Fall 2016

AMATH 732: Assignment 1.

Due Monday, September 26

1. Find approximate solutions of the following problems by finding the first three terms in a perturbation series solution (in an appropriate power of ϵ) using perturbation methods. For problem (e) explain whether the missing terms are $O_F(\epsilon^?)$ or $O(\epsilon^?)$. You should find all of the roots, including complex roots. In these cases you can leave the solutions in terms of x_0, where x_0 is the leading-order approximation of the complex roots.

   (a) $\epsilon x^3 + (x-1)(x-2) = 0$.

   (b) $x^2 + (2-\epsilon)x - 63 + 3\epsilon = 0$.

   (c) $x^2 - (8-2\epsilon)x + 16 + \epsilon = 0$.

   (d) $x^3 + 2\epsilon^2 x - 1 + \epsilon^2 = 0$.

   (e) $(x+1)^3(x-1) - \epsilon x^2 = 0$.

   (f) $\epsilon x^6 + (x+1)^4 = 0$.

   (g) $\epsilon x^5 + \epsilon x^4 + x^2 - 2x - 8 = 0$.

2. The equation

$$y = \cos\big(x + \epsilon \sin(y)\big)$$

   implicitely defines a function $y(x;\epsilon)$. Find $y(x;\epsilon)$ as a power series in $\epsilon \ll 1$. You should find the first three non-zero coefficients. [Hint: use $f(q(x)+\epsilon r(x)+\cdots) = f(q(x)) + f'(q(x))(\epsilon r(x)+\cdots) + \cdots$.]

3. (a) For the projectile problem discussed in class find the first two corrections to the solution of the reduced problem, i.e., solve to $O(\epsilon^3)$:

$$\tilde{x}(t,\epsilon) = x_o(t) + \epsilon x_1(t) + \epsilon^2 x_2(t) + O_F(\epsilon^3).$$

   (b) Find the solution using the method of parametric differentiation and show that the solution you obtained in part (a) is the Taylor series expansion of $x(t,\epsilon)$. Hence argue that the O_F symbol can be replaced by the O symbol given that you are only interested in the solution between the time the projectile leaves the surface and the time it returns to the surface.

   (c) Using your solution estimate the upward velocity $\tilde{v}(t,\epsilon)$ of the projectile. Let τ_m be the time the projectile reaches its maximum height $\tilde{x}_m$. Estimate the value of τ_m.