Guelph-Waterloo Physics Institute

Quantum Mechanics – Physics 701/7010 – Fall 2016 – Problem Set 4

Due Thursday, Nov. 17th, at the lecture in Waterloo, or at Guelph, in Kiley Rider's mailbox in the copy room.

(1) Problem 5.22 from Sakurai and Napolitano, *Modern Quantum Mechanics*, 2nd ed.
Consider a one-dimensional simple harmonic oscillator whose classical angular frequency is ω_0. For $t < 0$ it is known to be in the ground state. For $t > 0$ there is also a time-dependent potential

$$V(t) = F_0 x \cos \omega t,$$

where F_0 is constant in both space and time. Obtain an expression for the expectation value $\langle x \rangle$ as a function of time using time-dependent perturbation theory to lowest nonvanishing order. Is this procedure valid for $\omega \approx \omega_0$? [You may use $\langle n' | \, x \, | n \rangle = \sqrt{\hbar/(2m\omega_0)} \; (\sqrt{n+1}\delta_{n',n+1} + \sqrt{n}\delta_{n',n-1})$.]

(2) Problem 5.40 from Sakurai and Napolitano, *Modern Quantum Mechanics*, 2nd ed.
Linearly polarized light of angular frequency ω is incident on a one-electron "atom" whose wave function can be approximated by the ground state of a three dimensional isotropic harmonic oscillator of angular frequency ω_0. Show that the differential cross section for the ejection of a photoelectron is given by

$$\begin{aligned} \frac{d\sigma}{d\Omega} = & \frac{4\alpha\hbar^2 k_f^3}{m^2 \omega \omega_0} \sqrt{\frac{\pi\hbar}{m\omega_0}} \exp\left\{ -\frac{\hbar}{m\omega_0} \left[k_f^2 + \left(\frac{\omega}{c}\right)^2 \right] \right\} \\ & \times \sin^2\theta \cos^2\phi \exp\left[\left(\frac{2\hbar k_f \omega}{m\omega_0 c} \right) \cos\theta \right], \end{aligned}$$

provided the ejected electron of momentum $\hbar k_f$ can be regarded as being in a plane-wave state. (The coordinate system used is shown in Figure 5.12 of Sakurai.)

(3) The following problem fills in a gap in our derivation of the hydrogen photoionization cross-section. Problem 5.41 from Sakurai and Napolitano, *Modern Quantum Mechanics*, 2nd ed.
Find the probability $|\phi(\mathbf{p}')|^2 d^3\, p'$ of the particular momentum $\mathbf{p}'$ for the ground-state hydrogen atom. (This is a nice exercise in three-dimensional Fourier transforms. To perform the angular integration, choose the z-axis in the direction of $\mathbf{p}$.)