Guelph-Waterloo Physics Institute

Quantum Mechanics – Physics 701/7010 – Fall 2016 – Problem Set 5

Due Thursday, Dec. 1st, at the lecture in Waterloo, or at Guelph, in Kiley Rider's mailbox in the copy room.

(1) Problem 11.19 from Griffiths, *Introduction to Quantum Mechanics*, 2nd ed.
Prove the **optical theorem**, which relates the total cross-section to the imaginary part of the forward scattering amplitude:

$$\sigma = \frac{4\pi}{k}\,\mathrm{Im}(f(0)) \qquad \text{(Eq. 11.104 of Griffiths)}$$

using:

$$f(\theta) = \frac{1}{k}\sum_{\ell=0}^{\infty}(2\ell+1)e^{i\delta_\ell}\sin(\delta_\ell)P_\ell(\cos\theta) \qquad \text{(Eq. 11.47 of Griffiths)}$$

and

$$\sigma = \frac{4\pi}{k^2}\sum_{\ell=0}^{\infty}(2\ell+1)\sin^2(\delta_\ell). \qquad \text{(Eq. 11.48 of Griffiths)}$$

(2) Problem 11.20 from Griffiths, *Introduction to Quantum Mechanics*, 2nd ed.
Use the Born approximation to determine the total cross-section for scattering from a gaussian potential

$$V(\vec{r}) = Ae^{-\mu r^2}.$$

Express your answer in terms of the constants A, μ and m (the mass of the incident particle), and $k = \sqrt{2mE}/\hbar$, where E is the incident energy. Discuss limitations on validity of the Born approximation for this potential.

(3) (a) On the course web-site you can find some information on the low-energy elastic scattering cross-section for the H-Kr system. The cross-section exhibits a sharp resonance at low-energies. In Section 6.7 of Sakurai and Napolitano you will find a simple quantitative theory of these resonances (the "Breit-Wigner" formula), which shows that both the peak cross-section and the width of the resonance only depend on how rapidly the phase varies through the resonance. Check to see if the Breit-Wigner formula gives the same width and peak cross-section as the complete numerical calculation (see the slide "A closer look at first resonance" in the H-Kr presentation on the course web-site). Comment on discrepencies.

(b) Suppose that in the scattering of a spinless non-relativistic particle of mass μ by an unknown potential, a resonance is observed at energy E_R for which the elastic cross-section at the peak of the resonance is $\sigma_{\max}$. How would you use this data to give a value for the orbital angular momentum of the resonant state (based on a question in Weinberg's "Lectures on Quantum Mechanics").

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(4) Consider three-dimensional scattering of a particle of mass m from the radially symmetric potential:

$$V(r) = -V_0 \exp(-2r/b)$$

where V_0 and b are both positive constants.

We define a dimensionless characterization of the strength of the potential: $g^2 = V_0/(\hbar^2/(2mb^2))$, and a dimensionless total energy: $f = E/(\hbar^2/(2mb^2))$.

Shown below are three numerically calculated continuum wavefunctions for the radial part of the wavefunction: $u(r)$, where $\psi(r, \theta, \phi) = \{u(r)/r\}\, Y_{l,m}(\theta, \phi)$.

(a) Based on the calculated wavefunctions, estimate the total elastic scattering cross-section at $f = 0.2$.

(b) Estimate the total scattering cross-section for $f = 0$.

(c) Explain the difference in the relative contributions of s-wave and p-wave scattering to the total cross-section. Why can we ignore the d-wave (and higher) contributions?

(d) How many $\ell = 0$ bound states do you expect this potential to support? Estimate the binding energy of the most weakly bound state.