Homework 4
Due Friday, November 25

1. Consider a superconductor at a finite temperature T.

   (a) Show that the internal energy of the superconductor can be written as

   $$E(T) = \sum_{\mathbf{k}} \left[\xi_{\mathbf{k}} - \left(E_{\mathbf{k}} - \frac{\Delta^2}{2E_{\mathbf{k}}} \right) (1 - 2n_F(E_{\mathbf{k}})) \right],$$

   where $n_F(E) = \frac{1}{e^{E/T}+1}$ is the Fermi-Dirac distribution function.

   (b) Using the above expression for the internal energy, find explicitly the temperature dependence of the specific heat $C_V = \left(\frac{\partial E}{\partial T}\right)_V$ at low temperatures.

   (c) $C_V(T)$ for a superconductor is discontinuous at $T = T_c$. Find the magnitude of the discontinuity.

   (d) Using the above results, sketch $C_V(T)$ for all temperatures.

2. Consider a superconductor at a temperature near the critical temperature T_c.

   (a) Find the Helmholtz free energy as a Taylor expansion with respect to the order parameter Δ, keeping terms of up to Δ^4.

   (b) Minimizing the free energy with respect to Δ, rederive the temperature dependence of the order parameter near T_c:

   $$\Delta(T) \approx 3.1\, T_c \sqrt{\frac{T_c - T}{T_c}}.$$

   (c) Verify your result from Problem 1 on the specific heat discontinuity at T_c.

   (d) Sketch the free energy as a function of Δ for both $T < T_c$ and $T > T_c$ and discuss.