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H2a: Binary Alloy

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December 5, 2018

Task N ^o	Points	Avail. points
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Introduction

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Task 1: mean field theory

Fits: we obtained $\alpha \approx 0.494$

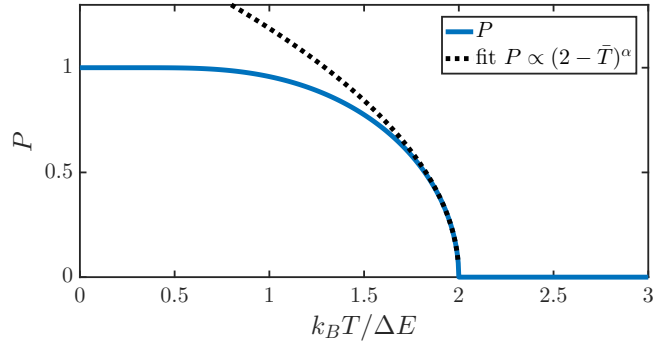


Figure 1:

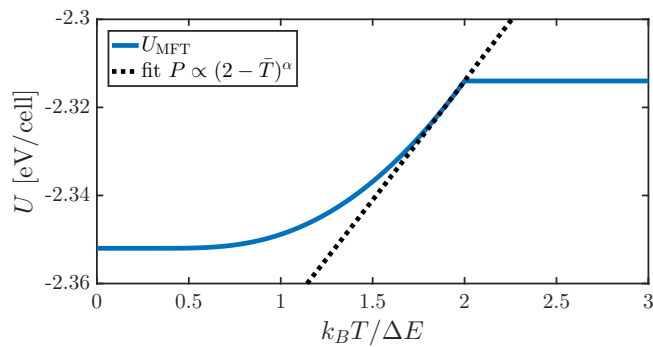


Figure 2:

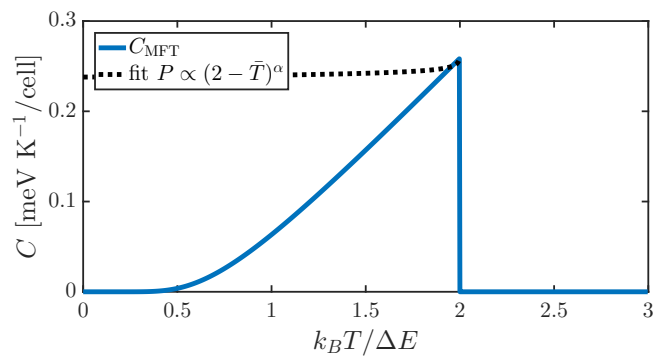


Figure 3:

Task 2: Ising model

We model the binary alloy with a static bcc lattice consisting of Cu and Zn atoms. The system size was $10 \times 10 \times 10$ cells and had periodic boundary conditions. Each atoms

has eight bonds to each nearest neighbors, with energies

$$\begin{aligned} E_{\text{CuZn}} &= -294 \text{ meV} \\ E_{\text{CuCu}} &= -436 \text{ meV} \\ E_{\text{ZnZn}} &= -133 \text{ meV} \end{aligned} \quad (1)$$

We use the Metropolis algorithm to estimate statistical properties of the system. In each simulation step, we swap two randomly selected atoms in the lattice, and determine the energy change ΔE . If $\Delta E \leq 0$, or if $(\exp[-\Delta E/(k_B T)]) > \xi$, where ξ is a random number between 0 and 1, the change is accepted; otherwise the lattice remains in the previous state for another timestep. In this way, the Metropolis algorithm allows us to sample the state space according to the probability function $p \propto \exp[-E/(k_B T)]$, and thus favor the most probable configurations.

Equilibration

To equilibrate the system, we started with an ordered system and performed $N_{\text{eq,long}} = 10^6$ Monte Carlo steps to equilibrate the system at $T = -200^\circ\text{C}$. At higher temperatures, we started with the final lattice state of the previous temperature, and therefore the number of equilibration steps was reduced to $N_{\text{eq,short}} = 5 \cdot 10^5$. For all temperatures, we used 10^7 Monte Carlo steps in the production run.

Figure 4 shows the equilibration of the energy at three different temperatures: significantly below, close to and significantly above the critical temperature $T = 440^\circ\text{C}$. By plotting the energy per bond, i.e. $E/(8N_{\text{Cu}})$, we can compare the energies to the binding energies in equation (1). We note that the energy per bond is in the range $E_{\text{CuZn}} \leq E \leq E_{\text{max}}$, where

$$E_{\text{max}} \equiv \frac{1}{2}(E_{\text{CuCu}} + E_{\text{ZnZn}}) = -284.5 \text{ meV}. \quad (2)$$

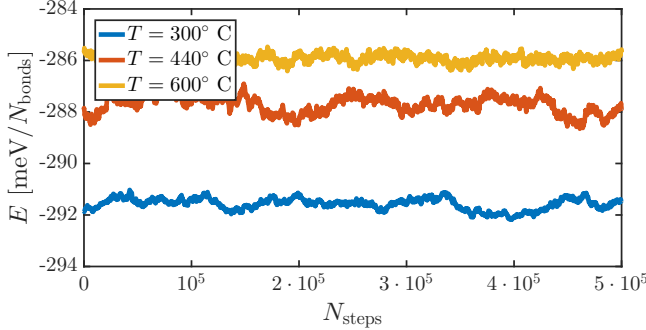


Figure 4: The energy normalized to the number of bonds in the system during the equilibration process.

Statistical properties

Figure 5 shows the equilibrium energy per cell as a function of temperature, and compared to the mean field theory. We also show the error bars of two standard deviations using the statistical inefficiency as calculated from the correlation function in section .

The metropolis simulation differs significantly from the mean free theory prediction; the critical temperature is significantly higher in the simulation and the mean energy continues to increase with temperature beyond the transition.

In mean field theory, the energy per cell never exceeds $8E_0 = -2.31 \text{ eV}$, but in the Monte Carlo simulation the system is allowed to develop clusters of Cu and Zn atoms, which gives a higher energy than the completely randomly ordered system. In the high temperature limit of an infinite system, the theoretical maximum energy is

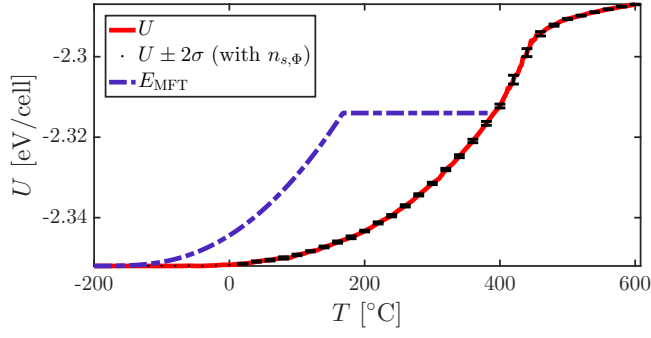


Figure 5: The average energy of the system normalized to the number of cells, as a function of temperature. Solid red: simulation, black: error bars at selected temperatures, dash-dotted blue: mean field theory prediction.

$8E_{\max} \approx -2.276$ eV (defined in equation (2)). With a limited system size with 10 cells in each direction, we estimate that the maximum energy should be approximately

$$8(0.9 \cdot E_{\max} + 0.1E_{\text{CuZn}}) \approx 2.284 \text{ eV} \quad (3)$$

Here we obtain $E \approx -2.287$ eV at 600 °C, which is slightly below this limit.

The heat capacity can be determined either by

$$C = \frac{dU}{dT}, \quad (4)$$

or as the variance in the energy:

$$C = \frac{1}{k_B T^2} (\langle E^2 \rangle - \langle E \rangle^2). \quad (5)$$

Since the latter method does not depend on the derivatives, it gives less noisy data. This can be seen from figure 6 by comparing the gray and the black lines. Again, we note that the mean field theory gives a lower critical temperature than the simulation.

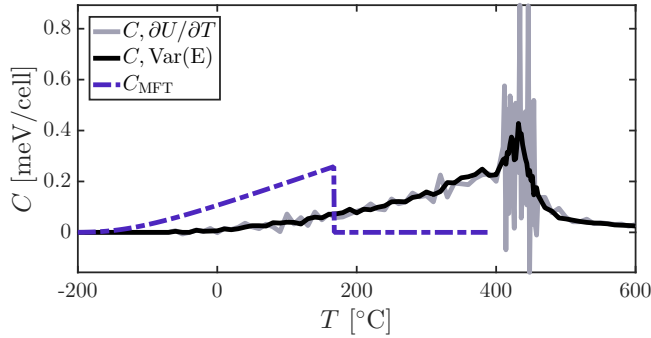


Figure 6: The specific heat of the system normalized to the number of cells, as a function of temperature. Solid gray: simulation using the derivative of U directly, black: simulation using the variance of E , dash-dotted blue: mean field theory prediction.

The order parameter P is shown in figure 7. Close to the phase transition, the data has high uncertainty, which is also reflected in the large error bars. Note that $P < 0$ at some temperature above the critical temperature – the system oscillates between a majority of the Cu atoms in the Cu lattice, and a majority in the Zn lattice.

Finally, the short range order parameter r is determined by

$$r = \frac{1}{4N} (q - 4N) \rightarrow \begin{cases} 1, & \text{perfect order} \\ 0, & \text{no order, homogeneous system} \end{cases} \quad (6)$$

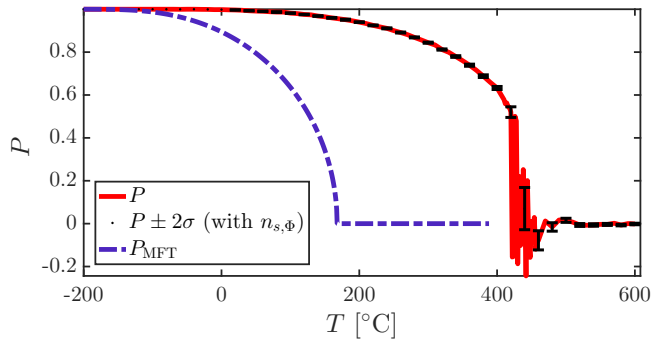


Figure 7: The order parameter P as a function of temperature. Solid red: simulation, black: error bars at selected temperatures, dash-dotted blue: mean field theory prediction.

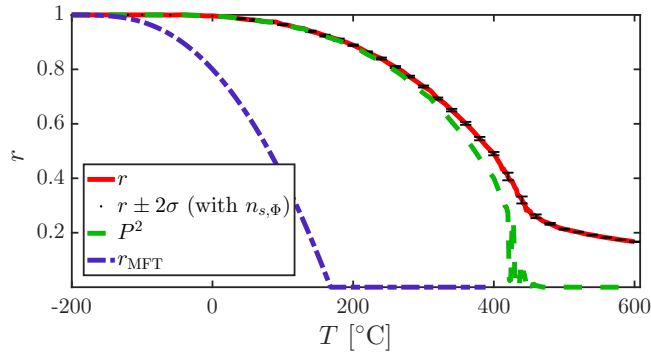


Figure 8: The short range order parameter as a function of temperature. Solid red: simulation, black: error bars at selected temperatures, dashed green: estimate $r \approx P^2$, dash-dotted blue: mean field theory prediction.

Statistical inefficiency

As described in the Lecture notes, the statistical inefficiency can be used to obtain error estimates of correlated data.

Suppose we want to measure a quantity I , as an average of $N \gg 1$ measurements:

$$I = \langle f \rangle \equiv \frac{1}{N} \sum_{i=1}^N f_i. \quad (7)$$

The variance is then given by

$$\text{Var}[I] = \frac{n_s}{N} \text{Var}[f], \quad \text{Var}[f] = \langle f^2 \rangle - \langle f \rangle^2, \quad (8)$$

where n_s is the statistical inefficiency. The statistical inefficiency can be determined either from the decay of the correlation function,

$$\Phi_{k=n_s} = e^{-2} \approx 0.1, \quad \frac{\langle f_i f_{i+k} \rangle - \langle f \rangle^2}{\langle f^2 \rangle - \langle f \rangle^2}, \quad (9)$$

or from block averaging

$$n_s = \lim_{B \rightarrow \infty} \frac{B \text{Var}[F]}{\text{Var}[f]}, \quad F_j = \frac{1}{B} \sum_{i=1}^B f_{i+(j-1)B}, \quad j \in [1, N_{\text{blocks}}]. \quad (10)$$

The two methods in equations (9) and (10) should give similar estimates of n_s , which they do in the simulations here. The obtained statistical inefficiency is shown in figures 9 and 10 at three different temperatures, calculated with the correlation function and block average respectively.

In the case of block average, we used a moving average of 100 points, as the data become noisy when the block size become comparable to the total number of steps. Alternatively, we could have made more blocks of the largest sizes by also using shifted blocks of data, but the results obtained here were considered accurate enough.

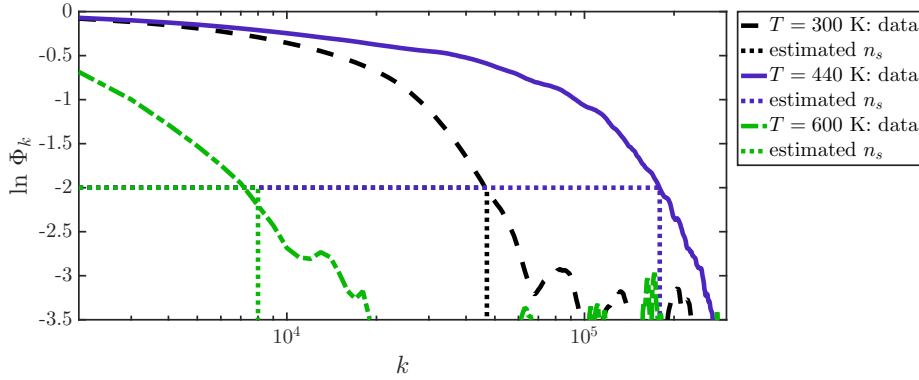


Figure 9: The logarithm of the correlation function $\Phi_k(k)$ for three different temperatures. Dotted lines mark the estimated value of $n_s = k(\ln \Phi_k = -2)$.

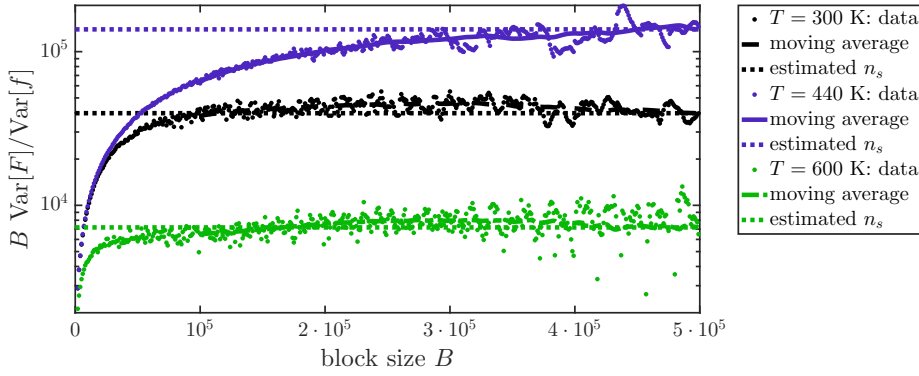


Figure 10: The statistical inefficiency determined with block averages for three different temperatures. Raw data is shown with dots, solid line show a moving average with 100 points, and the dotted lines show the estimated values of the statistical inefficiency.

Note in figures 9 and 10 that the statistical inefficiency is larger close to the phase transition at $T \approx 440^\circ\text{C}$ than at the lower and higher temperatures $T = 300^\circ\text{C}$ and $T = 600^\circ\text{C}$. We speculate that this is related to the diverging property of the correlation length close to the phase transition.

This peak in the statistical inefficiency close to the phase transition can be clearly identified also in figure 11, where n_s is plotted as a function of temperature using the two methods described above. We note that both methods give similar estimates of n_s , but the correlation function give larger fluctuations than the block average method. Moreover, we note that the statistical inefficiency diverges as $T \rightarrow 0\text{ K}$. This is because very few changes in the lattice will be accepted at low temperatures, which give highly correlated data. At low temperatures, the equilibrium system is almost completely ordered, and we note that the uncertainty of the quantities U , P and r is still small at low temperatures as their variance decrease rapidly with decreasing temperature.

Concluding discussion

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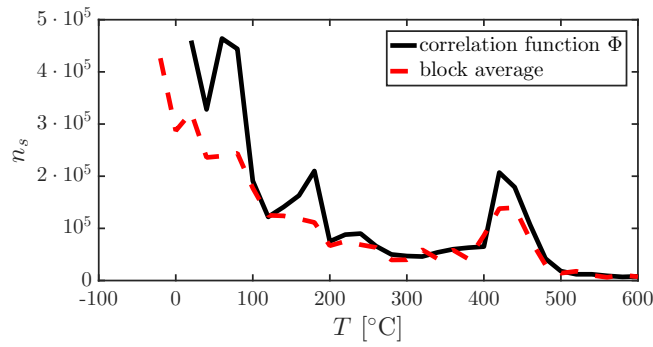


Figure 11: The statistical inefficiency n_s as a function of temperature using both the correlation function and block averages to determine n_s .

A Source Code

A.1 Main program task 2: main.T2.c

```

1  /*
2   H2a, Task 2
3  */
4  #include <stdio.h>
5  #include <math.h>
6  #include <stdlib.h>
7
8  #include "funcs.h"
9
10 #define Nc 10 //number of cells
11 #define N_neigh 8
12 #define degC_to_K 273.15
13 #define kB 8.61733e-5
14
15 /* Main program */
16 int main(){
17     int N_Cu = Nc*Nc*Nc;
18     int N_atoms = 2*N_Cu;
19     int N_bonds = 8*N_Cu;
20     double Etot, E_Var, r, P; // Macro parameters
21     gsl_rng *q = init_random(); // initialize random number generator
22
23     /* done for all saved steps: */
24     int N_MCsteps = 1e7;
25     int N_eq = 1e6;
26     int N_eq_short = 5e5;
27     double *E_eqilibration = malloc(sizeof(double[N_eq]));
28     double *P_eqilibration = malloc(sizeof(double[N_eq]));
29     double *E_production = malloc(sizeof(double[N_MCsteps]));
30
31     /* statistical inefficiency */
32     int N_k = 500;
33     int N_skip = 1000; // k_Max = N_k * N_skip;
34     double *phi = malloc(sizeof(double[N_k]));
35     double *var_F = malloc(sizeof(double[N_k]));
36
37     /* set Temperature steps */
38     double beta;
39     double dT_small = 2;
40     double dT_large = 10;
41     double T_start = -200;
42     double T_end = 600;
43     double T_start_fine = 410;
44     double T_end_fine = 460;
45     int nT;
46     double *T_degC = init_temps(&nT, dT_small, dT_large, T_start, T_end,
47                                T_start_fine, T_end_fine);
48     // save equilibration data and stat inefficiency at T%20 =0
49     int T_save_step = 20;
50     /* done for all temps: */
51     double *E_mean = malloc(sizeof(double[nT]));
52     double *E_mean_approx = malloc(sizeof(double[nT]));
53     double *E_sq_mean = malloc(sizeof(double[nT]));
54     double *P_mean = malloc(sizeof(double[nT]));
55     double *P_sq_mean = malloc(sizeof(double[nT]));
56     double *r_mean = malloc(sizeof(double[nT]));
57     double *r_sq_mean = malloc(sizeof(double[nT]));
58
59     /* allocate and initialize lattice and nearest neighbors */

```

```

60 int *lattice = malloc(sizeof(int)[N_atoms]);
61 init_ordered_lattice(N_atoms, N_Cu, lattice);
62 int (*nearest)[N_neigh] = malloc(sizeof(int)[N_atoms][N_neigh]);
63 init_nearestneighbor(Nc, nearest);
64
65 /* initialize macro parameters */
66 Etot = get_Etot(lattice, N_Cu, nearest);
67 P = get_order_parameter(lattice, N_Cu);
68 r = get_short_range_order_parameter(lattice, nearest, N_Cu);
69
70
71 /* ***** start simulation ***** */
72 for (int iT=0; iT<nT; iT++){
73     /* Loop over all temperatures */
74     printf("Now running T = %.0f degC\n", T_degC[iT]);
75     beta = 1/(kB*(T_degC[iT] + degC_to_K));
76
77     /* ***** Equilibration run ***** */
78     if (iT!=0){// First run needs longer equilibration
79         N_eq=N_eq_short;
80     }
81     /* Do the Monte Carlo stepping */
82     for( int i=0; i<N_eq; i++){
83         MC_step(&Etot, &r, &P, q, lattice, nearest, beta, N_Cu);
84         // Save the energy `Etot` and orerparameter `P`
85         E_eqilibration[i] = Etot;
86         P_eqilibration[i]= P;
87     }
88     //Write the equilibration run to file
89     if ( ((int)T_degC[iT]) % T_save_step==0){
90         write_equl_to_file(T_degC[iT],
91             E_eqilibration, N_bonds, P_eqilibration, N_eq);
92     }
93
94     /* ***** Production run ***** */
95     /*
96     The saved energies are shifted by this (semi-arbitrary) amount.
97     This helps to increase the accuracy when calculating the
98     (needed for the heat capacity).
99     */
100    E_mean_approx[iT] = Etot;
101    /* initialize at temperature[iT] */
102    E_mean[iT] = 0;    E_sq_mean[iT] = 0;
103    P_mean[iT] = 0;    P_sq_mean[iT] = 0;
104    r_mean[iT] = 0;    r_sq_mean[iT]=0;
105
106    /* Do the Monte Carlo stepping */
107    for( int i=0; i<N_MCsteps; i++){
108        MC_step( &Etot, &r, &P, q, lattice, nearest, beta, N_Cu);
109        E_production[i] = Etot- E_mean_approx[iT];
110        update_E_P_r(iT, Etot-E_mean_approx[iT], E_mean, E_sq_mean, P, P_mean,
111            P_sq_mean, r, r_mean, r_sq_mean, lattice, nearest, N_Cu);
112    }
113    /* Divide by number of Monte Carlo steps to get average */
114    E_mean[iT] *= 1/((double)N_MCsteps);
115    E_sq_mean[iT] *= 1/((double)N_MCsteps);
116    P_mean[iT] *= 1/((double)N_MCsteps);
117    P_sq_mean[iT] *= 1/((double)N_MCsteps);
118    r_mean[iT] *= 1/((double)N_MCsteps);
119    r_sq_mean[iT] *= 1/((double)N_MCsteps);
120
121    /*
122    We only calucluate the statistical inefficiency at some
123    temperatures to save on runtime.
124    */
125    if ( ((int)T_degC[iT]) % T_save_step==0 ){//calc stat ineff
126        // Calcualte the variance of the energy
127        E_Var = E_sq_mean[iT] - E_mean[iT]*E_mean[iT];
128
129        printf("Calculating statistical inefficiencies \n");
130        //Calcualte the auto-correlation
131        get_phi (phi, N_MCsteps, E_mean[iT], E_Var, E_production, N_k, N_skip);
132        //Calcualte the block-average variance
133        get_varF_block_average(var_F, N_MCsteps, E_mean[iT], E_Var,
134            E_production, N_k, N_skip);
135        //Write the stat ineff to file
136        write_stat_inefficiency_to_file(T_degC[iT], phi, var_F, N_k, N_skip);
137    }//END if calc stat ineff
138 }//END temp for
139
140 //Write the results of the production run to file
141 write_production(T_degC, nT, E_mean_approx, E_mean, E_sq_mean,
142     P_mean, P_sq_mean, r_mean, r_sq_mean);
143
144
145
146 //Don't forget to free all malloc's.
147 free(nearest);    nearest = NULL;
148 free(lattice);    lattice = NULL;
149 free(E_eqilibration);    E_eqilibration = NULL;
150 free(P_eqilibration);    P_eqilibration = NULL;

```



```

151 free(E_mean);           E_mean = NULL;
152 free(E_mean_approx);   E_mean_approx = NULL;
153 free(E_sq_mean);        E_sq_mean = NULL;
154 free(P_mean);           P_mean = NULL;
155 free(P_sq_mean);        P_sq_mean = NULL;
156 free(r_mean);           r_mean = NULL;
157 free(r_sq_mean);        r_sq_mean = NULL;
158 free(E_production);     E_production = NULL;
159 free(phi);              phi = NULL;
160 free(var_F);            var_F = NULL;
161 free(T_degC);           T_degC = NULL;
162
163 gsl_rng_free(q); // deallocate rng
164 return 0;
165 }

```

A.2 Misc functions : funcs.c

```

1  #include "funcs.h"
2
3  /***** get functions *****/
4  double get_bond_E(int site_1, int site_2){
5      /*
6       * The bond can be one of three types: ZnZn, CuZn=ZnCu, or CuCu.
7       * With the lattice encoding Cu=1 and Zn=0, we get
8       * Zn+Zn = 0, Zn+Cu = Cu+Zn = 1, Cu+Cu = 2.
9       * Hence the switch over the three cases: 0, 1, and 2.
10      */
11      double Ebond=0;
12      switch (site_1 + site_2){
13      case 0:
14          Ebond= -0.113;//E_ZnZn;
15          break;
16      case 1:
17          Ebond= -0.294;//E_CuZn;
18          break;
19      case 2:
20          Ebond= -0.436;//E_CuCu;
21          break;
22      }
23      return Ebond;
24  }
25
26  double get_order_parameter(int *lattice, int N_Cu){
27      /*
28       * The macro order parameter 'P' is given by the number of atoms in
29       * their respective sub-lattice (normalized and shifted to get a
30       * better physical interpretation), e.g. the number of Cu atoms in
31       * the Cu sub-lattice.
32      */
33      int N_Cu_in_Cu_lattice=0;
34      for(int i=0;i<N_Cu;i++){
35          /*
36           * Sum the atoms in the Cu sub-lattice (i=0,1,2,...,N_Cu-1), and
37           * with the encoding Cu=1 and Zn=0, we can simply add the values
38           * of the lattice encoding at each sub-lattice point.
39          */
40          N_Cu_in_Cu_lattice+=lattice[i];
41      }
42      return (double)N_Cu_in_Cu_lattice/N_Cu *2 -1;
43  }
44
45  double get_short_range_order_parameter(int *lattice, int(*nearest)[N_neigh],
46      int N_Cu){
47      /*
48       * The short range order parameter 'r' is given by the number of AB bonds
49       * (normalized and shifted to get a better physical interpretation).
50      */
51      int N_CuZnBonds=0;
52      for(int i=0;i<N_Cu;i++){
53          for( int j=0; j<N_neigh; j++){
54              /*
55               * With the encoding Cu=1 and Zn=0, we know that in order for a
56               * bond to be a CuZn/ZnCu the sum of a lattice point with its
57               * neighbour must be 1 (see 'get_bond_E' for more detail).
58              */
59              N_CuZnBonds+= ((lattice[i] + lattice[nearest[i][j]]) == 1);
60          }
61      }
62      return (double) N_CuZnBonds/(4*N_Cu)-1; // this is 'r'
63  }
64
65  double get_Etot(int *lattice, int N_Cu, int (*nearest)[N_neigh]){
66      /*
67       * The total energy of the system is given by looping over every atom
68       * in one of the sub-lattices (Cu) and summing the energies of its

```

```

69     bonds to every neighbour.
70     We only need to sum over every atom in one sub-lattice since there
71     are no bonds within a sub-lattice.
72 */
73 double Etot=0;
74 for(int i=0; i<N_Cu; i++){ // loop over atoms
75     for( int j=0; j<N_neigh; j++){ // loop over neighbours
76         Etot+= get_bond_E(lattice[i], lattice[nearest[i][j]]);
77     }
78 }
79 return Etot;
80 }
81
82 void get_phi (double *phi, int N_times, double f_mean,
83              double f_var, double *data, int N_k, int N_skip){
84     /*
85     Function for calculating the auto-correlation in a data set. The
86     rate at which the auto-correlation decay can be used to calculate
87     the statistical inefficiency in the data set.
88     Formula:
89      $\phi_k = (\langle f_{i+k} f_i \rangle - \langle f_i \rangle^2) / (\langle f_i^2 \rangle - \langle f_i \rangle^2)$ 
90
91     Note that, by definition,  $\phi_0 = 1$ .
92     */
93     int N_terms_in_avg; // helper variable
94     for (int k=0; k<N_k; k++){
95         /*
96         We loop over `k` in the formula above to get the auto-correlation
97         at the differnt times.
98         `phi[k]` is used to hold intermediary values, and only becomes the
99         auto-correlation at the last step in this loop.
100        */
101        phi[k] = 0;
102
103        /*
104        The number of terms in the sum to get  $\langle f_{i+k} f_i \rangle$  must be such
105        that i fulfills the relation:
106        `(i+k)*N_skip < N_times`,
107        which is equivalent to saying that
108        `i < N_times/N_skip - k`.
109        */
110        N_terms_in_avg = N_times/N_skip - k;
111        for (int i=0; i<N_terms_in_avg; i++){
112            /*
113            Add the products of the off-setted data points to get:
114            sum_{i} f_{i+k} f_{i}
115            */
116            phi[k] += data[i*N_skip]*data[(i+k)*N_skip];
117        }
118        /*
119        First:
120         $\langle f_{i+k} f_i \rangle = (1/N_{avg}) \sum_{i} \{N_{avg}\} f_{i+k} f_i$ ,
121        then we get the auto-correlation by subtracting ` $f_{mean}$ `^2
122        and dividing by the variance.
123        */
124        phi[k] = (phi[k]/N_terms_in_avg - f_mean*f_mean)/f_var;
125    }
126 }
127
128 void get_varF_block_average(double *var_F, int N_times, double f_mean,
129                             double f_var, double *data, int N_k, int N_skip){
130     /*
131     Function for calculating the variances of the blockaverages for `N_k`
132     different block sizes. This varaince can then be used to calculate the
133     statistical inefficiency in the data set.
134     */
135     int block_size;
136     double Fj; // help vaiable, holding each block average
137     int number_of_blocks; // The number of blocks depends on the block size
138
139     for (int k=0; k<N_k; k++) { // block size loop
140         /*
141         For every block size, we need to loop over every block,
142         and every element in that block
143         */
144         block_size = N_skip * (k+1);
145         number_of_blocks = N_times/block_size;
146
147         var_F[k] = 0; // start
148         for (int j=0; j<number_of_blocks; j++) {/// loop over all blocks
149             /* For every block, we loop over all elements in it to take average. */
150             Fj = 0; // reset to 0
151             for (int i=0; i<block_size; i++) {/// internal block loop
152                 /* Adding all elems in the block to get the average */
153                 Fj += data[j*block_size + i];
154             }
155             Fj *= 1/(double)block_size; // divide by block size to get average
156             var_F[k] += Fj*Fj; // will become the variance soon
157         }
158         /*
159         To get the varaince of F we use:

```

```

160     Var[F] = <F^2> - <F>^2 = <F^2> - <f>^2,
161     where f is the data set the block averages were taken from.
162     */
163     var_F[k] = var_F[k]/number_of_blocks - f_mean*f_mean;
164     var_F[k] *= block_size/f_var;
165 } // end block size loop
166 }
167
168 /***** Monte Carlo step functions *****/
169 void MC_step( double *Etot, double *r, double *P, gsl_rng *q,
170             int *lattice, int (*nearest)[N_neigh], double beta, int N_Cu){
171     /*
172     Function that takes a Monte Carlo step and updates the lattice points,
173     `Etot`, `r`, and `P` accordingly.
174     It is important to utilize the _chage_ in energy, `r` and `P` when
175     updating them as to not have to do a costly full calculation of either
176     every step in the Monte Carlo loop.
177     */
178     // Picks two random sites in the whole lattice.
179     int i1 = (int)(2*N_Cu*gsl_rng_uniform(q));
180     int i2 = (int)(2*N_Cu*gsl_rng_uniform(q));
181     // saves the original values
182     int old_1 = lattice[i1];
183     int old_2 = lattice[i2];
184     // Used to calculate the change in `Etot` and `r`
185     double dr = 0;
186     double dE = 0;
187     // We only need to do something if the two atoms are different
188     if (old_1 != old_2){
189         for( int j=0; j<N_neigh; j++){
190             /*
191             The change in `Etot` and `r` are first _minus_ the old energies and `r`
192             contributions.
193             */
194             dE -= get_bond_E(lattice[i1], lattice[nearest[i1][j]])
195                 + get_bond_E(lattice[i2], lattice[nearest[i2][j]]);
196
197             dr -= ((lattice[i1] + lattice[nearest[i1][j]]) == 1)
198                 + ((lattice[i2] + lattice[nearest[i2][j]]) == 1);
199         }
200         /* Then we do the change of the two atoms */
201         lattice[i1] = old_2;
202         lattice[i2] = old_1;
203         for( int j=0; j<N_neigh; j++){
204             /*
205             And _add_ the contributions to `Etot` and `r` from the updated lattice.
206             */
207             dE += +get_bond_E(lattice[i1], lattice[nearest[i1][j]])
208                 + get_bond_E(lattice[i2], lattice[nearest[i2][j]]);
209
210             dr += ((lattice[i1] + lattice[nearest[i1][j]]) == 1)
211                 + ((lattice[i2] + lattice[nearest[i2][j]]) == 1);
212         }
213
214         if ( (dE<=0) || (exp(-beta * dE) > gsl_rng_uniform(q)) ){
215             /*
216             The test is accepted if dE < 0 (accept immediately), OR
217             otherwise it's accepted with a probability of `exp(-beta * dE)`
218             */
219             // Updates P
220             if (i1 < N_Cu)
221                 *P += (double)(lattice[i1] - old_1 )/N_Cu *2;
222             if (i2 < N_Cu)
223                 *P += (double)(lattice[i2] - old_2 )/N_Cu *2;
224         }else{
225             /*
226             If the test failed, we change back to the old lattice configuration
227             and no change happens to `Etot` or `r`
228             */
229             lattice[i1] = old_1;
230             lattice[i2] = old_2;
231             dE = 0;
232             dr = 0;
233         } // end if step is accepted
234         *Etot += dE;
235         *r += dr/(4*N_Cu);
236     } // end if atoms are different
237 }
238
239 void update_E_P_r(int iT, double E_dev, double *E_mean, double *E_sq_mean,
240                 double P, double *P_mean, double *P_sq_mean,
241                 double r, double *r_mean, double *r_sq_mean,
242                 int *lattice, int (*nearest)[N_neigh], int N_Cu){
243     /*
244     Updates the macro parameters `E`, `P`, and `r`, as well as their squares.
245     Runs in every Monte Carlo step during the production run.
246     */
247     E_mean[iT] += E_dev;
248     E_sq_mean[iT] += E_dev * E_dev;
249
250     P_mean[iT] += P;

```

```

251 P_sq_mean[iT] += P*P;
252
253 r_mean[iT] += r;
254 r_sq_mean[iT] += r*r;
255 }
256
257 /***** initializing functions *****/
258 double * init_temps( int *nT, double dT_small, double dT_large,
259                     double T_start, double T_end, double T_start_fine,
260                     double T_end_fine){
261     /*
262     Creates an array `T_degC` with the temperatures to loop over in the main
263     function, given the fine temperature step range and the sizes of the
264     temperature steps.
265     */
266     *nT = (int) ((T_end_fine - T_start_fine)/dT_small
267                 +(T_start_fine-T_start + T_end-T_end_fine)/dT_large +1);
268     double *T_degC = malloc(sizeof(double)*nT);
269     T_degC[0] = T_start;
270     for (int iT=1; iT<*nT; iT++){ // loop over all temps
271         if (T_degC[iT-1]>=T_start_fine && T_degC[iT-1]<T_end_fine){
272             T_degC[iT] = T_degC[iT-1] + dT_small;
273         }else{
274             T_degC[iT] = T_degC[iT-1] + dT_large;
275         }
276     }
277     return T_degC;
278 }
279
280 void init_ordered_lattice(int N_atoms, int N_Cu, int *lattice){
281     /*
282     Initialize lattice with Cu atoms (1) in Cu lattice (i=0:N_Cu-1)
283     and Zn (0) in Zn lattice (i=N_Cu:N_atoms-1):
284     */
285     for( int i=0; i<N_Cu; i++){
286         lattice[i] = 1;
287     }
288     for( int i=N_Cu; i<N_atoms; i++){
289         lattice[i] = 0;
290     }
291 }
292
293 void init_random_lattice(int N_atoms, int N_Cu, int *lattice, gsl_rng *q){
294     /*
295     Initialize lattice with Cu and Zn atoms randomly distributed:
296     */
297     for( int i=0; i<N_Cu; i++){
298         lattice[i] = (int)(gsl_rng_uniform(q)+0.5);
299         lattice[i+N_Cu] = 1-lattice[i];
300     }
301 }
302
303 void init_nearestneighbor(int Nc, int (*nearest)[N_neigh]){
304     /*
305     Create a matrix `nearest[i][j]` with the index of the `j`th nearest
306     neighbors to site `i`.
307     N.B. Each site has `N_neigh` (8) nearest neighbors.
308     */
309     int i_atom;
310     int N_Cu = Nc*Nc*Nc;
311     for( int i=0; i<Nc; i++){
312         for( int j=0; j<Nc; j++){
313             for( int k=0; k<Nc; k++){
314                 i_atom = k + Nc*j + Nc*Nc*i;
315                 // k i j in one lattice <=> "k-0.5" "i-0.5" "j-0.5" in the other lattice
316                 // use mod to handle periodic boundary conditions
317                 nearest[i_atom][0] = k + Nc*j + Nc*Nc*i + N_Cu;
318                 nearest[i_atom][1] = k + Nc*j + Nc*Nc*((i+1)%Nc) + N_Cu;
319                 nearest[i_atom][2] = k + Nc*((j+1)%Nc) + Nc*Nc*i + N_Cu;
320                 nearest[i_atom][3] = k + Nc*((j+1)%Nc) + Nc*Nc*((i+1)%Nc) + N_Cu;
321                 nearest[i_atom][4] = (k+1)%Nc + Nc*j + Nc*Nc*i + N_Cu;
322                 nearest[i_atom][5] = (k+1)%Nc + Nc*j + Nc*Nc*((i+1)%Nc) + N_Cu;
323                 nearest[i_atom][6] = (k+1)%Nc + Nc*((j+1)%Nc) + Nc*Nc*i + N_Cu;
324                 nearest[i_atom][7] = (k+1)%Nc + Nc*((j+1)%Nc) + Nc*Nc*((i+1)%Nc) + N_Cu;
325
326                 // k i j in one lattice <=> "k+0.5" "i+0.5" "j+0.5" in the other lattice
327                 // use mod to handle periodic boundary conditions
328                 // note that mod([negative])<0 :
329                 i_atom += N_Cu;
330                 nearest[i_atom][0] = k + Nc*j + Nc*Nc*i;
331                 nearest[i_atom][1] = k + Nc*j + Nc*Nc*((i-1+Nc)%Nc) ←
332
333                 ;
334                 nearest[i_atom][2] = k + Nc*((j-1+Nc)%Nc) + Nc*Nc*i;
335                 nearest[i_atom][3] = k + Nc*((j-1+Nc)%Nc) + Nc*Nc*((i-1+Nc)%Nc) ←
336
337                 ;
338                 nearest[i_atom][4] = (k-1+Nc)%Nc + Nc*j + Nc*Nc*i;
339                 nearest[i_atom][5] = (k-1+Nc)%Nc + Nc*j + Nc*Nc*((i-1+Nc)%Nc) ←
340
341                 ;
342                 nearest[i_atom][6] = (k-1+Nc)%Nc + Nc*((j-1+Nc)%Nc) + Nc*Nc*i;

```

```

339     nearest[i_atom][7] =(k-1+Nc)%Nc + Nc*((j-1+Nc)%Nc) + Nc*Nc*((i-1+Nc)%Nc)↵
340     ;
341 }
342 }
343 }
344 }
345 gsl_rng* init_random(){
346     /*
347     Initializes a GSL random nubergenerator, and returns the pointer.
348     */
349     gsl_rng *q;
350     const gsl_rng_type *rng_T;    // static info about rngs
351     gsl_rng_env_setup ();         // setup the rngs
352     rng_T = gsl_rng_default;      // specify default rng
353     q = gsl_rng_alloc(rng_T);     // allocate default rng
354     gsl_rng_set(q,time(NULL));    // Initialize rng
355     return q;
356 }
357
358
359 /***** file I/O functions *****/
360 void write_equil_to_file(double T_degC, double *E_equilibration, int N_bonds,
361     double *P, int N_eq){
362     /*
363     Writes the energy per bond `E_equilibration`/`N_bonds` and order
364     parameter `P`, at each Monte Carlo step during the equilibration runs.
365     */
366     FILE *file_pointer;
367     char file_name[256];
368     sprintf(file_name, "../data/E_equilibration-T%d.tsv", (int) T_degC);
369     file_pointer = fopen(file_name, "w");
370     for (int i=0; i<N_eq; i++){
371         fprintf(file_pointer, "%.8f\t%.8f \n", E_equilibration[i]/N_bonds,P[i]);
372     }
373     fclose(file_pointer);
374 }
375
376 void write_production(double *T_degC, int nT, double *E_mean_approx,
377     double *E_mean, double *E_sq_mean,
378     double *P_mean, double *P_sq_mean,
379     double *r_mean, double *r_sq_mean){
380     /*
381     Writes the macro parameters `E_mean_approx`, `E_mean`, `E_sq_mean`,
382     `P_mean`, `P_sq_mean`, `r_mean`, and `r_sq_mean` for each temperature
383     to file.
384     */
385     FILE *file_pointer;
386     char file_name[256];
387     sprintf(file_name, "../data/E_production.tsv");
388     file_pointer = fopen(file_name, "w");
389     fprintf(file_pointer, "%s T[degC]\t E_approx\t<E-E_approx>\t<(E-E_approx)^2>\t↵
390         tP\ttr\n");
391     for (int iT=0; iT<nT; iT++){
392         fprintf(file_pointer, "%.2f\t%.8e\t%.8e\t%.8e\t%.8f\t%.8f\t %.8f\t%.8f \n",
393             T_degC[iT], E_mean_approx[iT], E_mean[iT], E_sq_mean[iT], P_mean[iT],
394             P_sq_mean[iT], r_mean[iT], r_sq_mean[iT]);
395     }
396     fclose(file_pointer);
397 }
398
399 void write_stat_inefficiency_to_file(double T_degC, double *phi, double *var_F,
400     int N_k, int N_skip){
401     /*
402     Writes the auto-correlation `phi` and block varaiances `var_F` for each
403     tested temperature to file.
404     */
405     FILE *file_pointer;
406     char file_name[256];
407     sprintf(file_name, "../data/stat_inefficiency-T%d.tsv", (int) T_degC);
408     file_pointer = fopen(file_name, "w");
409     for (int i=0; i<N_k; i++){
410         fprintf(file_pointer, "%d\t%.8f\t%.8f \n", i*N_skip, phi[i],var_F[i]);
411     }
412     fclose(file_pointer);
413 }

```

B Auxiliary

B.1 Makefile

```

1
2 CC = gcc
3 CFLAGS = -O3 -Wall

```

```

4
5 LIBS = -lm -lgsl -lgslcblas
6
7 HEADERS = funcs.h
8 OBJECTS = funcs.o
9
10
11 %.o: %.c $(HEADERS)
12     $(CC) -c -o $@ $< $(CFLAGS)
13
14 all: Task2
15
16
17
18 Task2: $(OBJECTS) main_T2.c
19     $(CC) -o $@ $^ $(CFLAGS) $(LIBS)
20
21 # $(PROGRAMS): $(OBJECTS) main_T1.c
22 #     $(CC) -o $@ $^ $(CFLAGS) $(LIBS)
23
24 clean:
25     rm -f *.o
26     touch *.c

```

C MATLAB scripts

C.1 Task 1 and analysis scripts for Task 2

```

1 %% initial
2
3 tmp = matlab.desktop.editor.getActive; %% cd to current path
4 cd(fileparts(tmp.Filename));
5 set(0,'DefaultFigureWindowStyle','docked');
6 warning('off','MATLAB:handle_graphics:exceptions:SceneNode'); % interpreter
7 GRAY = 0.7*[0.9 0.9 1];
8 kB = 8.61733e-5;
9 %% task 1: MFT
10 doSave = 0;
11 clc
12
13 Pmin = 0;
14 Pmax = 1;
15
16 E_CuCu = -.436;
17 E_ZnZn = -.133;
18 E_CuZn = -.294;
19
20 E0=2*(E_CuCu+E_ZnZn+2*E_CuZn);
21 Delta_E=(E_CuCu+E_ZnZn-2*E_CuZn);
22
23 E0_bar=E0/Delta_E;
24 E_MFT=@(P) E0 - 2*P.^2*Delta_E;
25 E_MFT_bar=@(P) E0_bar - 2*P.^2;
26 dE_MFTdP =@(P) - 4*P*Delta_E;
27
28 F_MFT = @(P,Tbar) E_MFT_bar(P) + Tbar*(-2*log(2) + (1+P).*log(1+P)+(1-P).*log(1-↵
P));
29 P_eq=@(Tbar) fminbnd(@(P)F_MFT(P, Tbar), Pmin, Pmax, optimset('TolX',1e-9));
30
31 Tbar = linspace(0,3,1000)';
32 T_MFT=Tbar*Delta_E/kB;
33 T_MFT_degC = T_MFT - 273.15;
34 Peq = zeros(size(Tbar));
35 for iT = 1:numel(Tbar)
36     Peq(iT) = P_eq(Tbar(iT));
37 end
38
39 % plot P(T) and make a fit
40 figure(1);clf
41 plot(Tbar, Peq);hold on
42
43 dT=2-Tbar(Tbar<2);
44 Peq_nonzero = Peq(Tbar<2);
45
46 I_good = (dT<0.1);
47 log_dT = log(dT(I_good));
48 log_P = log(Peq_nonzero(I_good));
49 A=[ones(size(log_dT)), log_dT]\log_P;
50 b = exp(A(1));
51 alpha = A(2);
52 fprintf('alpha = %.3f\n', alpha)
53
54 P_approx = @(alpha,b,Tbar) b*(2-Tbar).^alpha;
55 plot(Tbar(Tbar<2),P_approx(alpha,b,Tbar(Tbar<2)),'k:');

```

```

56 xlabel('$k_B T/ \Delta E$')
57 ylabel('$P$')
58 legend('$P$', 'fit $P \propto (2-\bar{T})^\beta$')
59 ylim([0 1.3]);
60 if doSave; setFigureSize(gcf, 300, 600); end
61
62 % plot E_MFT and the fit
63 figure(2);clf
64 plot(Tbar,E_MFT(Peq)); hold on
65 plot(Tbar,E_MFT(P_approx(alpha,b,Tbar)),'k:');
66 xlabel('$k_B T/ \Delta E$')
67 ylabel('$U$ [eV/cell]')
68 legend('$U_{\rm MFT}$', 'fit $P \propto (2-\bar{T})^\beta$', 'location', '↖
    NorthWest');
69 ylim([-2.36 -2.3]);
70 if doSave; setFigureSize(gcf, 300, 600); end
71
72 figure(3);clf
73 C_MFT=diff(E_MFT(Peq))./diff(T_MFT);
74 plot(Tbar(1:end-1), C_MFT*1e3); hold on
75 C_approx=4*b^2*kB*alpha*(2-Tbar).^(2*alpha-1);
76 plot(Tbar(Tbar<2),1e3*C_approx(Tbar<2),'k:');
77 xlabel('$k_B T/ \Delta E$')
78 ylabel('$C$ [meV K$^{-1}$]/cell]')
79 legend('$C_{\rm MFT}$', 'fit $P \propto (2-\bar{T})^\beta$', 'location', '↖
    NorthWest');
80 ylim([0 0.3]);
81 if doSave; setFigureSize(gcf, 300, 600); end
82
83 ImproveFigureCompPhys()
84 if doSave
85     saveas(1, '../figures/P_MFT.eps', 'epsc');
86     saveas(2, '../figures/E_MFT.eps', 'epsc');
87     saveas(3, '../figures/C_MFT.eps', 'epsc');
88 end
89
90
91 %% task 2: equilibration and statistical inefficiency
92 clc;
93 doSave = 0;
94 Ts=[-200:20:600]';
95 TsToPlot = [300 440 600]';
96 t_eq=0;
97
98 figure(1);clf;
99
100 for i=1:numel(TsToPlot)
101     data = load(sprintf('../data/E_equilibration-T%d.tsv',TsToPlot(i)));
102     E = data(:,1);
103     steps = 1:length(E);
104     %P = data(:,2);
105     plot(steps, E*1000); hold on
106 end
107 legstr = strcat({'$T=$', num2str(TsToPlot), '$\circ$ C'});
108 legend(legstr, 'location', 'NorthWest');
109 ylabel('$E$ [meV/$N_{\rm bonds}$]')
110 xlabel('$N_{\rm steps}$')
111 ax = gca;
112 ax.XTickLabel = {'0', '$10^5$', '$2\cdot 10^5$', '$3\cdot 10^5$', '$4\cdot 10^5$',
    '$5\cdot 10^5$'};
113
114 ImproveFigureCompPhys(1)
115
116 figure(3); clf;figure(2); clf;
117 [ns_Phi,ns_block] = deal(nan(size(Ts)));
118 Nskip = 10; % did not use all k's when calculating block averages
119 N_avg = 100; % moving average
120 for i=1:numel(Ts)
121     data = load(sprintf('../data/stat_inefficiency-T%d.tsv',Ts(i)));
122     k = data(:,1);
123     block_size = k+Nskip;
124     phi = data(:,2);
125     VarF_norm = data(:,3);
126     kstar = k(find(log(phi)<-2, 1, 'first'));
127     if ~isempty(kstar)
128         ns_Phi(i) = kstar;
129     end
130
131     filtereddata = movmean(VarF_norm,N_avg);
132     ns_block(i) = filtereddata(end);
133
134     if any(Ts(i) == TsToPlot)
135         figure(2)
136
137         semilogx(k, log(phi));hold on;
138         plot([0.1 kstar kstar], [-2 -2 -6],':k')
139
140         figure(3)
141         semilogy(block_size, VarF_norm, '.'); hold on;
142         plot(block_size(N_avg:end), filtereddata(N_avg:end));
143         plot(block_size, filtereddata(end)*ones(size(block_size)), ':k');

```

```

144     end
145 end
146
147 figure(4); clf;
148 plot(Ts, ns_Phi, 'k', Ts, ns_block, '--r')
149 ax = gca;
150 ax.YTickLabel = {'0', '$10^5$', '$2\cdot 10^5$', '$3\cdot 10^5$', '$4\cdot 10^5$', '$5\cdot 10^5$'};
151 ylabel('$n_s$');
152 legend('correlation function $\Phi_i$', 'block average');
153 xlabel('$Ts$ [$^\circ$C]');
154 ImproveFigureCompPhys(gcf)
155
156 legs_Phi = cell(6,1);
157 legs_block = cell(9,1);
158 for i = 1:numel(TsToPlot)
159     tt = ['$I=' num2str(TsToPlot(i)) '$ K: '];
160     legs_Phi{1 + 2*(i-1)} = [tt 'data'];
161     legs_Phi{2 + 2*(i-1)} = 'estimated $n_s$';
162     legs_block{1 + 3*(i-1)} = [tt 'data'];
163     legs_block{2 + 3*(i-1)} = 'moving average';
164     legs_block{3 + 3*(i-1)} = 'estimated $n_s$';
165 end
166
167 figure(2);
168
169 legend(legs_Phi, 'location', 'northeastoutside');
170 xlabel('$k$'); ylabel('$\ln \Phi_k$');
171 ylim([-3.5 0]);
172 xlim([2e3 3e5])
173 %ax = gca; ax.XTick = [3e3 1e4 3e4 1e5 3e5];
174 %ax.XTickLabel = {'$3\cdot 10^3$', '$10^4$', '$3\cdot 10^4$', '$10^5$', '$3\cdot 10^5$'};
175
176 figure(3);
177 ax = gca;
178 [ax.Children(:).MarkerSize] = deal(12);
179 legend(legs_block, 'location', 'northeastOutside');
180 xlabel('block size $B$');
181 ylabel('$B$ Var[$F$]/Var[$f$]');
182 ylim([2e3 2e5])
183 ax = gca;
184 ax.XTickLabel = {'0', '$10^5$', '$2\cdot 10^5$', '$3\cdot 10^5$', '$4\cdot 10^5$', '$5\cdot 10^5$'};
185
186 ImproveFigureCompPhys(2, 'LineColor', {'LINNEAGREEN', 'LINNEAGREEN', 'GERIBLUE', 'GERIBLUE', 'k', 'k'}, ...
187 'LineStyle', {':', '-.', '-', '-', ':', '--'})
188 ImproveFigureCompPhys(3, 'LineColor', {'LINNEAGREEN', 'LINNEAGREEN', 'LINNEAGREEN', 'GERIBLUE', 'GERIBLUE', 'GERIBLUE', 'k', 'k', 'k'}, ...
189 'LineStyle', {':', '-.', 'none', '-', '-', 'none', ':', '--', 'none'});
190
191 if doSave
192     figure(1);
193     setFigureSize(gcf, 300, 600);
194     saveas(gcf, '../figures/equilibration.eps', 'eps');
195     figure(2);
196     setFigureSize(gcf, 350, 900);
197     saveas(gcf, '../figures/stat_inefficiency_Phi.eps', 'eps');
198     figure(3);
199     setFigureSize(gcf, 350, 900);
200     saveas(gcf, '../figures/stat_inefficiency_block.eps', 'eps');
201     figure(4);
202     setFigureSize(gcf, 300, 600);
203     saveas(gcf, '../figures/stat_inefficiency_both.eps', 'eps');
204 end
205
206
207 %% task 2: U, C, P and r
208
209 doSave = 0;
210
211 data = load('../data/E_production.tsv');
212 T_degC = data(:,1);
213 N_Cu = 1e3;
214 N_timeSteps = 1e7;
215
216 Emean_approx = data(:,2)/N_Cu; % divide by N_Cu to get energy and Cv per cell
217 Emean_shifted = data(:,3)/N_Cu;
218 E_sq_mean_shifted = data(:,4)/N_Cu^2;
219
220 E_Var = (E_sq_mean_shifted - Emean_shifted.^2);
221
222 Cv = 1./(kB * (T_degC+273.15).^2).*E_Var*N_Cu;
223 U = (Emean_shifted + Emean_approx);
224 U_std = sqrt(E_Var/N_timeSteps);
225 P = data(:,5);
226 P_std = sqrt((data(:,6)-P.^2)/N_timeSteps); % without ns so far
227 r = data(:,7);
228 r_std = sqrt((data(:,8)- r.^2)/N_timeSteps);
229

```



```

230 ind = zeros(size(Ts));
231 for i = 1:numel(Ts)
232     ind(i) = find(Ts(i) == T_degC);
233 end
234
235 figure(1);clf;
236 plot(T_degC, U); hold on;
237 errorbar(Ts, U(ind), 2*U_std(ind).*sqrt(ns_Phi), '.k','linewidth', 2.5); hold on;
238 plot(T_MFT_degC, E_MFT(Peq), '-.'); hold on
239 ImproveFigureCompPhys(gcf, 'LineColor', {'GERIBLUE', 'r'}');
240 legend('$U$', '$U\pm 2 \sigma$ (with $n_{s, \rm \Phi}$)', '$E_{\rm MFT}$', '↔
    Location', 'NorthWest');
241 ylabel('$U$ [eV/cell]')
242 axis tight
243
244 figure(2); clf;
245 plot(T_degC(2:end), 1e3*diff(U)./diff(T_degC)); hold on;
246 plot(T_degC, 1e3*Cv);
247 plot(T_MFT_degC(1:end-1), 1e3*C_MFT, '-.');
248 ImproveFigureCompPhys(gcf, 'LineColor', {'GERIBLUE', 'k',GRAY});
249 legend('$C$, {\partial U / \partial T}$', '$C$, {\rm Var}(E)$', '$C_{\rm MFT}$', '↔
    Location', 'NorthWest');
250 ylabel('$C$ [meV/cell]')
251 ylim([0 0.6])
252
253 figure(3);clf;
254 plot(T_degC, P, 'r'); hold on;
255 errorbar(Ts, P(ind), 2*P_std(ind).*sqrt(ns_Phi), '.k', 'linewidth', 2.5); hold on;
256 plot(T_MFT_degC, Peq, '-.k');
257 ImproveFigureCompPhys(gcf, 'LineColor', {'GERIBLUE', 'r'}');
258 legend('$P$', '$P\pm 2 \sigma$ (with $n_{s, \rm \Phi}$)', '$P_{\rm MFT}$', '↔
    Location', 'SouthWest');
259 ylabel('$P$ ')
260 axis tight
261
262 figure(4);clf;
263 plot(T_degC, r, 'r');hold on;
264 errorbar(Ts, r(ind), 2*r_std(ind).*sqrt(ns_Phi), '.k','linewidth', 1.5);hold on;
265 plot(T_degC, P.^2, '--',T_MFT_degC, Peq.^2, '-.');
266 ImproveFigureCompPhys(gcf, 'LineColor', {'GERIBLUE', 'LINNEAGREEN','r'}');
267 legend('$r$', '$r\pm 2 \sigma$ (with $n_{s, \rm \Phi}$)', '$P^2$', '$r_{\rm MFT}$'↔
    ', Location', 'SouthWest');
268 ylabel('$r$ ')
269 axis tight
270 ImproveFigureCompPhys((2:4), 'linewidth', 2)
271
272 if doSave
273     for ifig = 1:4;
274         figure(ifig)
275         setFigureSize(gcf, 300, 600);
276         xlabel('$T$ [$^\circ$C]');
277         xlim([-200 Inf])
278     end
279     ImproveFigureCompPhys(1:4);
280     saveas(1, '../figures/U.eps', 'eps');
281     saveas(2, '../figures/C.eps', 'eps');
282     saveas(3, '../figures/P.eps', 'eps');
283     saveas(4, '../figures/r.eps', 'eps');
284 end
285
286 %%
287 Tcrit = 430;
288 dT=Tcrit-T_degC(T_degC<Tcrit);
289 P_nonzero = abs(P(T_degC<Tcrit));
290
291 I_good = (dT<30 & P_nonzero>0.4);
292 log_dT = log(dT(I_good));
293 log_P = log(P_nonzero(I_good));
294 A=[ones(size(log_dT)), log_dT]\log_P;
295 b = exp(A(1));
296 alpha = A(2);
297 fprintf('P: alpha = %.3f\n', alpha)
298 P_approx = @(alpha,b,T) b*(Tcrit-T).^alpha;
299
300 %figure(5);clf;
301 %loglog(dT,P_nonzero) ; hold on;
302 %plot(dT, P_approx(alpha, b, Tcrit-dT), 'g')
303
304 figure(3)
305 Tvec = linspace(300,Tcrit);
306 plot(Tvec, P_approx(alpha, b, Tvec), ':k')
307 ImproveFigureCompPhys(gcf)
308
309
310
311 Cv_good = abs(Cv(T_degC<Tcrit));
312 I_good = (dT<150);
313 log_dT = log(dT(I_good));
314 log_C = log(Cv_good(I_good));

```

```

315 A=[ones(size(log_dT)), log_dT]\log_C;
316 b = exp(A(1));
317 alpha = A(2);
318 fprintf('Cv: alpha = %.3f\n', alpha)
319 C_approx = @(alpha,b,T) b*(Tcrit-T).^alpha;
320
321 %figure(6);clf;
322 %loglog(dT,Cv_good) ; hold on;
323 %plot(dT, C_approx(alpha, b, Tcrit-dT), 'g')
324
325 figure(2);
326 plot(Tvec, 1e3*C_approx(alpha, b, Tvec), 'r')
327 ImproveFigureCompPhys(gcf)

```

C.2 Improve figure appearance: ImproveFigureCompPhys.m

```

1 function ImproveFigureCompPhys(varargin)
2 %ImproveFigureCompPhys Improves the figures of supplied handles
3 % Input:
4 % - none (improve all figures) or handles to figures to improve
5 % - optional:
6 %     LineWidth int
7 %     LineStyle column vector cell, e.g. {'-','--'}',
8 %     LineColor column vector cell, e.g. {'k',[0 1 1], 'MYBLUE'}'
9 %             colors: MYBLUE,MYORANGE,MYGREEN,MYPURPLE, MYYELLOW,
10 %             MYLIGHTBLUE, MYRED
11 %     Marker column vector cell, e.g. {'.', 'o', 'x'}'
12
13 % ImproveFigure was originally written by Adam Stahl, but has been heavily
14 % modified by Linnea Hesslow
15
16
17 %%% Handle inputs
18 % If no inputs or if the first argument is a string (a property rather than
19 % a handle), use all open figures
20 if nargin == 0 || ischar(varargin{1})
21     %Get all open figures
22     figHs = findobj('Type','figure');
23     nFigs = length(figHs);
24 else
25     % Check the supplied figure handles
26     figHs = varargin{1};
27     figHs = figHs(ishandle(figHs) == 1); %Keep only those handles that are ←
28     % proper graphics handles
29     nFigs = length(figHs);
30 end
31
32 % Define desired properties
33 titleSize = 24;
34 interpreter = 'latex';
35 lineWidth = 4;
36 axesWidth = 1.5;
37 labelSize = 22;
38 textSize = 20;
39 legTextSize = 18;
40 tickLabelSize = 18;
41 LineColor = {};
42 LineStyle = {};
43 Marker = {};
44
45 % define colors
46 co = [ 0      0.4470  0.7410
47       0.8500  0.3250  0.0980
48       0.9290  0.6940  0.1250
49       0.4940  0.1840  0.5560
50       0.4660  0.6740  0.1880
51       0.3010  0.7450  0.9330
52       0.6350  0.0780  0.1840 ];
53 colors = struct('MYBLUE', co(1,:),...
54 'MYORANGE', co(2,:),...
55 'MYYELLOW', co(3,:),...
56 'MYPURPLE', co(4,:),...
57 'MYGREEN', co(5,:),...
58 'MYLIGHTBLUE', co(6,:),...
59 'MYRED', co(7,:),...
60 'GERIBLUE', [0.3000  0.1500  0.7500],...
61 'GERIRED', [1.0000  0.2500  0.1500],...
62 'GERIYELLOW', [0.9000  0.7500  0.1000],...
63 'LIGHTGREEN', [0.4  0.85  0.4],...
64 'LINNEAGREEN', [7 184 4]/255);
65
66 % Loop through the supplied arguments and check for properties to set.
67 for i = 1:nargin
68     if ischar(varargin{i})
69         switch lower(varargin{i}) %Compare lower case strings
70             case 'linewidth'

```

```

70         lineWidth = varargin{i+1};
71         case 'linestyle'
72             LineStyle = varargin{i+1};
73         case 'linecolor'
74             LineColor = varargin{i+1};
75             for iLineColor = 1:numel(LineColor)
76                 if isfield(colors, LineColor{iLineColor})
77                     LineColor{iLineColor} = colors.(LineColor{iLineColor});
78             end
79         end
80         case 'marker'
81             Marker = varargin{i+1};
82         end
83     end
84 end
85 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
86
87 %% Improve the figure(s)
88
89 for iFig = 1:nFigs
90     fig = figHs(iFig);
91
92     lineObjects = findall(fig, 'Type', 'line');
93     textObjects = findall(fig, 'Type', 'text');
94     axesObjects = findall(fig, 'Type', 'axes');
95     legObjects = findall(fig, 'Type', 'legend');
96     contourObjects = findall(fig, 'Type', 'contour'); % not counted as lines
97
98     %% TEXT APPEARANCE: first set all to textSize and then change the ones
99     %% that need to be changed again
100
101     %Change size of any text objects in the plot
102     set(textObjects, 'FontSize', textSize);
103     set(legObjects, 'FontSize', legTextSize);
104
105     %% FIX LINSTYLE, COLOR ETC. FOR EACH PLOT SEPARATELY
106     for iAx = 1:numel(axesObjects)
107         lineObjInAx = findall(axesObjects(iAx), 'Type', 'line');
108
109         %set line style and color style (only works if all figs have some
110         %number of line plots..)
111         if ~isempty(LineStyle)
112             set(lineObjInAx, {'LineStyle'}, LineStyle)
113             set(contourObjects, {'LineStyle'}, LineStyle); %%%%%%%%%
114         end
115         if ~isempty(LineColor)
116             set(lineObjInAx, {'Color'}, LineColor)
117             set(contourObjects, {'LineColor'}, LineColor); %%%%%%%%%
118         end
119         if ~isempty(Marker)
120             set(lineObjInAx, {'Marker'}, Marker)
121             set(lineObjInAx, {'Markersize'}, num2cell(10+22*strcmp(Marker, '.'))↵
122             )
123         end
124
125         %% change font sizes.
126         % Tick label size
127         xLim = axesObjects(iAx).XLim;
128         axesObjects(iAx).FontSize = tickLabelSize;
129         axesObjects(iAx).XLim = xLim;
130         %Change label size
131         axesObjects(iAx).XLabel.FontSize = labelSize;
132         axesObjects(iAx).YLabel.FontSize = labelSize;
133
134         %Change title size
135         axesObjects(iAx).Title.FontSize = titleSize;
136     end
137
138     %% LINE APPEARANCE
139     %Change line thicknesses
140     set(lineObjects, 'LineWidth', lineWidth);
141     set(contourObjects, 'LineWidth', lineWidth);
142     set(axesObjects, 'LineWidth', axesWidth);
143
144     % set interpreter: latex or tex
145     set(textObjects, 'interpreter', interpreter)
146     set(legObjects, 'Interpreter', interpreter)
147     set(axesObjects, 'TickLabelInterpreter', interpreter);
148 end
149 end

```

C.3 Change size of figures: setFigureSize.m

```

1 function [ fig ] = setFigureSize( fig, H, W )
2 fig.Units = 'points';

```

```
3 fig.WindowStyle = 'normal'; % undock
4 fig.Position(3:4) = [W H];
5 end
```