Fall 2016

AMATH 732: Assignment 3.

Due Friday, October 21

1. Consider the DE

$$x^4 y'' = y.$$

   (a) Classify the points $x = 0$ and $x = \infty$. Try the method of Frobenius to find a series solution near $x = 0$. Why doesn't it work?

   (b) Use the method of Carlini-Liouville-Green to determine the leading asymptotic behaviour $e^{S_o(x)}$, i.e., find all the terms of $S(x)$ that do not vanish as $x \to 0$. Do so for two linearly independent solutions to this DE.

   (c) Show that linearly independent solutions $y_1(x)$ and $y_2(x)$ to the DE may now be constructed using the substitution $y = e^{S_o(x)} w(x)$. Determine the first three terms in the series expansions of the $w(x)$ and use them to construct the corresponding solutions $y_i(x)$.

2. Find the leading behaviour of two linearly independent solutions of the Airy equation

$$y'' = xy,$$

   as $x \to +\infty$, an irregular singular point.