**Homework 1**
**Due Friday, September 30**

1. Consider two subsystems in thermal equilibrium with each other, forming a closed system. Using additivity of energy and entropy, prove that the temperatures of the two subsystems are equal.

2. (Problem 1.9 of PB) Using additivity of entropy, show that

$$S = N\left(\frac{\partial S}{\partial N}\right)_{V,E} + V\left(\frac{\partial S}{\partial V}\right)_{N,E} + E\left(\frac{\partial S}{\partial E}\right)_{N,V}.$$

3. (Problem 3.15 of PB) Show that the partition function $Z$ of a gas of $N$ relativistic particles with energy-momentum relation $\epsilon = cp$, where $c$ is the speed of light, is given by

$$Z = \frac{1}{N!}\left[8\pi V\left(\frac{T}{hc}\right)^3\right]^N.$$

   Further demonstrate that in this system $PV = \frac{1}{3}E$, $E = 3NT$ and, for an adiabatic process, $PV^{4/3} = const.$

4. Consider a quantum rotor with a Hamiltonian

$$H = -\frac{\hbar^2}{2I}\frac{d^2}{d\theta^2},$$

   where $0 \le \theta \le 2\pi$ is an angular variable.

   (a) Find the eigenstates and energy eigenvalues of this system.

   (b) Find the density matrix in the $\theta$-representation, i.e. $\langle\theta'|\hat{\rho}|\theta\rangle$ at temperature $T$, and examine its low- and high-temperature limits.