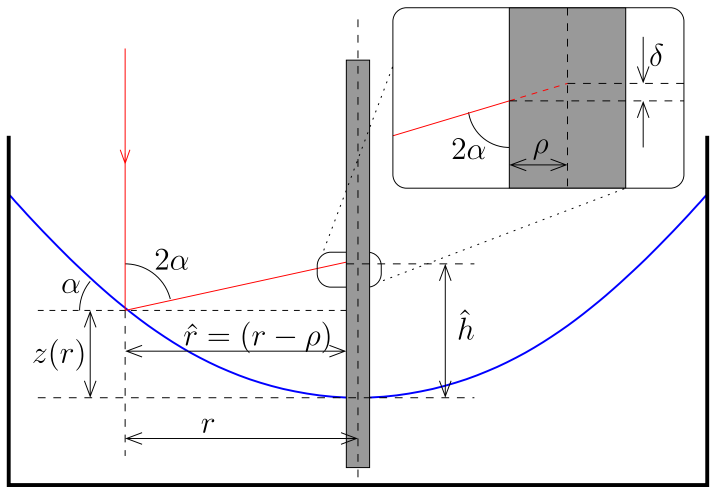

Figure 2: Schematic drawing of the setup in Figure 1 together with the definitions of some of the notation used in the calculations. Due to the focal height beeing measured slightly outside the exact center of the rod, by ρ, a small correction δ has to be made to the measured focal height.

ing the laser down on the watersurface when it's not rotating. The laserbeam will then be perfectly vertical when the reflection hits the source due to the watersurface beeing perfectly horisontal when still.

To get the central rod centered, a center mark was put in the rotating disc. A spirit level was used to ensure that the central rod was vertical. Both of these steps are essential, since deviation from the center will change the measure value of g, as we will see in next section.

2.2 Corrections

As shown in Figure 2 the measured focal height $\hat{h}$ is slightly of from the real focal height by

$$\delta = \rho \cot 2\alpha$$

due to a finite width ρ of the central rod. This width is easily measured with a pair of calipers.

We now have to determine α. First of we note that

$$\tan \alpha = \frac{\mathrm{d}z}{\mathrm{d}r} = \frac{2z(r)}{r}. \tag{2}$$

Then we see from Figure 2 that

$$\cot 2\alpha = \frac{\hat{h} - z(r)}{\hat{r}} = \frac{\hat{h}}{\hat{r}} - \frac{r}{2\hat{r}} \tan \alpha,$$

where we used (2) to substitute $z(r)$ in terms of $\tan \alpha$. By rewriting the last equation we get

$$\cot 2\alpha - \frac{\hat{h}}{\hat{r}} + \frac{\hat{r} + \rho}{2\hat{r}} \tan \alpha = 0.$$

only consisting of known or measured quantities and α. From this stage α can easily be obtained to a satisfying degree with regular numerical equation solvers.

3 Results

The results from the measurements together with the least square fitt are shown in Figure 3. The calculated value of the acceleration of gravity comes out as

$$g = 9.78 \pm 0.13\,\mathrm{m/s^2}$$

the uncertainty given is the standard deviation in the mean.

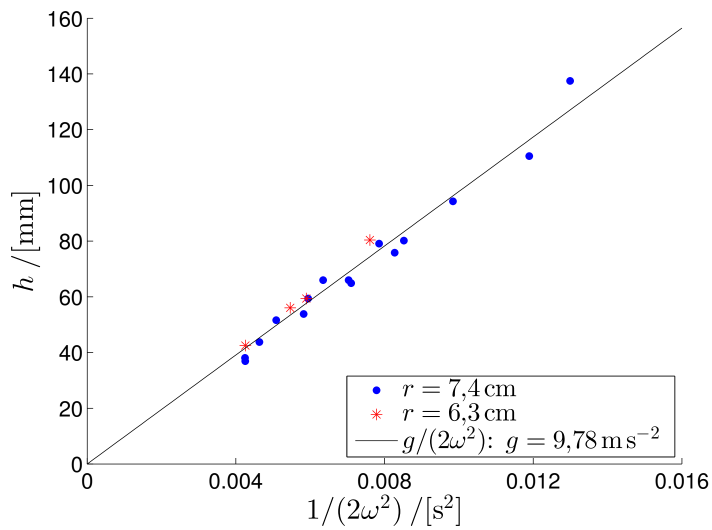

Figure 3: Measurements of the focal height h (with corrections for a center rod of non-zero width) plotted against $1/(2\omega^2)$. The acceleration of gravity g is now given as the slope of the least square fitt to these data points. Measurements were taken at two different radial distances from the center, denoted r in Figure 2.

4 Discussion

As we can see, the result comes out close to the familiar value of g as $9.8\,\mathrm{m/s^2}$. However, the error, of around 13 %, in this method doesn't make this the most accurate way to measure g, but as stated before this method is more of an educational experiment than a precision measurement. And as such, this experiment manages to combine both knowlege in mechanics and optics together with some careful thought needed to, for example, find the correction δ.

The significance of the correction due to finite width of the central rod, δ, is quite notable. The value of g comes out to be only about $9.2\,\mathrm{m/s^2}$ *without* these corrections. This demonstrates the necessity of the calculations made in section 2.2.

Most of the uncertainty in the result arose from small ripples on the water surface leading to a gittery motion of the reflected laserbeam. Both Berg[2] and IPhO[4] used glycerin as the liqud in thier experiments. Due to the high viscosity of glycerin, the ripples causing the uncertainties would have lessend. However as this experiment was primarily aimed as an experiment with a simple and inexpensive setup we used water instead.

Another way to increas accuracy would be to use a bigger bowl, preferably a bowl with vertical walls. Because the maximal rotational speed will be limited by the slope of the walls, since the walls have to have a steeper slope than the watersurface.

The experiment described here can be done as a shorter project in high school or lab exercise for first year undergraduate students. One sholud note though, that to set up this experiment will take around 2–4 hours from scratch to finished measurements, depending on the desired experimental accuracy. If this experiment is done as a project, the student could perhaps also try to compare this method to other more common methods.

Acknowledgments

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