Homework 3
Due Wednesday, November 9

1. Find the discontinuity in the derivative of the specific heat $(\partial C_V/\partial T)_V$ of a Bose gas at the Bose condensation transition temperature T_c.

2. Consider a system of weakly interacting bosons in one spatial dimension. Demonstrate that this system does not exhibit Bose condensation even in the limit of zero temperature. Show that if the interactions are turned off, the bosons will condense at $T = 0$, but not at any finite temperature.

3. (Problem 11.4 of PB)

   (a) Show that to first order in the scattering length a, the discontinuity in the specific heat C_V of an imperfect Bose gas at the transition temperature T_c is given by

   $$C_V(T = T_{c-}) - C_V(T = T_{c+}) = N\frac{9a}{2\lambda_c}\zeta(3/2),$$

   while the discontinuity in the bulk modulus K is given by

   $$K(T = T_{c-}) - K(T = T_{c+}) = -\frac{4\pi a\hbar^2}{mv_c^2}.$$

   (b) Examine the discontinuities in the quantities $(\partial^2 P/\partial T^2)_v$ and $(\partial^2 \mu/\partial T^2)_v$ as well, and show that your results are consistent with the thermodynamic relation

   $$C_V = VT\left(\frac{\partial^2 P}{\partial T^2}\right)_v - NT\left(\frac{\partial^2 \mu}{\partial T^2}\right)_v.$$

4. (Problem 11.1 of PB)

   (a) Show that, for bosons as well as fermions,

   $$[\Psi(\mathbf{r}_j), H] = \left(-\frac{\hbar^2}{2m}\nabla_j^2 + \int d^3r\,\Psi^\dagger(\mathbf{r})U(\mathbf{r}-\mathbf{r}_j)\Psi(\mathbf{r})\right)\Psi(\mathbf{r}_j),$$

   where H is the Hamiltonian of the system in the second quantized representation.

(b) Making use of the foregoing result, show that the equation

$$\frac{1}{\sqrt{N!}}\langle 0|\Psi(\mathbf{r}_1)\dots\Psi(\mathbf{r}_N)H|\Psi_{NE}\rangle = E\frac{1}{\sqrt{N!}}\langle 0|\Psi(\mathbf{r}_1)\dots\Psi(\mathbf{r}_N)|\Psi_{NE}\rangle = E\Psi_{NE}(\mathbf{r}_1,\dots,\mathbf{r}_N)$$

is equivalent to the Schrödinger equation for the function $\Psi_{NE}(\mathbf{r}_1,\dots,\mathbf{r}_N)$.