

MOSEK ApS

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# Chapter 1

# Introduction

The **MOSEK** Optimization Suite 9.3.18 is a powerful software package capable of solving large-scale optimization problems of the following kind:

- linear,
- conic:
  - conic quadratic (also known as second-order cone),
  - involving the exponential cone,
  - involving the power cone,
  - semidefinite.
- convex quadratic and quadratically constrained,
- integer.

In order to obtain an overview of features in the MOSEK Optimization Suite consult the product introduction guide.

The most widespread class of optimization problems is *linear optimization problems*, where all relations are linear. The tremendous success of both applications and theory of linear optimization can be ascribed to the following factors:

- The required data are simple, i.e. just matrices and vectors.
- Convexity is guaranteed since the problem is convex by construction.
- Linear functions are trivially differentiable.
- There exist very efficient algorithms and software for solving linear problems.
- Duality properties for linear optimization are nice and simple.

Even if the linear optimization model is only an approximation to the true problem at hand, the advantages of linear optimization may outweigh the disadvantages. In some cases, however, the problem formulation is inherently nonlinear and a linear approximation is either intractable or inadequate. *Conic optimization* has proved to be a very expressive and powerful way to introduce nonlinearities, while preserving all the nice properties of linear optimization listed above.

The fundamental expression in linear optimization is a linear expression of the form

$$Ax - b \ge 0$$
.

In conic optimization this is replaced with a wider class of constraints

$$Ax - b \in \mathcal{K}$$

where  $\mathcal{K}$  is a *convex cone*. For example in 3 dimensions  $\mathcal{K}$  may correspond to an ice cream cone. The conic optimizer in **MOSEK** supports a number of different types of cones  $\mathcal{K}$ , which allows a surprisingly large number of nonlinear relations to be modeled, as described in the **MOSEK** Modeling Cookbook, while preserving the nice algorithmic and theoretical properties of linear optimization.

## 1.1 Why the Optimizer API for C?

The Optimizer API for C provides low-level access to all functionalities of **MOSEK** from any C compatible language. It consists of a single header file and a set of library files which an application must link against when building. This interface has the smallest possible overhead, however other interfaces might be considered more convenient to use for the project at hand.

The Optimizer API for C provides access to:

- Linear Optimization (LO)
- Conic Quadratic (Second-Order Cone) Optimization (CQO, SOCO)
- Power Cone Optimization
- Conic Exponential Optimization (CEO)
- Convex Quadratic and Quadratically Constrained Optimization (QO, QCQO)
- Semidefinite Optimization (SDO)
- Mixed-Integer Optimization (MIO)

as well as additional interfaces for:

- problem analysis,
- sensitivity analysis,
- infeasibility analysis,
- BLAS/LAPACK linear algebra routines.

# Chapter 2

# **Contact Information**

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	Denmark	

You can get in touch with  $\mathbf{MOSEK}$  using popular social media as well:

Blogger	https://blog.mosek.com/
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Twitter	https://twitter.com/mosektw
Linkedin	https://www.linkedin.com/company/mosek-aps
Youtube	https://www.youtube.com/channel/UCvIyectEVLP31NXeD5mIbEw

In particular  $\mathbf{Twitter}$  is used for news, updates and release announcements.

## Chapter 3

# License Agreement

Before using the MOSEK software, please read the license agreement available in the distribution at MOSEK website MOSEK

MOSEK uses some third-party open-source libraries. Their license details follows.

#### zlib

**MOSEK** includes the *zlib* library obtained from the zlib website. The license agreement for *zlib* is shown in Listing 3.1.

### Listing 3.1: zlib license.

zlib.h -- interface of the 'zlib' general purpose compression library version 1.2.7, May 2nd, 2012

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### fplib

**MOSEK** includes the floating point formatting library developed by David M. Gay obtained from the netlib website. The license agreement for *fplib* is shown in Listing 3.2.

Listing 3.2: fplib license.

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For Zstandard software

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# Chapter 4

# Installation

In this section we discuss how to install and setup the MOSEK Optimizer API for C.

Important: Before running this MOSEK interface please make sure that you:

- Installed MOSEK correctly. Some operating systems require extra steps. See the Installation guide for instructions and common troubleshooting tips.
- Set up a license. See the Licensing guide for instructions.

### Compatibility

The Optimizer API for C is compatible with the following compiler tool chains:

Platform	Supported compiler	Framework
Linux 64 bit x86	$gcc (\geq 4.5)$	glibc ( $\geq 2.12$ )
Mac OS 64 bit	$X$ code ( $\geq 5$ )	MAC OS SDK ( $\geq 10.7$ )
Windows 32 and 64 bit	Visual Studio (≥ 2010)	Windows 8.1 SDK
Linux ARM64	$gcc (\geq 9.2)$	glibc ( $\geq 2.20$ )

In many cases older versions can also be used.

### Locating files in the MOSEK Optimization Suite

The relevant files of the Optimizer API for C are organized as reported in Table 4.1.

Table 4.1: Relevant files for the Optimizer API for C.

Relative Path	Description	Label
<pre><mskhome>/mosek/9.3/tools/platform/<platform>/h</platform></mskhome></pre>	Header files	<headerdir></headerdir>
<pre><mskhome>/mosek/9.3/tools/platform/<platform>/bin</platform></mskhome></pre>	Libraries and DLLs	<libdir></libdir>
<mskhome>/mosek/9.3/tools/examples/c</mskhome>	Examples	<exdir></exdir>
<mskhome>/mosek/9.3/tools/examples/data</mskhome>	Additional data	<miscdir></miscdir>

### where

- <MSKHOME> is the folder in which the MOSEK Optimization Suite has been installed,
- <PLATFORM> is the actual platform among those supported by MOSEK, i.e. win32x86, win64x86, linux64x86, osx64x86 or linuxaarch64.

### Setting up the paths

To compile and link C code using the Optimizer API for C, the relevant path to the header file and library must be included, and run-time dependencies must be resolved. Hence to compile, one should add appropriate compiler and linker options. Details vary depending on the operating system and compiler. See the Makefile included in the distribution under <MSKHOME>/mosek/9.3/tools/examples/c for a full working example. Examples are given below.

### Linux, 64bit x86

The shared library libmosek64.so.9.3 must be available at runtime.

#### Windows, 64bit

```
cl.exe /I<HEADERDIR> file.c /link /LIBPATH:<LIBDIR> /out:file.exe mosek64_9_3.lib

The shared library mosek64_9_3.dll must be available at runtime.
```

#### Windows, 32bit

```
cl.exe /I<HEADERDIR> file.c /link /LIBPATH:<LIBDIR> /out:file.exe mosek9_3.lib

The shared library mosek9_3.dll must be available at runtime.
```

#### Mac OS

```
clang file.c -o file -I<HEADERDIR> -L<LIBDIR> -W1,-headerpad,128 -lmosek64
install_name_tool -change libmosek64.9.3.dylib <LIBDIR>/libmosek64.9.3.dylib file
```

The shared library libmosek64.9.3.dylib must be registered and available at runtime.

### Linux, ARM64

```
gcc file.c -o file -I<HEADERDIR> -L<LIBDIR> -W1,-rpath-link,<LIBDIR> -W1,-rpath= _{\hookrightarrow}<LIBDIR> -lpthread -lmosek64
```

The shared library libmosek64.so.9.3 must be available at runtime.

## 4.1 Testing the Installation and Compiling Examples

This section describes how to verify that **MOSEK** has been installed correctly, and how to build and execute the C examples distributed with **MOSEK**.

### 4.1.1 Windows

### Compiling examples using NMake

The example directory <EXDIR> contains makefiles for use with Microsoft NMake. These makefiles requires that the Visual Studio tool chain is setup. Usually, the submenu containing Visual Studio also contains a Visual Studio Command Prompt which does the necessary setup.

To build the examples, open a DOS box and change directory to <EXDIR>. This directory contains a makefile named Makefile. To compile all examples, run the command

```
nmake /f Makefile all
```

To build only a single example instead of all examples, replace all by the corresponding executable name. For example, to build lol.exe type

### Compiling from command line

To compile and execute a distributed example, such as lol.c, do the following:

- 1. Compile the example into an executable lol.exe (we assume that the Visual Studio C compiler cl.exe is available). For 64-bit Windows:
  - cl <EXDIR>\lo1.c /I <HEADERDIR> /link <LIBDIR>\mosek64\_9\_3.lib
- 2. To run the compiled example, enter

lo1.exe

### Adding MOSEK to a Visual Studio Project

The following walk-through is specific for Microsoft Visual Studio 2012, but may work for other versions too. To compile a project linking to **MOSEK** in Visual Studio, the following steps are necessary:

- 1. Create a project or open an existing project in Visual Studio.
- 2. In the **Solution Explorer** right-click on the relevant project and select **Properties**. This will open the **Property pages** dialog.
- 3. In the selection box Configuration: select All Configurations.
- 4. In the tree-view open Configuration Properties  $\rightarrow$  C/C++  $\rightarrow$  General.
- 5. In the properties click the Additional Include Directories field and select edit.
- 6. Click on the **New Folder** button and write the *full path* to the h header file or browse for the file. For example, for 64-bit Windows use <HEADERDIR>.
- 7. Click **OK**.
- 8. Back in the **Property Pages** dialog select from the tree-view **Configuration Properties**  $\rightarrow$  **Linker**  $\rightarrow$  **Input**.
- 9. In the properties view click in the **Additional Dependencies** field and select edit. This will open the **Additional Dependencies** dialog.
- 10. Add the full path of the **MOSEK** lib. For example, for 64-bit Windows:

```
<LIBDIR>\mosek64_9_3.lib
```

- 11. Click **OK**.
- 12. Back in the **Property Pages** dialog click **OK**.

If you have selected to link with the 64 bit version of **MOSEK** you must also target the 64-bit platform. To do this follow the steps below:

- 1. Open the **property pages** for that project.
- 2. Click Configuration Manager to open the Configuration Manager Dialog Box.
- 3. Click the **Active Solution Platform** list, and then select the **New** option to open the New Solution Platform Dialog Box.
- 4. Click the Type or select the new platform drop-down arrow, and then select the x64 platform.
- 5. Click **OK**. The platform you selected in the preceding step will appear under Active Solution Platform in the Configuration Manager dialog box.

### 4.1.2 Mac OS and Linux

The example directory <EXDIR> contains makefiles for use with GNU Make. To build the examples enter

### make -f Makefile all

To build one example instead of all examples, replace all by the corresponding executable name. For example, to build the lo1 executable enter

make -f Makefile lo1

## Chapter 5

# Design Overview

## 5.1 Modeling

Optimizer API for C is an interface for specifying optimization problems directly in matrix form. It means that an optimization problem such as:

minimize 
$$c^T x$$
  
subject to  $Ax \leq b$ ,  
 $x \in \mathcal{K}$ 

is specified by describing the matrix A, vectors b, c and a list of cones K directly.

The main characteristics of this interface are:

- Simplicity: once the problem data is assembled in matrix form, it is straightforward to input it into the optimizer.
- Exploiting sparsity: data is entered in sparse format, enabling huge, sparse problems to be defined and solved efficiently.
- Efficiency: the Optimizer API incurs almost no overhead between the user's representation of the problem and MOSEK's internal one.

Optimizer API for C does not aid with modeling. It is the user's responsibility to express the problem in MOSEK's standard form, introducing, if necessary, auxiliary variables and constraints. See Sec. 12 for the precise formulations of problems MOSEK solves.

### 5.2 "Hello World!" in MOSEK

Here we present the most basic workflow pattern when using Optimizer API for C.

### Creating an environment and task

Every interaction with **MOSEK** using Optimizer API for C begins by creating a **MOSEK** environment. It coordinates the access to **MOSEK** from the current process.

In most cases the user does not interact directly with the environment, except for creating optimization **tasks**, which contain actual problem specifications and where optimization takes place. An environment can host multiple tasks.

### **Defining tasks**

After a task is created, the input data can be specified. An optimization problem consists of several components; objective, objective sense, constraints, variable bounds etc. See Sec. 6 for basic tutorials on how to specify and solve various types of optimization problems.

### Retrieving the solutions

When the model is set up, the optimizer is invoked with the call to  $MSK_optimize$ . When the optimization is over, the user can check the results and retrieve numerical values. See further details in Sec. 7.

We refer also to Sec. 7 for information about more advanced mechanisms of interacting with the solver.

### Source code example

Below is the most basic code sample that defines and solves a trivial optimization problem

```
minimize x
subject to 2.0 \le x \le 3.0.
```

For simplicity the example does not contain any error or status checks.

Listing 5.1: "Hello World!" in MOSEK

```
# include "mosek.h"
# include <stdio.h>
/* Error checking not included */
int main() {
 MSKrescodee
                  r, trmcode;
 MSKenv_t
                  env = NULL;
 MSKtask_t
                  task = NULL;
 double
                  xx = 0.0;
 MSK_makeenv(&env, NULL);
                                     // Create environment
 MSK_maketask(env, 0, 1, &task); // Create task
 MSK_appendvars(task, 1);
                                                      // 1 variable x
 MSK_putcj(task, 0, 1.0);
                                                      // c_0 = 1.0
 MSK_putvarbound(task, 0, MSK_BK_RA, 2.0, 3.0);
                                                      // 2.0 <= x <= 3.0
 MSK_putobjsense(task, MSK_OBJECTIVE_SENSE_MINIMIZE); // Minimize
 MSK_optimizetrm(task, &trmcode);
                                         // Optimize
 MSK_getxx(task, MSK_SOL_ITR, &xx);
                                         // Get solution
                                          // Print solution
 printf("Solution x = \frac{f}{n}, xx);
 MSK\_deletetask(\&task); // Clean up task
 MSK_deleteenv(&env); // Clean up environment
 return 0;
```

# Chapter 6

# Optimization Tutorials

In this section we demonstrate how to set up basic types of optimization problems. Each short tutorial contains a working example of formulating problems, defining variables and constraints and retrieving solutions.

## 6.1 Linear Optimization

The simplest optimization problem is a purely linear problem. A *linear optimization problem* is a problem of the following form:

Minimize or maximize the objective function

$$\sum_{j=0}^{n-1} c_j x_j + c^f$$

subject to the linear constraints

$$l_k^c \le \sum_{j=0}^{n-1} a_{kj} x_j \le u_k^c, \quad k = 0, \dots, m-1,$$

and the bounds

$$l_j^x \le x_j \le u_j^x, \quad j = 0, \dots, n - 1.$$

The problem description consists of the following elements:

- $\bullet$  m and n the number of constraints and variables, respectively,
- x the variable vector of length n,
- ullet c the coefficient vector of length n

$$c = \left[ \begin{array}{c} c_0 \\ \vdots \\ c_{n-1} \end{array} \right],$$

- $c^f$  fixed term in the objective,
- A an  $m \times n$  matrix of coefficients

$$A = \begin{bmatrix} a_{0,0} & \cdots & a_{0,(n-1)} \\ \vdots & \cdots & \vdots \\ a_{(m-1),0} & \cdots & a_{(m-1),(n-1)} \end{bmatrix},$$

- $l^c$  and  $u^c$  the lower and upper bounds on constraints,
- $l^x$  and  $u^x$  the lower and upper bounds on variables.

Please note that we are using 0 as the first index:  $x_0$  is the first element in variable vector x.

### 6.1.1 Example LO1

The following is an example of a small linear optimization problem:

maximize 
$$3x_0 + 1x_1 + 5x_2 + 1x_3$$
  
subject to  $3x_0 + 1x_1 + 2x_2 = 30$ ,  
 $2x_0 + 1x_1 + 3x_2 + 1x_3 \ge 15$ ,  
 $2x_1 + 3x_3 \le 25$ , (6.1)

under the bounds

### Solving the problem

To solve the problem above we go through the following steps:

- 1. Create an environment.
- 2. Create an optimization task.
- 3. Load a problem into the task object.
- 4. Optimization.
- 5. Extracting the solution.

Below we explain each of these steps.

### Create an environment.

Before setting up the optimization problem, a **MOSEK** environment must be created. All tasks in the program should share the same environment.

```
r = MSK_makeenv(&env, NULL);
```

### Create an optimization task.

Next, an empty task object is created:

```
/* Create the optimization task. */
r = MSK_maketask(env, numcon, numvar, &task);

/* Directs the log task stream to the 'printstr' function. */
if (r == MSK_RES_OK)
r = MSK_linkfunctotaskstream(task, MSK_STREAM_LOG, NULL, printstr);
```

We also connect a call-back function to the task log stream. Messages related to the task are passed to the call-back function. In this case the stream call-back function writes its messages to the standard output stream. See Sec. 7.3.

### Load a problem into the task object.

Before any problem data can be set, variables and constraints must be added to the problem via calls to the functions  $MSK\_appendcons$  and  $MSK\_appendcons$ .

```
/* Append 'numcon' empty constraints.
  The constraints will initially have no bounds. */
if (r == MSK_RES_OK)
  r = MSK_appendcons(task, numcon);

/* Append 'numvar' variables.
  The variables will initially be fixed at zero (x=0). */
if (r == MSK_RES_OK)
  r = MSK_appendvars(task, numvar);
```

New variables can now be referenced from other functions with indexes in  $0, \ldots, \mathtt{numvar} - 1$  and new constraints can be referenced with indexes in  $0, \ldots, \mathtt{numcon} - 1$ . More variables and/or constraints can be appended later as needed, these will be assigned indexes from  $\mathtt{numvar/numcon}$  and up.

Next step is to set the problem data. We loop over each variable index j = 0, ..., numvar - 1 calling functions to set problem data. We first set the objective coefficient  $c_j = c[j]$  by calling the function  $MSK\_putcj$ .

```
/* Set the linear term c_j in the objective.*/
if (r == MSK_RES_OK)
    r = MSK_putcj(task, j, c[j]);
```

### Setting bounds on variables

The bounds on variables are stored in the arrays

and are set with calls to MSK\_putvarbound.

The Bound key stored in bkx specifies the type of the bound according to Table 6.1.

Table 6.1: Bound keys as defined in the enum  ${\tt MSKboundkeye}.$ 

Bound key	Type of bound	Lower bound	Upper bound
$MSK\_BK\_FX$	$u_j = l_j$	Finite	Identical to the lower bound
MSK_BK_FR	Free	$-\infty$	$+\infty$
MSK_BK_LO	$l_j \leq \cdots$	Finite	$+\infty$
MSK_BK_RA	$l_j \leq \cdots \leq u_j$	Finite	Finite
MSK_BK_UP	$\cdots \leq u_j$	$-\infty$	Finite

For instance bkx[0] =  $MSK_BK_L0$  means that  $x_0 \ge l_0^x$ . Finally, the numerical values of the bounds on variables are given by

$$l_i^x = blx[j]$$

and

$$u_i^x = \text{bux}[j].$$

### Defining the linear constraint matrix.

Recall that in our example the A matrix is given by

$$A = \left[ \begin{array}{rrrr} 3 & 1 & 2 & 0 \\ 2 & 1 & 3 & 1 \\ 0 & 2 & 0 & 3 \end{array} \right].$$

This matrix is stored in sparse format in the arrays:

The aptrb, aptre, asub, and aval arguments define the constraint matrix A in the column ordered sparse format (for details, see Sec. 15.1.4).

Using the function  $MSK_putacol$  we set column j of A

There are many alternative formats for entering the A matrix. See functions such as  $MSK\_putarow$ ,  $MSK\_putarowlist$ ,  $MSK\_putaijlist$  and similar.

Finally, the bounds on each constraint are set by looping over each constraint index  $i=0,\ldots,\mathtt{numcon}-1$ 

### **Optimization**

After the problem is set-up the task can be optimized by calling the function MSK\_optimizetrm .

```
r = MSK_optimizetrm(task, &trmcode);
```

### Extracting the solution.

After optimizing the status of the solution is examined with a call to  $MSK\_getsolsta$ . If the solution status is reported as  $MSK\_SOL\_STA\_OPTIMAL$  the solution is extracted in the lines below:

```
MSK_getxx(task,

MSK_SOL_BAS, /* Request the basic solution. */

xx);
```

The  $MSK\_getxx$  function obtains the solution. **MOSEK** may compute several solutions depending on the optimizer employed. In this example the *basic solution* is requested by setting the first argument to  $MSK\_SOL\_BAS$ .

#### Source code

The complete source code lol.c of this example appears below. See also lol.c for a version where the A matrix is entered row-wise.

Listing 6.1: Linear optimization example.

```
# include <stdio.h>
# include "mosek.h"
/* This function prints log output from MOSEK to the terminal. */
static void MSKAPI printstr(void
                                       *handle,
                            const char str[])
 printf("%s", str);
} /* printstr */
int main(int argc, const char *argv[])
  const MSKint32t
                     numvar = 4,
                     numcon = 3;
 const double
                     c []
                             = \{3.0, 1.0, 5.0, 1.0\};
  /* Below is the sparse representation of the A
     matrix stored by column. */
  const MSKint32t
                     aptrb[] = \{0, 2, 5, 7\},
                     aptre[] = \{2, 5, 7, 9\},
                     asub[] = { 0, 1,}
                                  0, 1, 2,
                                  0, 1,
                                  1, 2
                                };
                     aval[] = { 3.0, 2.0, }
 const double
                                  1.0, 1.0, 2.0,
                                  2.0, 3.0,
                                  1.0, 3.0
                                };
  /* Bounds on constraints. */
  const MSKboundkeye bkc[] = {MSK_BK_FX, MSK_BK_LO,
                                                           MSK_BK_UP
                                                                        };
```

```
15.0,
 const double
                   blc[] = {30.0,}
                                                     -MSK_INFINITY};
                   buc[] = \{30.0,
                                      +MSK_INFINITY, 25.0
 const double
 /* Bounds on variables. */
 MSK BK LO
→};
                 blx[] = \{0.0,
const double
                                     0.0, 0.0,
                                                                   0.0
→};
                  bux[] = \{ +MSK_INFINITY, 10.0,
 const double
                                                     +MSK_INFINITY, +MSK_
→INFINITY };
                   env = NULL;
 MSKenv_t
 MSKtask_t
                   task = NULL;
 MSKrescodee
                   r;
 MSKint32t
                   i, j;
 /* Create the mosek environment. */
 r = MSK_makeenv(&env, NULL);
 if (r == MSK_RES_OK)
   /* Create the optimization task. */
   r = MSK_maketask(env, numcon, numvar, &task);
   /* Directs the log task stream to the 'printstr' function. */
   if (r == MSK_RES_OK)
    r = MSK_linkfunctotaskstream(task, MSK_STREAM_LOG, NULL, printstr);
   /* Append 'numcon' empty constraints.
    The constraints will initially have no bounds. */
   if (r == MSK_RES_OK)
    r = MSK_appendcons(task, numcon);
   /* Append 'numvar' variables.
    The variables will initially be fixed at zero (x=0). */
   if (r == MSK_RES_OK)
    r = MSK_appendvars(task, numvar);
   for (j = 0; j < numvar && r == MSK_RES_OK; ++j)
     /* Set the linear term c_j in the objective.*/
     if (r == MSK_RES_OK)
      r = MSK_putcj(task, j, c[j]);
     /* Set the bounds on variable j.
     blx[j] \leftarrow x_j \leftarrow bux[j] */
     if (r == MSK_RES_OK)
       r = MSK_putvarbound(task,
                                      /* Index of variable.*/
                          j,
                                    /* Bound key.*/
                          bkx[j],
                                      /* Numerical value of lower bound.*/
                          blx[j],
                          bux[j]);
                                      /* Numerical value of upper bound.*/
     /* Input column j of A */
     if (r == MSK_RES_OK)
       r = MSK_putacol(task,
                                        /* Variable (column) index.*/
                      j,
```

```
aptre[j] - aptrb[j], /* Number of non-zeros in column j.*/
                    asub + aptrb[j], /* Pointer to row indexes of column j.*/
                    aval + aptrb[j]); /* Pointer to Values of column j.*/
}
/* Set the bounds on constraints.
  for i=1, ...,numcon : blc[i] <= constraint i <= buc[i] */</pre>
for (i = 0; i < numcon && r == MSK_RES_OK; ++i)
 r = MSK_putconbound(task,
                                   /* Index of constraint.*/
                      bkc[i],
                                   /* Bound key.*/
                      blc[i],
                                 /* Numerical value of lower bound.*/
                      buc[i]);
                                  /* Numerical value of upper bound.*/
/* Maximize objective function. */
if (r == MSK_RES_OK)
  r = MSK_putobjsense(task, MSK_OBJECTIVE_SENSE_MAXIMIZE);
if (r == MSK_RES_OK)
{
 MSKrescodee trmcode;
  /* Run optimizer */
 r = MSK_optimizetrm(task, &trmcode);
  /* Print a summary containing information
     about the solution for debugging purposes. */
 MSK_solutionsummary(task, MSK_STREAM_LOG);
  if (r == MSK_RES_OK)
  {
   MSKsolstae solsta;
    if (r == MSK_RES_OK)
     r = MSK_getsolsta(task,
                        MSK_SOL_BAS,
                        &solsta);
    switch (solsta)
      case MSK_SOL_STA_OPTIMAL:
          double *xx = (double*) calloc(numvar, sizeof(double));
          if (xx)
            MSK_getxx(task,
                      MSK_SOL_BAS, /* Request the basic solution. */
                      xx);
            printf("Optimal primal solution\n");
            for (j = 0; j < numvar; ++j)
              printf("x[%d]: %e\n", j, xx[j]);
            free(xx);
          }
          else
            r = MSK_RES_ERR_SPACE;
```

```
break;
            }
          case MSK_SOL_STA_DUAL_INFEAS_CER:
          case MSK_SOL_STA_PRIM_INFEAS_CER:
            printf("Primal or dual infeasibility certificate found.\n");
          case MSK_SOL_STA_UNKNOWN:
            {
              char symname[MSK_MAX_STR_LEN];
              char desc[MSK_MAX_STR_LEN];
              /* If the solutions status is unknown, print the termination code
                 indicating why the optimizer terminated prematurely. */
              MSK_getcodedesc(trmcode,
                              symname,
                              desc);
              printf("The solution status is unknown.\n");
              printf("The optimizer terminitated with code: %s\n", symname);
              break;
            }
          default:
            printf("Other solution status.\n");
     }
   }
   if (r != MSK_RES_OK)
      /* In case of an error print error code and description. */
      char symname[MSK_MAX_STR_LEN];
      char desc[MSK_MAX_STR_LEN];
     printf("An error occurred while optimizing.\n");
     MSK_getcodedesc(r,
                      symname,
                      desc);
     printf("Error %s - '%s'\n", symname, desc);
    /* Delete the task and the associated data. */
   MSK_deletetask(&task);
 }
  /* Delete the environment and the associated data. */
 MSK_deleteenv(&env);
 return r;
}
```

### 6.2 Quadratic Optimization

**MOSEK** can solve quadratic and quadratically constrained problems, as long as they are convex. This class of problems can be formulated as follows:

minimize 
$$\frac{1}{2}x^{T}Q^{o}x + c^{T}x + c^{f}$$
subject to 
$$l_{k}^{c} \leq \frac{1}{2}x^{T}Q^{k}x + \sum_{j=0}^{n-1}a_{k,j}x_{j} \leq u_{k}^{c}, \quad k = 0, \dots, m-1,$$

$$l_{j}^{x} \leq x_{j} \leq u_{j}^{x}, \quad j = 0, \dots, n-1.$$

$$(6.2)$$

Without loss of generality it is assumed that  $Q^o$  and  $Q^k$  are all symmetric because

$$x^T Q x = \frac{1}{2} x^T (Q + Q^T) x.$$

This implies that a non-symmetric Q can be replaced by the symmetric matrix  $\frac{1}{2}(Q+Q^T)$ .

The problem is required to be convex. More precisely, the matrix  $Q^o$  must be positive semi-definite and the kth constraint must be of the form

$$l_k^c \le \frac{1}{2} x^T Q^k x + \sum_{j=0}^{n-1} a_{k,j} x_j \tag{6.3}$$

with a negative semi-definite  $Q^k$  or of the form

$$\frac{1}{2}x^T Q^k x + \sum_{j=0}^{n-1} a_{k,j} x_j \le u_k^c.$$

with a positive semi-definite  $Q^k$ . This implies that quadratic equalities are *not* allowed. Specifying a non-convex problem will result in an error when the optimizer is called.

A matrix is positive semidefinite if all the eigenvalues of Q are nonnegative. An alternative statement of the positive semidefinite requirement is

$$x^T Qx > 0, \quad \forall x.$$

If the convexity (i.e. semidefiniteness) conditions are not met **MOSEK** will not produce reliable results or work at all.

### 6.2.1 Example: Quadratic Objective

We look at a small problem with linear constraints and quadratic objective:

minimize 
$$x_1^2 + 0.1x_2^2 + x_3^2 - x_1x_3 - x_2$$
 subject to 
$$1 \le x_1 + x_2 + x_3$$
 
$$0 < x.$$
 (6.4)

The matrix formulation of (6.4) has:

$$Q^o = \begin{bmatrix} 2 & 0 & -1 \\ 0 & 0.2 & 0 \\ -1 & 0 & 2 \end{bmatrix}, c = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}, A = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix},$$

with the bounds:

$$l^c = 1, u^c = \infty, l^x = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$
 and  $u^x = \begin{bmatrix} \infty \\ \infty \\ \infty \end{bmatrix}$ 

Please note the explicit  $\frac{1}{2}$  in the objective function of (6.2) which implies that diagonal elements must be doubled in Q, i.e.  $Q_{11}=2$  even though 1 is the coefficient in front of  $x_1^2$  in (6.4).

### Setting up the linear part

The linear parts (constraints, variables, objective) are set up using exactly the same methods as for linear problems, and we refer to Sec. 6.1 for all the details. The same applies to technical aspects such as defining an optimization task, retrieving the solution and so on.

### Setting up the quadratic objective

The quadratic objective is specified using the function  $MSK_putqobj$ . Since  $Q^o$  is symmetric only the lower triangular part of  $Q^o$  is inputted. In fact entries from above the diagonal may not appear in the input.

The lower triangular part of the matrix  $Q^o$  is specified using an unordered sparse triplet format (for details, see Sec. 15.1.4):

```
qsubi[0] = 0; qsubj[0] = 0; qval[0] = 2.0;
qsubi[1] = 1; qsubj[1] = 1; qval[1] = 0.2;
qsubi[2] = 2; qsubj[2] = 0; qval[2] = -1.0;
qsubi[3] = 2; qsubj[3] = 2; qval[3] = 2.0;
```

Please note that

- only non-zero elements are specified (any element not specified is 0 by definition),
- the order of the non-zero elements is insignificant, and
- only the lower triangular part should be specified.

Finally, this definition of  $Q^o$  is loaded into the task:

```
r = MSK_putqobj(task, NUMQNZ, qsubi, qsubj, qval);
```

### Source code

Listing 6.2: Source code implementing problem (6.4).

```
# include <stdio.h>
#include "mosek.h" /* Include the MOSEK definition file. */
#define NUMCON 1 /* Number of constraints.
#define NUMVAR 3
                  /* Number of variables.
#define NUMANZ 3
                 /* Number of non-zeros in A.
                 /* Number of non-zeros in Q.
# define NUMQNZ 4
static void MSKAPI printstr(void *handle,
                            const char str[])
 printf("%s", str);
} /* printstr */
int main(int argc, const char *argv[])
                    = \{0.0, -1.0, 0.0\};
                c[]
 double
 MSKboundkeye bkc[] = {MSK_BK_LO};
 double
                blc[] = \{1.0\};
                buc[] = { +MSK_INFINITY };
 double
 MSKboundkeye bkx[] = {MSK_BK_LO,
```

```
MSK_BK_LO,
                       MSK_BK_LO
                      };
double
              blx[] = \{0.0,
                       0.0,
                       0.0
                      };
              bux[] = { +MSK_INFINITY,
double
                       +MSK_INFINITY,
                        +MSK_INFINITY
                      };
MSKint32t
              aptrb[] = \{0, 1, 2\},
              aptre[] = \{1, 2, 3\},
              asub[] = \{0, 0, 0\};
              aval[] = \{1.0, 1.0, 1.0\};
double
            qsubi[NUMQNZ];
MSKint32t
MSKint32t
            qsubj[NUMQNZ];
double
            qval[NUMQNZ];
MSKint32t
            i, j;
double
             xx[NUMVAR];
              env = NULL;
MSKenv_t
MSKtask_t
              task = NULL;
MSKrescodee
              r;
/* Create the mosek environment. */
r = MSK_makeenv(&env, NULL);
if (r == MSK_RES_OK)
  /* Create the optimization task. */
  r = MSK_maketask(env, NUMCON, NUMVAR, &task);
  if (r == MSK_RES_OK)
   r = MSK_linkfunctotaskstream(task, MSK_STREAM_LOG, NULL, printstr);
    /* Append 'NUMCON' empty constraints.
    The constraints will initially have no bounds. */
    if (r == MSK_RES_OK)
     r = MSK_appendcons(task, NUMCON);
    /* Append 'NUMVAR' variables.
    The variables will initially be fixed at zero (x=0). */
    if (r == MSK_RES_OK)
     r = MSK_appendvars(task, NUMVAR);
    /* Optionally add a constant term to the objective. */
    if (r == MSK_RES_OK)
     r = MSK_putcfix(task, 0.0);
    for (j = 0; j < NUMVAR && r == MSK_RES_OK; ++j)
      /* Set the linear term c_j in the objective.*/
```

```
if (r == MSK_RES_OK)
   r = MSK_putcj(task, j, c[j]);
  /* Set the bounds on variable j.
  blx[j] <= x_j <= bux[j] */
  if (r == MSK_RES_OK)
   r = MSK_putvarbound(task,
                                    /* Index of variable.*/
                        bkx[j],
                                    /* Bound key.*/
                                    /* Numerical value of lower bound.*/
                        blx[j],
                        bux[j]);
                                    /* Numerical value of upper bound.*/
  /* Input column j of A */
  if (r == MSK_RES_OK)
   r = MSK_putacol(task,
                                      /* Variable (column) index.*/
                    j,
                    aptre[j] - aptrb[j], /* Number of non-zeros in column j.*/
                    asub + aptrb[j], /* Pointer to row indexes of column j.*/
                    aval + aptrb[j]); /* Pointer to Values of column j.*/
}
/* Set the bounds on constraints.
  for i=1, ..., NUMCON : blc[i] <= constraint i <= buc[i] */</pre>
for (i = 0; i < NUMCON && r == MSK_RES_OK; ++i)
 r = MSK_putconbound(task,
                      i,
                                  /* Index of constraint.*/
                      bkc[i],
                                 /* Bound key.*/
                                 /* Numerical value of lower bound.*/
                      blc[i],
                      buc[i]);
                                 /* Numerical value of upper bound.*/
if (r == MSK_RES_OK)
{
   * The lower triangular part of the Q
   * matrix in the objective is specified.
  qsubi[0] = 0; qsubj[0] = 0; qval[0] = 2.0;
  qsubi[1] = 1; qsubj[1] = 1; qval[1] = 0.2;
                qsubj[2] = 0; qval[2] = -1.0;
  qsubi[2] = 2;
 qsubi[3] = 2; qsubj[3] = 2; qval[3] = 2.0;
 /* Input the Q for the objective. */
 r = MSK_putqobj(task, NUMQNZ, qsubi, qsubj, qval);
if (r == MSK_RES_OK)
 MSKrescodee trmcode;
 /* Run optimizer */
 r = MSK_optimizetrm(task, &trmcode);
  /* Print a summary containing information
```

```
about the solution for debugging purposes*/
       MSK_solutionsummary(task, MSK_STREAM_MSG);
       if (r == MSK_RES_OK)
         MSKsolstae solsta;
         int j;
         MSK_getsolsta(task, MSK_SOL_ITR, &solsta);
         switch (solsta)
           case MSK_SOL_STA_OPTIMAL:
             MSK_getxx(task,
                       MSK_SOL_ITR,
                                      /* Request the interior solution. */
                       xx);
             printf("Optimal primal solution\n");
             for (j = 0; j < NUMVAR; ++j)
               printf("x[%d]: %e\n", j, xx[j]);
             break;
           case MSK_SOL_STA_DUAL_INFEAS_CER:
           case MSK_SOL_STA_PRIM_INFEAS_CER:
             printf("Primal or dual infeasibility certificate found.\n");
             break;
           case MSK_SOL_STA_UNKNOWN:
             printf("The status of the solution could not be determined. Termination∪
break;
           default:
             printf("Other solution status.");
             break;
         }
       }
       else
         printf("Error while optimizing.\n");
     }
     if (r != MSK_RES_OK)
       /* In case of an error print error code and description. */
       char symname[MSK_MAX_STR_LEN];
       char desc[MSK_MAX_STR_LEN];
       printf("An error occurred while optimizing.\n");
       MSK_getcodedesc(r,
                       symname,
                       desc);
       printf("Error %s - '%s'\n", symname, desc);
```

```
}
   MSK_deletetask(&task);
}
MSK_deleteenv(&env);

return (r);
} /* main */
```

### 6.2.2 Example: Quadratic constraints

In this section we show how to solve a problem with quadratic constraints. Please note that quadratic constraints are subject to the convexity requirement (6.3).

Consider the problem:

This is equivalent to

$$\begin{array}{ll} \text{minimize} & \frac{1}{2}x^TQ^ox + c^Tx \\ \text{subject to} & \frac{1}{2}x^TQ^0x + Ax \geq b, \\ & x \geq 0, \end{array} \tag{6.5}$$

where

$$Q^o = \begin{bmatrix} 2 & 0 & -1 \\ 0 & 0.2 & 0 \\ -1 & 0 & 2 \end{bmatrix}, c = \begin{bmatrix} 0 & -1 & 0 \end{bmatrix}^T, A = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}, b = 1.$$
 
$$Q^0 = \begin{bmatrix} -2 & 0 & 0.2 \\ 0 & -2 & 0 \\ 0.2 & 0 & -0.2 \end{bmatrix}.$$

The linear parts and quadratic objective are set up the way described in the previous tutorial.

### Setting up quadratic constraints

To add quadratic terms to the constraints we use the function  $MSK\_putqconk$ .

While  $MSK_putqconk$  adds quadratic terms to a specific constraint, it is also possible to input all quadratic terms in one chunk using the  $MSK_putqcon$  function.

### Source code

Listing 6.3: Implementation of the quadratically constrained problem (6.5).

```
# include <stdio.h>
#include "mosek.h" /* Include the MOSEK definition file. */
#define NUMCON 1 /* Number of constraints.
                                                        */
#define NUMVAR 3 /* Number of variables.
                                                        */
#define NUMANZ 3 /* Number of non-zeros in A.
#define NUMQNZ 4 /* Number of non-zeros in Q.
static void MSKAPI printstr(void *handle,
                           const char str[])
 printf("%s", str);
} /* printstr */
int main(int argc, const char *argv[])
 MSKrescodee r;
 double
              c []
                   = \{0.0, -1.0, 0.0\};
 MSKboundkeye bkc[] = {MSK_BK_LO};
 double blc[] = {1.0};
              buc[] = { +MSK_INFINITY};
 double
 MSKboundkeye bkx[] = {MSK_BK_LO,
                        MSK_BK_LO,
                        MSK_BK_LO
                       };
              blx[] = \{0.0,
 double
                        0.0.
                        0.0
                       };
              bux[] = { +MSK_INFINITY,
 double
                         +MSK_INFINITY,
                         +MSK_INFINITY
                       };
              aptrb[] = {0, 1, 2},
 MSKint32t
              aptre[] = \{1, 2, 3\},\
              asub[] = { 0, 0, 0};
 double
              aval[] = { 1.0, 1.0, 1.0};
 MSKint32t
              qsubi[NUMQNZ],
              qsubj[NUMQNZ];
 double
              qval[NUMQNZ];
              j, i;
 MSKint32t
              xx[NUMVAR];
 double
 MSKenv_t
              env;
 MSKtask_t
              task;
```

```
/* Create the mosek environment. */
r = MSK_makeenv(&env, NULL);
if (r == MSK_RES_OK)
  /* Create the optimization task. */
  r = MSK_maketask(env, NUMCON, NUMVAR, &task);
  if (r == MSK_RES_OK)
   r = MSK_linkfunctotaskstream(task, MSK_STREAM_LOG, NULL, printstr);
    /* Append 'NUMCON' empty constraints.
    The constraints will initially have no bounds. */
    if (r == MSK_RES_OK)
      r = MSK_appendcons(task, NUMCON);
    /* Append 'NUMVAR' variables.
     The variables will initially be fixed at zero (x=0). */
    if (r == MSK_RES_OK)
     r = MSK_appendvars(task, NUMVAR);
    /* Optionally add a constant term to the objective. */
    if (r == MSK_RES_OK)
     r = MSK_putcfix(task, 0.0);
    for (j = 0; j < NUMVAR && r == MSK_RES_OK; ++j)
      /* Set the linear term c_j in the objective.*/
      if (r == MSK_RES_OK)
        r = MSK_putcj(task, j, c[j]);
      /* Set the bounds on variable j.
       blx[j] \leftarrow x_j \leftarrow bux[j] */
      if (r == MSK_RES_OK)
        r = MSK_putvarbound(task,
                                         /* Index of variable.*/
                            j,
                            bkx[j],
                                         /* Bound key.*/
                                         /* Numerical value of lower bound.*/
                            blx[j],
                            bux[j]);
                                         /* Numerical value of upper bound.*/
      /* Input column j of A */
      if (r == MSK_RES_OK)
        r = MSK_putacol(task,
                                           /* Variable (column) index.*/
                        aptre[j] - aptrb[j], /* Number of non-zeros in column j.*/
                        asub + aptrb[j], /* Pointer to row indexes of column j.*/
                        aval + aptrb[j]); /* Pointer to Values of column j.*/
    }
    /* Set the bounds on constraints.
      for i=1, ..., NUMCON : blc[i] <= constraint i <= buc[i] */
    for (i = 0; i < NUMCON && r == MSK_RES_OK; ++i)
      r = MSK_putconbound(task,
                                       /* Index of constraint.*/
                          i,
                                       /* Bound key.*/
                          bkc[i],
```

```
blc[i],
                                  /* Numerical value of lower bound.*/
                     buc[i]);
                                  /* Numerical value of upper bound.*/
if (r == MSK RES OK)
{
   * The lower triangular part of the Q^o
   * matrix in the objective is specified.
                qsubj[0] = 0; qval[0] = 2.0;
  qsubi[0] = 0;
  qsubi[1] = 1;
               qsubj[1] = 1; qval[1] = 0.2;
  qsubi[2] = 2; qsubj[2] = 0; qval[2] = -1.0;
  qsubi[3] = 2; qsubj[3] = 2; qval[3] = 2.0;
 /* Input the Q^o for the objective. */
 r = MSK_putqobj(task, NUMQNZ, qsubi, qsubj, qval);
if (r == MSK_RES_OK)
  * The lower triangular part of the Q^O
  * matrix in the first constraint is specified.
  This corresponds to adding the term
  -x_1^2 - x_2^2 - 0.1 x_3^2 + 0.2 x_1 x_3
  qsubi[0] = 0;
                qsubj[0] = 0; qval[0] = -2.0;
  qsubi[1] = 1; qsubj[1] = 1; qval[1] = -2.0;
                 qsubj[2] = 2; qval[2] = -0.2;
  qsubi[2] = 2;
                qsubj[3] = 0; qval[3] = 0.2;
 qsubi[3] = 2;
  /* Put Q^0 in constraint with index 0. */
 r = MSK_putqconk(task,
                   0,
                   4,
                   qsubi,
                   qsubj,
                  qval);
}
if (r == MSK_RES_OK)
 r = MSK_putobjsense(task, MSK_OBJECTIVE_SENSE_MINIMIZE);
if (r == MSK_RES_OK)
 MSKrescodee trmcode;
  /* Run optimizer */
 r = MSK_optimizetrm(task, &trmcode);
  /* Print a summary containing information
     about the solution for debugging purposes*/
```

```
MSK_solutionsummary(task, MSK_STREAM_LOG);
       if (r == MSK_RES_OK)
         MSKsolstae solsta;
         int j;
         MSK_getsolsta(task, MSK_SOL_ITR, &solsta);
         switch (solsta)
           case MSK_SOL_STA_OPTIMAL:
             MSK_getxx(task,
                       MSK_SOL_ITR, /* Request the interior solution. */
                       xx);
             printf("Optimal primal solution\n");
             for (j = 0; j < NUMVAR; ++j)
               printf("x[%d]: %e\n", j, xx[j]);
             break;
           case MSK_SOL_STA_DUAL_INFEAS_CER:
           case MSK_SOL_STA_PRIM_INFEAS_CER:
             printf("Primal or dual infeasibility certificate found.\n");
             break;
           case MSK_SOL_STA_UNKNOWN:
             printf("The status of the solution could not be determined. Termination∪
break;
             printf("Other solution status.");
             break:
         }
       }
       else
         printf("Error while optimizing.\n");
     }
     if (r != MSK_RES_OK)
       /* In case of an error print error code and description. */
       char symname[MSK_MAX_STR_LEN];
       char desc[MSK_MAX_STR_LEN];
       printf("An error occurred while optimizing.\n");
       MSK_getcodedesc(r,
                       symname,
                       desc);
       printf("Error %s - '%s'\n", symname, desc);
     }
   }
```

```
MSK_deletetask(&task);
}
MSK_deleteenv(&env);
return (r);
} /* main */
```

### 6.3 Conic Quadratic Optimization

Conic optimization is a generalization of linear optimization, allowing constraints of the type

$$x^t \in \mathcal{K}_t$$

where  $x^t$  is a subset of the problem variables and  $\mathcal{K}_t$  is a convex cone. Since the set  $\mathbb{R}^n$  of real numbers is also a convex cone, we can simply write a compound conic constraint  $x \in \mathcal{K}$  where  $\mathcal{K} = \mathcal{K}_1 \times \cdots \times \mathcal{K}_l$  is a product of smaller cones and x is the full problem variable.

MOSEK can solve conic quadratic optimization problems of the form

where the domain restriction,  $x \in \mathcal{K}$ , implies that all variables are partitioned into convex cones

$$x = (x^0, x^1, \dots, x^{p-1}), \text{ with } x^t \in \mathcal{K}_t \subseteq \mathbb{R}^{n_t}.$$

In this tutorial we describe how to use the two types of quadratic cones defined as:

• Quadratic cone:

$$Q^{n} = \left\{ x \in \mathbb{R}^{n} : x_{0} \ge \sqrt{\sum_{j=1}^{n-1} x_{j}^{2}} \right\}.$$

• Rotated quadratic cone:

$$Q_r^n = \left\{ x \in \mathbb{R}^n : 2x_0 x_1 \ge \sum_{j=2}^{n-1} x_j^2, \quad x_0 \ge 0, \quad x_1 \ge 0 \right\}.$$

For other types of cones supported by **MOSEK** see Sec. 6.4, Sec. 6.5, Sec. 6.6. See *MSK\_appendcone* for a list and definitions of available cone types. Different cone types can appear together in one optimization problem.

For example, the following constraint:

$$(x_4, x_0, x_2) \in \mathcal{Q}^3$$

describes a convex cone in  $\mathbb{R}^3$  given by the inequality:

$$x_4 \ge \sqrt{x_0^2 + x_2^2}.$$

Furthermore, each variable may belong to one cone at most. The constraint  $x_i - x_j = 0$  would however allow  $x_i$  and  $x_j$  to belong to different cones with same effect.

### 6.3.1 Example CQO1

Consider the following conic quadratic problem which involves some linear constraints, a quadratic cone and a rotated quadratic cone.

minimize 
$$x_4 + x_5 + x_6$$
  
subject to  $x_1 + x_2 + 2x_3 = 1$ ,  
 $x_1, x_2, x_3 \ge 0$ , (6.6)  
 $x_4 \ge \sqrt{x_1^2 + x_2^2}$ ,  
 $2x_5x_6 \ge x_3^2$ 

### Setting up the linear part

The linear parts (constraints, variables, objective) are set up using exactly the same methods as for linear problems, and we refer to Sec. 6.1 for all the details. The same applies to technical aspects such as defining an optimization task, retrieving the solution and so on.

#### Setting up the conic constraints

A cone is defined using the function MSK\_appendcone:

The first argument selects the type of quadratic cone, in this case either MSK\_CT\_QUAD for a quadratic cone or MSK\_CT\_RQUAD for a rotated quadratic cone. The second parameter is currently ignored and passing 0.0 will work.

The next argument denotes the number of variables in the cone, in this case 3, and the last argument is a list of indexes of the variables appearing in the cone.

Variants of this method are available to append multiple cones at a time.

#### Source code

Listing 6.4: Source code solving problem (6.6).

```
MSKboundkeye bkc[] = { MSK_BK_FX };
             blc[] = { 1.0 };
double
             buc[] = { 1.0 };
double
MSKboundkeye bkx[] = {MSK_BK_LO,
                      MSK_BK_LO,
                      MSK_BK_LO,
                      MSK_BK_FR,
                      MSK_BK_FR,
                      MSK_BK_FR
                     };
             blx[] = {0.0,}
double
                      0.0,
                      0.0,
                      -MSK_INFINITY,
                      -MSK_INFINITY,
                      -MSK_INFINITY
                     };
double
             bux[] = { +MSK_INFINITY,
                       +MSK_INFINITY,
                       +MSK_INFINITY,
                       +MSK_INFINITY,
                       +MSK_INFINITY,
                       +MSK_INFINITY
                     };
double
             c[]
                  = \{0.0,
                      0.0,
                      0.0,
                      1.0,
                      1.0,
                      1.0
                     };
MSKint32t
            aptrb[] = {0, 1, 2, 3, 3, 3},
            aptre[] = \{1, 2, 3, 3, 3, 3\},
            asub[] = \{0, 0, 0, 0\};
double
            aval[] = \{1.0, 1.0, 2.0\};
           i, j, csub[3];
MSKint32t
MSKenv_t
            env = NULL;
MSKtask_t
           task = NULL;
/* Create the mosek environment. */
r = MSK_makeenv(&env, NULL);
if (r == MSK_RES_OK)
  /* Create the optimization task. */
 r = MSK_maketask(env, numcon, numvar, &task);
  if (r == MSK_RES_OK)
   MSK_linkfunctotaskstream(task, MSK_STREAM_LOG, NULL, printstr);
```

```
/* Append 'numcon' empty constraints.
The constraints will initially have no bounds. */
if (r == MSK_RES_OK)
 r = MSK_appendcons(task, numcon);
/* Append 'numvar' variables.
The variables will initially be fixed at zero (x=0). */
if (r == MSK_RES_OK)
 r = MSK_appendvars(task, numvar);
for (j = 0; j < numvar && r == MSK_RES_OK; ++j)
  /* Set the linear term c_j in the objective.*/
 if (r == MSK_RES_OK)
   r = MSK_putcj(task, j, c[j]);
  /* Set the bounds on variable j.
  blx[j] \leftarrow x_j \leftarrow bux[j] */
  if (r == MSK_RES_OK)
   r = MSK_putvarbound(task,
                                    /* Index of variable.*/
                        j,
                        bkx[j],
                                     /* Bound key.*/
                                     /* Numerical value of lower bound.*/
                        blx[j],
                                     /* Numerical value of upper bound.*/
                        bux[j]);
  /* Input column j of A */
  if (r == MSK_RES_OK)
   r = MSK_putacol(task,
                                       /* Variable (column) index.*/
                    aptre[j] - aptrb[j], /* Number of non-zeros in column j.*/
                    asub + aptrb[j], /* Pointer to row indexes of column j.*/
                    aval + aptrb[j]); /* Pointer to Values of column j.*/
}
/* Set the bounds on constraints.
for i=1, ...,numcon : blc[i] <= constraint i <= buc[i] */</pre>
for (i = 0; i < numcon && r == MSK_RES_OK; ++i)
 r = MSK_putconbound(task,
                                   /* Index of constraint.*/
                                   /* Bound key.*/
                      bkc[i],
                      blc[i],
                                   /* Numerical value of lower bound.*/
                      buc[i]);
                                   /* Numerical value of upper bound.*/
if (r == MSK_RES_OK)
  /* Append the first cone. */
  csub[0] = 3;
  csub[1] = 0;
  csub[2] = 1;
  r = MSK_appendcone(task,
                     MSK_CT_QUAD,
                     0.0, /* Can be set to 0.0 */
```

```
csub);
}
if (r == MSK_RES_OK)
  /* Append the second cone. */
  csub[0] = 4;
 csub[1] = 5;
  csub[2] = 2;
 r = MSK_appendcone(task,
                     MSK_CT_RQUAD,
                     0.0,
                     3,
                     csub);
}
if (r == MSK_RES_OK)
 MSKrescodee trmcode;
  /* Run optimizer */
 r = MSK_optimizetrm(task, &trmcode);
  /* Print a summary containing information
     about the solution for debugging purposes*/
 MSK_solutionsummary(task, MSK_STREAM_MSG);
  if (r == MSK_RES_OK)
   MSKsolstae solsta;
   MSK_getsolsta(task, MSK_SOL_ITR, &solsta);
   switch (solsta)
    {
      case MSK_SOL_STA_OPTIMAL:
          double *xx = NULL;
          xx = calloc(numvar, sizeof(double));
          if (xx)
            MSK_getxx(task,
                      MSK_SOL_ITR, /* Request the interior solution. */
                      xx);
            printf("Optimal primal solution\n");
            for (j = 0; j < numvar; ++j)
              printf("x[%d]: %e\n", j, xx[j]);
          }
          else
          {
            r = MSK_RES_ERR_SPACE;
          }
```

 $({\rm continued\ from\ previous\ page})$ 

```
free(xx);
             }
             break;
            case MSK_SOL_STA_DUAL_INFEAS_CER:
            case MSK_SOL_STA_PRIM_INFEAS_CER:
             printf("Primal or dual infeasibility certificate found.\n");
             break;
           case MSK_SOL_STA_UNKNOWN:
             printf("The status of the solution could not be determined. Termination_{\sqcup}
break;
           default:
             printf("Other solution status.");
             break;
       }
       else
         printf("Error while optimizing.\n");
      if (r != MSK_RES_OK)
        /* In case of an error print error code and description. */
        char symname[MSK_MAX_STR_LEN];
        char desc[MSK_MAX_STR_LEN];
        printf("An error occurred while optimizing.\n");
       MSK_getcodedesc(r,
                       symname,
                       desc);
       printf("Error %s - '%s'\n", symname, desc);
    /* Delete the task and the associated data. */
   MSK_deletetask(&task);
 /* Delete the environment and the associated data. */
 MSK_deleteenv(&env);
 return (r);
} /* main */
```

## 6.4 Power Cone Optimization

Conic optimization is a generalization of linear optimization, allowing constraints of the type

$$x^t \in \mathcal{K}_t$$

where  $x^t$  is a subset of the problem variables and  $\mathcal{K}_t$  is a convex cone. Since the set  $\mathbb{R}^n$  of real numbers is also a convex cone, we can simply write a compound conic constraint  $x \in \mathcal{K}$  where  $\mathcal{K} = \mathcal{K}_1 \times \cdots \times \mathcal{K}_l$  is a product of smaller cones and x is the full problem variable.

MOSEK can solve conic optimization problems of the form

where the domain restriction,  $x \in \mathcal{K}$ , implies that all variables are partitioned into convex cones

$$x = (x^0, x^1, \dots, x^{p-1}), \text{ with } x^t \in \mathcal{K}_t \subseteq \mathbb{R}^{n_t}.$$

In this tutorial we describe how to use the power cone. The primal power cone of dimension n with parameter  $0 < \alpha < 1$  is defined as:

$$\mathcal{P}_n^{\alpha, 1 - \alpha} = \left\{ x \in \mathbb{R}^n : x_0^{\alpha} x_1^{1 - \alpha} \ge \sqrt{\sum_{i=2}^{n-1} x_i^2}, \ x_0, x_1 \ge 0 \right\}.$$

In particular, the most important special case is the three-dimensional power cone family:

$$\mathcal{P}_3^{\alpha, 1 - \alpha} = \left\{ x \in \mathbb{R}^3 : x_0^{\alpha} x_1^{1 - \alpha} \ge |x_2|, \ x_0, x_1 \ge 0 \right\}.$$

For example, the conic constraint  $(x, y, z) \in \mathcal{P}_3^{0.25, 0.75}$  is equivalent to  $x^{0.25}y^{0.75} \ge |z|$ , or simply  $xy^3 \ge z^4$  with  $x, y \ge 0$ .

**MOSEK** also supports the dual power cone:

$$\left(\mathcal{P}_n^{\alpha,1-\alpha}\right)^* = \left\{ x \in \mathbb{R}^n : \left(\frac{x_0}{\alpha}\right)^\alpha \left(\frac{x_1}{1-\alpha}\right)^{1-\alpha} \ge \sqrt{\sum_{i=2}^{n-1} x_i^2}, \ x_0, x_1 \ge 0 \right\}.$$

For other types of cones supported by **MOSEK** see Sec. 6.3, Sec. 6.5, Sec. 6.6. See *MSK\_appendcone* for a list and definitions of available cone types. Different cone types can appear together in one optimization problem.

Furthermore, each variable may belong to one cone at most. The constraint  $x_i - x_j = 0$  would however allow  $x_i$  and  $x_j$  to belong to different cones with same effect.

### 6.4.1 Example POW1

Consider the following optimization problem which involves powers of variables:

maximize 
$$x^{0.2}y^{0.8} + z^{0.4} - x$$
  
subject to  $x + y + \frac{1}{2}z = 2$ ,  $x, y, z \ge 0$ .  $(6.7)$ 

With  $(x, y, z) = (x_0, x_1, x_2)$  we convert it into conic form using auxiliary variables as bounds for the power expressions:

maximize 
$$x_3 + x_4 - x_0$$
  
subject to  $x_0 + x_1 + \frac{1}{2}x_2 = 2$ ,  
 $(x_0, x_1, x_3) \in \mathcal{P}_3^{0.2, 0.8}$ ,  
 $(x_2, x_5, x_4) \in \mathcal{P}_3^{0.4, 0.6}$ ,  
 $x_5 = 1$ . (6.8)

#### Setting up the linear part

The linear parts (constraints, variables, objective) are set up using exactly the same methods as for linear problems, and we refer to Sec. 6.1 for all the details. The same applies to technical aspects such as defining an optimization task, retrieving the solution and so on.

### Setting up the conic constraints

A cone is defined using the function MSK\_appendcone:

```
MSKint32t csub[2][3] = { {0, 1, 3}, {2, 5, 4} };
r = MSK_appendcone(task, MSK_CT_PPOW, 0.2, 3, csub[0]);
r = MSK_appendcone(task, MSK_CT_PPOW, 0.4, 3, csub[1]);
```

The first argument selects the type of power cone, that is  $MSK\_CT\_PPOW$ . The second argument is the cone parameter  $\alpha$ . The remaining arguments list the variables which form the cone. Variants of this method are available to append multiple cones at a time.

The code below produces the answer of (6.7) which is

#### Source code

Listing 6.5: Source code solving problem (6.7).

```
# include <stdio.h>
#include "mosek.h" /* Include the MOSEK definition file. */
static void MSKAPI printstr(void *handle,
                            const char str[])
 printf("%s", str);
} /* printstr */
int main(int argc, const char *argv[])
 MSKrescodee r;
 const MSKint32t numvar = 6,
                  numcon = 1;
 MSKboundkeye bkx[6];
 double
               blx[6], bux[6];
               val[] = { 1.0, 1.0, -1.0 };
 double
 MSKint32t
               sub[] = { 3, 4, 0 };
               aval[] = { 1.0, 1.0, 0.5 };
 double
               asub[] = { 0, 1, 2 };
 MSKint32t
 MSKint32t
              i, j;
 MSKenv_t
              env = NULL;
 MSKtask_t
              task = NULL;
  /* Create the mosek environment. */
 r = MSK_makeenv(&env, NULL);
```

```
if (r == MSK_RES_OK)
  /* Create the optimization task. */
  r = MSK_maketask(env, numcon, numvar, &task);
  if (r == MSK_RES_OK)
  {
   MSK_linkfunctotaskstream(task, MSK_STREAM_LOG, NULL, printstr);
    /* Append 'numcon' empty constraints.
    The constraints will initially have no bounds. */
    if (r == MSK_RES_OK)
     r = MSK_appendcons(task, numcon);
    /* Append 'numvar' variables.
    The variables will initially be fixed at zero (x=0). */
    if (r == MSK_RES_OK)
     r = MSK_appendvars(task, numvar);
    /* Set up the linear part */
   MSK_putclist(task, 3, sub, val);
    MSK_putarow(task, 0, 3, asub, aval);
   MSK_putconbound(task, 0, MSK_BK_FX, 2.0, 2.0);
    for(i=0;i<5;i++)
      bkx[i] = MSK_BK_FR, blx[i] = -MSK_INFINITY, bux[i] = MSK_INFINITY;
    bkx[5] = MSK_BK_FX, blx[5] = bux[5] = 1.0;
    MSK_putvarboundslice(task, 0, numvar, bkx, blx, bux);
    if (r == MSK_RES_OK)
      /* Append two power cones. */
     MSKint32t csub[2][3] = { {0, 1, 3}, {2, 5, 4} };
     r = MSK_appendcone(task, MSK_CT_PPOW, 0.2, 3, csub[0]);
      r = MSK_appendcone(task, MSK_CT_PPOW, 0.4, 3, csub[1]);
    MSK_putobjsense(task, MSK_OBJECTIVE_SENSE_MAXIMIZE);
    if (r == MSK_RES_OK)
      MSKrescodee trmcode;
      /* Run optimizer */
      r = MSK_optimizetrm(task, &trmcode);
      /* Print a summary containing information
         about the solution for debugging purposes*/
      MSK_solutionsummary(task, MSK_STREAM_MSG);
      if (r == MSK_RES_OK)
        MSKsolstae solsta;
        MSK_getsolsta(task, MSK_SOL_ITR, &solsta);
        switch (solsta)
```

```
{
           case MSK_SOL_STA_OPTIMAL:
             {
               double *xx = NULL;
               xx = calloc(numvar, sizeof(double));
               if (xx)
               {
                 MSK_getxx(task,
                                           /* Request the interior solution. */
                           MSK_SOL_ITR,
                           xx);
                 printf("Optimal primal solution\n");
                 for (j = 0; j < 3; ++j)
                   printf("x[%d]: %e\n", j, xx[j]);
               else
                 r = MSK_RES_ERR_SPACE;
               free(xx);
             }
             break;
           case MSK_SOL_STA_DUAL_INFEAS_CER:
           case MSK_SOL_STA_PRIM_INFEAS_CER:
             printf("Primal or dual infeasibility certificate found.\n");
             break;
           case MSK_SOL_STA_UNKNOWN:
             printf("The status of the solution could not be determined. Termination∪
break;
           default:
             printf("Other solution status.");
             break;
         }
       }
       else
         printf("Error while optimizing.\n");
     }
     if (r != MSK_RES_OK)
       /* In case of an error print error code and description. */
       char symname[MSK_MAX_STR_LEN];
       char desc[MSK_MAX_STR_LEN];
       printf("An error occurred while optimizing.\n");
       MSK_getcodedesc(r,
                       symname,
                       desc);
       printf("Error %s - '%s'\n", symname, desc);
     }
   }
   /* Delete the task and the associated data. */
```

```
MSK_deletetask(&task);
}

/* Delete the environment and the associated data. */
MSK_deleteenv(&env);

return (r);
} /* main */
```

## 6.5 Conic Exponential Optimization

Conic optimization is a generalization of linear optimization, allowing constraints of the type

$$x^t \in \mathcal{K}_t$$
,

where  $x^t$  is a subset of the problem variables and  $\mathcal{K}_t$  is a convex cone. Since the set  $\mathbb{R}^n$  of real numbers is also a convex cone, we can simply write a compound conic constraint  $x \in \mathcal{K}$  where  $\mathcal{K} = \mathcal{K}_1 \times \cdots \times \mathcal{K}_l$  is a product of smaller cones and x is the full problem variable.

MOSEK can solve conic optimization problems of the form

where the domain restriction,  $x \in \mathcal{K}$ , implies that all variables are partitioned into convex cones

$$x = (x^0, x^1, \dots, x^{p-1}), \text{ with } x^t \in \mathcal{K}_t \subseteq \mathbb{R}^{n_t}.$$

In this tutorial we describe how to use the primal exponential cone defined as:

$$K_{\text{exp}} = \{ x \in \mathbb{R}^3 : x_0 \ge x_1 \exp(x_2/x_1), \ x_0, x_1 \ge 0 \}.$$

 $\mathbf{MOSEK}$  also supports the dual exponential cone:

$$K_{\text{exp}}^* = \left\{ s \in \mathbb{R}^3 : s_0 \ge -s_2 e^{-1} \exp(s_1/s_2), \ s_2 \le 0, s_0 \ge 0 \right\}.$$

For other types of cones supported by **MOSEK** see Sec. 6.3, Sec. 6.4, Sec. 6.6. See *MSK\_appendcone* for a list and definitions of available cone types. Different cone types can appear together in one optimization problem.

For example, the following constraint:

$$(x_4, x_0, x_2) \in K_{\text{exp}}$$

describes a convex cone in  $\mathbb{R}^3$  given by the inequalities:

$$x_4 \ge x_0 \exp(x_2/x_0), \ x_0, x_4 \ge 0.$$

Furthermore, each variable may belong to one cone at most. The constraint  $x_i - x_j = 0$  would however allow  $x_i$  and  $x_j$  to belong to different cones with same effect.

### 6.5.1 Example CEO1

Consider the following basic conic exponential problem which involves some linear constraints and an exponential inequality:

minimize 
$$x_0 + x_1$$
  
subject to  $x_0 + x_1 + x_2 = 1$ ,  $x_0 \ge x_1 \exp(x_2/x_1)$ ,  $x_0, x_1 \ge 0$ . (6.9)

The conic form of (6.9) is:

minimize 
$$x_0 + x_1$$
  
subject to  $x_0 + x_1 + x_2 = 1,$   
 $(x_0, x_1, x_2) \in K_{\text{exp}},$   
 $x \in \mathbb{R}^3.$  (6.10)

#### Setting up the linear part

The linear parts (constraints, variables, objective) are set up using exactly the same methods as for linear problems, and we refer to Sec. 6.1 for all the details. The same applies to technical aspects such as defining an optimization task, retrieving the solution and so on.

### Setting up the conic constraints

A cone is defined using the function MSK\_appendcone:

The first argument selects the type of exponential cone, that is  $MSK\_CT\_PEXP$ . The second parameter is currently ignored and passing 0.0 will work.

The next argument denotes the number of variables in the cone, which must be 3 for the exponential cone, and the last argument is a list of indexes of the variables appearing in the cone.

Variants of this method are available to append multiple cones at a time.

### Source code

Listing 6.6: Source code solving problem (6.9).

```
MSKrescodee r;
const MSKint32t numvar = 3,
               numcon = 1;
MSKboundkeye bkc = MSK_BK_FX;
double
             blc = 1.0;
double
             buc = 1.0;
MSKboundkeye bkx[] = {MSK_BK_FR,
                     MSK_BK_FR,
                      MSK_BK_FR
                     };
double
             blx[] = {-MSK_INFINITY,
                      -MSK_INFINITY,
                      -MSK_INFINITY
                     };
             bux[] = { +MSK_INFINITY,
double
                       +MSK_INFINITY,
                      +MSK_INFINITY
                     };
double
             c[]
                  = \{1.0,
                     1.0,
                      0.0
                     };
              = \{1.0, 1.0, 1.0\};
double
          a[]
MSKint32t asub[] = \{0, 1, 2\};
MSKint32t i, j, csub[3];
           env = NULL;
MSKenv_t
MSKtask_t task = NULL;
/* Create the mosek environment. */
r = MSK_makeenv(&env, NULL);
if (r == MSK_RES_OK)
  /* Create the optimization task. */
 r = MSK_maketask(env, numcon, numvar, &task);
  if (r == MSK_RES_OK)
   MSK_linkfunctotaskstream(task, MSK_STREAM_LOG, NULL, printstr);
    /* Append 'numcon' empty constraints.
    The constraints will initially have no bounds. */
    if (r == MSK_RES_OK)
     r = MSK_appendcons(task, numcon);
    /* Append 'numvar' variables.
    The variables will initially be fixed at zero (x=0). */
    if (r == MSK_RES_OK)
     r = MSK_appendvars(task, numvar);
```

```
/* Set up the linear part */
if (r == MSK_RES_OK)
 r = MSK_putcslice(task, 0, numvar, c);
if (r == MSK_RES_OK)
 r = MSK_putarow(task, 0, numvar, asub, a);
if (r == MSK_RES_OK)
 r = MSK_putconbound(task, 0, bkc, blc, buc);
if (r == MSK_RES_OK)
 r = MSK_putvarboundslice(task, 0, numvar, bkx, blx, bux);
if (r == MSK_RES_OK)
  /* Append an exponential cone. */
 csub[0] = 0;
 csub[1] = 1;
 csub[2] = 2;
 r = MSK_appendcone(task,
                     MSK_CT_PEXP,
                     0.0, /* Can be set to 0.0 */
                     csub);
}
if (r == MSK_RES_OK)
 MSKrescodee trmcode;
  /* Run optimizer */
 r = MSK_optimizetrm(task, &trmcode);
  /* Print a summary containing information
     about the solution for debugging purposes*/
 MSK_solutionsummary(task, MSK_STREAM_MSG);
  if (r == MSK_RES_OK)
   MSKsolstae solsta;
   MSK_getsolsta(task, MSK_SOL_ITR, &solsta);
   switch (solsta)
      case MSK_SOL_STA_OPTIMAL:
        {
          double *xx = NULL;
          xx = calloc(numvar, sizeof(double));
          if (xx)
          {
            MSK_getxx(task,
                      MSK_SOL_ITR, /* Request the interior solution. */
                      xx);
            printf("Optimal primal solution\n");
```

```
for (j = 0; j < numvar; ++j)
                   printf("x[%d]: %e\n", j, xx[j]);
               }
               else
               {
                 r = MSK_RES_ERR_SPACE;
               free(xx);
             }
             break;
           case MSK_SOL_STA_DUAL_INFEAS_CER:
            case MSK_SOL_STA_PRIM_INFEAS_CER:
             printf("Primal or dual infeasibility certificate found.\n");
             break;
           case MSK_SOL_STA_UNKNOWN:
             printf("The status of the solution could not be determined. Termination ⊔
break;
           default:
             printf("Other solution status.");
             break;
         }
       }
       else
         printf("Error while optimizing.\n");
     }
      if (r != MSK_RES_OK)
        /* In case of an error print error code and description. */
        char symname[MSK_MAX_STR_LEN];
        char desc[MSK_MAX_STR_LEN];
       printf("An error occurred while optimizing.\n");
       MSK_getcodedesc(r,
                       symname,
                       desc);
       printf("Error %s - '%s'\n", symname, desc);
    /* Delete the task and the associated data. */
   MSK_deletetask(&task);
  /* Delete the environment and the associated data. */
 MSK_deleteenv(&env);
 return (r);
} /* main */
```

## 6.6 Semidefinite Optimization

Semidefinite optimization is a generalization of conic optimization, allowing the use of matrix variables belonging to the convex cone of positive semidefinite matrices

$$S_{\perp}^r = \left\{ X \in S^r : z^T X z \ge 0, \quad \forall z \in \mathbb{R}^r \right\},$$

where  $S^r$  is the set of  $r \times r$  real-valued symmetric matrices.

MOSEK can solve semidefinite optimization problems of the form

$$\begin{array}{lll} \text{minimize} & \sum_{j=0}^{n-1} c_j x_j + \sum_{j=0}^{p-1} \left\langle \overline{C}_j, \overline{X}_j \right\rangle + c^f \\ \text{subject to} & l_i^c & \leq & \sum_{j=0}^{n-1} a_{ij} x_j + \sum_{j=0}^{p-1} \left\langle \overline{A}_{ij}, \overline{X}_j \right\rangle & \leq & u_i^c, & i = 0, \dots, m-1, \\ & l_j^x & \leq & \underbrace{x_j} & \leq & u_j^x, & j = 0, \dots, n-1, \\ & x \in \mathcal{K}, \overline{X}_j \in \mathcal{S}_+^{r_j}, & j = 0, \dots, p-1 \end{array}$$

where the problem has p symmetric positive semidefinite variables  $\overline{X}_j \in \mathcal{S}_+^{r_j}$  of dimension  $r_j$  with symmetric coefficient matrices  $\overline{C}_j \in \mathcal{S}^{r_j}$  and  $\overline{A}_{i,j} \in \mathcal{S}^{r_j}$ . We use standard notation for the matrix inner product, i.e., for  $A, B \in \mathbb{R}^{m \times n}$  we have

$$\langle A, B \rangle := \sum_{i=0}^{m-1} \sum_{j=0}^{n-1} A_{ij} B_{ij}.$$

We demonstrate the setup of semidefinite variables and the matrices  $\overline{C}$ ,  $\overline{A}$  on the following examples:

- Sec. 6.6.1: A problem with one semidefinite variable and linear and conic constraints.
- Sec. 6.6.2: A problem with two semidefinite variables with a linear constraint and bound.

### 6.6.1 Example SDO1

We consider the simple optimization problem with semidefinite and conic quadratic constraints:

The problem description contains a 3-dimensional symmetric semidefinite variable which can be written explicitly as:

$$\overline{X} = \begin{bmatrix} \overline{X}_{00} & \overline{X}_{10} & \overline{X}_{20} \\ \overline{X}_{10} & \overline{X}_{11} & \overline{X}_{21} \\ \overline{X}_{20} & \overline{X}_{21} & \overline{X}_{22} \end{bmatrix} \in \mathcal{S}_{+}^{3},$$

and a conic quadratic variable  $(x_0, x_1, x_2) \in \mathcal{Q}^3$ . The objective is to minimize

$$2(\overline{X}_{00} + \overline{X}_{10} + \overline{X}_{11} + \overline{X}_{21} + \overline{X}_{22}) + x_0,$$

subject to the two linear constraints

$$\begin{array}{rcl} \overline{X}_{00} + \overline{X}_{11} + \overline{X}_{22} + x_0 & = & 1, \\ \overline{X}_{00} + \overline{X}_{11} + \overline{X}_{22} + 2(\overline{X}_{10} + \overline{X}_{20} + \overline{X}_{21}) + x_1 + x_2 & = & 1/2. \end{array}$$

### Setting up the linear and conic part

The linear and conic parts (constraints, variables, objective, cones) are set up using the methods described in the relevant tutorials; Sec. 6.1, Sec. 6.3, Sec. 6.5, Sec. 6.4. Here we only discuss the aspects directly involving semidefinite variables.

### Appending semidefinite variables

First, we need to declare the number of semidefinite variables in the problem, similarly to the number of linear variables and constraints. This is done with the function MSK\_appendbarvars.

```
r = MSK_appendbarvars(task, NUMBARVAR, DIMBARVAR);
```

### Appending coefficient matrices

Coefficient matrices  $\overline{C}_j$  and  $\overline{A}_{ij}$  are constructed as weighted combinations of sparse symmetric matrices previously appended with the function  $\texttt{MSK\_appendsparsesymmat}$ .

The arguments specify the dimension of the symmetric matrix, followed by its description in the sparse triplet format. Only lower-triangular entries should be included. The function produces a unique index of the matrix just entered in the collection of all coefficient matrices defined by the user.

After one or more symmetric matrices have been created using  $\textit{MSK\_appendsparsesymmat}$ , we can combine them to set up the objective matrix coefficient  $\overline{C}_j$  using  $\textit{MSK\_putbarcj}$ , which forms a linear combination of one or more symmetric matrices. In this example we form the objective matrix directly, i.e. as a weighted combination of a single symmetric matrix.

```
r = MSK_putbarcj(task, 0, 1, &idx, &falpha);
```

Similarly, a constraint matrix coefficient  $\overline{A}_{ij}$  is set up by the function  $MSK_putbaraij$ .

```
r = MSK_putbaraij(task, 0, 0, 1, &idx, &falpha);
```

### Retrieving the solution

After the problem is solved, we read the solution using MSK\_getbarxj:

The function returns the half-vectorization of  $\overline{X}_j$  (the lower triangular part stacked as a column vector), where the semidefinite variable index j is passed as an argument.

#### Source code

Listing 6.7: Source code solving problem (6.11).

```
# include <stdio.h>
# include "mosek.h"
                     /* Include the MOSEK definition file. */
               2 /* Number of constraints.
# define NUMCON
# define NUMVAR
               3 /* Number of conic quadratic variables */
#define NUMANZ 3 /* Number of non-zeros in A
#define NUMBARVAR 1 /* Number of semidefinite variables */
static void MSKAPI printstr(void *handle,
                          const char str[])
 printf("%s", str);
} /* printstr */
int main(int argc, const char *argv[])
 MSKrescodee r;
 MSKint32t
             DIMBARVAR[] = {3};  /* Dimension of semidefinite cone */
             LENBARVAR[] = \{3 * (3 + 1) / 2\}; /* Number of scalar SD variables */
 MSKint64t
 MSKboundkeye bkc[] = { MSK_BK_FX, MSK_BK_FX };
             blc[] = { 1.0, 0.5 };
 double
             buc[] = \{ 1.0, 0.5 \};
 double
 MSKint32t
             barc_i[] = \{0, 1, 1, 2, 2\},\
             barc_j[] = \{0, 0, 1, 1, 2\};
             barc_v[] = \{2.0, 1.0, 2.0, 1.0, 2.0\};
 double
             aptrb[] = \{0, 1\},
 MSKint32t
             aptre[] = \{1, 3\},
             asub[] = {0, 1, 2}; /* column subscripts of A */
             aval[] = \{1.0, 1.0, 1.0\};
 double
             bara_i[] = {0, 1, 2, 0, 1, 2, 1, 2, 2},
 MSKint32t
             bara_j[] = \{0, 1, 2, 0, 0, 0, 1, 1, 2\};
 double
             conesub[] = \{0, 1, 2\};
 MSKint32t
 MSKint32t
             i, j;
 MSKint64t
             idx;
 double
             falpha = 1.0;
 MSKrealt
             *xx;
 MSKrealt
             *barx;
 MSKenv_t
             env = NULL;
 MSKtask_t
             task = NULL;
 /* Create the mosek environment. */
 r = MSK_makeenv(&env, NULL);
 if (r == MSK_RES_OK)
```

```
{
  /* Create the optimization task. */
  r = MSK_maketask(env, NUMCON, 0, &task);
  if (r == MSK_RES_OK)
    MSK_linkfunctotaskstream(task, MSK_STREAM_LOG, NULL, printstr);
    /* Append 'NUMCON' empty constraints.
     The constraints will initially have no bounds. */
    if (r == MSK_RES_OK)
     r = MSK_appendcons(task, NUMCON);
    /* Append 'NUMVAR' variables.
    The variables will initially be fixed at zero (x=0). */
    if (r == MSK_RES_OK)
      r = MSK_appendvars(task, NUMVAR);
    /* Append 'NUMBARVAR' semidefinite variables. */
    if (r == MSK_RES_OK) {
     r = MSK_appendbarvars(task, NUMBARVAR, DIMBARVAR);
    /* Optionally add a constant term to the objective. */
    if (r == MSK_RES_OK)
      r = MSK_putcfix(task, 0.0);
    /* Set the linear term c_j in the objective.*/
    if (r == MSK_RES_OK)
      r = MSK_putcj(task, 0, 1.0);
    for (j = 0; j < NUMVAR && r == MSK_RES_OK; ++j)
      r = MSK_putvarbound(task,
                          j,
                          MSK BK FR.
                          -MSK_INFINITY,
                          MSK_INFINITY);
    /* Set the linear term barc_j in the objective.*/
    if (r == MSK_RES_OK)
      r = MSK_appendsparsesymmat(task,
                                 DIMBARVAR[0],
                                 5,
                                 barc_i,
                                 barc_j,
                                 barc_v,
                                 &idx);
    if (r == MSK_RES_OK)
      r = MSK_putbarcj(task, 0, 1, &idx, &falpha);
    /* Set the bounds on constraints.
      for i=1, ..., NUMCON : blc[i] <= constraint i <= buc[i] */
    for (i = 0; i < NUMCON && r == MSK_RES_OK; ++i)
      r = MSK_putconbound(task,
                                        /* Index of constraint.*/
```

```
bkc[i],
                                   /* Bound key.*/
                      blc[i],
                                   /* Numerical value of lower bound.*/
                                   /* Numerical value of upper bound.*/
                      buc[i]);
/* Input A row by row */
for (i = 0; i < NUMCON && r == MSK_RES_OK; ++i)
  r = MSK_putarow(task,
                  aptre[i] - aptrb[i],
                           + aptrb[i],
                  asub
                  aval
                           + aptrb[i]);
/* Append the conic quadratic cone */
if (r == MSK_RES_OK)
  r = MSK_appendcone(task,
                     MSK_CT_QUAD,
                     0.0,
                     3,
                     conesub);
/* Add the first row of barA */
if (r == MSK_RES_OK)
 r = MSK_appendsparsesymmat(task,
                             DIMBARVAR[0],
                             3,
                             bara_i,
                             bara_j,
                             bara_v,
                             &idx);
if (r == MSK_RES_OK)
  r = MSK_putbaraij(task, 0, 0, 1, &idx, &falpha);
/* Add the second row of barA */
if (r == MSK_RES_OK)
 r = MSK_appendsparsesymmat(task,
                             DIMBARVAR[0],
                             6,
                             bara_i + 3,
                             bara_j + 3,
                             bara_v + 3,
                             &idx);
if (r == MSK_RES_OK)
 r = MSK_putbaraij(task, 1, 0, 1, &idx, &falpha);
if (r == MSK_RES_OK)
  MSKrescodee trmcode;
  /* Run optimizer */
  r = MSK_optimizetrm(task, &trmcode);
  /* Print a summary containing information
     about the solution for debugging purposes*/
  MSK_solutionsummary(task, MSK_STREAM_MSG);
```

```
if (r == MSK_RES_OK)
         MSKsolstae solsta;
         MSK_getsolsta(task, MSK_SOL_ITR, &solsta);
         switch (solsta)
           case MSK_SOL_STA_OPTIMAL:
             xx = (MSKrealt *) MSK_calloctask(task, NUMVAR, sizeof(MSKrealt));
             barx = (MSKrealt *) MSK_calloctask(task, LENBARVAR[0],_
⇔sizeof(MSKrealt));
             MSK_getxx(task,
                        MSK_SOL_ITR,
                        xx);
             MSK_getbarxj(task,
                           MSK_SOL_ITR, /* Request the interior solution. */
                           0,
                           barx);
             printf("Optimal primal solution\n");
             for (i = 0; i < NUMVAR; ++i)
               printf("x[%d] : % e\n", i, xx[i]);
             for (i = 0; i < LENBARVAR[0]; ++i)</pre>
               printf("barx[%d]: % e\n", i, barx[i]);
             MSK_freetask(task, xx);
             MSK_freetask(task, barx);
             break;
           case MSK_SOL_STA_DUAL_INFEAS_CER:
           case MSK_SOL_STA_PRIM_INFEAS_CER:
             printf("Primal or dual infeasibility certificate found.\n");
             break;
           case MSK_SOL_STA_UNKNOWN:
             printf("The status of the solution could not be determined. Termination∪
\rightarrow code: %d.\n", trmcode);
             break;
           default:
             printf("Other solution status.");
             break;
         }
       }
       else
       {
         printf("Error while optimizing.\n");
       }
     if (r != MSK_RES_OK)
```

```
{
        /* In case of an error print error code and description. */
        char symname[MSK_MAX_STR_LEN];
        char desc[MSK_MAX_STR_LEN];
        printf("An error occurred while optimizing.\n");
        MSK_getcodedesc(r,
                        symname,
                        desc);
        printf("Error %s - '%s'\n", symname, desc);
   }
    /* Delete the task and the associated data. */
   MSK_deletetask(&task);
  /* Delete the environment and the associated data. */
 MSK_deleteenv(&env);
 return (r);
} /* main */
```

### 6.6.2 Example SDO2

We now demonstrate how to define more than one semidefinite variable using the following problem with two matrix variables and two types of constraints:

minimize 
$$\langle C_1, \overline{X}_1 \rangle + \langle C_2, \overline{X}_2 \rangle$$
  
subject to  $\langle A_1, \overline{X}_1 \rangle + \langle A_2, \overline{X}_2 \rangle = b,$   

$$\frac{(\overline{X}_2)_{01}}{\overline{X}_1, \overline{X}_2} \leq k,$$

$$\overline{X}_1, \overline{X}_2 \geq 0.$$
(6.12)

In our example  $\dim(\overline{X}_1) = 3$ ,  $\dim(\overline{X}_2) = 4$ , b = 23, k = -3 and

$$C_{1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 6 \end{bmatrix}, A_{1} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 2 \end{bmatrix},$$

$$C_{2} = \begin{bmatrix} 1 & -3 & 0 & 0 \\ -3 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, A_{2} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -3 \end{bmatrix},$$

are constant symmetric matrices.

Note that this problem does not contain any scalar variables, but they could be added in the same fashion as in Sec. 6.6.1.

Other than in Sec. 6.6.1 we don't append coefficient matrices separately but we directly input all nonzeros in each constraint and all nonzeros in the objective at once. Every term of the form  $(\overline{A}_{i,j})_{k,l}(\overline{X}_j)_{k,l}$  is determined by four indices (i,j,k,l) and a coefficient value  $v=(\overline{A}_{i,j})_{k,l}$ . Here i is the number of the constraint in which the term appears, j is the index of the semidefinite variable it involves and (k,l) is the position in that variable. This data is passed in the call to  $MSK\_putbarablocktriplet$ . Note that only the lower triangular part should be specified explicitly, that is one always has  $k \geq l$ . Semidefinite terms  $(\overline{C}_j)_{k,l}(\overline{X}_j)_{k,l}$  of the objective are specified in the same way in  $MSK\_putbarcblocktriplet$  but only include (j,k,l) and v.

For explanations of other data structures used in the example see Sec. 6.6.1.

The code representing the above problem is shown below.

Listing 6.8: Implementation of model (6.12).

```
# include <stdio.h>
# include "mosek.h"
                      /* Include the MOSEK definition file. */
static void MSKAPI printstr(void *handle,
                           const char str[])
 printf("%s", str);
} /* printstr */
int main(int argc, const char *argv[])
 MSKrescodee r;
 /* Input data */
 MSKint32t    numbarvar = 2;
 MSKint32t
              dimbarvar[] = \{3, 4\};
                                    /* Dimension of semidefinite variables⊔
→*/
 /* Objective coefficients concatenated */
 MSKint32t
              Cj[] = \{ 0, 0, 1, 1, 1, 1 \}; /* Which symmetric variable (j) */
 MSKint32t
              Ck[] = { 0, 2, 0, 1, 1, 2 };
                                           /* Which entry (k,l)->v */
 MSKint32t C1[] = \{ 0, 2, 0, 0, 1, 2 \};
             Cv[] = \{ 1.0, 6.0, 1.0, -3.0, 2.0, 1.0 \};
 MSKrealt
 /*\ \textit{Equality constraints coefficients concatenated */}
 MSKint32t Ai[] = { 0, 0, 0, 0, 0, 0 }; /* Which constraint (i = 0) */
 MSKint32t
              Aj[] = \{ 0, 0, 0, 1, 1, 1 \}; /* Which symmetric variable (j) */
             Ak[] = { 0, 2, 2, 1, 1, 3 };
                                            /* Which entry (k, l)->v */
 MSKint32t
 MSKint32t Al[] = \{0, 0, 2, 0, 1, 3\};
 MSKrealt
             Av[] = \{ 1.0, 1.0, 2.0, 1.0, -1.0, -3.0 \};
 /* The second constraint - one-term inequality */
 MSKint32t A2i = 1;
                                              /* Which constraint (i = 1) */
 MSKint32t
                                              /* Which symmetric variable (j = 1) */
             A2j = 1;
                                              /* Which entry A(1,0) = A(0,1) = 0.5 */
 MSKint32t
             A2k = 1;
 MSKint32t
            A21 = 0;
 MSKrealt
              A2v = 0.5;
 /* Constraint bounds and values */
 MSKint32t
             numcon = 2;
 MSKboundkeye bkc[] = { MSK_BK_FX, MSK_BK_UP };
 double
             blc[] = { 23.0, -MSK_INFINITY };
 double
              buc[] = \{ 23.0, -3.0 \};
 MSKint32t
            i, j, dim;
 MSKrealt
              *barx;
 MSKenv_t
              env = NULL;
 MSKtask_t
              task = NULL;
 /* Create the mosek environment. */
 r = MSK_makeenv(&env, NULL);
 if (r == MSK_RES_OK)
```

```
/* Create the optimization task. */
r = MSK_maketask(env, 0, 0, &task);
if (r == MSK_RES_OK)
{
 MSK_linkfunctotaskstream(task, MSK_STREAM_LOG, NULL, printstr);
  /* Append empty constraints.
  The constraints will initially have no bounds. */
  if (r == MSK_RES_OK)
   r = MSK_appendcons(task, numcon);
  /* Append semidefinite variables. */
  if (r == MSK_RES_OK)
   r = MSK_appendbarvars(task, numbarvar, dimbarvar);
  /* Set objective (6 nonzeros).*/
  if (r == MSK_RES_OK)
   r = MSK_putbarcblocktriplet(task, 6, Cj, Ck, Cl, Cv);
  /* Set the equality constraint (6 nonzeros).*/
  if (r == MSK_RES_OK)
   r = MSK_putbarablocktriplet(task, 6, Ai, Aj, Ak, Al, Av);
  /* Set the inequality constraint (1 nonzero).*/
  if (r == MSK_RES_OK)
   r = MSK_putbarablocktriplet(task, 1, &A2i, &A2j, &A2k, &A2l, &A2v);
  /* Set constraint bounds */
  if (r == MSK_RES_OK)
    r = MSK_putconboundslice(task, 0, 2, bkc, blc, buc);
  if (r == MSK_RES_OK)
    MSKrescodee trmcode;
    /* Run optimizer */
    r = MSK_optimizetrm(task, &trmcode);
    MSK_solutionsummary(task, MSK_STREAM_MSG);
    if (r == MSK_RES_OK)
    {
      MSKsolstae solsta;
      MSK_getsolsta(task, MSK_SOL_ITR, &solsta);
      switch (solsta)
        case MSK_SOL_STA_OPTIMAL:
          /* Retrieve the soution for all symmetric variables */
          printf("Solution (lower triangular part vectorized):\n");
          for(i = 0; i < numbarvar; i++) {</pre>
            dim = dimbarvar[i] * (dimbarvar[i] + 1) / 2;
            barx = (MSKrealt *) MSK_calloctask(task, dim, sizeof(MSKrealt));
```

```
MSK_getbarxj(task, MSK_SOL_ITR, i, barx);
               printf("X%d: ", i + 1);
               for (j = 0; j < dim; ++j)
                 printf("%.3f ", barx[j]);
               printf("\n");
               MSK_freetask(task, barx);
             }
             break;
           case MSK_SOL_STA_DUAL_INFEAS_CER:
            case MSK_SOL_STA_PRIM_INFEAS_CER:
             printf("Primal or dual infeasibility certificate found.\n");
             break;
            case MSK_SOL_STA_UNKNOWN:
             printf("The status of the solution could not be determined. Termination ∪
break;
           default:
             printf("Other solution status.");
             break;
         }
        }
        else
        {
         printf("Error while optimizing.\n");
       }
      }
      if (r != MSK_RES_OK)
        /* In case of an error print error code and description. */
        char symname[MSK_MAX_STR_LEN];
        char desc[MSK_MAX_STR_LEN];
       printf("An error occurred while optimizing.\n");
       MSK_getcodedesc(r,
                        symname,
                       desc);
       printf("Error %s - '%s'\n", symname, desc);
     }
    /* Delete the task and the associated data. */
   MSK_deletetask(&task);
 \slash * Delete the environment and the associated data. */
 MSK_deleteenv(&env);
 return (r);
} /* main */
```

## 6.7 Integer Optimization

An optimization problem where one or more of the variables are constrained to integer values is called a (mixed) integer optimization problem. **MOSEK** supports integer variables in combination with linear, quadratic and quadratically constrained and conic problems (except semidefinite). See the previous tutorials for an introduction to how to model these types of problems.

### 6.7.1 Example MILO1

We use the example

to demonstrate how to set up and solve a problem with integer variables. It has the structure of a linear optimization problem (see Sec. 6.1) except for integrality constraints on the variables. Therefore, only the specification of the integer constraints requires something new compared to the linear optimization problem discussed previously.

First, the integrality constraints are imposed using the function MSK\_putvartype:

```
for (j = 0; j < numvar && r == MSK_RES_OK; ++j)
r = MSK_putvartype(task, j, MSK_VAR_TYPE_INT);
```

Next, the example demonstrates how to set various useful parameters of the mixed-integer optimizer. See Sec. 13.4 for details.

The complete source for the example is listed Listing 6.9. Please note that when  $MSK\_getsolutionslice$  is called, the integer solution is requested by using  $MSK\_SOL\_ITG$ . No dual solution is defined for integer optimization problems.

Listing 6.9: Source code implementing problem (6.13).

```
# include <stdio.h>
#include "mosek.h" /* Include the MOSEK definition file. */
static void MSKAPI printstr(void *handle,
                            const char str[])
 printf("%s", str);
} /* printstr */
int main(int argc, char *argv[])
 const MSKint32t numvar = 2,
                  numcon = 2;
               c[] = { 1.0, 0.64 };
 double
 MSKboundkeye bkc[] = { MSK_BK_UP,
                                       MSK_BK_LO };
 double
               blc[] = { -MSK_INFINITY, -4.0 };
 double
               buc[] = { 250.0,}
                                       MSK_INFINITY };
 MSKboundkeye bkx[] = { MSK_BK_LO,
                                       MSK_BK_LO };
               blx[] = { 0.0,}
                                       0.0 \};
```

```
double
             bux[] = { MSK_INFINITY, MSK_INFINITY };
MSKint32t
             aptrb[] = { 0, 2 },
             aptre[] = { 2, 4 },
             asub[] = \{ 0, 1, 0, 
                                       1 };
double
             aval[] = { 50.0, 3.0, 31.0, -2.0 };
MSKint32t
             i, j;
             env = NULL;
MSKenv_t
MSKtask_t
             task = NULL;
MSKrescodee r;
/* Create the mosek environment. */
r = MSK_makeenv(&env, NULL);
/* Check if return code is ok. */
if (r == MSK_RES_OK)
  /* Create the optimization task. */
 r = MSK_maketask(env, 0, 0, &task);
  if (r == MSK_RES_OK)
   r = MSK_linkfunctotaskstream(task, MSK_STREAM_LOG, NULL, printstr);
  /* Append 'numcon' empty constraints.
   The constraints will initially have no bounds. */
  if (r == MSK_RES_OK)
   r = MSK_appendcons(task, numcon);
  /* Append 'numvar' variables.
  The variables will initially be fixed at zero (x=0). */
  if (r == MSK_RES_OK)
   r = MSK_appendvars(task, numvar);
  /* Optionally add a constant term to the objective. */
  if (r == MSK_RES_OK)
   r = MSK_putcfix(task, 0.0);
  for (j = 0; j < numvar && r == MSK_RES_OK; ++j)
    /* Set the linear term c_j in the objective.*/
    if (r == MSK_RES_OK)
     r = MSK_putcj(task, j, c[j]);
    /* Set the bounds on variable j.
    blx[j] <= x_j <= bux[j] */
    if (r == MSK_RES_OK)
     r = MSK_putvarbound(task,
                                       /* Index of variable.*/
                          j,
                          bkx[j],
                                       /* Bound key.*/
                          blx[j],
                                       /* Numerical value of lower bound.*/
                          bux[j]);
                                       /* Numerical value of upper bound.*/
    /* Input column j of A */
    if (r == MSK_RES_OK)
     r = MSK_putacol(task,
```

```
/* Variable (column) index.*/
                    aptre[j] - aptrb[j], /* Number of non-zeros in column j.*/
                    asub + aptrb[j], /* Pointer to row indexes of column j.*/
                    aval + aptrb[j]); /* Pointer to Values of column j.*/
}
/* Set the bounds on constraints.
  for i=1, ...,numcon : blc[i] <= constraint i <= buc[i] */</pre>
for (i = 0; i < numcon && r == MSK_RES_OK; ++i)
 r = MSK_putconbound(task,
                                   /* Index of constraint.*/
                      i,
                                  /* Bound key.*/
                      bkc[i],
                      blc[i],
                                  /* Numerical value of lower bound.*/
                      buc[i]);
                                  /* Numerical value of upper bound.*/
/* Specify integer variables. */
for (j = 0; j < numvar && r == MSK_RES_OK; ++j)
 r = MSK_putvartype(task, j, MSK_VAR_TYPE_INT);
if (r == MSK_RES_OK)
 r = MSK_putobjsense(task,
                       MSK_OBJECTIVE_SENSE_MAXIMIZE);
if (r == MSK_RES_OK)
 /* Set max solution time */
 r = MSK_putdouparam(task,
                      MSK_DPAR_MIO_MAX_TIME,
                      60.0);
if (r == MSK_RES_OK)
 MSKrescodee trmcode;
  /* Run optimizer */
 r = MSK_optimizetrm(task, &trmcode);
  /* Print a summary containing information
     about the solution for debugging purposes*/
  MSK_solutionsummary(task, MSK_STREAM_MSG);
  if (r == MSK_RES_OK)
   MSKint32t j;
    MSKsolstae solsta;
    double
             *xx = NULL;
    MSK_getsolsta(task, MSK_SOL_ITG, &solsta);
    xx = calloc(numvar, sizeof(double));
    if (xx)
    {
      switch (solsta)
        case MSK_SOL_STA_INTEGER_OPTIMAL:
          MSK_getxx(task,
```

```
MSK_SOL_ITG,
                                    /* Request the integer solution. */
                    xx);
          printf("Optimal solution.\n");
          for (j = 0; j < numvar; ++j)
            printf("x[%d]: %e\n", j, xx[j]);
          break;
        case MSK_SOL_STA_PRIM_FEAS:
          /* A feasible but not necessarily optimal solution was located. */
          MSK_getxx(task, MSK_SOL_ITG, xx);
          printf("Feasible solution.\n");
          for (j = 0; j < numvar; ++j)
            printf("x[%d]: %e\n", j, xx[j]);
          break;
        case MSK_SOL_STA_UNKNOWN:
            MSKprostae prosta;
            MSK_getprosta(task, MSK_SOL_ITG, &prosta);
            switch (prosta)
              case MSK_PRO_STA_PRIM_INFEAS_OR_UNBOUNDED:
                printf("Problem status Infeasible or unbounded\n");
                break;
              case MSK_PRO_STA_PRIM_INFEAS:
                printf("Problem status Infeasible.\n");
                break;
              case MSK_PRO_STA_UNKNOWN:
                printf("Problem status unknown. Termination code %d.\n", trmcode);
              default:
                printf("Other problem status.");
                break;
          }
          break;
        default:
          printf("Other solution status.");
          break;
      }
    }
    else
    {
      r = MSK_RES_ERR_SPACE;
    }
    free(xx);
  }
}
if (r != MSK_RES_OK)
  /* In case of an error print error code and description. */
  char symname[MSK_MAX_STR_LEN];
  char desc[MSK_MAX_STR_LEN];
  printf("An error occurred while optimizing.\n");
```

### 6.7.2 Specifying an initial solution

It is a common strategy to provide a starting feasible point (if one is known in advance) to the mixed-integer solver. This can in many cases reduce solution time.

It is not necessary to specify the whole solution. **MOSEK** will attempt to use it to speed up the computation. **MOSEK** will first try to construct a feasible solution by fixing integer variables to the values provided by the user (rounding if necessary) and optimizing over the continuous variables. The outcome of this process can be inspected via information items <code>MSK\_IINF\_MIO\_CONSTRUCT\_SOLUTION</code> and <code>MSK\_DINF\_MIO\_CONSTRUCT\_SOLUTION\_OBJ</code>, and via the Construct solution objective entry in the log. We concentrate on a simple example below.

maximize 
$$7x_0 + 10x_1 + x_2 + 5x_3$$
  
subject to  $x_0 + x_1 + x_2 + x_3 \le 2.5$   
 $x_0, x_1, x_2 \in \mathbb{Z}$   
 $x_0, x_1, x_2, x_3 \ge 0$  (6.14)

Solution values can be set using  $MSK\_putsolution$ .

Listing 6.10: Implementation of problem (6.14) specifying an initial solution

The log output from the optimizer will in this case indicate that the inputted values were used to construct an initial feasible solution:

```
Construct solution objective : 1.950000000000e+01
```

The same information can be obtained from the API:

Listing 6.11: Retrieving information about usage of initial solution

```
MSK_getintinf(task, MSK_IINF_MIO_CONSTRUCT_SOLUTION, &constr);
MSK_getdouinf(task, MSK_DINF_MIO_CONSTRUCT_SOLUTION_OBJ, &constr_obj);
printf("Initial solution utilization: %d\nInitial solution objective: %.3f\n", \_
\topconstr, constr_obj);
```

### 6.7.3 Example MICO1

Integer variables can also be used arbitrarily in conic problems (except semidefinite). We refer to the previous tutorials for how to set up a conic optimization problem. Here we present sample code that sets up a simple optimization problem:

minimize 
$$x^2 + y^2$$
  
subject to  $x \ge e^y + 3.8$ ,  
 $x, y$  integer. (6.15)

The canonical conic formulation of (6.15) suitable for Optimizer API for C is

minimize 
$$x_0$$
  
subject to  $(x_0, x_1, x_2) \in \mathcal{Q}^3$   $(x_0 \ge \sqrt{x_1^2 + x_2^2})$   
 $(x_3, x_4, x_5) \in K_{\text{exp}}$   $(x_3 \ge x_4 \exp(x_5/x_4))$   
 $-x_1 + x_3 = -3.8$   $x_4 = 1$   
 $x_2 - x_5 = 0$   
 $x_1, x_2$  integer. (6.16)

Listing 6.12: Implementation of problem (6.16).

```
int main(int argc, char *argv[])
 MSKboundkeye bkc[] = { MSK_BK_FX, MSK_BK_FX, MSK_BK_FX };
               bc[] = { -3.8, 1, 0 };
 MSKvariabletypee vart[] = { MSK_VAR_TYPE_INT, MSK_VAR_TYPE_INT };
                  intsub[] = { 1, 2 };
 MSKint32t
 MSKint32t
               asubi[] = \{0, 0, 1, 2, 2\},
               asubj[] = \{1, 3, 4, 2, 5\};
               aval[] = \{-1, 1, 1, 1, -1\};
 MSKrealt
 MSKint32t
               i, j;
 MSKenv_t
               env = NULL;
               task = NULL;
 MSKtask_t
 MSKrescodee r, trm;
 r = MSK_makeenv(&env, NULL);
 if (r == MSK_RES_OK)
   r = MSK_maketask(env, 0, 0, &task);
   if (r == MSK_RES_OK)
     r = MSK_linkfunctotaskstream(task, MSK_STREAM_LOG, NULL, printstr);
   if (r == MSK_RES_OK)
     r = MSK_appendvars(task, 6);
```

```
if (r == MSK_RES_OK)
   r = MSK_appendcons(task, 3);
  if (r == MSK_RES_OK)
   r = MSK_putvarboundsliceconst(task, 0, 6, MSK_BK_FR, -0.0, 0.0);
  /* Integrality constraints */
  if (r == MSK_RES_OK)
   r = MSK_putvartypelist(task, 2, intsub, vart);
  /* Linear part of the problem */
  if (r == MSK_RES_OK)
   r = MSK_putaijlist(task, 5, asubi, asubj, aval);
  if (r == MSK_RES_OK)
   r = MSK_putconboundslice(task, 0, 3, bkc, bc, bc);
  /* Objective */
  if (r == MSK_RES_OK)
   r = MSK_putobjsense(task, MSK_OBJECTIVE_SENSE_MINIMIZE);
  if (r == MSK_RES_OK)
   r = MSK_putcj(task, 0, 1);
  /* Conic part of the problem */
  if (r == MSK_RES_OK)
   r = MSK_appendconeseq(task, MSK_CT_QUAD, 0, 3, 0);
  if (r == MSK_RES_OK)
   r = MSK_appendconeseq(task, MSK_CT_PEXP, 0, 3, 3);
  /* Optimize the problem */
  if (r == MSK_RES_OK)
   r = MSK_optimizetrm(task, &trm);
  if (r == MSK_RES_OK)
   r = MSK_solutionsummary(task, MSK_STREAM_MSG);
  if (r == MSK_RES_OK)
   MSKrealt xx[] = \{0, 0\};
   r = MSK_getxxslice(task, MSK_SOL_ITG, 1, 3, xx);
   if (r == MSK_RES_OK)
      printf("x = \%.2f, y = \%.2f\n", xx[0], xx[1]);
  if (task) MSK_deletetask(&task);
if (env) MSK_deleteenv(&env);
return r;
```

Error and solution status handling were omitted for readability.

## 6.8 Geometric Programming

Geometric programs (GP) are a particular class of optimization problems which can be expressed in special polynomial form as positive sums of generalized monomials. More precisely, a geometric problem in canonical form is

minimize 
$$f_0(x)$$
  
subject to  $f_i(x) \le 1$ ,  $i = 1, ..., m$ ,  $x_j > 0$ ,  $j = 1, ..., n$ , 
$$(6.17)$$

where each  $f_0, \ldots, f_m$  is a posynomial, that is a function of the form

$$f(x) = \sum_{k} c_{k} x_{1}^{\alpha_{k1}} x_{2}^{\alpha_{k2}} \cdots x_{n}^{\alpha_{kn}}$$

with arbitrary real  $\alpha_{ki}$  and  $c_k > 0$ . The standard way to formulate GPs in convex form is to introduce a variable substitution

$$x_i = \exp(y_i).$$

Under this substitution all constraints in a GP can be reduced to the form

$$\log(\sum_{k} \exp(a_k^T y + b_k)) \le 0 \tag{6.18}$$

involving a log-sum-exp bound. Moreover, constraints involving only a single monomial in x can be even more simply written as a linear inequality:

$$a_k^T y + b_k < 0$$

We refer to the **MOSEK** Modeling Cookbook and to [BKVH07] for more details on this reformulation. A geometric problem formulated in convex form can be entered into **MOSEK** with the help of exponential cones.

### 6.8.1 Example GP1

The following problem comes from [BKVH07]. Consider maximizing the volume of a  $h \times w \times d$  box subject to upper bounds on the area of the floor and of the walls and bounds on the ratios h/w and d/w:

maximize 
$$hwd$$
  
subject to  $2(hw + hd) \le A_{\text{wall}},$   
 $wd \le A_{\text{floor}},$   
 $\alpha \le h/w \le \beta,$   
 $\gamma \le d/w \le \delta.$  (6.19)

The decision variables in the problem are h, w, d. We make a substitution

$$h = \exp(x), w = \exp(y), d = \exp(z)$$

after which (6.19) becomes

maximize 
$$x + y + z$$
  
subject to  $\log(\exp(x + y + \log(2/A_{\text{wall}})) + \exp(x + z + \log(2/A_{\text{wall}}))) \le 0,$   
 $y + z \le \log(A_{\text{floor}}),$  (6.20)  
 $\log(\alpha) \le x - y \le \log(\beta),$   
 $\log(\gamma) \le z - y \le \log(\delta).$ 

Next, we demonstrate how to implement a log-sum-exp constraint (6.18). It can be written as:

$$u_k \ge \exp(a_k^T y + b_k), \quad (\text{equiv. } (u_k, 1, a_k^T y + b_k) \in K_{\text{exp}}),$$

$$\sum_k u_k = 1. \tag{6.21}$$

This presentation requires one extra variable  $u_k$  for each monomial appearing in the original posynomial constraint. Another fixed variable  $t_k = 1$  stands for the second entry in the exponential cone.

Listing 6.13: Implementation of log-sum-exp as in (6.21).

```
// Add a single log-sum-exp constraint sum(log(exp(z_i))) \le 0
   // Assume numExp variable triples are ordered as (u0, t0, z0, u1, t1, z1...)
   // starting from variable with index expStart
                *val = (double*) calloc(numExp, sizeof(double));
   double
                *sub = (int*) calloc(numExp, sizeof(int));
   MSKconetypee *ct = (MSKconetypee*) calloc(numExp, sizeof(MSKconetypee));
                *len = (int*) calloc(numExp, sizeof(int));
   // sum(u_i) = 1 as constraint number c, u_i unbounded
   for(i = 0; i < numExp; i++)
     { sub[i] = expStart + 3*i; val[i] = 1.0; }
   if (r == MSK_RES_OK)
     r = MSK_putarow(task, c, numExp, sub, val);
   if (r == MSK_RES_OK)
     r = MSK_putconbound(task, c, MSK_BK_FX, 1.0, 1.0);
   if (r == MSK_RES_OK)
     r = MSK_putvarboundlistconst(task, numExp, sub, MSK_BK_FR, -MSK_INFINITY, MSK_
→INFINITY);
   // z_i unbounded
   for(i = 0; i < numExp; i++) sub[i] = expStart + 3*i + 2;
   if (r == MSK_RES_OK)
     r = MSK_putvarboundlistconst(task, numExp, sub, MSK_BK_FR, -MSK_INFINITY, MSK_
→INFINITY);
   // t_i = 1
   for(i = 0; i < numExp; i++) sub[i] = expStart + 3*i + 1;</pre>
   if (r == MSK_RES_OK)
     r = MSK_putvarboundlistconst(task, numExp, sub, MSK_BK_FX, 1, 1);
   // Every triple is in an exponential cone
   for(i = 0; i < numExp; i++)
     { ct[i] = MSK_CT_PEXP; val[i] = 0.0; len[i] = 3; }
   if (r == MSK_RES_OK)
     r = MSK_appendconesseq(task, numExp, ct, val, len, expStart);
   free(val); free(sub); free(ct); free(len);
```

We can now use this function to assemble all constraints in the model. The linear part of the problem is entered as in Sec. 6.1.

Listing 6.14: Source code solving problem (6.20).

```
asubj[] = \{1, 2, 0, 1, 2, 1\};
 const int
 const int
                         = 6;
                    alen
                   aval[] = \{1.0, 1.0, 1.0, -1.0, 1.0, -1.0\};
 const double
 const MSKboundkeye bkc[] = {MSK_BK_UP, MSK_BK_RA, MSK_BK_RA};
 const double blc[]
                           = {-MSK_INFINITY, log(alpha), log(gamma)};
 const double
                   buc[] = {log(Af), log(beta), log(delta)};
 // Linear part setting up slack variables
 // for the linear expressions appearing inside exps
 // x_5 - x - y = log(2/Awall)
 // x_8 - x - z = log(2/Awall)
 // The slack indexes are convenient for defining exponential cones, see later
 const int a2subi[] = {3, 3, 3, 4, 4, 4};
 const int
                   a2subj[] = \{5, 0, 1, 8, 0, 2\};
                            = 6;
 const int
                   a21en
 const double a2val[] = {1.0, -1.0, -1.0, 1.0, -1.0, -1.0};
 const MSKboundkeye b2kc[] = {MSK_BK_FX, MSK_BK_FX};
                  b2luc[] = \{log(2/Aw), log(2/Aw)\};
 const double
 MSKenv_t
                  env = NULL;
 MSKtask_t
                   task = NULL;
 MSKrescodee
                  r, trmcode;
 MSKsolstae
                   solsta;
 MSKint32t
                   i:
                    *xyz = (double*) calloc(numvar, sizeof(double));
 double
 r = MSK_makeenv(&env, NULL);
 if (r == MSK_RES_OK)
   r = MSK_maketask(env, 0, 0, &task);
 if (r == MSK_RES_OK)
   r = MSK_linkfunctotaskstream(task, MSK_STREAM_LOG, NULL, printstr);
 if (r == MSK_RES_OK)
   r = MSK_appendvars(task, numvar + 3*numExp);
 if (r == MSK_RES_OK)
   r = MSK_appendcons(task, numLinCon + numExp + 1);
 // Objective is the sum of three first variables
 if (r == MSK_RES_OK)
   r = MSK_putobjsense(task, MSK_OBJECTIVE_SENSE_MAXIMIZE);
 if (r == MSK_RES_OK)
   r = MSK_putcslice(task, 0, numvar, cval);
 if (r == MSK_RES_OK)
   r = MSK_putvarboundsliceconst(task, 0, numvar, MSK_BK_FR, -MSK_INFINITY, +MSK_
→INFINITY);
 // Add the three linear constraints
 if (r == MSK_RES_OK)
  r = MSK_putaijlist(task, alen, asubi, asubj, aval);
 if (r == MSK_RES_OK)
   r = MSK_putconboundslice(task, 0, numvar, bkc, blc, buc);
 // Add linear constraints for the expressions appearing in exp(...)
```

```
if (r == MSK_RES_OK)
   r = MSK_putaijlist(task, a2len, a2subi, a2subj, a2val);
 if (r == MSK_RES_OK)
   r = MSK_putconboundslice(task, numLinCon, numLinCon+numExp, b2kc, b2luc, b2luc);
   int c = numLinCon + numExp;
   int expStart = numvar;
   // Add a single log-sum-exp constraint sum(log(exp(z_i))) \le 0
   // Assume numExp variable triples are ordered as (u0, t0, z0, u1, t1, z1...)
   // starting from variable with index expStart
   double
                *val = (double*) calloc(numExp, sizeof(double));
                *sub = (int*) calloc(numExp, sizeof(int));
   int
   MSKconetypee *ct = (MSKconetypee*) calloc(numExp, sizeof(MSKconetypee));
                *len = (int*) calloc(numExp, sizeof(int));
   // sum(u_i) = 1 as constraint number c, u_i unbounded
   for(i = 0; i < numExp; i++)
     { sub[i] = expStart + 3*i; val[i] = 1.0; }
   if (r == MSK_RES_OK)
     r = MSK_putarow(task, c, numExp, sub, val);
   if (r == MSK_RES_OK)
     r = MSK_putconbound(task, c, MSK_BK_FX, 1.0, 1.0);
   if (r == MSK_RES_OK)
     r = MSK_putvarboundlistconst(task, numExp, sub, MSK_BK_FR, -MSK_INFINITY, MSK_
→INFINITY);
   // z_i unbounded
   for(i = 0; i < numExp; i++) sub[i] = expStart + 3*i + 2;</pre>
   if (r == MSK_RES_OK)
     r = MSK_putvarboundlistconst(task, numExp, sub, MSK_BK_FR, -MSK_INFINITY, MSK_
→INFINITY);
   // t_i = 1
   for(i = 0; i < numExp; i++) sub[i] = expStart + 3*i + 1;
   if (r == MSK_RES_OK)
    r = MSK_putvarboundlistconst(task, numExp, sub, MSK_BK_FX, 1, 1);
   // Every triple is in an exponential cone
   for(i = 0; i < numExp; i++)
     { ct[i] = MSK_CT_PEXP; val[i] = 0.0; len[i] = 3; }
   if (r == MSK_RES_OK)
     r = MSK_appendconesseq(task, numExp, ct, val, len, expStart);
   free(val); free(sub); free(ct); free(len);
 }
 // Solve and map to original h, w, d
 if (r == MSK_RES_OK)
   r = MSK_optimizetrm(task, &trmcode);
 if (r == MSK_RES_OK)
   MSK_getsolsta(task, MSK_SOL_ITR, &solsta);
 if (solsta == MSK_SOL_STA_OPTIMAL)
```

```
if (r == MSK_RES_OK)
    r = MSK_getxxslice(task, MSK_SOL_ITR, 0, numvar, xyz);
    for(i = 0; i < numvar; i++) hwd[i] = exp(xyz[i]);
}
else
{
    printf("Solution not optimal, termination code %d.\n", trmcode);
    r = trmcode;
}
free(xyz);
return r;
}</pre>
```

Given sample data we obtain the solution h, w, d as follows:

Listing 6.15: Sample data for problem (6.19).

```
int main()
{
 const double Aw
                     = 200.0;
                     = 50.0;
 const double Af
 const double alpha = 2.0;
 const double beta = 10.0;
  const double gamma = 2.0;
 const double delta = 10.0;
 MSKrescodee r;
 double
               hwd[3];
 r = max_volume_box(Aw, Af, alpha, beta, gamma, delta, hwd);
 printf("Response code: %d\n", r);
 if (r == MSK_RES_OK)
   printf("Solution h=%.4f w=%.4f d=%.4f\n", hwd[0], hwd[1], hwd[2]);
 return r;
```

# 6.9 Library of basic functions

This section contains a library of small models of basic functions frequently appearing in optimization models. It is essentially an implementation of the mathematical models from the **MOSEK** Modeling Cookbook using Optimizer API for C. These short code snippets can be seen as illustrative examples, can be copy-pasted to other code, and can even be directly called when assembling optimization models as we show in Sec. 6.9.6 (although this may be more suitable for prototyping; also note that additional variables and constraints will be introduced and there is no error checking).

## 6.9.1 Variable and constraint management

### **Append variables**

Adds a number of new variables. Returns the index of the first variable in the sequence.

Listing 6.16: New variables.

```
int msk_newvar(MSKtask_t task, int num) { // free
 int i, v;
 MSK_getnumvar(task, &v);
 MSK_appendvars(task, num);
 for(i=0; i<num; i++)</pre>
    MSK_putvarbound(task, v+i, MSK_BK_FR, -inf, inf);
 return v;
int msk_newvar_fx(MSKtask_t task, int num, double val) { // fixed
 int i, v;
 MSK_getnumvar(task, &v);
 MSK_appendvars(task, num);
 for(i=0; i<num; i++)</pre>
    MSK_putvarbound(task, v+i, MSK_BK_FX, val, val);
 return v;
int msk_newvar_bin(MSKtask_t task, int num) { // binary
 int i, v;
 MSK_getnumvar(task, &v);
 MSK_appendvars(task, num);
 for(i=0; i<num; i++) {</pre>
    MSK_putvarbound(task, v+i, MSK_BK_RA, 0.0, 1.0);
    MSK_putvartype(task, v+i, MSK_VAR_TYPE_INT);
 }
 return v;
```

### Variable duplication

Declares equality of two variables, or returns an index of a new duplicate of an existing variable.

Listing 6.17: Duplicate variables.

```
// x = y
void msk_equal(MSKtask_t task, int x, int y) {
  int c;
  c = msk_newcon(task, 1);
  MSK_putaij(task, c, x, 1.0);
  MSK_putaij(task, c, y, -1.0);
  MSK_putconbound(task, c, MSK_BK_FX, 0.0, 0.0);
}
int msk_dup(MSKtask_t task, int x) {
  int y;
  y = msk_newvar(task, 1);
  msk_equal(task, x, y);
  return y;
}
```

### **Append constraints**

Adds a number of new constraints. Returns the index of the first constraint in the sequence.

Listing 6.18: New constraints.

```
int msk_newcon(MSKtask_t task, int num) {
  int c;
  MSK_getnumcon(task, &c);
  MSK_appendcons(task, num);
  return c;
}
```

## 6.9.2 Linear operations

### Absolute value

 $t \ge |x|$ 

Listing 6.19: Absolute value.

```
// t >= /x/
void msk_abs(MSKtask_t task, int t, int x) {
   int c;
   c = msk_newcon(task, 2);
   MSK_putaij(task, c, t, 1.0);
   MSK_putaij(task, c, x, 1.0);
   MSK_putconbound(task, c, MSK_BK_LO, 0.0, inf);
   MSK_putaij(task, c+1, t, 1.0);
   MSK_putaij(task, c+1, x, -1.0);
   MSK_putaij(task, c+1, x, -1.0);
   MSK_putconbound(task, c+1, MSK_BK_LO, 0.0, inf);
}
```

#### 1-norm

 $t \geq \sum_{i} |x_i|$ 

### Listing 6.20: 1-norm.

```
// t >= sum( |x_i| ), x is a list of variables
void msk_norm1(MSKtask_t task, int t, int n, int* x) {
   int i, c, u;
   u = msk_newvar(task, n);
   for(i=0; i<n; i++) msk_abs(task, u+i, x[i]);
   c = msk_newcon(task, 1);
   for(i=0; i<n; i++) MSK_putaij(task, c, u+i, -1.0);
   MSK_putaij(task, c, t, 1.0);
   MSK_putconbound(task, c, MSK_BK_LO, 0.0, inf);
}</pre>
```

## 6.9.3 Quadratic and power operations

### Square

 $t > x^2$ 

### Listing 6.21: Square.

```
// t >= x^2
void msk_sq(MSKtask_t task, int t, int x) {
  int submem[3];
  submem[0] = msk_newvar_fx(task, 1, 0.5), submem[1] = t, submem[2] = x;
  MSK_appendcone(task, MSK_CT_RQUAD, 0.0, 3, submem);
}
```

### 2-norm

 $t \ge \sqrt{\sum_i x_i^2}$ 

### Listing 6.22: 2-norm.

```
// t >= sqrt(x_1^2 + ... + x_n^2) where x is a list of variables
void msk_norm2(MSKtask_t task, int t, int n, int* x) {
  int* submem;
  int i;
  submem = (int*) MSK_calloctask(task, n+1, sizeof(int));
```

```
submem[0] = t;
for(i=0; i<n; i++) submem[i+1] = x[i];
MSK_appendcone(task, MSK_CT_QUAD, 0.0, n+1, submem);
MSK_freetask(task, (void*)submem);
}</pre>
```

### **Powers**

 $t \ge |x|^p, p > 1$ 

## Listing 6.23: Power.

```
// t >= /x/^p (where p>1)
void msk_pow(MSKtask_t task, int t, int x, double p) {
  int submem[3];
  submem[0] = t, submem[1] = msk_newvar_fx(task, 1, 1.0), submem[2] = x;
  MSK_appendcone(task, MSK_CT_PPOW, 1.0/p, 3, submem);
}
```

 $t \ge 1/x^p, \ x > 0, \ p > 0$ 

## Listing 6.24: Power reciprocal.

```
// t >= 1/x^p, x>0 (where p>0)
void msk_pow_inv(MSKtask_t task, int t, int x, double p) {
  int submem[3];
  submem[0] = t, submem[1] = x, submem[2] = msk_newvar_fx(task, 1, 1.0);
  MSK_appendcone(task, MSK_CT_PPOW, 1.0/(1.0+p), 3, submem);
}
```

### p-norm

```
t \ge (\sum_i |x_i|^p)^{1/p}, \, p > 1
```

## Listing 6.25: p-norm.

```
// t >= \/x\/_p (where p>1), x is a list of variables
void msk_pnorm(MSKtask_t task, int t, int n, int* x, double p) {
  int i, r, c, submem[3];
  r = msk_newvar(task, n);
  for(i=0; i<n; i++) {
    submem[0] = t, submem[1] = r+i, submem[2] = x[i];
    MSK_appendcone(task, MSK_CT_PPOW, 1.0-1.0/p, 3, submem);
  }
  c = msk_newcon(task, 1);
  for(i=0; i<n; i++)
    MSK_putaij(task, c, r+i, -1.0);
  MSK_putaij(task, c, t, 1.0);
  MSK_putconbound(task, c, MSK_BK_FX, 0.0, 0.0);
}</pre>
```

### Geometric mean

```
t \leq (x_1 \cdot \dots \cdot x_n)^{1/n}, x_i > 0
```

Listing 6.26: Geometric mean.

```
// /t/ <= (x_1...x_n)^(1/n), x_i>=0, x is a list of variables of Length >= 1
void msk_geo_mean(MSKtask_t task, int t, int n, int* x) {
   if (n==1) msk_abs(task, x[0], t);
   else {
     int t2, submem[3];
     t2 = msk_newvar(task, 1);
     submem[0] = t2, submem[1] = x[n-1], submem[2] = t;
     MSK_appendcone(task, MSK_CT_PPOW, 1.0-1.0/n, 3, submem);
     msk_geo_mean(task, msk_dup(task, t2), n-1, x);
   }
}
```

## 6.9.4 Exponentials and logarithms

### log

```
t \le \log x, \ x > 0
```

### Listing 6.27: Logarithm.

```
// t <= log(x), x>=0
void msk_log(MSKtask_t task, int t, int x) {
  int submem[3];
  submem[0] = x, submem[1] = msk_newvar_fx(task, 1, 1.0), submem[2] = t;
  MSK_appendcone(task, MSK_CT_PEXP, 0.0, 3, submem);
}
```

### exp

 $t \ge e^x$ 

### Listing 6.28: Exponential.

```
// t >= exp(x)
void msk_exp(MSKtask_t task, int t, int x) {
  int submem[3];
  submem[0] = t, submem[1] = msk_newvar_fx(task, 1, 1.0), submem[2] = x;
  MSK_appendcone(task, MSK_CT_PEXP, 0.0, 3, submem);
}
```

### **Entropy**

 $t \ge x \log x, \ x > 0$ 

### Listing 6.29: Entropy.

```
// t >= x * log(x), x>=0
void msk_ent(MSKtask_t task, int t, int x) {
   int v, c, submem[3];
   v = msk_newvar(task, 1);
   c = msk_newcon(task, 1);
   MSK_putaij(task, c, v, 1.0);
   MSK_putaij(task, c, t, 1.0);
   MSK_putconbound(task, c, MSK_BK_FX, 0.0, 0.0);
   submem[0] = msk_newvar_fx(task, 1, 1.0), submem[1] = x, submem[2] = v;
   MSK_appendcone(task, MSK_CT_PEXP, 0.0, 3, submem);
}
```

### Relative entropy

 $t \ge x \log x/y, \ x, y > 0$ 

Listing 6.30: Relative entropy.

```
// t >= x * log(x/y), x,y>=0
void msk_relent(MSKtask_t task, int t, int x, int y) {
   int v, c, submem[3];
   v = msk_newvar(task, 1);
   c = msk_newcon(task, 1);
   MSK_putaij(task, c, v, 1.0);
   MSK_putaij(task, c, t, 1.0);
   MSK_putconbound(task, c, MSK_BK_FX, 0.0, 0.0);
   submem[0] = y, submem[1] = x, submem[2] = v;
   MSK_appendcone(task, MSK_CT_PEXP, 0.0, 3, submem);
}
```

### Log-sum-exp

```
\log \sum_{i} e^{x_i} \le t
```

Listing 6.31: Log-sum-exp.

```
// log(sum_i(exp(x_i))) \le t, where x is a list of variables
void msk_logsumexp(MSKtask_t task, int t, int n, int* x) {
 int z, u, c, s, i;
 u = msk_newvar(task, n);
 z = msk_newvar(task, n);
 for(i=0; i<n; i++) msk_exp(task, u+i, z+i);</pre>
 c = msk_newcon(task, n);
 for(i=0; i<n; i++) {
    MSK_putaij(task, c+i, x[i], 1.0);
    MSK_putaij(task, c+i, t,
    MSK_putaij(task, c+i, z+i, -1.0);
    MSK_putconbound(task, c+i, MSK_BK_FX, 0.0, 0.0);
 }
 s = msk_newcon(task, 1);
 for(i=0; i<n; i++) MSK_putaij(task, s, u+i, 1.0);</pre>
 MSK_putconbound(task, s, MSK_BK_UP, -inf, 1.0);
```

## 6.9.5 Integer Modeling

### Semicontinuous variable

```
x \in \{0\} \cup [a, b], b > a > 0
```

Listing 6.32: Semicontinuous variable.

```
// x = 0 or a <= x <= b
void msk_semicontinuous(MSKtask_t task, int x, double a, double b) {
  int u, c;
  u = msk_newvar_bin(task, 1);
  c = msk_newcon(task, 2);
  MSK_putaij(task, c, x, 1.0);
  MSK_putaij(task, c, u, -a);
  MSK_putconbound(task, c, MSK_BK_LO, 0.0, inf);
  MSK_putaij(task, c+1, x, 1.0);
  MSK_putaij(task, c+1, u, -b);
  MSK_putaij(task, c+1, u, -b);
  MSK_putconbound(task, c+1, MSK_BK_UP, -inf, 0.0);
}</pre>
```

### Indicator variable

 $x \neq 0 \implies t = 1$ . We assume x is a priori normalized so  $|x_i| \leq 1$ .

### Listing 6.33: Indicator variable.

```
// x!=0 implies t=1. Assumes that |x|<=1 in advance.
int msk_indicator(MSKtask_t task, int x) {
  int t;
  t = msk_newvar_bin(task, 1);
  msk_abs(task, t, x);
  return t;
}</pre>
```

### Logical OR

At least one of the conditions is true.

Listing 6.34: Logical OR.

```
// x OR y, where x, y are binary
void msk_logic_or(MSKtask_t task, int x, int y) {
  int c;
  c = msk_newcon(task, 1);
  MSK_putaij(task, c, x, 1.0);
  MSK_putaij(task, c, y, 1.0);
  MSK_putconbound(task, c, MSK_BK_LO, 1.0, inf);
}
// x_1 OR ... OR x_n, where x is sequence of variables
void msk_logic_or_vect(MSKtask_t task, int n, int* x) {
  int c, i;
  c = msk_newcon(task, 1);
  for(i=0; i<n; i++) MSK_putaij(task, c, x[i], 1.0);
  MSK_putconbound(task, c, MSK_BK_LO, 1.0, inf);
}</pre>
```

### **Logical NAND**

At most one of the conditions is true (also known as SOS1).

### Listing 6.35: Logical NAND.

```
// at most one of x_1,...,x_n, where x is a binary vector (SOS1 constraint)
void msk_logic_sos1(MSKtask_t task, int n, int* x) {
   int c, i;
   c = msk_newcon(task, 1);
   for(i=0; i<n; i++) MSK_putaij(task, c, x[i], 1.0);
   MSK_putconbound(task, c, MSK_BK_UP, -inf, 1.0);
}
// NOT(x AND y), where x, y are binary
void msk_logic_nand(MSKtask_t task, int x, int y) {
   int c;
   c = msk_newcon(task, 1);
   MSK_putaij(task, c, x, 1.0);
   MSK_putaij(task, c, y, 1.0);
   MSK_putconbound(task, c, MSK_BK_UP, -inf, 1.0);
}</pre>
```

### Cardinality bound

At most k of the continuous variables are nonzero. We assume x is a priori normalized so  $|x_i| \leq 1$ .

### Listing 6.36: Cardinality bound.

```
// At most k of entries in x are nonzero, assuming in advance that |x_i|<=1.
void msk_card(MSKtask_t task, int n, int* x, int k) {
  int t, i, c;
  t = msk_newvar_bin(task, n);
  for(i=0; i<n; i++) msk_abs(task, t+i, x[i]);
  c = msk_newcon(task, 1);
  for(i=0; i<n; i++) MSK_putaij(task, c, t+i, 1.0);
  MSK_putconbound(task, c, MSK_BK_UP, -inf, k);
}</pre>
```

## 6.9.6 Model assembly example

We now demonstrate how to quickly build a simple optimization model for the problem

maximize 
$$-\sqrt{x^2 + y^2} + \log y - x^{1.5}$$
, subject to  $x \ge y + 3$ , (6.22)

or equivalently

$$\begin{array}{ll} \text{maximize} & -t_0+t_1-t_2, \\ \text{subject to} & x \geq y+3, \\ & t_0 \geq \sqrt{x^2+y^2}, \\ & t_1 \leq \log y, \\ & t_2 \geq x^{1.5}. \end{array}$$

Listing 6.37: Modeling (6.22).

```
MSK_makeemptytask(env, &task);
x = msk_newvar(task, 1);
y = msk_newvar(task, 1);
t = msk_newvar(task, 3);
c = msk_newcon(task, 1);
MSK_putaij(task, c, x, 1.0);
MSK_putaij(task, c, y, -1.0);
MSK_putconbound(task, c, MSK_BK_LO, 3.0, inf);
sub[0] = x, sub[1] = y;
msk_norm2(task, t+0, 2, sub);
msk_log (task, t+1, msk_dup(task, y));
msk_pow (task, t+2, msk_dup(task, x), 1.5);
sub[0] = t, sub[1] = t+1, sub[2] = t+2;
cj[0] = -1.0, cj[1] = 1.0, cj[2] = -1.0;
MSK_putclist(task, 3, sub, cj);
MSK_putobjsense(task, MSK_OBJECTIVE_SENSE_MAXIMIZE);
```

# 6.10 Problem Modification and Reoptimization

Often one might want to solve not just a single optimization problem, but a sequence of problems, each differing only slightly from the previous one. This section demonstrates how to modify and re-optimize an existing problem.

The example we study is a simple production planning model.

Problem modifications regarding variables, cones, objective function and constraints can be grouped in categories:

- add/remove,
- coefficient modifications,
- bounds modifications.

Especially removing variables and constraints can be costly. Special care must be taken with respect to constraints and variable indexes that may be invalidated.

Depending on the type of modification, **MOSEK** may be able to optimize the modified problem more efficiently exploiting the information and internal state from the previous execution. After optimization, the solution is always stored internally, and is available before next optimization. The former optimal solution may be still feasible, but no longer optimal; or it may remain optimal if the modification of the objective function was small. This special case is discussed in Sec. 14.3.

In general, **MOSEK** exploits dual information and availability of an optimal basis from the previous execution. The simplex optimizer is well suited for exploiting an existing primal or dual feasible solution. Restarting capabilities for interior-point methods are still not as reliable and effective as those for the simplex algorithm. More information can be found in Chapter 10 of the book [Chyatal83].

Parameter settings (see Sec. 7.4) can also be changed between optimizations.

## 6.10.1 Example: Production Planning

A company manufactures three types of products. Suppose the stages of manufacturing can be split into three parts: Assembly, Polishing and Packing. In the table below we show the time required for each stage as well as the profit associated with each product.

	Product no.	Assembly (minutes)	Polishing (minutes)	Packing (minutes)	Profit (\$)
	0	2	3	2	1.50
	1	4	2	3	2.50
ĺ	2	3	3	2	3.00

With the current resources available, the company has 100,000 minutes of assembly time, 50,000 minutes of polishing time and 60,000 minutes of packing time available per year. We want to know how many items of each product the company should produce each year in order to maximize profit?

Denoting the number of items of each type by  $x_0, x_1$  and  $x_2$ , this problem can be formulated as a linear optimization problem:

maximize 
$$1.5x_0 + 2.5x_1 + 3.0x_2$$
  
subject to  $2x_0 + 4x_1 + 3x_2 \le 100000$ ,  
 $3x_0 + 2x_1 + 3x_2 \le 50000$ ,  
 $2x_0 + 3x_1 + 2x_2 \le 60000$ , (6.23)

and

$$x_0, x_1, x_2 \ge 0.$$

Code in Listing 6.38 loads and solves this problem.

Listing 6.38: Setting up and solving problem (6.23)

```
numvar = 3.
MSKint32t
                 numcon = 3;
MSKint32t
double
                        = \{1.5, 2.5, 3.0\};
                 ptrb[] = \{0, 3, 6\},\
MSKint32t
                 ptre[] = {3, 6, 9},
                 asub[] = \{ 0, 1, 2, \}
                             0, 1, 2,
                             0, 1, 2
                 aval[] = { 2.0, 3.0, 2.0, }
double
                            4.0, 2.0, 3.0,
                             3.0, 3.0, 2.0
                           };
MSKboundkeye
                        = {MSK_BK_UP, MSK_BK_UP, MSK_BK_UP
                 bkc[]
                                                                 };
double
                 blc[]
                        = { -MSK_INFINITY, -MSK_INFINITY, -MSK_INFINITY};
double
                 buc[]
                        = \{100000, 50000, 60000\};
                        = {MSK_BK_LO,
                                            MSK_BK_LO,
                                                           MSK_BK_LO};
MSKboundkeye
                 bkx[]
double
                 blx[]
                        = \{0.0,
                                            0.0,
                                                           0.0,\};
double
                 bux[]
                        = { +MSK_INFINITY, +MSK_INFINITY, +MSK_INFINITY};
```

```
double
               *xx = NULL;
MSKenv_t
               env;
MSKtask t
               task;
               varidx, conidx;
MSKint32t
MSKrescodee
               r,lr;
/* Create the mosek environment. */
r = MSK_makeenv(&env,DEBUG ? "" : NULL);
if (r == MSK_RES_OK)
  /* Create the optimization task. */
  r = MSK_maketask(env, numcon, numvar, &task);
  /* Append the constraints. */
  if (r == MSK_RES_OK)
   r = MSK_appendcons(task, numcon);
  /* Append the variables. */
  if (r == MSK_RES_OK)
   r = MSK_appendvars(task, numvar);
  /* Put C. */
  if (r == MSK_RES_OK)
   r = MSK_putcfix(task, 0.0);
  if (r == MSK_RES_OK)
    for (j = 0; j < numvar; ++j)
      r = MSK_putcj(task, j, c[j]);
  /* Put constraint bounds. */
  if (r == MSK_RES_OK)
    for (i = 0; i < numcon; ++i)
      r = MSK_putconbound(task, i, bkc[i], blc[i], buc[i]);
  /* Put variable bounds. */
  if (r == MSK_RES_OK)
   for (j = 0; j < numvar; ++j)
      r = MSK_putvarbound(task, j, bkx[j], blx[j], bux[j]);
  /* Put A. */
  if (r == MSK_RES_OK)
    if (numcon > 0)
     for (j = 0; j < numvar; ++j)
        r = MSK_putacol(task,
                        ptre[j] - ptrb[j],
                        asub + ptrb[j],
                        aval + ptrb[j]);
  if (r == MSK_RES_OK)
   r = MSK_putobjsense(task,
                        MSK_OBJECTIVE_SENSE_MAXIMIZE);
  if (r == MSK_RES_OK)
```

## 6.10.2 Changing the Linear Constraint Matrix

Suppose we want to change the time required for assembly of product 0 to 3 minutes. This corresponds to setting  $a_{0,0} = 3$ , which is done by calling the function  $MSK_putaij$  as shown below.

```
if (r == MSK_RES_OK)
r = MSK_putaij(task, 0, 0, 3.0);
```

The problem now has the form:

maximize 
$$1.5x_0 + 2.5x_1 + 3.0x_2$$
  
subject to  $3x_0 + 4x_1 + 3x_2 \le 100000$ ,  
 $3x_0 + 2x_1 + 3x_2 \le 50000$ ,  
 $2x_0 + 3x_1 + 2x_2 \le 60000$ , (6.24)

and

$$x_0, x_1, x_2 \geq 0.$$

After this operation we can reoptimize the problem.

## 6.10.3 Appending Variables

We now want to add a new product with the following data:

Product no.	Assembly (minutes)	Polishing (minutes)	Packing (minutes)	Profit (\$)
3	4	0	1	1.00

This corresponds to creating a new variable  $x_3$ , appending a new column to the A matrix and setting a new term in the objective. We do this in Listing 6.39

Listing 6.39: How to add a new variable (column)

```
/************************* Add a new variable **************
/* Get index of new variable, this should be 3 */
if (r == MSK_RES_OK)
    r = MSK_getnumvar(task, &varidx);
/* Append a new variable x_3 to the problem */
if (r == MSK_RES_OK)
{
    r = MSK_RES_OK)
{
        r = MSK_appendvars(task, 1);
        numvar++;
}
/* Set bounds on new variable */
```

```
if (r == MSK_RES_OK)
  r = MSK_putvarbound(task,
                       varidx,
                      MSK_BK_LO,
                      0,
                      +MSK_INFINITY);
/* Change objective */
if (r == MSK_RES_OK)
 r = MSK_putcj(task, varidx, 1.0);
/* Put new values in the A matrix */
if (r == MSK_RES_OK)
 MSKint32t acolsub[] = \{0, 2\};
            acolval[] = \{4.0, 1.0\};
  double
 r = MSK_putacol(task,
                  varidx, /* column index */
                  2, /* num nz in column*/
                  acolsub,
                  acolval);
}
```

After this operation the new problem is:

maximize 
$$1.5x_0 + 2.5x_1 + 3.0x_2 + 1.0x_3$$
  
subject to  $3x_0 + 4x_1 + 3x_2 + 4x_3 \le 100000$ ,  
 $3x_0 + 2x_1 + 3x_2 \le 50000$ ,  
 $2x_0 + 3x_1 + 2x_2 + 1x_3 \le 60000$ , (6.25)

and

$$x_0, x_1, x_2, x_3 \ge 0.$$

## 6.10.4 Appending Constraints

Now suppose we want to add a new stage to the production process called *Quality control* for which 30000 minutes are available. The time requirement for this stage is shown below:

Product no.	Quality control (minutes)
0	1
1	2
2	1
3	1

This corresponds to adding the constraint

$$x_0 + 2x_1 + x_2 + x_3 \le 30000$$

to the problem. This is done as follows.

Listing 6.40: Adding a new constraint.

```
/* *********** Add a new constraint ********** */
/* Get index of new constraint*/
if (r == MSK_RES_OK)
```

```
r = MSK_getnumcon(task, &conidx);
/* Append a new constraint */
if (r == MSK_RES_OK)
{
 r = MSK_appendcons(task, 1);
 numcon++;
/* Set bounds on new constraint */
if (r == MSK_RES_OK)
 r = MSK_putconbound(task,
                      conidx,
                      MSK_BK_UP,
                      -MSK_INFINITY,
                      30000);
/* Put new values in the A matrix */
if (r == MSK_RES_OK)
{
 MSKint32t arowsub[] = {0, 1, 2, 3};
            arowval[] = \{1.0, 2.0, 1.0, 1.0\};
 r = MSK_putarow(task,
                  conidx, /* row index */
                          /* num nz in row*/
                  arowsub,
                  arowval);
}
```

Again, we can continue with re-optimizing the modified problem.

## 6.10.5 Changing bounds

One typical reoptimization scenario is to change bounds. Suppose for instance that we must operate with limited time resources, and we must change the upper bounds in the problem as follows:

Operation	Time available (before)	Time available (new)
Assembly	100000	80000
Polishing	50000	40000
Packing	60000	50000
Quality control	30000	22000

That means we would like to solve the problem:

maximize 
$$1.5x_0 + 2.5x_1 + 3.0x_2 + 1.0x_3$$
  
subject to  $3x_0 + 4x_1 + 3x_2 + 4x_3 \le 80000$ ,  
 $3x_0 + 2x_1 + 3x_2 \le 40000$ ,  
 $2x_0 + 3x_1 + 2x_2 + 1x_3 \le 50000$ ,  
 $x_0 + 2x_1 + x_2 + x_3 \le 22000$ . (6.26)

In this case all we need to do is redefine the upper bound vector for the constraints, as shown in the next listing.

Listing 6.41: Change constraint bounds.

```
/* ************* Change constraint bounds ************ */
if (r == MSK_RES_OK)
```

Again, we can continue with re-optimizing the modified problem.

### 6.10.6 Advanced hot-start

If the optimizer used the data from the previous run to hot-start the optimizer for reoptimization, this will be indicated in the log:

```
Optimizer - hotstart : yes
```

When performing re-optimizations, instead of removing a basic variable it may be more efficient to fix the variable at zero and then remove it when the problem is re-optimized and it has left the basis. This makes it easier for **MOSEK** to restart the simplex optimizer.

## 6.11 Parallel optimization

In this section we demonstrate the simplest possible multi-threading setup to run multiple MOSEK optimizations in parallel. All tasks must be created using the same MOSEK environment. One license token checked out by the environment will be shared by the tasks.

We first define a simple method that runs a number of optimization tasks in parallel, using the standard multi-threading setup available in the language.

Listing 6.42: Parallel optimization of a list of tasks. (C++).

```
/** Runs a single task */
void runTask(int
                         num,
             MSKtask_t
                         task
             MSKrescodee *res.
             MSKrescodee *trm)
  *res = MSK_optimizetrm(task, trm);
/** Takes a list of tasks and optimizes then in parallel. The
    response code and termination code from each optimization is
    stored in ``res`` and ``trm``.
void paropt(int
            MSKtask_t
                        *tasks,
            MSKrescodee *res.
            MSKrescodee *trm)
  int i;
 std::thread * jobs = new std::thread[n];
  // Initialize
 for(i = 0; i < n; ++i) {
   res[i] = trm[i] = MSK_RES_ERR_UNKNOWN;
    jobs[i] = std::thread(runTask, i, tasks[i], &(res[i]), &(trm[i]));
```

```
// Join all threads
for(i = 0; i < n; ++i) jobs[i].join();
delete[] jobs;
}
</pre>
```

It remains to call the method with a few different tasks. When optimizing many task in parallel it usually makes sense to solve each task using one thread to avoid additional multitasking overhead. When all tasks complete we access the solutions in the standard way.

Listing 6.43: Calling the parallel optimizer (C++).

```
/** Example of how to use ``paropt``.
    Optimizes tasks whose names were read from command line.
int main(int argc, char **argv)
 MSKenv_t env;
 int n = argc - 1;
 MSKtask_t *tasks = new MSKtask_t[n];
                   = new MSKrescodee[n];
 MSKrescodee *res
 MSKrescodee *trm = new MSKrescodee[n];
 MSK_makeenv(&env, NULL);
 for (int i = 0; i < n; i++) {
   MSK_makeemptytask(env, &(tasks[i]));
   MSK_readdata(tasks[i], argv[i+1]);
    // Each task will be single-threaded
   MSK_putintparam(tasks[i], MSK_IPAR_INTPNT_MULTI_THREAD, MSK_OFF);
 }
 paropt(n, tasks, res, trm);
 for(int i = 0; i < n; i++) {
   double obj, tm;
   MSK_getdouinf(tasks[i], MSK_DINF_INTPNT_PRIMAL_OBJ, &obj);
   MSK_getdouinf(tasks[i], MSK_DINF_OPTIMIZER_TIME, &tm);
   printf("Task %d res %d trm %d obj_val %.5f time %.5f\n",
            i,
            res[i],
            trm[i],
            obj,
            tm);
 }
 for(int i = 0; i < n; i++)
   MSK_deletetask(&(tasks[i]));
 delete[] tasks;
 delete[] res;
 delete[] trm;
 MSK_deleteenv(&env);
 return 0;
```

Another, slightly more advanced application of the parallel optimizer is presented in Sec. 11.3.

For a more in-depth treatment see the following sections:

- $\bullet$   $\it Case\ studies$  for more advanced and complicated optimization examples.
- ullet Problem Formulation and Solutions for formal mathematical formulations of problems MOSEK can solve, dual problems and infeasibility certificates.

# Chapter 7

# Solver Interaction Tutorials

In this section we cover the interaction with the solver.

## 7.1 Accessing the solution

This section contains important information about the status of the solver and the status of the solution, which must be checked in order to properly interpret the results of the optimization.

### 7.1.1 Solver termination

The optimizer provides two status codes relevant for error handling:

- Response code of type MSKrescodee. It indicates if any unexpected error (such as an out of memory error, licensing error etc.) has occurred. The expected value for a successful optimization is MSK\_RES\_OK.
- **Termination code**: It provides information about why the optimizer terminated, for instance if a predefined time limit has been reached. These are not errors, but ordinary events that can be expected (depending on parameter settings and the type of optimizer used).

To obtain both codes separately call the function  $MSK\_optimizetrm$  to optimize the problem. When using the simplified  $MSK\_optimize$  the response code or termination code most relevant for the user will be returned.

If a runtime error causes the program to crash during optimization, the first debugging step is to enable logging and check the log output. See Sec. 7.3.

If the optimization completes successfully, the next step is to check the solution status, as explained below.

### 7.1.2 Available solutions

 $\mathbf{MOSEK}$  uses three kinds of optimizers and provides three types of solutions:

- basic solution from the simplex optimizer,
- interior-point solution from the interior-point optimizer,
- integer solution from the mixed-integer optimizer.

Under standard parameters settings the following solutions will be available for various problem types:

Table 7.1: Types of solutions available from MOSEK

| Simplex opti- Interior-point opti- Normal mizer | Normal Nor

	Simplex opti-	Interior-point opti-	Mixed-integer opti-
	mizer	mizer	mizer
Linear problem	MSK_SOL_BAS	MSK_SOL_ITR	
Nonlinear continuous prob-		MSK_SOL_ITR	
lem			
Problem with integer vari-			MSK_SOL_ITG
ables			

For linear problems the user can force a specific optimizer choice making only one of the two solutions available. For example, if the user disables basis identification, then only the interior point solution will be available for a linear problem. Numerical issues may cause one of the solutions to be unknown even if another one is feasible.

Not all components of a solution are always available. For example, there is no dual solution for integer problems and no dual conic variables from the simplex optimizer.

The user will always need to specify which solution should be accessed.

### 7.1.3 Problem and solution status

Assuming that the optimization terminated without errors, the next important step is to check the problem and solution status. There is one for every type of solution, as explained above.

### **Problem status**

Problem status (MSKprostae) determines whether the problem is certified as feasible. Its values can roughly be divided into the following broad categories:

- **feasible** the problem is feasible. For continuous problems and when the solver is run with default parameters, the feasibility status should ideally be MSK\_PRO\_STA\_PRIM\_AND\_DUAL\_FEAS.
- **primal/dual infeasible** the problem is infeasible or unbounded or a combination of those. The exact problem status will indicate the type of infeasibility.
- unknown the solver was unable to reach a conclusion, most likely due to numerical issues.

#### Solution status

Solution status (MSKsolstae) provides the information about what the solution values actually contain. The most important broad categories of values are:

- optimal (MSK\_SOL\_STA\_OPTIMAL) the solution values are feasible and optimal.
- **certificate** the solution is in fact a certificate of infeasibility (primal or dual, depending on the solution).
- unknown/undefined the solver could not solve the problem or this type of solution is not available for a given problem.

Problem and solution status for each solution can be retrieved with  $MSK\_getprosta$  and  $MSK\_getsolsta$ , respectively.

The solution status determines the action to be taken. For example, in some cases a suboptimal solution may still be valuable and deserve attention. It is the user's responsibility to check the status and quality of the solution.

### Typical status reports

Here are the most typical optimization outcomes described in terms of the problem and solution statuses. Note that these do not cover all possible situations that can occur.

Table 7.2: Continuous problems (solution status for interior-point and basic solution)

Outcome	Problem status	Solution status
Optimal	MSK_PRO_STA_PRIM_AND_DUAL_FEAMSK_SOL_STA_OPTIMAL	
Primal infeasible	MSK_PRO_STA_PRIM_INFEAS	MSK_SOL_STA_PRIM_INFEAS_CER
Dual infeasible (unbounded)	MSK_PRO_STA_DUAL_INFEAS	MSK_SOL_STA_DUAL_INFEAS_CER
Uncertain (stall, numerical issues,	MSK_PRO_STA_UNKNOWN	MSK_SOL_STA_UNKNOWN
etc.)		

Table 7.3: Integer problems (solution status for integer solution, others undefined)

Outcome	Problem status	Solution status
Integer optimal	MSK_PRO_STA_PRIM_FEAS	MSK_SOL_STA_INTEGER_OPTIMAL
Infeasible	MSK_PRO_STA_PRIM_INFEAS	MSK_SOL_STA_UNKNOWN
Integer feasible point	MSK_PRO_STA_PRIM_FEAS	MSK_SOL_STA_PRIM_FEAS
No conclusion	MSK_PRO_STA_UNKNOWN	MSK_SOL_STA_UNKNOWN

## 7.1.4 Retrieving solution values

After the meaning and quality of the solution (or certificate) have been established, we can query for the actual numerical values. They can be accessed using:

- $MSK\_getprimalobj$ ,  $MSK\_getdualobj$  the primal and dual objective value.
- MSK\_getxx solution values for the variables.
- MSK\_getsolution a full solution with primal and dual values

and many more specialized methods, see the API reference.

### 7.1.5 Source code example

Below is a source code example with a simple framework for assessing and retrieving the solution to a conic optimization problem.

Listing 7.1: Sample framework for checking optimization result.

```
# include <stdio.h>
# include "mosek.h"
/* Log handler */
void MSKAPI printlog(void *ptr,
                     const char s[])
 printf("%s", s);
int main(int argc, char const *argv[])
 MSKenv_t
              env;
 MSKtask_t
             task;
 MSKrescodee r;
  char
             symname[MSK_MAX_STR_LEN];
              desc[MSK_MAX_STR_LEN];
  char
 int
             i, numvar;
             *xx = NULL;
 double
 const char *filename;
 if (argc >= 2) filename = argv[1];
                 filename = "../data/cqo1.mps";
  // Create the environment
 r = MSK_makeenv(&env, NULL);
 if (r == MSK_RES_OK)
    // Create the task
```

```
r = MSK_makeemptytask(env, &task);
   // (Optionally) attach the log handler to receive log information
   // if ( r == MSK_RES_OK ) MSK_linkfunctotaskstream(task, MSK_STREAM_LOG, NULL,
\rightarrow printlog);
   // (Optionally) uncomment this line to most likely see solution status Unknown
   // MSK_putintparam(task, MSK_IPAR_INTPNT_MAX_ITERATIONS, 1);
   // In this example we read an optimization problem from a file
   r = MSK_readdata(task, filename);
   if (r == MSK_RES_OK)
     MSKrescodee trmcode;
     MSKsolstae solsta;
     // Do the optimization, and exit in case of error
     r = MSK_optimizetrm(task, &trmcode);
     if (r != MSK_RES_OK) {
       MSK_getcodedesc(r, symname, desc);
       printf("Error during optimization: %s %s\n", symname, desc);
       exit(r);
     MSK_solutionsummary(task, MSK_STREAM_LOG);
     /* Expected result: The solution status of the interiot-point solution is ____
→optimal. */
     if (MSK_RES_OK == MSK_getsolsta(task, MSK_SOL_ITR, &solsta))
       switch (solsta)
         case MSK_SOL_STA_OPTIMAL:
           printf("An optimal interior-point solution is located.\n");
           /* Read and print the variable values in the solution */
           MSK_getnumvar(task, &numvar);
           xx = calloc(numvar, sizeof(double));
           MSK_getxx(task, MSK_SOL_ITR, xx);
           for (i = 0; i < numvar; i++)
             printf("xx[%d] = %.4lf\n", i, xx[i]);
           free(xx);
           break;
         case MSK_SOL_STA_DUAL_INFEAS_CER:
           printf("Dual infeasibility certificate found.\n");
           break;
         case MSK_SOL_STA_PRIM_INFEAS_CER:
           printf("Primal infeasibility certificate found.\n");
           break;
         case MSK_SOL_STA_UNKNOWN:
```

```
/* The solutions status is unknown. The termination code
               indicating why the optimizer terminated prematurely. */
           printf("The solution status is unknown.\n");
           /* No-error cause of termination e.g. an iteration limit is reached. */
           MSK_getcodedesc(trmcode, symname, desc);
           printf(" Termination code: %s %s\n", symname, desc);
           break;
         default:
           MSK_solstatostr(task, solsta, desc);
           printf("An unexpected solution status %s with code %d is obtained.\n",
→desc, solsta);
           break;
       }
     }
     else
       printf("Could not obtain the solution status for the requested solution.\n");
   }
   else {
     MSK_getcodedesc(r, symname, desc);
     printf("Optimization was not started because of error %s(%d): %s\n", symname, r,
→ desc);
   MSK_deletetask(&task);
 MSK_deleteenv(&env);
 return r;
```

# 7.2 Errors and exceptions

### Response codes

Almost every function in Optimizer API for C returns a **response code**, which is an integer (implemented as the enum MSKrescodee), informing if the requested operation was performed correctly, and if not, what error occurred. The expected response, indicating successful execution, is always  $MSK_RES_OK$ . It is a good idea to check the response code every time to avoid silent fails such as for instance:

- referencing a nonexisting variable (for example with too large index),
- defining an invalid value for a parameter,
- accessing an undefined solution,
- repeating a variable name, etc.

The one case where it is *extremely important* to check the response code is during optimization, when  $MSK_optimizetrm$  is invoked. We will say more about this in Sec. 7.1.

A numerical response code can be converted into a human-readable description using  $MSK\_getcodedesc$ . A full list of response codes, error, warning and termination codes can be found in the API reference. For example, the following code

```
res = MSK_putdouparam(task, MSK_DPAR_INTPNT_CO_TOL_REL_GAP, -1.0e-7);
if (res != MSK_RES_OK) {
   MSK_getcodedesc(res, symb, str);
```

```
printf("Error %s(%d): %s\n", symb, res, str);
}
```

will produce as output:

```
Error MSK_RES_ERR_PARAM_IS_TOO_SMALL(1216): A parameter value is too small.
```

### Optimizer errors and warnings

The optimizer may also produce warning messages. They indicate non-critical but important events, that will not prevent solver execution, but may be an indication that something in the optimization problem might be improved. Warning messages are normally printed to a log stream (see Sec. 7.3). A typical warning is, for example:

```
MOSEK warning 53: A numerically large upper bound value 6.6e+09 is specified for \Box \Box constraint 'C69200' (46020).
```

Warnings can also be suppressed by setting the  $MSK\_IPAR\_MAX\_NUM\_WARNINGS$  parameter to zero, if they are well-understood.

The user can also register a dedicated callback function to handle all errors and warnings. This is done with  $MSK\_putresponsefunc$ .

## Error and solution status handling example

Below is a source code example with a simple framework for handling major errors when assessing and retrieving the solution to a conic optimization problem.

Listing 7.2: Sample framework for checking optimization result.

```
# include <stdio.h>
# include "mosek.h"
/* Log handler */
void MSKAPI printlog(void *ptr,
                     const char s[])
 printf("%s", s);
int main(int argc, char const *argv[])
 MSKenv_t
             env;
 MSKtask_t
             task;
 MSKrescodee r;
             symname [MSK_MAX_STR_LEN];
  char
  char
              desc[MSK_MAX_STR_LEN];
 int
             i, numvar;
           *xx = NULL;
 double
 const char *filename;
 if (argc >= 2) filename = argv[1];
                 filename = "../data/cqo1.mps";
  // Create the environment
 r = MSK_makeenv(&env, NULL);
 if (r == MSK_RES_OK)
  {
```

```
// Create the task
   r = MSK_makeemptytask(env, &task);
   // (Optionally) attach the log handler to receive log information
   // if ( r == MSK_RES_OK ) MSK_linkfunctotaskstream(task, MSK_STREAM_LOG, NULL, _{\sqcup}
\rightarrow printlog);
   // (Optionally) uncomment this line to most likely see solution status Unknown
   // MSK_putintparam(task, MSK_IPAR_INTPNT_MAX_ITERATIONS, 1);
   // In this example we read an optimization problem from a file
   r = MSK_readdata(task, filename);
   if (r == MSK_RES_OK)
     MSKrescodee trmcode;
     MSKsolstae solsta;
     // Do the optimization, and exit in case of error
     r = MSK_optimizetrm(task, &trmcode);
     if (r != MSK_RES_OK) {
       MSK_getcodedesc(r, symname, desc);
       printf("Error during optimization: %s %s\n", symname, desc);
       exit(r);
     MSK_solutionsummary(task, MSK_STREAM_LOG);
     /* Expected result: The solution status of the interiot-point solution is ____
→optimal. */
     if (MSK_RES_OK == MSK_getsolsta(task, MSK_SOL_ITR, &solsta))
       switch (solsta)
         case MSK_SOL_STA_OPTIMAL:
           printf("An optimal interior-point solution is located.\n");
           /* Read and print the variable values in the solution */
           MSK_getnumvar(task, &numvar);
           xx = calloc(numvar, sizeof(double));
           MSK_getxx(task, MSK_SOL_ITR, xx);
           for (i = 0; i < numvar; i++)
             printf("xx[%d] = %.4lf\n", i, xx[i]);
           free(xx);
           break;
         case MSK_SOL_STA_DUAL_INFEAS_CER:
           printf("Dual infeasibility certificate found.\n");
           break;
         case MSK_SOL_STA_PRIM_INFEAS_CER:
           printf("Primal infeasibility certificate found.\n");
           break;
```

```
case MSK_SOL_STA_UNKNOWN:
            /* The solutions status is unknown. The termination code
               indicating why the optimizer terminated prematurely. */
            printf("The solution status is unknown.\n");
            /* No-error cause of termination e.g. an iteration limit is reached. */
            MSK_getcodedesc(trmcode, symname, desc);
            printf(" Termination code: %s %s\n", symname, desc);
            break;
          default:
            MSK_solstatostr(task, solsta, desc);
            printf("An unexpected solution status %s with code %d is obtained.\n", __
→desc, solsta);
            break;
      }
      else
        printf("Could not obtain the solution status for the requested solution.\n");
   else {
     MSK_getcodedesc(r, symname, desc);
     printf("Optimization was not started because of error %s(%d): %s\n", symname, r,
→ desc);
   MSK_deletetask(&task);
 MSK_deleteenv(&env);
 return r;
}
```

# 7.3 Input/Output

The logging and I/O features are provided mainly by the **MOSEK** task and to some extent by the **MOSEK** environment objects.

### 7.3.1 Stream logging

By default the solver runs silently and does not produce any output to the console or otherwise. However, the log output can be redirected to a user-defined output stream or stream callback function. The log output is analogous to the one produced by the command-line version of **MOSEK**.

The log messages are partitioned in three streams:

```
• messages, MSK_STREAM_MSG
```

- $\bullet$  warnings,  $MSK\_STREAM\_WRN$
- $\bullet$  errors,  $\textit{MSK\_STREAM\_ERR}$

These streams are aggregated in the  $MSK\_STREAM\_LOG$  stream. A stream handler can be defined for each stream separately.

A stream handler is simply a user-defined function of type MSKstreamfunc that accepts a string, for example:

It is attached to a stream as follows:

```
MSK_linkfunctotaskstream(task, MSK_STREAM_LOG, NULL, printstr);
```

The stream can be detached by calling

```
MSK_linkfunctotaskstream(task, MSK_STREAM_LOG, NULL, NULL);
```

A log stream can also be redirected to a file:

```
MSK_linkfiletotaskstream(task, MSK_STREAM_LOG, "mosek.log", 0);
```

After optimization is completed an additional short summary of the solution and optimization process can be printed to any stream using the method MSK\_solutionsummary.

## 7.3.2 Log verbosity

The logging verbosity can be controlled by setting the relevant parameters, as for instance

- MSK\_IPAR\_LOG,
- MSK\_IPAR\_LOG\_INTPNT,
- MSK\_IPAR\_LOG\_MIO,
- MSK\_IPAR\_LOG\_CUT\_SECOND\_OPT,
- MSK\_IPAR\_LOG\_SIM, and
- MSK\_IPAR\_LOG\_SIM\_MINOR.

Each parameter controls the output level of a specific functionality or algorithm. The main switch is  $MSK\_IPAR\_LOG$  which affect the whole output. The actual log level for a specific functionality is determined as the minimum between  $MSK\_IPAR\_LOG$  and the relevant parameter. For instance, the log level for the output produce by the interior-point algorithm is tuned by the  $MSK\_IPAR\_LOG\_INTPNT$ ; the actual log level is defined by the minimum between  $MSK\_IPAR\_LOG$  and  $MSK\_IPAR\_LOG\_INTPNT$ .

Tuning the solver verbosity may require adjusting several parameters. It must be noticed that verbose logging is supposed to be of interest during debugging and tuning. When output is no more of interest, the user can easily disable it globally with <code>MSK\_IPAR\_LOG</code>. Larger values of <code>MSK\_IPAR\_LOG</code> do not necessarily result in increased output.

By default  $\mathbf{MOSEK}$  will reduce the amount of log information after the first optimization on a given problem. To get full log output on subsequent re-optimizations set  $MSK\_IPAR\_LOG\_CUT\_SECOND\_OPT$  to zero.

## 7.3.3 Saving a problem to a file

An optimization problem can be dumped to a file using the method MSK\_writedata. The file format will be determined from the extension of the filename. Supported formats are listed in Sec. 16 together with a table of problem types supported by each.

For instance the problem can be written to an OPF file with

```
MSK_writedata(task,"data.opf");
```

All formats can be compressed with gzip by appending the .gz extension, for example

```
MSK_writedata(task, "data.task.gz");
```

Some remarks:

- Unnamed variables are given generic names. It is therefore recommended to use meaningful variable names if the problem file is meant to be human-readable.
- The task format is MOSEK's native file format which contains all the problem data as well as solver settings.

## 7.3.4 Reading a problem from a file

A problem saved in any of the supported file formats can be read directly into a task using MSK\_readdata. The task must be created in advance. Afterwards the problem can be optimized, modified, etc. If the file contained solutions, then are also imported, but the status of any solution will be set to MSK\_SOL\_STA\_UNKNOWN (solutions can also be read separately using MSK\_readsolution). If the file contains parameters, they will be set accordingly.

```
res = MSK_maketask(env, 0,0, &task);
if (res == MSK_RES_OK)
  res = MSK_readdata(task, "file.task.gz");
if (res == MSK_RES_OK)
  res = MSK_optimize(task);
```

## 7.4 Setting solver parameters

**MOSEK** comes with a large number of parameters that allows the user to tune the behavior of the optimizer. The typical settings which can be changed with solver parameters include:

- choice of the optimizer for linear problems,
- choice of primal/dual solver,
- turning presolve on/off,
- turning heuristics in the mixed-integer optimizer on/off,
- level of multi-threading,
- feasibility tolerances,
- solver termination criteria,
- behaviour of the license manager,

and more. All parameters have default settings which will be suitable for most typical users. The API reference contains:

- Full list of parameters
- List of parameters grouped by topic

### **Setting parameters**

Each parameter is identified by a unique name. There are three types of parameters depending on the values they take:

- Integer parameters. They take either either simple integer values or values from an enumeration provided for readability and compatibility of the code. Set with MSK\_putintparam.
- Double (floating point) parameters. Set with MSK\_putdouparam.
- String parameters. Set with MSK\_putstrparam.

There are also parameter setting functions which operate fully on symbolic strings containing generic command-line style names of parameters and their values. See the example below. The optimizer will try to convert the given argument to the exact expected type, and will error if that fails.

If an incorrect value is provided then the parameter is left unchanged.

For example, the following piece of code sets up parameters which choose and tune the interior point optimizer before solving a problem.

Listing 7.3: Parameter setting example.

```
// Set log level (integer parameter)
res = MSK_putintparam(task, MSK_IPAR_LOG, 1);
// Select interior-point optimizer... (integer parameter)
res = MSK_putintparam(task, MSK_IPAR_OPTIMIZER, MSK_OPTIMIZER_INTPNT);
// ... without basis identification (integer parameter)
res = MSK_putintparam(task, MSK_IPAR_INTPNT_BASIS, MSK_BI_NEVER);
// Set relative gap tolerance (double parameter)
res = MSK_putdouparam(task, MSK_DPAR_INTPNT_CO_TOL_REL_GAP, 1.0e-7);

// The same using explicit string names
res = MSK_putparam(task, "MSK_DPAR_INTPNT_CO_TOL_REL_GAP", "1.0e-7");
res = MSK_putparam(task, "MSK_DPAR_INTPNT_CO_TOL_REL_GAP", 1.0e-7);

// Incorrect value
res = MSK_putdouparam(task, MSK_DPAR_INTPNT_CO_TOL_REL_GAP, -1.0);
if (res != MSK_RES_OK)
    printf("Wrong parameter value\n");
```

### Reading parameter values

The functions  $MSK\_getintparam$ ,  $MSK\_getdouparam$ ,  $MSK\_getstrparam$  can be used to inspect the current value of a parameter, for example:

```
res = MSK_getdouparam(task, MSK_DPAR_INTPNT_CO_TOL_REL_GAP, &param);
printf("Current value for parameter MSK_DPAR_INTPNT_CO_TOL_REL_GAP = %e\n", 
→param);
```

# 7.5 Retrieving information items

After the optimization the user has access to the solution as well as to a report containing a large amount of additional *information items*. For example, one can obtain information about:

- **timing**: total optimization time, time spent in various optimizer subroutines, number of iterations, etc.
- solution quality: feasibility measures, solution norms, constraint and bound violations, etc.
- problem structure: counts of variables of different types, constraints, nonzeros, etc.
- integer optimizer: integrality gap, objective bound, number of cuts, etc.

and more. Information items are numerical values of integer, long integer or double type. The full list can be found in the API reference:

- Double
- Integer
- Long

Certain information items make sense, and are made available, also during the optimization process. They can be accessed from a callback function, see Sec. 7.6 for details.

### Remark

For efficiency reasons, not all information items are automatically computed after optimization. To force all information items to be updated use the parameter  $MSK\_IPAR\_AUTO\_UPDATE\_SOL\_INFO$ .

### Retrieving the values

Values of information items are fetched using one of the methods

- MSK\_getdouinf for a double information item,
- MSK\_getintinf for an integer information item,
- MSK\_getlintinf for a long integer information item.

Each information item is identified by a unique name. The example below reads two pieces of data from the solver: total optimization time and the number of interior-point iterations.

Listing 7.4: Information items example.

## 7.6 Progress and data callback

Callbacks are a very useful mechanism that allow the caller to track the progress of the MOSEK optimizer. A callback function provided by the user is regularly called during the optimization and can be used to

- obtain a customized log of the solver execution,
- collect information for debugging purposes or
- ask the solver to terminate.

### Warning

The callbacks functions *must not* invoke any functions of the solver, environment or task. Otherwise the state of the solver and its outcome are undefined. The only exception is the possibility to retrieve an integer solution, see below.

### Retrieving mixed-integer solutions

If the mixed-integer optimizer is used, the callback will take place, in particular, every time an improved integer solution is found. In that case it is possible to retrieve the current values of the best integer solution from within the callback function. It can be useful for implementing complex termination criteria for integer optimization. The example in Listing 7.5 shows how to do it by handling the callback code  $MSK\_CALLBACK\_NEW\_INT\_MIO$ .

### 7.6.1 Data callback

In the data callback **MOSEK** passes a callback code and values of all information items to a user-defined function. The callback function is called, in particular, at the beginning of each iteration of the interior-point optimizer. For the simplex optimizers  $MSK\_IPAR\_LOG\_SIM\_FREQ$  controls how frequently the call-back is called.

The callback is set by calling the function  $MSK\_putcallbackfunc$  and providing a handle to a user-defined function MSKcallbackfunc.

Non-zero return value of the callback function indicates that the optimizer should be terminated.

## 7.6.2 Working example: Data callback

The following example defines a data callback function that prints out some of the information items. It interrupts the solver after a certain time limit.

Listing 7.5: An example of a data callback function.

```
static int MSKAPI usercallback(MSKtask_t
                               MSKuserhandle t
                                                    handle.
                               MSKcallbackcodee
                                                     caller,
                               const MSKrealt * douinf,
                               const MSKint32t * intinf,
                               const MSKint64t * lintinf)
 cbdata_t data = (cbdata_t) handle;
 double maxtime = data->maxtime;
 MSKrescodee r;
 switch (caller)
   case MSK_CALLBACK_BEGIN_INTPNT:
     printf("Starting interior-point optimizer\n");
    case MSK_CALLBACK_INTPNT:
      printf("Iterations: %-3d Time: %6.2f(%.2f) ",
             intinf[MSK_IINF_INTPNT_ITER],
             douinf[MSK_DINF_OPTIMIZER_TIME],
             douinf[MSK_DINF_INTPNT_TIME]);
      printf("Primal obj.: %-18.6e Dual obj.: %-18.6e\n",
             douinf[MSK_DINF_INTPNT_PRIMAL_OBJ],
             douinf[MSK_DINF_INTPNT_DUAL_OBJ]);
      break:
    case MSK_CALLBACK_END_INTPNT:
      printf("Interior-point optimizer finished.\n");
    case MSK_CALLBACK_BEGIN_PRIMAL_SIMPLEX:
      printf("Primal simplex optimizer started.\n");
      break:
    case MSK_CALLBACK_UPDATE_PRIMAL_SIMPLEX:
      printf("Iterations: %-3d ",
             intinf[MSK_IINF_SIM_PRIMAL_ITER]);
     printf(" Elapsed time: \%6.2f(\%.2f)\n",
             douinf[MSK_DINF_OPTIMIZER_TIME],
             douinf[MSK_DINF_SIM_TIME]);
      printf("Obj.: %-18.6e\n",
             douinf[MSK_DINF_SIM_OBJ]);
      break:
    case MSK_CALLBACK_END_PRIMAL_SIMPLEX:
      printf("Primal simplex optimizer finished.\n");
    case MSK_CALLBACK_BEGIN_DUAL_SIMPLEX:
      printf("Dual simplex optimizer started.\n");
    case MSK_CALLBACK_UPDATE_DUAL_SIMPLEX:
      printf("Iterations: %-3d ", intinf[MSK_IINF_SIM_DUAL_ITER]);
      printf(" Elapsed time: \%6.2f(\%.2f)\n",
             douinf[MSK_DINF_OPTIMIZER_TIME],
```

```
douinf[MSK_DINF_SIM_TIME]);
      printf("Obj.: %-18.6e\n", douinf[MSK_DINF_SIM_OBJ]);
    case MSK_CALLBACK_END_DUAL_SIMPLEX:
      printf("Dual simplex optimizer finished.\n");
      break;
    case MSK_CALLBACK_NEW_INT_MIO:
      printf("New integer solution has been located.\n");
      r = MSK_getxx(task, MSK_SOL_ITG, data->xx);
      if (r == MSK_RES_OK) {
        int i;
        printf("xx = ");
        for (i = 0; i < data->numvars; i++) printf("%lf ", data->xx[i]);
        printf("\n0bj.: %f\n", douinf[MSK_DINF_MIO_OBJ_INT]);
   default:
      break;
 if (douinf[MSK_DINF_OPTIMIZER_TIME] >= maxtime)
    /* mosek is spending too much time.
       Terminate it. */
   return (1);
 }
 return (0);
} /* usercallback */
```

Assuming that we have defined a task task and a time limit maxtime, the callback function is attached as follows:

Listing 7.6: Attaching the data callback function to the model.

# 7.7 MOSEK OptServer

MOSEK provides an easy way to offload optimization problem to a remote server. This section demonstrates related functionalities from the client side, i.e. sending optimization tasks to the remote server and retrieving solutions.

Setting up and configuring the remote server is described in a separate manual for the OptServer.

## 7.7.1 Synchronous Remote Optimization

In synchronous mode the client sends an optimization problem to the server and blocks, waiting for the optimization to end. Once the result has been received, the program can continue. This is the simplest mode all it takes is to provide the address of the server before starting optimization. The rest of the code remains untouched.

Note that it is impossible to recover the job in case of a broken connection.

### Source code example

Listing 7.7: Using the OptServer in synchronous mode.

```
# include "mosek.h"
static void MSKAPI printstr(void *handle, const char str[])
 printf("%s", str);
int main(int argc, const char * argv[])
 MSKenv_t
             env = NULL;
 MSKtask_t task = NULL;
 MSKrescodee res = MSK_RES_OK;
 MSKrescodee trm = MSK_RES_OK;
 if (argc <= 2)
   printf("Missing argument, syntax is:\n");
   printf(" opt_server_sync inputfile serveraddr\n");
 }
 else
    // Create the mosek environment.
    // The `NULL' arguments here, are used to specify customized
   // memory allocators and a memory debug file. These can
   // safely be ignored for now.
   res = MSK_makeenv(&env, NULL);
   // Create a task object linked with the environment env.
    // We create it with 0 variables and 0 constraints initially,
    // since we do not know the size of the problem.
   if (res == MSK_RES_OK)
     res = MSK_maketask(env, 0, 0, &task);
    // Direct the task log stream to a user specified function
   if (res == MSK_RES_OK)
     res = MSK_linkfunctotaskstream(task, MSK_STREAM_LOG, NULL, printstr);
    // We assume that a problem file was given as the first command
    // line argument (received in `argv')
   if (res == MSK_RES_OK)
     res = MSK_readdata(task, argv[1]);
   // Set OptServer URL
   if (res == MSK_RES_OK)
      res = MSK_putoptserverhost(task, argv[2]);
```

```
// Solve the problem remotely
if (res == MSK_RES_OK)
   res = MSK_optimizetrm(task, &trm);

// Print a summary of the solution.
if (res == MSK_RES_OK)
   res = MSK_solutionsummary(task, MSK_STREAM_LOG);

// Delete task and environment
   MSK_deletetask(&task);
   MSK_deleteenv(&env);
}
return res;
}
```

## 7.7.2 Asynchronous Remote Optimization

In asynchronous mode the client sends a job to the remote server and the execution of the client code continues. In particular, it is the client's responsibility to periodically check the optimization status and, when ready, fetch the results. The client can also interrupt optimization. The most relevant methods are:

- $\bullet$  MSK\_asyncoptimize: Offload the optimization task to a solver server.
- MSK\_asyncpoll: Request information about the status of the remote job.
- $\bullet$  MSK\_asyncgetresult: Request the results from a completed remote job.
- MSK\_asyncstop: Terminate a remote job.

### Source code example

In the example below the program enters in a polling loop that regularly checks whether the result of the optimization is available.

Listing 7.8: Using the OptServer in asynchronous mode.

```
MSKbooleant respavailable;
 MSKenv_t env = NULL;
 MSKtask_t task = NULL;
 MSKrescodee res = MSK_RES_OK;
 MSKrescodee trm;
 MSKrescodee resp;
 const char * filename = "../data/25fv47.mps";
 const char * host = "karise";
const char * port = "30080";
 if (argc < 5)
   fprintf(stderr, "Syntax: opt_server_async filename host port numpolls\n");
   return 0;
 }
 if (argc > 1) filename = argv[1];
 if (argc > 2) host = argv[2];
if (argc > 2) port = argv[3];
 if (argc > 4) numpolls = atoi(argv[4]);
 res = MSK_makeenv(&env, NULL);
 if (res == MSK_RES_OK)
   res = MSK_maketask(env, 0, 0, &task);
 if (res == MSK_RES_OK)
   res = MSK_linkfunctotaskstream(task, MSK_STREAM_LOG, NULL, printstr);
 if (res == MSK_RES_OK)
   res = MSK_readdata(task, filename);
 res = MSK_asyncoptimize(task,
                          host,
                           port,
                           token);
 MSK_deletetask(&task);
 printf("token = %s\n", token);
 if (res == MSK_RES_OK)
   res = MSK_maketask(env, 0, 0, &task);
 if (res == MSK_RES_OK)
   res = MSK_readdata(task, filename);
 if (res == MSK_RES_OK)
   res = MSK_linkfunctotaskstream(task, MSK_STREAM_LOG, NULL, printstr);
 for (i = 0; i < numpolls && res == MSK_RES_OK; i++)
# if __linux__
   sleep(1);
#elif defined(_WIN32)
   Sleep(1000);
```

```
#endif
    printf("poll %d\n ", i);
    res = MSK_asyncpoll(task,
                        host,
                        port,
                        token,
                        &respavailable,
                        &resp,
                        &trm);
    puts("polling done");
    if (respavailable)
      puts("solution available!");
     res = MSK_asyncgetresult(task,
                               host,
                               port,
                               token,
                                &respavailable,
                                &resp,
                                &trm);
      MSK_solutionsummary(task, MSK_STREAM_LOG);
      break;
    }
  }
  if (i == numpolls)
    printf("max num polls reached, stopping %s", host);
    MSK_asyncstop(task, host, port, token);
  MSK_deletetask(&task);
  MSK_deleteenv(&env);
  printf("%s:%d: Result = %d\n", __FILE__, __LINE__, res); fflush(stdout);
  return res;
}
```

# Chapter 8

# **Debugging Tutorials**

This collection of tutorials contains basic techniques for debugging optimization problems using tools available in MOSEK: optimizer log, solution summary, infeasibility report, command-line tools. It is intended as a first line of technical help for issues such as: Why do I get solution status *unknown* and how can I fix it? Why is my model infeasible while it shouldn't be? Should I change some parameters? Can the model solve faster? etc.

### The major steps when debugging a model are always:

• Enable log output. See Sec. 7.3.1 for how to do it. In the simplest case: Create a log handler function:

attach it to the log stream:

```
MSK_linkfunctotaskstream(task, MSK_STREAM_LOG, NULL, printstr);
```

and include solution summary after the call to optimize:

```
res = MSK_optimize(task);
MSK_solutionsummary(task, MSK_STREAM_LOG);
```

- Run the optimization and analyze the log output, see Sec. 8.1. In particular:
  - check if the problem setup (number of constraints/variables etc.) matches your expectation.
  - check solution summary and solution status.
- Dump the problem to disk if necessary to continue analysis. See Sec. 7.3.3.
  - use a human-readable text format, such as \*.opf if you want to check the problem structure by hand. Assign names to variables and constraints to make them easier to identify.

```
MSK_writedata(task,"data.opf");
```

- use the MOSEK native format \*.task.gz when submitting a bug report or support question.

```
MSK_writedata(task,"data.task.gz");
```

• Fix problem setup, improve the model, locate infeasibility or adjust parameters, depending on the diagnosis.

See the following sections for details.

# 8.1 Understanding optimizer log

The optimizer produces a log which splits roughly into four sections:

- 1. summary of the input data,
- 2. presolve and other pre-optimize problem setup stages,
- 3. actual optimizer iterations,
- 4. solution summary.

In this tutorial we show how to analyze the most important parts of the log when initially debugging a model: input data (1) and solution summary (4). For the iterations log (3) see Sec. 13.3.4 or Sec. 13.4.8.

### 8.1.1 Input data

If **MOSEK** behaves very far from expectations it may be due to errors in problem setup. The log file will begin with a summary of the structure of the problem, which looks for instance like:

```
Problem

Name :

Objective sense : max

Type : CONIC (conic optimization problem)

Constraints : 20413

Cones : 2508

Scalar variables : 20414

Matrix variables : 0

Integer variables : 0
```

This can be consulted to eliminate simple errors: wrong objective sense, wrong number of variables etc. Note that Fusion, and third-party modeling tools can introduce additional variables and constraints to the model. In the remaining **MOSEK** APIs the problem dimensions should match exactly what the user specified.

If this is not sufficient a bit more information can be obtained by dumping the problem to a file (see Sec. 8) and using the anapro option of any of the command line tools. It can also be done directly with the function MSK\_analyzeproblem. This will produce a longer summary similar to:

```
** Variables
scalar: 20414
                   integer: 0
                                      matrix: 0
                                                                             fixed: 426
low: 2082
                   up: 5014
                                      ranged: 0
                                                         free: 12892
** Constraints
all: 20413
low: 10028
                   up: 0
                                      ranged: 0
                                                         free: 0
                                                                            fixed: 10385
** Cones
QUAD: 1
                   dims: 2865: 1
RQUAD: 2507
                   dims: 3: 2507
** Problem data (numerics)
|c|
                   nnz: 10028
                                      min=2.09e-05
                                                         max=1.00e+00
| A |
                                      min=1.17e-10
                                                         max=1.00e+00
                   nnz: 597023
blx
                   fin: 2508
                                      min=-3.60e+09
                                                         max=2.75e+05
bux
                   fin: 5440
                                      min=0.00e+00
                                                         \max=2.94e+08
blc
                   fin: 20413
                                      min=-7.61e+05
                                                         \max = 7.61e + 05
buc
                   fin: 10385
                                      min=-5.00e-01
                                                         max=0.00e+00
```

Again, this can be used to detect simple errors, such as:

• Wrong type of cone was used or it has wrong dimension.

- The bounds for variables or constraints are incorrect or incomplete. Check if you defined bound keys for all variables. A variable for which no bound was defined is by default fixed at 0.
- The model is otherwise incomplete.
- Suspicious values of coefficients.
- For various data sizes the model does not scale as expected.

Finally saving the problem in a human-friendly text format such as LP or OPF (see Sec. 8) and analyzing it by hand can reveal if the model is correct.

#### Warnings and errors

At this stage the user can encounter warnings which should not be ignored, unless they are well-understood. They can also serve as hints as to numerical issues with the problem data. A typical warning of this kind is

```
MOSEK warning 53: A numerically large upper bound value 2.9e+08 is specified for variable 'absh[107]' (2613).
```

Warnings do not stop the problem setup. If, on the other hand, an error occurs then the model will become invalid. The user should make sure to test for errors/exceptions from all API calls that set up the problem and validate the data. See Sec. 7.2 for more details.

### 8.1.2 Solution summary

The last item in the log is the solution summary. In the Optimizer API it is only printed by invoking the function  $MSK\_solutionsummary$ .

#### Continuous problem

#### **Optimal solution**

A typical solution summary for a continuous (linear, conic, quadratic) problem looks like:

It contains the following elements:

- Problem and solution status. For details see Sec. 7.1.3.
- A summary of the primal solution: objective value, infinity norm of the solution vector xx, maximal violations of constraints, variable bounds and cones. The violation of a linear constraint such as  $a^Tx \leq b$  is  $\max(a^Tx b, 0)$ . The violation of a conic constraint  $x \in \mathcal{K}$  is the distance  $\mathrm{dist}(x, \mathcal{K})$ .
- The same for the dual solution.

The features of the solution summary which characterize a very good and accurate solution and a well-posed model are:

- Status: The solution status is OPTIMAL.
- Duality gap: The primal and dual objective values are (almost) identical, which proves the solution is (almost) optimal.
- Norms: Ideally the norms of the solution and the objective values should not be too large. This of course depends on the input data, but a huge solution norm can be an indicator of issues with the scaling, conditioning and/or well-posedness of the model. It may also indicate that the problem is borderline between feasibility and infeasibility and sensitive to small perturbations in this respect.

• Violations: The violations are close to zero, which proves the solution is (almost) feasible. Observe that due to rounding errors it can be expected that the violations are proportional to the norm (nrm:) of the solution. It is rarely the case that violations are exactly zero.

#### Solution status UNKNOWN

A typical example with solution status UNKNOWN due to numerical problems will look like:

#### Note that:

- The primal and dual objective are very different.
- The dual solution has very large norm.
- There are considerable violations so the solution is likely far from feasible.

Follow the hints in Sec. 8.2 to resolve the issue.

#### Solution status UNKNOWN with a potentially useful solution

Solution status UNKNOWN does not necessarily mean that the solution is completely useless. It only means that the solver was unable to make any more progress due to numerical difficulties, and it was not able to reach the accuracy required by the termination criteria (see Sec. 13.3.2). Consider for instance:

```
Problem status
                : UNKNOWN
Solution status : UNKNOWN
Primal. obj: 3.4531019648e+04
                                                 Viol.
                                   nrm: 1e+05
                                                        con: 7e-02
                                                                       var: 0e+00
⇔cones: 0e+00
Dual.
         obj: 3.4529720645e+04
                                   nrm: 8e+03
                                                 Viol.
                                                        con: 1e-04
                                                                       var: 2e-04
→cones: 0e+00
```

Such a solution may still be useful, and it is always up to the user to decide. It may be a good enough approximation of the optimal point. For example, the large constraint violation may be due to the fact that one constraint contained a huge coefficient.

#### Infeasibility certificate

A primal infeasibility certificate is stored in the dual variables:

```
Problem status : PRIMAL_INFEASIBLE
Solution status : PRIMAL_INFEASIBLE_CER
Dual. obj: 2.9238975853e+02 nrm: 6e+02 Viol. con: 0e+00 var: 1e-11 ucones: 0e+00
```

It is a Farkas-type certificate as described in Sec. 12.2.2. In particular, for a good certificate:

- The dual objective is positive for a minimization problem, negative for a maximization problem. Ideally it is well bounded away from zero.
- The norm is not too big and the violations are small (as for a solution).

If the model was not expected to be infeasible, the likely cause is an error in the problem formulation. Use the hints in Sec. 8.1.1 and Sec. 8.3 to locate the issue.

Just like a solution, the infeasibility certificate can be of better or worse quality. The infeasibility certificate above is very solid. However, there can be less clear-cut cases, such as for example:

```
Problem status : PRIMAL_INFEASIBLE
Solution status : PRIMAL_INFEASIBLE_CER
Dual. obj: 1.6378689238e-06 nrm: 6e+05 Viol. con: 7e-03 var: 2e-04 u

cones: 0e+00
```

This infeasibility certificate is more dubious because the dual objective is positive, but barely so in comparison with the large violations. It also has rather large norm. This is more likely an indication that the problem is borderline between feasibility and infeasibility or simply ill-posed and sensitive to tiny variations in input data. See Sec. 8.3 and Sec. 8.2.

The same remarks apply to dual infeasibility (i.e. unboundedness) certificates. Here the primal objective should be negative a minimization problem and positive for a maximization problem.

### 8.1.3 Mixed-integer problem

#### Optimal integer solution

For a mixed-integer problem there is no dual solution and a typical optimal solution report will look as follows:

The interpretation of all elements is as for a continuous problem. The additional field itg denotes the maximum violation of an integer variable from being an exact integer.

#### Feasible integer solution

If the solver found an integer solution but did not prove optimality, for instance because of a time limit, the solution status will be PRIMAL\_FEASIBLE:

```
Problem status : PRIMAL_FEASIBLE
Solution status : PRIMAL_FEASIBLE
Primal. obj: 6.0114607792e+06 nrm: 1e+03 Viol. con: 2e-13 var: 2e-13 u
itg: 4e-15
```

In this case it is valuable to go back to the optimizer summary to see how good the best solution is:

In this case the best integer solution found has objective value 6.011460779193e+06, the best proved lower bound is 6.007896089225e+06 and so the solution is guaranteed to be within 0.06% from optimum. The same data can be obtained as information items through an API. See also Sec. 13.4 for more details.

### Infeasible problem

If the problem is declared infeasible the summary is simply

```
Problem status : PRIMAL_INFEASIBLE
Solution status : UNKNOWN
Primal. obj: 0.00000000000e+00 nrm: 0e+00 Viol. con: 0e+00 var: 0e+00

itg: 0e+00
```

If infeasibility was not expected, consult Sec. 8.3.

# 8.2 Addressing numerical issues

The suggestions in this section should help diagnose and solve issues with numerical instability, in particular UNKNOWN solution status or solutions with large violations. Since numerically stable models tend to solve faster, following these hints can also dramatically shorten solution times.

We always recommend that issues of this kind are addressed by reformulating or rescaling the model, since it is the modeler who has the best insight into the structure of the problem and can fix the cause of the issue.

### 8.2.1 Formulating problems

### **Scaling**

Make sure that all the data in the problem are of comparable orders of magnitude. This applies especially to the linear constraint matrix. Use Sec. 8.1.1 if necessary. For example a report such as

means that the ratio of largest to smallest elements in A is  $10^{11}$ . In this case the user should rescale or reformulate the model to avoid such spread which makes it difficult for **MOSEK** to scale the problem internally. In many cases it may be possible to change the units, i.e. express the model in terms of rescaled variables (for instance work with millions of dollars instead of dollars, etc.).

Similarly, if the objective contains very different coefficients, say

maximize 
$$10^{10}x + y$$

then it is likely to lead to inaccuracies. The objective will be dominated by the contribution from x and y will become insignificant.

#### Removing huge bounds

**Never** use a very large number as replacement for  $\infty$ . Instead define the variable or constraint as unbounded from below/above. Similarly, avoid artificial huge bounds if you expect they will not become tight in the optimal solution.

#### **Avoiding linear dependencies**

As much as possible try to avoid linear dependencies and near-linear dependencies in the model. See Example 8.3.

#### **Avoiding ill-posedness**

Avoid continuous models which are ill-posed: the solution space is degenerate, for example consists of a single point (technically, the Slater condition is not satisfied). In general, this refers to problems which are borderline between feasible and infeasible. See Example 8.1.

## Scaling the expected solution

Try to formulate the problem in such a way that the expected solution (both primal and dual) is not very large. Consult the solution summary Sec. 8.1.2 to check the objective values or solution norms.

### 8.2.2 Further suggestions

Here are other simple suggestions that can help locate the cause of the issues. They can also be used as hints for how to tune the optimizer if fixing the root causes of the issue is not possible.

- Remove the objective and solve the feasibility problem. This can reveal issues with the objective.
- Change the objective or change the objective sense from minimization to maximization (if applicable). If the two objective values are almost identical, this may indicate that the feasible set is very small, possibly degenerate.
- Perturb the data, for instance bounds, very slightly, and compare the results.
- $\bullet$  For linear problems: solve the problem using a different optimizer by setting the parameter  $MSK\_IPAR\_OPTIMIZER$  and compare the results.
- Force the optimizer to solve the primal/dual versions of the problem by setting the parameter <code>MSK\_IPAR\_INTPNT\_SOLVE\_FORM</code> or <code>MSK\_IPAR\_SIM\_SOLVE\_FORM</code>. **MOSEK** has a heuristic to decide whether to dualize, but for some problems the guess is wrong an explicit choice may give better results.
- Solve the problem without presolve or some of its parts by setting the parameter MSK\_IPAR\_PRESOLVE\_USE, see Sec. 13.1.
- Use different numbers of threads (MSK\_IPAR\_NUM\_THREADS) and compare the results. Very different results indicate numerical issues resulting from round-off errors.

If the problem was dumped to a file, experimenting with various parameters is facilitated with the **MOSEK** Command Line Tool or **MOSEK** Python Console Sec. 8.4.

### 8.2.3 Typical pitfalls

Example 8.1 (Ill-posedness). A toy example of this situation is the feasibility problem

$$(x-1)^2 < 1, (x+1)^2 < 1$$

whose only solution is x=0 and moreover replacing any 1 on the right hand side by  $1-\varepsilon$  makes the problem infeasible and replacing it by  $1+\varepsilon$  yields a problem whose solution set is an interval (fully-dimensional). This is an example of ill-posedness.

**Example 8.2** (Huge solution). If the norm of the expected solution is very large it may lead to numerical issues or infeasibility. For example the problem

$$(10^{-4}, x, 10^3) \in \mathcal{Q}_x^3$$

may be declared infeasible because the expected solution must satisfy  $x \ge 5 \cdot 10^9$ .

**Example 8.3** (Near linear dependency). Consider the following problem:

If we add the equalities together we obtain:

$$0 = \varepsilon$$

which is infeasible for any  $\varepsilon \neq 0$ . Here infeasibility is caused by a linear dependency in the constraint matrix coupled with a precision error represented by the  $\varepsilon$ . Indeed if a problem contains linear dependencies then the problem is either infeasible or contains redundant constraints. In the above case any of the equality constraints can be removed while not changing the set of feasible solutions. To summarize linear dependencies in the constraints can give rise to infeasible problems and therefore it is better to avoid them.

Example 8.4 (Presolving very tight bounds). Next consider the problem

Now the  $\mathbf{MOSEK}$  presolve will, for the sake of efficiency, fix variables (and constraints) that have tight bounds where tightness is controlled by the parameter  $MSK\_DPAR\_PRESOLVE\_TOL\_X$ . Since the bounds

$$-10^{-9} < x_1 < 10^{-9}$$

are tight, presolve will set  $x_1 = 0$ . It easy to see that this implies  $x_4 = 0$ , which leads to the incorrect conclusion that the problem is infeasible. However a tiny change of the value  $10^{-9}$  makes the problem feasible. In general it is recommended to avoid ill-posed problems, but if that is not possible then one solution is to reduce parameters such as  $MSK_DPAR_PRESOLVE_TOL_X$  to say  $10^{-10}$ . This will at least make sure that presolve does not make the undesired reduction.

# 8.3 Debugging infeasibility

This section contains hints for debugging problems that are unexpectedly infeasible. It is always a good idea to remove the objective, i.e. only solve a feasibility problem when debugging such issues.

## 8.3.1 Numerical issues

Infeasible problem status may be just an artifact of numerical issues appearing when the problem is badly-scaled, barely feasible or otherwise ill-conditioned so that it is unstable under small perturbations of the data or round-off errors. This may be visible in the solution summary if the infeasibility certificate has poor quality. See Sec. 8.1.2 for how to diagnose that and Sec. 8.2 for possible hints. Sec. 8.2.3 contains examples of situations which may lead to infeasibility for numerical reasons.

We refer to Sec. 8.2 for further information on dealing with those sort of issues. For the rest of this section we concentrate on the case when the solution summary leaves little doubt that the problem solved by the optimizer actually is infeasible.

### 8.3.2 Locating primal infeasibility

As an example of a primal infeasible problem consider minimizing the cost of transportation between a number of production plants and stores: Each plant produces a fixed number of goods, and each store has a fixed demand that must be met. Supply, demand and cost of transportation per unit are given in Fig. 8.1.

The problem represented in Fig. 8.1 is infeasible, since the total demand

$$2300 = 1100 + 200 + 500 + 500$$

exceeds the total supply

$$2200 = 200 + 1000 + 1000$$

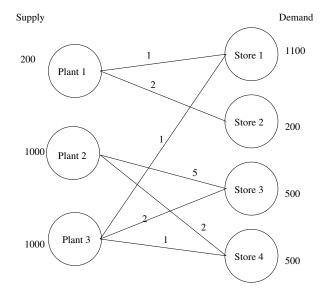


Fig. 8.1: Supply, demand and cost of transportation.

If we denote the number of transported goods from plant i to store j by  $x_{ij}$ , the problem can be formulated as the LP:

minimize 
$$x_{11} + 2x_{12} + 5x_{23} + 2x_{24} + x_{31} + 2x_{33} + x_{34}$$
 subject to  $s_0$ :  $x_{11} + x_{12}$   $\leq 200$ ,  $s_1$ :  $s_2$ :  $s_2$ :  $s_3$ :  $s_4$ :  $s_5$ :

Solving problem (8.1) using **MOSEK** will result in an infeasibility status. The infeasibility certificate is contained in the dual variables an can be accessed from an API. The variables and constraints with nonzero solution values form an infeasible subproblem, which frequently is very small. See Sec. 12.1.2 or Sec. 12.2.2 for detailed specifications of infeasibility certificates.

A short infeasibility report can also be printed to the log stream. It can be turned on by setting the parameter <code>MSK\_IPAR\_INFEAS\_REPORT\_AUTO</code> to <code>MSK\_ON</code>. This causes <code>MOSEK</code> to print a report on variables and constraints which are involved in infeasibility in the above sense, i.e. have nonzero values in the certificate. The parameter <code>MSK\_IPAR\_INFEAS\_REPORT\_LEVEL</code> controls the amount of information presented in the infeasibility report. The default value is 1. For the above example the report is

Problem status: The problem is primal infeasible						
The following constraints are involved in the primal infeasibility.						
Index	Name	Lower bound	Upper bound	Dual lower	Dual upper	
Index						
	s0	NONE	2.000000e+002	0.000000e+000	1.000000e+000	
)	s0 s2	NONE NONE	2.000000e+002 1.000000e+003	0.000000e+000 0.000000e+000	1.000000e+000 1.000000e+000	
nidex 0 2 3						

Index	Name	Lower bound	Upper bound	Dual lower	Dual upper
8	x33	0.000000e+000	NONE	1.000000e+000	0.000000e+000
10	x34	0.000000e+000	NONE	1.000000e+000	0.000000e+000

The infeasibility report is divided into two sections corresponding to constraints and variables. It is a selection of those lines from the problem solution which are important in understanding primal infeasibility. In this case the constraints s0, s2, d1, d2 and variables x33, x34 are of importance because of nonzero dual values. The columns Dual lower and Dual upper contain the values of dual variables  $s_t^c$ ,  $s_u^c$ ,  $s_u^t$ , and  $s_u^x$  in the primal infeasibility certificate (see Sec. 12.1.2).

In our example the certificate means that an appropriate linear combination of constraints s0, s1 with coefficient  $s_u^c = 1$ , constraints d1 and d2 with coefficient  $s_u^c - s_l^c = 0 - 1 = -1$  and lower bounds on x33 and x34 with coefficient  $-s_l^x = -1$  gives a contradiction. Indeed, the combination of the four involved constraints is  $x_{33} + x_{34} \le -100$  (as indicated in the introduction, the difference between supply and demand).

It is also possible to extract the infeasible subproblem with the command-line tool. For an infeasible problem called infeas.lp the command:

```
mosek -d MSK_IPAR_INFEAS_REPORT_AUTO MSK_ON infeas.lp -info rinfeas.lp
```

will produce the file rinfeas.bas.inf.lp which contains the infeasible subproblem. Because of its size it may be easier to work with than the original problem file.

Returning to the transportation example, we discover that removing the fifth constraint  $x_{12} = 200$  makes the problem feasible. Almost all undesired infeasibilities should be fixable at the modeling stage.

### 8.3.3 Locating dual infeasibility

A problem may also be  $dual\ infeasible$ . In this case the primal problem is usually unbounded, meaning that feasible solutions exists such that the objective tends towards infinity. For example, consider the problem

maximize 
$$\begin{array}{ll} \text{maximize} & 200y_1 + 1000y_2 + 1000y_3 + 1100y_4 + 200y_5 + 500y_6 + 500y_7 \\ \text{subject to} & y_1 + y_4 \leq 1, \ y_1 + y_5 \leq 2, \ y_2 + y_6 \leq 5, \ y_2 + y_7 \leq 2, \\ & y_3 + y_4 \leq 1, \ y_3 + y_6 \leq 2, \ y_3 + y_7 \leq 1 \\ & y_1, y_2, y_3 \leq 0 \end{array}$$

which is dual to (8.1) (and therefore is dual infeasible). The dual infeasibility report may look as follows:

MOSEK DUAL INFEASIBILITY REPORT.

Problem status: The problem is dual infeasible

The following constraints are involved in the infeasibility.

Index	Name	Activity	Objective	Lower bound	Upper⊔
⊶bound					
5	x33	-1.000000e+00		NONE	2.
<b>→</b> 000000€	e+00				
6	x34	-1.000000e+00		NONE	1.
<u></u> 000000€	e+00				

The following variables are involved in the infeasibility.

Index Name  ⇒bound	Activity	Objective	Lower bound	Upper <mark>⊔</mark>
0 y1 →000000e+00	-1.000000e+00	2.000000e+02	NONE	0.
2 y3 →000000e+00	-1.000000e+00	1.000000e+03	NONE	0.

3	у4	1.000000e	+00	1.100	000e+03	NO	NE		NONE	
4	у5	1.000000e	+00	2.000	000e+02	NO	NE		NONE	
Inte	Interior-point solution summary									
	Problem status : DUAL_INFEASIBLE									
Sol	Solution status : DUAL_INFEASIBLE_CER									
Pr	imal. obj: 1.00	000000000e+02	nrm:	1e+00	Viol.	con:	0e+00	var:	0e+00	

In the report we see that the variables y1, y3, y4, y5 and two constraints contribute to infeasibility with non-zero values in the Activity column. Therefore

$$(y_1,\ldots,y_7)=(-1,0,-1,1,1,0,0)$$

is the dual infeasibility certificate as in Sec. 12.1.2. This just means, that along the ray

$$(0,0,0,0,0,0,0) + t(y_1,\ldots,y_7) = (-t,0,-t,t,t,0,0), t>0,$$

which belongs to the feasible set, the objective value 100t can be arbitrarily large, i.e. the problem is unbounded.

In the example problem we could

- Add a lower bound on y3. This will directly invalidate the certificate of dual infeasibility.
- Increase the objective coefficient of y3. Changing the coefficients sufficiently will invalidate the inequality  $c^T y^* > 0$  and thus the certificate.

## 8.3.4 Suggestions

#### Primal infeasibility

When trying to understand what causes the unexpected primal infeasible status use the following hints:

- Remove the objective function. This does not change the infeasibility status but simplifies the problem, eliminating any possibility of issues related to the objective function.
- Remove cones, semidefinite variables and integer constraints. Solve only the linear part of the problem. Typical simple modeling errors will lead to infeasibility already at this stage.
- Consider whether your problem has some obvious necessary conditions for feasibility and examine if these are satisfied, e.g. total supply should be greater than or equal to total demand.
- Verify that coefficients and bounds are reasonably sized in your problem.
- See if there are any obvious contradictions, for instance a variable is bounded both in the variables and constraints section, and the bounds are contradictory.
- Consider replacing suspicious equality constraints by inequalities. For instance, instead of  $x_{12} = 200$  see what happens for  $x_{12} \ge 200$  or  $x_{12} \le 200$ .
- Relax bounds of the suspicious constraints or variables.
- For integer problems, remove integrality constraints on some/all variables and see if the problem solves
- Remember that variables without explicitly initialized bounds are fixed at zero.
- Form an **elastic model**: allow to violate constraints at a cost. Introduce slack variables and add them to the objective as penalty. For instance, suppose we have a constraint

minimize 
$$c^T x$$
, subject to  $a^T x \le b$ .

which might be causing infeasibility. Then create a new variable y and form the problem which contains:

Solving this problem will reveal by how much the constraint needs to be relaxed in order to become feasible. This is equivalent to inspecting the infeasibility certificate but may be more intuitive.

- If you think you have a feasible solution or its part, fix all or some of the variables to those values. Presolve will propagate them through the model and potentially reveal more localized sources of infeasibility.
- Dump the problem in OPF or LP format and verify that the problem that was passed to the optimizer corresponds to the problem expressed in the high-level modeling language, if any such was used.

#### **Dual infeasibility**

When trying to understand what causes the unexpected dual infeasible status use the following hints:

- Verify that the objective coefficients are reasonably sized.
- Check if no bounds and constraints are missing, for example if all variables that should be nonnegative have been declared as such etc.
- Strengthen bounds of the suspicious constraints or variables.
- Remember that constraints without explicitly initialized bounds are free (no bound).
- Form an series of models with decreasing bounds on the objective, that is, instead of objective

minimize 
$$c^T x$$

solve the problem with an additional constraint such as

$$c^T r = -10^5$$

and inspect the solution to figure out the mechanism behind arbitrarily decreasing objective values. This is equivalent to inspecting the infeasibility certificate but may be more intuitive.

• Dump the problem in OPF or LP format and verify that the problem that was passed to the optimizer corresponds to the problem expressed in the high-level modeling language, if any such was used.

Please note that modifying the problem to invalidate the reported certificate does *not* imply that the problem becomes feasible — the reason for infeasibility may simply *move*, resulting a problem that is still infeasible, but for a different reason. More often, the reported certificate can be used to give a hint about errors or inconsistencies in the model that produced the problem.

# 8.4 Python Console

The MOSEK Python Console is an alternative to the MOSEK Command Line Tool. It can be used for interactive loading, solving and debugging optimization problems stored in files, for example MOSEK task files. It facilitates debugging techniques described in Sec. 8.

### 8.4.1 Usage

The tool requires Python 2 or 3. The **MOSEK** interface for Python must be installed following the installation instructions for Python API or Python Fusion API. In the basic case it should be sufficient to execute the script

```
python setup.py install --user
```

in the directory containing the MOSEK Python module.

The Python Console is contained in the file mosekconsole.py in the folder with MOSEK binaries. It can be copied to an arbitrary location. The file is also available for download here (mosekconsole.py).

To run the console in interactive mode use

```
python mosekconsole.py
```

To run the console in batch mode provide a semicolon-separated list of commands as the second argument of the script, for example:

```
python mosekconsole.py "read data.task.gz; solve form=dual; writesol data"
```

The script is written using the **MOSEK** Python API and can be extended by the user if more specific functionality is required. We refer to the documentation of the Python API.

### 8.4.2 Examples

To read a problem from data.task.gz, solve it, and write solutions to data.sol, data.bas or data.itg:

```
read data.task.gz; solve; writesol data
```

To convert between file formats:

```
read data.task.gz; write data.mps
```

To set a parameter before solving:

```
read data.task.gz; param INTPNT_CO_TOL_DFEAS 1e-9; solve"
```

To list parameter values related to the mixed-integer optimizer in the task file:

```
read data.task.gz; param MIO
```

To print a summary of problem structure:

```
read data.task.gz; anapro
```

To solve a problem forcing the dual and switching off presolve:

```
read data.task.gz; solve form=dual presolve=no
```

To write an infeasible subproblem to a file for debugging purposes:

```
read data.task.gz; solve; infsub; write inf.opf
```

#### 8.4.3 Full list of commands

Below is a brief description of all the available commands. Detailed information about a specific command cmd and its options can be obtained with

```
help cmd
```

Table 8.1: List of commands of the MOSEK Python Console.

Command	Description
help [command]	Print list of commands or info about a specific command
log filename	Save the session to a file
intro	Print MOSEK splashscreen
testlic	Test the license system
read filename	Load problem from file
reread	Reload last problem file
solve	Solve current problem
[options]	
write filename	Write current problem to file
param [name	Set a parameter or get parameter values
[value]]	
paramdef	Set all parameters to default values
paramdiff	Show parameters with non-default values
info [name]	Get an information item
anapro	Analyze problem data
hist	Plot a histogram of problem data
histsol	Plot a histogram of the solutions
spy	Plot the sparsity pattern of the A matrix
truncate	Truncate small coefficients down to 0
epsilon	
resobj [fac]	Rescale objective by a factor
anasol	Analyze solutions
removeitg	Remove integrality constraints
removecones	Remove all cones and leave just the linear part
infsub	Replace current problem with its infeasible subproblem
writesol	Write solution(s) to file(s) with given basename
basename	
delsol	Remove all solutions from the task
optserver	Use an OptServer to optimize
[url]	
exit	Leave

# Chapter 9

# **Advanced Numerical Tutorials**

# 9.1 Solving Linear Systems Involving the Basis Matrix

A linear optimization problem always has an optimal solution which is also a basic solution. In an optimal basic solution there are exactly m basic variables where m is the number of rows in the constraint matrix A. Define

$$B \in \mathbb{R}^{m \times m}$$

as a matrix consisting of the columns of A corresponding to the basic variables. The basis matrix B is always non-singular, i.e.

$$det(B) \neq 0$$

or, equivalently,  $B^{-1}$  exists. This implies that the linear systems

$$B\bar{x} = w \tag{9.1}$$

and

$$B^T \bar{x} = w \tag{9.2}$$

each have a unique solution for all w.

**MOSEK** provides functions for solving the linear systems (9.1) and (9.2) for an arbitrary w. In the next sections we will show how to use **MOSEK** to

- identify the solution basis,
- solve arbitrary linear systems.

#### 9.1.1 Basis identification

To use the solutions to (9.1) and (9.2) it is important to know how the basis matrix B is constructed. Internally **MOSEK** employs the linear optimization problem

maximize 
$$c^{T}x$$
subject to 
$$Ax - x^{c} = 0,$$

$$l^{x} \leq x \leq u^{x},$$

$$l^{c} \leq x^{c} \leq u^{c}.$$

$$(9.3)$$

where

$$x^c \in \mathbb{R}^m$$
 and  $x \in \mathbb{R}^n$ .

The basis matrix is constructed of m columns taken from

$$\begin{bmatrix} A & -I \end{bmatrix}$$
.

If variable  $x_j$  is a basis variable, then the j-th column of A, denoted  $a_{:,j}$ , will appear in B. Similarly, if  $x_i^c$  is a basis variable, then the i-th column of -I will appear in the basis. The ordering of the basis variables and therefore the ordering of the columns of B is arbitrary. The ordering of the basis variables may be retrieved by calling the function  $MSK\_initbasissolve$ . This function initializes data structures for later use and returns the indexes of the basic variables in the array basis. The interpretation of the basis is as follows. If we have

then the i-th basis variable is

$$x_{\mathtt{basis}[i]}^c$$
.

Moreover, the *i*-th column in B will be the *i*-th column of -I. On the other hand if

$$basis[i] \ge numcon$$
,

then the *i*-th basis variable is the variable

$$x_{\mathtt{basis}[i]-\mathtt{numcon}}$$

and the i-th column of B is the column

$$A_{:,(\mathtt{basis}[i]-\mathtt{numcon})}$$
.

For instance if basis[0] = 4 and numcon = 5, then since basis[0] < numcon, the first basis variable is  $x_4^c$ . Therefore, the first column of B is the fourth column of -I. Similarly, if basis[1] = 7, then the second variable in the basis is  $x_{basis[1]-numcon} = x_2$ . Hence, the second column of B is identical to  $a_{:,2}$ .

#### An example

Consider the linear optimization problem:

minimize 
$$x_0 + x_1$$
  
subject to  $x_0 + 2x_1 \le 2$ ,  
 $x_0 + x_1 \le 6$ ,  
 $x_0, x_1 \ge 0$ . (9.4)

Suppose a call to MSK\_initbasissolve returns an array basis so that

```
basis[0] = 1,
basis[1] = 2.
```

Then the basis variables are  $x_1^c$  and  $x_0$  and the corresponding basis matrix B is

$$\left[\begin{array}{cc} 0 & 1 \\ -1 & 1 \end{array}\right].$$

Please note the ordering of the columns in B.

Listing 9.1: A program showing how to identify the basis.

```
{
 MSKenv_t
              env;
 MSKtask_t
              task;
 MSKint32t
              numcon = 2, numvar = 2;
 double
              c[] = \{1.0, 1.0\};
 MSKint32t
               ptrb[] = \{0, 2\},\
                       ptre[] = {2, 3};
               asub[] = \{0, 1, 0, 1\};
 MSKint32t
               aval[] = \{1.0, 1.0, 2.0, 1.0\};
 double
 MSKboundkeye bkc[] = { MSK_BK_UP, MSK_BK_UP };
 double
               blc[] = { -MSK_INFINITY, -MSK_INFINITY };
 double
               buc[] = \{2.0, 6.0\};
 MSKboundkeye bkx[] = { MSK_BK_LO, MSK_BK_LO };
               blx[] = \{0.0, 0.0\};
               bux[] = { +MSK_INFINITY, +MSK_INFINITY};
 double
 MSKrescodee r
                      = MSK_RES_OK;
              i, nz;
 MSKint32t
              w[] = \{2.0, 6.0\};
 double
             sub[] = \{0, 1\};
 MSKint32t
 MSKint32t *basis;
 if (r == MSK_RES_OK)
   r = MSK_makeenv(&env, NULL);
 if (r == MSK_RES_OK)
   r = MSK_makeemptytask(env, &task);
 if (r == MSK_RES_OK)
   MSK_linkfunctotaskstream(task, MSK_STREAM_LOG, NULL, printstr);
 if (r == MSK_RES_OK)
   r = MSK_inputdata(task, numcon, numvar, numcon, numvar,
                      c, 0.0,
                      ptrb, ptre, asub, aval, bkc, blc, buc, bkx, blx, bux);
 if (r == MSK_RES_OK)
   r = MSK_putobjsense(task, MSK_OBJECTIVE_SENSE_MAXIMIZE);
 if (r == MSK_RES_OK)
   r = MSK_optimizetrm(task, NULL);
 if (r == MSK_RES_OK)
   basis = MSK_calloctask(task, numcon, sizeof(MSKint32t));
 if (r == MSK_RES_OK)
   r = MSK_initbasissolve(task, basis);
  /* List basis variables corresponding to columns of B */
 for (i = 0; i < numcon && r == MSK_RES_OK; ++i)
   printf("basis[%d] = %d\n", i, basis[i]);
   if (basis[sub[i]] < numcon)</pre>
     printf("Basis variable no %d is xc%d.\n", i, basis[i]);
   else
     printf("Basis variable no %d is x%d.\n", i, basis[i] - numcon);
```

```
}
 nz = 2;
  /* solve Bx = w */
  /* sub contains index of non-zeros in w.
     On return w contains the solution x and sub
     the index of the non-zeros in x.
 if (r == MSK_RES_OK)
   r = MSK_solvewithbasis(task, 0, &nz, sub, w);
 if (r == MSK_RES_OK)
   printf("\nSolution to Bx = w:\n\n");
   /* Print solution and b. */
   for (i = 0; i < nz; ++i)
      if (basis[sub[i]] < numcon)</pre>
        printf("xc%d = %e\n", basis[sub[i]] , w[sub[i]]);
      else
        printf("x\%d = \%e\n", basis[sub[i]] - numcon, w[sub[i]]);
   }
 }
  /* Solve B^T y = w */
       = 1; /* Only one element in sub is nonzero. */
 sub[0] = 1;
                 /* Only w[1] is nonzero. */
 w[0]
       = 0.0;
 w[1]
       = 1.0;
 if (r == MSK_RES_OK)
   r = MSK_solvewithbasis(task, 1, &nz, sub, w);
 if (r == MSK_RES_OK)
   printf("\nSolution to B^T y = w: \ln n);
   /* Print solution and y. */
   for (i = 0; i < nz; ++i)
     printf("y%d = %e\n", sub[i], w[sub[i]]);
 return (r);
}/* main */
```

In the example above the linear system is solved using the optimal basis for (9.4) and the original right-hand side of the problem. Thus the solution to the linear system is the optimal solution to the problem. When running the example program the following output is produced.

```
basis[0] = 1
Basis variable no 0 is xc1.
basis[1] = 2
Basis variable no 1 is x0.
Solution to Bx = b:
```

```
x0 = 2.000000e+00

xc1 = -4.000000e+00

Solution to B^Tx = c:

x1 = -1.000000e+00

x0 = 1.000000e+00
```

Please note that the ordering of the basis variables is

$$\left[\begin{array}{c} x_1^c \\ x_0 \end{array}\right]$$

and thus the basis is given by:

$$B = \left[ \begin{array}{cc} 0 & 1 \\ -1 & 1 \end{array} \right]$$

It can be verified that

$$\left[\begin{array}{c} x_1^c \\ x_0 \end{array}\right] = \left[\begin{array}{c} -4 \\ 2 \end{array}\right]$$

is a solution to

$$\left[\begin{array}{cc} 0 & 1 \\ -1 & 1 \end{array}\right] \left[\begin{array}{c} x_1^c \\ x_0 \end{array}\right] = \left[\begin{array}{c} 2 \\ 6 \end{array}\right].$$

## 9.1.2 Solving arbitrary linear systems

MOSEK can be used to solve an arbitrary (rectangular) linear system

$$Ax = b$$

using the  $MSK\_solvewithbasis$  function without optimizing the problem as in the previous example. This is done by setting up an A matrix in the task, setting all variables to basic and calling the  $MSK\_solvewithbasis$  function with the b vector as input. The solution is returned by the function.

#### An example

Below we demonstrate how to solve the linear system

$$\begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$
 (9.5)

with two inputs b = (1, -2) and b = (7, 0).

```
MSKint32t *ptre,
                        MSKint32t numvar,
                         MSKint32t *basis)
{
 MSKint32t
            i, j;
 MSKrescodee r = MSK_RES_OK;
 MSKstakeye *skx = NULL , *skc = NULL;
 skx = (MSKstakeye *) calloc(numvar, sizeof(MSKstakeye));
 if (skx == NULL && numvar)
   r = MSK_RES_ERR_SPACE;
 skc = (MSKstakeye *) calloc(numvar, sizeof(MSKstakeye));
 if (skc == NULL && numvar)
   r = MSK_RES_ERR_SPACE;
 for (i = 0; i < numvar && r == MSK_RES_OK; ++i)
   skx[i] = MSK_SK_BAS;
   skc[i] = MSK_SK_FIX;
 }
 /* Create a coefficient matrix and right hand
    side with the data from the linear system */
 if (r == MSK_RES_OK)
   r = MSK_appendvars(task, numvar);
 if (r == MSK_RES_OK)
   r = MSK_appendcons(task, numvar);
 for (i = 0; i < numvar && r == MSK_RES_OK; ++i)
   r = MSK_putacol(task, i, ptre[i] - ptrb[i], asub + ptrb[i], aval + ptrb[i]);
 for (i = 0; i < numvar && r == MSK_RES_OK; ++i)
   r = MSK_putconbound(task, i, MSK_BK_FX, 0, 0);
 for (i = 0; i < numvar && r == MSK_RES_OK; ++i)
   r = MSK_putvarbound(task, i, MSK_BK_FR, -MSK_INFINITY);
  /* Allocate space for the solution and set status to unknown */
 if (r == MSK_RES_OK)
   r = MSK_deletesolution(task, MSK_SOL_BAS);
 /* Setup status keys. That is all that is needed. */
 if (r == MSK_RES_OK)
   r = MSK_putskcslice(task, MSK_SOL_BAS, 0, numvar, skc);
 if (r == MSK_RES_OK)
   r = MSK_putskxslice(task, MSK_SOL_BAS, 0, numvar, skx);
 if (r == MSK_RES_OK)
   r = MSK_initbasissolve(task, basis);
 free(skx);
```

```
free(skc);
  return (r);
}
#define NUMCON 2
#define NUMVAR 2
int main(int argc, const char *argv[])
 const MSKint32t numvar = NUMCON,
                 numcon = NUMVAR; /* we must have numvar == numcon */
  MSKenv_t
                 env;
  MSKtask_t
                 task;
  MSKrescodee
                 r = MSK_RES_OK;
  MSKint32t
                 i, nz;
                  aval[] = \{-1.0, 1.0, 1.0\};
  double
 MSKint32t asub[] = {1, 0, 1};

MSKint32t ptrb[] = {0, 1};

MSKint32t ptre[] = {1, 3};
  MSKint32t
                 bsub[NUMCON];
                 b[NUMCON];
  MSKint32t
                  *basis = NULL;
  if (r == MSK_RES_OK)
   r = MSK_makeenv(&env, NULL);
  if (r == MSK_RES_OK)
   r = MSK_makeemptytask(env, &task);
  if (r == MSK_RES_OK)
    MSK_linkfunctotaskstream(task, MSK_STREAM_LOG, NULL, printstr);
  basis = (MSKint32t *) calloc(numcon, sizeof(MSKint32t));
  if (basis == NULL && numvar)
   r = MSK_RES_ERR_SPACE;
  /* setup: Put A matrix and factor A.
            Call this function only once for a given task. */
  if (r == MSK_RES_OK)
   r = setup(task,
              aval,
              asub,
              ptrb,
              ptre,
              numvar,
              basis
             );
  /* now solve rhs */
  b[0] = 1;
  b[1] = -2;
  bsub[0] = 0;
  bsub[1] = 1;
  nz = 2;
```

```
if (r == MSK_RES_OK)
    r = MSK_solvewithbasis(task, 0, &nz, bsub, b);
 if (r == MSK_RES_OK)
    printf("\nSolution to Bx = b: \n\n");
    /* Print solution and show correspondents
       to original variables in the problem */
    for (i = 0; i < nz; ++i)
      if (basis[bsub[i]] < numcon)</pre>
        printf("This should never happen\n");
        printf("x\%d = \%e\n", basis[bsub[i]] - numcon, b[bsub[i]]);
    }
 }
 b[0] = 7;
 bsub[0] = 0;
 nz = 1;
 if (r == MSK_RES_OK)
    r = MSK_solvewithbasis(task, 0, &nz, bsub, b);
 if (r == MSK_RES_OK)
    printf("\nSolution to Bx = b: \n\n");
    /* Print solution and show correspondents
       to original variables in the problem */
    for (i = 0; i < nz; ++i)
      if (basis[bsub[i]] < numcon)</pre>
        printf("This should never happen\n");
      else
        printf("x\%d = \%e\n", basis[bsub[i]] - numcon , b[bsub[i]]);
    }
 }
 free(basis);
 return r;
}
```

The most important step in the above example is the definition of the basic solution, where we define the status key for each variable. The actual values of the variables are not important and can be selected arbitrarily, so we set them to zero. All variables corresponding to columns in the linear system we want to solve are set to basic and the slack variables for the constraints, which are all non-basic, are set to their bound.

The program produces the output:

```
Solution to Bx = b:

x1 = 1
x0 = 3

Solution to Bx = b:
```

```
\begin{array}{rcl}
x1 &=& 7 \\
x0 &=& 7
\end{array}
```

# 9.2 Calling BLAS/LAPACK Routines from MOSEK

Sometimes users need to perform linear algebra operations that involve dense matrices and vectors. Also **MOSEK** extensively uses high-performance linear algebra routines from the BLAS and LAPACK packages and some of these routines are included in the package shipped to the users.

The **MOSEK** versions of BLAS/LAPACK routines:

- use MOSEK data types and return value conventions,
- preserve the BLAS/LAPACK naming convention.

Therefore the user can leverage on efficient linear algebra routines, with a simplified interface, with no need for additional packages.

#### List of available routines

BLAS Name	MOSEK function	Math Expression
AXPY	$MSK\_axpy$	$y = \alpha x + y$
DOT	MSK_dot	$x^Ty$
GEMV	MSK_gemv	$y = \alpha Ax + \beta y$
GEMM	MSK_gemm	$C = \alpha AB + \beta C$
SVRK	MCK sumb	$C - \alpha A A^T + \beta C$

Table 9.1: BLAS routines available.

Table 9.2: LAPACK routines available.

LAPACK Name	MOSEK function	Description
POTRF	MSK_potrf	Cholesky factorization of a semidefinite symmetric matrix
SYEVD	MSK_syevd	Eigenvalues and eigenvectors of a symmetric matrix
SYEIG	MSK_syeig	Eigenvalues of a symmetric matrix

#### Source code examples

In Listing 9.2 we provide a simple working example. It has no practical meaning except showing how to organize the input and call the methods.

Listing 9.2: Calling BLAS and LAPACK routines from Optimizer API for  $\mathcal C.$ 

```
#include "mosek.h"
void print_matrix(MSKrealt* x, MSKint32t r, MSKint32t c)
{
    MSKint32t i, j;
    for (i = 0; i < r; i++)
    {
        for (j = 0; j < c; j++)
            printf("%f ", x[j * r + i]);
    }
}</pre>
```

```
int main(int argc, char* argv[])
 MSKrescodee r = MSK_RES_OK;
 MSKenv_t env = NULL;
 const MSKint32t n = 3, m = 2, k = 3;
 MSKrealt alpha = 2.0, beta = 0.5;
 MSKrealt x[]
                = \{1.0, 1.0, 1.0\};
 MSKrealt y[]
                = \{1.0, 2.0, 3.0\};
 MSKrealt z[] = \{1.0, 1.0\};
               = \{1.0, 1.0, 2.0, 2.0, 3.0, 3.0\};
 MSKrealt A[]
 MSKrealt B[]
               = \{1.0, 2.0, 3.0, 4.0, 5.0, 6.0\};
 MSKrealt C[]
 MSKrealt D[]
               = \{1.0, 1.0, 1.0, 1.0\};
              = {1.0, 0.0, 0.0, 2.0};
 MSKrealt Q[]
 MSKrealt v[]
                = \{0.0, 0.0, 0.0\};
 MSKrealt xy;
 /* BLAS routines*/
 r = MSK_makeenv(&env, NULL);
 printf("n=%d m=%d k=%d\n", m, n, k);
 printf("alpha=%f\n", alpha);
 printf("beta=%f\n", beta);
 r = MSK_dot(env, n, x, y, &xy);
 printf("dot results= %f r=%d\n", xy, r);
 print_matrix(x, 1, n);
 print_matrix(y, 1, n);
 r = MSK_axpy(env, n, alpha, x, y);
 puts("axpy results is");
 print_matrix(y, 1, n);
 r = MSK_gemv(env, MSK_TRANSPOSE_NO, m, n, alpha, A, x, beta, z);
 printf("gemv results is (r=\%d) \n", r);
 print_matrix(z, 1, m);
 r = MSK_gemm(env, MSK_TRANSPOSE_NO, MSK_TRANSPOSE_NO, m, n, k, alpha, A, B, beta, u
C);
 printf("gemm results is (r=%d) \n", r);
 print_matrix(C, m, n);
 r = MSK_syrk(env, MSK_UPLO_LO, MSK_TRANSPOSE_NO, m, k, 1., A, beta, D);
 printf("syrk results is (r=%d) \n", r);
 print_matrix(D, m, m);
 /* LAPACK routines*/
 r = MSK_potrf(env, MSK_UPLO_LO, m, Q);
```

```
printf("potrf results is (r=%d) \n", r);
print_matrix(Q, m, m);

r = MSK_syeig(env, MSK_UPLO_LO, m, Q, v);
printf("syeig results is (r=%d) \n", r);
print_matrix(v, 1, m);

r = MSK_syevd(env, MSK_UPLO_LO, m, Q, v);
printf("syevd results is (r=%d) \n", r);
print_matrix(v, 1, m);
print_matrix(Q, m, m);

/* Delete the environment and the associated data. */
MSK_deleteenv(&env);

return r;
}
```

## 9.3 Computing a Sparse Cholesky Factorization

Given a positive semidefinite symmetric (PSD) matrix

$$A \in \mathbb{R}^{n \times n}$$

it is well known there exists a matrix L such that

$$A = LL^T.$$

If the matrix L is lower triangular then it is called a *Cholesky factorization*. Given A is positive definite (nonsingular) then L is also nonsingular. A Cholesky factorization is useful for many reasons:

- A system of linear equations Ax = b can be solved by first solving the lower triangular system Ly = b followed by the upper triangular system  $L^Tx = y$ .
- A quadratic term  $x^T A x$  in a constraint or objective can be replaced with  $y^T y$  for  $y = L^T x$ , potentially leading to a more robust formulation (see [And13]).

Therefore, **MOSEK** provides a function that can compute a Cholesky factorization of a PSD matrix. In addition a function for solving linear systems with a nonsingular lower or upper triangular matrix is available.

In practice A may be very large with n is in the range of millions. However, then A is typically sparse which means that most of the elements in A are zero, and sparsity can be exploited to reduce the cost of computing the Cholesky factorization. The computational savings depend on the positions of zeros in A. For example, below a matrix A is given together with a Cholesky factor up to 5 digits of accuracy:

$$A = \begin{bmatrix} 4 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}, \quad L = \begin{bmatrix} 2.0000 & 0 & 0 & 0 \\ 0.5000 & 0.8660 & 0 & 0 \\ 0.5000 & -0.2887 & 0.8165 & 0 \\ 0.5000 & -0.2887 & -0.4082 & 0.7071 \end{bmatrix}.$$
(9.6)

However, if we symmetrically permute the rows and columns of A using a permutation matrix P

$$P = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}, \quad A' = PAP^T = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 4 \end{bmatrix},$$

then the Cholesky factorization of  $A' = L'L'^T$  is

$$L' = \left[ \begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{array} \right]$$

which is sparser than L.

Computing a permutation matrix that leads to the sparsest Cholesky factorization or the minimal amount of work is NP-hard. Good permutations can be chosen by using heuristics, such as the minimum degree heuristic and variants. The function  $MSK\_computesparsecholesky$  provided by MOSEK for computing a Cholesky factorization has a build in permutation aka. reordering heuristic. The following code illustrates the use of  $MSK\_computesparsecholesky$  and  $MSK\_sparsetriangularsolvedense$ .

Listing 9.3: How to use the sparse Cholesky factorization routine available in **MOSEK**.

```
r = MSK_computesparsecholesky(env,
                                             /* Disable multithreading */
                                            /* Apply a reordering heuristic */
                                           /* Singularity tolerance */
                                 n, anzc, aptrc, asubc, avalc,
                                  &perm, &diag, &lnzc, &lptrc, &lensubnval, &lsubc, &
→lvalc);
   if (r == MSK_RES_OK)
     MSKint32t i, j;
     MSKrealt *x;
     printsparse(n, perm, diag, lnzc, lptrc, lensubnval, lsubc, lvalc);
     x = MSK_callocenv(env, n, sizeof(MSKrealt));
     if (x)
       /* Permuted b is stored as x. */
       for (i = 0; i < n; ++i) x[i] = b[perm[i]];
       /* Compute inv(L)*x. */
       r = MSK_sparsetriangularsolvedense(env, MSK_TRANSPOSE_NO, n,
                                           lnzc, lptrc, lensubnval, lsubc, lvalc, x);
       if (r == MSK_RES_OK) {
         /* Compute inv(L^T)*x. */
         r = MSK_sparsetriangularsolvedense(env, MSK_TRANSPOSE_YES, n,
                                             lnzc, lptrc, lensubnval, lsubc, lvalc,
→x);
         printf("\nSolution A x = b, x = [ ");
         for (i = 0; i < n; i++)
           for (j = 0; j < n; j++) if (perm[j] == i) printf("%.2f ", x[j]);
         printf("]\n");
       }
       MSK_freeenv(env, x);
     }
     else
       printf("Out of space while creating x.\n");
   }
   else
     printf("Cholesky computation failed: %d\n", (int) r);
```

We can set up the data to recreate the matrix A from (9.6):

```
// Data from the example in the text
   const MSKint32t n
   //Observe that anzc, aptrc, asubc and avalc only specify the lower triangularu
\rightarrow part.
   const MSKint32t anzc[] = \{4, 1, 1, 1\},
                    asubc[] = {0, 1, 2, 3, 1, 2, 3};
   const MSKint64t aptrc[] = \{0, 4, 5, 6\};
   const MSKrealt avalc[] = {4.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0},
                    b[]
                           = \{13.0, 3.0, 4.0, 5.0\};
                    *perm = NULL, *lnzc = NULL, *lsubc = NULL;
   MSKint32t
   MSKint64t
                    lensubnval, *lptrc = NULL;
                    *diag = NULL, *lvalc = NULL;
   MSKrealt
```

and we obtain the following output:

The output indicates that with the permutation matrix

$$P = \left[ \begin{array}{cccc} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{array} \right]$$

there is a Cholesky factorization  $PAP^T = LL^T$ , where

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1.4142 & 0 \\ 0 & 0 & 0.7071 & 0.7071 \end{bmatrix}$$

The remaining part of the code solvers the linear system Ax = b for  $b = [13, 3, 4, 5]^T$ . The solution is reported to be  $x = [1, 2, 3, 4]^T$ , which is correct.

The second example shows what happens when we compute a sparse Cholesky factorization of a singular matrix. In this example A is a rank 1 matrix

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}^{T}$$

$$(9.7)$$

Now we get the output

```
P = [ 0 2 1 ]
diag(D) = [ 0.00e+00 1.00e-14 1.00e-14 ]
L=
1.00e+00 0.00e+00 0.00e+00
1.00e+00 1.00e-07 0.00e+00
1.00e+00 0.00e+00 1.00e-07
```

which indicates the decomposition

$$PAP^T = LL^T - D$$

where

$$P = \left[ \begin{array}{ccc} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{array} \right], \quad L = \left[ \begin{array}{ccc} 1 & 0 & 0 \\ 1 & 10^{-7} & 0 \\ 1 & 0 & 10^{-7} \end{array} \right], \quad D = \left[ \begin{array}{ccc} 1 & 0 & 0 \\ 0 & 10^{-14} & 0 \\ 0 & 0 & 10^{-14} \end{array} \right].$$

Since A is only positive semdefinite, but not of full rank, some of diagonal elements of A are boosted to make it truely positive definite. The amount of boosting is passed as an argument to  $MSK\_computesparsecholesky$ , in this case  $10^{-14}$ . Note that

$$PAP^T = LL^T - D$$

where D is a small matrix so the computed Cholesky factorization is exact of slightly perturbed A. In general this is the best we can hope for in finite precision and when A is singular or close to being singular.

We will end this section by a word of caution. Computing a Cholesky factorization of a matrix that is not of full rank and that is not suffciently well conditioned may lead to incorrect results i.e. a matrix that is indefinite may declared positive semidefinite and vice versa.

# 9.4 Converting a quadratically constrained problem to conic form

MOSEK employs the following form of quadratic problems:

A conic quadratic constraint has the form

$$x \in \mathcal{Q}^n$$

in its most basic form where

$$Q^{n} = \left\{ x \in \mathbb{R}^{n} : x_{1} \ge \sqrt{\sum_{j=2}^{n} x_{j}^{2}} \right\}.$$

A quadratic problem such as (9.8), if convex, can be reformulated in conic form. This is in fact the reformulation **MOSEK** performs internally. It has many advantages:

- elegant duality theory for conic problems,
- reporting accurate dual information for quadratic inequalities is hard and/or computational expensive,
- it certifies that the original quadratic problem is indeed convex,
- modeling directly in conic form usually leads to a better model [And13] i.e. a faster solution time and better numerical properties.

In addition, there are more types of conic constraints that can be combined with a quadratic cone, for example semidefinite cones.

**MOSEK** offers a function that performs the conversion from quadratic to conic quadratic form explicitly. Note that the reformulation is not unique. The approach followed by **MOSEK** is to introduce additional variables, linear constraints and quadratic cones to obtain a larger but equivalent problem in which the original variables are preserved.

In particular:

- all variables and constraints are kept in the problem,
- each quadratic constraint and quadratic terms in the objective generate one rotated quadratic cone,
- each quadratic constraint will contain no coefficients and upper/lower bounds will be set to  $\infty, -\infty$  respectively.

This allows the user to recover the original variable and constraint values, as well as their dual values, with no conversion or additional effort.

**Note:** *MSK\_toconic* modifies the input task in-place: this means that if the reformulation is not possible, i.e. the problem is not conic representable, the state of the task is in general undefined. The user should consider cloning the original task.

### 9.4.1 Quadratic Constraint Reformulation

Let us assume we want to convert the following quadratic constraint

$$l \le \frac{1}{2}x^TQx + \sum_{j=0}^{n-1} a_j x_j \le u$$

to conic form. We first check whether  $l = -\infty$  or  $u = \infty$ , otherwise either the constraint can be dropped, or the constraint is not convex. Thus let us consider the case

$$\frac{1}{2}x^T Q x + \sum_{j=0}^{n-1} a_j^T x_j \le u. \tag{9.9}$$

Introducing an additional variable w such that

$$w = u - \sum_{j=0}^{n-1} a_j^T x_j \tag{9.10}$$

we obtain the equivalent form

$$\begin{array}{rcl} \frac{1}{2}x^TQx & \leq & w, \\ u - \sum_{j=0}^{n-1} a_j x_j & = & w. \end{array}$$

If Q is positive semidefinite, then there exists a matrix F such that

$$Q = FF^T (9.11)$$

and therefore we can write

$$||Fx||^2 \le 2w,$$
  
 $u - \sum_{j=0}^{n-1} a_j^T x_j = w.$ 

Introducing an additional variable z = 1, and setting y = Fx we obtain the conic formulation

$$(w, z, y) \in \mathcal{Q}_r,$$

$$z = 1$$

$$y = Fx$$

$$w = y - a^T x.$$

$$(9.12)$$

Summarizing, for each quadratic constraint involving t variables, MOSEK introduces

- 1. a rotated quadratic cone of dimension t+2,
- 2. two additional variables for the cone roots,
- $3. \ t$  additional variables to map the remaining part of the cone,
- 4. t linear constraints.

A quadratic term in the objective is reformulated in a similar fashion. We refer to [And13] for a more thorough discussion.

#### **Example**

Next we consider a simple problem with quadratic objective function:

```
minimize \frac{1}{2}(13x_0^2 + 17x_1^2 + 12x_2^2 + 24x_0x_1 + 12x_1x_2 - 4x_0x_2) - 22x_0 - 14.5x_1 + 12x_2 + 1 subject to -1 \le x_0, x_1, x_2 \le 1
```

We can specify it in the human-readable OPF format.

```
[comment]
An example of small QO problem from Boyd and Vandenberghe, "Convex Optimization", page 189 ex 4.3
The solution is (1,0.5,-1)
[/comment]
[variables]
x0 x1 x2
[/variables]
[objective min]
0.5 (13 x0^2 + 17 x1^2 + 12 x2^2 + 24 x0 * x1 + 12 x1 * x2 - 4 x0 * x2 ) - 22 x0 - 14.5 x1 + 12 x2 + 1
[/objective]

[bounds]
[b] -1 <= * <= 1 [/b]
[/bounds]
```

The objective function is convex, the minimum is attained for  $x^* = (1, 0.5, -1)$ . The conversion will introduce first a variable  $x_3$  in the objective function such that  $x_3 \ge 1/2x^TQx$  and then convert the latter directly in conic form. The converted problem follows:

```
\begin{array}{ll} \text{minimize} & -22x_0 - 14.5x_1 + 12x_2 + x_3 + 1 \\ \text{subject to} & 3.61x_0 + 3.33x_1 - 0.55x_2 - x_6 = 0 \\ & +2.29x_1 + 3.42x_2 - x_7 = 0 \\ & 0.81x_1 - x_8 = 0 \\ & -x_3 + x_4 = 0 \\ & x_5 = 1 \\ & (x_4, x_5, x_6, x_7, x_8) \in \mathcal{Q}_\nabla \\ & -1 \leq x_0, x_1, x_2 \leq 1 \end{array}
```

The model generated by  $MSK\_toconic$  is

```
[comment]
  Written by MOSEK version 8.1.0.19
  Date 21-08-17
  Time 10:53:36
[/comment]
[hints]
```

```
[hint NUMVAR] 9 [/hint]
  [hint NUMCON] 4 [/hint]
  [hint NUMANZ] 11 [/hint]
  [hint NUMQNZ] 0 [/hint]
  [hint NUMCONE] 1 [/hint]
[/hints]
[variables disallow_new_variables]
 x0000_x0 x0001_x1 x0002_x2 x0003 x0004
 x0005 x0006 x0007 x0008
[/variables]
[objective minimize]
   - 2.2e+01 x0000_x0 - 1.45e+01 x0001_x1 + 1.2e+01 x0002_x2 + x0003
[/objective]
[constraints]
 [con c0000] 3.605551275463989e+00 x0000_x0 - 5.547001962252291e-01 x0002_x2 + 3.
\Rightarrow328201177351375e+00 x0001_x1 - x0006 = 0e+00 [/con]
 [con c0001] 3.419401657060442e+00 x0002_x2 + 2.294598480395823e+00 x0001_x1 -__
\rightarrowx0007 = 0e+00 [/con]
 [con c0002] 8.111071056538127e-01 \times 0001_x1 - x0008 = 0e+00 [/con]
 [con c0003] - x0003 + x0004 = 0e+00 [/con]
[/constraints]
[bounds]
 [b] -1e+00
                 <= x0000_x0,x0001_x1,x0002_x2 <= 1e+00 [/b]</pre>
 [b]
                    x0003,x0004 free [/b]
  [b]
                    x0005 = 1e+00 [/b]
                    x0006,x0007,x0008 free [/b]
  [cone rquad k0000] x0004, x0005, x0006, x0007, x0008 [/cone]
[/bounds]
```

We can clearly see that constraints c0000, c0001 and c0002 represent the original linear constraints as in (9.11), while c0003 corresponds to (9.10). The cone roots are x0005 and x0004.

# Chapter 10

# Technical guidelines

This section contains some more in-depth technical guidelines for Optimizer API for C, not strictly necessary for basic use of MOSEK.

# 10.1 Memory management and garbage collection

Users who experience memory leaks, especially:

- memory usage not decreasing after the solver terminates,
- memory usage increasing when solving a sequence of problems,

should make sure that the memory used by the task is released when the task is no longer needed. This is done with the method  $MSK\_deletetask$ . The same applies to the environment when it is no longer needed.

```
MSK_deletetask(&task);
MSK_deleteenv(&env);
```

#### **10.2** Names

All elements of an optimization problem in **MOSEK** (objective, constraints, variables, etc.) can be given names. Assigning meaningful names to variables and constraints makes it much easier to understand and debug optimization problems dumped to a file. On the other hand, note that assigning names can substantially increase setup time, so it should be avoided in time-critical applications.

Names of various elements of the problem can be set and retrieved using various functions listed in the **Names** section of Sec. 15.2.

Note that file formats impose various restrictions on names, so not all names can be written verbatim to each type of file. If at least one name cannot be written to a given format then generic names and substitutions of offending characters will be used when saving to a file, resulting in a transformation of all names in the problem. See Sec. 16.

# 10.3 Multithreading

#### Thread safety

Sharing a task between threads is safe, as long as it is not accessed from more than one thread at a time. Multiple tasks can be created and used in parallel without any problems.

#### **Parallelization**

The interior-point and mixed-integer optimizers in **MOSEK** are parallelized. By default **MOSEK** will automatically select the number of threads. However, the maximum number of threads allowed can be changed by setting the parameter <code>MSK\_IPAR\_NUM\_THREADS</code> and related parameters. This should never exceed the number of cores. See Sec. 13 and Sec. 13.4 for more details.

The speed-up obtained when using multiple threads is highly problem and hardware dependent. We recommend experimenting with various thread numbers to determine the optimal settings. For small problems using multiple threads may be counter-productive because of the associated overhead.

#### **Determinism**

By default the optimizer is run-to-run deterministic, which means that it will return the same answer each time it is run on the same machine with the same input, the same parameter settings (including number of threads) and no time limits.

#### Setting the number of threads

The number of threads the optimizer uses can be changed with the parameter  $MSK\_IPAR\_NUM\_THREADS$ . For conic problems (when the conic optimizer is used) the value set at the first optimization will remain fixed through the lifetime of the process. The thread pool will be reserved once for all and subsequent changes to  $MSK\_IPAR\_NUM\_THREADS$  will have no effect. The only possibility at that point is to switch between multi-threaded and single-threaded interior-point optimization using the parameter  $MSK\_IPAR\_INTPNT\_MULTI\_THREAD$ .

# 10.4 Efficiency

Although **MOSEK** is implemented to handle memory efficiently, the user may have valuable knowledge about a problem, which could be used to improve the performance of **MOSEK** This section discusses some tricks and general advice that hopefully make **MOSEK** process your problem faster.

#### Reduce the number of function calls and avoid input loops

For example, instead of setting the entries in the linear constraint matrix one by one (MSK\_putaij) define them all at once (MSK\_putaijlist) or in convenient large chunks (MSK\_putacollist etc.)

#### Use one environment only

If possible share the environment between several tasks. For most applications you need to create only a single environment.

#### Read part of the solution

When fetching the solution, data has to be copied from the optimizer to the user's data structures. Instead of fetching the whole solution, consider fetching only the interesting part (see for example  $MSK\_getxxslice$  and similar).

#### **Avoiding memory fragmentation**

MOSEK stores the optimization problem in internal data structures in the memory. Initially MOSEK will allocate structures of a certain size, and as more items are added to the problem the structures are reallocated. For large problems the same structures may be reallocated many times causing memory fragmentation. One way to avoid this is to give MOSEK an estimated size of your problem using the functions:

- MSK\_putmaxnumvar. Estimate for the number of variables.
- MSK\_putmaxnumcon. Estimate for the number of constraints.
- MSK\_putmaxnumcone. Estimate for the number of cones.

- MSK\_putmaxnumbarvar. Estimate for the number of semidefinite matrix variables.
- $MSK_putmaxnumanz$ . Estimate for the number of non-zeros in A.
- MSK\_putmaxnumqnz. Estimate for the number of non-zeros in the quadratic terms.

None of these functions changes the problem, they only serve as hints. If the problem ends up growing larger, the estimates are automatically increased.

#### Do not mix put- and get- functions

MOSEK will queue put- requests internally until a get- function is called. If put- and get- calls are interleaved, the queue will have to be flushed more frequently, decreasing efficiency.

In general get- commands should not be called often (or at all) during problem setup.

#### Use the LIFO principle

When removing constraints and variables, try to use a LIFO (Last In First Out) approach. MOSEK can more efficiently remove constraints and variables with a high index than a small index.

An alternative to removing a constraint or a variable is to fix it at 0, and set all relevant coefficients to 0. Generally this will not have any impact on the optimization speed.

### Add more constraints and variables than you need (now)

The cost of adding one constraint or one variable is about the same as adding many of them. Therefore, it may be worthwhile to add many variables instead of one. Initially fix the unused variable at zero, and then later unfix them as needed. Similarly, you can add multiple free constraints and then use them as needed.

#### Do not remove basic variables

When performing re-optimizations, instead of removing a basic variable it may be more efficient to fix the variable at zero and then remove it when the problem is re-optimized and it has left the basis. This makes it easier for **MOSEK** to restart the simplex optimizer.

# 10.5 The license system

MOSEK is a commercial product that **always** needs a valid license to work. MOSEK uses a third party license manager to implement license checking. The number of license tokens provided determines the number of optimizations that can be run simultaneously.

By default a license token remains checked out from the first optimization until the end of the **MOSEK** session, i.e.

- a license token is checked out when MSK\_optimize is first called, and
- it is returned when the MOSEK environment is deleted.

Calling  $MSK\_optimize$  from different threads using the same  $\mathbf{MOSEK}$  environment only consumes one license token.

Starting the optimization when no license tokens are available will result in an error.

Default behaviour of the license system can be changed in several ways:

- Setting the parameter  $MSK\_IPAR\_CACHE\_LICENSE$  to  $MSK\_OFF$  will force  $\mathbf{MOSEK}$  to return the license token immediately after the optimization completed.
- Setting the license wait flag with the parameter MSK\_IPAR\_LICENSE\_WAIT will force MOSEK to wait until a license token becomes available instead of returning with an error. The wait time between checks can be set with MSK\_putlicensewait.
- ullet Additional license checkouts and checkins can be performed with the functions  $MSK\_checkinlicense$  and  $MSK\_checkoutlicense$ .

• Usually the license system is stopped automatically when the MOSEK library is unloaded. However, when the user explicitly unloads the library (using e.g. FreeLibrary), the license system must be stopped before the library is unloaded. This can be done by calling the function <code>MSK\_licensecleanup</code> as the last function call to MOSEK.

## 10.6 Deployment

When redistributing a C application using the **MOSEK** Optimizer API for C 9.3.18, the following libraries must be included:

64-bit x86 Linux	64-bit Windows	32-bit Win-	64-bit Mac OS	Linux ARM64
		dows		
libmosek64.so.	mosek64_9_3.	mosek9_3.	libmosek64.9.3.	libmosek64.so.
9.3	dll	dll	dylib	9.3
libcilkrts.so.5	cilkrts20.dll	cilkrts20.	libcilkrts.5.dylib	
		dll		

# 10.7 Strings

MOSEK supports Unicode strings. All arguments of type char \* are allowed to be UTF8 encoded strings (http://en.wikipedia.org/wiki/UTF-8). Please note that

- an ASCII string is always a valid UTF8 string, and
- an UTF8 string is stored in a char array.

It is possible to convert between wchar\_t strings and UTF8 strings using the functions MSK\_wchartoutf8 and MSK\_utf8towchar. A working example is provided in the example file unicode.c.

# Chapter 11

# Case Studies

In this section we present some case studies in which the Optimizer API for C is used to solve real-life applications. These examples involve some more advanced modeling skills and possibly some input data. The user is strongly recommended to first read the basic tutorials of Sec. 6 before going through these advanced case studies.

- Portfolio Optimization
  - Keywords: Markowitz model, variance, risk, efficient frontier, transaction cost, market impact cost
  - Type: Conic Quadratic, Power Cone, Mixed-Integer Optimization
- Logistic regression
  - Keywords: machine learning, logistic regression, classifier, log-sum-exp, softplus, regularization
  - **Type:** Exponential Cone, Quadratic Cone
- Concurrent Optimizer
  - **Keywords:** Concurrent optimization
  - Type: Linear Optimization, Mixed-Integer Optimization

# 11.1 Portfolio Optimization

In this section the Markowitz portfolio optimization problem and variants are implemented using the MOSEK optimizer API.

- Basic Markowitz model
- Efficient frontier
- Factor model and efficiency
- Market impact costs
- Transaction costs
- Cardinality constraints

## 11.1.1 A Basic Portfolio Optimization Model

The classical Markowitz portfolio optimization problem considers investing in n stocks or assets held over a period of time. Let  $x_j$  denote the amount invested in asset j, and assume a stochastic model where the return of the assets is a random variable r with known mean

$$\mu = \mathbf{E}r$$

and covariance

$$\Sigma = \mathbf{E}(r - \mu)(r - \mu)^T.$$

The return of the investment is also a random variable  $y = r^T x$  with mean (or expected return)

$$\mathbf{E}y = \mu^T x$$

and variance

$$\mathbf{E}(y - \mathbf{E}y)^2 = x^T \Sigma x.$$

The standard deviation

$$\sqrt{x^T \Sigma x}$$

is usually associated with risk.

The problem facing the investor is to rebalance the portfolio to achieve a good compromise between risk and expected return, e.g., maximize the expected return subject to a budget constraint and an upper bound (denoted  $\gamma$ ) on the tolerable risk. This leads to the optimization problem

$$\begin{array}{lll} \text{maximize} & \mu^T x \\ \text{subject to} & e^T x & = & w + e^T x^0, \\ & x^T \Sigma x & \leq & \gamma^2, \\ & x & \geq & 0. \end{array} \tag{11.1}$$

The variables x denote the investment i.e.  $x_j$  is the amount invested in asset j and  $x_j^0$  is the initial holding of asset j. Finally, w is the initial amount of cash available.

A popular choice is  $x^0 = 0$  and w = 1 because then  $x_j$  may be interpreted as the relative amount of the total portfolio that is invested in asset j.

Since e is the vector of all ones then

$$e^T x = \sum_{j=1}^n x_j$$

is the total investment. Clearly, the total amount invested must be equal to the initial wealth, which is

$$w + e^T x^0$$
.

This leads to the first constraint

$$e^T x = w + e^T x^0.$$

The second constraint

$$x^T \Sigma x \leq \gamma^2$$

ensures that the variance, is bounded by the parameter  $\gamma^2$ . Therefore,  $\gamma$  specifies an upper bound of the standard deviation (risk) the investor is willing to undertake. Finally, the constraint

$$x_j \geq 0$$

excludes the possibility of short-selling. This constraint can of course be excluded if short-selling is allowed.

The covariance matrix  $\Sigma$  is positive semidefinite by definition and therefore there exist a matrix G such that

$$\Sigma = GG^T. \tag{11.2}$$

In general the choice of G is **not** unique and one possible choice of G is the Cholesky factorization of  $\Sigma$ . However, in many cases another choice is better for efficiency reasons as discussed in Sec. 11.1.3. For a given G we have that

$$x^{T} \Sigma x = x^{T} G G^{T} x$$
$$= \|G^{T} x\|^{2}.$$

Hence, we may write the risk constraint as

$$\gamma \ge \|G^T x\|$$

or equivalently

$$(\gamma, G^T x) \in \mathcal{Q}^{n+1},$$

where  $Q^{n+1}$  is the (n+1)-dimensional quadratic cone. Therefore, problem (11.1) can be written as

$$\begin{array}{lll} \text{maximize} & \mu^T x \\ \text{subject to} & e^T x & = & w + e^T x^0, \\ & (\gamma, G^T x) & \in & \mathcal{Q}^{n+1}, \\ & x & \geq & 0, \end{array} \tag{11.3}$$

which is a conic quadratic optimization problem that can easily be formulated and solved with Optimizer API for C. Subsequently we will use the example data

$$\mu = \left[ \begin{array}{c} 0.1073 \\ 0.0737 \\ 0.0627 \end{array} \right]$$

and

$$\Sigma = 0.1 \cdot \left[ \begin{array}{ccc} 0.2778 & 0.0387 & 0.0021 \\ 0.0387 & 0.1112 & -0.0020 \\ 0.0021 & -0.0020 & 0.0115 \end{array} \right].$$

This implies

$$G^T = \sqrt{0.1} \begin{bmatrix} 0.5271 & 0.0734 & 0.0040 \\ 0 & 0.3253 & -0.0070 \\ 0 & 0 & 0.1069 \end{bmatrix}$$

# Why a Conic Formulation?

Problem (11.1) is a convex quadratically constrained optimization problem that can be solved directly using MOSEK. Why then reformulate it as a conic quadratic optimization problem (11.3)? The main reason for choosing a conic model is that it is more robust and usually solves faster and more reliably. For instance it is not always easy to numerically validate that the matrix  $\Sigma$  in (11.1) is positive semidefinite

due to the presence of rounding errors. It is also very easy to make a mistake so  $\Sigma$  becomes indefinite. These problems are completely eliminated in the conic formulation.

Moreover, observe the constraint

$$||G^Tx|| \le \gamma$$

more numerically robust than

$$x^T \Sigma x \le \gamma^2$$

for very small and very large values of  $\gamma$ . Indeed, if say  $\gamma \approx 10^4$  then  $\gamma^2 \approx 10^8$ , which introduces a scaling issue in the model. Hence, using conic formulation we work with the standard deviation instead of variance, which usually gives rise to a better scaled model.

## Implementing the Portfolio Model

## Creating a matrix formulation

The Optimizer API for C requires that an optimization problem is entered in the following standard form:

We refer to  $\hat{x}$  as the API variable. It means we need to reformulate (11.3). The first step is to introduce auxiliary variables so that the conic constraint involves only unique variables:

maximize 
$$\mu^T x$$
  
subject to  $e^T x = w + e^T x^0$ ,  
 $G^T x - t = 0$ ,  
 $[s;t] \in \mathcal{Q}^{n+1}$ ,  
 $x \geq 0$ ,  
 $s = \gamma$ . (11.5)

Here s is an additional scalar variable and t is a vector variable of dimension n. The next step is to concatenate all the variables into one long variable vector:

$$\hat{x} = [x; s; t] = \begin{bmatrix} x \\ s \\ t \end{bmatrix}$$
 (11.6)

The details of the concatenation are specified below.

Table 11.1: Storage layout of the  $\hat{x}$  variable.

Variable	Length	Offset
x	$\mid n \mid$	0
s	1	n
t	n	n+1

The offset determines where the variable starts. (Note that all variables are indexed from 0). For instance

$$\hat{x}_{n+1+i} = t_i.$$

because the offset of the t variable is n+1.

Given the ordering of the variables specified by (11.6) it is useful to visualize the linear constraints (11.4) in an explicit block matrix form:

$$\begin{bmatrix}
 & 1 & 0 & 0 \\
\hline
 & G^T & 0 & -1 \\
\hline
 & & & -1
\end{bmatrix} \cdot \begin{bmatrix} x \\
\hline
 & s \\
\hline
 & t \end{bmatrix} = \begin{bmatrix} w + e^T x_0 \\
\hline
 & 0 \end{bmatrix}.$$
(11.7)

In other words, we should define the specific components of the problem description as follows:

$$c = \begin{bmatrix} \mu^{T} & 0 & 0_{n} \end{bmatrix}^{T},$$

$$A = \begin{bmatrix} e^{T} & 0 & 0_{n} \\ G^{T} & 0_{n} & -I_{n} \end{bmatrix},$$

$$l^{c} = \begin{bmatrix} w + e^{T}x^{0} & 0_{n} \end{bmatrix}^{T},$$

$$u^{c} = \begin{bmatrix} w + e^{T}x^{0} & 0_{n} \end{bmatrix}^{T},$$

$$l^{x} = \begin{bmatrix} 0_{n} & \gamma & -\infty_{n} \end{bmatrix}^{T},$$

$$u^{x} = \begin{bmatrix} \infty_{n} & \gamma & \infty_{n} \end{bmatrix}^{T}.$$

$$(11.8)$$

## Source code example

From the block matrix form (11.7) and the explicit specification (11.8), using the offset information in Table 11.1 it is easy to calculate the index and value of each entry of the linear constraint matrix. The code below sets up the general optimization problem (11.5) and solves it for the example data. Of course it is only necessary to set non-zero entries of the linear constraint matrix.

Listing 11.1: Code implementing model (11.5).

```
# include <math.h>
# include <stdio.h>
# include "mosek.h"
# define MOSEKCALL(\_r,\_call) if ((\_r) = MSK\_RES\_OK) (\_r) = (\_call)
static void MSKAPI printstr(void *handle,
                            const char str[])
 printf("%s", str);
} /* printstr */
int main(int argc, const char **argv)
{
                  buf[128];
  char
                  expret = 0.0,
 double
                  stddev = 0.0,
 const MSKint32t n
                          = 3;
                          = 0.05;
  const MSKrealt gamma
  const MSKrealt mu[]
                          = \{0.1073, 0.0737, 0.0627\};
  const MSKrealt GT[][3] = \{\{0.1667, 0.0232, 0.0013\},
                             \{0.0000, 0.1033, -0.0022\},\
                             {0.0000, 0.0000, 0.0338}};
                          = \{0.0, 0.0, 0.0\};
  const MSKrealt x0[3]
  const MSKrealt w
                          = 1.0;
```

```
MSKrealt
               rtemp;
MSKenv_t
                env;
               k, i, j, offsetx, offsets, offsett, *sub;
MSKint32t
               res = MSK_RES_OK, trmcode;
MSKrescodee
MSKtask_t
               task;
/* Initial setup. */
env = NULL;
task = NULL;
MOSEKCALL(res, MSK_makeenv(&env, NULL));
MOSEKCALL(res, MSK_maketask(env, 0, 0, &task));
MOSEKCALL(res, MSK_linkfunctotaskstream(task, MSK_STREAM_LOG, NULL, printstr));
/* Total budget */
rtemp = w;
for (j = 0; j < n; ++j)
 rtemp += x0[j];
/* Constraints. */
MOSEKCALL(res, MSK_appendcons(task, 1 + n));
/* Total budget constraint - set bounds l^c = u^c */
MOSEKCALL(res, MSK_putconbound(task, 0, MSK_BK_FX, rtemp, rtemp));
sprintf(buf, "%s", "budget");
MOSEKCALL(res, MSK_putconname(task, 0, buf));
/* The remaining constraints GT * x - t = 0 - set bounds l^c = u^c * /
for (i = 0; i < n; ++i)
  MOSEKCALL(res, MSK_putconbound(task, 1 + i, MSK_BK_FX, 0.0, 0.0));
  sprintf(buf, "GT[%d]", 1 + i);
  MOSEKCALL(res, MSK_putconname(task, 1 + i, buf));
}
/* Variables. */
MOSEKCALL(res, MSK_appendvars(task, 1 + 2 * n));
offsetx = 0; /* Offset of variable x into the API variable. */
offsets = n; /* Offset of variable x into the API variable. */
offsett = n + 1; /* Offset of variable t into the API variable. */
/* x variables. */
for (j = 0; j < n; ++j)
  /* Return of asset j in the objective */
  MOSEKCALL(res, MSK_putcj(task, offsetx + j, mu[j]));
  /* Coefficients in the first row of A */
  MOSEKCALL(res, MSK_putaij(task, 0, offsetx + j, 1.0));
  /* No short-selling - x^l = 0, x^u = inf */
  MOSEKCALL(res, MSK_putvarbound(task, offsetx + j, MSK_BK_LO, 0.0, MSK_INFINITY));
  sprintf(buf, "x[%d]", 1 + j);
  MOSEKCALL(res, MSK_putvarname(task, offsetx + j, buf));
}
/* s variable is a constant equal to gamma. */
MOSEKCALL(res, MSK_putvarbound(task, offsets + 0, MSK_BK_FX, gamma));
```

```
sprintf(buf, "s");
 MOSEKCALL(res, MSK_putvarname(task, offsets + 0, buf));
 /* t variables (t = GT*x). */
 for (j = 0; j < n; ++j)
    /* Copying the GT matrix in the appropriate block of A */
   for (k = 0; k < n; ++k)
     if ( GT[k][j] != 0.0 )
       MOSEKCALL(res, MSK_putaij(task, 1 + k, offsetx + j, GT[k][j]));
    /* Diagonal -1 entries in a block of A */
   MOSEKCALL(res, MSK_putaij(task, 1 + j, offsett + j, -1.0));
    /* Free - no bounds */
   MOSEKCALL(res, MSK_putvarbound(task, offsett + j, MSK_BK_FR, -MSK_INFINITY, MSK_
→INFINITY));
   sprintf(buf, "t[%d]", 1 + j);
   MOSEKCALL(res, MSK_putvarname(task, offsett + j, buf));
 }
 if ( res == MSK_RES_OK )
    /* Define the cone spanned by variables (s, t), i.e. dimension = n + 1 */
   MSKint32t *sub = (MSKint32t *) MSK_calloctask(task, n + 1, sizeof(MSKint32t));
   if (sub)
      /* Copy indices of variables involved in the conic constraint */
     sub[0] = offsets + 0;
     for (j = 0; j < n; ++j)
       sub[j + 1] = offsett + j;
     MOSEKCALL(res, MSK_appendcone(task, MSK_CT_QUAD, 0.0, n + 1, sub));
     MOSEKCALL(res, MSK_putconename(task, 0, "stddev"));
     MSK_freetask(task, sub);
   }
   else
     res = MSK_RES_ERR_SPACE;
 MOSEKCALL(res, MSK_putobjsense(task, MSK_OBJECTIVE_SENSE_MAXIMIZE));
#if 0
 /* No log output */
 MOSEKCALL(res, MSK_putintparam(task, MSK_IPAR_LOG, 0));
#endif
#if 0
 /* Dump the problem to a human readable OPF file. */
 MOSEKCALL(res, MSK_writedata(task, "dump.opf"));
#endif
 MOSEKCALL(res, MSK_optimizetrm(task, &trmcode));
# if 1
 /* Display the solution summary for quick inspection of results. */
```

```
MSK_solutionsummary(task, MSK_STREAM_MSG);
# endif
 if ( res == MSK_RES_OK )
   expret = 0.0;
   stddev = 0.0;
   /* Read the x variables one by one and compute expected return. */
   /* Can also be obtained as value of the objective. */
   for (j = 0; j < n; ++j)
     MOSEKCALL(res, MSK_getxxslice(task, MSK_SOL_ITR, offsetx + j, offsetx + j + 1, &
\rightarrow xj));
     expret += mu[j] * xj;
    /* Read the value of s. This should be gamma. */
   MOSEKCALL(res, MSK_getxxslice(task, MSK_SOL_ITR, offsets + 0, offsets + 1, &

stddev));
   printf("\nExpected return %e for gamma %e\n", expret, stddev);
 if ( task != NULL )
   MSK_deletetask(&task);
 if ( env != NULL )
   MSK_deleteenv(&env);
 return ( 0 );
}
```

The above code produces the result:

Listing 11.2: Output from the solver.

```
Interior-point solution summary
Problem status : PRIMAL_AND_DUAL_FEASIBLE
Solution status : OPTIMAL
Primal. obj: 7.4766507287e-02  nrm: 1e+00  Viol. con: 2e-08  var: 0e+00  Ucones: 2e-08
Dual. obj: 7.4766554102e-02  nrm: 3e-01  Viol. con: 0e+00  var: 3e-08  Ucones: 0e+00

Expected return 7.476651e-02 for gamma 5.000000e-02
```

#### Source code comments

The source code is a direct translation of the model (11.5) using the explicit block matrix specification (11.8) but a few comments are nevertheless in place.

The code uses a macro which can be defined as follows:

```
# define MOSEKCALL(\_r, \_call) ( (\_r) == MSK\_RES\_OK ? ( (\_r) = (\_call) ) : ( (\_r) = (\_r) _{\hookrightarrow}) );
```

and then used as a method of hiding multiple if statements and hence making the code more compact:

```
MOSEKCALL(res, MSK_optimize(task));

//is the same as
if ( res == MSK_RES_OK )
  res = MSK_optimize(task);
```

In the lines

```
offsetx = 0; /* Offset of variable x into the API variable. */
offsets = n; /* Offset of variable x into the API variable. */
offsett = n + 1; /* Offset of variable t into the API variable. */
```

offsets into the MOSEK API variable are stored as in Table 11.1. The code

```
for (j = 0; j < n; ++j)
{
    /* Return of asset j in the objective */
    MOSEKCALL(res, MSK_putcj(task, offsetx + j, mu[j]));
    /* Coefficients in the first row of A */
    MOSEKCALL(res, MSK_putaij(task, 0, offsetx + j, 1.0));
    /* No short-selling - x^l = 0, x^u = inf */
    MOSEKCALL(res, MSK_putvarbound(task, offsetx + j, MSK_BK_LO, 0.0, MSK_INFINITY));
    sprintf(buf, "x[%d]", 1 + j);
    MOSEKCALL(res, MSK_putvarname(task, offsetx + j, buf));
}</pre>
```

sets up the data for x variables. For instance

```
/* Return of asset j in the objective */
MOSEKCALL(res, MSK_putcj(task, offsetx + j, mu[j]));
```

inputs the objective coefficients for the x variables. Moreover, the code

```
sprintf(buf, "x[%d]", 1 + j);
MOSEKCALL(res, MSK_putvarname(task, offsetx + j, buf));
```

assigns meaningful names to the API variables. This is not needed but it makes debugging easier. Note that the solution values are only accessed for the interesting variables; for instance the auxiliary variable t is omitted from this process.

#### **Debugging Tips**

Implementing an optimization model in Optimizer API for C can be error-prone. In order to check the code for accidental errors it is very useful to dump the problem to a file in a human readable form for visual inspection. The line

```
/* Dump the problem to a human readable OPF file. */
MOSEKCALL(res, MSK_writedata(task, "dump.opf"));
```

does that and it produces a file with the content:

Listing 11.3: Problem (11.5) stored in OPF format.

```
[comment]
  Written by MOSEK version 8.1.0.24
  Date 11-09-17
  Time 14:34:24
[/comment]
[hints]
 [hint NUMVAR] 7 [/hint]
 [hint NUMCON] 4 [/hint]
 [hint NUMANZ] 12 [/hint]
 [hint NUMQNZ] O [/hint]
 [hint NUMCONE] 1 [/hint]
[/hints]
[variables disallow_new_variables]
  'x[1]' 'x[2]' 'x[3]' s 't[1]'
  't[2]' 't[3]'
[/variables]
[objective maximize]
  1.073e-01 \ 'x[1]' + 7.37e-02 \ 'x[2]' + 6.27000000000001e-02 \ 'x[3]'
[/objective]
[constraints]
 [con 'budget'] 'x[1]' + 'x[2]' + 'x[3]' = 1e+00 [/con]
 [con 'GT[1]'] 1.667e-01 'x[1]' + 2.32e-02 'x[2]' + 1.3e-03 'x[3]' - 't[1]' = 0e+00_0
 [con 'GT[2]'] 1.033e-01 'x[2]' - 2.2e-03 'x[3]' - 't[2]' = 0e+00 [/con]
 [con 'GT[3]'] 3.38e-02 'x[3]' - 't[3]' = 0e+00 [/con]
[/constraints]
[bounds]
                 <= 'x[1]','x[2]','x[3]' [/b]
 [b] 0e+00
                    s = 5e-02 [/b]
 ГъЪ
                    't[1]','t[2]','t[3]' free [/b]
  [cone quad 'stddev'] s, 't[1]', 't[2]', 't[3]' [/cone]
[/bounds]
```

Since the API variables have been given meaningful names it is easy to verify by hand that the model is correct.

## 11.1.2 The efficient Frontier

The portfolio computed by the Markowitz model is efficient in the sense that there is no other portfolio giving a strictly higher return for the same amount of risk. An efficient portfolio is also sometimes called a Pareto optimal portfolio. Clearly, an investor should only invest in efficient portfolios and therefore it may be relevant to present the investor with all efficient portfolios so the investor can choose the portfolio that has the desired tradeoff between return and risk.

Given a nonnegative  $\alpha$  the problem

$$\begin{array}{ll} \text{maximize} & \mu^T x - \alpha x^T \Sigma x \\ \text{subject to} & e^T x = w + e^T x^0, \\ & x \geq 0. \end{array}$$

is one standard way to trade the expected return against penalizing variance. Note that, in contrast to the previous example, we explicitly use the variance ( $\|G^Tx\|_2^2$ ) rather than standard deviation ( $\|G^Tx\|_2^2$ ),

therefore the conic model includes a rotated quadratic cone:

```
maximize \mu^T x - \alpha s

subject to e^T x = w + e^T x^0,

G^T x - t = 0,

u = 0.5,

(s, u, t) \in Q_r^{n+2} (evaluates to s \ge \|G^T x\|_2^2 = x^T \Sigma x),

x \ge 0. (11.10)
```

Ideally the problem (11.9) should be solved for all values  $\alpha \geq 0$  but in practice it is impossible. Using the example data as before, the optimal values of return and variance for several values of  $\alpha$  are shown below:

Listing 11.4: Results obtained solving problem (11.9) for different values of  $\alpha$ .

```
alpha
              exp ret
                             variance
0.000e+00
              1.073e-01
                             2.779e-02
2.500e-01
              1.073e-01
                             2.779e-02
5.000e-01
              1.073e-01
                             2.779e-02
                             2.554e-02
7.500e-01
              1.057e-01
                             1.851e-02
1.000e+00
              9.965e-02
              8.802e-02
                             8.850e-03
1.500e+00
2.000e+00
              8.213e-02
                             5.415e-03
2.500e+00
              7.860e-02
                             3.826e-03
3.000e+00
              7.625e-02
                             2.963e-03
3.500e+00
              7.457e-02
                             2.442e-03
4.000e+00
              7.331e-02
                             2.104e-03
                             1.873e-03
4.500e+00
              7.232e-02
```

#### Source code example

The example code in Listing 11.5 demonstrates how to compute the efficient portfolios for several values of  $\alpha$ . The code is mostly similar to the one in Sec. 11.1.1, except the problem is re-optimized in a loop for varying  $\alpha$ .

Listing 11.5: Code implementing model (11.9).

```
# include <math.h>
# include <stdio.h>
# include "mosek.h"
 \# \ define \ \ \textit{MOSEKCALL}(\_r,\_call) \quad if \ (\ (\_r) == \textit{MSK\_RES\_OK}\ )\ (\_r) \ = \ (\_call) 
#define LOGLEVEL
static void MSKAPI printstr(void
                                             *handle,
                                const char str[])
  printf("%s", str);
} /* printstr */
int main(int argc, const char **argv)
{
  char
                     buf[128];
  const MSKint32t n
                                  = 3,
                                 = 12;
                     numalpha
                                 = \{0.1073, 0.0737, 0.0627\},
  const double
                     mu[]
                     x0[3]
                                 = \{0.0, 0.0, 0.0\},\
```

```
= 1.0.
                 alphas[12] = \{0.0, 0.25, 0.5, 0.75, 1.0, 1.5, 2.0, 2.5, 3.0, 3.5, 4.
-0, 4.5
                 GT[][3]
                            = \{\{0.1667, 0.0232, 0.0013\},\
                               \{0.0000, 0.1033, -0.0022\},\
                               {0.0000, 0.0000, 0.0338}
                              };
 MSKenv_t
                 env:
 MSKint32t
                 k, i, j, offsetx, offsets, offsett;
                 res = MSK_RES_OK, lres;
 MSKrescodee
 MSKtask_t
                 task;
 MSKrealt
                 хj;
 MSKsolstae
                 solsta;
 /* Initial setup. */
 env = NULL;
 task = NULL;
 /* Replace "" with NULL in production. */
 MOSEKCALL(res, MSK_makeenv(&env, ""));
 MOSEKCALL(res, MSK_maketask(env, 0, 0, &task));
 MOSEKCALL(res, MSK_linkfunctotaskstream(task, MSK_STREAM_LOG, NULL, printstr));
 /* Constraints. */
 MOSEKCALL(res, MSK_appendcons(task, 1 + n));
 MOSEKCALL(res, MSK_putconbound(task, 0, MSK_BK_FX, 1.0, 1.0));
 sprintf(buf, "%s", "budget");
 MOSEKCALL(res, MSK_putconname(task, 0, buf));
 for (i = 0; i < n; ++i)
   MOSEKCALL(res, MSK_putconbound(task, 1 + i, MSK_BK_FX, 0.0, 0.0));
   sprintf(buf, "GT[%d]", 1 + i);
   MOSEKCALL(res, MSK_putconname(task, 1 + i, buf));
 }
 /* Variables. */
 MOSEKCALL(res, MSK_appendvars(task, 2 + 2 * n));
 offsetx = 0; /* Offset of variable x into the API variable. */
              /* Offset of variable s into the API variable. */
 offsets = n;
 offsett = n + 1; /* Offset of variable t into the API variable. */
 offsetu = 2*n + 1; /* Offset of variable u into the API variable. */
 /* x variables. */
 for (j = 0; j < n; ++j)
   MOSEKCALL(res, MSK_putcj(task, offsetx + j, mu[j]));
   MOSEKCALL(res, MSK_putaij(task, 0, offsetx + j, 1.0));
   for (k = 0; k < n; ++k)
     if (GT[k][j] != 0.0)
       MOSEKCALL(res, MSK_putaij(task, 1 + k, offsetx + j, GT[k][j]));
   MOSEKCALL(res, MSK_putvarbound(task, offsetx + j, MSK_BK_LO, 0.0, MSK_INFINITY));
   sprintf(buf, "x[%d]", 1 + j);
```

```
MOSEKCALL(res, MSK_putvarname(task, offsetx + j, buf));
 }
 /* s variable. */
 MOSEKCALL(res, MSK_putvarbound(task, offsets + 0, MSK_BK_FR, -MSK_INFINITY, MSK_
→INFINITY));
 sprintf(buf, "s");
 MOSEKCALL(res, MSK_putvarname(task, offsets + 0, buf));
 /* t variables. */
 for (j = 0; j < n; ++j)
   MOSEKCALL(res, MSK_putaij(task, 1 + j, offsett + j, -1.0));
   MOSEKCALL(res, MSK_putvarbound(task, offsett + j, MSK_BK_FR, -MSK_INFINITY, MSK_
→INFINITY));
   sprintf(buf, "t[%d]", 1 + j);
   MOSEKCALL(res, MSK_putvarname(task, offsett + j, buf));
 }
 /* u variable. */
 MOSEKCALL(res, MSK_putvarbound(task, offsetu + 0, MSK_BK_FX, 0.5, 0.5));
 sprintf(buf, "u");
 MOSEKCALL(res, MSK_putvarname(task, offsetu + 0, buf));
 if (res == MSK_RES_OK)
   /* sub should be n+2 long i.e. the dimmension of the cone. */
   MSKint32t *sub = (MSKint32t *) MSK_calloctask(task, n + 2, sizeof(MSKint32t));
   if (sub)
     sub[0] = offsets + 0;
     sub[1] = offsetu + 0;
     for (j = 0; j < n; ++j)
       sub[j + 2] = offsett + j;
     MOSEKCALL(res, MSK_appendcone(task, MSK_CT_RQUAD, 0.0, n + 2, sub));
     MOSEKCALL(res, MSK_putconename(task, 0, "variance"));
     MSK_freetask(task, sub);
   }
   else
     res = MSK_RES_ERR_SPACE;
 }
 MOSEKCALL(res, MSK_putobjsense(task, MSK_OBJECTIVE_SENSE_MAXIMIZE));
 /* Set the log level */
 MOSEKCALL(res, MSK_putintparam(task, MSK_IPAR_LOG, LOGLEVEL));
 printf("%-12s %-12s %-12s\n", "alpha", "exp ret", "variance");
 for (k = 0; k < numalpha && res==MSK_RES_OK; ++k)</pre>
   const double alpha = alphas[k];
   MSKrescodee trmcode;
```

```
/* Sets the objective function coefficient for s. */
   MOSEKCALL(res, MSK_putcj(task, offsets + 0, -alpha));
   MOSEKCALL(res, MSK_optimizetrm(task, &trmcode));
   MOSEKCALL(res, MSK_getsolsta(task, MSK_SOL_ITR, &solsta));
   if (solsta == MSK_SOL_STA_OPTIMAL)
     double expret = 0.0,
            stddev;
     for (j = 0; j < n; ++j)
       MOSEKCALL(res, MSK_getxxslice(task, MSK_SOL_ITR, offsetx + j, offsetx + j + 1,
→ &xj));
       expret += mu[j] * xj;
     MOSEKCALL(res, MSK_getxxslice(task, MSK_SOL_ITR, offsets + 0, offsets + 1, &
→stddev));
     printf("%-12.3e %-12.3e \n", alpha, expret, stddev);
   }
   else
     printf("An error occurred when solving for alpha=%e\n", alpha);
   }
 }
 lres = MSK_deletetask(&task);
 res = res==MSK_RES_OK ? lres : res;
 lres = MSK_deleteenv(&env);
 res = res==MSK_RES_OK ? lres : res;
 return ( res );
```

## 11.1.3 Factor model and efficiency

In practice it is often important to solve the portfolio problem very quickly. Therefore, in this section we discuss how to improve computational efficiency at the modeling stage.

The computational cost is of course to some extent dependent on the number of constraints and variables in the optimization problem. However, in practice a more important factor is the sparsity: the number of nonzeros used to represent the problem. Indeed it is often better to focus on the number of nonzeros in G see (11.2) and try to reduce that number by for instance changing the choice of G.

In other words if the computational efficiency should be improved then it is always good idea to start with focusing at the covariance matrix. As an example assume that

$$\Sigma = D + VV^T$$

where D is a positive definite diagonal matrix. Moreover, V is a matrix with n rows and p columns. Such a model for the covariance matrix is called a factor model and usually p is much smaller than n. In practice p tends to be a small number independent of n, say less than 100.

One possible choice for G is the Cholesky factorization of  $\Sigma$  which requires storage proportional to

n(n+1)/2. However, another choice is

$$G^T = \left[ \begin{array}{c} D^{1/2} \\ V^T \end{array} \right]$$

because then

$$GG^T = D + VV^T.$$

This choice requires storage proportional to n + pn which is much less than for the Cholesky choice of G. Indeed assuming p is a constant storage requirements are reduced by a factor of n.

The example above exploits the so-called factor structure and demonstrates that an alternative choice of G may lead to a significant reduction in the amount of storage used to represent the problem. This will in most cases also lead to a significant reduction in the solution time.

The lesson to be learned is that it is important to investigate how the covariance matrix is formed. Given this knowledge it might be possible to make a special choice for G that helps reducing the storage requirements and enhance the computational efficiency. More details about this process can be found in [And13].

# 11.1.4 Slippage Cost

The basic Markowitz model assumes that there are no costs associated with trading the assets and that the returns of the assets are independent of the amount traded. Neither of those assumptions is usually valid in practice. Therefore, a more realistic model is

maximize 
$$\mu^{T}x$$
subject to  $e^{T}x + \sum_{j=1}^{n} C_{j}|x_{j} - x_{j}^{0}| = w + e^{T}x^{0},$ 

$$x^{T}\sum x \leq \gamma^{2},$$

$$x \geq 0,$$
(11.11)

where the function

$$C_j|x_j-x_j^0|$$

specifies the transaction costs when the holding of asset j is changed from its initial value. In the next two sections we show two different variants of this problem with two nonlinear cost functions T.

# 11.1.5 Market Impact Costs

If the initial wealth is fairly small and no short selling is allowed, then the holdings will be small and the traded amount of each asset must also be small. Therefore, it is reasonable to assume that the prices of the assets are independent of the amount traded. However, if a large volume of an asset is sold or purchased, the price, and hence return, can be expected to change. This effect is called market impact costs. It is common to assume that the market impact cost for asset j can be modeled by

$$C_j = m_j \sqrt{|x_j - x_j^0|}$$

where  $m_j$  is a constant that is estimated in some way by the trader. See [GK00] [p. 452] for details. Hence, we have

$$C_j(x_j - x_j^0) = m_j |x_j - x_j^0| \sqrt{|x_j - x_j^0|} = m_j |x_j - x_j^0|^{3/2}.$$

From the Modeling Cookbook we know that  $c \geq z^{3/2}$  can be modeled directly using the power cone  $\mathcal{P}_3^{2/3,1/3}$ :

$$\{(c,z):c\geq z^{3/2},z\geq 0\}=\{(c,z):(c,1,z)\in \mathcal{P}_3^{2/3,1/3}\}$$

Hence, it follows that we can write the model as

$$\begin{aligned}
z_j &= |x_j - x_j^0|, \\
(c_j, 1, z_j) &\in \mathcal{P}_3^{2/3, 1/3}, \\
\sum_{j=1}^n C_j |x_j - x_j^0| &= \sum_{j=1}^n c_j m_j.
\end{aligned}$$

Unfortunately this set of constraints is nonconvex due to the constraint

$$z_j = |x_j - x_j^0| (11.12)$$

but in many cases the constraint may be replaced by the relaxed constraint

$$z_j \ge |x_j - x_j^0|,\tag{11.13}$$

which is equivalent to

$$z_{j} \ge x_{j} - x_{j}^{0}, z_{j} \ge -(x_{j} - x_{j}^{0}).$$
(11.14)

For instance if the universe of assets contains a risk free asset then

$$z_j > |x_j - x_j^0| \tag{11.15}$$

cannot hold for an optimal solution.

If the optimal solution has the property (11.15) then the market impact cost within the model is larger than the true market impact cost and hence money are essentially considered garbage and removed by generating transaction costs. This may happen if a portfolio with very small risk is requested because the only way to obtain a small risk is to get rid of some of the assets by generating transaction costs. We generally assume that this is not the case and hence the models (11.12) and (11.13) are equivalent.

The above observations lead to

maximize 
$$\mu^{T}x$$
  
subject to  $e^{T}x + m^{T}c = w + e^{T}x^{0}$ ,  
 $[\gamma; G^{T}x] \in \mathcal{Q}^{n+1}$ ,  
 $z_{j} \geq x_{j} - x_{j}^{0}$ ,  $j = 1, \dots, n$ ,  
 $z_{j} \geq x_{j}^{0} - x_{j}$ ,  $j = 1, \dots, n$ ,  
 $(c_{j}, 1, z_{j}) \in \mathcal{P}_{3}^{2/3, 1/3}$ ,  $j = 1, \dots, n$ ,  
 $x \geq 0$ . (11.16)

The revised budget constraint

$$e^T x + m^T c = w + e^T x^0$$

specifies that the initial wealth covers the investment and the transaction costs. It should be mentioned that transaction costs of the form

$$t_j \geq z_j^p$$

where p > 1 is a real number can be modeled with the power cone as

$$(t_i, 1, z_i) \in \mathcal{P}_3^{1/p, 1 - 1/p}$$
.

See Modeling Cookbook for details.

#### Creating a matrix formulation

One more reformulation of (11.16) is needed to bring it to the standard form (11.4).

maximize 
$$\mu^{T}x$$
  
subject to  $e^{T}x + m^{T}c = w + e^{T}x^{0}$ ,  
 $G^{T}x - t = 0$ ,  
 $z_{j} - x_{j} \geq -x_{j}^{0}$ ,  $j = 1, ..., n$ ,  
 $z_{j} + x_{j} \geq x_{j}^{0}$ ,  $j = 1, ..., n$ ,  
 $(s,t) \in \mathcal{Q}^{n+1}$ ,  
 $(c_{j}, f_{j}, z_{j}) \in \mathcal{P}_{3}^{2/3, 1/3}$ ,  $j = 1, ..., n$ ,  
 $x \geq 0$ ,  
 $f_{j} = 1$ ,  $j = 1, ..., n$ ,  
 $s = \gamma$ . (11.17)

where  $f \in \mathbb{R}^n$  is an additional variable representing the unused coordinate in the power cone. The formulation (11.17) is not the most compact possible, but it is easy to implement. **MOSEK** presolve will automatically simplify it.

The first step in developing the implementation is to chose an ordering of the variables. We will choose the following ordering:

$$\hat{x} = [x; s; t; c; z; f]$$

Table 11.2 shows the mapping between the  $\hat{x}$  vector and the model variables.

Table 11.2: Storage layout for  $\hat{x}$ 

Variable	Length	Offset
x	n	0
s	1	n
t	n	n+1
c	n	2n + 1
z	n	3n+1
f	n	4n + 1

The next step is to consider how the linear constraint matrix A and the remaining data vectors are laid out. Reusing the idea in Sec. 11.1.1 we can write the data in block matrix form and read off all the required coordinates. This extension of the code setting up the constraint  $G^Tx - t = 0$  from Sec. 11.1.1 is shown below.

# Source code example

The example code in Listing 11.6 demonstrates how to implement the model (11.17).

Listing 11.6: Code implementing model (11.17).

```
# include <math.h>
#include <stdio.h>
# include "mosek.h"
\# define MOSEKCALL(\_r,\_call) if ((\_r)==MSK\_RES\_OK)(\_r)=(\_call)
static void MSKAPI printstr(void *handle,
                             const char str[])
 printf("%s", str);
} /* printstr */
int main(int argc, const char **argv)
                   buf [128];
  char
  const MSKint32t n
                           = 3;
  const double
                           = 1.0,
                           = \{0.0, 0.0, 0.0\},\
                   x0[]
                           = 0.05,
                   gamma
                   mu []
                           = \{0.1073, 0.0737, 0.0627\},
                           = \{0.01, 0.01, 0.01\},\
                   m[]
                   GT[][3] = \{\{0.1667, 0.0232, 0.0013\},
                              {0.0000, 0.1033, -0.0022},
                              {0.0000, 0.0000, 0.0338}
                             };
 double
                   expret,
                   stddev,
```

```
xj;
 MSKenv_t
                 env;
 MSKint32t
                 k, i, j,
                 offsetx, offsets, offsett, offsetc,
                 offsetz, offsetf;
                 res = MSK_RES_OK, trmcode;
 MSKrescodee
 MSKtask_t
                 task;
 /* Initial setup. */
 env = NULL;
 task = NULL;
 MOSEKCALL(res, MSK_makeenv(&env, NULL));
 MOSEKCALL(res, MSK_maketask(env, 0, 0, &task));
 MOSEKCALL(res, MSK_linkfunctotaskstream(task, MSK_STREAM_LOG, NULL, printstr));
 /* Constraints. */
 MOSEKCALL(res, MSK_appendcons(task, 1 + 3 * n));
 MOSEKCALL(res, MSK_putconbound(task, 0, MSK_BK_FX, w, w));
 sprintf(buf, "%s", "budget");
 MOSEKCALL(res, MSK_putconname(task, 0, buf));
 for (i = 0; i < n; ++i)
   MOSEKCALL(res, MSK_putconbound(task, 1 + i, MSK_BK_FX, 0.0, 0.0));
   sprintf(buf, "GT[%d]", 1 + i);
   MOSEKCALL(res, MSK_putconname(task, 1 + i, buf));
 for (i = 0; i < n; ++i)
   MOSEKCALL(res, MSK_putconbound(task, 1 + n + i, MSK_BK_LO, -x0[i], MSK_INFINITY));
   sprintf(buf, "zabs1[%d]", 1 + i);
   MOSEKCALL(res, MSK_putconname(task, 1 + n + i, buf));
 for (i = 0; i < n; ++i)
   MOSEKCALL(res, MSK_putconbound(task, 1 + 2 * n + i, MSK_BK_LO, x0[i], MSK_
→INFINITY));
   sprintf(buf, "zabs2[%d]", 1 + i);
   MOSEKCALL(res, MSK_putconname(task, 1 + 2 * n + i, buf));
 }
 /* Offsets of variables into the (serialized) API variable. */
 offsetx = 0;
 offsets = n;
 offsett = n + 1;
 offsetc = 2 * n + 1;
 offsetz = 3 * n + 1;
 offsetf = 4 * n + 1;
 /* Variables. */
 MOSEKCALL(res, MSK_appendvars(task, 5 * n + 1));
 /* x variables. */
 for (j = 0; j < n; ++j)
```

```
MOSEKCALL(res, MSK_putcj(task, offsetx + j, mu[j]));
   MOSEKCALL(res, MSK_putaij(task, 0, offsetx + j, 1.0));
   for (k = 0; k < n; ++k)
     if (GT[k][j] != 0.0)
       MOSEKCALL(res, MSK_putaij(task, 1 + k, offsetx + j, GT[k][j]));
   MOSEKCALL(res, MSK_putaij(task, 1 + n + j, offsetx + j, -1.0));
   MOSEKCALL(res, MSK_putaij(task, 1 + 2 * n + j, offsetx + j, 1.0));
   MOSEKCALL(res, MSK_putvarbound(task, offsetx + j, MSK_BK_LO, 0.0, MSK_INFINITY));
   sprintf(buf, "x[%d]", 1 + j);
   MOSEKCALL(res, MSK_putvarname(task, offsetx + j, buf));
 }
 /* s variable. */
 MOSEKCALL(res, MSK_putvarbound(task, offsets + 0, MSK_BK_FX, gamma, gamma));
 sprintf(buf, "s");
 MOSEKCALL(res, MSK_putvarname(task, offsets + 0, buf));
 /* t variables. */
 for (j = 0; j < n; ++j)
   MOSEKCALL(res, MSK_putaij(task, 1 + j, offsett + j, -1.0));
   MOSEKCALL(res, MSK_putvarbound(task, offsett + j, MSK_BK_FR, -MSK_INFINITY, MSK_
→INFINITY));
   sprintf(buf, "t[%d]", 1 + j);
   MOSEKCALL(res, MSK_putvarname(task, offsett + j, buf));
 /* c variables. */
 for (j = 0; j < n; ++j)
   MOSEKCALL(res, MSK_putaij(task, 0, offsetc + j, m[j]));
   MOSEKCALL(res, MSK_putvarbound(task, offsetc + j, MSK_BK_FR, -MSK_INFINITY, MSK_
→INFINITY)):
   sprintf(buf, "c[%d]", 1 + j);
   MOSEKCALL(res, MSK_putvarname(task, offsetc + j, buf));
 /* z variables. */
 for (j = 0; j < n; ++j)
   MOSEKCALL(res, MSK_putaij(task, 1 + 1 * n + j, offsetz + j, 1.0));
   MOSEKCALL(res, MSK_putaij(task, 1 + 2 * n + j, offsetz + j, 1.0));
   MOSEKCALL(res, MSK_putvarbound(task, offsetz + j, MSK_BK_FR, -MSK_INFINITY, MSK_
→INFINITY));
   sprintf(buf, "z[%d]", 1 + j);
   MOSEKCALL(res, MSK_putvarname(task, offsetz + j, buf));
 }
 /* f variables. */
 for (j = 0; j < n; ++j)
   MOSEKCALL(res, MSK_putvarbound(task, offsetf + j, MSK_BK_FX, 1.0, 1.0));
   sprintf(buf, "f[%d]", 1 + j);
   MOSEKCALL(res, MSK_putvarname(task, offsetf + j, buf));
```

```
}
 if (res == MSK_RES_OK)
    /st sub should be n+1 long i.e. the dimmension of the cone. st/
   MSKint32t *sub = (MSKint32t *) MSK_calloctask(task, 3 >= n + 1 ? 3 : n + 1, ...
⇔sizeof(MSKint32t));
   if (sub)
      /* Add quadratic cone */
      sub[0] = offsets + 0;
     for (j = 0; j < n; ++j)
       sub[j + 1] = offsett + j;
     MOSEKCALL(res, MSK_appendcone(task, MSK_CT_QUAD, 0.0, n + 1, sub));
     MOSEKCALL(res, MSK_putconename(task, 0, "stddev"));
      /* Add power cones */
     for (k = 0; k < n; ++k)
        sub[0] = offsetc + k, sub[1] = offsetf + k, sub[2] = offsetz + k;
       MOSEKCALL(res, MSK_appendcone(task, MSK_CT_PPOW, 2.0/3.0, 3, sub));
        sprintf(buf, "trans[%d]", 1 + k);
        MOSEKCALL(res, MSK_putconename(task, 1 + k, buf));
     MSK_freetask(task, sub);
   }
     res = MSK_RES_ERR_SPACE;
 MOSEKCALL(res, MSK_putobjsense(task, MSK_OBJECTIVE_SENSE_MAXIMIZE));
#if 0
 /* no log output. */
 MOSEKCALL(res, MSK_putintparam(task, MSK_IPAR_LOG, 0));
#endif
#if 0
 /* Dump the problem to a human readable OPF file. */
 MOSEKCALL(res, MSK_writedata(task, "dump.opf"));
#endif
 MOSEKCALL(res, MSK_optimizetrm(task, &trmcode));
 /* Display the solution summary for quick inspection of results. */
 MSK_solutionsummary(task, MSK_STREAM_MSG);
# endif
 if (res == MSK_RES_OK)
   expret = 0.0;
   stddev = 0.0;
```

The example code above produces the result

```
Interior-point solution summary
Problem status: PRIMAL_AND_DUAL_FEASIBLE
Solution status: OPTIMAL
Primal. obj: 7.4390639578e-02 nrm: 1e+00 Viol. con: 1e-08 var: 0e+00 u
cones: 7e-09
Dual. obj: 7.4390755614e-02 nrm: 3e-01 Viol. con: 1e-19 var: 3e-08 u
cones: 0e+00

Expected return 7.439064e-02 for gamma 5.000000e-02
```

If the problem is dumped to an OPF file, it has the following content.

Listing 11.7: OPF file for problem (11.17).

```
[comment]
  Written by MOSEK version 9.0.0.31
  Date 10-01-18
  Time 12:10:24
[/comment]
[hints]
  [hint NUMVAR] 16 [/hint]
  [hint NUMCON] 10 [/hint]
  [hint NUMANZ] 27 [/hint]
  [hint NUMQNZ] 0 [/hint]
  [hint NUMCONE] 4 [/hint]
[/hints]
[variables disallow_new_variables]
 'x[1]' 'x[2]' 'x[3]' s 't[1]'
  't[2]' 't[3]' 'c[1]' 'c[2]' 'c[3]'
  'z[1]' 'z[2]' 'z[3]' 'f[1]' 'f[2]'
 'f[3]'
[/variables]
[objective maximize]
```

```
1.073e-01 'x[1]' + 7.37e-02 'x[2]' + 6.270000000000000000e-02 'x[3]'
[/objective]
[constraints]
  [con 'budget'] 'x[1]' + 'x[2]' + 'x[3]' + 1e-02 'c[1]' + 1e-02 'c[2]'
     + 1e-02 'c[3]' = 1e+00 [/con]
  [con 'GT[1]'] 1.667e-01 'x[1]' + 2.32e-02 'x[2]' + 1.3e-03 'x[3]' - 't[1]' = 0e+00_1
\rightarrow [/con]
  [con 'GT[2]'] 1.033e-01 'x[2]' - 2.2e-03 'x[3]' - 't[2]' = 0e+00 [/con]
  [con 'GT[3]'] 3.38e-02 'x[3]' - 't[3]' = 0e+00 [/con]
  [con 'zabs1[1]'] Oe+00 \leftarrow 'x[1]' + 'z[1]' [/con]
  [con 'zabs1[2]'] 0e+00 \le - 'x[2]' + 'z[2]' [/con]
  [con 'zabs1[3]'] 0e+00 \le - 'x[3]' + 'z[3]' [/con]
  [con 'zabs2[1]'] Oe+OO <= 'x[1]' + 'z[1]' [/con]
  [con 'zabs2[2]'] 0e+00 \le 'x[2]' + 'z[2]' [/con]
  [con 'zabs2[3]'] 0e+00 \le 'x[3]' + 'z[3]' [/con]
[/constraints]
[bounds]
  [b] 0e+00
                = 'x[1]', 'x[2]', 'x[3]' [/b]
  [b]
                     s = 5e-02 [/b]
  [b]
                      't[1]','t[2]','t[3]','c[1]','c[2]','c[3]' free [/b]
                      'z[1]','z[2]','z[3]' free [/b]
  [b]
                      'f[1]', 'f[2]', 'f[3]' = 1e+00 [/b]
  ГЪЪ
  [cone quad 'stddev'] s, 't[1]', 't[2]', 't[3]' [/cone]
  [cone ppow '6.66666666666666666e-01' 'trans[1]'] 'c[1]', 'f[1]', 'z[1]' [/cone] [cone ppow '6.66666666666666666060e-01' 'trans[2]'] 'c[2]', 'f[2]', 'z[2]' [/cone]
  [cone ppow '6.666666666666666e-01' 'trans[3]'] 'c[3]', 'f[3]', 'z[3]' [/cone]
[/bounds]
```

The file verifies that the correct problem has been set up.

#### 11.1.6 Transaction Costs

Now assume there is a cost associated with trading asset j given by

$$T_{j}(\Delta x_{j}) = \begin{cases} 0, & \Delta x_{j} = 0, \\ f_{j} + g_{j} |\Delta x_{j}|, & \text{otherwise.} \end{cases}$$

Here  $\Delta x_j$  is the change in the holding of asset j i.e.

$$\Delta x_j = x_j - x_j^0.$$

Hence, whenever asset j is traded we pay a fixed setup cost  $f_j$  and a variable cost of  $g_j$  per unit traded. This sort of cost function can be modeled using mixed-integer optimization, in particular using a binary variable  $y_j$  to indicate if asset j is traded. Given the assumptions about transaction costs in this section problem (11.11) may be formulated as

maximize 
$$\mu^{T}x$$
  
subject to  $e^{T}x + f^{T}y + g^{T}z = w + e^{T}x^{0}$ ,  
 $[\gamma; G^{T}x] \in \mathcal{Q}^{n+1}$ ,  
 $z_{j} \geq x_{j} - x_{j}^{0}$ ,  $j = 1, \dots, n$ ,  
 $z_{j} \geq x_{j}^{0} - x_{j}$ ,  $j = 1, \dots, n$ ,  
 $z_{j} \leq U_{j}y_{j}$ ,  $j = 1, \dots, n$ ,  
 $y_{j} \in \{0, 1\}$ ,  $j = 1, \dots, n$ ,  
 $x \geq 0$ . (11.18)

First observe that

$$z_j \ge |x_j - x_j^0| = |\Delta x_j|.$$

Here  $U_j$  is some a priori chosen upper bound on the amount of trading in asset j and therefore if  $z_j > 0$ then  $y_i = 1$  has to be the case. This implies that the transaction cost for asset j is given by

$$f_i y_i + g_i z_i$$
.

In our problem a safe bound for each  $U_j$  is the total initial wealth  $w + e^T x^0$ , however knowing a tighter bound may lead to shorter solution times.

#### Creating a matrix formulation

One more reformulation of (11.18) is needed to bring it to the standard form (11.4).

maximize 
$$\mu^T x$$
  
subject to  $e^T x + f^T y + g^T z = w + e^T x^0$ ,  
 $G^T x - t = 0$ ,  
 $z_j - x_j \geq -x_j^0$ ,  $j = 1, \dots, n$ ,  
 $z_j + x_j \geq x_j^0$ ,  $j = 1, \dots, n$ ,  
 $(s,t) \in \mathcal{Q}^{n+1}$ ,  $(11.19)$   
 $z_j - U_j y_j \leq 0$ ,  $j = 1, \dots, n$ ,  
 $x \geq 0$ ,  
 $y_j \in [0,1]$ ,  $j = 1, \dots, n$ ,  
 $y_j \in \mathbb{Z}$ ,  $j = 1, \dots, n$ ,  
 $s = \gamma$ .

We will choose the following ordering of variables:

$$\hat{x} = [x; s; t; z; y]$$

Table 11.3 shows the mapping between the  $\hat{x}$  vector and the model variables.

Table 11.3: Storage layout for  $\hat{x}$ 

Variable	Length	Offset
x	n	0
s	1	n
t	n	n+1
z	n	2n + 1
y	n	3n + 1

The next step is to consider how the linear constraint matrix A and the remaining data vectors are laid out. Reusing the idea in Sec. 11.1.1 we can write the data in block matrix form and read off all the required coordinates. This extension of the code setting up the constraint  $G^Tx - t = 0$  from Sec. 11.1.1 is shown below.

#### Example code

The following example code demonstrates how to compute an optimal portfolio when transaction setup costs are included. Note that we are now solving a problem with integer variables, and therefore the solution must be retrieved from  $MSK\_SOL\_ITG$  rather than  $MSK\_SOL\_ITR$ .

Listing 11.8: Code solving problem (11.18).

```
# include <math.h>
# include <stdio.h>
#include "mosek.h"
```

```
# define MOSEKCALL(\_r,\_call) if ((\_r)==MSK\_RES\_OK)(\_r) = (\_call)
static void MSKAPI printstr(void *handle,
                            const char str[])
 printf("%s", str);
} /* printstr */
int main(int argc, const char **argv)
 char
                  buf[128];
 const MSKint32t n
                         = 3;
 const double
                          = 1.0,
                  x0[]
                         = \{0.0, 0.0, 0.0\},
                         = 0.05,
                  gamma
                          = \{0.1073, 0.0737, 0.0627\},
                  mu[]
                          = \{0.01, 0.01, 0.01\},\
                  f[]
                         = \{0.001, 0.001, 0.001\},\
                  g[]
                  GT[][3] = \{\{0.1667, 0.0232, 0.0013\},
                             \{0.0000, 0.1033, -0.0022\},\
                             {0.0000, 0.0000, 0.0338}
                            };
 double
                  U,
                  expret,
                  stddev,
                  хj;
 MSKenv_t
                  env;
 MSKint32t
                  k, i, j,
                  offsetx, offsets, offsetz, offsety;
 MSKrescodee
                 res = MSK_RES_OK, trmcode;
 MSKtask_t
                 task;
  /* Initial setup. */
 env = NULL:
 task = NULL;
 MOSEKCALL(res, MSK_makeenv(&env, NULL));
 MOSEKCALL(res, MSK_maketask(env, 0, 0, &task));
 MOSEKCALL(res, MSK_linkfunctotaskstream(task, MSK_STREAM_LOG, NULL, printstr));
  /* Compute total wealth */
 U = w;
 for (k = 0; k < n; ++k) U += x0[k];
  /* Constraints. */
 MOSEKCALL(res, MSK_appendcons(task, 1 + 4 * n));
 MOSEKCALL(res, MSK_putconbound(task, 0, MSK_BK_FX, w, w));
 sprintf(buf, "%s", "budget");
 MOSEKCALL(res, MSK_putconname(task, 0, buf));
 for (i = 0; i < n; ++i)
 {
   MOSEKCALL(res, MSK_putconbound(task, 1 + i, MSK_BK_FX, 0.0, 0.0));
   sprintf(buf, "GT[%d]", 1 + i);
   MOSEKCALL(res, MSK_putconname(task, 1 + i, buf));
 }
```

```
for (i = 0; i < n; ++i)
   MOSEKCALL(res, MSK_putconbound(task, 1 + n + i, MSK_BK_LO, -x0[i], MSK_INFINITY));
   sprintf(buf, "zabs1[%d]", 1 + i);
   MOSEKCALL(res, MSK_putconname(task, 1 + n + i, buf));
 for (i = 0; i < n; ++i)
   MOSEKCALL(res, MSK_putconbound(task, 1 + 2 * n + i, MSK_BK_LO, x0[i], MSK_
→INFINITY));
   sprintf(buf, "zabs2[%d]", 1 + i);
   MOSEKCALL(res, MSK_putconname(task, 1 + 2 * n + i, buf));
 for (i = 0; i < n; ++i)
   MOSEKCALL(res, MSK_putconbound(task, 1 + 3 * n + i, MSK_BK_UP, -MSK_INFINITY, 0.
→0));
   sprintf(buf, "indicator[%d]", 1 + i);
   MOSEKCALL(res, MSK_putconname(task, 1 + 3 * n + i, buf));
 /* Offsets of variables into the (serialized) API variable. */
 offsetx = 0;
 offsets = n;
 offsett = n + 1;
 offsetz = 2 * n + 1;
 offsety = 3 * n + 1;
 /* Variables. */
 MOSEKCALL(res, MSK_appendvars(task, 4 * n + 1));
 /* x variables. */
 for (j = 0; j < n; ++j)
   MOSEKCALL(res, MSK_putcj(task, offsetx + j, mu[j]));
   MOSEKCALL(res, MSK_putaij(task, 0, offsetx + j, 1.0));
   for (k = 0; k < n; ++k)
     if (GT[k][j] != 0.0)
       MOSEKCALL(res, MSK_putaij(task, 1 + k, offsetx + j, GT[k][j]));
   MOSEKCALL(res, MSK_putaij(task, 1 + n + j, offsetx + j, -1.0));
   MOSEKCALL(res, MSK_putaij(task, 1 + 2 * n + j, offsetx + j, 1.0));
   MOSEKCALL(res, MSK_putvarbound(task, offsetx + j, MSK_BK_LO, 0.0, MSK_INFINITY));
   sprintf(buf, "x[%d]", 1 + j);
   MOSEKCALL(res, MSK_putvarname(task, offsetx + j, buf));
 }
 /* s variable. */
 MOSEKCALL(res, MSK_putvarbound(task, offsets + 0, MSK_BK_FX, gamma, gamma));
 sprintf(buf, "s");
 MOSEKCALL(res, MSK_putvarname(task, offsets + 0, buf));
 /* t variables. */
```

```
for (j = 0; j < n; ++j)
   MOSEKCALL(res, MSK_putaij(task, 1 + j, offsett + j, -1.0));
   MOSEKCALL(res, MSK_putvarbound(task, offsett + j, MSK_BK_FR, -MSK_INFINITY, MSK_
→INFINITY));
   sprintf(buf, "t[%d]", 1 + j);
   MOSEKCALL(res, MSK_putvarname(task, offsett + j, buf));
 /* z variables. */
 for (j = 0; j < n; ++j)
   MOSEKCALL(res, MSK_putaij(task, 0, offsetz + j, g[j]));
   MOSEKCALL(res, MSK_putaij(task, 1 + 1 * n + j, offsetz + j, 1.0));
   MOSEKCALL(res, MSK_putaij(task, 1 + 2 * n + j, offsetz + j, 1.0));
   MOSEKCALL(res, MSK_putaij(task, 1 + 3 * n + j, offsetz + j, 1.0));
   MOSEKCALL(res, MSK_putvarbound(task, offsetz + j, MSK_BK_FR, -MSK_INFINITY, MSK_
→INFINITY));
   sprintf(buf, "z[%d]", 1 + j);
   MOSEKCALL(res, MSK_putvarname(task, offsetz + j, buf));
 }
 /* y variables. */
 for (j = 0; j < n; ++j)
   MOSEKCALL(res, MSK_putaij(task, 0, offsety + j, f[j]));
   MOSEKCALL(res, MSK_putaij(task, 1 + 3 * n + j, offsety + j, -U));
   MOSEKCALL(res, MSK_putvarbound(task, offsety + j, MSK_BK_RA, 0.0, 1.0));
   MOSEKCALL(res, MSK_putvartype(task, offsety + j, MSK_VAR_TYPE_INT));
   sprintf(buf, "y[%d]", 1 + j);
   MOSEKCALL(res, MSK_putvarname(task, offsety + j, buf));
 }
 if (res == MSK_RES_OK)
   /* sub should be n+1 long i.e. the dimmension of the cone. */
   MSKint32t *sub = (MSKint32t *) MSK_calloctask(task, 3 >= n + 1 ? 3 : n + 1, 
if (sub)
     /* Add quadratic cone */
     sub[0] = offsets + 0;
     for (j = 0; j < n; ++j)
       sub[j + 1] = offsett + j;
     MOSEKCALL(res, MSK_appendcone(task, MSK_CT_QUAD, 0.0, n + 1, sub));
     MOSEKCALL(res, MSK_putconename(task, 0, "stddev"));
     MSK_freetask(task, sub);
   }
   else
     res = MSK_RES_ERR_SPACE;
 MOSEKCALL(res, MSK_putobjsense(task, MSK_OBJECTIVE_SENSE_MAXIMIZE));
```

```
#if 0
 /* no log output. */
 MOSEKCALL(res, MSK_putintparam(task, MSK_IPAR_LOG, 0));
#endif
#if 0
 /* Dump the problem to a human readable OPF file. */
 MOSEKCALL(res, MSK_writedata(task, "dump.opf"));
#endif
 MOSEKCALL(res, MSK_optimizetrm(task, &trmcode));
# if 1
 /* Display the solution summary for quick inspection of results. */
 MSK_solutionsummary(task, MSK_STREAM_MSG);
# endif
 if (res == MSK_RES_OK)
    expret = 0.0;
   stddev = 0.0;
   for (j = 0; j < n; ++j)
     MOSEKCALL(res, MSK_getxxslice(task, MSK_SOL_ITG, offsetx + j, offsetx + j + 1, &

    xj));
     expret += mu[j] * xj;
   MOSEKCALL(res, MSK_getxxslice(task, MSK_SOL_ITG, offsets + 0, offsets + 1, &
→stddev));
   printf("\nExpected return %e for gamma %e\n", expret, stddev);
 MSK_deletetask(&task);
 MSK_deleteenv(&env);
 return (0);
```

# 11.1.7 Cardinality constraints

Another method to reduce costs involved with processing transactions is to only change positions in a small number of assets. In other words, at most k of the differences  $|\Delta x_j| = |x_j - x_j^0|$  are allowed to be non-zero, where k is (much) smaller than the total number of assets n.

This type of constraint can be again modeled by introducing a binary variable  $y_j$  which indicates if  $\Delta x_j \neq 0$  and bounding the sum of  $y_j$ . The basic Markowitz model then gets updated as follows:

maximize 
$$\mu^T x$$
  
subject to  $e^T x = w + e^T x^0$ ,  
 $[\gamma; G^T x] \in \mathcal{Q}^{n+1}$ ,  
 $U_j y_j \geq |x_j - x_j^0|$ ,  $j = 1, \dots, n$ ,  
 $y_j \in \{0, 1\}$ ,  $j = 1, \dots, n$ ,  
 $e^T y \leq k$ ,  
 $x \geq 0$ ,

Ori chosen upper bound on the amount of trading in asset  $i$ . This quark

were  $U_j$  is some a priori chosen upper bound on the amount of trading in asset j. This guarantees that  $|x_j - x_j^0|$  forces  $y_j = 1$  and therefore  $e^T y$  counts the number of assets in which we trade. In our problem a safe bound for each  $U_j$  is the total initial wealth  $w + e^T x^0$ , however knowing a tighter bound may lead to shorter solution times.

## Creating a matrix formulation

One more reformulation of (11.20) is needed to bring it to the standard form (11.4).

maximize 
$$\mu^{T}x$$
  
subject to  $e^{T}x = w + e^{T}x^{0}$ ,  
 $G^{T}x - t = 0$ ,  
 $z_{j} - x_{j} \geq -x_{j}^{0}$ ,  $j = 1, ..., n$ ,  
 $z_{j} + x_{j} \geq x_{j}^{0}$ ,  $j = 1, ..., n$ ,  
 $(s,t) \in \mathcal{Q}^{n+1}$ ,  
 $z_{j} - U_{j}y_{j} \leq 0$ ,  $j = 1, ..., n$ ,  
 $e^{T}y \leq k$ ,  
 $x \geq 0$ ,  
 $y_{j} \in [0,1]$ ,  $j = 1, ..., n$ ,  
 $y_{j} \in \mathbb{Z}$ ,  $j = 1, ..., n$ ,  
 $s = \gamma$ . (11.21)

We will choose the following ordering of variables:

$$\hat{x} = [x; s; t; z; y]$$

Table 11.4 shows the mapping between the  $\hat{x}$  vector and the model variables.

Table 11.4: Storage layout for  $\hat{x}$ 

	_	•
Variable	Length	Offset
x	n	0
s	1	n
t	$\mid n \mid$	n+1
z	$\mid n \mid$	2n+1
y	n	3n + 1

The next step is to consider how the linear constraint matrix A and the remaining data vectors are laid out. Reusing the idea in Sec. 11.1.1 we can write the data in block matrix form and read off all the required coordinates. This extension of the code setting up the constraint  $G^T x - t = 0$  from Sec. 11.1.1 is shown below.

#### Example code

The following example code demonstrates how to compute an optimal portfolio with cardinality bounds. Note that we are now solving a problem with integer variables, and therefore the solution must be retrieved from  $MSK\_SOL\_ITG$ .

Listing 11.9: Code solving problem (11.20).

```
MSKrescodee markowitz_with_card(const int
                                                  n.
                                const double
                                                  x0[],
                                const double
                                                  W,
                                const double
                                                  gamma,
                                const double
                                                  mu[],
                                                  GT[],
                                const double
                                const int
                                                  k,
                                      double
                                                  *xx)
{
                  buf[128];
 char
  double
                  U;
 MSKenv_t
                  env;
 MSKint32t
                  i, j,
                  offsetx, offsets, offsett, offsetz, offsety;
                  res = MSK_RES_OK, trmcode;
 MSKrescodee
 MSKtask_t
                  task;
 /* Initial setup. */
 env = NULL;
 task = NULL:
 MOSEKCALL(res, MSK_makeenv(&env, NULL));
 MOSEKCALL(res, MSK_maketask(env, 0, 0, &task));
 MOSEKCALL(res, MSK_linkfunctotaskstream(task, MSK_STREAM_LOG, NULL, printstr));
 /* Compute total wealth */
 U = w;
 for (i = 0; i < n; ++i) U += x0[i];
  /* Constraints. */
 MOSEKCALL(res, MSK_appendcons(task, 2 + 4 * n));
 MOSEKCALL(res, MSK_putconbound(task, 0, MSK_BK_FX, w, w));
 sprintf(buf, "%s", "budget");
 MOSEKCALL(res, MSK_putconname(task, 0, buf));
 MOSEKCALL(res, MSK_putconbound(task, 1 + 4 * n, MSK_BK_UP, -MSK_INFINITY,_

    double)k));
 sprintf(buf, "%s", "cardinality");
 MOSEKCALL(res, MSK_putconname(task, 1 + 4 * n, buf));
 for (i = 0; i < n; ++i)
   MOSEKCALL(res, MSK_putconbound(task, 1 + i, MSK_BK_FX, 0.0, 0.0));
    sprintf(buf, "GT[%d]", 1 + i);
   MOSEKCALL(res, MSK_putconname(task, 1 + i, buf));
 }
 for (i = 0; i < n; ++i)
   MOSEKCALL(res, MSK_putconbound(task, 1 + n + i, MSK_BK_LO, -x0[i], MSK_INFINITY));
```

```
sprintf(buf, "zabs1[%d]", 1 + i);
   MOSEKCALL(res, MSK_putconname(task, 1 + n + i, buf));
 }
 for (i = 0; i < n; ++i)
   MOSEKCALL(res, MSK_putconbound(task, 1 + 2 * n + i, MSK_BK_LO, x0[i], MSK_
→INFINITY));
   sprintf(buf, "zabs2[%d]", 1 + i);
   MOSEKCALL(res, MSK_putconname(task, 1 + 2 * n + i, buf));
 }
 for (i = 0; i < n; ++i)
   MOSEKCALL(res, MSK_putconbound(task, 1 + 3 * n + i, MSK_BK_UP, -MSK_INFINITY, 0.
\rightarrow 0));
   sprintf(buf, "indicator[%d]", 1 + i);
   MOSEKCALL(res, MSK_putconname(task, 1 + 3 * n + i, buf));
 /* Offsets of variables into the (serialized) API variable. */
 offsetx = 0;
 offsets = n;
 offsett = n + 1;
 offsetz = 2 * n + 1;
 offsety = 3 * n + 1;
 /* Variables. */
 MOSEKCALL(res, MSK_appendvars(task, 4 * n + 1));
 /* x variables. */
 for (j = 0; j < n; ++j)
   MOSEKCALL(res, MSK_putcj(task, offsetx + j, mu[j]));
   MOSEKCALL(res, MSK_putaij(task, 0, offsetx + j, 1.0));
   for (i = 0; i < n; ++i)
     if (GT[i*n+j] != 0.0)
       MOSEKCALL(res, MSK_putaij(task, 1 + i, offsetx + j, GT[i*n+j]));
   MOSEKCALL(res, MSK_putaij(task, 1 + n + j, offsetx + j, -1.0));
   MOSEKCALL(res, MSK_putaij(task, 1 + 2 * n + j, offsetx + j, 1.0));
   MOSEKCALL(res, MSK_putvarbound(task, offsetx + j, MSK_BK_LO, 0.0, MSK_INFINITY));
   sprintf(buf, "x[%d]", 1 + j);
   MOSEKCALL(res, MSK_putvarname(task, offsetx + j, buf));
 }
 /* s variable. */
 MOSEKCALL(res, MSK_putvarbound(task, offsets + 0, MSK_BK_FX, gamma, gamma));
 sprintf(buf, "s");
 MOSEKCALL(res, MSK_putvarname(task, offsets + 0, buf));
 /* t variables. */
 for (j = 0; j < n; ++j)
   MOSEKCALL(res, MSK_putaij(task, 1 + j, offsett + j, -1.0));
   MOSEKCALL(res, MSK_putvarbound(task, offsett + j, MSK_BK_FR, -MSK_INFINITY, MSK_
→ INFINITY));
                                                                      (continues on next page)
```

```
sprintf(buf, "t[%d]", 1 + j);
   MOSEKCALL(res, MSK_putvarname(task, offsett + j, buf));
 }
 /* z variables. */
 for (j = 0; j < n; ++j)
   MOSEKCALL(res, MSK\_putaij(task, 1 + 1 * n + j, offsetz + j, 1.0));
   MOSEKCALL(res, MSK_putaij(task, 1 + 2 * n + j, offsetz + j, 1.0));
   MOSEKCALL(res, MSK_putaij(task, 1 + 3 * n + j, offsetz + j, 1.0));
   MOSEKCALL(res, MSK_putvarbound(task, offsetz + j, MSK_BK_FR, -MSK_INFINITY, MSK_
→INFINITY));
   sprintf(buf, "z[%d]", 1 + j);
   MOSEKCALL(res, MSK_putvarname(task, offsetz + j, buf));
 /* y variables. */
 for (j = 0; j < n; ++j)
   MOSEKCALL(res, MSK_putaij(task, 1 + 4 * n, offsety + j, 1.0));
   MOSEKCALL(res, MSK_putaij(task, 1 + 3 * n + j, offsety + j, -U));
   MOSEKCALL(res, MSK_putvarbound(task, offsety + j, MSK_BK_RA, 0.0, 1.0));
   MOSEKCALL(res, MSK_putvartype(task, offsety + j, MSK_VAR_TYPE_INT));
   sprintf(buf, "y[%d]", 1 + j);
   MOSEKCALL(res, MSK_putvarname(task, offsety + j, buf));
 }
 if (res == MSK_RES_OK)
    /* sub should be n+1 long i.e. the dimmension of the cone. */
   MSKint32t *sub = (MSKint32t *) MSK_calloctask(task, 3 >= n + 1 ? 3 : n + 1, ...
⇔sizeof(MSKint32t));
   if (sub)
     /* Add quadratic cone */
     sub[0] = offsets + 0;
     for (j = 0; j < n; ++j)
       sub[j + 1] = offsett + j;
     MOSEKCALL(res, MSK_appendcone(task, MSK_CT_QUAD, 0.0, n + 1, sub));
     MOSEKCALL(res, MSK_putconename(task, 0, "stddev"));
     MSK_freetask(task, sub);
   }
   else
     res = MSK_RES_ERR_SPACE;
 MOSEKCALL(res, MSK_putobjsense(task, MSK_OBJECTIVE_SENSE_MAXIMIZE));
# if 1
 /* no log output. */
 MOSEKCALL(res, MSK_putintparam(task, MSK_IPAR_LOG, 0));
# endif
```

```
#if 0
    /* Dump the problem to a human readable OPF file. */
    MOSEKCALL(res, MSK_writedata(task, "dump.opf"));
#endif

MOSEKCALL(res, MSK_optimizetrm(task, &trmcode));

#if 1
    /* Display the solution summary for quick inspection of results. */
    MSK_solutionsummary(task, MSK_STREAM_MSG);
#endif

if (res == MSK_RES_OK)
    MOSEKCALL(res, MSK_getxxslice(task, MSK_SOL_ITG, offsetx, offsetx + n, xx));

MSK_deletetask(&task);
    MSK_deleteenv(&env);
    return res;
}
```

If we solve our running example with k = 1, 2, 3 then we get the following solutions, with increasing expected returns:

```
Bound 1: x = 0.00000 0.00000 1.00000 Return: x = 0.06270
Bound 2: x = 0.25286 0.00000 0.74714 Return: x = 0.07398
Bound 3: x = 0.23639 0.13850 0.62511 Return: x = 0.07477
```

# 11.2 Logistic regression

Logistic regression is an example of a binary classifier, where the output takes one two values 0 or 1 for each data point. We call the two values *classes*.

## Formulation as an optimization problem

Define the sigmoid function

$$S(x) = \frac{1}{1 + \exp(-x)}.$$

Next, given an observation  $x \in \mathbb{R}^d$  and a weights  $\theta \in \mathbb{R}^d$  we set

$$h_{\theta}(x) = S(\theta^T x) = \frac{1}{1 + \exp(-\theta^T x)}.$$

The weights vector  $\theta$  is part of the setup of the classifier. The expression  $h_{\theta}(x)$  is interpreted as the probability that x belongs to class 1. When asked to classify x the returned answer is

$$x \mapsto \begin{cases} 1 & h_{\theta}(x) \ge 1/2, \\ 0 & h_{\theta}(x) < 1/2. \end{cases}$$

When training a logistic regression algorithm we are given a sequence of training examples  $x_i$ , each labelled with its class  $y_i \in \{0,1\}$  and we seek to find the weights  $\theta$  which maximize the likelihood function

$$\prod_{i} h_{\theta}(x_{i})^{y_{i}} (1 - h_{\theta}(x_{i}))^{1 - y_{i}}.$$

Of course every single  $y_i$  equals 0 or 1, so just one factor appears in the product for each training data point. By taking logarithms we can define the logistic loss function:

$$J(\theta) = -\sum_{i:y_i=1} \log(h_{\theta}(x_i)) - \sum_{i:y_i=0} \log(1 - h_{\theta}(x_i)).$$

The training problem with regularization (a standard technique to prevent overfitting) is now equivalent to

$$\min_{\theta} J(\theta) + \lambda \|\theta\|_2.$$

This can equivalently be phrased as

minimize 
$$\sum_{i} t_{i} + \lambda r$$
subject to 
$$\begin{aligned} t_{i} &\geq -\log(h_{\theta}(x)) &= \log(1 + \exp(-\theta^{T} x_{i})) & \text{if } y_{i} = 1, \\ t_{i} &\geq -\log(1 - h_{\theta}(x)) &= \log(1 + \exp(\theta^{T} x_{i})) & \text{if } y_{i} = 0, \\ r &\geq \|\theta\|_{2}. \end{aligned}$$

$$(11.22)$$

#### Implementation

As can be seen from (11.22) the key point is to implement the softplus bound  $t \ge \log(1 + e^u)$ , which is the simplest example of a log-sum-exp constraint for two scalar variables t, u. This is equivalent to

$$\exp(u-t) + \exp(-t) \le 1$$

and further to

$$(z_1, 1, u - t) \in K_{\exp} (z_1 \ge \exp(u - t)),$$
  
 $(z_2, 1, -t) \in K_{\exp} (z_2 \ge \exp(-t)),$   
 $z_1 + z_2 \le 1.$  (11.23)

To feed these constraints into **MOSEK** we add more auxiliary variables  $q_1, q_2, v_1, v_2$  with constraints  $(z_1, q_1, v_1) \in K_{\text{exp}}, (z_2, q_2, v_2) \in K_{\text{exp}}, q_1 = q_2 = 1, v_1 = u - t \text{ and } v_2 = -t.$ 

Listing 11.10: Implementation of  $t \ge \log(1 + e^u)$  as in (11.23).

```
//t \geq \log(1 + \exp(u))
// t_i >= log(1 + exp(u_i)), i = 0..n-1
// Adds auxiliary variables and constraints
MSKrescodee softplus(MSKtask_t task, MSKint32t t, MSKint32t u, int n)
 MSKint32t nvar, ncon;
 MSKint32t z1, z2, v1, v2, q1, q2;
 MSKint32t zcon, v1con, v2con;
 MSKint32t *subi = (MSKint32t*) calloc(7*n, sizeof(MSKint32t));
              *subj = (MSKint32t*) calloc(7*n, sizeof(MSKint32t));
 MSKint32t
               *aval = (MSKrealt*) calloc(7*n, sizeof(MSKrealt));
 MSKrealt
             k = 0, i = 0;
 MSKrescodee res = MSK_RES_OK;
 MSKCALL(MSK_getnumvar(task, &nvar));
 MSKCALL(MSK_getnumcon(task, &ncon));
 z1 = nvar, z2 = nvar+n, v1 = nvar+2*n, v2 = nvar+3*n, q1 = nvar+4*n, q2 = nvar+5*n;
 zcon = ncon, v1con = ncon+n, v2con=ncon+2*n;
 MSKCALL(MSK_appendvars(task, 6*n));
 MSKCALL(MSK_appendcons(task, 3*n));
  // Linear constraints
  for(i = 0; i < n; i++)
```

```
{
  // z1 + z2 = 1
  subi[k] = zcon+i; subj[k] = z1+i; aval[k] = 1; k++;
  subi[k] = zcon+i; subj[k] = z2+i; aval[k] = 1; k++;
  // u - t - v1 = 0
  subi[k] = v1con+i; subj[k] = u+i; aval[k] = 1; k++;
  subi[k] = v1con+i; subj[k] = t+i; aval[k] = -1; k++;
  subi[k] = v1con+i; subj[k] = v1+i; aval[k] = -1; k++;
  // - t - v2 = 0
  subi[k] = v2con+i; subj[k] = t+i; aval[k] = -1; k++;
  subi[k] = v2con+i; subj[k] = v2+i; aval[k] = -1; k++;
MSKCALL(MSK_putaijlist(task, 7*n, subi, subj, aval));
MSKCALL(MSK_putconboundsliceconst(task, ncon, ncon+n, MSK_BK_FX, 1, 1));
MSKCALL(MSK_putconboundsliceconst(task, ncon+n, ncon+3*n, MSK_BK_FX, 0, 0));
// Bounds for variables
MSKCALL(MSK_putvarboundsliceconst(task, nvar, nvar+4*n, MSK_BK_FR, -inf, inf));
MSKCALL(MSK_putvarboundsliceconst(task, nvar+4*n, nvar+6*n, MSK_BK_FX, 1, 1));
// Cones
for(i = 0; i < n && res == MSK_RES_OK; i++)</pre>
  MSKint32t csub[3];
  csub[0] = z1+i; csub[1] = q1+i; csub[2] = v1+i;
  MSKCALL(MSK_appendcone(task, MSK_CT_PEXP, 0.0, 3, csub));
  csub[0] = z2+i; csub[1] = q2+i; csub[2] = v2+i;
  MSKCALL(MSK_appendcone(task, MSK_CT_PEXP, 0.0, 3, csub));
}
free(subi); free(subj); free(aval);
return res;
```

Once we have this subroutine, it is easy to implement a function that builds the regularized loss function model (11.22).

Listing 11.11: Implementation of (11.22).

```
// Model logistic regression (regularized with full 2-norm of theta)
// X - n x d matrix of data points
// y - length n vector classifying training points
// lamb - regularization parameter
MSKrescodee logisticRegression(MSKenv_t
                                                    // num samples
                               int
                                             n,
                                                    // dimension
                               int.
                                             d,
                                            *X,
                               double
                               int
                                             *y,
                               double
                                             lamb,
                               double
                                            *thetaVal) // result
 MSKrescodee res = MSK_RES_OK;
 MSKrescodee trm = MSK_RES_OK;
 MSKtask_t task = NULL;
 MSKint32t nvar = 1+d+2*n;
 MSKint32t r = 0, theta = 1, t = 1+d, u = 1+d+n;
 int i = 0;
```

```
MSKCALL(MSK_maketask(env, 0, 0, &task));
MSKCALL(MSK_linkfunctotaskstream(task, MSK_STREAM_LOG, NULL, printstr));
// Variables [r; theta; t; u]
MSKCALL(MSK_appendvars(task, nvar));
MSKCALL(MSK_putvarboundsliceconst(task, 0, nvar, MSK_BK_FR, -inf, inf));
// Constraints: theta'*X +/- u = 0
MSKCALL(MSK_appendcons(task, n));
MSKCALL(MSK_putconboundsliceconst(task, 0, n, MSK_BK_FX, 0, 0));
// Objective lambda*r + sum(t)
MSKCALL(MSK_putcj(task, r, lamb));
for(i = 0; i < n && res == MSK_RES_OK; i++)</pre>
  MSKCALL(MSK_putcj(task, t+i, 1.0));
// The X block in theta'*X +/- u = 0
  MSKint32t *subi = (MSKint32t*) calloc(d*n+n, sizeof(MSKint32t));
  MSKint32t *subj = (MSKint32t*) calloc(d*n+n, sizeof(MSKint32t));
  MSKrealt *aval = (MSKrealt*) calloc(d*n+n, sizeof(MSKrealt));
  int j, k;
  for(i = 0; i < n; i++)
   for(j = 0; j < d; j++)
     k = i * d + j;
      subi[k] = i; subj[k] = theta+j; aval[k] = X[k];
    subi[d*n+i] = i; subj[d*n+i] = u+i;
    if (y[i]) aval[d*n+i] = 1; else aval[d*n+i] = -1;
  MSKCALL(MSK_putaijlist(task, n*d+n, subi, subj, aval));
  free(subi); free(subj); free(aval);
// Softplus function constraints
MSKCALL(softplus(task, t, u, n));
// Regularization
MSKCALL(MSK_appendconeseq(task, MSK_CT_QUAD, 0.0, 1+d, r));
// Solution
MSKCALL(MSK_optimizetrm(task, &trm));
MSKCALL(MSK_solutionsummary(task, MSK_STREAM_MSG));
MSKCALL(MSK_getxxslice(task, MSK_SOL_ITR, theta, theta+d, thetaVal));
return res;
```

### Example: 2D dataset fitting

In the next figure we apply logistic regression to the training set of 2D points taken from the example ex2data2.txt. The two-dimensional dataset was converted into a feature vector  $x \in \mathbb{R}^{28}$  using monomial coordinates of degrees at most 6.

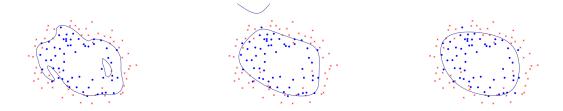


Fig. 11.1: Logistic regression example with none, medium and strong regularization (small, medium, large  $\lambda$ ). Without regularization we get obvious overfitting.

# 11.3 Concurrent optimizer

The idea of the concurrent optimizer is to run multiple optimizations of **the same problem** simultaneously, and pick the one that provides the fastest or best answer. This approach is especially useful for problems which require a very long time and it is hard to say in advance which optimizer or algorithm will perform best.

The major applications of concurrent optimization we describe in this section are:

- Using the interior-point and simplex optimizers simultaneously on a linear problem. Note that any solution present in the task will also be used for hot-starting the simplex algorithms. One possible scenario would therefore be running a hot-start simplex in parallel with interior point, taking advantage of both the stability of the interior-point method and the ability of the simplex method to use an initial solution.
- Using multiple instances of the mixed-integer optimizer to solve many copies of one mixed-integer problem. This is not in contradiction with the run-to-run determinism of **MOSEK** if a different value of the MIO seed parameter <code>MSK\_IPAR\_MIO\_SEED</code> is set in each instance. As a result each setting leads to a different optimizer run (each of them being deterministic in its own right).

The downloadable file contains usage examples of both kinds.

## 11.3.1 Common setup

We first define a method that runs a number of optimization tasks in parallel, using the standard multithreading setup available in the language. All tasks register for a callback function which will signal them to interrupt as soon as the first task completes successfully (with response code MSK\_RES\_OK).

Listing 11.12: Simple callback function which signals the optimizer to stop (C++).

```
{
  return stop;
}
```

When all remaining tasks respond to the stop signal, response codes and statuses are returned to the caller, together with the index of the task which won the race.

Listing 11.13: A routine for parallel task race (C++).

```
void runTask(int
                         num,
             MSKtask_t
                         task,
             MSKrescodee *res,
             MSKrescodee *trm)
{
  *res = MSK_optimizetrm(task, trm);
  if (*res == MSK_RES_OK) {
    if (!stop) firstStop = num;
    stop = 1;
}
int optimize(int
             MSKtask_t
                         *tasks,
             MSKrescodee *res,
             MSKrescodee *trm)
{
  int i;
  std::thread * jobs = new std::thread[n];
  // Set a callback function and start optimization
 for(i = 0; i < n; ++i) {
    MSK_putcallbackfunc(tasks[i], callback, NULL);
    res[i] = trm[i] = MSK_RES_ERR_UNKNOWN;
    jobs[i] = std::thread(runTask, i, tasks[i], &(res[i]), &(trm[i]));
  // Join all threads
  for(i = 0; i < n; ++i) jobs[i].join();</pre>
  delete[] jobs;
  // For debugging, print res and trm codes for all optimizers
  for(i = 0; i < n; ++i)
    printf("Optimizer %d res %d trm %d\n", i, res[i], trm[i]);
  return firstStop;
```

## 11.3.2 Linear optimization

We use the multithreaded setup to run the interior-point and simplex optimizers simultaneously on a linear problem. The next methods simply clones the given task and sets a different optimizer for each. The result is the clone which finished first.

Listing 11.14: Concurrent optimization with different optimizers (C++).

```
int optimizeconcurrent(MSKtask_t
                                            task,
                       MSKoptimizertypee
                                            *optimizers,
                       MSKtask_t
                                            *winTask,
                       MSKrescodee
                                            *winTrm.
                       MSKrescodee
                                            *winRes)
            *tasks = new MSKtask_t[n];
 MSKtask_t
 MSKrescodee *res = new MSKrescodee[n];
 MSKrescodee *trm = new MSKrescodee[n];
 // Clone tasks and choose various optimizers
 for (int i = 0; i < n; ++i)
   MSK_clonetask(task, &(tasks[i]));
   MSK_putintparam(tasks[i], MSK_IPAR_OPTIMIZER, optimizers[i]);
 }
 // Solve tasks in parallel
 int firstOK = optimize(n, tasks, res, trm);
 if (firstOK >= 0)
    *winTask = tasks[firstOK];
    *winTrm = trm[firstOK];
    *winRes = res[firstOK];
 }
 else
    *winTask = NULL;
    *winTrm = MSK_RES_ERR_UNKNOWN;
    *winRes = MSK_RES_ERR_UNKNOWN;
 }
 // Cleanup
 for (int i = 0; i < n; ++i)
    if (i != firstOK) MSK_deletetask(&(tasks[i]));
 delete[] tasks; delete[] res; delete[] trm;
 return firstOK;
```

It remains to call the method with a choice of optimizers, for example:

Listing 11.15: Calling concurrent linear optimization (C++).

```
MSKoptimizertypee optimizers[3] = {
    MSK_OPTIMIZER_CONIC,
    MSK_OPTIMIZER_DUAL_SIMPLEX,
    MSK_OPTIMIZER_PRIMAL_SIMPLEX
```

```
};
idx = optimizeconcurrent(task, 3, optimizers, &t, &trm, &res);
```

### 11.3.3 Mixed-integer optimization

We use the multithreaded setup to run many, differently seeded copies of the mixed-integer optimizer. This approach is most useful for hard problems where we don't expect an optimal solution in reasonable time. The input task would typically contain a time limit. It is possible that all the cloned tasks reach the time limit, in which case it doesn't really mater which one terminated first. Instead we examine all the task clones for the best objective value.

Listing 11.16: Concurrent optimization of a mixed-integer problem (C++).

```
int optimizeconcurrentMIO(MSKtask_t
                                               task,
                          int
                                               n,
                          int
                                               *seeds,
                          MSKtask_t
                                               *winTask,
                          MSKrescodee
                                               *winTrm,
                          MSKrescodee
                                               *winRes)
{
 MSKtask_t *tasks = new MSKtask_t[n];
 MSKrescodee *res = new MSKrescodee[n];
                    = new MSKrescodee[n];
 MSKrescodee *trm
  *winTask = NULL;
  *winTrm = MSK_RES_ERR_UNKNOWN;
  *winRes
          = MSK_RES_ERR_UNKNOWN;
 int bestPos = -1;
 // Clone tasks and choose various optimizers
 for (int i = 0; i < n; ++i)
   MSK_clonetask(task, &(tasks[i]));
   MSK_putintparam(tasks[i], MSK_IPAR_MIO_SEED, seeds[i]);
 // Solve tasks in parallel
 int firstOK = optimize(n, tasks, res, trm);
 if (firstOK >= 0)
    // Pick the task that ended with res = ok
   // and contains an integer solution with best objective value
   MSKobjsensee sense;
   double bestObj;
   MSK_getobjsense(task, &sense);
   best0bj = (sense == MSK_OBJECTIVE_SENSE_MINIMIZE) ? 1.0e+10 : -1.0e+10;
   for (int i = 0; i < n; ++i) {
      double priObj;
     MSK_getprimalobj(tasks[i], MSK_SOL_ITG, &priObj);
     printf("%d
                    %f\n", i, priObj);
   }
```

```
for (int i = 0; i < n; ++i) {
    double priObj;
    MSKsolstae solsta;
   MSK_getprimalobj(tasks[i], MSK_SOL_ITG, &priObj);
   MSK_getsolsta(tasks[i], MSK_SOL_ITG, &solsta);
    if ((res[i] == MSK_RES_OK) &&
        (solsta == MSK_SOL_STA_PRIM_FEAS | |
         solsta == MSK_SOL_STA_INTEGER_OPTIMAL) &&
        ((sense == MSK_OBJECTIVE_SENSE_MINIMIZE) ?
            (priObj < bestObj) : (priObj > bestObj) ) )
      bestObj = priObj;
      bestPos = i;
  if (bestPos != -1)
    *winTask = tasks[bestPos];
    *winTrm = trm[bestPos];
    *winRes = res[bestPos];
}
// Cleanup
for (int i = 0; i < n; ++i)
  if (i != bestPos) MSK_deletetask(&(tasks[i]));
delete[] tasks; delete[] res; delete[] trm;
return bestPos;
```

It remains to call the method with a choice of seeds, for example:

Listing 11.17: Calling concurrent integer optimization (C++).

```
int seeds[3] = { 42, 13, 71749373 };
idx = optimizeconcurrentMIO(task, 3, seeds, &t, &trm, &res);
```

# Chapter 12

# Problem Formulation and Solutions

In this chapter we will discuss the following issues:

- The formal, mathematical formulations of the problem types that MOSEK can solve and their duals.
- The solution information produced by **MOSEK**.
- The infeasibility certificate produced by MOSEK if the problem is infeasible.

For the underlying mathematical concepts, derivations and proofs see the Modeling Cookbook or any book on convex optimization. This chapter explains how the related data is organized specifically within the **MOSEK** API.

# 12.1 Linear Optimization

MOSEK accepts linear optimization problems of the form

where

- *m* is the number of constraints.
- $\bullet$  *n* is the number of decision variables.
- $x \in \mathbb{R}^n$  is a vector of decision variables.
- $c \in \mathbb{R}^n$  is the linear part of the objective function.
- $c^f \in \mathbb{R}$  is a constant term in the objective
- $A \in \mathbb{R}^{m \times n}$  is the constraint matrix.
- $l^c \in \mathbb{R}^m$  is the lower limit on the activity for the constraints.
- $u^c \in \mathbb{R}^m$  is the upper limit on the activity for the constraints.
- $l^x \in \mathbb{R}^n$  is the lower limit on the activity for the variables.
- $u^x \in \mathbb{R}^n$  is the upper limit on the activity for the variables.

Lower and upper bounds can be infinite, or in other words the corresponding bound may be omitted. A primal solution (x) is (primal) feasible if it satisfies all constraints in (12.1). If (12.1) has at least one primal feasible solution, then (12.1) is said to be (primal) feasible. In case (12.1) does not have a feasible solution, the problem is said to be (primal) infeasible

## 12.1.1 Duality for Linear Optimization

Corresponding to the primal problem (12.1), there is a dual problem

maximize 
$$(l^c)^T s_l^c - (u^c)^T s_u^c + (l^x)^T s_l^x - (u^x)^T s_u^x + c^f$$

$$A^T y + s_l^x - s_u^x = c,$$
subject to 
$$-y + s_l^c - s_u^c = 0,$$

$$s_l^c, s_u^c, s_l^x, s_u^x \geq 0.$$
 (12.2)

If a bound in the primal problem is plus or minus infinity, the corresponding dual variable is fixed at 0, and we use the convention that the product of the bound value and the corresponding dual variable is 0. This is equivalent to removing variable  $(s_l^x)_j$  from the dual problem. In other words:

$$l_i^x = -\infty \quad \Rightarrow \quad (s_l^x)_j = 0 \text{ and } l_i^x \cdot (s_l^x)_j = 0.$$

A solution

$$(y, s_l^c, s_u^c, s_l^x, s_u^x)$$

to the dual problem is feasible if it satisfies all the constraints in (12.2). If (12.2) has at least one feasible solution, then (12.2) is (dual) feasible, otherwise the problem is (dual) infeasible.

A solution

$$(x^*,y^*,(s^c_l)^*,(s^c_u)^*,(s^x_l)^*,(s^x_u)^*)$$

is denoted a primal-dual feasible solution, if  $(x^*)$  is a solution to the primal problem (12.1) and  $(y^*, (s_l^c)^*, (s_u^c)^*, (s_u^c)^*, (s_u^c)^*, (s_u^c)^*, (s_u^c)^*$  is a solution to the corresponding dual problem (12.2). We also define an auxiliary vector

$$(x^c)^* := Ax^*$$

containing the activities of linear constraints.

For a primal-dual feasible solution we define the  $duality\ gap$  as the difference between the primal and the dual objective value,

$$c^{T}x^{*} + c^{f} - \left\{ (l^{c})^{T}(s_{l}^{c})^{*} - (u^{c})^{T}(s_{u}^{c})^{*} + (l^{x})^{T}(s_{l}^{x})^{*} - (u^{x})^{T}(s_{u}^{x})^{*} + c^{f} \right\}$$

$$= \sum_{i=0}^{m-1} \left[ (s_{l}^{c})_{i}^{*}((x_{i}^{c})^{*} - l_{i}^{c}) + (s_{u}^{c})_{i}^{*}(u_{i}^{c} - (x_{i}^{c})^{*}) \right]$$

$$+ \sum_{j=0}^{m-1} \left[ (s_{l}^{x})_{j}^{*}(x_{j} - l_{j}^{x}) + (s_{u}^{x})_{j}^{*}(u_{j}^{x} - x_{j}^{*}) \right] \ge 0$$

$$(12.3)$$

where the first relation can be obtained by transposing and multiplying the dual constraints (12.2) by  $x^*$  and  $(x^c)^*$  respectively, and the second relation comes from the fact that each term in each sum is nonnegative. It follows that the primal objective will always be greater than or equal to the dual objective.

It is well-known that a linear optimization problem has an optimal solution if and only if there exist feasible primal-dual solution so that the duality gap is zero, or, equivalently, that the *complementarity* conditions

$$\begin{array}{rclcrcl} (s_l^c)_i^*((x_i^c)^*-l_i^c) & = & 0, & i=0,\dots,m-1, \\ (s_u^c)_i^*(u_i^c-(x_i^c)^*) & = & 0, & i=0,\dots,m-1, \\ (s_l^x)_j^*(x_j^*-l_j^x) & = & 0, & j=0,\dots,n-1, \\ (s_u^x)_j^*(u_j^x-x_j^*) & = & 0, & j=0,\dots,n-1, \end{array}$$

are satisfied.

If (12.1) has an optimal solution and **MOSEK** solves the problem successfully, both the primal and dual solution are reported, including a status indicating the exact state of the solution.

#### 12.1.2 Infeasibility for Linear Optimization

#### **Primal Infeasible Problems**

If the problem (12.1) is infeasible (has no feasible solution), **MOSEK** will report a certificate of primal infeasibility: The dual solution reported is the certificate of infeasibility, and the primal solution is undefined.

A certificate of primal infeasibility is a feasible solution to the modified dual problem

maximize 
$$(l^c)^T s_l^c - (u^c)^T s_u^c + (l^x)^T s_l^x - (u^x)^T s_u^x$$
 subject to 
$$A^T y + s_l^x - s_u^x = 0,$$
 
$$-y + s_l^c - s_u^c = 0,$$
 
$$s_l^c, s_u^c, s_l^x, s_u^x \geq 0,$$
 
$$(12.4)$$

such that the objective value is strictly positive, i.e. a solution

$$(y^*, (s_l^c)^*, (s_u^c)^*, (s_l^x)^*, (s_u^x)^*)$$

to (12.4) so that

$$(l^c)^T (s_l^c)^* - (u^c)^T (s_u^c)^* + (l^x)^T (s_l^x)^* - (u^x)^T (s_u^x)^* > 0.$$

Such a solution implies that (12.4) is unbounded, and that (12.1) is infeasible.

#### **Dual Infeasible Problems**

If the problem (12.2) is infeasible (has no feasible solution), **MOSEK** will report a certificate of dual infeasibility: The primal solution reported is the certificate of infeasibility, and the dual solution is undefined.

A certificate of dual infeasibility is a feasible solution to the modified primal problem

minimize 
$$c^T x$$
  
subject to  $\hat{l}^c \leq Ax \leq \hat{u}^c$ ,  $\hat{l}^x \leq x \leq \hat{u}^x$ , (12.5)

where

$$\hat{l}_i^c = \left\{ \begin{array}{ll} 0 & \text{if } l_i^c > -\infty, \\ -\infty & \text{otherwise,} \end{array} \right\} \quad \text{and} \quad \hat{u}_i^c := \left\{ \begin{array}{ll} 0 & \text{if } u_i^c < \infty, \\ \infty & \text{otherwise,} \end{array} \right\}$$

and

$$\hat{l}_{j}^{x} = \left\{ \begin{array}{ll} 0 & \text{if } l_{j}^{x} > -\infty, \\ -\infty & \text{otherwise,} \end{array} \right\} \quad \text{and} \quad \hat{u}_{j}^{x} := \left\{ \begin{array}{ll} 0 & \text{if } u_{j}^{x} < \infty, \\ \infty & \text{otherwise,} \end{array} \right\}$$

such that

$$c^T x < 0.$$

Such a solution implies that (12.5) is unbounded, and that (12.2) is infeasible.

In case that both the primal problem (12.1) and the dual problem (12.2) are infeasible, **MOSEK** will report only one of the two possible certificates — which one is not defined (**MOSEK** returns the first certificate found).

#### 12.1.3 Minimalization vs. Maximalization

When the objective sense of problem (12.1) is maximization, i.e.

the objective sense of the dual problem changes to minimization, and the domain of all dual variables changes sign in comparison to (12.2). The dual problem thus takes the form

This means that the duality gap, defined in (12.3) as the primal minus the dual objective value, becomes nonpositive. It follows that the dual objective will always be greater than or equal to the primal objective. The primal infeasibility certificate will be reported by **MOSEK** as a solution to the system

$$\begin{split} A^T y + s_l^x - s_u^x &= 0, \\ - y + s_l^c - s_u^c &= 0, \\ s_l^c, s_u^x, s_l^x, s_u^x &\leq 0, \end{split} \tag{12.6}$$

such that the objective value is strictly negative

$$(l^c)^T (s_l^c)^* - (u^c)^T (s_u^c)^* + (l^x)^T (s_l^x)^* - (u^x)^T (s_u^x)^* < 0.$$

Similarly, the certificate of dual infeasibility is an x satisfying the requirements of (12.5) such that  $c^T x > 0$ .

# 12.2 Conic Optimization

Conic optimization is an extension of linear optimization (see Sec. 12.1) allowing conic domains to be specified for subsets of the problem variables. A conic optimization problem to be solved by **MOSEK** can be written as

minimize 
$$c^T x + c^f$$
  
subject to  $l^c \le Ax \le u^c$ ,  
 $l^x \le x \le u^x$ ,  
 $x \in \mathcal{K}$ , (12.7)

where

- *m* is the number of constraints.
- $\bullet$  *n* is the number of decision variables.
- $x \in \mathbb{R}^n$  is a vector of decision variables.
- $c \in \mathbb{R}^n$  is the linear part of the objective function.
- $c^f \in \mathbb{R}$  is a constant term in the objective
- $A \in \mathbb{R}^{m \times n}$  is the constraint matrix.
- $l^c \in \mathbb{R}^m$  is the lower limit on the activity for the constraints.
- $u^c \in \mathbb{R}^m$  is the upper limit on the activity for the constraints.
- $l^x \in \mathbb{R}^n$  is the lower limit on the activity for the variables.
- $u^x \in \mathbb{R}^n$  is the upper limit on the activity for the variables.

Lower and upper bounds can be infinite, or in other words the corresponding bound may be omitted. The set  $\mathcal{K}$  is a Cartesian product of convex cones, namely  $\mathcal{K} = \mathcal{K}_1 \times \cdots \times \mathcal{K}_p$ . Having the domain restriction  $x \in \mathcal{K}$ , is thus equivalent to

$$x^t \in \mathcal{K}_t \subseteq \mathbb{R}^{n_t}$$
,

where  $x = (x^1, ..., x^p)$  is a partition of the problem variables. Please note that the *n*-dimensional Euclidean space  $\mathbb{R}^n$  is a cone itself, so simple linear variables are still allowed. The user only needs to specify subsets of variables which belong to non-trivial cones.

In this section we discuss the formulations which apply to the following cones supported by MOSEK:

- The set  $\mathbb{R}^n$ .
- The zero cone  $\{(0, ..., 0)\}.$
- Quadratic cone

$$Q^{n} = \left\{ x \in \mathbb{R}^{n} : x_{1} \ge \sqrt{\sum_{j=2}^{n} x_{j}^{2}} \right\}.$$

• Rotated quadratic cone

$$Q_r^n = \left\{ x \in \mathbb{R}^n : 2x_1 x_2 \ge \sum_{j=3}^n x_j^2, \quad x_1 \ge 0, \quad x_2 \ge 0 \right\}.$$

• Primal exponential cone

$$K_{\text{exp}} = \left\{ x \in \mathbb{R}^3 : x_1 \ge x_2 \exp(x_3/x_2), \quad x_1, x_2 \ge 0 \right\}$$

as well as its dual

$$K_{\text{exp}}^* = \left\{ x \in \mathbb{R}^3 : x_1 \ge -x_3 e^{-1} \exp(x_2/x_3), \quad x_3 \le 0, x_1 \ge 0 \right\}.$$

• Primal power cone (with parameter  $0 < \alpha < 1$ )

$$\mathcal{P}_n^{\alpha, 1 - \alpha} = \left\{ x \in \mathbb{R}^n : x_1^{\alpha} x_2^{1 - \alpha} \ge \sqrt{\sum_{j=3}^n x_j^2}, \quad x_1, x_2 \ge 0 \right\}$$

as well as its dual

$$(\mathcal{P}_n^{\alpha,1-\alpha})^* = \left\{ x \in \mathbb{R}^n : \left(\frac{x_1}{\alpha}\right)^\alpha \left(\frac{x_2}{1-\alpha}\right)^{1-\alpha} \ge \sqrt{\sum_{j=3}^n x_j^2}, \quad x_1, x_2 \ge 0 \right\}.$$

**MOSEK** supports also the cone of positive semidefinite matrices. Since that is handled through a separate interface, we discuss it in Sec. 12.3.

#### 12.2.1 Duality for Conic Optimization

Corresponding to the primal problem (12.7), there is a dual problem

$$\begin{array}{ll} \text{maximize} & (l^c)^T s_l^c - (u^c)^T s_u^c + (l^x)^T s_l^x - (u^x)^T s_u^x + c^f \\ \text{subject to} & \\ & A^T y + s_l^x - s_u^x + s_n^x = c \\ & - y + s_l^c - s_u^c = 0, \\ & s_l^c, s_u^c, s_l^x, s_u^x \geq 0, \\ & s_n^x \in \mathcal{K}^*, \end{array}$$

where the dual cone  $\mathcal{K}^*$  is a Cartesian product of the cones dual to  $\mathcal{K}_t$ . In practice this means that  $s_n^x$  has one entry for each entry in x. Please note that the dual problem of the dual problem is identical to the original primal problem.

If a bound in the primal problem is plus or minus infinity, the corresponding dual variable is fixed at 0, and we use the convention that the product of the bound value and the corresponding dual variable is 0. This is equivalent to removing variable  $(s_l^x)_j$  from the dual problem. In other words:

$$l_i^x = -\infty \quad \Rightarrow \quad (s_l^x)_j = 0 \text{ and } l_i^x \cdot (s_l^x)_j = 0.$$

A solution

$$(y, s_l^c, s_u^c, s_l^x, s_u^x, s_n^x)$$

to the dual problem is feasible if it satisfies all the constraints in (12.8). If (12.8) has at least one feasible solution, then (12.8) is (dual) feasible, otherwise the problem is (dual) infeasible.

A solution

$$(x^*, y^*, (s_l^c)^*, (s_u^c)^*, (s_l^x)^*, (s_u^x)^*, (s_u^x)^*)$$

is denoted a primal-dual feasible solution, if  $(x^*)$  is a solution to the primal problem (12.7) and  $(y^*, (s_l^c)^*, (s_u^c)^*, (s_u^c)^*, (s_u^c)^*, (s_u^c)^*, (s_u^c)^*, (s_u^c)^*, (s_u^c)^*$  is a solution to the corresponding dual problem (12.8). We also define an auxiliary vector

$$(x^c)^* := Ax^*$$

containing the activities of linear constraints.

For a primal-dual feasible solution we define the *duality gap* as the difference between the primal and the dual objective value,

$$c^{T}x^{*} + c^{f} - \left\{ (l^{c})^{T}(s_{l}^{c})^{*} - (u^{c})^{T}(s_{u}^{c})^{*} + (l^{x})^{T}(s_{l}^{x})^{*} - (u^{x})^{T}(s_{u}^{x})^{*} + c^{f} \right\}$$

$$= \sum_{i=0}^{m-1} \left[ (s_{l}^{c})_{i}^{*}((x_{i}^{c})^{*} - l_{i}^{c}) + (s_{u}^{c})_{i}^{*}(u_{i}^{c} - (x_{i}^{c})^{*}) \right]$$

$$+ \sum_{j=0}^{m-1} \left[ (s_{l}^{x})_{j}^{*}(x_{j} - l_{j}^{x}) + (s_{u}^{x})_{j}^{*}(u_{j}^{x} - x_{j}^{*}) \right] + \sum_{j=0}^{m-1} (s_{n}^{x})_{j}^{*}x_{j}^{*} \ge 0$$

$$(12.9)$$

where the first relation can be obtained by transposing and multiplying the dual constraints (12.2) by  $x^*$  and  $(x^c)^*$  respectively, and the second relation comes from the fact that each term in each sum is nonnegative. It follows that the primal objective will always be greater than or equal to the dual objective.

It is well-known that, under some non-degeneracy assumptions that exclude ill-posed cases, a conic optimization problem has an optimal solution if and only if there exist feasible primal-dual solution so that the duality gap is zero, or, equivalently, that the *complementarity conditions* 

$$(s_l^c)_i^*((x_i^c)^* - l_i^c) = 0, \quad i = 0, \dots, m - 1,$$

$$(s_u^c)_i^*(u_i^c - (x_i^c)^*) = 0, \quad i = 0, \dots, m - 1,$$

$$(s_l^x)_j^*(x_j^* - l_j^x) = 0, \quad j = 0, \dots, n - 1,$$

$$(s_u^x)_j^*(u_j^x - x_j^*) = 0, \quad j = 0, \dots, n - 1,$$

$$\sum_{j=0}^{n-1} (s_n^x)_j^* x_j^* = 0.$$

$$(12.10)$$

are satisfied.

If (12.7) has an optimal solution and **MOSEK** solves the problem successfully, both the primal and dual solution are reported, including a status indicating the exact state of the solution.

#### 12.2.2 Infeasibility for Conic Optimization

#### **Primal Infeasible Problems**

If the problem (12.7) is infeasible (has no feasible solution), **MOSEK** will report a certificate of primal infeasibility: The dual solution reported is the certificate of infeasibility, and the primal solution is undefined.

A certificate of primal infeasibility is a feasible solution to the modified dual problem

$$\begin{array}{ll} \text{maximize} & (l^c)^T s_l^c - (u^c)^T s_u^c + (l^x)^T s_l^x - (u^x)^T s_u^x \\ \text{subject to} & \\ & A^T y + s_l^x - s_u^x + s_n^x = 0, \\ & -y + s_l^c - s_u^c = 0, \\ & s_l^c, s_u^c, s_l^x, s_u^x \geq 0, \\ & s_n^x \in \mathcal{K}^*, \end{array}$$

such that the objective value is strictly positive, i.e. a solution

$$(y^*, (s_l^c)^*, (s_u^c)^*, (s_l^x)^*, (s_u^x)^*, (s_u^x)^*)$$

to (12.11) so that

$$(l^c)^T (s_l^c)^* - (u^c)^T (s_u^c)^* + (l^x)^T (s_l^x)^* - (u^x)^T (s_u^x)^* > 0.$$

Such a solution implies that (12.11) is unbounded, and that (12.7) is infeasible.

#### **Dual Infeasible Problems**

If the problem (12.8) is infeasible (has no feasible solution), **MOSEK** will report a certificate of dual infeasibility: The primal solution reported is the certificate of infeasibility, and the dual solution is undefined.

A certificate of dual infeasibility is a feasible solution to the modified primal problem

minimize 
$$c^T x$$
  
subject to  $\hat{l}^c \leq Ax \leq \hat{u}^c$ ,  
 $\hat{l}^x \leq x \leq \hat{u}^x$ ,  
 $x \in K$ , (12.12)

where

$$\hat{l}_{i}^{c} = \left\{ \begin{array}{ll} 0 & \text{if } l_{i}^{c} > -\infty, \\ -\infty & \text{otherwise,} \end{array} \right\} \quad \text{and} \quad \hat{u}_{i}^{c} := \left\{ \begin{array}{ll} 0 & \text{if } u_{i}^{c} < \infty, \\ \infty & \text{otherwise,} \end{array} \right\}$$
 (12.13)

and

$$\hat{l}_{j}^{x} = \left\{ \begin{array}{cc} 0 & \text{if } l_{j}^{x} > -\infty, \\ -\infty & \text{otherwise,} \end{array} \right\} \quad \text{and} \quad \hat{u}_{j}^{x} := \left\{ \begin{array}{cc} 0 & \text{if } u_{j}^{x} < \infty, \\ \infty & \text{otherwise,} \end{array} \right\}$$
 (12.14)

such that

$$c^T x < 0.$$

Such a solution implies that (12.12) is unbounded, and that (12.8) is infeasible.

In case that both the primal problem (12.7) and the dual problem (12.8) are infeasible, **MOSEK** will report only one of the two possible certificates — which one is not defined (**MOSEK** returns the first certificate found).

#### 12.2.3 Minimalization vs. Maximalization

When the objective sense of problem (12.7) is maximization, i.e.

the objective sense of the dual problem changes to minimization, and the domain of all dual variables changes sign in comparison to (12.2). The dual problem thus takes the form

$$\begin{array}{ll} \text{minimize} & (l^c)^T s_l^c - (u^c)^T s_u^c + (l^x)^T s_l^x - (u^x)^T s_u^x + c^f \\ \text{subject to} & A^T y + s_l^x - s_u^x + s_n^x = c, \\ & -y + s_l^c - s_u^c = 0, \\ & s_l^c, s_u^c, s_l^x, s_u^x \leq 0, \\ & -s_n^x \in \mathcal{K}^* \\ \end{array}$$

This means that the duality gap, defined in (12.9) as the primal minus the dual objective value, becomes nonpositive. It follows that the dual objective will always be greater than or equal to the primal objective. The primal infeasibility certificate will be reported by **MOSEK** as a solution to the system

$$\begin{split} A^T y + s_l^x - s_u^x + s_n^x &= 0, \\ - y + s_l^c - s_u^c &= 0, \\ s_l^c, s_u^c, s_l^x, s_u^x &\leq 0, \\ - s_n^x &\in \mathcal{K}^* \end{split} \tag{12.15}$$

such that the objective value is strictly negative

$$(l^c)^T (s_l^c)^* - (u^c)^T (s_u^c)^* + (l^x)^T (s_l^x)^* - (u^x)^T (s_u^x)^* < 0.$$

Similarly, the certificate of dual infeasibility is an x satisfying the requirements of (12.12) such that  $c^T x > 0$ .

# 12.3 Semidefinite Optimization

Semidefinite optimization is an extension of conic optimization (see Sec. 12.2) allowing positive semidefinite matrix variables to be used in addition to the usual scalar variables. All the other parts of the input are defined exactly as in Sec. 12.2, and the discussion from that section applies verbatim to all properties of problems with semidefinite variables. We only briefly indicate how the corresponding formulae should be modified with semidefinite terms.

A semidefinite optimization problem can be written as

minimize 
$$\sum_{j=0}^{n-1} c_j x_j + \sum_{j=0}^{p-1} \left\langle \overline{C}_j, \overline{X}_j \right\rangle + c^f$$
subject to  $l_i^c \leq \sum_{j=0}^{n-1} a_{ij} x_j + \sum_{j=0}^{p-1} \left\langle \overline{A}_{ij}, \overline{X}_j \right\rangle \leq u_i^c, \quad i = 0, \dots, m-1$ 

$$l_j^x \leq x_j \leq u_j^x, \quad j = 0, \dots, n-1$$

$$x \in \mathcal{K},$$

$$\overline{X}_j \in \mathcal{S}_+^{r_j}, \qquad j = 0, \dots, p-1$$

$$(12.16)$$

where the problem has p symmetric positive semidefinite variables  $\overline{X}_j \in \mathcal{S}_+^{r_j}$  of dimension  $r_j$  with symmetric coefficient matrices  $\overline{C}_j \in \mathcal{S}^{r_j}$  and  $\overline{A}_{i,j} \in \mathcal{S}^{r_j}$ . We use standard notation for the matrix inner product, i.e., for  $U, V \in \mathbb{R}^{m \times n}$  we have

$$\langle U, V \rangle := \sum_{i=0}^{m-1} \sum_{j=0}^{n-1} U_{ij} V_{ij}.$$

As always we write  $A = (a_{i,j})$  for the linear coefficient matrix.

#### **Duality**

The definition of the dual problem (12.8) becomes:

$$\begin{array}{ll} \text{maximize} & (l^c)^T s_l^c - (u^c)^T s_u^c + (l^x)^T s_l^x - (u^x)^T s_u^x + c^f \\ \text{subject to} & \\ & A^T y + s_l^x - s_u^x + s_n^x = c \\ & -y + s_l^c - s_u^c = 0, \\ & \overline{C}_j - \sum_{i=0}^{m-1} y_i \overline{A}_{ij} = \overline{S}_j, & j = 0, \dots, p-1 \\ & s_l^c, s_u^c, s_l^x, s_u^x \geq 0, \\ & s_n^x \in \mathcal{K}^*, \\ & \overline{S}_j \in \mathcal{S}_+^r, & j = 0, \dots, p-1. \end{array}$$

The duality gap (12.9) is computed as:

$$c^{T}x^{*} + c^{f} - \left\{ (l^{c})^{T}(s_{l}^{c})^{*} - (u^{c})^{T}(s_{u}^{c})^{*} + (l^{x})^{T}(s_{l}^{x})^{*} - (u^{x})^{T}(s_{u}^{x})^{*} + c^{f} \right\}$$

$$= \sum_{i=0}^{m-1} \left[ (s_{l}^{c})_{i}^{*}((x_{i}^{c})^{*} - l_{i}^{c}) + (s_{u}^{c})_{i}^{*}(u_{i}^{c} - (x_{i}^{c})^{*}) \right]$$

$$+ \sum_{j=0}^{m-1} \left[ (s_{l}^{x})_{j}^{*}(x_{j} - l_{j}^{x}) + (s_{u}^{x})_{j}^{*}(u_{j}^{x} - x_{j}^{*}) \right] + \sum_{j=0}^{m-1} (s_{n}^{x})_{j}^{*}x_{j}^{*} + \sum_{j=0}^{p-1} \langle \overline{X}_{j}, \overline{S}_{j} \rangle \ge 0.$$

$$(12.18)$$

Complementarity conditions (12.10) include the additional relation:

$$\langle \overline{X}_i, \overline{S}_i \rangle = 0 \quad j = 0, \dots, p - 1.$$
 (12.19)

#### Infeasibility

A certificate of primal infeasibility (12.11) is now a feasible solution to:

maximize 
$$(l^c)^T s_l^c - (u^c)^T s_u^c + (l^x)^T s_l^x - (u^x)^T s_u^x$$
  
subject to
$$A^T y + s_l^x - s_u^x + s_n^x = 0,$$

$$-y + s_l^c - s_u^c = 0,$$

$$\sum_{i=0}^{m-1} y_i \overline{A}_{ij} + \overline{S}_j = 0,$$

$$s_l^c, s_u^c, s_l^x, s_u^x \ge 0,$$

$$s_l^x \in \mathcal{K}^*,$$

$$\overline{S}_j \in \mathcal{S}_+^{r_j},$$
 $j = 0, \dots, p-1.$ 

$$(12.20)$$

such that the objective value is strictly positive.

Similarly, a dual infeasibility certificate (12.12) is a feasible solution to

minimize 
$$\sum_{j=0}^{n-1} c_j x_j + \sum_{j=0}^{p-1} \left\langle \overline{C}_j, \overline{X}_j \right\rangle$$
subject to  $\hat{l}_i^c \leq \sum_{j=0}^{n-1} a_{ij} x_j + \sum_{j=0}^{p-1} \left\langle \overline{A}_{ij}, \overline{X}_j \right\rangle \leq \hat{u}_i^c, \quad i = 0, \dots, m-1$ 

$$\hat{l}_j^x \leq x_j \leq \hat{u}_j^x, \quad j = 0, \dots, n-1$$

$$x \in \mathcal{K},$$

$$\overline{X}_j \in \mathcal{S}_+^{r_j}, \qquad j = 0, \dots, p-1$$

$$(12.21)$$

where the modified bounds are as in (12.13) and (12.14) and the objective value is strictly negative.

# 12.4 Quadratic and Quadratically Constrained Optimization

A convex quadratic and quadratically constrained optimization problem has the form

where all variables and bounds have the same meaning as for linear problems (see Sec. 12.1) and  $Q^o$  and all  $Q^k$  are symmetric matrices. Moreover, for convexity,  $Q^o$  must be a positive semidefinite matrix and  $Q^k$  must satisfy

$$\begin{array}{rcl} -\infty < l_k^c & \Rightarrow & Q^k \text{ is negative semidefinite,} \\ u_k^c < \infty & \Rightarrow & Q^k \text{ is positive semidefinite,} \\ -\infty < l_k^c \leq u_k^c < \infty & \Rightarrow & Q^k = 0. \end{array}$$

The convexity requirement is very important and MOSEK checks whether it is fulfilled.

#### 12.4.1 A Recommendation

Any convex quadratic optimization problem can be reformulated as a conic quadratic optimization problem, see Modeling Cookbook and [And13]. In fact **MOSEK** does such conversion internally as a part of the solution process for the following reasons:

- the conic optimizer is numerically more robust than the one for quadratic problems.
- the conic optimizer is usually faster because quadratic cones are simpler than quadratic functions, even though the conic reformulation usually has more constraints and variables than the original quadratic formulation.
- it is easy to dualize the conic formulation if deemed worthwhile potentially leading to (huge) computational savings.

However, instead of relying on the automatic reformulation we recommend to formulate the problem as a conic problem from scratch because:

- it saves the computational overhead of the reformulation including the convexity check. A conic problem is convex by construction and hence no convexity check is needed for conic problems.
- usually the modeler can do a better reformulation than the automatic method because the modeler can exploit the knowledge of the problem at hand.

To summarize we recommend to formulate quadratic problems and in particular quadratically constrained problems directly in conic form.

# 12.4.2 Duality for Quadratic and Quadratically Constrained Optimization

The dual problem corresponding to the quadratic and quadratically constrained optimization problem (12.22) is given by

$$\begin{array}{ll} \text{maximize} & (l^c)^T s_l^c - (u^c)^T s_u^c + (l^x)^T s_l^x - (u^x)^T s_u^x + \frac{1}{2} x^T \left\{ \sum_{k=0}^{m-1} y_k Q^k - Q^o \right\} x + c^f \\ \text{subject to} & A^T y + s_l^x - s_u^x + \left\{ \sum_{k=0}^{m-1} y_k Q^k - Q^o \right\} x = c, \\ & - y + s_l^c - s_u^c = 0, \\ & s_l^c, s_u^c, s_l^x, s_u^x \geq 0. \end{array}$$

The dual problem is related to the dual problem for linear optimization (see Sec. 12.1.1), but depends on the variable x which in general can not be eliminated. In the solutions reported by **MOSEK**, the value of x is the same for the primal problem (12.22) and the dual problem (12.23).

## 12.4.3 Infeasibility for Quadratic Optimization

In case **MOSEK** finds a problem to be infeasible it reports a certificate of infeasibility. We write them out explicitly for quadratic problems, that is when  $Q^k = 0$  for all k and quadratic terms appear only in the objective  $Q^o$ . In this case the constraints both in the primal and dual problem are linear, and **MOSEK** produces for them the same infeasibility certificate as for linear problems.

The certificate of primal infeasibility is a solution to the problem (12.4) such that the objective value is strictly positive.

The certificate of dual infeasibility is a solution to the problem (12.5) together with an additional constraint

$$Q^o x = 0$$

such that the objective value is strictly negative.

# Chapter 13

# **Optimizers**

The most essential part of MOSEK are the optimizers:

- primal simplex (linear problems),
- dual simplex (linear problems),
- interior-point (linear, quadratic and conic problems),
- mixed-integer (problems with integer variables).

The structure of a successful optimization process is roughly:

#### • Presolve

- 1. Elimination: Reduce the size of the problem.
- 2. Dualizer: Choose whether to solve the primal or the dual form of the problem.
- 3. Scaling: Scale the problem for better numerical stability.

#### • Optimization

- 1. Optimize: Solve the problem using selected method.
- 2. Terminate: Stop the optimization when specific termination criteria have been met.
- 3. Report: Return the solution or an infeasibility certificate.

The preprocessing stage is transparent to the user, but useful to know about for tuning purposes. The purpose of the preprocessing steps is to make the actual optimization more efficient and robust. We discuss the details of the above steps in the following sections.

#### 13.1 Presolve

Before an optimizer actually performs the optimization the problem is preprocessed using the so-called presolve. The purpose of the presolve is to

- 1. remove redundant constraints,
- 2. eliminate fixed variables,
- 3. remove linear dependencies,
- 4. substitute out (implied) free variables, and
- 5. reduce the size of the optimization problem in general.

After the presolved problem has been optimized the solution is automatically postsolved so that the returned solution is valid for the original problem. Hence, the presolve is completely transparent. For further details about the presolve phase, please see [AA95] and [AGMeszarosX96].

It is possible to fine-tune the behavior of the presolve or to turn it off entirely. If presolve consumes too much time or memory compared to the reduction in problem size gained it may be disabled. This is done by setting the parameter  $MSK\_IPAR\_PRESOLVE\_USE$  to  $MSK\_PRESOLVE\_MODE\_OFF$ . The two most time-consuming steps of the presolve are

- the eliminator, and
- the linear dependency check.

Therefore, in some cases it is worthwhile to disable one or both of these.

#### Numerical issues in the presolve

During the presolve the problem is reformulated so that it hopefully solves faster. However, in rare cases the presolved problem may be harder to solve then the original problem. The presolve may also be infeasible although the original problem is not. If it is suspected that presolved problem is much harder to solve than the original, we suggest to first turn the eliminator off by setting the parameter  $MSK\_IPAR\_PRESOLVE\_ELIMINATOR\_MAX\_NUM\_TRIES$  to 0. If that does not help, then trying to turn entire presolve off may help.

Since all computations are done in finite precision, the presolve employs some tolerances when concluding a variable is fixed or a constraint is redundant. If it happens that  $\mathbf{MOSEK}$  incorrectly concludes a problem is primal or dual infeasible, then it is worthwhile to try to reduce the parameters  $MSK\_DPAR\_PRESOLVE\_TOL\_X$  and  $MSK\_DPAR\_PRESOLVE\_TOL\_S$ . However, if reducing the parameters actually helps then this should be taken as an indication that the problem is badly formulated.

#### Eliminator

The purpose of the eliminator is to eliminate free and implied free variables from the problem using substitution. For instance, given the constraints

$$\begin{array}{rcl} y & = & \sum_j x_j, \\ y, x & \geq & 0, \end{array}$$

y is an implied free variable that can be substituted out of the problem, if deemed worthwhile. If the eliminator consumes too much time or memory compared to the reduction in problem size gained it may be disabled. This can be done by setting the parameter  $MSK\_IPAR\_PRESOLVE\_ELIMINATOR\_MAX\_NUM\_TRIES$  to 0. In rare cases the eliminator may cause that the problem becomes much hard to solve.

#### Linear dependency checker

The purpose of the linear dependency check is to remove linear dependencies among the linear equalities. For instance, the three linear equalities

$$\begin{array}{rcl} x_1 + x_2 + x_3 & = & 1, \\ x_1 + 0.5x_2 & = & 0.5, \\ 0.5x_2 + x_3 & = & 0.5. \end{array}$$

contain exactly one linear dependency. This implies that one of the constraints can be dropped without changing the set of feasible solutions. Removing linear dependencies is in general a good idea since it reduces the size of the problem. Moreover, the linear dependencies are likely to introduce numerical problems in the optimization phase. It is best practice to build models without linear dependencies, but that is not always easy for the user to control. If the linear dependencies are removed at the modeling stage, the linear dependency check can safely be disabled by setting the parameter  $MSK\_IPAR\_PRESOLVE\_LINDEP\_USE$  to  $MSK\_OFF$ .

### Dualizer

All linear, conic, and convex optimization problems have an equivalent dual problem associated with them. **MOSEK** has built-in heuristics to determine if it is more efficient to solve the primal or dual problem. The form (primal or dual) is displayed in the **MOSEK** log and available as an information item from the solver. Should the internal heuristics not choose the most efficient form of the problem it may be worthwhile to set the dualizer manually by setting the parameters:

- MSK\_IPAR\_INTPNT\_SOLVE\_FORM: In case of the interior-point optimizer.
- MSK\_IPAR\_SIM\_SOLVE\_FORM: In case of the simplex optimizer.

Note that currently only linear and conic (but not semidefinite) problems may be automatically dualized.

#### **Scaling**

Problems containing data with large and/or small coefficients, say 1.0e + 9 or 1.0e - 7, are often hard to solve. Significant digits may be truncated in calculations with finite precision, which can result in the optimizer relying on inaccurate data. Since computers work in finite precision, extreme coefficients should be avoided. In general, data around the same *order of magnitude* is preferred, and we will refer to a problem, satisfying this loose property, as being *well-scaled*. If the problem is not well scaled, **MOSEK** will try to scale (multiply) constraints and variables by suitable constants. **MOSEK** solves the scaled problem to improve the numerical properties.

The scaling process is transparent, i.e. the solution to the original problem is reported. It is important to be aware that the optimizer terminates when the termination criterion is met on the scaled problem, therefore significant primal or dual infeasibilities may occur after unscaling for badly scaled problems. The best solution of this issue is to reformulate the problem, making it better scaled.

By default **MOSEK** heuristically chooses a suitable scaling. The scaling for interior-point and simplex optimizers can be controlled with the parameters <code>MSK\_IPAR\_INTPNT\_SCALING</code> and <code>MSK\_IPAR\_SIM\_SCALING</code> respectively.

# 13.2 Linear Optimization

### 13.2.1 Optimizer Selection

Two different types of optimizers are available for linear problems: The default is an interior-point method, and the alternative is the simplex method (primal or dual). The optimizer can be selected using the parameter  $MSK\_IPAR\_OPTIMIZER$ .

#### The Interior-point or the Simplex Optimizer?

Given a linear optimization problem, which optimizer is the best: the simplex or the interior-point optimizer? It is impossible to provide a general answer to this question. However, the interior-point optimizer behaves more predictably: it tends to use between 20 and 100 iterations, almost independently of problem size, but cannot perform warm-start. On the other hand the simplex method can take advantage of an initial solution, but is less predictable from cold-start. The interior-point optimizer is used by default.

#### The Primal or the Dual Simplex Variant?

MOSEK provides both a primal and a dual simplex optimizer. Predicting which simplex optimizer is faster is impossible, however, in recent years the dual optimizer has seen several algorithmic and computational improvements, which, in our experience, make it faster on average than the primal version. Still, it depends much on the problem structure and size. Setting the MSK\_IPAR\_OPTIMIZER parameter to MSK\_OPTIMIZER\_FREE\_SIMPLEX instructs MOSEK to choose one of the simplex variants automatically.

To summarize, if you want to know which optimizer is faster for a given problem type, it is best to try all the options.

### 13.2.2 The Interior-point Optimizer

The purpose of this section is to provide information about the algorithm employed in the **MOSEK** interior-point optimizer for linear problems and about its termination criteria.

#### The homogeneous primal-dual problem

In order to keep the discussion simple it is assumed that  $\mathbf{MOSEK}$  solves linear optimization problems of standard form

$$\begin{array}{lll} \text{minimize} & c^T x \\ \text{subject to} & Ax & = & b, \\ & x \geq 0. & \end{array} \tag{13.1}$$

This is in fact what happens inside **MOSEK**; for efficiency reasons **MOSEK** converts the problem to standard form before solving, then converts it back to the input form when reporting the solution.

Since it is not known beforehand whether problem (13.1) has an optimal solution, is primal infeasible or is dual infeasible, the optimization algorithm must deal with all three situations. This is the reason why **MOSEK** solves the so-called homogeneous model

$$Ax - b\tau = 0,$$

$$A^{T}y + s - c\tau = 0,$$

$$-c^{T}x + b^{T}y - \kappa = 0,$$

$$x, s, \tau, \kappa \geq 0,$$

$$(13.2)$$

where y and s correspond to the dual variables in (13.1), and  $\tau$  and  $\kappa$  are two additional scalar variables. Note that the homogeneous model (13.2) always has solution since

$$(x, y, s, \tau, \kappa) = (0, 0, 0, 0, 0)$$

is a solution, although not a very interesting one. Any solution

$$(x^*, y^*, s^*, \tau^*, \kappa^*)$$

to the homogeneous model (13.2) satisfies

$$x_i^* s_i^* = 0 \text{ and } \tau^* \kappa^* = 0.$$

Moreover, there is always a solution that has the property  $\tau^* + \kappa^* > 0$ .

First, assume that  $\tau^* > 0$ . It follows that

$$\begin{array}{rcl} A\frac{x^*}{\tau^*} & = & b, \\ A^T\frac{y^*}{\tau^*} + \frac{s^*}{\tau^*} & = & c, \\ -c^T\frac{x^*}{\tau^*} + b^T\frac{y^*}{\tau^*} & = & 0, \\ x^*, s^*, \tau^*, \kappa^* & \geq & 0. \end{array}$$

This shows that  $\frac{x^*}{\tau^*}$  is a primal optimal solution and  $(\frac{y^*}{\tau^*}, \frac{s^*}{\tau^*})$  is a dual optimal solution; this is reported as the optimal interior-point solution since

$$(x, y, s) = \left\{ \frac{x^*}{\tau^*}, \frac{y^*}{\tau^*}, \frac{s^*}{\tau^*} \right\}$$

is a primal-dual optimal solution (see Sec. 12.1 for the mathematical background on duality and optimality).

On other hand, if  $\kappa^* > 0$  then

$$\begin{array}{rcl} Ax^* & = & 0, \\ A^Ty^* + s^* & = & 0, \\ -c^Tx^* + b^Ty^* & = & \kappa^*, \\ x^*, s^*, \tau^*, \kappa^* & \geq & 0. \end{array}$$

This implies that at least one of

$$c^T x^* < 0 \tag{13.3}$$

or

$$b^T y^* > 0 \tag{13.4}$$

is satisfied. If (13.3) is satisfied then  $x^*$  is a certificate of dual infeasibility, whereas if (13.4) is satisfied then  $y^*$  is a certificate of primal infeasibility.

In summary, by computing an appropriate solution to the homogeneous model, all information required for a solution to the original problem is obtained. A solution to the homogeneous model can be computed using a primal-dual interior-point algorithm [And09].

#### **Interior-point Termination Criterion**

For efficiency reasons it is not practical to solve the homogeneous model exactly. Hence, an exact optimal solution or an exact infeasibility certificate cannot be computed and a reasonable termination criterion has to be employed.

In the k-th iteration of the interior-point algorithm a trial solution

$$(x^k, y^k, s^k, \tau^k, \kappa^k)$$

to homogeneous model is generated, where

$$x^k, s^k, \tau^k, \kappa^k > 0.$$

#### **Optimal** case

Whenever the trial solution satisfies the criterion

$$\left\| A \frac{x^{k}}{\tau^{k}} - b \right\|_{\infty} \leq \epsilon_{p} (1 + \|b\|_{\infty}),$$

$$\left\| A^{T} \frac{y^{k}}{\tau^{k}} + \frac{s^{k}}{\tau^{k}} - c \right\|_{\infty} \leq \epsilon_{d} (1 + \|c\|_{\infty}), \text{ and}$$

$$\min \left( \frac{(x^{k})^{T} s^{k}}{(\tau^{k})^{2}}, \left| \frac{c^{T} x^{k}}{\tau^{k}} - \frac{b^{T} y^{k}}{\tau^{k}} \right| \right) \leq \epsilon_{g} \max \left( 1, \frac{\min(\left| c^{T} x^{k} \right|, \left| b^{T} y^{k} \right|)}{\tau^{k}} \right), \tag{13.5}$$

the interior-point optimizer is terminated and

$$\frac{(x^k, y^k, s^k)}{\tau^k}$$

is reported as the primal-dual optimal solution. The interpretation of (13.5) is that the optimizer is terminated if

- $\frac{x^k}{\tau^k}$  is approximately primal feasible,
- $\left\{\frac{y^k}{\tau^k}, \frac{s^k}{\tau^k}\right\}$  is approximately dual feasible, and
- the duality gap is almost zero.

#### **Dual infeasibility certificate**

On the other hand, if the trial solution satisfies

$$-\epsilon_i c^T x^k > \frac{\|c\|_{\infty}}{\max\left(1, \|b\|_{\infty}\right)} \|Ax^k\|_{\infty}$$

then the problem is declared dual infeasible and  $x^k$  is reported as a certificate of dual infeasibility. The motivation for this stopping criterion is as follows: First assume that  $\|Ax^k\|_{\infty} = 0$ ; then  $x^k$  is an exact certificate of dual infeasibility. Next assume that this is not the case, i.e.

$$\left\|Ax^k\right\|_{\infty} > 0,$$

and define

$$\bar{x} := \epsilon_i \frac{\max\left(1, \|b\|_{\infty}\right)}{\|Ax^k\|_{\infty} \|c\|_{\infty}} x^k.$$

It is easy to verify that

$$||A\bar{x}||_{\infty} = \epsilon_i \frac{\max(1, ||b||_{\infty})}{||c||_{\infty}} \text{ and } -c^T \bar{x} > 1,$$

which shows  $\bar{x}$  is an approximate certificate of dual infeasibility, where  $\varepsilon_i$  controls the quality of the approximation. A smaller value means a better approximation.

#### Primal infeasibility certificate

Finally, if

$$\epsilon_i b^T y^k > \frac{\|b\|_{\infty}}{\max(1, \|c\|_{\infty})} \|A^T y^k + s^k\|_{\infty}$$

then  $y^k$  is reported as a certificate of primal infeasibility.

#### Adjusting optimality criteria

It is possible to adjust the tolerances  $\varepsilon_p$ ,  $\varepsilon_d$ ,  $\varepsilon_g$  and  $\varepsilon_i$  using parameters; see table for details.

Table 13.1: Parameters employed in termination criterion

ToleranceParameter	name
$arepsilon_p$	MSK_DPAR_INTPNT_TOL_PFEAS
$arepsilon_d$	MSK_DPAR_INTPNT_TOL_DFEAS
$\varepsilon_g$	MSK_DPAR_INTPNT_TOL_REL_GAP
$\varepsilon_i$	MSK_DPAR_INTPNT_TOL_INFEAS

The default values of the termination tolerances are chosen such that for a majority of problems appearing in practice it is not possible to achieve much better accuracy. Therefore, tightening the tolerances usually is not worthwhile. However, an inspection of (13.5) reveals that the quality of the solution depends on  $\|b\|_{\infty}$  and  $\|c\|_{\infty}$ ; the smaller the norms are, the better the solution accuracy.

The interior-point method as implemented by **MOSEK** will converge toward optimality and primal and dual feasibility at the same rate [And09]. This means that if the optimizer is stopped prematurely then it is very unlikely that either the primal or dual solution is feasible. Another consequence is that in most cases all the tolerances,  $\varepsilon_p$ ,  $\varepsilon_d$ ,  $\varepsilon_g$  and  $\varepsilon_i$ , have to be relaxed together to achieve an effect.

The basis identification discussed in Sec. 13.2.2 requires an optimal solution to work well; hence basis identification should be turned off if the termination criterion is relaxed.

To conclude the discussion in this section, relaxing the termination criterion is usually not worthwhile.

#### **Basis Identification**

An interior-point optimizer does not return an optimal basic solution unless the problem has a unique primal and dual optimal solution. Therefore, the interior-point optimizer has an optional post-processing step that computes an optimal basic solution starting from the optimal interior-point solution. More information about the basis identification procedure may be found in [AY96]. In the following we provide an overall idea of the procedure.

There are some cases in which a basic solution could be more valuable:

- a basic solution is often more accurate than an interior-point solution,
- a basic solution can be used to warm-start the simplex algorithm in case of reoptimization,
- a basic solution is in general more sparse, i.e. more variables are fixed to zero. This is particularly appealing when solving continuous relaxations of mixed integer problems, as well as in all applications in which sparser solutions are preferred.

To illustrate how the basis identification routine works, we use the following trivial example:

$$\begin{array}{lll} \mbox{minimize} & x+y \\ \mbox{subject to} & x+y & = & 1, \\ & x,y \geq 0. \end{array}$$

It is easy to see that all feasible solutions are also optimal. In particular, there are two basic solutions, namely

$$\begin{array}{rcl} (x_1^*,y_1^*) & = & (1,0), \\ (x_2^*,y_2^*) & = & (0,1). \end{array}$$

The interior point algorithm will actually converge to the center of the optimal set, i.e. to  $(x^*, y^*) = (1/2, 1/2)$  (to see this in **MOSEK** deactivate *Presolve*).

In practice, when the algorithm gets close to the optimal solution, it is possible to construct in polynomial time an initial basis for the simplex algorithm from the current interior point solution. This basis is used to warm-start the simplex algorithm that will provide the optimal basic solution. In most cases the constructed basis is optimal, or very few iterations are required by the simplex algorithm to make it optimal and hence the final *clean-up* phase be short. However, for some cases of ill-conditioned problems the additional simplex clean up phase may take of lot a time.

By default **MOSEK** performs a basis identification. However, if a basic solution is not needed, the basis identification procedure can be turned off. The parameters

- MSK\_IPAR\_INTPNT\_BASIS,
- MSK\_IPAR\_BI\_IGNORE\_MAX\_ITER, and
- MSK\_IPAR\_BI\_IGNORE\_NUM\_ERROR

control when basis identification is performed.

The type of simplex algorithm to be used (primal/dual) can be tuned with the parameter  $MSK\_IPAR\_BI\_CLEAN\_OPTIMIZER$ , and the maximum number of iterations can be set with  $MSK\_IPAR\_BI\_MAX\_ITERATIONS$ .

Finally, it should be mentioned that there is no guarantee on which basic solution will be returned.

#### The Interior-point Log

Below is a typical log output from the interior-point optimizer:

```
Optimizer
           - threads
                                     : 1
Optimizer
           - solved problem
                                     : the dual
Optimizer
           - Constraints
                                     : 2
          - Cones
                                     : 0
Optimizer
Optimizer
           - Scalar variables
                                     : 6
                                                                                 : 0
                                                          conic
           - Semi-definite variables: 0
Optimizer
                                                          scalarized
                                                                                 : 0
Factor
           - setup time
                                     : 0.00
                                                          dense det. time
                                                                                 : 0.00
Factor
                                     : 0.00
                                                                                 : 0.00
           - ML order time
                                                          GP order time
Factor
           - nonzeros before factor : 3
                                                          after factor
                                                                                 : 3
Factor
           - dense dim.
                                     : 0
                                                          flops
                                                                                 : 7.
→00e+001
ITE PFEAS
                                                              DOBJ
             DFFAS
                      GFEAS
                               PRSTATUS
                                           POB.I
                                                                                MU
→ TIME
    1.0e+000 8.6e+000 6.1e+000 1.00e+000
                                           0.000000000e+000 -2.208000000e+003 1.
→0e+000 0.00
    1.1e+000 2.5e+000 1.6e-001 0.00e+000
                                           -7.901380925e+003 -7.394611417e+003 2.
-5e+000 0.00
    1.4e-001 3.4e-001 2.1e-002 8.36e-001
                                           -8.113031650e+003 -8.055866001e+003 3.3e-
-001 0.00
    2.4e-002 5.8e-002 3.6e-003 1.27e+000
                                           -7.777530698e+003 -7.766471080e+003 5.7e-
→002 0.01
   1.3e-004 3.2e-004 2.0e-005 1.08e+000
                                           -7.668323435e+003 -7.668207177e+003 3.2e-
→004 0.01
   1.3e-008 3.2e-008 2.0e-009 1.00e+000
                                          -7.668000027e+003 -7.668000015e+003 3.2e-
→008 0.01
    1.3e-012 3.2e-012 2.0e-013 1.00e+000 -7.667999994e+003 -7.667999994e+003 3.2e-
→012 0.01
```

The first line displays the number of threads used by the optimizer and the second line indicates if the optimizer chose to solve the primal or dual problem (see MSK\_IPAR\_INTPNT\_SOLVE\_FORM). The next lines display the problem dimensions as seen by the optimizer, and the Factor... lines show various statistics. This is followed by the iteration log.

Using the same notation as in Sec. 13.2.2 the columns of the iteration log have the following meaning:

• ITE: Iteration index k.

- PFEAS:  $||Ax^k b\tau^k||_{\infty}$ . The numbers in this column should converge monotonically towards zero but may stall at low level due to rounding errors.
- DFEAS:  $\|A^Ty^k + s^k c\tau^k\|_{\infty}$ . The numbers in this column should converge monotonically towards zero but may stall at low level due to rounding errors.
- GFEAS:  $|-c^Tx^k+b^Ty^k-\kappa^k|$ . The numbers in this column should converge monotonically towards zero but may stall at low level due to rounding errors.
- PRSTATUS: This number converges to 1 if the problem has an optimal solution whereas it converges to -1 if that is not the case.
- POBJ:  $c^T x^k / \tau^k$ . An estimate for the primal objective value.
- DOBJ:  $b^T y^k / \tau^k$ . An estimate for the dual objective value.
- $\bullet$  MU:  $\frac{(x^k)^Ts^k+\tau^k\kappa^k}{n+1}$  . The numbers in this column should always converge to zero.
- TIME: Time spent since the optimization started.

## 13.2.3 The Simplex Optimizer

An alternative to the interior-point optimizer is the simplex optimizer. The simplex optimizer uses a different method that allows exploiting an initial guess for the optimal solution to reduce the solution time. Depending on the problem it may be faster or slower to use an initial guess; see Sec. 13.2.1 for a discussion. **MOSEK** provides both a primal and a dual variant of the simplex optimizer.

#### **Simplex Termination Criterion**

The simplex optimizer terminates when it finds an optimal basic solution or an infeasibility certificate. A basic solution is optimal when it is primal and dual feasible; see Sec. 12.1 for a definition of the primal and dual problem. Due to the fact that computations are performed in finite precision **MOSEK** allows violations of primal and dual feasibility within certain tolerances. The user can control the allowed primal and dual tolerances with the parameters <code>MSK\_DPAR\_BASIS\_TOL\_X</code> and <code>MSK\_DPAR\_BASIS\_TOL\_S</code>.

Setting the parameter MSK\_IPAR\_OPTIMIZER to MSK\_OPTIMIZER\_FREE\_SIMPLEX instructs MOSEK to select automatically between the primal and the dual simplex optimizers. Hence, MOSEK tries to choose the best optimizer for the given problem and the available solution. The same parameter can also be used to force one of the variants.

#### Starting From an Existing Solution

When using the simplex optimizer it may be possible to reuse an existing solution and thereby reduce the solution time significantly. When a simplex optimizer starts from an existing solution it is said to perform a *warm-start*. If the user is solving a sequence of optimization problems by solving the problem, making modifications, and solving again, **MOSEK** will warm-start automatically.

By default **MOSEK** uses presolve when performing a warm-start. If the optimizer only needs very few iterations to find the optimal solution it may be better to turn off the presolve.

#### Numerical Difficulties in the Simplex Optimizers

Though MOSEK is designed to minimize numerical instability, completely avoiding it is impossible when working in finite precision. MOSEK treats a "numerically unexpected behavior" event inside the optimizer as a *set-back*. The user can define how many set-backs the optimizer accepts; if that number is exceeded, the optimization will be aborted. Set-backs are a way to escape long sequences where the optimizer tries to recover from an unstable situation.

Examples of set-backs are: repeated singularities when factorizing the basis matrix, repeated loss of feasibility, degeneracy problems (no progress in objective) and other events indicating numerical difficulties. If the simplex optimizer encounters a lot of set-backs the problem is usually badly scaled; in such a situation try to reformulate it into a better scaled problem. Then, if a lot of set-backs still occur, trying one or more of the following suggestions may be worthwhile:

• Raise tolerances for allowed primal or dual feasibility: increase the value of

- MSK\_DPAR\_BASIS\_TOL\_X, and
- MSK\_DPAR\_BASIS\_TOL\_S.
- Raise or lower pivot tolerance: Change the MSK\_DPAR\_SIMPLEX\_ABS\_TOL\_PIV parameter.
- Switch optimizer: Try another optimizer.
- Switch off crash: Set both MSK\_IPAR\_SIM\_PRIMAL\_CRASH and MSK\_IPAR\_SIM\_DUAL\_CRASH to 0.
- Experiment with other pricing strategies: Try different values for the parameters
  - MSK\_IPAR\_SIM\_PRIMAL\_SELECTION and
  - MSK\_IPAR\_SIM\_DUAL\_SELECTION.
- If you are using warm-starts, in rare cases switching off this feature may improve stability. This is controlled by the  $MSK\_IPAR\_SIM\_HOTSTART$  parameter.
- Increase maximum number of set-backs allowed controlled by MSK\_IPAR\_SIM\_MAX\_NUM\_SETBACKS.
- If the problem repeatedly becomes infeasible try switching off the special degeneracy handling. See the parameter  $MSK\_IPAR\_SIM\_DEGEN$  for details.

#### The Simplex Log

Below is a typical log output from the simplex optimizer:

Optim		olved problem	: the	primal		
Optimizer - Constraints			: 667			
Optim	nizer - Sc	alar variables	: 1424	conic		: 0
Optim	nizer - ho	tstart	: no			
ITER	DEGIT	ER(%) PFEAS	DFEAS	POBJ	DOBJ	ш
$\hookrightarrow$	TIME	TOTTIME				
0	0.00	1.43e+05	NA	6.5584140832e+03	NA	ш
$\hookrightarrow$	0.00	0.02				
1000	1.10	0.00e+00	NA	1.4588289726e+04	NA	ш
$\hookrightarrow$	0.13	0.14				
2000	0.75	0.00e+00	NA	7.3705564855e+03	NA	ш
$\hookrightarrow$	0.21	0.22				
3000	0.67	0.00e+00	NA	6.0509727712e+03	NA	ш
$\hookrightarrow$	0.29	0.31				
4000	0.52	0.00e+00	NA	5.5771203906e+03	NA	ш
$\hookrightarrow$	0.38	0.39				
4533	0.49	0.00e+00	NA	5.5018458883e+03	NA	ш
$\hookrightarrow$	0.42	0.44				

The first lines summarize the problem the optimizer is solving. This is followed by the iteration log, with the following meaning:

- ITER: Number of iterations.
- DEGITER(%): Ratio of degenerate iterations.
- PFEAS: Primal feasibility measure reported by the simplex optimizer. The numbers should be 0 if the problem is primal feasible (when the primal variant is used).
- DFEAS: Dual feasibility measure reported by the simplex optimizer. The number should be 0 if the problem is dual feasible (when the dual variant is used).
- POBJ: An estimate for the primal objective value (when the primal variant is used).
- DOBJ: An estimate for the dual objective value (when the dual variant is used).
- TIME: Time spent since this instance of the simplex optimizer was invoked (in seconds).
- TOTTIME: Time spent since optimization started (in seconds).

# 13.3 Conic Optimization - Interior-point optimizer

For conic optimization problems only an interior-point type optimizer is available.

## 13.3.1 The homogeneous primal-dual problem

The interior-point optimizer is an implementation of the so-called homogeneous and self-dual algorithm. For a detailed description of the algorithm, please see [ART03]. In order to keep our discussion simple we will assume that **MOSEK** solves a conic optimization problem of the form:

$$\begin{array}{lll} \text{minimize} & c^T x \\ \text{subject to} & Ax & = & b, \\ & x \in \mathcal{K} & \end{array} \tag{13.6}$$

where K is a convex cone. The corresponding dual problem is

$$\begin{array}{lll} \text{maximize} & b^T y \\ \text{subject to} & A^T y + s & = & c, \\ & s \in \mathcal{K}^* \end{array} \tag{13.7}$$

where  $\mathcal{K}^*$  is the dual cone of  $\mathcal{K}$ . See Sec. 12.2 for definitions.

Since it is not known beforehand whether problem (13.6) has an optimal solution, is primal infeasible or is dual infeasible, the optimization algorithm must deal with all three situations. This is the reason that **MOSEK** solves the so-called homogeneous model

$$Ax - b\tau = 0,$$

$$A^{T}y + s - c\tau = 0,$$

$$-c^{T}x + b^{T}y - \kappa = 0,$$

$$x \in \mathcal{K},$$

$$s \in \mathcal{K}^{*},$$

$$\tau, \kappa \geq 0,$$

$$(13.8)$$

where y and s correspond to the dual variables in (13.6), and  $\tau$  and  $\kappa$  are two additional scalar variables. Note that the homogeneous model (13.8) always has a solution since

$$(x, y, s, \tau, \kappa) = (0, 0, 0, 0, 0)$$

is a solution, although not a very interesting one. Any solution

$$(x^*, y^*, s^*, \tau^*, \kappa^*)$$

to the homogeneous model (13.8) satisfies

$$(x^*)^T s^* + \tau^* \kappa^* = 0$$

i.e. complementarity. Observe that  $x^* \in \mathcal{K}$  and  $s^* \in \mathcal{K}^*$  implies

$$(x^*)^T s^* > 0$$

and therefore

$$\tau^* \kappa^* = 0.$$

since  $\tau^*, \kappa^* \geq 0$ . Hence, at least one of  $\tau^*$  and  $\kappa^*$  is zero.

First, assume that  $\tau^* > 0$  and hence  $\kappa^* = 0$ . It follows that

$$\begin{array}{rcl} A\frac{x^*}{\tau^*} & = & b, \\ A^T\frac{y^*}{\tau^*} + \frac{s^*}{\tau^*} & = & c, \\ -c^T\frac{x^*}{\tau^*} + b^T\frac{y^*}{\tau^*} & = & 0, \\ x^*/\tau^* & \in & \mathcal{K}, \\ s^*/\tau^* & \in & \mathcal{K}^*. \end{array}$$

This shows that  $\frac{x^*}{\tau^*}$  is a primal optimal solution and  $(\frac{y^*}{\tau^*}, \frac{s^*}{\tau^*})$  is a dual optimal solution; this is reported as the optimal interior-point solution since

$$(x, y, s) = \left(\frac{x^*}{\tau^*}, \frac{y^*}{\tau^*}, \frac{s^*}{\tau^*}\right)$$

is a primal-dual optimal solution.

On other hand, if  $\kappa^* > 0$  then

$$\begin{array}{rcl} Ax^* & = & 0, \\ A^Ty^* + s^* & = & 0, \\ -c^Tx^* + b^Ty^* & = & \kappa^*, \\ x^* & \in & \mathcal{K}, \\ s^* & \in & \mathcal{K}^*. \end{array}$$

This implies that at least one of

$$c^T x^* < 0 \tag{13.9}$$

or

$$b^T y^* > 0 (13.10)$$

holds. If (13.9) is satisfied, then  $x^*$  is a certificate of dual infeasibility, whereas if (13.10) holds then  $y^*$  is a certificate of primal infeasibility.

In summary, by computing an appropriate solution to the homogeneous model, all information required for a solution to the original problem is obtained. A solution to the homogeneous model can be computed using a primal-dual interior-point algorithm [And09].

# 13.3.2 Interior-point Termination Criterion

Since computations are performed in finite precision, and for efficiency reasons, it is not possible to solve the homogeneous model exactly in general. Hence, an exact optimal solution or an exact infeasibility certificate cannot be computed and a reasonable termination criterion has to be employed.

In every iteration k of the interior-point algorithm a trial solution

$$(x^k, y^k, s^k, \tau^k, \kappa^k)$$

to the homogeneous model is generated, where

$$x^k \in \mathcal{K}, s^k \in \mathcal{K}^*, \tau^k, \kappa^k > 0.$$

Therefore, it is possible to compute the values:

$$\begin{array}{lll} \rho_p^k &=& \arg\min_{\rho} \left\{ \rho \mid \left\| A \frac{x^k}{\tau^k} - b \right\|_{\infty} \leq \rho \varepsilon_p (1 + \|b\|_{\infty}) \right\}, \\ \rho_d^k &=& \arg\min_{\rho} \left\{ \rho \mid \left\| A \frac{y^k}{\tau^k} + \frac{s^k}{\tau^k} - c \right\|_{\infty} \leq \rho \varepsilon_d (1 + \|c\|_{\infty}) \right\}, \\ \rho_g^k &=& \arg\min_{\rho} \left\{ \rho \mid \left( \frac{(x^k)^T s^k}{(\tau^k)^2}, \left| \frac{c^T x^k}{\tau^k} - \frac{b^T y^k}{\tau^k} \right| \right) \leq \rho \varepsilon_g \max \left( 1, \frac{\min\left(\left| c^T x^k \right|, \left| b^T y^k \right| \right)}{\tau^k} \right) \right\}, \\ \rho_{pi}^k &=& \arg\min_{\rho} \left\{ \rho \mid \left\| A^T y^k + s^k \right\|_{\infty} \leq \rho \varepsilon_i b^T y^k, \, b^T y^k > 0 \right\} \text{ and } \\ \rho_{di}^k &=& \arg\min_{\rho} \left\{ \rho \mid \left\| A x^k \right\|_{\infty} \leq -\rho \varepsilon_i c^T x^k, \, c^T x^k < 0 \right\}. \end{array}$$

Note  $\varepsilon_p, \varepsilon_d, \varepsilon_q$  and  $\varepsilon_i$  are nonnegative user specified tolerances.

#### **Optimal Case**

Observe  $\rho_p^k$  measures how far  $x^k/\tau^k$  is from being a good approximate primal feasible solution. Indeed if  $\rho_p^k \leq 1$ , then

$$\left\| A \frac{x^k}{\tau^k} - b \right\|_{\infty} \le \varepsilon_p (1 + \|b\|_{\infty}). \tag{13.11}$$

This shows the violations in the primal equality constraints for the solution  $x^k/\tau^k$  is small compared to the size of b given  $\varepsilon_p$  is small.

Similarly, if  $\rho_d^k \leq 1$ , then  $(y^k, s^k)/\tau^k$  is an approximate dual feasible solution. If in addition  $\rho_g \leq 1$ , then the solution  $(x^k, y^k, s^k)/\tau^k$  is approximate optimal because the associated primal and dual objective values are almost identical.

In other words if  $\max(\rho_p^k, \rho_d^k, \rho_g^k) \leq 1$ , then

$$\frac{(x^k, y^k, s^k)}{\tau^k}$$

is an approximate optimal solution.

#### **Dual Infeasibility Certificate**

Next assume that  $\rho_{di}^k \leq 1$  and hence

$$||Ax^k||_{\infty} \le -\varepsilon_i c^T x^k$$
 and  $-c^T x^k > 0$ 

holds. Now in this case the problem is declared dual infeasible and  $x^k$  is reported as a certificate of dual infeasibility. The motivation for this stopping criterion is as follows. Let

$$\bar{x} := \frac{x^k}{-c^T x^k}$$

and it is easy to verify that

$$||A\bar{x}||_{\infty} \leq \varepsilon_i$$
 and  $c^T\bar{x} = -1$ 

which shows  $\bar{x}$  is an approximate certificate of dual infeasibility, where  $\varepsilon_i$  controls the quality of the approximation.

## **Primal Infeasiblity Certificate**

Next assume that  $\rho_{pi}^k \leq 1$  and hence

$$||A^T y^k + s^k||_{\infty} \le \varepsilon_i b^T y^k$$
 and  $b^T y^k > 0$ 

holds. Now in this case the problem is declared primal infeasible and  $(y^k, s^k)$  is reported as a certificate of primal infeasibility. The motivation for this stopping criterion is as follows. Let

$$\bar{y} := \frac{y^k}{b^T y^k}$$
 and  $\bar{s} := \frac{s^k}{b^T y^k}$ 

and it is easy to verify that

$$||A^T \bar{y} + \bar{s}||_{\infty} \le \varepsilon_i \text{ and } b^T \bar{y} = 1$$

which shows  $(y^k, s^k)$  is an approximate certificate of dual infeasibility, where  $\varepsilon_i$  controls the quality of the approximation.

#### 13.3.3 Adjusting optimality criteria

It is possible to adjust the tolerances  $\varepsilon_p$ ,  $\varepsilon_d$ ,  $\varepsilon_g$  and  $\varepsilon_i$  using parameters; see table for details.

Table 13.2: Parameters employed in termination criterion

ToleranceParameter	name
$\varepsilon_p$	MSK_DPAR_INTPNT_CO_TOL_PFEAS
$arepsilon_d$	MSK_DPAR_INTPNT_CO_TOL_DFEAS
$\varepsilon_g$	MSK_DPAR_INTPNT_CO_TOL_REL_GAP
$arepsilon_i$	MSK_DPAR_INTPNT_CO_TOL_INFEAS

The default values of the termination tolerances are chosen such that for a majority of problems appearing in practice it is not possible to achieve much better accuracy. Therefore, tightening the tolerances usually is not worthwhile. However, an inspection of (13.11) reveals that the quality of the solution depends on  $||b||_{\infty}$  and  $||c||_{\infty}$ ; the smaller the norms are, the better the solution accuracy.

The interior-point method as implemented by **MOSEK** will converge toward optimality and primal and dual feasibility at the same rate [And09]. This means that if the optimizer is stopped prematurely then it is very unlikely that either the primal or dual solution is feasible. Another consequence is that in most cases all the tolerances,  $\varepsilon_p$ ,  $\varepsilon_d$ ,  $\varepsilon_g$  and  $\varepsilon_i$ , have to be relaxed together to achieve an effect.

If the optimizer terminates without locating a solution that satisfies the termination criteria, for example because of a stall or other numerical issues, then it will check if the solution found up to that point satisfies the same criteria with all tolerances multiplied by the value of  $MSK_DPAR_INTPNT_CO_TOL_NEAR_REL$ . If this is the case, the solution is still declared as optimal.

To conclude the discussion in this section, relaxing the termination criterion is usually not worthwhile.

### 13.3.4 The Interior-point Log

Below is a typical log output from the interior-point optimizer:

```
Optimizer
           - threads
                                      : 20
Optimizer
            - solved problem
                                      : the primal
Optimizer
           - Constraints
                                        1
Optimizer
           - Cones
Optimizer
           - Scalar variables
                                                                                     : 6
                                      : 6
                                                            conic
                                                                                     : 0
Optimizer
           - Semi-definite variables: 0
                                                            scalarized
                                      : 0.00
                                                                                     : 0.00
Factor
            - setup time
                                                            dense det. time
            - ML order time
                                                                                     : 0.00
Factor
                                                            GP order time
Factor
            - nonzeros before factor : 1
                                                            after factor
                                                                                     : 1
Factor
            - dense dim.
                                                            flops
                                                                                     : 1.
⊶70e+01
             DFEAS
                                 PRSTATUS
                                             POBJ
                                                                DOBJ
ITE PFEAS
                       GFEAS
                                                                                   MU
→ TIME
                                                                0.00000000e+00
0
    1.0e+00
             2.9e-01
                       3.4e + 00
                                 0.00e+00
                                             2.414213562e+00
                                                                                    1.0e+00<sub>L</sub>
   0.01
                                                                -9.685901771e-03
                                                                                   2.7e-01
    2.7e-01
             7.9e-02
                       2.2e+00
                                 8.83e-01
                                             6.969257574e-01
1
- 0.01
2
    6.5e-02
             1.9e-02
                       1.2e+00
                                 1.16e+00
                                             7.606090061e-01
                                                                6.046141322e-01
                                                                                   6.5e-02
→ 0.01
3
    1.7e-03
             5.0e-04
                       2.2e-01
                                 1.12e+00
                                             7.084385672e-01
                                                                7.045122560e-01
                                                                                    1.7e-03
- 0.01
                                             7.071067941e-01
                                                                7.071067599e-01
4
    1.4e-08
             4.2e-09
                       4.9e-08
                                1.00e+00
                                                                                    1.4e-08
```

The first line displays the number of threads used by the optimizer and the second line indicates if the optimizer chose to solve the primal or dual problem (see MSK\_IPAR\_INTPNT\_SOLVE\_FORM). The next lines display the problem dimensions as seen by the optimizer, and the Factor... lines show various statistics. This is followed by the iteration log.

Using the same notation as in Sec. 13.3.1 the columns of the iteration log have the following meaning:

- ITE: Iteration index k.
- PFEAS:  $||Ax^k b\tau^k||_{\infty}$ . The numbers in this column should converge monotonically towards zero but may stall at low level due to rounding errors.
- DFEAS:  $||A^Ty^k + s^k c\tau^k||_{\infty}$ . The numbers in this column should converge monotonically towards zero but may stall at low level due to rounding errors.
- GFEAS:  $|-c^Tx^k+b^Ty^k-\kappa^k|$ . The numbers in this column should converge monotonically towards zero but may stall at low level due to rounding errors.
- PRSTATUS: This number converges to 1 if the problem has an optimal solution whereas it converges to −1 if that is not the case.

- POBJ:  $c^T x^k / \tau^k$ . An estimate for the primal objective value.
- DOBJ:  $b^T y^k / \tau^k$ . An estimate for the dual objective value.
- MU:  $\frac{(x^k)^T s^k + \tau^k \kappa^k}{n+1}$  . The numbers in this column should always converge to zero.
- TIME: Time spent since the optimization started (in seconds).

# 13.4 The Optimizer for Mixed-integer Problems

A problem is a mixed-integer optimization problem when one or more of the variables are constrained to be integer valued. Readers unfamiliar with integer optimization are recommended to consult some relevant literature, e.g. the book [Wol98] by Wolsey.

## 13.4.1 The Mixed-integer Optimizer Overview

MOSEK can solve mixed-integer

- linear,
- quadratic and quadratically constrained, and
- conic

problems, except for mixed-integer semidefinite problems. The mixed-integer optimizer is specialized for solving linear and conic optimization problems. Pure quadratic and quadratically constrained problems are automatically converted to conic form.

By default the mixed-integer optimizer is run-to-run deterministic. This means that if a problem is solved twice on the same computer with identical parameter settings and no time limit then the obtained solutions will be identical. If a time limit is set then this may not be case since the time taken to solve a problem is not deterministic. The mixed-integer optimizer is parallelized i.e. it can exploit multiple cores during the optimization.

The solution process can be split into these phases:

- 1. Presolve: See Sec. 13.1.
- 2. Cut generation: Valid inequalities (cuts) are added to improve the lower bound.
- 3. **Heuristic:** Using heuristics the optimizer tries to guess a good feasible solution. Heuristics can be controlled by the parameter MSK\_IPAR\_MIO\_HEURISTIC\_LEVEL.
- 4. **Search:** The optimal solution is located by branching on integer variables.

#### 13.4.2 Relaxations and bounds

It is important to understand that, in a worst-case scenario, the time required to solve integer optimization problems grows exponentially with the size of the problem (solving mixed-integer problems is NP-hard). For instance, a problem with n binary variables, may require time proportional to  $2^n$ . The value of  $2^n$  is huge even for moderate values of n.

In practice this implies that the focus should be on computing a near-optimal solution quickly rather than on locating an optimal solution. Even if the problem is only solved approximately, it is important to know how far the approximate solution is from an optimal one. In order to say something about the quality of an approximate solution the concept of *relaxation* is important.

Consider for example a mixed-integer optimization problem

$$z^* = \text{minimize} \quad c^T x$$
subject to  $Ax = b$ ,
 $x \ge 0$ 
 $x_j \in \mathbb{Z}, \quad \forall j \in \mathcal{J}.$ 

$$(13.12)$$

It has the continuous relaxation

$$\underline{z} = \text{minimize} \quad c^T x$$
subject to  $Ax = b$ , (13.13)
 $x \ge 0$ 

obtained simply by ignoring the integrality restrictions. The relaxation is a continuous problem, and therefore much faster to solve to optimality with a linear (or, in the general case, conic) optimizer. We call the optimal value  $\underline{z}$  the *objective bound*. The objective bound  $\underline{z}$  normally increases during the solution search process when the continuous relaxation is gradually refined.

Moreover, if  $\hat{x}$  is any feasible solution to (13.12) and

$$\bar{z} := c^T \hat{x}$$

then

$$\underline{z} \le z^* \le \bar{z}.$$

These two inequalities allow us to estimate the quality of the integer solution: it is no further away from the optimum than  $\bar{z} - \underline{z}$  in terms of the objective value. Whenever a mixed-integer problem is solved **MOSEK** reports this lower bound so that the quality of the reported solution can be evaluated.

## 13.4.3 Outer approximation for mixed-integer conic problems

The relaxations of mixed integer conic problems can be solved either as a nonlinear problem with the interior point algorithm (default) or with a linear outer approximation algorithm. The type of relaxation used can be set with <code>MSK\_IPAR\_MIO\_CONIC\_OUTER\_APPROXIMATION</code>. The best value for this option is highly problem dependent.

#### 13.4.4 Randomization

A number of internal algorithms of the mixed-integer solver are dependend on random tie-breaking. The random tie-breaking can have a significant impact on the path taken by the algorithm and the optimal solution returned. The random seed can be set with the parameter <code>MSK\_IPAR\_MIO\_SEED</code>.

### 13.4.5 Termination Criterion

In general, it is time consuming to find an exact feasible and optimal solution to an integer optimization problem, though in many practical cases it may be possible to find a sufficiently good solution. The issue of terminating the mixed-integer optimizer is rather delicate and the user has numerous possibilities of influencing it with various parameters. The mixed-integer optimizer employs a relaxed feasibility and optimality criterion to determine when a satisfactory solution is located.

A candidate solution that is feasible for the continuous relaxation is said to be an *integer feasible* solution if the criterion

$$\min(x_j - \lfloor x_j \rfloor, \lceil x_j \rceil - x_j) \le \delta_1 \quad \forall j \in \mathcal{J}$$

is satisfied, meaning that  $x_j$  is at most  $\delta_1$  from the nearest integer.

Whenever the integer optimizer locates an integer feasible solution it will check if the criterion

$$\bar{z} - \underline{z} \leq \max(\delta_2, \delta_3 \max(\delta_4, |\bar{z}|))$$

is satisfied. If this is the case, the integer optimizer terminates and reports the integer feasible solution as an optimal solution.

All the  $\delta$  tolerances discussed above can be adjusted using suitable parameters — see Table 13.3.

Table 13.3: Tolerances for the mixed-integer optimizer.

Tolerance	Parameter name
$\delta_1$	MSK_DPAR_MIO_TOL_ABS_RELAX_INT
$\delta_2$	MSK_DPAR_MIO_TOL_ABS_GAP
$\delta_3$	MSK_DPAR_MIO_TOL_REL_GAP
$\delta_4$	MSK_DPAR_MIO_REL_GAP_CONST

In Table 13.4 some other common parameters affecting the integer optimizer termination criterion are shown.

Table 13.4: Other parameters affecting the integer optimizer termination criterion.

Parameter name	Explanation
MSK_IPAR_MIO_MAX_NUM_BRANCHES	Maximum number of branches allowed.
MSK_IPAR_MIO_MAX_NUM_RELAXS	Maximum number of relaxations allowed.
MSK_IPAR_MIO_MAX_NUM_SOLUTIONS	Maximum number of feasible integer solutions allowed.

## 13.4.6 Speeding Up the Solution Process

As mentioned previously, in many cases it is not possible to find an optimal solution to an integer optimization problem in a reasonable amount of time. Some suggestions to reduce the solution time are:

- Relax the termination criterion: In case the run time is not acceptable, the first thing to do is to relax the termination criterion see Sec. 13.4.5 for details.
- Specify a good initial solution: In many cases a good feasible solution is either known or easily computed using problem-specific knowledge. If a good feasible solution is known, it is usually worthwhile to use this as a starting point for the integer optimizer. See Sec. 6.7.2.
- Improve the formulation: A mixed-integer optimization problem may be impossible to solve in one form and quite easy in another form. However, it is beyond the scope of this manual to discuss good formulations for mixed-integer problems. For discussions on this topic see for example [Wol98].

# 13.4.7 Understanding Solution Quality

To determine the quality of the solution one should check the following:

- The problem status and solution status returned by MOSEK, as well as constraint violations in case of suboptimal solutions.
- The optimality gap defined as

$$\epsilon = |(\text{objective value of feasible solution}) - (\text{objective bound})| = |\bar{z} - \underline{z}|.$$

which measures how much the located solution can deviate from the optimal solution to the problem. The optimality gap can be retrieved through the information item  $MSK\_DINF\_MIO\_OBJ\_ABS\_GAP$ . Often it is more meaningful to look at the relative optimality gap normalized against the magnitude of the solution.

$$\epsilon_{\rm rel} = \frac{|\bar{z} - \underline{z}|}{\max(\delta_4, |\bar{z}|)}.$$

The relative optimality gap is available in the information item  $MSK\_DINF\_MIO\_OBJ\_REL\_GAP$ .

## 13.4.8 The Mixed-integer Log

Below is a typical log output from the mixed-integer optimizer:

Presolved problem: 6573 variables, 35728 constraints, 101258 non-zeros Presolved problem: 0 general integer, 4294 binary, 2279 continuous Clique table size: 1636						
BRANC	CHES RELAXS		DEPTH	BEST_INT_OBJ	BEST_RELAX_OBJ	REL_GAP(
→%) 0	TIME 1	0	0	NA	1.8218819866e+07	NA 👝
<b>↔</b> 0	1.6	0	0	1.8331557950e+07	1.8218819866e+07	0.61
$\hookrightarrow$	3.5	· ·		1100010010000		0.01

					(continued from	i previous p	age)		
0	1	0	0	1.8300507546e+07	1.8218819866e+07	0.45	⊔		
$\hookrightarrow$	4.3								
Cut	generation	started.							
0	2	0	0	1.8300507546e+07	1.8218819866e+07	0.45	ш		
$\hookrightarrow$	5.3								
Cut	generation	terminat	ed. Time =	1.43					
0	3	0	0	1.8286893047e+07	1.8231580587e+07	0.30	ш		
$\hookrightarrow$	7.5						_		
15	18	1	0	1.8286893047e+07	1.8231580587e+07	0.30	ш		
$\hookrightarrow$	10.5						_		
31	34	1	0	1.8286893047e+07	1.8231580587e+07	0.30	ш		
$\hookrightarrow$	11.1						_		
51	54	1	0	1.8286893047e+07	1.8231580587e+07	0.30	ш		
$\hookrightarrow$	11.6						- u		
91	94	1	0	1.8286893047e+07	1.8231580587e+07	0.30	ш		
	12.4								
171	174	1	0	1.8286893047e+07	1.8231580587e+07	0.30	ш		
$\hookrightarrow$	14.3						_		
331	334	1	0	1.8286893047e+07	1.8231580587e+07	0.30	ш		
$\hookrightarrow$	17.9						_		
Γ.	]								
_									
Obi	ective of b	est integ	er solutio	on: 1.825846762609e+07	7				
_	t objective	_	,01 0014010	: 1.823311032986e+07					
	•		ctive	: Not employed					
Construct solution objective Construct solution # roundings				: 0					
User objective cut value				: 0					
Number of cuts generated				: 117					
Number of Gomory cuts				: 108					
Number of CMIR cuts				: 9					
Number of branches				: 4425					
				: 4410					
Number of interior point iterations									
Number of simplex iterations				: 221131					
ıv uili	Number of simplex iterations : 221131								

The first lines contain a summary of the problem as seen by the optimizer. This is followed by the iteration log. The columns have the following meaning:

- BRANCHES: Number of branches generated.
- RELAXS: Number of relaxations solved.
- ACT\_NDS: Number of active branch bound nodes.
- DEPTH: Depth of the recently solved node.
- BEST\_INT\_OBJ: The best integer objective value,  $\bar{z}$ .
- BEST\_RELAX\_OBJ: The best objective bound,  $\underline{z}$ .
- REL\_GAP(%): Relative optimality gap,  $100\% \cdot \epsilon_{\rm rel}$
- TIME: Time (in seconds) from the start of optimization.

Following that a summary of the optimization process is printed.

# Chapter 14

# Additional features

In this section we describe additional features and tools which enable more detailed analysis of optimization problems with  $\mathbf{MOSEK}$ .

# 14.1 Problem Analyzer

The problem analyzer prints a survey of the structure of the problem, with information about linear constraints and objective, quadratic constraints, conic constraints and variables.

In the initial stages of model formulation the problem analyzer may be used as a quick way of verifying that the model has been built or imported correctly. In later stages it can help revealing special structures within the model that may be used to tune the optimizer's performance or to identify the causes of numerical difficulties.

The problem analyzer is run using MSK\_analyzeproblem. It prints its output to a log stream. The output is similar to the one below (this is the problem survey of the aflow30a problem from the MIPLIB 2003 collection).

Analyzing the problem							
*** Structural	report Dimensions						
	Constraints 479	Variables 842	Matrix var. O	Cones O			
	Constraint ar	nd bound types					
	Free	Lower	Upper	Ranged	Fixed		
Constraints:	0	0	421	0	58		
Variables:	0	0	0	842	0		
	Integer constraint types						
	Binary	General					
	421	0					
*** Data repor	rt						
	Nonzeros	Min	Max				
lcjl:	421	1.1e+01	5.0e+02				
Aij :		1.0e+00	1.0e+02				
	# finite	Min	Max				
blci :	58	1.0e+00	1.0e+01				
buci :		0.0e+00	1.0e+01				
blxj :		0.0e+00	0.0e+00				
buxj :		1.0e+00	1.0e+02				
					(continues on next page)		

#### \*\*\* Done analyzing the problem

The survey is divided into a structural and numerical report. The content should be self-explanatory.

# 14.2 Automatic Repair of Infeasible Problems

**MOSEK** provides an automatic repair tool for infeasible linear problems which we cover in this section. Note that most infeasible models are so due to bugs which can (and should) be more reliably fixed manually, using the knowledge of the model structure. We discuss this approach in Sec. 8.3.

## 14.2.1 Automatic repair

The main idea can be described as follows. Consider the linear optimization problem with m constraints and n variables

which is assumed to be infeasible.

One way of making the problem feasible is to reduce the lower bounds and increase the upper bounds. If the change is sufficiently large the problem becomes feasible. Now an obvious idea is to compute the optimal relaxation by solving an optimization problem. The problem

minimize 
$$p(v_l^c, v_u^c, v_l^x, v_u^x)$$
  
subject to  $l^c - v_l^c \le Ax \le u^c + v_u^c,$   
 $l^x - v_l^x \le x \le u^x + v_u^x,$   
 $v_l^c, v_u^c, v_l^x, v_u^x \ge 0$  (14.1)

does exactly that. The additional variables  $(v_l^c)_i$ ,  $(v_u^c)_i$ ,  $(v_l^x)_j$  and  $(v_u^c)_j$  are elasticity variables because they allow a constraint to be violated and hence add some elasticity to the problem. For instance, the elasticity variable  $(v_l^c)_i$  controls how much the lower bound  $(l^c)_i$  should be relaxed to make the problem feasible. Finally, the so-called penalty function

$$p(v_l^c, v_u^c, v_l^x, v_u^x)$$

is chosen so it penalizes changes to bounds. Given the weights

- $w_l^c \in \mathbb{R}^m$  (associated with  $l^c$ ),
- $w_u^c \in \mathbb{R}^m$  (associated with  $u^c$ ),
- $w_l^x \in \mathbb{R}^n$  (associated with  $l^x$ ),
- $w_u^x \in \mathbb{R}^n$  (associated with  $u^x$ ),

a natural choice is

$$p(v_l^c, v_u^c, v_l^x, v_u^x) = (w_l^c)^T v_l^c + (w_u^c)^T v_u^c + (w_l^x)^T v_l^x + (w_u^x)^T v_u^x.$$

Hence, the penalty function p() is a weighted sum of the elasticity variables and therefore the problem (14.1) keeps the amount of relaxation at a minimum. Please observe that

- the problem (14.1) is always feasible.
- a negative weight implies problem (14.1) is unbounded. For this reason if the value of a weight is negative **MOSEK** fixes the associated elasticity variable to zero. Clearly, if one or more of the weights are negative, it may imply that it is not possible to repair the problem.

A simple choice of weights is to set them all to 1, but of course that does not take into account that constraints may have different importance.

#### **Caveats**

Observe if the infeasible problem

```
minimize x + z

subject to x = -1,

x > 0
```

is repaired then it will become unbounded. Hence, a repaired problem may not have an optimal solution. Another and more important caveat is that only a minimal repair is performed i.e. the repair that barely makes the problem feasible. Hence, the repaired problem is barely feasible and that sometimes makes the repaired problem hard to solve.

#### Using the automatic repair tool

In this subsection we consider an infeasible linear optimization example:

minimize 
$$-10x_1$$
  $-9x_2$ ,  
subject to  $7/10x_1$  +  $1x_2$   $\leq 630$ ,  
 $1/2x_1$  +  $5/6x_2$   $\leq 600$ ,  
 $1x_1$  +  $2/3x_2$   $\leq 708$ ,  
 $1/10x_1$  +  $1/4x_2$   $\leq 135$ ,  
 $x_1$ ,  $x_2$   $\geq 0$ ,  
 $x_2$   $\geq 650$ .

The function  $MSK\_primalrepair$  can be used to repair an infeasible problem. This can be used for linear and conic optimization problems, possibly with integer variables.

Listing 14.1: An example of feasibility repair applied to problem (14.2).

```
# include <math.h>
# include <stdio.h>
# include "mosek.h"
static void MSKAPI printstr(void *handle,
                             const char str[])
 fputs(str, stdout);
} /* printstr */
static const char * feasrepair_lp =
"minimize
                                               n''
" obj: - 10 x1 - 9 x2
                                               n''
"st
                                               \n"
" c1: + 7e-01 \times 1 + \times 2 \le 630
                                               \n"
" c2: + 5e-01 x1 + 8.33333333e-01 x2 <= 600 \n"
" c3: + x1 + 6.6666667e-01 x2 <= 708
" c4: + 1e-01 x1 + 2.5e-01 x2 <= 135
                                               n''
                                               \n"
"bounds
"x2 >= 650
                                               \n"
"end
                                               n";
int main(int argc, const char *argv[])
 MSKenv_t
              env:
 MSKrescodee r, trmcode;
 MSKtask_t task;
```

```
r = MSK_makeenv(&env, NULL);
if (r == MSK_RES_OK)
  r = MSK_makeemptytask(env, &task);
if (r == MSK_RES_OK)
  MSK_linkfunctotaskstream(task, MSK_STREAM_LOG, NULL, printstr);
if (r == MSK_RES_OK)
  r = MSK_readlpstring(task,feasrepair_lp); /* Read problem from string */
if (r == MSK_RES_OK)
 r = MSK_putintparam(task, MSK_IPAR_LOG_FEAS_REPAIR, 3);
if (r == MSK_RES_OK)
  /* Weights are NULL implying all weights are 1. */
  r = MSK_primalrepair(task, NULL, NULL, NULL, NULL);
if (r == MSK_RES_OK)
  double sum_viol;
 r = MSK_getdouinf(task, MSK_DINF_PRIMAL_REPAIR_PENALTY_OBJ, &sum_viol);
  if (r == MSK_RES_OK)
   printf("Minimized sum of violations = %e\n", sum_viol);
   r = MSK_optimizetrm(task, &trmcode); /* Optimize the repaired task. */
   MSK_solutionsummary(task, MSK_STREAM_MSG);
}
printf("Return code: %d\n", r);
return (r);
```

The above code will produce the following log report:

```
MOSEK Version 9.0.0.25(ALPHA) (Build date: 2017-11-7 16:11:50)
Copyright (c) MOSEK ApS, Denmark. WWW: mosek.com
Platform: Linux/64-X86

Open file 'feasrepair.lp'
Reading started.
Reading terminated. Time: 0.00

Read summary
Type : LO (linear optimization problem)
Objective sense : min
Scalar variables : 2
Matrix variables : 0
Constraints : 4
```

```
Cones
                 : 0
                 : 0.0
 Time
Problem
 Name
 Objective sense
                     : min
                     : LO (linear optimization problem)
 Constraints
                     : 4
 Cones
                      : 0
 Scalar variables
                      : 2
 Matrix variables
                      : 0
 Integer variables
                     : 0
Primal feasibility repair started.
Optimizer started.
Presolve started.
Linear dependency checker started.
Linear dependency checker terminated.
Eliminator started.
Freed constraints in eliminator : 2
Eliminator terminated.
Eliminator - tries
                                : 1
                                                   time
                                                                        : 0.00
Lin. dep. - tries
                                                   time
                                                                        : 0.00
                                : 1
Lin. dep. - number
                                : 0
Presolve terminated. Time: 0.00
Problem
 Name
 Objective sense : min
 Type
                     : LO (linear optimization problem)
 Constraints
                     : 8
 Cones
                      : 0
 Scalar variables : 14 Matrix variables : 0
 Integer variables
                      : 0
                                : 20
Optimizer - threads
Optimizer - solved problem
                              : the primal
Optimizer - Constraints
                                : 2
Optimizer - Cones
                                : 0
Optimizer - Scalar variables : 5
                                                   conic
                                                                        : 0
                                                   scalarized
Optimizer - Semi-definite variables: 0
                                                                        : 0
                                                 dense det. time
Factor - setup time : 0.00
                                                                        : 0.00
Factor
         - ML order time
                               : 0.00
                                                 GP order time
                                                                        : 0.00
Factor - nonzeros before factor : 3
                                                   after factor
                                                                        : 3
Factor
        - dense dim.
                                                   flops
                                                                        : 5.
                                : 0
→00e+01
ITE PFEAS DFEAS GFEAS PRSTATUS
                                      POBJ
                                                       DOBJ
                                                                        MU
→ TIME
0 2.7e+01 1.0e+00 4.0e+00 1.00e+00
                                      3.000000000e+00
                                                       0.00000000e+00
                                                                        1.0e+00<sub>L</sub>
→ 0.00
1 2.5e+01 9.1e-01 1.4e+00 0.00e+00
                                                                        2.4e+00<sub>L</sub>
                                      8.711262850e+00
                                                       1.115287830e+01
→ 0.00
2 2.4e+00 8.8e-02 1.4e-01 -7.33e-01 4.062505701e+01
                                                       4.422203730e+01
                                                                        2.3e-01
3 9.4e-02 3.4e-03 5.5e-03 1.33e+00 4.250700434e+01 4.258548510e+01
                                                                        9.1e-03
→ 0.00
```

```
2.0e-05 7.2e-07 1.1e-06 1.02e+00
                                        4.249996599e+01
                                                          4.249998669e+01
                                                                            1.9e-06
→ 0.00
  2.0e-09 7.2e-11 1.1e-10 1.00e+00 4.250000000e+01
                                                          4.250000000e+01
                                                                            1.9e-10
Basis identification started.
Basis identification terminated. Time: 0.00
Optimizer terminated. Time: 0.01
Basic solution summary
 Problem status : PRIMAL_AND_DUAL_FEASIBLE
 Solution status : OPTIMAL
 Primal. obj: 4.2500000000e+01 nrm: 6e+02
                                                Viol. con: 1e-13
                                                                     var: 0e+00
          obj: 4.249999999e+01
                                  nrm: 2e+00
                                                Viol. con: 0e+00
                                                                     var: 9e-11
 Dual.
Optimal objective value of the penalty problem: 4.250000000000e+01
Repairing bounds.
Increasing the upper bound 1.35e+02 on constraint 'c4' (3) with 2.25e+01.
Decreasing the lower bound 6.50e+02 on variable 'x2' (4) with 2.00e+01.
Primal feasibility repair terminated.
Optimizer started.
Optimizer terminated. Time: 0.00
Interior-point solution summary
 Problem status : PRIMAL_AND_DUAL_FEASIBLE
 Solution status : OPTIMAL
 Primal. obj: -5.6700000000e+03 nrm: 6e+02
                                                Viol. con: 0e+00
                                                                     var: 0e+00
          obj: -5.6700000000e+03 nrm: 1e+01
                                                Viol. con: 0e+00
                                                                     var: 0e+00
 Dual.
Basic solution summary
 Problem status : PRIMAL_AND_DUAL_FEASIBLE
 Solution status : OPTIMAL
 Primal. obj: -5.6700000000e+03 nrm: 6e+02
                                                Viol. con: 0e+00
                                                                     var: 0e+00
          obj: -5.6700000000e+03 nrm: 1e+01
                                                Viol. con: 0e+00
                                                                     var: 0e+00
Optimizer summary
 Optimizer
                                                   time: 0.00
   Interior-point
                           - iterations : 0
                                                   time: 0.00
     Basis identification -
                                                   time: 0.00
                                                   time: 0.00
       Primal
                           - iterations : 0
                                                   time: 0.00
       Dual
                           - iterations : 0
       Clean primal
                          - iterations : 0
                                                   time: 0.00
       Clean dual
                           - iterations : 0
                                                   time: 0.00
   Simplex
                                                   time: 0.00
     Primal simplex
                           - iterations : 0
                                                   time: 0.00
     Dual simplex
                           - iterations : 0
                                                   time: 0.00
   Mixed integer
                           - relaxations: 0
                                                   time: 0.00
```

It will also modify the task according to the optimal elasticity variables found. In this case the optimal repair it is to increase the upper bound on constraint c4 by 22.5 and decrease the lower bound on variable x2 by 20.

# 14.3 Sensitivity Analysis

Given an optimization problem it is often useful to obtain information about how the optimal objective value changes when the problem parameters are perturbed. E.g, assume that a bound represents the capacity of a machine. Now, it may be possible to expand the capacity for a certain cost and hence it is worthwhile knowing what the value of additional capacity is. This is precisely the type of questions the sensitivity analysis deals with.

Analyzing how the optimal objective value changes when the problem data is changed is called sensitivity analysis.

#### References

The book [Chvatal83] discusses the classical sensitivity analysis in Chapter 10 whereas the book [RTV97] presents a modern introduction to sensitivity analysis. Finally, it is recommended to read the short paper [Wal00] to avoid some of the pitfalls associated with sensitivity analysis.

Warning: Currently, sensitivity analysis is only available for continuous linear optimization problems. Moreover, MOSEK can only deal with perturbations of bounds and objective function coefficients.

# 14.3.1 Sensitivity Analysis for Linear Problems

#### The Optimal Objective Value Function

Assume that we are given the problem

$$z(l^{c}, u^{c}, l^{x}, u^{x}, c) = \underset{\text{subject to}}{\text{minimize}} c^{T}x$$

$$l^{c} \leq Ax \leq u^{c},$$

$$l^{x} \leq x \leq u^{x},$$

$$(14.3)$$

and we want to know how the optimal objective value changes as  $l_i^c$  is perturbed. To answer this question we define the perturbed problem for  $l_i^c$  as follows

$$\begin{array}{lcl} f_{l_i^c}(\beta) & = & \text{minimize} & & c^T x \\ & & \text{subject to} & l^c + \beta e_i & \leq & Ax & \leq u^c, \\ & & l^x & \leq & x \leq & u^x, \end{array}$$

where  $e_i$  is the *i*-th column of the identity matrix. The function

$$f_{l_{\cdot}^{c}}(\beta) \tag{14.4}$$

shows the optimal objective value as a function of  $\beta$ . Please note that a change in  $\beta$  corresponds to a perturbation in  $l_i^c$  and hence (14.4) shows the optimal objective value as a function of varying  $l_i^c$  with the other bounds fixed.

It is possible to prove that the function (14.4) is a piecewise linear and convex function, i.e. its graph may look like in Fig. 14.1 and Fig. 14.2.

Clearly, if the function  $f_{l_i^c}(\beta)$  does not change much when  $\beta$  is changed, then we can conclude that the optimal objective value is insensitive to changes in  $l_i^c$ . Therefore, we are interested in the rate of change in  $f_{l_i^c}(\beta)$  for small changes in  $\beta$ —specifically the gradient

$$f_{l_i^c}'(0),$$

which is called the *shadow price* related to  $l_i^c$ . The shadow price specifies how the objective value changes for small changes of  $\beta$  around zero. Moreover, we are interested in the *linearity interval* 

$$\beta \in [\beta_1, \beta_2]$$

for which

$$f'_{l_i^c}(\beta) = f'_{l_i^c}(0).$$

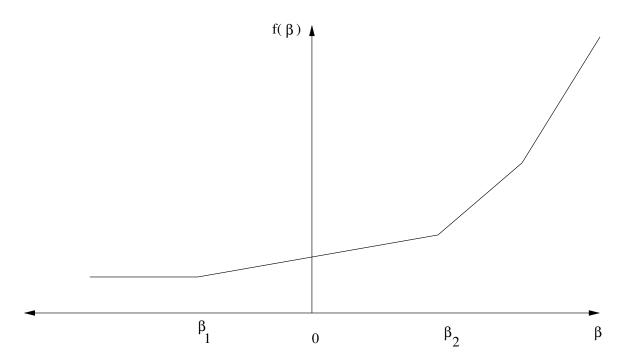


Fig. 14.1:  $\beta=0$  is in the interior of linearity interval.

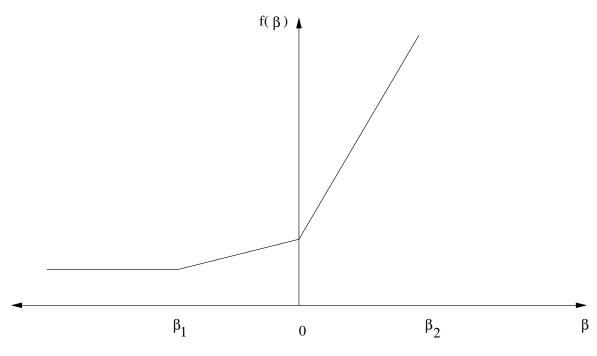


Fig. 14.2:  $\beta = 0$  is a breakpoint.

Since  $f_{l_i^c}$  is not a smooth function  $f'_{l_i^c}$  may not be defined at 0, as illustrated in Fig. 14.2. In this case we can define a left and a right shadow price and a left and a right linearity interval.

The function  $f_{l_i^c}$  considered only changes in  $l_i^c$ . We can define similar functions for the remaining parameters of the z defined in (14.3) as well:

$$f_{l_i^c}(\beta) = z(l^c + \beta e_i, u^c, l^x, u^x, c), \quad i = 1, \dots, m,$$

$$f_{u_i^c}(\beta) = z(l^c, u^c + \beta e_i, l^x, u^x, c), \quad i = 1, \dots, m,$$

$$f_{l_j^x}(\beta) = z(l^c, u^c, l^x + \beta e_j, u^x, c), \quad j = 1, \dots, n,$$

$$f_{u_j^x}(\beta) = z(l^c, u^c, l^x, u^x + \beta e_j, c), \quad j = 1, \dots, n,$$

$$f_{c_j}(\beta) = z(l^c, u^c, l^x, u^x, c + \beta e_j), \quad j = 1, \dots, n.$$

Given these definitions it should be clear how linearity intervals and shadow prices are defined for the parameters  $u_i^c$  etc.

# **Equality Constraints**

In **MOSEK** a constraint can be specified as either an equality constraint or a ranged constraint. If some constraint  $e_i^c$  is an equality constraint, we define the optimal value function for this constraint as

$$f_{e_i^c}(\beta) = z(l^c + \beta e_i, u^c + \beta e_i, l^x, u^x, c)$$

Thus for an equality constraint the upper and the lower bounds (which are equal) are perturbed simultaneously. Therefore, **MOSEK** will handle sensitivity analysis differently for a ranged constraint with  $l_i^c = u_i^c$  and for an equality constraint.

## The Basis Type Sensitivity Analysis

The classical sensitivity analysis discussed in most textbooks about linear optimization, e.g. [Chvatal83], is based on an optimal basis. This method may produce misleading results [RTV97] but is computationally cheap. This is the type of sensitivity analysis implemented in **MOSEK**.

We will now briefly discuss the basis type sensitivity analysis. Given an optimal basic solution which provides a partition of variables into basic and non-basic variables, the basis type sensitivity analysis computes the linearity interval  $[\beta_1, \beta_2]$  so that the basis remains optimal for the perturbed problem. A shadow price associated with the linearity interval is also computed. However, it is well-known that an optimal basic solution may not be unique and therefore the result depends on the optimal basic solution employed in the sensitivity analysis. If the optimal objective value function has a breakpoint for  $\beta = 0$  then the basis type sensitivity method will only provide a subset of either the left or the right linearity interval.

In summary, the basis type sensitivity analysis is computationally cheap but does not provide complete information. Hence, the results of the basis type sensitivity analysis should be used with care.

#### **Example: Sensitivity Analysis**

As an example we will use the following transportation problem. Consider the problem of minimizing the transportation cost between a number of production plants and stores. Each plant supplies a number of goods and each store has a given demand that must be met. Supply, demand and cost of transportation per unit are shown in Fig. 14.3.

If we denote the number of transported goods from location i to location j by  $x_{ij}$ , problem can be formulated as the linear optimization problem of minimizing

$$1x_{11} + 2x_{12} + 5x_{23} + 2x_{24} + 1x_{31} + 2x_{33} + 1x_{34}$$

subject to

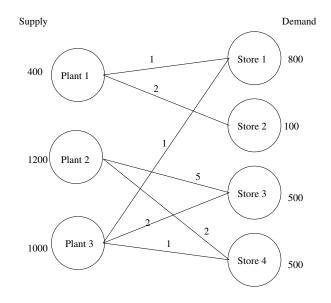


Fig. 14.3: Supply, demand and cost of transportation.

The sensitivity parameters are shown in Table 14.1 and Table 14.2.

Table 14.1: Ranges and shadow prices related to bounds on constraints and variables.

Con.	$\beta_1$	$\beta_2$	$\sigma_1$	$\sigma_2$
1	-300.00	0.00	3.00	3.00
2	-700.00	$+\infty$	0.00	0.00
3	-500.00	0.00	3.00	3.00
4	-0.00	500.00	4.00	4.00
5	-0.00	300.00	5.00	5.00
6	-0.00	700.00	5.00	5.00
7	-500.00	700.00	2.00	2.00
Var.	$\beta_1$	$\beta_2$	$\sigma_1$	$\sigma_2$
$x_{11}$	$-\infty$	300.00	0.00	0.00
	🏎	000.00	0.00	0.00
$x_{12}$	$-\infty$	100.00	0.00	0.00
$x_{12} \\ x_{23}$				
	$-\infty$	100.00	0.00	0.00
$x_{23}$	$-\infty$ $-\infty$	100.00	0.00	0.00
$x_{23} \\ x_{24}$	$-\infty$ $-\infty$ $-\infty$	100.00 0.00 500.00	0.00 0.00 0.00	0.00 0.00 0.00

Table 14.2: Ranges and shadow prices related to the objective coefficients.

Var.	$\beta_1$	$\beta_2$	$\sigma_1$	$\sigma_2$
$c_1$	$-\infty$	3.00	300.00	300.00
$c_2$	$-\infty$	$\infty$	100.00	100.00
$c_3$	-2.00	$\infty$	0.00	0.00
$c_4$	$-\infty$	2.00	500.00	500.00
$c_5$	-3.00	$\infty$	500.00	500.00
$c_6$	$-\infty$	2.00	500.00	500.00
$c_7$	-2.00	$\infty$	0.00	0.00

Examining the results from the sensitivity analysis we see that for constraint number 1 we have  $\sigma_1 = 3$  and  $\beta_1 = -300$ ,  $\beta_2 = 0$ .

If the upper bound on constraint 1 is decreased by

$$\beta \in [0, 300]$$

then the optimal objective value will increase by the value

$$\sigma_1\beta=3\beta$$
.

# 14.3.2 Sensitivity Analysis with MOSEK

MOSEK provides the functions  $MSK\_primalsensitivity$  and  $MSK\_dualsensitivity$  for performing sensitivity analysis. The code in Listing 14.2 gives an example of its use.

Listing 14.2: Example of sensitivity analysis with the **MOSEK** Optimizer API for C.

```
# include <stdio.h>
#include "mosek.h" /* Include the MOSEK definition file. */
static void MSKAPI printstr(void *handle,
                           const char str[])
 printf("%s", str);
} /* printstr */
int main(int argc, char *argv[])
 const MSKint32t numcon = 7,
                 numvar = 7;
 {\tt MSKint32t}
                 i, j;
                 bkc[] = {MSK_BK_UP, MSK_BK_UP, MSK_BK_UP, MSK_BK_FX,
 MSKboundkeye
                          MSK_BK_FX, MSK_BK_FX, MSK_BK_FX
                         };
                 bkx[] = {MSK_BK_LO, MSK_BK_LO, MSK_BK_LO,
 MSKboundkeye
                          MSK_BK_LO, MSK_BK_LO, MSK_BK_LO, MSK_BK_LO
                         };
 MSKint32t
                 ptrb[] = {0, 2, 4, 6, 8, 10, 12};
                 ptre[] = {2, 4, 6, 8, 10, 12, 14};
 MSKint32t
                 sub[] = \{0, 3, 0, 4, 1, 5, 1, 6, 2, 3, 2, 5, 2, 6\};
 MSKint32t
                 blc[] = { -MSK_INFINITY, -MSK_INFINITY, -MSK_INFINITY, 800, 100, __
 MSKrealt
500, 500;
 {	t MSKrealt}
                 buc[] = \{400,
                                        1200,
                                                      1000.
                                                                   800, 100, 500,<sub>L</sub>
→500};
 MSKrealt
                     = \{1.0, 2.0, 5.0, 2.0, 1.0, 2.0, 1.0\};
                 blx[] = {0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0};
 MSKrealt
                 bux[] = {MSK_INFINITY, MSK_INFINITY, MSK_INFINITY, MSK_INFINITY,
 MSKrealt
                          MSK_INFINITY, MSK_INFINITY, MSK_INFINITY
                 MSKrealt
→ 1.0, 1.0};
 MSKrescodee
                 r;
 MSKenv_t
                 env;
 MSKtask_t
                 task;
```

```
/* Create mosek environment. */
r = MSK_makeenv(&env, NULL);
if (r == MSK_RES_OK)
  /* Make the optimization task. */
  r = MSK_makeemptytask(env, &task);
  if (r == MSK_RES_OK)
    /* Directs the log task stream to the user
       specified procedure 'printstr'. */
   MSK_linkfunctotaskstream(task, MSK_STREAM_LOG, NULL, printstr);
   MSK_echotask(task,
                 MSK_STREAM_MSG,
                 "Defining the problem data.\n");
  /* Append the constraints. */
  if (r == MSK_RES_OK)
   r = MSK_appendcons(task, numcon);
  /* Append the variables. */
  if (r == MSK_RES_OK)
   r = MSK_appendvars(task, numvar);
  /* Put C. */
  if (r == MSK_RES_OK)
   r = MSK_putcfix(task, 0.0);
  if (r == MSK_RES_OK)
   r = MSK_putcslice(task, 0, numvar, c);
  /* Put constraint bounds. */
  if (r == MSK_RES_OK)
   r = MSK_putconboundslice(task, 0, numcon, bkc, blc, buc);
  /* Put variable bounds. */
  if (r == MSK_RES_OK)
   r = MSK_putvarboundslice(task, 0, numvar, bkx, blx, bux);
  /* Put A. */
  if (r == MSK_RES_OK)
   r = MSK_putacolslice(task, 0, numvar, ptrb, ptre, sub, val);
  if (r == MSK_RES_OK)
   r = MSK_putobjsense(task, MSK_OBJECTIVE_SENSE_MINIMIZE);
  if (r == MSK_RES_OK)
   r = MSK_optimizetrm(task, NULL);
  if (r == MSK_RES_OK)
    /* Analyze upper bound on c1 and the equality constraint on c4 */
```

```
MSKint32t subi[] = {0, 3};
  MSKmarke marki[] = {MSK_MARK_UP, MSK_MARK_UP};
  /* Analyze lower bound on the variables x12 and x31 */
  MSKint32t subj[] = \{1, 4\};
  MSKmarke markj[] = {MSK_MARK_LO, MSK_MARK_LO};
  MSKrealt leftpricei[2];
  MSKrealt rightpricei[2];
  MSKrealt leftrangei[2];
  MSKrealt rightrangei[2];
  MSKrealt leftpricej[2];
  MSKrealt rightpricej[2];
  MSKrealt leftrangej[2];
  MSKrealt rightrangej[2];
  r = MSK_primalsensitivity(task,
                            2,
                            subi,
                            marki.
                            2,
                            subj,
                            markj,
                            leftpricei,
                            rightpricei,
                            leftrangei,
                            rightrangei,
                            leftpricej,
                            rightpricej,
                            leftrangej,
                            rightrangej);
  printf("Results from sensitivity analysis on bounds:\n");
 printf("For constraints:\n");
  for (i = 0; i < 2; ++i)
    printf("leftprice = %e, rightprice = %e,leftrange = %e, rightrange = %e\n",
           leftpricei[i], rightpricei[i], leftrangei[i], rightrangei[i]);
  printf("For variables:\n");
  for (i = 0; i < 2; ++i)
    printf("leftprice = %e, rightprice = %e,leftrange = %e, rightrange = %e\n",
           leftpricej[i], rightpricej[i], leftrangej[i], rightrangej[i]);
}
if (r == MSK_RES_OK)
 MSKint32t subj[] = \{2, 5\};
  MSKrealt leftprice[2];
  MSKrealt rightprice[2];
  MSKrealt leftrange[2];
  MSKrealt rightrange[2];
 r = MSK_dualsensitivity(task,
                          2,
                          subj,
```

# Chapter 15

# **API** Reference

This section contains the complete reference of the **MOSEK** Optimizer API for C. It is organized as follows:

- General API conventions.
- Functions:
  - Full list
  - Browse by topic
- Optimizer parameters:
  - Double, Integer, String
  - Full list
  - Browse by topic
- Optimizer information items:
  - Double, Integer, Long
- Optimizer response codes
- Constants
- User-defined function types
- Simple data types
- Nonlinear API (SCopt, DGopt, EXPopt)

# 15.1 API Conventions

# 15.1.1 Function arguments

# **Naming Convention**

In the definition of the **MOSEK** Optimizer API for C a consistent naming convention has been used. This implies that whenever for example numcon is an argument in a function definition it indicates the number of constraints. In Table 15.1 the variable names used to specify the problem parameters are listed.

Table 15.1: Naming conventions used in the **MOSEK** Optimizer API for C.

API name	API type	Dimension	Related problem parameter
numcon	int		m
numvar	int		n
numcone	int		t
aptrb	int[]	numvar	$a_{ij}$
aptre	int[]	numvar	$a_{ij}$
asub	int[]	aptre[numvar-1]	$a_{ij}$
aval	double[]	aptre[numvar-1]	$a_{ij}$
С	double[]	numvar	$c_j$ $c^f$
cfix	double		$c^f$
blc	double[]	numcon	$l_k^c$
buc	double[]	numcon	$u_k^c$
blx	double[]	numvar	$l_k^x$
bux	double[]	numvar	$u_k^x$
numqonz	int		$q_{ij}^o$
qosubi	int[]	numqonz	$q_{ij}^o$
qosubj	int[]	numqonz	$q_{ij}^{o}$
qoval	double*	numqonz	$q_{ij}^{o}$
numqcnz	int		$q_{ij}^{ec{k}}$
qcsubk	int[]	numqcnz	$q_{ij}^{\vec{k}}$
qcsubi	int[]	numqcnz	$q_{ij}^{ec{k}}$
qcsubj	int[]	numqcnz	$q_{ij}^{ec{k}}$
qcval	double*	numqcnz	$q_{ij}^{\vec{k}}$
bkc	MSKboundkeye*	numcon	$l_k^c$ and $u_k^c$
bkx	MSKboundkeye*	numvar	$l_k^x$ and $u_k^x$

The relation between the variable names and the problem parameters is as follows:

- $\bullet \ \ \text{The quadratic terms in the objective:} \ \ q^o_{\texttt{qosubi}[\texttt{t}],\texttt{qosubj}[\texttt{t}]} = \texttt{qoval}[\texttt{t}], \quad \ t = 0, \dots, \texttt{numqonz} 1.$
- The linear terms in the objective :  $c_j = c[j], \quad j = 0, \dots, numvar 1$
- The fixed term in the objective :  $c^f = cfix$ .
- $\bullet \ \ \text{The quadratic terms in the constraints:} \ \ q_{\mathtt{qcsubi[t]},\mathtt{qcsubj[t]}}^{\mathtt{qcsubk[t]}} = \mathtt{qcval[t]}, \quad t = 0, \ldots, \mathtt{numqcnz} 1$
- The linear terms in the constraints:  $a_{\tt asub[t],j} = \tt aval[t], \quad t = \tt ptrb[j], \ldots, ptre[j] 1, \quad j = 0, \ldots, numvar 1$

### Passing arguments by reference

An argument described as  $\mathbf{T}$  by reference indicates that the function interprets its given argument as a reference to a variable of type  $\mathbf{T}$ . This usually means that the argument is used to output or update a value of type  $\mathbf{T}$ . For example, suppose we have a function documented as

```
MSKrescodee MSK_foo (..., int * nzc, ...)
```

• nzc (int by reference) – The number of nonzero elements in the matrix. (output)

Then it could be called as follows.

```
int nzc;
MSK_foo (..., &nzc, ...)
printf("The number of nonzero elements: %d\n", nzc)
```

# Information about input/output arguments

The following are purely informational tags which indicate how MOSEK treats a specific function argument.

- (input) An input argument. It is used to input data to MOSEK.
- (output) An output argument. It can be a user-preallocated data structure, a reference, a string buffer etc. where **MOSEK** will output some data.
- (input/output) An input/output argument. **MOSEK** will read the data and overwrite it with new/updated information.

# 15.1.2 Bounds

The bounds on the constraints and variables are specified using the variables bkc, blc, and buc. The components of the integer array bkc specify the bound type according to Table 15.2

v				
Symbolic constant	Lower bound	Upper bound		
MSK_BK_FX	finite	identical to the lower bound		
MSK_BK_FR	minus infinity	plus infinity		
MSK_BK_LO	finite	plus infinity		
MSK_BK_RA	finite	finite		
MSK_BK_UP	minus infinity	finite		

Table 15.2: Symbolic key for variable and constraint bounds.

For instance bkc[2]= $MSK_BK_L0$  means that  $-\infty < l_2^c$  and  $u_2^c = \infty$ . Even if a variable or constraint is bounded only from below, e.g.  $x \ge 0$ , both bounds are inputted or extracted; the irrelevant value is ignored.

Finally, the numerical values of the bounds are given by

$$l_k^c = \texttt{blc}[\texttt{k}], \quad k = 0, \dots, \texttt{numcon} - 1$$
 
$$u_k^c = \texttt{buc}[\texttt{k}], \quad k = 0, \dots, \texttt{numcon} - 1.$$

The bounds on the variables are specified using the variables bkx, blx, and bux in the same way. The numerical values for the lower bounds on the variables are given by

$$egin{aligned} l_j^x &= \mathtt{blx}[\mathtt{j}], \quad j = 0, \dots, \mathtt{numvar} - 1. \end{aligned}$$
  $u_i^x &= \mathtt{bux}[\mathtt{j}], \quad j = 0, \dots, \mathtt{numvar} - 1. \end{aligned}$ 

# 15.1.3 Vector Formats

Three different vector formats are used in the **MOSEK** API:

# Full (dense) vector

This is simply an array where the first element corresponds to the first item, the second element to the second item etc. For example to get the linear coefficients of the objective in task with number variables, one would write

```
MSKrealt * c = MSK_calloctask(task, numvar, sizeof(MSKrealt));

if ( c )
   res = MSK_getc(task,c);
else
   printf("Out of space\n");
```

### **Vector slice**

A vector slice is a range of values from first up to and **not including last** entry in the vector, i.e. for the set of indices i such that first <= i < last. For example, to get the bounds associated with constrains 2 through 9 (both inclusive) one would write

### Sparse vector

A sparse vector is given as an array of indexes and an array of values. The indexes need not be ordered. For example, to input a set of bounds associated with constraints number 1, 6, 3, and 9, one might write

```
MSKint32t
             bound_index[] = {
                                                    6,
                                                                3,
                                                                           9 };
                                         1,
                            = { MSK_BK_FR, MSK_BK_LO, MSK_BK_UP, MSK_BK_FX };
MSKboundkeye bound_key[]
             lower_bound[] = {
                                                              0.0,
MSKrealt
                                       0.0,
                                                -10.0,
                                                                         5.0 };
             upper_bound[] = {
                                       0.0,
                                                  0.0,
MSKrealt
                                                              6.0,
                                                                         5.0 };
res = MSK_putconboundlist(task, 4, bound_index,
                           bound_key,lower_bound,upper_bound);
```

### 15.1.4 Matrix Formats

The coefficient matrices in a problem are inputted and extracted in a sparse format. That means only the nonzero entries are listed.

# **Unordered Triplets**

In unordered triplet format each entry is defined as a row index, a column index and a coefficient. For example, to input the A matrix coefficients for  $a_{1,2}=1.1, a_{3,3}=4.3$ , and  $a_{5,4}=0.2$ , one would write as follows:

Please note that in some cases (like MSK\_putaijlist) only the specified indexes are modified — all other are unchanged. In other cases (such as MSK\_putqconk) the triplet format is used to modify all entries — entries that are not specified are set to 0.

# Column or Row Ordered Sparse Matrix

In a sparse matrix format only the non-zero entries of the matrix are stored. **MOSEK** uses a sparse packed matrix format ordered either by columns or rows. Here we describe the column-wise format. The row-wise format is based on the same principle.

# Column ordered sparse format

A sparse matrix in column ordered format is essentially a list of all non-zero entries read column by column from left to right and from top to bottom within each column. The exact representation uses four arrays:

- asub: Array of size equal to the number of nonzeros. List of row indexes.
- aval: Array of size equal to the number of nonzeros. List of non-zero entries of A ordered by columns
- ptrb: Array of size numcol, where ptrb[j] is the position of the first value/index in aval/ asub for the j-th column.
- ptre: Array of size numcol, where ptre[j] is the position of the last value/index plus one in aval / asub for the j-th column.

With this representation the values of a matrix A with numcol columns are assigned using:

$$a_{\mathtt{asub}[k],j} = \mathtt{aval}[k] \quad \text{for} \quad j = 0, \dots, \mathtt{numcol} - 1, \ k = \mathtt{ptrb}[j], \dots, \mathtt{ptre}[j] - 1.$$

As an example consider the matrix

$$A = \begin{bmatrix} 1.1 & 1.3 & 1.4 \\ & 2.2 & & 2.5 \\ 3.1 & & 3.4 \\ & & 4.4 \end{bmatrix}$$
 (15.1)

which can be represented in the column ordered sparse matrix format as

$$\begin{array}{lll} \mathtt{ptrb} &=& [0,2,3,5,7], \\ \mathtt{ptre} &=& [2,3,5,7,8], \\ \mathtt{asub} &=& [0,2,1,0,3,0,2,1], \\ \mathtt{aval} &=& [1.1,3.1,2.2,1.3,4.4,1.4,3.4,2.5]. \end{array}$$

Fig. 15.1 illustrates how the matrix A in (15.1) is represented in column ordered sparse matrix format.

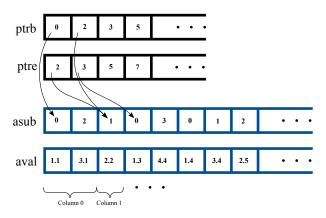


Fig. 15.1: The matrix A (15.1) represented in column ordered packed sparse matrix format.

#### Column ordered sparse format with nonzeros

Note that nzc[j] := ptre[j]-ptrb[j] is exactly the number of nonzero elements in the j-th column of A. In some functions a sparse matrix will be represented using the equivalent dataset asub, aval, ptrb, nzc. The matrix A (15.1) would now be represented as:

```
\begin{array}{lll} \mathtt{ptrb} &=& [0,2,3,5,7], \\ \mathtt{nzc} &=& [2,1,2,2,1], \\ \mathtt{asub} &=& [0,2,1,0,3,0,2,1], \\ \mathtt{aval} &=& [1.1,3.1,2.2,1.3,4.4,1.4,3.4,2.5]. \end{array}
```

## Row ordered sparse matrix

The matrix A (15.1) can also be represented in the row ordered sparse matrix format as:

```
\begin{array}{lll} \mathtt{ptrb} &=& [0,3,5,7],\\ \mathtt{ptre} &=& [3,5,7,8],\\ \mathtt{asub} &=& [0,2,3,1,4,0,3,2],\\ \mathtt{aval} &=& [1.1,1.3,1.4,2.2,2.5,3.1,3.4,4.4]. \end{array}
```

# 15.2 Functions grouped by topic

### **Callback**

- MSK\_linkfunctotaskstream Connects a user-defined function to a task stream.
- MSK\_putcallbackfunc Input the progress callback function.
- $\begin{array}{lll} \bullet & \textit{Infrequent:} & \textit{MSK\_getcallbackfunc}, & \textit{MSK\_linkfunctoenvstream}, & \textit{MSK\_putexitfunc}, \\ & \textit{MSK\_putresponsefunc}, & \textit{MSK\_unlinkfuncfromenvstream}, & \textit{MSK\_unlinkfuncfromtaskstream} \end{array}$

### **Environment and task management**

- MSK\_clonetask Creates a clone of an existing task.
- MSK\_deleteenv Delete a MOSEK environment.
- MSK\_deletetask Deletes a task.
- MSK\_makeenv Creates a new MOSEK environment.
- MSK\_maketask Creates a new task.
- MSK\_puttaskname Assigns a new name to the task.
- $\begin{tabular}{ll} \hline \bullet & Infrequent: & MSK\_commitchinges, & MSK\_delete solution, & MSK\_getenv, & MSK\_makeemptytask, \\ & MSK\_makeenvalloc, & MSK\_putmaxnumanz, & MSK\_putmaxnumbarvar, & MSK\_putmaxnumcon, \\ & MSK\_putmaxnumcone, & MSK\_putmaxnumqnz, & MSK\_putmaxnumvar, & MSK\_resizetask \\ \hline \end{tabular}$

# Infeasibility diagnostic

- MSK\_getinfeasiblesubproblem Obtains an infeasible subproblem.
- MSK\_primalrepair Repairs a primal infeasible optimization problem by adjusting the bounds on the constraints and variables.

# Information items and statistics

- MSK\_getdouinf Obtains a double information item.
- MSK\_getintinf Obtains an integer information item.
- MSK\_getlintinf Obtains a long integer information item.
- $\bullet$  MSK\_updatesolutioninfo Update the information items related to the solution.
- $\begin{array}{lll} \bullet & \mathit{Infrequent:} & \mathit{MSK\_getinfindex}, & \mathit{MSK\_getinfmax}, & \mathit{MSK\_getinfname}, & \mathit{MSK\_getnadouinf}, \\ & \mathit{MSK\_getnaintinf} \end{array}$

# Input/Output

- MSK\_readdata Reads problem data from a file.
- MSK\_writedata Writes problem data to a file.
- MSK\_writesolution Write a solution to a file.
- $\begin{array}{llll} \bullet & Infrequent: & \textit{MSK\_echotask}, & \textit{MSK\_readdataautoformat}, & \textit{MSK\_readdataformat}, \\ & \textit{MSK\_readjsonstring}, & \textit{MSK\_readlpstring}, & \textit{MSK\_readopfstring}, & \textit{MSK\_readparamfile}, \\ & \textit{MSK\_readptfstring}, & \textit{MSK\_readsolution}, & \textit{MSK\_readsummary}, & \textit{MSK\_readtask}, \\ & \textit{MSK\_writejsonsol}, & \textit{MSK\_writeparamfile}, & \textit{MSK\_writetask} \\ \end{array}$

#### Inspecting the task

- MSK\_analyzeproblem Analyze the data of a task.
- MSK\_getnumcon Obtains the number of constraints.
- MSK\_getnumcone Obtains the number of cones.
- MSK\_getnumvar Obtains the number of variables.
- $\bullet \ \textit{Infrequent:} \quad \textit{MSK\_analyze solution}, \ \textit{MSK\_getacol}, \ \textit{MSK\_getacolnumnz}, \ \textit{MSK\_getacolslice}, \\$ MSK\_getacolslice64, MSK\_getacolslicenumnz, MSK\_getacolslicenumnz64, MSK\_getacolslicetrip,  $MSK_{qetaij}$  $\mathit{MSK\_getapiecenumnz}$ , MSK\_getarow.  $MSK\_getarownumnz$ ,  $MSK\_getarowslice$ ,  $MSK\_getarowslice64$ ,  $MSK\_getarowslicenumnz$ , MSK\_getbarablocktriplet,  $MSK\_getbaraidx$ ,  $MSK\_getbaraidxij$ ,  $MSK\_getbaraidxinfo$ ,  $MSK_getbarasparsity$ ,  $MSK\_getbarcidx$ ,  $MSK\_getbarcidx$ ,  $MSK\_getbarcidxinfo$ ,  $MSK\_getbarcidxj$ , MSK\_getbarcsparsity,  $MSK\_getbarvarname$ ,  $MSK\_getbarvarnameindex$ ,  $\mathit{MSK\_getcj}$ , MSK\_getbarvarnamelen,  $\mathit{MSK\_getc}$ ,  $extit{MSK\_getcfix},$ MSK\_getclist, MSK\_getconbound, MSK\_getconboundslice, MSK\_getcone, MSK\_getconeinfo, MSK\_getconename, MSK\_getconenameindex,  $MSK\_getconenamelen$ ,  $MSK_getconname$ ,  $\mathit{MSK}\_getconnameindex$ ,  $MSK\_getconnamelen$ ,  $\mathit{MSK\_getcslice}$ ,  $MSK\_getdimbarvarj$ , MSK\_getlenbarvarj, MSK\_getmaxnamelen, MSK\_getmaxnumanz,  $MSK\_getmaxnumanz64$ , MSK\_qetmaxnumbarvar, MSK\_qetmaxnumcon, MSK\_qetmaxnumcone, MSK\_qetmaxnumqnz,  $MSK\_getmaxnumqnz64$ ,  $\mathit{MSK\_getmaxnumvar}$ ,  $MSK\_getnumanz$ ,  $MSK\_getnumanz64$ ,  $MSK\_getnumbarablocktriplets$ ,  $MSK\_getnumbaranz$ ,  $MSK\_getnumbarcblocktriplets$ ,  $MSK\_getnumbarcnz$ ,  $MSK\_getnumbarvar$ ,  $MSK\_getnumconemem$ ,  $MSK\_getnumintvar$ ,  $\textit{MSK\_getnumqconknz64}$ , MSK\_getnumqobjnz, MSK\_getnumparam,  $MSK\_getnumgconknz$ ,  $MSK\_getnumqobjnz64$ ,  $MSK\_getnumsymmat$ ,  $MSK\_getobjname$ ,  $MSK\_getobjnamelen$ , MSK\_getprobtype, MSK\_getqconk, MSK\_getqconk64, MSK\_getqobj, MSK\_getqobj64, MSK\_getqobjij,  $MSK\_getsparsesymmat$ ,  ${\it MSK\_getsymmatinfo}$ ,  $\mathit{MSK\_gettaskname}$ , MSK\_gettasknamelen, MSK\_getvarbound, MSK\_getvarboundslice, MSK\_getvarname, MSK\_getvarnameindex, MSK\_getvarnamelen, MSK\_getvartype, MSK\_getvartypelist, MSK\_printparam, MSK\_probtypetostr, MSK\_readsummary

# License system

- MSK\_checkoutlicense Check out a license feature from the license server ahead of time.
- MSK\_putlicensedebug Enables debug information for the license system.
- MSK\_putlicensepath Set the path to the license file.
- *MSK\_putlicensewait* Control whether mosek should wait for an available license if no license is available.
- $\begin{array}{ll} \bullet \ \, \textit{Infrequent:} & \textit{MSK\_checkinall}\,, & \textit{MSK\_checkinlicense}\,, & \textit{MSK\_licensecleanup}\,, \\ \textit{MSK\_putlicensecode} & \\ \end{array}$

# Linear algebra

• Infrequent: MSK\_axpy, MSK\_computesparsecholesky, MSK\_dot, MSK\_gemm, MSK\_gemv, MSK\_potrf, MSK\_sparsetriangularsolvedense, MSK\_syeiq, MSK\_syevd, MSK\_syrk

## Logging

- MSK\_linkfiletotaskstream Directs all output from a task stream to a file.
- MSK\_linkfunctotaskstream Connects a user-defined function to a task stream.
- MSK\_onesolutionsummary Prints a short summary of a specified solution.
- MSK\_optimizersummary Prints a short summary with optimizer statistics from last optimization.
- MSK\_solutionsummary Prints a short summary of the current solutions.
- $\begin{array}{lll} \bullet & Infrequent: & \textit{MSK\_echoenv}, & \textit{MSK\_echointro}, & \textit{MSK\_linkfiletoenvstream}, \\ \textit{MSK\_linkfunctoenvstream}, & \textit{MSK\_printparam}, & \textit{MSK\_putexitfunc}, & \textit{MSK\_putresponsefunc}, \\ \textit{MSK\_unlinkfuncfromenvstream}, & \textit{MSK\_unlinkfuncfromtaskstream} \end{array}$

#### Names

- MSK\_getcodedesc Obtains a short description of a response code.
- MSK\_putbarvarname Sets the name of a semidefinite variable.
- MSK\_putconename Sets the name of a cone.
- MSK\_putconname Sets the name of a constraint.
- MSK\_putobjname Assigns a new name to the objective.
- MSK\_puttaskname Assigns a new name to the task.
- MSK\_putvarname Sets the name of a variable.
- Infrequent:  $MSK_analyzenames$ ,  $MSK_bktostr$ ,  $MSK_callbackcodetostr$ ,  $MSK_conetypetostr$ ,  $MSK_generateconenames$ ,  $MSK_generateconnames$ ,  $MSK_generateconnames$ ,  $MSK_generatevarnames$ ,  $MSK_getbarvarnamelen$ ,  $MSK_getbarvarnamelen$ ,  $MSK_getconename$ ,  $MSK_getconnamelen$ ,  $MSK_getconnamelen$ ,  $MSK_getconnamelen$ ,  $MSK_getconnamelen$ ,  $MSK_getconnamelen$ ,  $MSK_getnastrparam$ ,  $MSK_getobjname$ ,  $MSK_getobjnamelen$ ,  $MSK_getparamname$ ,  $MSK_getstrparam$ ,  $MSK_getstrparamlen$ ,  $MSK_getstrparamlen$ ,  $MSK_getstrparamlen$ ,  $MSK_getvarnamelen$ ,  $MSK_getvarname$ ,  $MSK_ge$

#### **Optimization**

- MSK\_optimize Optimizes the problem.
- MSK\_optimizetrm Optimizes the problem.

#### **Parameters**

- MSK\_putdouparam Sets a double parameter.
- MSK\_putintparam Sets an integer parameter.
- MSK\_putparam Modifies the value of parameter.
- MSK\_putstrparam Sets a string parameter.
- MSK\_setdefaults Resets all parameter values.

#### Problem data - bounds

- MSK\_putconbound Changes the bound for one constraint.
- MSK\_putconboundslice Changes the bounds for a slice of the constraints.
- MSK\_putvarbound Changes the bounds for one variable.
- MSK\_putvarboundslice Changes the bounds for a slice of the variables.
- $\begin{tabular}{ll} \bf & Infrequent: MSK\_chgconbound, MSK\_chgvarbound, MSK\_getconbound, MSK\_getconboundslice, \\ MSK\_getvarbound, & MSK\_getvarboundslice, & MSK\_inputdata, & MSK\_inputdata64, \\ MSK\_putconboundlist, & MSK\_putconboundlistconst, & MSK\_putconboundsliceconst, \\ MSK\_putvarboundlist, & MSK\_putvarboundlistconst, & MSK\_putvarboundsliceconst \\ \end{tabular}$

#### Problem data - cones

- MSK\_appendcone Appends a new conic constraint to the problem.
- MSK\_appendconesseq Appends multiple conic constraints to the problem.
- MSK\_getnumcone Obtains the number of cones.
- MSK\_putcone Replaces a conic constraint.
- MSK\_putconename Sets the name of a cone.
- MSK\_removecones Removes a number of conic constraints from the problem.
- $\begin{tabular}{ll} \bf MSK\_appendconeseq, & MSK\_generate conenames, & MSK\_getcone, & MSK\_getconeinfo, \\ & MSK\_getconename, & MSK\_getconenameindex, & MSK\_getconenamelen, & MSK\_getmaxnumcone, \\ & MSK\_getnumconemem, & MSK\_putmaxnumcone \\ \end{tabular}$

## Problem data - constraints

- MSK\_appendcons Appends a number of constraints to the optimization task.
- MSK\_getnumcon Obtains the number of constraints.
- MSK\_putconbound Changes the bound for one constraint.
- MSK\_putconboundslice Changes the bounds for a slice of the constraints.
- MSK\_putconname Sets the name of a constraint.

- MSK\_removecons Removes a number of constraints.

#### Problem data - linear part

- MSK\_appendcons Appends a number of constraints to the optimization task.
- MSK\_appendvars Appends a number of variables to the optimization task.
- MSK\_getnumcon Obtains the number of constraints.
- MSK\_putacol Replaces all elements in one column of the linear constraint matrix.
- MSK\_putacolslice Replaces all elements in a sequence of columns the linear constraint matrix.
- MSK\_putacolslice64 Replaces all elements in a sequence of columns the linear constraint matrix.
- MSK\_putaij Changes a single value in the linear coefficient matrix.
- MSK\_putaijlist Changes one or more coefficients in the linear constraint matrix.
- MSK\_putaijlist64 Changes one or more coefficients in the linear constraint matrix.
- MSK\_putarow Replaces all elements in one row of the linear constraint matrix.
- MSK\_putarowslice Replaces all elements in several rows the linear constraint matrix.
- MSK\_putarowslice64 Replaces all elements in several rows the linear constraint matrix.
- MSK\_putcfix Replaces the fixed term in the objective.
- MSK\_putcj Modifies one linear coefficient in the objective.
- MSK\_putconbound Changes the bound for one constraint.
- MSK\_putconboundslice Changes the bounds for a slice of the constraints.
- MSK\_putconname Sets the name of a constraint.
- MSK\_putcslice Modifies a slice of the linear objective coefficients.
- MSK\_putobjname Assigns a new name to the objective.
- MSK\_putobjsense Sets the objective sense.
- MSK\_putvarbound Changes the bounds for one variable.
- MSK\_putvarboundslice Changes the bounds for a slice of the variables.
- MSK\_putvarname Sets the name of a variable.
- MSK\_removecons Removes a number of constraints.
- MSK\_removevars Removes a number of variables.
- Infrequent: MSK\_chgconbound, MSK\_chgvarbound, MSK\_generateconnames, MSK\_generatevarnames, MSK\_getacol, MSK\_getacolnumnz, MSK\_getacolslice, MSK\_getacolslice64, MSK\_getacolslicenumnz, MSK\_getacolslicenumnz64, MSK\_getacolslicetrip, MSK\_getaij, MSK\_getapiecenumnz, MSK\_getarow, MSK\_getarownumnz, MSK\_getarowslice, MSK\_getarowslice64, MSK\_getarowslicenumnz, MSK\_getarowslicenumnz64, MSK\_getarowslicetrip, MSK\_getaruncatetol, MSK\_getc, MSK\_getcfix, MSK\_getcj, MSK\_getclist, MSK\_getconbound, MSK\_getconboundslice,

```
MSK\_getconname,
                      \mathit{MSK\_getconnameindex},
                                                      \mathit{MSK\_getconnamelen},
                                                                                    MSK_getcslice,
\textit{MSK\_getmaxnumanz}, \qquad \textit{MSK\_getmaxnumanz64}, \qquad \textit{MSK\_getmaxnumcon}, \qquad \textit{MSK\_getmaxnumvar},
MSK\_getnumanz,
                         MSK\_getnumanz64, MSK\_getobjsense,
                                                                                MSK\_getvarbound,
MSK\_getvarboundslice, MSK\_getvarname, MSK\_getvarnameindex, MSK\_getvarnamelen,
\mathit{MSK\_inputdata}, \mathit{MSK\_inputdata64}, \mathit{MSK\_putacollist}, \mathit{MSK\_putacollist64}, \mathit{MSK\_putarowlist},
MSK_putarowlist64, MSK_putatruncatetol,
                                                       {\it MSK\_putclist}\,, \qquad {\it MSK\_putconboundlist}\,,
                                                                            \textit{MSK\_putvarboundlist},
                                  \mathit{MSK\_putconboundsliceconst},
MSK_putconboundlistconst,
\mathit{MSK\_putvarboundlistconst}, \mathit{MSK\_putvarboundsliceconst}
```

## Problem data - objective

- MSK\_putbarcj Changes one element in barc.
- MSK\_putcfix Replaces the fixed term in the objective.
- MSK\_putcj Modifies one linear coefficient in the objective.
- MSK\_putcslice Modifies a slice of the linear objective coefficients.
- MSK\_putobjname Assigns a new name to the objective.
- MSK\_putobjsense Sets the objective sense.
- MSK\_putqobj Replaces all quadratic terms in the objective.
- MSK\_putqobjij Replaces one coefficient in the quadratic term in the objective.
- Infrequent: MSK\_putclist

# Problem data - quadratic part

- MSK\_putqcon Replaces all quadratic terms in constraints.
- MSK\_putqconk Replaces all quadratic terms in a single constraint.
- MSK\_putqobj Replaces all quadratic terms in the objective.
- MSK\_putqobjij Replaces one coefficient in the quadratic term in the objective.
- $\begin{array}{lll} \bullet & Infrequent: & \textit{MSK\_getmaxnumqnz}, & \textit{MSK\_getmaxnumqnz64}, & \textit{MSK\_getnumqconknz}, \\ \textit{MSK\_getnumqconknz64}, & \textit{MSK\_getnumqobjnz}, & \textit{MSK\_getnumqobjnz64}, & \textit{MSK\_getqconk}, \\ \textit{MSK\_getqconk64}, & \textit{MSK\_getqobj}, & \textit{MSK\_getqobj64}, & \textit{MSK\_getqobjij}, & \textit{MSK\_putmaxnumqnz}, \\ \textit{MSK\_toconic} \end{array}$

# Problem data - semidefinite

- MSK\_appendbarvars Appends semidefinite variables to the problem.
- MSK\_appendsparsesymmat Appends a general sparse symmetric matrix to the storage of symmetric matrices.
- MSK\_appendsparsesymmatlist Appends a general sparse symmetric matrix to the storage of symmetric matrices.
- MSK\_putbaraij Inputs an element of barA.
- MSK\_putbaraijlist Inputs list of elements of barA.
- MSK\_putbararowlist Replace a set of rows of barA
- MSK\_putbarcj Changes one element in barc.
- MSK\_putbarvarname Sets the name of a semidefinite variable.

• Infrequent: MSK\_getbarablocktriplet, MSK\_getbaraidx, MSK\_getbaraidxij, MSK\_getbaraidxinfo, MSK\_getbarasparsity, MSK\_getbarcblocktriplet, MSK\_getbarcidx, MSK\_getbarcidxinfo, MSK\_getbarcidxj, MSK\_getbarcsparsity, MSK\_getdimbarvarj, MSK\_getlenbarvarj, MSK\_getmaxnumbarvar, MSK\_getnumbarablocktriplets, MSK\_getnumbaranz, MSK\_getnumbarcblocktriplets, MSK\_getnumbarcnz, MSK\_getnumbarvar, MSK\_getnumsymmat, MSK\_getsparsesymmat, MSK\_getsymmatinfo, MSK\_putbarablocktriplet, MSK\_putbarcblocktriplet, MSK\_putmaxnumanz, MSK\_putmaxnumbarvar, MSK\_removebarvars

#### Problem data - variables

- MSK\_appendvars Appends a number of variables to the optimization task.
- MSK\_getnumvar Obtains the number of variables.
- MSK\_putvarbound Changes the bounds for one variable.
- MSK\_putvarboundslice Changes the bounds for a slice of the variables.
- MSK\_putvarname Sets the name of a variable.
- MSK\_putvartype Sets the variable type of one variable.
- MSK\_removevars Removes a number of variables.
- Infrequent: MSK\_chgvarbound, MSK\_generatevarnames, MSK\_getc, MSK\_getcj, MSK\_getmaxnumvar, MSK\_getnumintvar, MSK\_getvarbound, MSK\_getvarboundslice, MSK\_getvarname, MSK\_getvarnameindex, MSK\_getvarnamelen, MSK\_getvartype, MSK\_getvartypelist, MSK\_putclist, MSK\_putmaxnumvar, MSK\_putvarboundlist, MSK\_putvarboundlistconst, MSK\_putvarboundsliceconst, MSK\_putvartypelist

# Remote optimization

- $MSK\_asyncgetresult$  Request a response from a remote job.
- $\bullet$   ${\it MSK\_asyncoptimize}$  Offload the optimization task to a solver server.
- MSK\_asyncpoll Requests information about the status of the remote job.
- MSK\_asyncstop Request that the job identified by the token is terminated.
- $\bullet$  MSK\_optimizermt Offload the optimization task to a solver server.
- MSK\_putoptserverhost Specify an OptServer for remote calls.

# Responses, errors and warnings

- MSK\_getcodedesc Obtains a short description of a response code.
- $\bullet \ \textit{Infrequent: MSK\_getlasterror}, \ \textit{MSK\_getlasterror64}, \ \textit{MSK\_getresponseclass}$

# Sensitivity analysis

- MSK\_dualsensitivity Performs sensitivity analysis on objective coefficients.
- MSK\_primalsensitivity Perform sensitivity analysis on bounds.
- MSK\_sensitivityreport Creates a sensitivity report.

### Solution - dual

- MSK\_getdualobj Computes the dual objective value associated with the solution.
- MSK\_gety Obtains the y vector for a solution.
- MSK\_getyslice Obtains a slice of the y vector for a solution.
- Infrequent: MSK\_getreducedcosts, MSK\_getslc, MSK\_getslcslice, MSK\_getslx, MSK\_getslxslice, MSK\_getsnx, MSK\_getsnxslice, MSK\_getsolution, MSK\_getsolutionslice, MSK\_getsuc, MSK\_getsucslice, MSK\_getsux, MSK\_getsuxslice, MSK\_putconsolutioni, MSK\_putslc, MSK\_putslcslice, MSK\_putslx, MSK\_putslxslice, MSK\_putsnx, MSK\_putsnxslice, MSK\_putsolution, MSK\_putsolutionyi, MSK\_putsuc, MSK\_putsucslice, MSK\_putsux, MSK\_putsuxslice, MSK\_putvarsolutionj, MSK\_putyslice

#### Solution - primal

- MSK\_getprimalobj Computes the primal objective value for the desired solution.
- MSK\_qetxx Obtains the xx vector for a solution.
- MSK\_getxxslice Obtains a slice of the xx vector for a solution.
- MSK\_putxx Sets the xx vector for a solution.
- MSK\_putxxslice Sets a slice of the xx vector for a solution.
- $\begin{array}{lll} \bullet & Infrequent: & \textit{MSK\_getsolution}, & \textit{MSK\_getsolutionslice}, & \textit{MSK\_getxc}, & \textit{MSK\_getxcslice}, \\ & \textit{MSK\_putconsolutioni}, & \textit{MSK\_putsolution}, & \textit{MSK\_putvarsolutionj}, & \textit{MSK\_putxc}, \\ & \textit{MSK\_putxcslice}, & \textit{MSK\_puty} \end{array}$

# Solution - semidefinite

- MSK\_getbarsj Obtains the dual solution for a semidefinite variable.
- MSK\_getbarsslice Obtains the dual solution for a sequence of semidefinite variables.
- MSK\_qetbarxj Obtains the primal solution for a semidefinite variable.
- MSK\_getbarxslice Obtains the primal solution for a sequence of semidefinite variables.
- Infrequent: MSK\_putbarsj, MSK\_putbarxj

# Solution information

- MSK\_getdualobj Computes the dual objective value associated with the solution.
- MSK\_getprimalobj Computes the primal objective value for the desired solution.
- MSK\_getprosta Obtains the problem status.
- MSK\_getpviolcon Computes the violation of a primal solution associated to a constraint.
- MSK\_qetpviolvar Computes the violation of a primal solution for a list of scalar variables.
- MSK\_getsolsta Obtains the solution status.
- $\bullet$   $\textit{MSK\_getsolutioninfo}$  Obtains information about of a solution.
- MSK\_onesolutionsummary Prints a short summary of a specified solution.
- MSK\_solutiondef Checks whether a solution is defined.
- MSK\_solutionsummary Prints a short summary of the current solutions.

• Infrequent: MSK\_analyzesolution, MSK\_deletesolution, MSK\_getdualsolutionnorms, MSK\_getduiolbarvar, MSK\_getduiolcon, MSK\_getduiolcones, MSK\_getduiolvar, MSK\_getprimalsolutionnorms, MSK\_getpviolbarvar, MSK\_getpviolcones, MSK\_getskc, MSK\_getskcslice, MSK\_getskn, MSK\_getskx, MSK\_getskxslice, MSK\_getsolution, MSK\_getsolutionslice, MSK\_prostatostr, MSK\_putconsolutioni, MSK\_putskc, MSK\_putskcslice, MSK\_putskx, MSK\_putskxslice, MSK\_putsolutionyi, MSK\_putvarsolutionj

# Solving systems with basis matrix

• Infrequent: MSK\_basiscond, MSK\_initbasissolve, MSK\_solvewithbasis

### System, memory and debugging

 $\begin{tabular}{ll} \begin{tabular}{ll} & & & & & & & & & \\ & & & & & & & & \\ & & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & \\ & & & \\$ 

#### **Versions**

- MSK\_getversion Obtains MOSEK version information.
- ullet Infrequent: MSK\_checkversion, MSK\_getbuildinfo

#### Other

• Infrequent: MSK\_isinfinity

# 15.3 Functions in alphabetical order

MSK\_analyzenames

```
MSKrescodee (MSKAPI MSK_analyzenames) (
MSKtask_t task,
MSKstreamtypee whichstream,
MSKnametypee nametype)
```

The function analyzes the names and issues an error if a name is invalid.

#### **Parameters**

- task  $(\textit{MSKtask\_t})$  An optimization task. (input)
- whichstream (MSKstreamtypee) Index of the stream. (input)
- nametype (MSKnametypee) The type of names e.g. valid in MPS or LP files. (input)

Return (MSKrescodee) - The function response code. Groups Names

MSK\_analyzeproblem

```
MSKrescodee (MSKAPI MSK_analyzeproblem) (
MSKtask_t task,
MSKstreamtypee whichstream)
```

The function analyzes the data of a task and writes out a report.

#### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- whichstream (MSKstreamtypee) Index of the stream. (input)

Return (MSKrescodee) - The function response code.

Groups Inspecting the task

MSK\_analyzesolution

```
MSKrescodee (MSKAPI MSK_analyzesolution) (
MSKtask_t task,
MSKstreamtypee whichstream,
MSKsoltypee whichsol)
```

Print information related to the quality of the solution and other solution statistics.

By default this function prints information about the largest infeasibilities in the solution, the primal (and possibly dual) objective value and the solution status.

Following parameters can be used to configure the printed statistics:

- MSK\_IPAR\_ANA\_SOL\_BASIS enables or disables printing of statistics specific to the basis solution (condition number, number of basic variables etc.). Default is on.
- MSK\_IPAR\_ANA\_SOL\_PRINT\_VIOLATED enables or disables listing names of all constraints (both primal and dual) which are violated by the solution. Default is off.
- MSK\_DPAR\_ANA\_SOL\_INFEAS\_TOL is the tolerance defining when a constraint is considered violated. If a constraint is violated more than this, it will be listed in the summary.

#### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- whichstream (MSKstreamtypee) Index of the stream. (input)
- whichsol (MSKsoltypee) Selects a solution. (input)

Return (MSKrescodee) - The function response code.

**Groups** Solution information, Inspecting the task

MSK\_appendbarvars

```
MSKrescodee (MSKAPI MSK_appendbarvars) (
MSKtask_t task,
MSKint32t num,
const MSKint32t * dim)
```

Appends positive semidefinite matrix variables of dimensions given by dim to the problem.

# **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- num (MSKint32t) Number of symmetric matrix variables to be appended. (input)
- dim (MSKint32t\*) Dimensions of symmetric matrix variables to be added. (input)

Return (MSKrescodee) - The function response code.

Groups Problem data - semidefinite

MSK\_appendcone

```
MSKrescodee (MSKAPI MSK_appendcone) (
MSKtask_t task,
MSKconetypee ct,
MSKrealt conepar,
MSKint32t nummem,
const MSKint32t * submem)
```

Appends a new conic constraint to the problem. Hence, add a constraint

$$\hat{x} \in \mathcal{K}$$

to the problem, where K is a convex cone.  $\hat{x}$  is a subset of the variables which will be specified by the argument submem. Cone type is specified by ct.

Define

$$\hat{x} = x_{\mathtt{submem}[0]}, \dots, x_{\mathtt{submem}[\mathtt{nummem}-1]}.$$

Depending on the value of ct this function appends one of the constraints:

• Quadratic cone ( $MSK\_CT\_QUAD$ , requires nummem  $\geq 1$ ):

$$\hat{x}_0 \geq \sqrt{\sum_{i=1}^{i < \text{nummem}} \hat{x}_i^2}$$

• Rotated quadratic cone ( $MSK\_CT\_RQUAD$ , requires nummem  $\geq 2$ ):

$$2\hat{x}_0\hat{x}_1 \geq \sum_{i=2}^{i<\text{nummem}} \hat{x}_i^2, \quad \hat{x}_0, \hat{x}_1 \geq 0$$

• Primal exponential cone (MSK\_CT\_PEXP, requires nummem = 3):

$$\hat{x}_0 \ge \hat{x}_1 \exp(\hat{x}_2/\hat{x}_1), \quad \hat{x}_0, \hat{x}_1 \ge 0$$

• Primal power cone ( $MSK\_CT\_PPOW$ , requires nummem  $\geq 2$ ):

$$\hat{x}_0^\alpha \hat{x}_1^{1-\alpha} \geq \sqrt{\sum_{i=2}^{i < \text{nummem}} \hat{x}_i^2}, \quad \hat{x}_0, \hat{x}_1 \geq 0$$

where  $\alpha$  is the cone parameter specified by conepar.

• Dual exponential cone (MSK\_CT\_DEXP, requires nummem = 3):

$$\hat{x}_0 \ge -\hat{x}_2 e^{-1} \exp(\hat{x}_1/\hat{x}_2), \quad \hat{x}_2 \le 0, \hat{x}_0 \ge 0$$

• Dual power cone (MSK\_CT\_DPOW, requires nummem > 2):

$$\left(\frac{\hat{x}_0}{\alpha}\right)^{\alpha} \left(\frac{\hat{x}_1}{1-\alpha}\right)^{1-\alpha} \geq \sqrt{\sum_{i=2}^{i < \text{nummem}} \hat{x}_i^2}, \quad \hat{x}_0, \hat{x}_1 \geq 0$$

where  $\alpha$  is the cone parameter specified by conepar.

• Zero cone  $(MSK\_CT\_ZERO)$ :

$$\hat{x}_i = 0$$
 for all  $i$ 

Please note that the sets of variables appearing in different conic constraints must be disjoint. For an explained code example see Sec. 6.3, Sec. 6.5 or Sec. 6.4.

# Parameters

- task (MSKtask\_t) An optimization task. (input)
- ct (MSKconetypee) Specifies the type of the cone. (input)
- conepar (MSKrealt) For the power cone it denotes the exponent alpha. For other cone types it is unused and can be set to 0. (input)
- nummem (MSKint32t) Number of member variables in the cone. (input)
- submem  $(\mathit{MSKint32t*})$  Variable subscripts of the members in the cone. (input)

 ${\bf Return}~({\it MSKrescodee}\,)$  — The function response code.

Groups Problem data - cones

# MSK\_appendconeseq

```
MSKrescodee (MSKAPI MSK_appendconeseq) (
MSKtask_t task,
MSKconetypee ct,
MSKrealt conepar,
MSKint32t nummem,
MSKint32t j)
```

Appends a new conic constraint to the problem, as in MSK\_appendcone. The function assumes the members of cone are sequential where the first member has index j and the last j+nummem-1.

#### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- ct (MSKconetypee) Specifies the type of the cone. (input)
- conepar (MSKrealt) For the power cone it denotes the exponent alpha. For other cone types it is unused and can be set to 0. (input)
- nummem (MSKint32t) Number of member variables in the cone. (input)
- j (MSKint32t) Index of the first variable in the conic constraint. (input)

Return (MSKrescodee) - The function response code.

Groups Problem data - cones

# MSK\_appendconesseq

```
MSKrescodee (MSKAPI MSK_appendconesseq) (
   MSKtask_t task,
   MSKint32t num,
   const MSKconetypee * ct,
   const MSKrealt * conepar,
   const MSKint32t * nummem,
   MSKint32t j)
```

Appends a number of conic constraints to the problem, as in  $MSK\_appendcone$ . The kth cone is assumed to be of dimension nummem[k]. Moreover, it is assumed that the first variable of the first cone has index j and starting from there the sequentially following variables belong to the first cone, then to the second cone and so on.

#### Parameters

- task (MSKtask\_t) An optimization task. (input)
- num (MSKint32t) Number of cones to be added. (input)
- ct (MSKconetypee\*) Specifies the type of the cone. (input)
- conepar (MSKrealt\*) For the power cone it denotes the exponent alpha. For other cone types it is unused and can be set to 0. (input)
- nummem (MSKint32t\*) Numbers of member variables in the cones. (input)
- j (MSKint32t) Index of the first variable in the first cone to be appended. (input)

```
Return (MSKrescodee) - The function response code.
Groups Problem data - cones
```

MSK\_appendcons

```
MSKrescodee (MSKAPI MSK_appendcons) (
MSKtask_t task,
MSKint32t num)
```

Appends a number of constraints to the model. Appended constraints will be declared free. Please note that **MOSEK** will automatically expand the problem dimension to accommodate the additional constraints.

#### Parameters

- task (MSKtask\_t) An optimization task. (input)
- num (MSKint32t) Number of constraints which should be appended. (input)

Return (MSKrescodee) - The function response code.

Groups Problem data - linear part, Problem data - constraints

MSK\_appendsparsesymmat

```
MSKrescodee (MSKAPI MSK_appendsparsesymmat) (
   MSKtask_t task,
   MSKint32t dim,
   MSKint64t nz,
   const MSKint32t * subi,
   const MSKint32t * subj,
   const MSKrealt * valij,
   MSKint64t * idx)
```

**MOSEK** maintains a storage of symmetric data matrices that is used to build  $\overline{C}$  and  $\overline{A}$ . The storage can be thought of as a vector of symmetric matrices denoted E. Hence,  $E_i$  is a symmetric matrix of certain dimension.

This function appends a general sparse symmetric matrix on triplet form to the vector E of symmetric matrices. The vectors  $\mathtt{subi}$ ,  $\mathtt{subj}$ , and  $\mathtt{valij}$  contains the row subscripts, column subscripts and values of each element in the symmetric matrix to be appended. Since the matrix that is appended is symmetric, only the lower triangular part should be specified. Moreover, duplicates are not allowed.

Observe the function reports the index (position) of the appended matrix in E. This index should be used for later references to the appended matrix.

#### Parameters

- task (MSKtask\_t) An optimization task. (input)
- dim (MSKint32t) Dimension of the symmetric matrix that is appended. (input)
- nz (MSKint64t) Number of triplets. (input)
- subi (MSKint32t\*) Row subscript in the triplets. (input)
- subj (MSKint32t\*) Column subscripts in the triplets. (input)
- valij (MSKrealt\*) Values of each triplet. (input)
- idx (MSKint64t by reference) Unique index assigned to the inputted matrix that can be used for later reference. (output)

Return (MSKrescodee) - The function response code.

Groups Problem data - semidefinite

MSK\_appendsparsesymmatlist

```
MSKrescodee (MSKAPI MSK_appendsparsesymmatlist) (
   MSKtask_t task,
   MSKint32t num,
   const MSKint32t * dims,
   const MSKint64t * nz,
   const MSKint32t * subi,
   const MSKint32t * subj,
   const MSKrealt * valij,
   MSKint64t * idx)
```

**MOSEK** maintains a storage of symmetric data matrices that is used to build  $\overline{C}$  and  $\overline{A}$ . The storage can be thought of as a vector of symmetric matrices denoted E. Hence,  $E_i$  is a symmetric matrix of certain dimension.

This function appends general sparse symmetric matrixes on triplet form to the vector E of symmetric matrices. The vectors  $\mathtt{subi}$ ,  $\mathtt{subj}$ , and  $\mathtt{valij}$  contains the row subscripts, column subscripts and values of each element in the symmetric matrix to be appended. Since the matrix that is appended is symmetric, only the lower triangular part should be specified. Moreover, duplicates are not allowed.

Observe the function reports the index (position) of the appended matrix in E. This index should be used for later references to the appended matrix.

#### Parameters

- task (MSKtask\_t) An optimization task. (input)
- num (MSKint32t) Number of matrixes to append. (input)
- dims (MSKint32t\*) Dimensions of the symmetric matrixes. (input)
- nz (MSKint64t\*) Number of nonzeros for each matrix. (input)
- subi (MSKint32t\*) Row subscript in the triplets. (input)
- subj (MSKint32t\*) Column subscripts in the triplets. (input)
- valij (MSKrealt\*) Values of each triplet. (input)
- idx (MSKint64t\*) Unique index assigned to the inputted matrix that can be used for later reference. (output)

Return (MSKrescodee) - The function response code.

Groups Problem data - semidefinite

#### MSK\_appendvars

```
MSKrescodee (MSKAPI MSK_appendvars) (
MSKtask_t task,
MSKint32t num)
```

Appends a number of variables to the model. Appended variables will be fixed at zero. Please note that **MOSEK** will automatically expand the problem dimension to accommodate the additional variables.

#### Parameters

- task (MSKtask\_t) An optimization task. (input)
- num (MSKint32t) Number of variables which should be appended. (input)

Return (MSKrescodee) - The function response code.

Groups Problem data - linear part, Problem data - variables

MSK\_asyncgetresult

```
MSKrescodee (MSKAPI MSK_asyncgetresult) (
   MSKtask_t task,
   const char * server,
   const char * port,
   const char * token,
   MSKbooleant * respavailable,
   MSKrescodee * resp,
   MSKrescodee * trm)
```

Request a response from a remote job. If successful, solver response, termination code and solutions are retrieved.

### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- server (MSKstring\_t) Name or IP address of the solver server. (input)
- port (MSKstring\_t) Network port of the solver service. (input)
- token  $(MSKstring_t)$  The task token. (input)
- respavailable (MSKbooleant by reference) Indicates if a remote response is available. If this is not true, resp and trm should be ignored. (output)
- resp (MSKrescodee by reference) Is the response code from the remote solver. (output)
- trm (MSKrescodee by reference) Is either MSK\_RES\_OK or a termination response code. (output)

Return (MSKrescodee) - The function response code.

Groups Remote optimization

MSK\_asyncoptimize

```
MSKrescodee (MSKAPI MSK_asyncoptimize) (
   MSKtask_t task,
   const char * server,
   const char * port,
   char * token)
```

Offload the optimization task to a solver server defined by server:port. The call will return immediately and not wait for the result.

If the string parameter  $MSK\_SPAR\_REMOTE\_ACCESS\_TOKEN$  is not blank, it will be passed to the server as authentication.

#### Parameters

- task (MSKtask\_t) An optimization task. (input)
- server (MSKstring\_t) Name or IP address of the solver server (input)
- port (MSKstring\_t) Network port of the solver service (input)
- token (MSKstring\_t) Returns the task token (output)

Return (MSKrescodee) - The function response code.

Groups Remote optimization

MSK\_asyncpoll

```
MSKrescodee (MSKAPI MSK_asyncpoll) (
   MSKtask_t task,
   const char * server,
   const char * port,
   const char * token,
```

```
MSKbooleant * respavailable,
MSKrescodee * resp,
MSKrescodee * trm)
```

Requests information about the status of the remote job.

#### Parameters

- task (MSKtask\_t) An optimization task. (input)
- server (MSKstring\_t) Name or IP address of the solver server (input)
- port (MSKstring\_t) Network port of the solver service (input)
- token (MSKstring\_t) The task token (input)
- respavailable (MSKbooleant by reference) Indicates if a remote response is available. If this is not true, resp and trm should be ignored. (output)
- resp (MSKrescodee by reference) Is the response code from the remote solver. (output)
- trm (MSKrescodee by reference) Is either MSK\_RES\_OK or a termination response code. (output)

Return (MSKrescodee) - The function response code.

Groups Remote optimization

MSK\_asyncstop

```
MSKrescodee (MSKAPI MSK_asyncstop) (
   MSKtask_t task,
   const char * server,
   const char * port,
   const char * token)
```

Request that the job identified by the token is terminated.

# **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- server (MSKstring\_t) Name or IP address of the solver server (input)
- port (MSKstring\_t) Network port of the solver service (input)
- token (MSKstring\_t) The task token (input)

Return (MSKrescodee) - The function response code.

Groups Remote optimization

MSK\_axpy

```
MSKrescodee (MSKAPI MSK_axpy) (
   MSKenv_t env,
   MSKint32t n,
   MSKrealt alpha,
   const MSKrealt * x,
   MSKrealt * y)
```

Adds  $\alpha x$  to y, i.e. performs the update

```
y := \alpha x + y.
```

Note that the result is stored overwriting y. It must not overlap with the other input arrays.

#### **Parameters**

 $\bullet$  env (MSKenv\_t) - The MOSEK environment. (input)

```
n (MSKint32t) - Length of the vectors. (input)
alpha (MSKrealt) - The scalar that multiplies x. (input)
x (MSKrealt*) - The x vector. (input)
y (MSKrealt*) - The y vector. (input/output)
Return (MSKrescodee) - The function response code.
Groups Linear algebra
```

MSK\_basiscond

```
MSKrescodee (MSKAPI MSK_basiscond) (
    MSKtask_t task,
    MSKrealt * nrmbasis,
    MSKrealt * nrminvbasis)
```

If a basic solution is available and it defines a nonsingular basis, then this function computes the 1-norm estimate of the basis matrix and a 1-norm estimate for the inverse of the basis matrix. The 1-norm estimates are computed using the method outlined in [Ste98], pp. 388-391.

By definition the 1-norm condition number of a matrix B is defined as

$$\kappa_1(B) := \|B\|_1 \|B^{-1}\|_1.$$

Moreover, the larger the condition number is the harder it is to solve linear equation systems involving B. Given estimates for  $||B||_1$  and  $||B^{-1}||_1$  it is also possible to estimate  $\kappa_1(B)$ .

#### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- nrmbasis (MSKrealt by reference) An estimate for the 1-norm of the basis. (output)
- nrminvbasis (MSKrealt by reference) An estimate for the 1-norm of the inverse of the basis. (output)

Return (MSKrescodee) - The function response code.

Groups Solving systems with basis matrix

MSK\_bktostr

```
MSKrescodee (MSKAPI MSK_bktostr) (
MSKtask_t task,
MSKboundkeye bk,
char * str)
```

Obtains an identifier string corresponding to a bound key.

#### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- bk (MSKboundkeye) Bound key. (input)
- str (MSKstring\_t) String corresponding to the bound key code bk. (output)

Return (MSKrescodee) - The function response code.

Groups Names

MSK\_callbackcodetostr

```
MSKrescodee (MSKAPI MSK_callbackcodetostr) (
MSKcallbackcodee code,
char * callbackcodestr)
```

Obtains the string representation of a callback code.

#### **Parameters**

- code (MSKcallbackcodee) A callback code. (input)
- $\bullet$  callbackcodestr (MSKstring\_t) String corresponding to the callback code. (output)

 $\textbf{Return} \ \ (\textit{MSKrescodee}) - \textbf{The function response code}.$ 

Groups Names

MSK\_callocdbgenv

```
void * (MSKAPI MSK_callocdbgenv) (
   MSKenv_t env,
   const size_t number,
   const size_t size,
   const char * file,
   const unsigned line)
```

Debug version of MSK\_callocenv.

### **Parameters**

- env (MSKenv\_t) The MOSEK environment. (input)
- number (size\_t) Number of elements. (input)
- size (size\_t) Size of each individual element. (input)
- file (MSKstring\_t) File from which the function is called. (input)
- line (unsigned) Line in the file from which the function is called. (input)

Return (void\*) - A pointer to the memory allocated through the environment.

Groups System, memory and debugging

MSK\_callocdbgtask

```
void * (MSKAPI MSK_callocdbgtask) (
   MSKtask_t task,
   const size_t number,
   const size_t size,
   const char * file,
   const unsigned line)
```

Debug version of MSK\_calloctask.

# Parameters

- task (MSKtask\_t) An optimization task. (input)
- number (size\_t) Number of elements. (input)
- size (size\_t) Size of each individual element. (input)
- file (MSKstring\_t) File from which the function is called. (input)
- line (unsigned) Line in the file from which the function is called. (input)

Return (void\*) - A pointer to the memory allocated through the task.

Groups System, memory and debugging

MSK\_callocenv

```
void * (MSKAPI MSK_callocenv) (
   MSKenv_t env,
   const size_t number,
   const size_t size)
```

Equivalent to calloc i.e. allocate space for an array of length number where each element is of size size.

#### **Parameters**

- env (MSKenv\_t) The MOSEK environment. (input)
- number (size\_t) Number of elements. (input)
- size (size\_t) Size of each individual element. (input)

Return (void\*) - A pointer to the memory allocated through the environment.

Groups System, memory and debugging

#### MSK\_calloctask

```
void * (MSKAPI MSK_calloctask) (
   MSKtask_t task,
   const size_t number,
   const size_t size)
```

Equivalent to calloc i.e. allocate space for an array of length number where each element is of size size.

#### Parameters

- task (MSKtask\_t) An optimization task. (input)
- number (size\_t) Number of elements. (input)
- size (size\_t) Size of each individual element. (input)

Return (void\*) - A pointer to the memory allocated through the task.

**Groups** System, memory and debugging

#### MSK\_checkinall

```
MSKrescodee (MSKAPI MSK_checkinall) (
MSKenv_t env)
```

Check in all unused license features to the license token server.

```
Parameters env (MSKenv_t) – The MOSEK environment. (input) Return (MSKrescodee) – The function response code. Groups License system
```

#### MSK\_checkinlicense

```
MSKrescodee (MSKAPI MSK_checkinlicense) (
MSKenv_t env,
MSKfeaturee feature)
```

Check in a license feature to the license server. By default all licenses consumed by functions using a single environment are kept checked out for the lifetime of the **MOSEK** environment. This function checks in a given license feature back to the license server immediately.

If the given license feature is not checked out at all, or it is in use by a call to  $MSK\_optimize$ , calling this function has no effect.

Please note that returning a license to the license server incurs a small overhead, so frequent calls to this function should be avoided.

# Parameters

- env (MSKenv\_t) The MOSEK environment. (input)
- feature (MSK featuree) Feature to check in to the license system. (input)

Return (MSKrescodee) - The function response code.

Groups License system

```
MSKrescodee (MSKAPI MSK_checkmemenv) (
MSKenv_t env,
const char * file,
MSKint32t line)
```

Checks the memory allocated by the environment.

# **Parameters**

- env (MSKenv\_t) The MOSEK environment. (input)
- file (MSKstring\_t) File from which the function is called. (input)
- line (MSKint32t) Line in the file from which the function is called. (input)

Return (MSKrescodee) - The function response code.

Groups System, memory and debugging

 $MSK\_checkmemtask$ 

```
MSKrescodee (MSKAPI MSK_checkmemtask) (
   MSKtask_t task,
   const char * file,
   MSKint32t line)
```

Checks the memory allocated by the task.

#### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- file (MSKstring\_t) File from which the function is called. (input)
- line (MSKint32t) Line in the file from which the function is called. (input)

Return (MSKrescodee) - The function response code.

Groups System, memory and debugging

MSK\_checkoutlicense

```
MSKrescodee (MSKAPI MSK_checkoutlicense) (
MSKenv_t env,
MSKfeaturee feature)
```

Checks out a license feature from the license server. Normally the required license features will be automatically checked out the first time they are needed by the function  $MSK\_optimize$ . This function can be used to check out one or more features ahead of time.

The feature will remain checked out until the environment is deleted or the function <code>MSK\_checkinlicense</code> is called.

If a given feature is already checked out when this function is called, the call has no effect.

#### **Parameters**

- env (MSKenv\_t) The MOSEK environment. (input)
- feature (MSKfeaturee) Feature to check out from the license system. (input)

Return (MSKrescodee) - The function response code.

Groups License system

MSK\_checkversion

```
MSKrescodee (MSKAPI MSK_checkversion) (
   MSKenv_t env,
   MSKint32t major,
   MSKint32t minor,
   MSKint32t revision)
```

Compares the version of the MOSEK DLL with a specified version. Returns  $MSK\_RES\_OK$  if the versions match and one of  $MSK\_RES\_ERR\_NEWER\_DLL$ ,  $MSK\_RES\_ERR\_OLDER\_DLL$  otherwise.

# Parameters

- env (MSKenv\_t) The MOSEK environment. (input)
- major (MSKint32t) Major version number. (input)
- minor (MSKint32t) Minor version number. (input)
- revision (MSKint32t) Revision number. (input)

 ${\bf Return} \ \ ({\tt MSKrescodee}) - {\bf The} \ {\bf function} \ {\bf response} \ {\bf code}.$ 

Groups Versions

MSK\_chgconbound

```
MSKrescodee (MSKAPI MSK_chgconbound) (
MSKtask_t task,
MSKint32t i,
MSKint32t lower,
MSKint32t finite,
MSKrealt value)
```

Changes a bound for one constraint.

If lower is non-zero, then the lower bound is changed as follows:

$$\text{new lower bound} = \left\{ \begin{array}{ll} -\infty, & \texttt{finite} = 0, \\ \texttt{value} & \text{otherwise}. \end{array} \right.$$

Otherwise if lower is zero, then

$$\text{new upper bound} = \left\{ \begin{array}{ll} \infty, & \text{finite} = 0, \\ \text{value} & \text{otherwise}. \end{array} \right.$$

Please note that this function automatically updates the bound key for the bound, in particular, if the lower and upper bounds are identical, the bound key is changed to fixed.

### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- i  $(\mathit{MSKint32t})$  Index of the constraint for which the bounds should be changed. (input)
- lower (MSKint32t) If non-zero, then the lower bound is changed, otherwise the upper bound is changed. (input)
- finite (MSKint32t) If non-zero, then value is assumed to be finite. (input)
- value (MSKrealt) New value for the bound. (input)

Return (MSKrescodee) - The function response code.

Groups Problem data - bounds, Problem data - constraints, Problem data - linear part

MSK\_chgvarbound

```
MSKrescodee (MSKAPI MSK_chgvarbound) (
   MSKtask_t task,
   MSKint32t j,
   MSKint32t lower,
   MSKint32t finite,
   MSKrealt value)
```

Changes a bound for one variable.

If lower is non-zero, then the lower bound is changed as follows:

$$\text{new lower bound} = \left\{ \begin{array}{ll} -\infty, & \text{finite} = 0, \\ \text{value} & \text{otherwise}. \end{array} \right.$$

Otherwise if lower is zero, then

$$\text{new upper bound} = \left\{ \begin{array}{ll} \infty, & \text{finite} = 0, \\ \text{value} & \text{otherwise}. \end{array} \right.$$

Please note that this function automatically updates the bound key for the bound, in particular, if the lower and upper bounds are identical, the bound key is changed to fixed.

#### Parameters

- task (MSKtask\_t) An optimization task. (input)
- j (MSKint32t) Index of the variable for which the bounds should be changed. (input)
- lower (MSKint32t) If non-zero, then the lower bound is changed, otherwise the upper bound is changed. (input)
- finite (MSKint32t) If non-zero, then value is assumed to be finite. (input)
- value (MSKrealt) New value for the bound. (input)

Return (MSKrescodee) - The function response code.

Groups Problem data - bounds, Problem data - variables, Problem data - linear part

MSK\_clonetask

```
MSKrescodee (MSKAPI MSK_clonetask) (
MSKtask_t task,
MSKtask_t * clonedtask)
```

Creates a clone of an existing task copying all problem data and parameter settings to a new task. Callback functions are not copied.

#### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- clonedtask (MSKtask\_t by reference) The cloned task. (output)

Return (MSKrescodee) - The function response code.

Groups Environment and task management

MSK\_commitchanges

```
MSKrescodee (MSKAPI MSK_commitchanges) (
MSKtask_t task)
```

Commits all cached problem changes to the task. It is usually not necessary to call this function explicitly since changes will be committed automatically when required.

```
Parameters task (MSKtask_t) - An optimization task. (input) Return (MSKrescodee) - The function response code.
```

### **Groups** Environment and task management

MSK\_computesparsecholesky

```
MSKrescodee (MSKAPI MSK_computesparsecholesky) (
 MSKenv_t env,
 MSKint32t multithread,
 MSKint32t ordermethod,
 MSKrealt tolsingular,
 MSKint32t n,
 const MSKint32t * anzc,
 const MSKint64t * aptrc,
 const MSKint32t * asubc,
 const MSKrealt * avalc,
 MSKint32t ** perm,
 MSKrealt ** diag,
 MSKint32t ** lnzc,
 MSKint64t ** lptrc,
 MSKint64t * lensubnval,
 MSKint32t ** lsubc,
 MSKrealt ** lvalc)
```

The function computes a Cholesky factorization of a sparse positive semidefinite matrix. Sparsity is exploited during the computations to reduce the amount of space and work required. Both the input and output matrices are represented using the sparse format.

To be precise, given a symmetric matrix  $A \in \mathbb{R}^{n \times n}$  the function computes a nonsingular lower triangular matrix L, a diagonal matrix D and a permutation matrix P such that

$$LL^T - D = PAP^T$$
.

If ordermethod is zero then reordering heuristics are not employed and P is the identity.

If a pivot during the computation of the Cholesky factorization is less than

$$-\rho \cdot \max((PAP^T)_{jj}, 1.0)$$

then the matrix is declared negative semidefinite. On the hand if a pivot is smaller than

$$\rho \cdot \max((PAP^T)_{ij}, 1.0),$$

then  $D_{ij}$  is increased from zero to

$$\rho \cdot \max((PAP^T)_{jj}, 1.0).$$

Therefore, if A is sufficiently positive definite then D will be the zero matrix. Here  $\rho$  is set equal to value of tolsingular.

The function allocates memory for the output arrays. It must be freed by the user with  $MSK\_freeenv$ .

# **Parameters**

- env (MSKenv\_t) The MOSEK environment. (input)
- multithread (MSKint32t) If nonzero then the function may exploit multiple threads. (input)
- ordermethod (MSKint32t) If nonzero, then a sparsity preserving ordering will be employed. (input)
- tolsingular (MSKrealt) A positive parameter controlling when a pivot is declared zero. (input)
- n (MSKint32t) Specifies the order of A. (input)

- anzc  $(\mathit{MSKint32t*})$  anzc[j] is the number of nonzeros in the j-th column of A. (input)
- aptrc (MSKint64t\*) aptrc[j] is a pointer to the first element in column j of A. (input)
- asubc (MSKint32t\*) Row indexes for each column stored in increasing order. (input)
- avalc (MSKrealt\*) The value corresponding to row indexed stored in asubc. (input)
- perm (MSKint32t\* by reference) Permutation array used to specify the permutation matrix P computed by the function. (output)
- diag(MSKrealt\*by reference) The diagonal elements of matrix D. (output)
- lnzc (MSKint32t\* by reference) lnzc[j] is the number of non zero elements in column j of L. (output)
- lptrc (MSKint64t \* by reference) lptrc[j] is a pointer to the first row index and value in column j of L. (output)
- lensubnval (MSKint64t by reference) Number of elements in lsubc and lvalc. (output)
- lsubc (MSKint32t\* by reference) Row indexes for each column stored in increasing order. (output)
- lvalc (MSKrealt\* by reference) The values corresponding to row indexed stored in lsubc. (output)

Return (MSKrescodee) - The function response code.

Groups Linear algebra

MSK\_conetypetostr

```
MSKrescodee (MSKAPI MSK_conetypetostr) (
MSKtask_t task,
MSKconetypee ct,
char * str)
```

Obtains the cone string identifier corresponding to a cone type.

### Parameters

- task (MSKtask\_t) An optimization task. (input)
- ct (MSKconetypee) Specifies the type of the cone. (input)
- str (MSKstring\_t) String corresponding to the cone type code ct. (output)

Return (MSKrescodee) - The function response code.

Groups Names

MSK\_deleteenv

```
MSKrescodee (MSKAPI MSK_deleteenv) (
MSKenv_t * env)
```

Deletes a MOSEK environment and all the data associated with it.

Before calling this function it is a good idea to call the function MSK\_unlinkfuncfromenvstream for each stream that has had a function linked to it.

```
Parameters env (MSKenv_t by reference) - The MOSEK environment. (input/output)
Return (MSKrescodee) - The function response code.
Groups Environment and task management
```

MSK\_deletesolution

```
MSKrescodee (MSKAPI MSK_deletesolution) (
MSKtask_t task,
MSKsoltypee whichsol)
```

Undefine a solution and free the memory it uses.

#### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- whichsol (MSKsoltypee) Selects a solution. (input)

Return (MSKrescodee) - The function response code.

Groups Environment and task management, Solution information

MSK\_deletetask

```
MSKrescodee (MSKAPI MSK_deletetask) (
MSKtask_t * task)
```

Deletes a task.

Parameters task (MSKtask\_t by reference) - An optimization task. (input/output)
Return (MSKrescodee) - The function response code.

Groups Environment and task management

MSK\_dot

```
MSKrescodee (MSKAPI MSK_dot) (
   MSKenv_t env,
   MSKint32t n,
   const MSKrealt * x,
   const MSKrealt * y,
   MSKrealt * xty)
```

Computes the inner product of two vectors x, y of length  $n \geq 0$ , i.e

$$x \cdot y = \sum_{i=1}^{n} x_i y_i.$$

Note that if n = 0, then the result of the operation is 0.

# Parameters

- env (MSKenv\_t) The MOSEK environment. (input)
- n (MSKint32t) Length of the vectors. (input)
- x (MSKrealt\*) The x vector. (input)
- y (MSKrealt\*) The y vector. (input)
- xty (MSKrealt by reference) The result of the inner product between x and y. (output)

 $\mathbf{Return} \ \ (\textit{MSKrescodee}) - \mathbf{The} \ \mathbf{function} \ \mathbf{response} \ \mathbf{code}.$ 

Groups Linear algebra

MSK\_dualsensitivity

```
MSKrescodee (MSKAPI MSK_dualsensitivity) (
MSKtask_t task,
MSKint32t numj,
const MSKint32t * subj,
```

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```
MSKrealt * leftpricej,
MSKrealt * rightpricej,
MSKrealt * leftrangej,
MSKrealt * rightrangej)
```

Calculates sensitivity information for objective coefficients. The indexes of the coefficients to analyze are

```
\{ \text{subj}[i] \mid i = 0, \dots, \text{numj} - 1 \}
```

The type of sensitivity analysis to perform (basis or optimal partition) is controlled by the parameter  $MSK\_IPAR\_SENSITIVITY\_TYPE$ .

For an example, please see Section Example: Sensitivity Analysis.

#### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- numj (MSKint32t) Number of coefficients to be analyzed. Length of subj. (input)
- subj (MSKint32t\*) Indexes of objective coefficients to analyze. (input)
- leftpricej (MSKrealt\*) leftpricej[j] is the left shadow price for the coefficient with index subj[j]. (output)
- rightpricej (MSKrealt\*) rightpricej[j] is the right shadow price for the coefficient with index subj[j]. (output)
- leftrangej (MSKrealt\*) leftrangej[j] is the left range  $\beta_1$  for the coefficient with index subj[j]. (output)
- rightrangej (MSKrealt\*) rightrangej[j] is the right range  $\beta_2$  for the coefficient with index subj[j]. (output)

Return (MSKrescodee) - The function response code.

Groups Sensitivity analysis

MSK\_echoenv

```
MSKrescodee (MSKAPIVA MSK_echoenv) (
MSKenv_t env,
MSKstreamtypee whichstream,
const char * format,
...)
```

Prints a formatted message to the environment stream.

#### Parameters

- env (MSKenv\_t) The MOSEK environment. (input)
- whichstream (MSKstreamtypee) Index of the stream. (input)
- format (MSKstring\_t) Is a valid C format string which matches the arguments in .... (input)
- varnumarg (...) A variable argument list. (input)

 ${\bf Return} \ \ ({\it MSKrescodee}\,) - {\bf The} \ {\bf function} \ {\bf response} \ {\bf code}.$ 

Groups Logging

MSK\_echointro

```
MSKrescodee (MSKAPI MSK_echointro) (
    MSKenv_t env,
    MSKint32t longver)
```

Prints an intro to message stream.

#### Parameters

- env (MSKenv\_t) The MOSEK environment. (input)
- longver (MSKint32t) If non-zero, then the intro is slightly longer. (input)

Return (MSKrescodee) - The function response code.

Groups Logging

MSK\_echotask

```
MSKrescodee (MSKAPIVA MSK_echotask) (
MSKtask_t task,
MSKstreamtypee whichstream,
const char * format,
...)
```

Prints a formatted string to a task stream.

#### Parameters

- task (MSKtask\_t) An optimization task. (input)
- whichstream (MSKstreamtypee) Index of the stream. (input)
- format (MSKstring\_t) Is a valid C format string which matches the arguments in . . . . (input)
- ullet varnumarg (...) Additional arguments (input)

Return (MSKrescodee) - The function response code.

Groups Input/Output

MSK\_freedbgenv

```
void (MSKAPI MSK_freedbgenv) (
   MSKenv_t env,
   void * buffer,
   const char * file,
   const unsigned line)
```

Frees space allocated by MOSEK. Debug version of MSK\_freeenv.

#### **Parameters**

- env (MSKenv\_t) The MOSEK environment. (input)
- buffer (void\*) A pointer. (input/output)
- file (MSKstring\_t) File from which the function is called. (input)
- line (unsigned) Line in the file from which the function is called. (input)

Return (void)

Groups System, memory and debugging

MSK\_freedbgtask

```
void (MSKAPI MSK_freedbgtask) (
   MSKtask_t task,
   void * buffer,
   const char * file,
   const unsigned line)
```

Frees space allocated by MOSEK. Debug version of MSK\_freetask.

### Parameters

```
• task (MSKtask_t) - An optimization task. (input)
```

- buffer (void\*) A pointer. (input/output)
- file (MSKstring\_t) File from which the function is called. (input)
- line (unsigned) Line in the file from which the function is called. (input)

Return (void)

Groups System, memory and debugging

MSK\_freeenv

```
void (MSKAPI MSK_freeenv) (
   MSKenv_t env,
   void * buffer)
```

Frees space allocated by a **MOSEK** function. Must not be applied to the **MOSEK** environment and task.

# Parameters

- env (MSKenv\_t) The MOSEK environment. (input)
- buffer (void\*) A pointer. (input/output)

Return (void)

Groups System, memory and debugging

MSK\_freetask

```
void (MSKAPI MSK_freetask) (
    MSKtask_t task,
    void * buffer)
```

Frees space allocated by a **MOSEK** function. Must not be applied to the **MOSEK** environment and task.

# **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- buffer (void\*) A pointer. (input/output)

Return (void)

Groups System, memory and debugging

MSK\_gemm

```
MSKrescodee (MSKAPI MSK_gemm) (
   MSKenv_t env,
   MSKtransposee transa,
   MSKtransposee transb,
   MSKint32t m,
   MSKint32t k,
   MSKint32t k,
   MSKrealt alpha,
   const MSKrealt * a,
   const MSKrealt * b,
   MSKrealt beta,
   MSKrealt * c)
```

Performs a matrix multiplication plus addition of dense matrices. Given A, B and C of compatible dimensions, this function computes

$$C := \alpha op(A)op(B) + \beta C$$

where  $\alpha, \beta$  are two scalar values. The function op(X) denotes X if transX is  $\textit{MSK\_TRANSPOSE\_NO}$ , or  $X^T$  if set to  $\textit{MSK\_TRANSPOSE\_YES}$ . The matrix C has m rows and n columns, and the other matrices must have compatible dimensions.

The result of this operation is stored in C. It must not overlap with the other input arrays.

### **Parameters**

- env (MSKenv\_t) The MOSEK environment. (input)
- transa (MSKtransposee) Indicates whether the matrix A must be transposed. (input)
- transb (MSKtransposee) Indicates whether the matrix B must be transposed. (input)
- m (MSKint32t) Indicates the number of rows of matrix C. (input)
- n (MSKint32t) Indicates the number of columns of matrix C. (input)
- k (MSKint32t) Specifies the common dimension along which op(A) and op(B) are multiplied. For example, if neither A nor B are transposed, then this is the number of columns in A and also the number of rows in B. (input)
- alpha (MSKrealt) A scalar value multiplying the result of the matrix multiplication. (input)
- a (MSKrealt\*) The pointer to the array storing matrix A in a column-major format. (input)
- b (MSKrealt\*) The pointer to the array storing matrix B in a column-major format. (input)
- beta (MSKrealt) A scalar value that multiplies C. (input)
- c (MSKrealt\*) The pointer to the array storing matrix C in a column-major format. (input/output)

Return (MSKrescodee) - The function response code.

Groups Linear algebra

MSK\_gemv

```
MSKrescodee (MSKAPI MSK_gemv) (
   MSKenv_t env,
   MSKtransposee transa,
   MSKint32t m,
   MSKint32t n,
   MSKrealt alpha,
   const MSKrealt * a,
   const MSKrealt * x,
   MSKrealt beta,
   MSKrealt * y)
```

Computes the multiplication of a scaled dense matrix times a dense vector, plus a scaled dense vector. Precisely, if trans is MSK\_TRANSPOSE\_NO then the update is

$$y := \alpha Ax + \beta y,$$

and if trans is MSK\_TRANSPOSE\_YES then

$$y := \alpha A^T x + \beta y$$
,

where  $\alpha, \beta$  are scalar values and A is a matrix with m rows and n columns.

Note that the result is stored overwriting y. It must not overlap with the other input arrays.

### **Parameters**

- env (MSKenv\_t) The MOSEK environment. (input)
- transa (MSKtransposee) Indicates whether the matrix A must be transposed. (input)

- m (MSKint32t) Specifies the number of rows of the matrix A. (input)
- n (MSKint32t) Specifies the number of columns of the matrix A. (input)
- alpha (MSKrealt) A scalar value multiplying the matrix A. (input)
- a (MSKrealt\*) A pointer to the array storing matrix A in a column-major format. (input)
- x (MSKrealt\*) A pointer to the array storing the vector x. (input)
- beta (MSKrealt) A scalar value multiplying the vector y. (input)
- y (MSKrealt\*) A pointer to the array storing the vector y. (input/output)

Return (MSKrescodee) - The function response code.

Groups Linear algebra

## MSK\_generateconenames

```
MSKrescodee (MSKAPI MSK_generateconenames) (
   MSKtask_t task,
   MSKint32t num,
   const MSKint32t * subk,
   const char * fmt,
   MSKint32t ndims,
   const MSKint32t * dims,
   const MSKint64t * sp)
```

Generates systematic names for cone.

#### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- num (MSKint32t) Number of cone indexes. (input)
- subk (MSKint32t\*) Indexes of the cone. (input)
- fmt  $(MSKstring_t)$  The cone name formatting string. (input)
- ndims (MSKint32t) Number of dimensions in the shape. (input)
- dims (MSKint32t\*) Dimensions in the shape. (input)
- sp (MSKint64t\*) Items that should be named. (input)

Return (MSKrescodee) - The function response code.

Groups Names, Problem data - cones

# MSK\_generateconnames

```
MSKrescodee (MSKAPI MSK_generateconnames) (
   MSKtask_t task,
   MSKint32t num,
   const MSKint32t * subi,
   const char * fmt,
   MSKint32t ndims,
   const MSKint32t * dims,
   const MSKint64t * sp)
```

Generates systematic names for constraints.

### Parameters

- task (MSKtask\_t) An optimization task. (input)
- num (MSKint32t) Number of constraint indexes. (input)
- subi (MSKint32t\*) Indexes of the constraints. (input)
- fmt (MSKstring\_t) The constraint name formatting string. (input)
- ndims (MSKint32t) Number of dimensions in the shape. (input)
- dims (MSKint32t\*) Dimensions in the shape. (input)

```
ullet sp (\mathit{MSKint64t*}) – Items that should be named. (input) Return (\mathit{MSKrescodee}) – The function response code. Groups Names,\ Problem\ data - constraints,\ Problem\ data - linear\ part
```

MSK\_generatevarnames

```
MSKrescodee (MSKAPI MSK_generatevarnames) (
    MSKtask_t task,
    MSKint32t num,
    const MSKint32t * subj,
    const char * fmt,
    MSKint32t ndims,
    const MSKint32t * dims,
    const MSKint64t * sp)
```

Generates systematic names for variables.

#### Parameters

- task (MSKtask\_t) An optimization task. (input)
- num (MSKint32t) Number of variable indexes. (input)
- subj (MSKint32t\*) Indexes of the variables. (input)
- fmt  $(MSKstring_t)$  The variable name formatting string. (input)
- ndims (MSKint32t) Number of dimensions in the shape. (input)
- dims (MSKint32t\*) Dimensions in the shape. (input)
- sp(MSKint64t\*) Items that should be named. (input)

Return (MSKrescodee) - The function response code.

Groups Names, Problem data - variables, Problem data - linear part

MSK\_getacol

```
MSKrescodee (MSKAPI MSK_getacol) (
   MSKtask_t task,
   MSKint32t j,
   MSKint32t * nzj,
   MSKint32t * subj,
   MSKrealt * valj)
```

Obtains one column of A in a sparse format.

## Parameters

- task (MSKtask\_t) An optimization task. (input)
- j (MSKint32t) Index of the column. (input)
- nzj (MSKint32t by reference) Number of non-zeros in the column obtained. (output)
- subj (MSKint32t\*) Row indices of the non-zeros in the column obtained. (output)
- valj (MSKrealt\*) Numerical values in the column obtained. (output)

Return (MSKrescodee) - The function response code.

Groups Problem data - linear part, Inspecting the task

MSK\_getacolnumnz

```
MSKrescodee (MSKAPI MSK_getacolnumnz) (
   MSKtask_t task,
   MSKint32t i,
   MSKint32t * nzj)
```

Obtains the number of non-zero elements in one column of A.

#### Parameters

- task (MSKtask\_t) An optimization task. (input)
- i (MSKint32t) Index of the column. (input)
- nzj (MSKint32t by reference) Number of non-zeros in the j-th column of A. (output)

Return (MSKrescodee) - The function response code.

Groups Problem data - linear part, Inspecting the task

MSK\_getacolslice

```
MSKrescodee (MSKAPI MSK_getacolslice) (
   MSKtask_t task,
   MSKint32t first,
   MSKint32t last,
   MSKint32t maxnumnz,
   MSKint32t * surp,
   MSKint32t * ptrb,
   MSKint32t * ptre,
   MSKint32t * sub,
   MSKrealt * val)
```

Obtains a sequence of columns from A in sparse format.

#### Parameters

- task (MSKtask\_t) An optimization task. (input)
- first (MSKint32t) Index of the first column in the sequence. (input)
- last (MSKint32t) Index of the last column in the sequence plus one. (input)
- maxnumnz (MSKint32t) Denotes the length of the arrays sub and val. (input)
- surp (MSKint32t by reference) Surplus of subscript and coefficient arrays. The required entries are stored sequentially in sub and val starting from position surp away from the end of the arrays. Upon return surp will be decremented by the total number of non-zeros written. (input/output)
- ptrb (MSKint32t\*) ptrb[t] is an index pointing to the first element in the t-th column obtained. (output)
- ptre (MSKint32t\*) ptre[t] is an index pointing to the last element plus one in the t-th column obtained. (output)
- sub (MSKint32t\*) Contains the row subscripts. (output)
- val (MSKrealt\*) Contains the coefficient values. (output)

Return (MSKrescodee) - The function response code.

Groups Problem data - linear part, Inspecting the task

MSK\_getacolslice64

```
MSKrescodee (MSKAPI MSK_getacolslice64) (
MSKtask_t task,
MSKint32t first,
MSKint32t last,
MSKint64t maxnumnz,
MSKint64t * surp,
MSKint64t * ptrb,
MSKint64t * ptre,
MSKint32t * sub,
MSKrealt * val)
```

Obtains a sequence of columns from A in sparse format.

#### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- first (MSKint32t) Index of the first column in the sequence. (input)
- last (MSKint32t) Index of the last column in the sequence plus one. (input)
- maxnumnz (MSKint64t) Denotes the length of the arrays sub and val. (input)
- surp (MSKint64t by reference) Surplus of subscript and coefficient arrays. The required entries are stored sequentially in sub and val starting from position surp away from the end of the arrays. Upon return surp will be decremented by the total number of non-zeros written. (input/output)
- ptrb (MSKint64t\*) ptrb[t] is an index pointing to the first element in the t-th column obtained. (output)
- ptre (MSKint64t\*) ptre[t] is an index pointing to the last element plus one
  in the t-th column obtained. (output)
- sub (MSKint32t\*) Contains the row subscripts. (output)
- ullet val (MSKrealt\*) Contains the coefficient values. (output)

Return (MSKrescodee) - The function response code.

Groups Problem data - linear part, Inspecting the task

## MSK\_getacolslicenumnz

```
MSKrescodee (MSKAPI MSK_getacolslicenumnz) (
   MSKtask_t task,
   MSKint32t first,
   MSKint32t last,
   MSKint32t * numnz)
```

Obtains the number of non-zeros in a slice of columns of A.

### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- first (MSKint32t) Index of the first column in the sequence. (input)
- last (MSKint32t) Index of the last column plus one in the sequence. (input)
- numnz (MSKint32t by reference) Number of non-zeros in the slice. (output)

Return (MSKrescodee) - The function response code.

Groups Problem data - linear part, Inspecting the task

# MSK\_getacolslicenumnz64

```
MSKrescodee (MSKAPI MSK_getacolslicenumnz64) (
MSKtask_t task,
MSKint32t first,
MSKint32t last,
MSKint64t * numnz)
```

Obtains the number of non-zeros in a slice of columns of A.

#### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- first (MSKint32t) Index of the first column in the sequence. (input)
- last (MSKint32t) Index of the last column plus one in the sequence. (input)
- numnz (MSKint64t by reference) Number of non-zeros in the slice. (output)

Return (MSKrescodee) - The function response code.

**Groups** Problem data - linear part, Inspecting the task

```
MSKrescodee (MSKAPI MSK_getacolslicetrip) (
   MSKtask_t task,
   MSKint32t first,
   MSKint32t last,
   MSKint64t maxnumnz,
   MSKint64t * surp,
   MSKint32t * subi,
   MSKint32t * subj,
   MSKrealt * val)
```

Obtains a sequence of columns from A in sparse triplet format. The function returns the content of all columns whose index j satisfies first  $\leq j \leq last$ . The triplets corresponding to nonzero entries are stored in the arrays subj, subj and val.

#### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- first (MSKint32t) Index of the first column in the sequence. (input)
- last (MSKint32t) Index of the last column in the sequence plus one. (input)
- maxnumnz (MSKint64t) Denotes the length of the arrays subi, subj, and val. (input)
- surp (MSKint64t by reference) Surplus of subscript and coefficient arrays. The required entries are stored sequentially in subi, subj and val starting from position surp away from the end of the arrays. On return surp will be decremented by the total number of non-zeros written. (input/output)
- subi (MSKint32t\*) Constraint subscripts. (output)
- subj (MSKint32t\*) Column subscripts. (output)
- val (MSKrealt\*) Values. (output)

Return (MSKrescodee) - The function response code.

Groups Problem data - linear part, Inspecting the task

MSK\_getaij

```
MSKrescodee (MSKAPI MSK_getaij) (
    MSKtask_t task,
    MSKint32t i,
    MSKint32t j,
    MSKrealt * aij)
```

Obtains a single coefficient in A.

# Parameters

- task (MSKtask\_t) An optimization task. (input)
- i (MSKint32t) Row index of the coefficient to be returned. (input)
- j (MSKint32t) Column index of the coefficient to be returned. (input)
- aij (MSKrealt by reference) The required coefficient  $a_{i,j}$ . (output)

Return (MSKrescodee) - The function response code.

Groups Problem data - linear part, Inspecting the task

MSK\_getapiecenumnz

```
MSKrescodee (MSKAPI MSK_getapiecenumnz) (
   MSKtask_t task,
   MSKint32t firsti,
   MSKint32t lasti,
   MSKint32t firstj,
   MSKint32t lastj,
   MSKint32t * numnz)
```

Obtains the number non-zeros in a rectangular piece of A, i.e. the number of elements in the set

```
\{(i,j): a_{i,j} \neq 0, \text{ firsti} \leq i \leq \text{lasti} - 1, \text{ firstj} \leq j \leq \text{lastj} - 1\}
```

This function is not an efficient way to obtain the number of non-zeros in one row or column. In that case use the function  $MSK\_getarownumnz$  or  $MSK\_getacolnumnz$ .

#### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- firsti (MSKint32t) Index of the first row in the rectangular piece. (input)
- lasti (MSKint32t) Index of the last row plus one in the rectangular piece. (input)
- firstj (MSKint32t) Index of the first column in the rectangular piece. (input)
- lastj (MSKint32t) Index of the last column plus one in the rectangular piece. (input)
- numnz (MSKint32t by reference) Number of non-zero A elements in the rectangular piece. (output)

Return (MSKrescodee) - The function response code.

Groups Problem data - linear part, Inspecting the task

MSK\_getarow

```
MSKrescodee (MSKAPI MSK_getarow) (
MSKtask_t task,
MSKint32t i,
MSKint32t * nzi,
MSKint32t * subi,
MSKrealt * vali)
```

Obtains one row of A in a sparse format.

### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- i (MSKint32t) Index of the row. (input)
- nzi (MSKint32t by reference) Number of non-zeros in the row obtained. (output)
- subi (MSKint32t\*) Column indices of the non-zeros in the row obtained. (output)
- vali (MSKrealt\*) Numerical values of the row obtained. (output)

Return (MSKrescodee) - The function response code.

Groups Problem data - linear part, Inspecting the task

MSK\_getarownumnz

```
MSKrescodee (MSKAPI MSK_getarownumnz) (
   MSKtask_t task,
   MSKint32t i,
   MSKint32t * nzi)
```

Obtains the number of non-zero elements in one row of A.

#### Parameters

- task (MSKtask\_t) An optimization task. (input)
- i (MSKint32t) Index of the row. (input)
- nzi (MSKint32t by reference) Number of non-zeros in the i-th row of A. (output)

Return (MSKrescodee) - The function response code.

Groups Problem data - linear part, Inspecting the task

MSK\_getarowslice

```
MSKrescodee (MSKAPI MSK_getarowslice) (
   MSKtask_t task,
   MSKint32t first,
   MSKint32t last,
   MSKint32t maxnumnz,
   MSKint32t * surp,
   MSKint32t * ptrb,
   MSKint32t * ptre,
   MSKint32t * sub,
   MSKrealt * val)
```

Obtains a sequence of rows from A in sparse format.

#### Parameters

- task (MSKtask\_t) An optimization task. (input)
- first (MSKint32t) Index of the first row in the sequence. (input)
- last (MSKint32t) Index of the last row in the sequence plus one. (input)
- maxnumnz (MSKint32t) Denotes the length of the arrays sub and val. (input)
- surp (MSKint32t by reference) Surplus of subscript and coefficient arrays. The required entries are stored sequentially in sub and val starting from position surp away from the end of the arrays. Upon return surp will be decremented by the total number of non-zeros written. (input/output)
- ptrb (MSKint32t\*) ptrb[t] is an index pointing to the first element in the t-th row obtained. (output)
- ptre (MSKint32t\*) ptre[t] is an index pointing to the last element plus one in the t-th row obtained. (output)
- sub (MSKint32t\*) Contains the column subscripts. (output)
- val (MSKrealt\*) Contains the coefficient values. (output)

Return (MSKrescodee) - The function response code.

Groups Problem data - linear part, Inspecting the task

MSK\_getarowslice64

```
MSKrescodee (MSKAPI MSK_getarowslice64) (
MSKtask_t task,
MSKint32t first,
MSKint32t last,
MSKint64t maxnumnz,
MSKint64t * surp,
MSKint64t * ptrb,
MSKint64t * ptre,
MSKint32t * sub,
MSKrealt * val)
```

Obtains a sequence of rows from A in sparse format.

#### Parameters

- task (MSKtask\_t) An optimization task. (input)
- first (MSKint32t) Index of the first row in the sequence. (input)
- last (MSKint32t) Index of the last row in the sequence plus one. (input)
- maxnumnz (MSKint64t) Denotes the length of the arrays sub and val. (input)
- surp (MSKint64t by reference) Surplus of subscript and coefficient arrays. The required entries are stored sequentially in sub and val starting from position surp away from the end of the arrays. Upon return surp will be decremented by the total number of non-zeros written. (input/output)
- ptrb (MSKint64t\*) ptrb[t] is an index pointing to the first element in the t-th row obtained. (output)
- ptre (MSKint64t\*) ptre[t] is an index pointing to the last element plus one in the t-th row obtained. (output)
- sub (MSKint32t\*) Contains the column subscripts. (output)
- val (MSKrealt\*) Contains the coefficient values. (output)

Return (MSKrescodee) - The function response code.

Groups Problem data - linear part, Inspecting the task

## MSK\_getarowslicenumnz

```
MSKrescodee (MSKAPI MSK_getarowslicenumnz) (
    MSKtask_t task,
    MSKint32t first,
    MSKint32t last,
    MSKint32t * numnz)
```

Obtains the number of non-zeros in a slice of rows of A.

### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- first (MSKint32t) Index of the first row in the sequence. (input)
- last (MSKint32t) Index of the last row plus one in the sequence. (input)
- numnz (MSKint32t by reference) Number of non-zeros in the slice. (output)

Return (MSKrescodee) - The function response code.

**Groups** Problem data - linear part, Inspecting the task

# MSK\_getarowslicenumnz64

```
MSKrescodee (MSKAPI MSK_getarowslicenumnz64) (
MSKtask_t task,
MSKint32t first,
MSKint32t last,
MSKint64t * numnz)
```

Obtains the number of non-zeros in a slice of rows of A.

#### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- first (MSKint32t) Index of the first row in the sequence. (input)
- last (MSKint32t) Index of the last row plus one in the sequence. (input)
- numnz (MSKint64t by reference) Number of non-zeros in the slice. (output)

Return (MSKrescodee) - The function response code.

**Groups** Problem data - linear part, Inspecting the task

### MSK\_getarowslicetrip

```
MSKrescodee (MSKAPI MSK_getarowslicetrip) (
   MSKtask_t task,
   MSKint32t first,
   MSKint32t last,
   MSKint64t maxnumnz,
   MSKint64t * surp,
   MSKint32t * subi,
   MSKint32t * subj,
   MSKrealt * val)
```

Obtains a sequence of rows from A in sparse triplet format. The function returns the content of all rows whose index i satisfies first <= i < last. The triplets corresponding to nonzero entries are stored in the arrays subi, subj and val.

#### Parameters

- task (MSKtask\_t) An optimization task. (input)
- first (MSKint32t) Index of the first row in the sequence. (input)
- last (MSKint32t) Index of the last row in the sequence plus one. (input)
- maxnumnz (MSKint64t) Denotes the length of the arrays subi, subj, and val. (input)
- surp (MSKint64t by reference) Surplus of subscript and coefficient arrays. The required entries are stored sequentially in subi, subj and val starting from position surp away from the end of the arrays. On return surp will be decremented by the total number of non-zeros written. (input/output)
- subi (MSKint32t\*) Constraint subscripts. (output)
- subj (MSKint32t\*) Column subscripts. (output)
- val (MSKrealt\*) Values. (output)

Return (MSKrescodee) - The function response code.

Groups Problem data - linear part, Inspecting the task

# MSK\_getatruncatetol

```
MSKrescodee (MSKAPI MSK_getatruncatetol) (
MSKtask_t task,
MSKrealt * tolzero)
```

Obtains the tolerance value set with  $\textit{MSK\_putatruncatetol}$ .

# Parameters

- task (MSKtask\_t) An optimization task. (input)
- tolzero (MSKrealt\*) All elements  $|a_{i,j}|$  less than this tolerance is truncated to zero. (output)

Return (MSKrescodee) - The function response code.

Groups Parameters, Problem data - linear part

# ${\tt MSK\_getbarablocktriplet}$

```
MSKrescodee (MSKAPI MSK_getbarablocktriplet) (
   MSKtask_t task,
   MSKint64t maxnum,
   MSKint64t * num,
   MSKint32t * subi,
   MSKint32t * subj,
```

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```
MSKint32t * subk,
MSKint32t * subl,
MSKrealt * valijkl)
```

Obtains  $\overline{A}$  in block triplet form.

#### Parameters

- task (MSKtask\_t) An optimization task. (input)
- maxnum (MSKint64t) subi, subj, subk, subl and valijkl must be maxnum long. (input)
- num (MSKint64t by reference) Number of elements in the block triplet form. (output)
- subi (MSKint32t\*) Constraint index. (output)
- subj (MSKint32t\*) Symmetric matrix variable index. (output)
- subk (MSKint32t\*) Block row index. (output)
- subl (MSKint32t\*) Block column index. (output)
- • valijkl (MSKrealt\*) – The numerical value associated with each block triplet. (output)

Return (MSKrescodee) - The function response code.

Groups Problem data - semidefinite, Inspecting the task

MSK\_getbaraidx

```
MSKrescodee (MSKAPI MSK_getbaraidx) (
MSKtask_t task,
MSKint64t idx,
MSKint64t maxnum,
MSKint32t * i,
MSKint32t * j,
MSKint64t * num,
MSKint64t * sub,
MSKrealt * weights)
```

Obtains information about an element in  $\overline{A}$ . Since  $\overline{A}$  is a sparse matrix of symmetric matrices, only the nonzero elements in  $\overline{A}$  are stored in order to save space. Now  $\overline{A}$  is stored vectorized i.e. as one long vector. This function makes it possible to obtain information such as the row index and the column index of a particular element of the vectorized form of  $\overline{A}$ .

Please observe if one element of  $\overline{A}$  is inputted multiple times then it may be stored several times in vectorized form. In that case the element with the highest index is the one that is used.

### Parameters

- task (MSKtask\_t) An optimization task. (input)
- idx (MSKint64t) Position of the element in the vectorized form. (input)
- maxnum (MSKint64t) sub and weights must be at least maxnum long. (input)
- i (MSKint32t by reference) Row index of the element at position idx. (output)
- j (MSKint32t by reference) Column index of the element at position idx. (output)
- num (MSKint64t by reference) Number of terms in weighted sum that forms the element. (output)
- sub (MSKint64t\*) A list indexes of the elements from symmetric matrix storage that appear in the weighted sum. (output)
- weights (MSKrealt\*) The weights associated with each term in the weighted sum. (output)

Return (MSKrescodee) - The function response code.

# **Groups** Problem data - semidefinite, Inspecting the task

MSK\_getbaraidxij

```
MSKrescodee (MSKAPI MSK_getbaraidxij) (
   MSKtask_t task,
   MSKint64t idx,
   MSKint32t * i,
   MSKint32t * j)
```

Obtains information about an element in  $\overline{A}$ . Since  $\overline{A}$  is a sparse matrix of symmetric matrices, only the nonzero elements in  $\overline{A}$  are stored in order to save space. Now  $\overline{A}$  is stored vectorized i.e. as one long vector. This function makes it possible to obtain information such as the row index and the column index of a particular element of the vectorized form of  $\overline{A}$ .

Please note that if one element of  $\overline{A}$  is inputted multiple times then it may be stored several times in vectorized form. In that case the element with the highest index is the one that is used.

#### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- idx (MSKint64t) Position of the element in the vectorized form. (input)
- i (MSKint32t by reference) Row index of the element at position idx. (output)
- j (MSKint32t by reference) Column index of the element at position idx. (output)

Return (MSKrescodee) - The function response code.

Groups Problem data - semidefinite, Inspecting the task

MSK\_getbaraidxinfo

```
MSKrescodee (MSKAPI MSK_getbaraidxinfo) (
MSKtask_t task,
MSKint64t idx,
MSKint64t * num)
```

Each nonzero element in  $\overline{A}_{ij}$  is formed as a weighted sum of symmetric matrices. Using this function the number of terms in the weighted sum can be obtained. See description of  $MSK\_appendsparsesymmat$  for details about the weighted sum.

#### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- idx (MSKint64t) The internal position of the element for which information should be obtained. (input)
- num  $(MSKint64t \ by \ reference)$  Number of terms in the weighted sum that form the specified element in  $\overline{A}$ . (output)

 ${\bf Return} \ \ ({\it MSKrescodee}\,) - {\bf The} \ {\bf function} \ {\bf response} \ {\bf code}.$ 

Groups Problem data - semidefinite, Inspecting the task

MSK\_getbarasparsity

```
MSKrescodee (MSKAPI MSK_getbarasparsity) (
   MSKtask_t task,
   MSKint64t maxnumnz,
   MSKint64t * numnz,
   MSKint64t * idxij)
```

The matrix  $\overline{A}$  is assumed to be a sparse matrix of symmetric matrices. This implies that many of the elements in  $\overline{A}$  are likely to be zero matrices. Therefore, in order to save space, only nonzero elements in  $\overline{A}$  are stored on vectorized form. This function is used to obtain the sparsity pattern of  $\overline{A}$  and the position of each nonzero element in the vectorized form of  $\overline{A}$ . From the index detailed information about each nonzero  $\overline{A}_{i,j}$  can be obtained using  $MSK\_getbaraidxinfo$  and  $MSK\_getbaraidx$ .

### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- maxnumnz (MSKint64t) The array idxij must be at least maxnumnz long. (input)
- numnz (MSKint64t by reference) Number of nonzero elements in  $\overline{A}$ . (output)
- idxij (MSKint64t\*) Position of each nonzero element in the vectorized form of  $\overline{A}$ . (output)

 ${\bf Return} \ \ ({\it MSKrescodee}\,) - {\bf The} \ {\bf function} \ {\bf response} \ {\bf code}.$ 

**Groups** Problem data - semidefinite, Inspecting the task

### MSK\_getbarcblocktriplet

```
MSKrescodee (MSKAPI MSK_getbarcblocktriplet) (
   MSKtask_t task,
   MSKint64t maxnum,
   MSKint64t * num,
   MSKint32t * subj,
   MSKint32t * subk,
   MSKint32t * subl,
   MSKrealt * valjkl)
```

Obtains  $\overline{C}$  in block triplet form.

#### Parameters

- task (MSKtask\_t) An optimization task. (input)
- maxnum (MSKint64t) subj, subk, subl and valjkl must be maxnum long. (input)
- num (MSKint64t by reference) Number of elements in the block triplet form. (output)
- subj (MSKint32t\*) Symmetric matrix variable index. (output)
- subk (MSKint32t\*) Block row index. (output)
- subl (MSKint32t\*) Block column index. (output)
- valjkl (MSKrealt\*) The numerical value associated with each block triplet. (output)

Return (MSKrescodee) - The function response code.

 ${\bf Groups}\ {\it Problem\ data\ -\ semidefinite,\ Inspecting\ the\ task}$ 

# ${\tt MSK\_getbarcidx}$

```
MSKrescodee (MSKAPI MSK_getbarcidx) (
MSKtask_t task,
MSKint64t idx,
MSKint64t maxnum,
MSKint32t * j,
MSKint64t * num,
MSKint64t * sub,
MSKrealt * weights)
```

Obtains information about an element in  $\overline{C}$ .

#### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- idx (MSKint64t) Index of the element for which information should be obtained. (input)
- maxnum (MSKint64t) sub and weights must be at least maxnum long. (input)
- j (MSKint32t by reference) Row index in  $\overline{C}$ . (output)
- num (MSKint64t by reference) Number of terms in the weighted sum. (output)
- sub (MSKint64t\*) Elements appearing the weighted sum. (output)
- weights (MSKrealt\*) Weights of terms in the weighted sum. (output)

Return (MSKrescodee) - The function response code.

Groups Problem data - semidefinite, Inspecting the task

MSK\_getbarcidxinfo

```
MSKrescodee (MSKAPI MSK_getbarcidxinfo) (
MSKtask_t task,
MSKint64t idx,
MSKint64t * num)
```

Obtains the number of terms in the weighted sum that forms a particular element in  $\overline{C}$ .

#### Parameters

- task (MSKtask\_t) An optimization task. (input)
- idx (MSKint64t) Index of the element for which information should be obtained. The value is an index of a symmetric sparse variable. (input)
- num (MSKint64t by reference) Number of terms that appear in the weighted sum that forms the requested element. (output)

Return (MSKrescodee) - The function response code.

**Groups** Problem data - semidefinite, Inspecting the task

MSK\_getbarcidxj

```
MSKrescodee (MSKAPI MSK_getbarcidxj) (
   MSKtask_t task,
   MSKint64t idx,
   MSKint32t * j)
```

Obtains the row index of an element in  $\overline{C}$ .

#### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- idx (MSKint64t) Index of the element for which information should be obtained. (input)
- j (MSKint32t by reference) Row index in  $\overline{C}$ . (output)

Return (MSKrescodee) - The function response code.

Groups Problem data - semidefinite, Inspecting the task

MSK\_getbarcsparsity

```
MSKrescodee (MSKAPI MSK_getbarcsparsity) (
MSKtask_t task,
MSKint64t maxnumnz,
MSKint64t * numnz,
MSKint64t * idxj)
```

Internally only the nonzero elements of  $\overline{C}$  are stored in a vector. This function is used to obtain the nonzero elements of  $\overline{C}$  and their indexes in the internal vector representation (in idx). From the index detailed information about each nonzero  $\overline{C}_j$  can be obtained using  $MSK\_getbarcidxinfo$  and  $MSK\_getbarcidx$ .

#### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- maxnumnz (MSKint64t) idxj must be at least maxnumnz long. (input)
- numnz (MSKint64t by reference) Number of nonzero elements in  $\overline{C}$ . (output)
- idxj (MSKint64t\*) Internal positions of the nonzeros elements in  $\overline{C}$ . (output)

Return (MSKrescodee) - The function response code.

Groups Problem data - semidefinite, Inspecting the task

MSK\_getbarsj

```
MSKrescodee (MSKAPI MSK_getbarsj) (
MSKtask_t task,
MSKsoltypee whichsol,
MSKint32t j,
MSKrealt * barsj)
```

Obtains the dual solution for a semidefinite variable. Only the lower triangular part of  $\overline{S}_j$  is returned because the matrix by construction is symmetric. The format is that the columns are stored sequentially in the natural order.

#### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- whichsol (MSKsoltypee) Selects a solution. (input)
- j (MSKint32t) Index of the semidefinite variable. (input)
- barsj (MSKrealt\*) Value of  $\overline{S}_i$ . (output)

Return (MSKrescodee) - The function response code.

Groups Solution - semidefinite

MSK\_getbarsslice

```
MSKrescodee (MSKAPI MSK_getbarsslice) (
   MSKtask_t task,
   MSKsoltypee whichsol,
   MSKint32t first,
   MSKint32t last,
   MSKint64t slicesize,
   MSKrealt * barsslice)
```

Obtains the dual solution for a sequence of semidefinite variables. The format is that matrices are stored sequentially, and in each matrix the columns are stored as in <code>MSK\_getbarsj</code>.

#### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- whichsol (MSKsoltypee) Selects a solution. (input)
- first (MSKint32t) Index of the first semidefinite variable in the slice. (input)
- last (MSKint32t) Index of the last semidefinite variable in the slice plus one. (input)
- slicesize (MSKint64t) Denotes the length of the array barsslice. (input)
- barsslice (MSKrealt\*) Dual solution values of symmetric matrix variables in the slice, stored sequentially. (output)

Return (MSKrescodee) - The function response code.

## Groups Solution - semidefinite

# MSK\_getbarvarname

```
MSKrescodee (MSKAPI MSK_getbarvarname) (
   MSKtask_t task,
   MSKint32t i,
   MSKint32t sizename,
   char * name)
```

Obtains the name of a semidefinite variable.

#### Parameters

- task (MSKtask\_t) An optimization task. (input)
- i (MSKint32t) Index of the variable. (input)
- sizename (MSKint32t) Length of the name buffer. (input)
- name (MSKstring\_t) The requested name is copied to this buffer. (output)

Return (MSKrescodee) - The function response code.

Groups Names, Inspecting the task

## MSK\_getbarvarnameindex

```
MSKrescodee (MSKAPI MSK_getbarvarnameindex) (
   MSKtask_t task,
   const char * somename,
   MSKint32t * asgn,
   MSKint32t * index)
```

Obtains the index of semidefinite variable from its name.

# Parameters

- task (MSKtask\_t) An optimization task. (input)
- somename (MSKstring\_t) The name of the variable. (input)
- asgn (MSKint32t by reference) Non-zero if the name somename is assigned to some semidefinite variable. (output)
- index (MSKint32t by reference) The index of a semidefinite variable with the name somename (if one exists). (output)

Return (MSKrescodee) - The function response code.

Groups Names, Inspecting the task

# MSK\_getbarvarnamelen

```
MSKrescodee (MSKAPI MSK_getbarvarnamelen) (
MSKtask_t task,
MSKint32t i,
MSKint32t * len)
```

Obtains the length of the name of a semidefinite variable.

# Parameters

- task (MSKtask\_t) An optimization task. (input)
- i (MSKint32t) Index of the variable. (input)
- len (MSKint32t by reference) Returns the length of the indicated name. (output)

Return (MSKrescodee) - The function response code.

**Groups** Names, Inspecting the task

```
MSKrescodee (MSKAPI MSK_getbarxj) (
    MSKtask_t task,
    MSKsoltypee whichsol,
    MSKint32t j,
    MSKrealt * barxj)
```

Obtains the primal solution for a semidefinite variable. Only the lower triangular part of  $\overline{X}_j$  is returned because the matrix by construction is symmetric. The format is that the columns are stored sequentially in the natural order.

#### Parameters

- task (MSKtask\_t) An optimization task. (input)
- whichsol (MSKsoltypee) Selects a solution. (input)
- j (MSKint32t) Index of the semidefinite variable. (input)
- barxj (MSKrealt\*) Value of  $\overline{X}_i$ . (output)

Return (MSKrescodee) - The function response code.

**Groups** Solution - semidefinite

MSK\_getbarxslice

```
MSKrescodee (MSKAPI MSK_getbarxslice) (
   MSKtask_t task,
   MSKsoltypee whichsol,
   MSKint32t first,
   MSKint32t last,
   MSKint64t slicesize,
   MSKrealt * barxslice)
```

Obtains the primal solution for a sequence of semidefinite variables. The format is that matrices are stored sequentially, and in each matrix the columns are stored as in MSK\_getbarxj.

### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- whichsol (MSKsoltypee) Selects a solution. (input)
- first (MSKint32t) Index of the first semidefinite variable in the slice. (input)
- last (MSKint32t) Index of the last semidefinite variable in the slice plus one. (input)
- slicesize (MSKint64t) Denotes the length of the array barxslice. (input)
- barxslice (MSKrealt\*) Solution values of symmetric matrix variables in the slice, stored sequentially. (output)

Return (MSKrescodee) - The function response code.

Groups Solution - semidefinite

MSK\_getbuildinfo

```
MSKrescodee (MSKAPI MSK_getbuildinfo) (
   char * buildstate,
   char * builddate)
```

Obtains build information.

#### Parameters

• buildstate (MSKstring\_t) - State of binaries, i.e. a debug, release candidate or final release. (output)

```
• builddate (\textit{MSKstring\_t}) – Date when the binaries were built. (output) Return (\textit{MSKrescodee}) – The function response code. Groups Versions
```

MSK\_getc

```
MSKrescodee (MSKAPI MSK_getc) (
MSKtask_t task,
MSKrealt * c)
```

Obtains all objective coefficients c.

### Parameters

- task (MSKtask\_t) An optimization task. (input)
- c (MSKrealt\*) Linear terms of the objective as a dense vector. The length is the number of variables. (output)

Return (MSKrescodee) - The function response code.

Groups Problem data - linear part, Inspecting the task, Problem data - variables

MSK\_getcallbackfunc

```
MSKrescodee (MSKAPI MSK_getcallbackfunc) (
MSKtask_t task,
MSKcallbackfunc * func,
MSKuserhandle_t * handle)
```

Obtains the current user-defined callback function and associated user handle.

#### Parameters

- task (MSKtask\_t) An optimization task. (input)
- func (MSKcallbackfunc by reference) Get the user-defined progress callback function MSKcallbackfunc associated with task. If func is identical to NULL, then no callback function is associated with the task. (output)
- handle (MSKuserhandle\_t by reference) The user-defined pointer associated with the user-defined callback function. (output)

Return (MSKrescodee) - The function response code.

Groups Callback

 ${ t MSK\_getcfix}$ 

```
MSKrescodee (MSKAPI MSK_getcfix) (
MSKtask_t task,
MSKrealt * cfix)
```

Obtains the fixed term in the objective.

# Parameters

- task (MSKtask\_t) An optimization task. (input)
- $\bullet$  cfix (MSKrealt by reference) Fixed term in the objective. (output)

Return (MSKrescodee) - The function response code.

Groups Problem data - linear part, Inspecting the task

MSK\_getcj

```
MSKrescodee (MSKAPI MSK_getcj) (
   MSKtask_t task,
   MSKint32t j,
   MSKrealt * cj)
```

Obtains one coefficient of c.

#### Parameters

- task (MSKtask\_t) An optimization task. (input)
- j  $(\mathit{MSKint32t})$  Index of the variable for which the c coefficient should be obtained. (input)
- cj (MSKrealt by reference) The value of  $c_i$ . (output)

Return (MSKrescodee) - The function response code.

Groups Problem data - linear part, Inspecting the task, Problem data - variables

MSK\_getclist

```
MSKrescodee (MSKAPI MSK_getclist) (
    MSKtask_t task,
    MSKint32t num,
    const MSKint32t * subj,
    MSKrealt * c)
```

Obtains a sequence of elements in c.

### Parameters

- $task (MSKtask_t)$  An optimization task. (input)
- $\bullet$  num (MSKint32t) Number of variables for which the c values should be obtained. (input)
- subj (MSKint32t\*) A list of variable indexes. (input)
- $\bullet$  c (MSKrealt\*) Linear terms of the requested list of the objective as a dense vector. (output)

**Return** (MSKrescodee) - The function response code.

Groups Inspecting the task, Problem data - linear part

MSK\_getcodedesc

```
MSKrescodee (MSKAPI MSK_getcodedesc) (
MSKrescodee code,
char * symname,
char * str)
```

Obtains a short description of the meaning of the response code given by code.

# Parameters

- code (MSKrescodee) A valid MOSEK response code. (input)
- symname (MSKstring\_t) Symbolic name corresponding to code. (output)
- $str(MSKstring_t)$  Obtains a short description of a response code. (output)

Return (MSKrescodee) - The function response code.

Groups Names, Responses, errors and warnings

MSK\_getconbound

```
MSKrescodee (MSKAPI MSK_getconbound) (
   MSKtask_t task,
   MSKint32t i,
   MSKboundkeye * bk,
   MSKrealt * bl,
   MSKrealt * bu)
```

Obtains bound information for one constraint.

#### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- i (MSKint32t) Index of the constraint for which the bound information should be obtained. (input)
- bk (MSKboundkeye by reference) Bound keys. (output)
- bl (MSKrealt by reference) Values for lower bounds. (output)
- bu (MSKrealt by reference) Values for upper bounds. (output)

Return (MSKrescodee) - The function response code.

**Groups** Problem data - linear part, Inspecting the task, Problem data - bounds, Problem data - constraints

MSK\_getconboundslice

```
MSKrescodee (MSKAPI MSK_getconboundslice) (
MSKtask_t task,
MSKint32t first,
MSKint32t last,
MSKboundkeye * bk,
MSKrealt * bl,
MSKrealt * bu)
```

Obtains bounds information for a slice of the constraints.

## Parameters

- task (MSKtask\_t) An optimization task. (input)
- first (MSKint32t) First index in the sequence. (input)
- last (MSKint32t) Last index plus 1 in the sequence. (input)
- bk (MSKboundkeye\*) Bound keys. (output)
- bl (MSKrealt\*) Values for lower bounds. (output)
- bu (MSKrealt\*) Values for upper bounds. (output)

Return (MSKrescodee) - The function response code.

**Groups** Problem data - linear part, Inspecting the task, Problem data - bounds, Problem data - constraints

MSK\_getcone

```
MSKrescodee (MSKAPI MSK_getcone) (
   MSKtask_t task,
   MSKint32t k,
   MSKconetypee * ct,
   MSKrealt * conepar,
   MSKint32t * nummem,
   MSKint32t * submem)
```

Obtains a cone.

#### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- k (MSKint32t) Index of the cone. (input)
- ct (MSKconetypee by reference) Specifies the type of the cone. (output)
- conepar (MSKrealt by reference) For the power cone it denotes the exponent alpha. For other cone types it is unused and can be set to 0. (output)
- nummem (MSKint32t by reference) Number of member variables in the cone. (output)
- submem (MSKint32t\*) Variable subscripts of the members in the cone. (output)

Return (MSKrescodee) - The function response code.

**Groups** Inspecting the task, Problem data - cones

#### MSK\_getconeinfo

```
MSKrescodee (MSKAPI MSK_getconeinfo) (
    MSKtask_t task,
    MSKint32t k,
    MSKconetypee * ct,
    MSKrealt * conepar,
    MSKint32t * nummem)
```

Obtains information about a cone.

### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- k (MSKint32t) Index of the cone. (input)
- ct (MSKconetypee by reference) Specifies the type of the cone. (output)
- conepar (MSKrealt by reference) For the power cone it denotes the exponent alpha. For other cone types it is unused and can be set to 0. (output)
- nummem (MSKint32t by reference) Number of member variables in the cone. (output)

Return (MSKrescodee) - The function response code.

Groups Inspecting the task, Problem data - cones

### MSK\_getconename

```
MSKrescodee (MSKAPI MSK_getconename) (
   MSKtask_t task,
   MSKint32t i,
   MSKint32t sizename,
   char * name)
```

Obtains the name of a cone.

#### Parameters

- task (MSKtask\_t) An optimization task. (input)
- i (MSKint32t) Index of the cone. (input)
- sizename (MSKint32t) Maximum length of a name that can be stored in name. (input)
- name (MSKstring\_t) The required name. (output)

Return (MSKrescodee) - The function response code.

Groups Names, Problem data - cones, Inspecting the task

### MSK\_getconenameindex

```
MSKrescodee (MSKAPI MSK_getconenameindex) (
MSKtask_t task,
const char * somename,
MSKint32t * asgn,
MSKint32t * index)
```

Checks whether the name somename has been assigned to any cone. If it has been assigned to a cone, then the index of the cone is reported.

### Parameters

- task (MSKtask\_t) An optimization task. (input)
- somename (MSKstring\_t) The name which should be checked. (input)
- asgn (MSKint32t by reference) Is non-zero if the name somename is assigned to some cone. (output)
- index (MSKint32t by reference) If the name somename is assigned to some cone, then index is the index of the cone. (output)

Return (MSKrescodee) - The function response code.

Groups Names, Problem data - cones, Inspecting the task

#### MSK\_getconenamelen

```
MSKrescodee (MSKAPI MSK_getconenamelen) (
   MSKtask_t task,
   MSKint32t i,
   MSKint32t * len)
```

Obtains the length of the name of a cone.

# Parameters

- task (MSKtask\_t) An optimization task. (input)
- i (MSKint32t) Index of the cone. (input)
- len (MSKint32t by reference) Returns the length of the indicated name. (output)

Return (MSKrescodee) - The function response code.

Groups Names, Problem data - cones, Inspecting the task

# MSK\_getconname

```
MSKrescodee (MSKAPI MSK_getconname) (
MSKtask_t task,
MSKint32t i,
MSKint32t sizename,
char * name)
```

Obtains the name of a constraint.

# **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- i (MSKint32t) Index of the constraint. (input)
- sizename (MSKint32t) Maximum length of name that can be stored in name. (input)
- name  $(MSKstring_t)$  The required name. (output)

Return (MSKrescodee) - The function response code.

**Groups** Names, Problem data - linear part, Problem data - constraints, Inspecting the task

```
MSKrescodee (MSKAPI MSK_getconnameindex) (
   MSKtask_t task,
   const char * somename,
   MSKint32t * asgn,
   MSKint32t * index)
```

Checks whether the name somename has been assigned to any constraint. If so, the index of the constraint is reported.

# Parameters

- task (MSKtask\_t) An optimization task. (input)
- somename (MSKstring\_t) The name which should be checked. (input)
- asgn (MSKint32t by reference) Is non-zero if the name somename is assigned to some constraint. (output)
- index (MSKint32t by reference) If the name somename is assigned to a constraint, then index is the index of the constraint. (output)

Return (MSKrescodee) - The function response code.

**Groups** Names, Problem data - linear part, Problem data - constraints, Inspecting the task

# MSK\_getconnamelen

```
MSKrescodee (MSKAPI MSK_getconnamelen) (
MSKtask_t task,
MSKint32t i,
MSKint32t * len)
```

Obtains the length of the name of a constraint.

# **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- i (MSKint32t) Index of the constraint. (input)
- len (MSKint32t by reference) Returns the length of the indicated name. (output)

Return (MSKrescodee) - The function response code.

**Groups** Names, Problem data - linear part, Problem data - constraints, Inspecting the task

## MSK\_getcslice

```
MSKrescodee (MSKAPI MSK_getcslice) (
    MSKtask_t task,
    MSKint32t first,
    MSKint32t last,
    MSKrealt * c)
```

Obtains a sequence of elements in c.

## Parameters

- task (MSKtask\_t) An optimization task. (input)
- first (MSKint32t) First index in the sequence. (input)
- last (MSKint32t) Last index plus 1 in the sequence. (input)
- c (MSKrealt\*) Linear terms of the requested slice of the objective as a dense vector. The length is last-first. (output)

```
Return (MSKrescodee) - The function response code.
Groups Inspecting the task, Problem data - linear part
```

MSK\_getdimbarvarj

```
MSKrescodee (MSKAPI MSK_getdimbarvarj) (
MSKtask_t task,
MSKint32t j,
MSKint32t * dimbarvarj)
```

Obtains the dimension of a symmetric matrix variable.

#### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- j (MSKint32t) Index of the semidefinite variable whose dimension is requested. (input)
- dimbarvarj (MSKint32t by reference) The dimension of the j-th semidefinite variable. (output)

 $\mathbf{Return} \ \ (\textit{MSKrescodee}) - \mathbf{The} \ \mathbf{function} \ \mathbf{response} \ \mathbf{code}.$ 

Groups Inspecting the task, Problem data - semidefinite

MSK\_getdouinf

```
MSKrescodee (MSKAPI MSK_getdouinf) (
MSKtask_t task,
MSKdinfiteme whichdinf,
MSKrealt * dvalue)
```

Obtains a double information item from the task information database.

# Parameters

- task (MSKtask\_t) An optimization task. (input)
- whichdinf (MSKdinfiteme) Specifies a double information item. (input)
- dvalue (MSKrealt by reference) The value of the required double information item. (output)

Return (MSKrescodee) - The function response code.

**Groups** Information items and statistics

MSK\_getdouparam

```
MSKrescodee (MSKAPI MSK_getdouparam) (
MSKtask_t task,
MSKdparame param,
MSKrealt * parvalue)
```

Obtains the value of a double parameter.

# Parameters

- task (MSKtask\_t) An optimization task. (input)
- param (MSKdparame) Which parameter. (input)
- parvalue (MSKrealt by reference) Parameter value. (output)

Return (MSKrescodee) - The function response code.

Groups Parameters

MSK\_getdualobj

```
MSKrescodee (MSKAPI MSK_getdualobj) (
MSKtask_t task,
MSKsoltypee whichsol,
MSKrealt * dualobj)
```

Computes the dual objective value associated with the solution. Note that if the solution is a primal infeasibility certificate, then the fixed term in the objective value is not included.

Moreover, since there is no dual solution associated with an integer solution, an error will be reported if the dual objective value is requested for the integer solution.

#### Parameters

- task (MSKtask\_t) An optimization task. (input)
- whichsol (MSKsoltypee) Selects a solution. (input)
- dualobj (MSKrealt by reference) Objective value corresponding to the dual solution. (output)

Return (MSKrescodee) - The function response code.

Groups Solution information, Solution - dual

#### MSK\_getdualsolutionnorms

```
MSKrescodee (MSKAPI MSK_getdualsolutionnorms) (
   MSKtask_t task,
   MSKsoltypee whichsol,
   MSKrealt * nrmy,
   MSKrealt * nrmslc,
   MSKrealt * nrmsuc,
   MSKrealt * nrmsux,
   MSKrealt * nrmsux,
   MSKrealt * nrmsux,
   MSKrealt * nrmsnx,
   MSKrealt * nrmsnx)
```

Compute norms of the dual solution.

### Parameters

- task (MSKtask\_t) An optimization task. (input)
- whichsol (MSKsoltypee) Selects a solution. (input)
- nrmy (MSKrealt by reference) The norm of the y vector. (output)
- nrmslc (MSKrealt by reference) The norm of the  $s_l^c$  vector. (output)
- nrmsuc (MSKrealt by reference) The norm of the  $s_n^c$  vector. (output)
- ullet nrmslx (MSKrealt by reference) The norm of the  $s_l^x$  vector. (output)
- nrmsux (MSKrealt by reference) The norm of the  $s_u^x$  vector. (output)
- ullet nrmsnx (MSKrealt by reference) The norm of the  $s_n^x$  vector. (output)
- nrmbars (MSKrealt by reference) The norm of the  $\overline{S}$  vector. (output) Return (MSKrescodee) The function response code.

Groups Solution information

### MSK\_getdviolbarvar

```
MSKrescodee (MSKAPI MSK_getdviolbarvar) (
   MSKtask_t task,
   MSKsoltypee whichsol,
   MSKint32t num,
   const MSKint32t * sub,
   MSKrealt * viol)
```

Let  $(\overline{S}_j)^*$  be the value of variable  $\overline{S}_j$  for the specified solution. Then the dual violation of the solution associated with variable  $\overline{S}_j$  is given by

$$\max(-\lambda_{\min}(\overline{S}_j), 0.0).$$

Both when the solution is a certificate of primal infeasibility and when it is dual feasible solution the violation should be small.

#### Parameters

- task (MSKtask\_t) An optimization task. (input)
- whichsol (MSKsoltypee) Selects a solution. (input)
- num (MSKint32t) Length of sub and viol. (input)
- sub (MSKint32t\*) An array of indexes of  $\overline{X}$  variables. (input)
- viol (MSKrealt\*) viol[k] is the violation of the solution for the constraint  $\overline{S}_{\text{sub}[k]} \in \mathcal{S}_+$ . (output)

 ${\bf Return} \ \ ({\it MSKrescodee}\,) - {\bf The} \ {\bf function} \ {\bf response} \ {\bf code}.$ 

**Groups** Solution information

MSK\_getdviolcon

```
MSKrescodee (MSKAPI MSK_getdviolcon) (
    MSKtask_t task,
    MSKsoltypee whichsol,
    MSKint32t num,
    const MSKint32t * sub,
    MSKrealt * viol)
```

The violation of the dual solution associated with the i-th constraint is computed as follows

$$\max(\rho((s_l^c)_i^*, (b_l^c)_i), \ \rho((s_u^c)_i^*, -(b_u^c)_i), \ |-y_i + (s_l^c)_i^* - (s_u^c)_i^*|)$$

where

$$\rho(x,l) = \left\{ \begin{array}{ll} -x, & l > -\infty, \\ |x|, & \text{otherwise.} \end{array} \right.$$

Both when the solution is a certificate of primal infeasibility or it is a dual feasible solution the violation should be small.

# Parameters

- task (MSKtask\_t) An optimization task. (input)
- whichsol (MSKsoltypee) Selects a solution. (input)
- num (MSKint32t) Length of sub and viol. (input)
- sub (MSKint32t\*) An array of indexes of constraints. (input)
- viol (MSKrealt\*) viol[k] is the violation of dual solution associated with the constraint sub[k]. (output)

Return (MSKrescodee) - The function response code.

Groups Solution information

MSK\_getdviolcones

```
MSKrescodee (MSKAPI MSK_getdviolcones) (
MSKtask_t task,
MSKsoltypee whichsol,
MSKint32t num,
const MSKint32t * sub,
MSKrealt * viol)
```

Let  $(s_n^x)^*$  be the value of variable  $(s_n^x)$  for the specified solution. For simplicity let us assume that  $s_n^x$  is a member of a quadratic cone, then the violation is computed as follows

$$\begin{cases} \max(0, (\|s_n^x\|_{2:n}^* - (s_n^x)_1^*)/\sqrt{2}, & (s_n^x)^* \ge -\|(s_n^x)_{2:n}^*\|, \\ \|(s_n^x)^*\|, & \text{otherwise.} \end{cases}$$

Both when the solution is a certificate of primal infeasibility or when it is a dual feasible solution the violation should be small.

#### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- whichsol (MSKsoltypee) Selects a solution. (input)
- num (MSKint32t) Length of sub and viol. (input)
- sub (MSKint32t\*) An array of indexes of conic constraints. (input)
- viol (MSKrealt\*) viol[k] is the violation of the dual solution associated with the conic constraint sub[k]. (output)

Return (MSKrescodee) - The function response code.

Groups Solution information

MSK\_getdviolvar

```
MSKrescodee (MSKAPI MSK_getdviolvar) (
   MSKtask_t task,
   MSKsoltypee whichsol,
   MSKint32t num,
   const MSKint32t * sub,
   MSKrealt * viol)
```

The violation of the dual solution associated with the j-th variable is computed as follows

$$\max \left( \rho((s_l^x)_j^*, (b_l^x)_j), \ \rho((s_u^x)_j^*, -(b_u^x)_j), \ | \sum_{i=0}^{numcon-1} a_{ij} y_i + (s_l^x)_j^* - (s_u^x)_j^* - \tau c_j | \right)$$

where

$$\rho(x,l) = \begin{cases} -x, & l > -\infty, \\ |x|, & \text{otherwise} \end{cases}$$

and  $\tau=0$  if the solution is a certificate of primal infeasibility and  $\tau=1$  otherwise. The formula for computing the violation is only shown for the linear case but is generalized appropriately for the more general problems. Both when the solution is a certificate of primal infeasibility or when it is a dual feasible solution the violation should be small.

# Parameters

- task (MSKtask\_t) An optimization task. (input)
- whichsol (MSKsoltypee) Selects a solution. (input)
- num (MSKint32t) Length of sub and viol. (input)
- sub (MSKint32t\*) An array of indexes of x variables. (input)
- viol (MSKrealt\*) viol[k] is the violation of dual solution associated with the variable sub[k]. (output)

Return (MSKrescodee) - The function response code.

Groups Solution information

MSK\_getenv

```
MSKrescodee (MSKAPI MSK_getenv) (
MSKtask_t task,
MSKenv_t * env)
```

Obtains the environment used to create the task.

#### Parameters

- task (MSKtask\_t) An optimization task. (input)
- env (MSKenv\_t by reference) The MOSEK environment. (output)

Return (MSKrescodee) - The function response code.

Groups Environment and task management

MSK\_getinfeasiblesubproblem

```
MSKrescodee (MSKAPI MSK_getinfeasiblesubproblem) (
MSKtask_t task,
MSKsoltypee whichsol,
MSKtask_t * inftask)
```

Given the solution is a certificate of primal or dual infeasibility then a primal or dual infeasible subproblem is obtained respectively. The subproblem tends to be much smaller than the original problem and hence it is easier to locate the infeasibility inspecting the subproblem than the original problem.

For the procedure to be useful it is important to assign meaningful names to constraints, variables etc. in the original task because those names will be duplicated in the subproblem.

The function is only applicable to linear and conic quadratic optimization problems.

For more information see Sec. 8.3 and Sec. 14.2.

## Parameters

- task (MSKtask\_t) An optimization task. (input)
- whichsol (MSKsoltypee) Which solution to use when determining the infeasible subproblem. (input)
- inftask (MSKtask\_t by reference) A new task containing the infeasible subproblem. (output)

Return (MSKrescodee) - The function response code.

Groups Infeasibility diagnostic

MSK\_getinfindex

```
MSKrescodee (MSKAPI MSK_getinfindex) (
MSKtask_t task,
MSKinftypee inftype,
const char * infname,
MSKint32t * infindex)
```

Obtains the index of a named information item.

# Parameters

- task  $(\textit{MSKtask\_t})$  An optimization task. (input)
- inftype (MSKinftypee) Type of the information item. (input)
- infname (MSKstring\_t) Name of the information item. (input)
- infindex (MSKint32t by reference) The item index. (output)

Return (MSKrescodee) - The function response code.

Groups Information items and statistics

```
MSKrescodee (MSKAPI MSK_getinfmax) (
MSKtask_t task,
MSKinftypee inftype,
MSKint32t * infmax)
```

Obtains the maximum index of an information item of a given type inftype plus 1.

### Parameters

- task (MSKtask\_t) An optimization task. (input)
- inftype (MSKinftypee) Type of the information item. (input)
- infmax (MSKint32t\*) The maximum index (plus 1) requested. (output)

Return (MSKrescodee) - The function response code.

Groups Information items and statistics

MSK\_getinfname

```
MSKrescodee (MSKAPI MSK_getinfname) (
MSKtask_t task,
MSKinftypee inftype,
MSKint32t whichinf,
char * infname)
```

Obtains the name of an information item.

### Parameters

- task (MSKtask\_t) An optimization task. (input)
- inftype (MSKinftypee) Type of the information item. (input)
- whichinf (MSKint32t) An information item. (input)
- $infname (MSKstring_t)$  Name of the information item. (output)

Return (MSKrescodee) - The function response code.

Groups Information items and statistics, Names

MSK\_getintinf

```
MSKrescodee (MSKAPI MSK_getintinf) (
MSKtask_t task,
MSKiinfiteme whichiinf,
MSKint32t * ivalue)
```

Obtains an integer information item from the task information database.

# Parameters

- task (MSKtask\_t) An optimization task. (input)
- whichiinf (MSKiinfiteme) Specifies an integer information item. (input)
- ivalue (MSKint32t by reference) The value of the required integer information item. (output)

Return (MSKrescodee) - The function response code.

**Groups** Information items and statistics

MSK\_getintparam

```
MSKrescodee (MSKAPI MSK_getintparam) (
MSKtask_t task,
MSKiparame param,
MSKint32t * parvalue)
```

Obtains the value of an integer parameter.

#### Parameters

- task (MSKtask\_t) An optimization task. (input)
- param (MSKiparame) Which parameter. (input)
- parvalue (MSKint32t by reference) Parameter value. (output)

 ${\bf Return} \ ({\tt MSKrescodee})$  – The function response code.

Groups Parameters

MSK\_getlasterror

```
MSKrescodee (MSKAPI MSK_getlasterror) (
MSKtask_t task,
MSKrescodee * lastrescode,
MSKint32t sizelastmsg,
MSKint32t * lastmsglen,
char * lastmsg)
```

Obtains the last response code and corresponding message reported in MOSEK.

If there is no previous error, warning or termination code for this task, lastrescode returns  $MSK\_RES\_OK$  and lastmsg returns an empty string, otherwise the last response code different from  $MSK\_RES\_OK$  and the corresponding message are returned.

## Parameters

- task (MSKtask\_t) An optimization task. (input)
- lastrescode (MSKrescodee by reference) Returns the last error code reported in the task. (output)
- sizelastmsg (MSKint32t) The length of the lastmsg buffer. (input)
- lastmsglen (MSKint32t by reference) Returns the length of the last error message reported in the task. (output)
- lastmsg (MSKstring\_t) Returns the last error message reported in the task. (output)

Return (MSKrescodee) - The function response code.

**Groups** Responses, errors and warnings

MSK\_getlasterror64

```
MSKrescodee (MSKAPI MSK_getlasterror64) (
MSKtask_t task,
MSKrescodee * lastrescode,
MSKint64t sizelastmsg,
MSKint64t * lastmsglen,
char * lastmsg)
```

Obtains the last response code and corresponding message reported in MOSEK.

If there is no previous error, warning or termination code for this task, lastrescode returns  $MSK\_RES\_OK$  and lastmsg returns an empty string, otherwise the last response code different from  $MSK\_RES\_OK$  and the corresponding message are returned.

# **Parameters**

• task (MSKtask\_t) - An optimization task. (input)

- lastrescode (MSKrescodee by reference) Returns the last error code reported in the task. (output)
- sizelastmsg (MSKint64t) The length of the lastmsg buffer. (input)
- lastmsglen (MSKint64t by reference) Returns the length of the last error message reported in the task. (output)
- lastmsg (MSKstring\_t) Returns the last error message reported in the task. (output)

Return (MSKrescodee) - The function response code.

Groups Responses, errors and warnings

MSK\_getlenbarvarj

```
MSKrescodee (MSKAPI MSK_getlenbarvarj) (
MSKtask_t task,
MSKint32t j,
MSKint64t * lenbarvarj)
```

Obtains the length of the j-th semidefinite variable i.e. the number of elements in the lower triangular part.

# **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- j (MSKint32t) Index of the semidefinite variable whose length if requested. (input)
- lenbarvarj (MSKint64t by reference) Number of scalar elements in the lower triangular part of the semidefinite variable. (output)

Return (MSKrescodee) - The function response code.

Groups Inspecting the task, Problem data - semidefinite

MSK\_getlintinf

```
MSKrescodee (MSKAPI MSK_getlintinf) (
MSKtask_t task,
MSKliinfiteme whichliinf,
MSKint64t * ivalue)
```

Obtains a long integer information item from the task information database.

#### Parameters

- task (MSKtask\_t) An optimization task. (input)
- whichliinf (MSKliinfiteme) Specifies a long information item. (input)
- ivalue (MSKint64t by reference) The value of the required long integer information item. (output)

Return (MSKrescodee) - The function response code.

**Groups** Information items and statistics

MSK\_getmaxnamelen

```
MSKrescodee (MSKAPI MSK_getmaxnamelen) (
MSKtask_t task,
MSKint32t * maxlen)
```

Obtains the maximum length (not including terminating zero character) of any objective, constraint, variable or cone name.

## Parameters

- task (MSKtask\_t) An optimization task. (input)
- maxlen (MSKint32t by reference) The maximum length of any name. (output)

Return (MSKrescodee) - The function response code.

Groups Inspecting the task, Names

MSK\_getmaxnumanz

```
MSKrescodee (MSKAPI MSK_getmaxnumanz) (
MSKtask_t task,
MSKint32t * maxnumanz)
```

Obtains number of preallocated non-zeros in A. When this number of non-zeros is reached **MOSEK** will automatically allocate more space for A.

#### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- maxnumanz (MSKint32t by reference) Number of preallocated non-zero linear matrix elements. (output)

Return (MSKrescodee) - The function response code.

Groups Inspecting the task, Problem data - linear part

MSK\_getmaxnumanz64

```
MSKrescodee (MSKAPI MSK_getmaxnumanz64) (
MSKtask_t task,
MSKint64t * maxnumanz)
```

Obtains number of preallocated non-zeros in A. When this number of non-zeros is reached **MOSEK** will automatically allocate more space for A.

#### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- maxnumanz (MSKint64t by reference) Number of preallocated non-zero linear matrix elements. (output)

 ${\bf Return} \ \ ({\it MSKrescodee}\,) - {\bf The} \ {\bf function} \ {\bf response} \ {\bf code}.$ 

Groups Inspecting the task, Problem data - linear part

MSK\_getmaxnumbarvar

```
MSKrescodee (MSKAPI MSK_getmaxnumbarvar) (
MSKtask_t task,
MSKint32t * maxnumbarvar)
```

Obtains maximum number of symmetric matrix variables for which space is currently preallocated.

## **Parameters**

- $task (MSKtask_t)$  An optimization task. (input)
- maxnumbarvar (MSKint32t by reference) Maximum number of symmetric matrix variables for which space is currently preallocated. (output)

Return (MSKrescodee) - The function response code.

Groups Inspecting the task, Problem data - semidefinite

MSK\_getmaxnumcon

```
MSKrescodee (MSKAPI MSK_getmaxnumcon) (
MSKtask_t task,
MSKint32t * maxnumcon)
```

Obtains the number of preallocated constraints in the optimization task. When this number of constraints is reached **MOSEK** will automatically allocate more space for constraints.

### Parameters

- task (MSKtask\_t) An optimization task. (input)
- maxnumcon (MSKint32t by reference) Number of preallocated constraints in the optimization task. (output)

Return (MSKrescodee) - The function response code.

Groups Inspecting the task, Problem data - linear part, Problem data - constraints

MSK\_getmaxnumcone

```
MSKrescodee (MSKAPI MSK_getmaxnumcone) (
MSKtask_t task,
MSKint32t * maxnumcone)
```

Obtains the number of preallocated cones in the optimization task. When this number of cones is reached **MOSEK** will automatically allocate space for more cones.

### Parameters

- task (MSKtask\_t) An optimization task. (input)
- maxnumcone (MSKint32t by reference) Number of preallocated conic constraints in the optimization task. (output)

Return (MSKrescodee) - The function response code.

Groups Inspecting the task, Problem data - cones

MSK\_getmaxnumqnz

```
MSKrescodee (MSKAPI MSK_getmaxnumqnz) (
MSKtask_t task,
MSKint32t * maxnumqnz)
```

Obtains the number of preallocated non-zeros for Q (both objective and constraints). When this number of non-zeros is reached **MOSEK** will automatically allocate more space for Q.

# Parameters

- task (MSKtask\_t) An optimization task. (input)
- maxnumqnz (MSKint32t by reference) Number of non-zero elements preallocated in quadratic coefficient matrices. (output)

Return (MSKrescodee) - The function response code.

Groups Inspecting the task, Problem data - quadratic part

 ${\tt MSK\_getmaxnumqnz64}$ 

```
MSKrescodee (MSKAPI MSK_getmaxnumqnz64) (
MSKtask_t task,
MSKint64t * maxnumqnz)
```

Obtains the number of preallocated non-zeros for Q (both objective and constraints). When this number of non-zeros is reached **MOSEK** will automatically allocate more space for Q.

- task (MSKtask\_t) An optimization task. (input)
- maxnumqnz (MSKint64t by reference) Number of non-zero elements preallocated in quadratic coefficient matrices. (output)

Groups Inspecting the task, Problem data - quadratic part

MSK\_getmaxnumvar

```
MSKrescodee (MSKAPI MSK_getmaxnumvar) (
MSKtask_t task,
MSKint32t * maxnumvar)
```

Obtains the number of preallocated variables in the optimization task. When this number of variables is reached **MOSEK** will automatically allocate more space for variables.

#### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- maxnumvar (MSKint32t by reference) Number of preallocated variables in the optimization task. (output)

Return (MSKrescodee) - The function response code.

Groups Inspecting the task, Problem data - linear part, Problem data - variables

 ${\tt MSK\_getmemusagetask}$ 

```
MSKrescodee (MSKAPI MSK_getmemusagetask) (
MSKtask_t task,
MSKint64t * meminuse,
MSKint64t * maxmemuse)
```

Obtains information about the amount of memory used by a task.

## **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- meminuse (MSKint64t by reference) Amount of memory currently used by the task. (output)
- maxmemuse (MSKint64t by reference) Maximum amount of memory used by the task until now. (output)

Return (MSKrescodee) - The function response code.

Groups System, memory and debugging

MSK\_getnadouinf

```
MSKrescodee (MSKAPI MSK_getnadouinf) (
   MSKtask_t task,
   const char * infitemname,
   MSKrealt * dvalue)
```

Obtains a named double information item from task information database.

## **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- infitemname (MSKstring\_t) The name of a double information item. (input)
- dvalue (MSKrealt by reference) The value of the required double information item. (output)

Return (MSKrescodee) - The function response code.

**Groups** Information items and statistics

```
MSKrescodee (MSKAPI MSK_getnadouparam) (
   MSKtask_t task,
   const char * paramname,
   MSKrealt * parvalue)
```

Obtains the value of a named double parameter.

### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- paramname (MSKstring\_t) Name of a parameter. (input)
- parvalue (MSKrealt by reference) Parameter value. (output)

Return (MSKrescodee) - The function response code.

Groups Parameters

MSK\_getnaintinf

```
MSKrescodee (MSKAPI MSK_getnaintinf) (
   MSKtask_t task,
   const char * infitemname,
   MSKint32t * ivalue)
```

Obtains a named integer information item from the task information database.

#### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- infitemname (MSKstring\_t) The name of an integer information item. (input)
- ivalue (MSKint32t by reference) The value of the required integer information item. (output)

Return (MSKrescodee) - The function response code.

Groups Information items and statistics

 ${\tt MSK\_getnaintparam}$ 

```
MSKrescodee (MSKAPI MSK_getnaintparam) (
    MSKtask_t task,
    const char * paramname,
    MSKint32t * parvalue)
```

Obtains the value of a named integer parameter.

# Parameters

- task (MSKtask\_t) An optimization task. (input)
- paramname (MSKstring\_t) Name of a parameter. (input)
- parvalue (MSKint32t by reference) Parameter value. (output)

**Return** (MSKrescodee) - The function response code.

Groups Parameters

MSK\_getnastrparam

```
MSKrescodee (MSKAPI MSK_getnastrparam) (
   MSKtask_t task,
   const char * paramname,
   MSKint32t sizeparamname,
   MSKint32t * len,
   char * parvalue)
```

Obtains the value of a named string parameter.

### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- paramname (MSKstring\_t) Name of a parameter. (input)
- sizeparamname (MSKint32t) Size of the name buffer parvalue. (input)
- len (MSKint32t by reference) Length of the string in parvalue. (output)
- parvalue (MSKstring\_t) Parameter value. (output)

Return (MSKrescodee) - The function response code.

Groups Parameters, Names

MSK\_getnastrparamal

```
MSKrescodee (MSKAPI MSK_getnastrparamal) (
    MSKtask_t task,
    const char * paramname,
    MSKint32t numaddchr,
    MSKstring_t * value)
```

Obtains the value of a named string parameter.

#### Parameters

- task (MSKtask\_t) An optimization task. (input)
- paramname (MSKstring\_t) Name of a parameter. (input)
- numaddchr (MSKint32t) Number of additional characters for which room is left in value. (input)
- value (MSKstring\_t\*) Parameter value. MOSEK will allocate this char buffer of size equal to the actual length of the string parameter plus numaddchr. This memory must be freed by MSK\_freetask. (input/output)

Return (MSKrescodee) - The function response code.

Groups Parameters

MSK\_getnumanz

```
MSKrescodee (MSKAPI MSK_getnumanz) (
MSKtask_t task,
MSKint32t * numanz)
```

Obtains the number of non-zeros in A.

## Parameters

- task (MSKtask\_t) An optimization task. (input)
- numanz (MSKint32t by reference) Number of non-zero elements in the linear constraint matrix. (output)

Return (MSKrescodee) - The function response code.

Groups Inspecting the task, Problem data - linear part

 $MSK\_getnumanz64$ 

```
MSKrescodee (MSKAPI MSK_getnumanz64) (
MSKtask_t task,
MSKint64t * numanz)
```

Obtains the number of non-zeros in A.

- task (MSKtask\_t) An optimization task. (input)
- numanz (MSKint64t by reference) Number of non-zero elements in the linear constraint matrix. (output)

Groups Inspecting the task, Problem data - linear part

MSK\_getnumbarablocktriplets

```
MSKrescodee (MSKAPI MSK_getnumbarablocktriplets) (
MSKtask_t task,
MSKint64t * num)
```

Obtains an upper bound on the number of elements in the block triplet form of  $\overline{A}$ .

### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- num (MSKint64t by reference) An upper bound on the number of elements in the block triplet form of  $\overline{A}$ . (output)

Return (MSKrescodee) - The function response code.

Groups Problem data - semidefinite, Inspecting the task

MSK\_getnumbaranz

```
MSKrescodee (MSKAPI MSK_getnumbaranz) (
MSKtask_t task,
MSKint64t * nz)
```

Get the number of nonzero elements in  $\overline{A}$ .

# Parameters

- task (MSKtask\_t) An optimization task. (input)
- nz  $(MSKint64t \ by \ reference)$  The number of nonzero block elements in  $\overline{A}$  i.e. the number of  $\overline{A}_{ij}$  elements that are nonzero. (output)

Return (MSKrescodee) - The function response code.

**Groups** Problem data - semidefinite, Inspecting the task

MSK\_getnumbarcblocktriplets

```
MSKrescodee (MSKAPI MSK_getnumbarcblocktriplets) (
MSKtask_t task,
MSKint64t * num)
```

Obtains an upper bound on the number of elements in the block triplet form of  $\overline{C}$ .

### Parameters

- task (MSKtask\_t) An optimization task. (input)
- num (MSKint64t by reference) An upper bound on the number of elements in the block triplet form of  $\overline{C}$ . (output)

Return (MSKrescodee) - The function response code.

**Groups** Problem data - semidefinite, Inspecting the task

MSK\_getnumbarcnz

```
MSKrescodee (MSKAPI MSK_getnumbarcnz) (
MSKtask_t task,
MSKint64t * nz)
```

Obtains the number of nonzero elements in  $\overline{C}$ .

### Parameters

- task (MSKtask\_t) An optimization task. (input)
- nz (MSKint64t by reference) The number of nonzeros in  $\overline{C}$  i.e. the number of elements  $\overline{C}_i$  that are nonzero. (output)

 ${\bf Return} \ \ ({\it MSKrescodee}\,) - {\bf The} \ {\bf function} \ {\bf response} \ {\bf code}.$ 

Groups Problem data - semidefinite, Inspecting the task

MSK\_getnumbarvar

```
MSKrescodee (MSKAPI MSK_getnumbarvar) (
   MSKtask_t task,
   MSKint32t * numbarvar)
```

Obtains the number of semidefinite variables.

#### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- numbarvar (MSKint32t by reference) Number of semidefinite variables in the problem. (output)

Return (MSKrescodee) - The function response code.

Groups Inspecting the task, Problem data - semidefinite

MSK\_getnumcon

```
MSKrescodee (MSKAPI MSK_getnumcon) (
MSKtask_t task,
MSKint32t * numcon)
```

Obtains the number of constraints.

# Parameters

- task (MSKtask\_t) An optimization task. (input)
- numcon (MSKint32t by reference) Number of constraints. (output)

 $\mathbf{Return} \ \ (\textit{MSKrescodee}) - \mathbf{The} \ \mathbf{function} \ \mathbf{response} \ \mathbf{code}.$ 

Groups Problem data - linear part, Problem data - constraints, Inspecting the task

MSK\_getnumcone

```
MSKrescodee (MSKAPI MSK_getnumcone) (
MSKtask_t task,
MSKint32t * numcone)
```

Obtains the number of cones.

### Parameters

- task  $(\textit{MSKtask\_t})$  An optimization task. (input)
- numcone (MSKint32t by reference) Number of conic constraints. (output)

Return (MSKrescodee) - The function response code.

**Groups** Problem data - cones, Inspecting the task

MSK\_getnumconemem

```
MSKrescodee (MSKAPI MSK_getnumconemem) (
MSKtask_t task,
MSKint32t k,
MSKint32t * nummem)
```

Obtains the number of members in a cone.

### Parameters

- task (MSKtask\_t) An optimization task. (input)
- k (MSKint32t) Index of the cone. (input)
- nummem (MSKint32t by reference) Number of member variables in the cone. (output)

Return (MSKrescodee) - The function response code.

Groups Problem data - cones, Inspecting the task

MSK\_getnumintvar

```
MSKrescodee (MSKAPI MSK_getnumintvar) (
MSKtask_t task,
MSKint32t * numintvar)
```

Obtains the number of integer-constrained variables.

### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- numintvar (MSKint32t by reference) Number of integer variables. (output)

 ${\bf Return} \ ({\tt MSKrescodee})$  – The function response code.

Groups Inspecting the task, Problem data - variables

MSK\_getnumparam

```
MSKrescodee (MSKAPI MSK_getnumparam) (
MSKtask_t task,
MSKparametertypee partype,
MSKint32t * numparam)
```

Obtains the number of parameters of a given type.

# Parameters

- task  $(\textit{MSKtask\_t})$  An optimization task. (input)
- partype (MSKparametertypee) Parameter type. (input)
- numparam (MSKint32t by reference) The number of parameters of type partype. (output)

 ${\bf Return} \ \ ({\it MSKrescodee}\,) - {\bf The} \ {\bf function} \ {\bf response} \ {\bf code}.$ 

**Groups** Inspecting the task, Parameters

MSK\_getnumqconknz

```
MSKrescodee (MSKAPI MSK_getnumqconknz) (
MSKtask_t task,
MSKint32t k,
MSKint32t * numqcnz)
```

Obtains the number of non-zero quadratic terms in a constraint.

- task (MSKtask\_t) An optimization task. (input)
- $k \, (\textit{MSKint32t})$  Index of the constraint for which the number of non-zero quadratic terms should be obtained. (input)
- numqcnz (MSKint32t by reference) Number of quadratic terms. (output)

**Groups** Inspecting the task, Problem data - constraints, Problem data - quadratic part

MSK\_getnumqconknz64

```
MSKrescodee (MSKAPI MSK_getnumqconknz64) (
MSKtask_t task,
MSKint32t k,
MSKint64t * numqcnz)
```

Obtains the number of non-zero quadratic terms in a constraint.

### Parameters

- task (MSKtask\_t) An optimization task. (input)
- k (MSKint32t) Index of the constraint for which the number quadratic terms should be obtained. (input)
- numqcnz (MSKint64t by reference) Number of quadratic terms. (output)

Return (MSKrescodee) - The function response code.

Groups Inspecting the task, Problem data - constraints, Problem data - quadratic part

MSK\_getnumqobjnz

```
MSKrescodee (MSKAPI MSK_getnumqobjnz) (
MSKtask_t task,
MSKint32t * numqonz)
```

Obtains the number of non-zero quadratic terms in the objective.

### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- numqonz (MSKint32t by reference) Number of non-zero elements in the quadratic objective terms. (output)

Return (MSKrescodee) - The function response code.

Groups Inspecting the task, Problem data - quadratic part

 $MSK\_getnumqobjnz64$ 

```
MSKrescodee (MSKAPI MSK_getnumqobjnz64) (
MSKtask_t task,
MSKint64t * numqonz)
```

Obtains the number of non-zero quadratic terms in the objective.

# Parameters

- task (MSKtask\_t) An optimization task. (input)
- numqonz (MSKint64t by reference) Number of non-zero elements in the quadratic objective terms. (output)

**Return** (*MSKrescodee*) – The function response code.

Groups Inspecting the task, Problem data - quadratic part

MSK\_getnumsymmat

```
MSKrescodee (MSKAPI MSK_getnumsymmat) (
MSKtask_t task,
MSKint64t * num)
```

Obtains the number of symmetric matrices stored in the vector E.

#### Parameters

- task (MSKtask\_t) An optimization task. (input)
- num (MSKint64t by reference) The number of symmetric sparse matrices. (output)

Return (MSKrescodee) - The function response code.

Groups Problem data - semidefinite, Inspecting the task

MSK\_getnumvar

```
MSKrescodee (MSKAPI MSK_getnumvar) (
MSKtask_t task,
MSKint32t * numvar)
```

Obtains the number of variables.

### Parameters

- task (MSKtask\_t) An optimization task. (input)
- numvar (MSKint32t by reference) Number of variables. (output)

Return (MSKrescodee) - The function response code.

Groups Inspecting the task, Problem data - variables

MSK\_getobjname

```
MSKrescodee (MSKAPI MSK_getobjname) (
    MSKtask_t task,
    MSKint32t sizeobjname,
    char * objname)
```

Obtains the name assigned to the objective function.

### Parameters

- task (MSKtask\_t) An optimization task. (input)
- sizeobjname (MSKint32t) Length of objname. (input)
- objname (MSKstring\_t) Assigned the objective name. (output)

Return (MSKrescodee) - The function response code.

Groups Inspecting the task, Names

MSK\_getobjnamelen

```
MSKrescodee (MSKAPI MSK_getobjnamelen) (
MSKtask_t task,
MSKint32t * len)
```

Obtains the length of the name assigned to the objective function.

### Parameters

- task (MSKtask\_t) An optimization task. (input)
- $\bullet$  len (MSKint32t by reference) Assigned the length of the objective name. (output)

Return (MSKrescodee) - The function response code.

## Groups Inspecting the task, Names

MSK\_getobjsense

```
MSKrescodee (MSKAPI MSK_getobjsense) (
MSKtask_t task,
MSKobjsensee * sense)
```

Gets the objective sense of the task.

### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- sense (MSKobjsensee by reference) The returned objective sense. (output)

Return (MSKrescodee) - The function response code.

Groups Problem data - linear part

MSK\_getparammax

```
MSKrescodee (MSKAPI MSK_getparammax) (
MSKtask_t task,
MSKparametertypee partype,
MSKint32t * parammax)
```

Obtains the maximum index of a parameter of type partype plus 1.

### Parameters

- task (MSKtask\_t) An optimization task. (input)
- partype (MSKparametertypee) Parameter type. (input)
- parammax (MSKint32t by reference) The maximum index (plus 1) of the given parameter type. (output)

Return (MSKrescodee) - The function response code.

Groups Parameters

MSK\_getparamname

```
MSKrescodee (MSKAPI MSK_getparamname) (
MSKtask_t task,
MSKparametertypee partype,
MSKint32t param,
char * parname)
```

Obtains the name for a parameter param of type partype.

### Parameters

- $\bullet$  task (MSKtask\_t) An optimization task. (input)
- partype (MSKparametertypee) Parameter type. (input)
- param (MSKint32t) Which parameter. (input)
- parname (MSKstring\_t) Parameter name. (output)

Return (MSKrescodee) - The function response code.

Groups Names, Parameters

MSK\_getprimalobj

```
MSKrescodee (MSKAPI MSK_getprimalobj) (
MSKtask_t task,
MSKsoltypee whichsol,
MSKrealt * primalobj)
```

Computes the primal objective value for the desired solution. Note that if the solution is an infeasibility certificate, then the fixed term in the objective is not included.

#### Parameters

- task (MSKtask\_t) An optimization task. (input)
- whichsol (MSKsoltypee) Selects a solution. (input)
- primalobj (MSKrealt by reference) Objective value corresponding to the primal solution. (output)

 ${\bf Return} \ \ ({\it MSKrescodee}\,) - {\bf The} \ {\bf function} \ {\bf response} \ {\bf code}.$ 

Groups Solution information, Solution - primal

MSK\_getprimalsolutionnorms

```
MSKrescodee (MSKAPI MSK_getprimalsolutionnorms) (
MSKtask_t task,
MSKsoltypee whichsol,
MSKrealt * nrmxc,
MSKrealt * nrmxx,
MSKrealt * nrmxx,
```

Compute norms of the primal solution.

#### Parameters

- task (MSKtask\_t) An optimization task. (input)
- whichsol (MSKsoltypee) Selects a solution. (input)
- nrmxc (MSKrealt by reference) The norm of the  $x^c$  vector. (output)
- nrmxx (MSKrealt by reference) The norm of the x vector. (output)
- nrmbarx (MSKrealt by reference) The norm of the  $\overline{X}$  vector. (output)

Return (MSKrescodee) - The function response code.

**Groups** Solution information

MSK\_getprobtype

```
MSKrescodee (MSKAPI MSK_getprobtype) (
MSKtask_t task,
MSKproblemtypee * probtype)
```

Obtains the problem type.

# Parameters

- task (MSKtask\_t) An optimization task. (input)
- probtype (MSKproblemtypee by reference) The problem type. (output)

Return (MSKrescodee) - The function response code.

Groups Inspecting the task

MSK\_getprosta

```
MSKrescodee (MSKAPI MSK_getprosta) (
MSKtask_t task,
MSKsoltypee whichsol,
MSKprostae * prosta)
```

Obtains the problem status.

### Parameters

• task (MSKtask\_t) - An optimization task. (input)

```
• whichsol (MSKsoltypee) - Selects a solution. (input)
```

• prosta (MSKprostae by reference) - Problem status. (output)

Return (MSKrescodee) - The function response code.

Groups Solution information

MSK\_getpviolbarvar

```
MSKrescodee (MSKAPI MSK_getpviolbarvar) (
MSKtask_t task,
MSKsoltypee whichsol,
MSKint32t num,
const MSKint32t * sub,
MSKrealt * viol)
```

Computes the primal solution violation for a set of semidefinite variables. Let  $(\overline{X}_j)^*$  be the value of the variable  $\overline{X}_j$  for the specified solution. Then the primal violation of the solution associated with variable  $\overline{X}_j$  is given by

$$\max(-\lambda_{\min}(\overline{X}_i), 0.0).$$

Both when the solution is a certificate of dual infeasibility or when it is primal feasible the violation should be small.

#### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- whichsol (MSKsoltypee) Selects a solution. (input)
- num (MSKint32t) Length of sub and viol. (input)
- sub (MSKint32t\*) An array of indexes of  $\overline{X}$  variables. (input)
- viol (MSKrealt\*) viol[k] is how much the solution violates the constraint  $\overline{X}_{\text{sub}[k]} \in \mathcal{S}_+$ . (output)

Return (MSKrescodee) - The function response code.

Groups Solution information

MSK\_getpviolcon

```
MSKrescodee (MSKAPI MSK_getpviolcon) (
   MSKtask_t task,
   MSKsoltypee whichsol,
   MSKint32t num,
   const MSKint32t * sub,
   MSKrealt * viol)
```

Computes the primal solution violation for a set of constraints. The primal violation of the solution associated with the i-th constraint is given by

$$\max(\tau l_i^c - (x_i^c)^*, \ (x_i^c)^* - \tau u_i^c), \ |\sum_{j=0}^{numvar-1} a_{ij} x_j^* - x_i^c|)$$

where  $\tau=0$  if the solution is a certificate of dual infeasibility and  $\tau=1$  otherwise. Both when the solution is a certificate of dual infeasibility and when it is primal feasible the violation should be small. The above formula applies for the linear case but is appropriately generalized in other cases.

- task (MSKtask\_t) An optimization task. (input)
- whichsol (MSKsoltypee) Selects a solution. (input)

- num (MSKint32t) Length of sub and viol. (input)
- sub (MSKint32t\*) An array of indexes of constraints. (input)
- viol (MSKrealt\*) viol[k] is the violation associated with the solution for the constraint sub[k]. (output)

Groups Solution information

MSK\_getpviolcones

```
MSKrescodee (MSKAPI MSK_getpviolcones) (
   MSKtask_t task,
   MSKsoltypee whichsol,
   MSKint32t num,
   const MSKint32t * sub,
   MSKrealt * viol)
```

Computes the primal solution violation for a set of conic constraints. Let  $x^*$  be the value of the variable x for the specified solution. For simplicity let us assume that x is a member of a quadratic cone, then the violation is computed as follows

$$\begin{cases} \max(0, ||x_{2:n}|| - x_1)/\sqrt{2}, & x_1 \ge -||x_{2:n}||, \\ ||x||, & \text{otherwise.} \end{cases}$$

Both when the solution is a certificate of dual infeasibility or when it is primal feasible the violation should be small.

### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- whichsol (MSKsoltypee) Selects a solution. (input)
- num (MSKint32t) Length of sub and viol. (input)
- sub (MSKint32t\*) An array of indexes of conic constraints. (input)
- viol (MSKrealt\*) viol[k] is the violation of the solution associated with the conic constraint number sub[k]. (output)

 $\mathbf{Return} \ \ (\textit{MSKrescodee}) - \mathbf{The} \ \mathbf{function} \ \mathbf{response} \ \mathbf{code}.$ 

**Groups** Solution information

MSK\_getpviolvar

```
MSKrescodee (MSKAPI MSK_getpviolvar) (
MSKtask_t task,
MSKsoltypee whichsol,
MSKint32t num,
const MSKint32t * sub,
MSKrealt * viol)
```

Computes the primal solution violation associated to a set of variables. Let  $x_j^*$  be the value of  $x_j$  for the specified solution. Then the primal violation of the solution associated with variable  $x_j$  is given by

$$\max(\tau l_i^x - x_i^*, \ x_i^* - \tau u_i^x, \ 0).$$

where  $\tau = 0$  if the solution is a certificate of dual infeasibility and  $\tau = 1$  otherwise. Both when the solution is a certificate of dual infeasibility and when it is primal feasible the violation should be small.

# Parameters

• task (MSKtask\_t) - An optimization task. (input)

- whichsol (MSKsoltypee) Selects a solution. (input)
- num (MSKint32t) Length of sub and viol. (input)
- sub (MSKint32t\*) An array of indexes of x variables. (input)
- viol (MSKrealt\*) viol[k] is the violation associated with the solution for the variable  $x_{\text{sub[k]}}$ . (output)

Groups Solution information

## MSK\_getqconk

```
MSKrescodee (MSKAPI MSK_getqconk) (
   MSKtask_t task,
   MSKint32t k,
   MSKint32t maxnumqcnz,
   MSKint32t * qcsurp,
   MSKint32t * numqcnz,
   MSKint32t * qcsubi,
   MSKint32t * qcsubi,
   MSKint32t * qcsubj,
   MSKrealt * qcval)
```

Obtains all the quadratic terms in a constraint. The quadratic terms are stored sequentially in qcsubi, qcsubj, and qcval.

### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- k (MSKint32t) Which constraint. (input)
- maxnumqcnz (MSKint32t) Length of the arrays qcsubi, qcsubj, and qcval. (input)
- qcsurp (MSKint32t by reference) Surplus of subscript and coefficient arrays. The required entries are stored sequentially in qssubi, qssubj and qsval starting from position qcsurp away from the end of the arrays. On return qcsurp will be decremented by the total number of non-zeros written. (input/output)
- numqcnz (MSKint32t by reference) Number of quadratic terms. (output)
- qcsubi (MSKint32t\*) Row subscripts for quadratic constraint matrix. (output)
- qcsubj (MSKint32t\*) Column subscripts for quadratic constraint matrix. (output)
- qcval (MSKrealt\*) Quadratic constraint coefficient values. (output)

Return (MSKrescodee) - The function response code.

Groups Inspecting the task, Problem data - quadratic part, Problem data - constraints

# MSK\_getqconk64

```
MSKrescodee (MSKAPI MSK_getqconk64) (
MSKtask_t task,
MSKint32t k,
MSKint64t maxnumqcnz,
MSKint64t * qcsurp,
MSKint64t * numqcnz,
MSKint32t * qcsubi,
MSKint32t * qcsubj,
MSKrealt * qcval)
```

Obtains all the quadratic terms in a constraint. The quadratic terms are stored sequentially in qcsubi, qcsubj, and qcval.

### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- k (MSKint32t) Which constraint. (input)
- maxnumqcnz (MSKint64t) Length of the arrays qcsubi, qcsubj, and qcval. (input)
- qcsurp (MSKint64t by reference) Surplus of subscript and coefficient arrays. The required entries are stored sequentially in qcsubi, qcsubj and qcval starting from position qcsurp away from the end of the arrays. On return qcsurp will be decremented by the total number of non-zeros written. (input/output)
- numqcnz (MSKint64t by reference) Number of quadratic terms. (output)
- qcsubi (MSKint32t\*) Row subscripts for quadratic constraint matrix. (output)
- qcsubj (MSKint32t\*) Column subscripts for quadratic constraint matrix. (output)
- qcval (MSKrealt\*) Quadratic constraint coefficient values. (output)

Return (MSKrescodee) - The function response code.

**Groups** Inspecting the task, Problem data - quadratic part, Problem data - constraints

MSK\_getqobj

```
MSKrescodee (MSKAPI MSK_getqobj) (
   MSKtask_t task,
   MSKint32t maxnumqonz,
   MSKint32t * qosurp,
   MSKint32t * numqonz,
   MSKint32t * qosubi,
   MSKint32t * qosubi,
   MSKint32t * qosubj,
   MSKrealt * qoval)
```

Obtains the quadratic terms in the objective. The required quadratic terms are stored sequentially in qosubi, qosubj, and qoval.

### Parameters

- $task (MSKtask_t)$  An optimization task. (input)
- maxnumqonz ( $\mathit{MSKint32t}$ ) The length of the arrays qosubi, qosubj, and qoval. (input)
- qosurp (MSKint32t by reference) Surplus of subscript and coefficient arrays. The required entries are stored sequentially in qosubi, qosubj and qoval starting from position qosurp away from the end of the arrays. On return qosurp will be decremented by the total number of non-zeros written. (input/output)
- $\bullet$  numqonz (MSKint32t by reference) Number of non-zero elements in the quadratic objective terms. (output)
- qosubj (MSKint32t\*) Column subscripts for quadratic objective coefficients. (output)
- qoval (MSKrealt\*) Quadratic objective coefficient values. (output)

Return (MSKrescodee) - The function response code.

Groups Inspecting the task, Problem data - quadratic part

MSK\_getqobj64

```
MSKrescodee (MSKAPI MSK_getqobj64) (
   MSKtask_t task,
   MSKint64t maxnumqonz,
   MSKint64t * qosurp,
   MSKint64t * numqonz,
   MSKint32t * qosubi,
   MSKint32t * qosubj,
   MSKrealt * qoval)
```

Obtains the quadratic terms in the objective. The required quadratic terms are stored sequentially in qosubj, and qoval.

### Parameters

- task (MSKtask\_t) An optimization task. (input)
- maxnumqonz (MSKint64t) The length of the arrays qosubi, qosubj, and qoval. (input)
- qosurp (MSKint64t by reference) Surplus of subscript and coefficient arrays. The required entries are stored sequentially in qosubi, qosubj and qoval starting from position qosurp away from the end of the arrays. On return qosurp will be decremented by the total number of non-zeros written. (input/output)
- numqonz (MSKint64t by reference) Number of non-zero elements in the quadratic objective terms. (output)
- qosubi (MSKint32t\*) Row subscripts for quadratic objective coefficients. (output)
- qosubj (MSKint32t\*) Column subscripts for quadratic objective coefficients.
   (output)
- qoval (MSKrealt\*) Quadratic objective coefficient values. (output)

Return (MSKrescodee) - The function response code.

Groups Inspecting the task, Problem data - quadratic part

MSK\_getqobjij

```
MSKrescodee (MSKAPI MSK_getqobjij) (
    MSKtask_t task,
    MSKint32t i,
    MSKint32t j,
    MSKrealt * qoij)
```

Obtains one coefficient  $q_{ij}^o$  in the quadratic term of the objective.

# **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- i (MSKint32t) Row index of the coefficient. (input)
- j (MSKint32t) Column index of coefficient. (input)
- qoij (MSKrealt by reference) The required coefficient. (output)

Return (MSKrescodee) - The function response code.

**Groups** Inspecting the task, Problem data - quadratic part

MSK\_getreducedcosts

```
MSKrescodee (MSKAPI MSK_getreducedcosts) (
   MSKtask_t task,
   MSKsoltypee whichsol,
   MSKint32t first,
   MSKint32t last,
   MSKrealt * redcosts)
```

Computes the reduced costs for a slice of variables and returns them in the array redcosts i.e.

$$redcosts[j-first] = (s_l^x)_j - (s_u^x)_j, \ j = first, \dots, last - 1$$
 (15.2)

### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- whichsol (MSKsoltypee) Selects a solution. (input)
- first (MSKint32t) The index of the first variable in the sequence. (input)
- last (MSKint32t) The index of the last variable in the sequence plus 1. (input)
- redcosts (MSKrealt\*) The reduced costs for the required slice of variables. (output)

 ${\bf Return} \ \ ({\it MSKrescodee}\,) - {\bf The} \ {\bf function} \ {\bf response} \ {\bf code}.$ 

Groups Solution - dual

 ${\tt MSK\_getresponseclass}$ 

```
MSKrescodee (MSKAPI MSK_getresponseclass) (
MSKrescodee r,
MSKrescodetypee * rc)
```

Obtain the class of a response code.

# Parameters

- $\bullet$  r (MSKrescodee) A response code indicating the result of function call. (input)
- rc (MSKrescodetypee by reference) The response class. (output)

Return (MSKrescodee) - The function response code.

Groups Responses, errors and warnings

MSK\_getskc

```
MSKrescodee (MSKAPI MSK_getskc) (
MSKtask_t task,
MSKsoltypee whichsol,
MSKstakeye * skc)
```

Obtains the status keys for the constraints.

# Parameters

- task (MSKtask\_t) An optimization task. (input)
- whichsol (MSKsoltypee) Selects a solution. (input)
- skc (MSKstakeye\*) Status keys for the constraints. (output)

Return (MSKrescodee) - The function response code.

**Groups** Solution information

MSK\_getskcslice

```
MSKrescodee (MSKAPI MSK_getskcslice) (
MSKtask_t task,
MSKsoltypee whichsol,
MSKint32t first,
MSKint32t last,
MSKstakeye * skc)
```

Obtains the status keys for a slice of the constraints.

- task (MSKtask\_t) An optimization task. (input)
- whichsol (MSKsoltypee) Selects a solution. (input)
- first (MSKint32t) First index in the sequence. (input)
- last (MSKint32t) Last index plus 1 in the sequence. (input)
- skc (MSKstakeye\*) Status keys for the constraints. (output)

**Groups** Solution information

# MSK\_getskn

```
MSKrescodee (MSKAPI MSK_getskn) (
MSKtask_t task,
MSKsoltypee whichsol,
MSKstakeye * skn)
```

Obtains the status keys for the conic constraints.

#### Parameters

- task (MSKtask\_t) An optimization task. (input)
- whichsol (MSKsoltypee) Selects a solution. (input)
- skn (MSKstakeye\*) Status keys for the conic constraints. (output)

Return (MSKrescodee) - The function response code.

**Groups** Solution information

## MSK\_getskx

```
MSKrescodee (MSKAPI MSK_getskx) (
   MSKtask_t task,
   MSKsoltypee whichsol,
   MSKstakeye * skx)
```

Obtains the status keys for the scalar variables.

## Parameters

- task (MSKtask\_t) An optimization task. (input)
- whichsol (MSKsoltypee) Selects a solution. (input)
- skx (MSKstakeye\*) Status keys for the variables. (output)

 ${\bf Return} \ ({\it MSKrescodee}\,)$  — The function response code.

Groups Solution information

# MSK\_getskxslice

```
MSKrescodee (MSKAPI MSK_getskxslice) (
MSKtask_t task,
MSKsoltypee whichsol,
MSKint32t first,
MSKint32t last,
MSKstakeye * skx)
```

Obtains the status keys for a slice of the scalar variables.

- task (MSKtask\_t) An optimization task. (input)
- whichsol (MSKsoltypee) Selects a solution. (input)
- first (MSKint32t) First index in the sequence. (input)

```
• last (MSKint32t) - Last index plus 1 in the sequence. (input)
```

• skx (MSKstakeye\*) - Status keys for the variables. (output)

Return (MSKrescodee) - The function response code.

**Groups** Solution information

MSK\_getslc

```
MSKrescodee (MSKAPI MSK_getslc) (
MSKtask_t task,
MSKsoltypee whichsol,
MSKrealt * slc)
```

Obtains the  $s_l^c$  vector for a solution.

#### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- whichsol (MSKsoltypee) Selects a solution. (input)
- slc (MSKrealt\*) Dual variables corresponding to the lower bounds on the constraints. (output)

Return (MSKrescodee) - The function response code.

Groups Solution - dual

MSK\_getslcslice

```
MSKrescodee (MSKAPI MSK_getslcslice) (
   MSKtask_t task,
   MSKsoltypee whichsol,
   MSKint32t first,
   MSKint32t last,
   MSKrealt * slc)
```

Obtains a slice of the  $s_l^c$  vector for a solution.

### Parameters

- task (MSKtask\_t) An optimization task. (input)
- whichsol (MSKsoltypee) Selects a solution. (input)
- $\bullet$  first (MSKint32t) First index in the sequence. (input)
- last (MSKint32t) Last index plus 1 in the sequence. (input)
- slc (MSKrealt\*) Dual variables corresponding to the lower bounds on the constraints. (output)

Return (MSKrescodee) - The function response code.

Groups Solution - dual

MSK\_getslx

```
MSKrescodee (MSKAPI MSK_getslx) (
MSKtask_t task,
MSKsoltypee whichsol,
MSKrealt * slx)
```

Obtains the  $s_l^x$  vector for a solution.

- task (MSKtask\_t) An optimization task. (input)
- whichsol (MSKsoltypee) Selects a solution. (input)

• slx (MSKrealt\*) – Dual variables corresponding to the lower bounds on the variables. (output)

Return (MSKrescodee) - The function response code.

Groups Solution - dual

MSK\_getslxslice

```
MSKrescodee (MSKAPI MSK_getslxslice) (
MSKtask_t task,
MSKsoltypee whichsol,
MSKint32t first,
MSKint32t last,
MSKrealt * slx)
```

Obtains a slice of the  $s_l^x$  vector for a solution.

### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- whichsol (MSKsoltypee) Selects a solution. (input)
- first (MSKint32t) First index in the sequence. (input)
- last (MSKint32t) Last index plus 1 in the sequence. (input)
- slx (MSKrealt\*) Dual variables corresponding to the lower bounds on the variables. (output)

Return (MSKrescodee) - The function response code.

Groups Solution - dual

MSK\_getsnx

```
MSKrescodee (MSKAPI MSK_getsnx) (
   MSKtask_t task,
   MSKsoltypee whichsol,
   MSKrealt * snx)
```

Obtains the  $s_n^x$  vector for a solution.

# Parameters

- task (MSKtask\_t) An optimization task. (input)
- whichsol (MSKsoltypee) Selects a solution. (input)
- snx (MSKrealt\*) Dual variables corresponding to the conic constraints on the variables. (output)

Return (MSKrescodee) - The function response code.

Groups Solution - dual

MSK\_getsnxslice

```
MSKrescodee (MSKAPI MSK_getsnxslice) (
MSKtask_t task,
MSKsoltypee whichsol,
MSKint32t first,
MSKint32t last,
MSKrealt * snx)
```

Obtains a slice of the  $s_n^x$  vector for a solution.

# **Parameters**

• task (MSKtask\_t) - An optimization task. (input)

- whichsol (MSKsoltypee) Selects a solution. (input)
- first (MSKint32t) First index in the sequence. (input)
- last (MSKint32t) Last index plus 1 in the sequence. (input)
- snx (MSKrealt\*) Dual variables corresponding to the conic constraints on the variables. (output)

Groups Solution - dual

MSK\_getsolsta

```
MSKrescodee (MSKAPI MSK_getsolsta) (
MSKtask_t task,
MSKsoltypee whichsol,
MSKsolstae * solsta)
```

Obtains the solution status.

### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- whichsol (MSKsoltypee) Selects a solution. (input)
- solsta (MSKsolstae by reference) Solution status. (output)

Return (MSKrescodee) - The function response code.

Groups Solution information

MSK\_getsolution

```
MSKrescodee (MSKAPI MSK_getsolution) (
 MSKtask_t task,
 MSKsoltypee whichsol,
 MSKprostae * prosta,
 MSKsolstae * solsta,
 MSKstakeye * skc,
 MSKstakeye * skx,
 MSKstakeye * skn,
 MSKrealt * xc,
 MSKrealt * xx,
 MSKrealt * y,
 MSKrealt * slc,
 MSKrealt * suc,
 MSKrealt * slx,
 MSKrealt * sux,
 MSKrealt * snx)
```

Obtains the complete solution.

Consider the case of linear programming. The primal problem is given by

and the corresponding dual problem is

A conic optimization problem has the same primal variables as in the linear case. Recall that the dual of a conic optimization problem is given by:

$$\begin{array}{lll} \text{maximize} & (l^c)^T s_l^c - (u^c)^T s_u^c \\ & + (l^x)^T s_l^x - (u^x)^T s_u^x + c^f \\ \text{subject to} & A^T y + s_l^x - s_u^x + s_n^x & = & c, \\ & -y + s_l^c - s_u^c & = & 0, \\ & s_l^c, s_u^c, s_l^x, s_u^x & \geq & 0, \\ & s_n^x \in \mathcal{K}^* & \end{array}$$

The mapping between variables and arguments to the function is as follows:

- xx: Corresponds to variable x (also denoted  $x^x$ ).
- xc : Corresponds to  $x^c := Ax$ .
- y: Corresponds to variable y.
- slc: Corresponds to variable  $s_i^c$ .
- suc: Corresponds to variable  $s_u^c$ .
- slx: Corresponds to variable  $s_l^x$ .
- sux: Corresponds to variable  $s_u^x$ .
- snx: Corresponds to variable  $s_n^x$ .

The meaning of the values returned by this function depend on the *solution status* returned in the argument solsta. The most important possible values of solsta are:

- MSK\_SOL\_STA\_OPTIMAL : An optimal solution satisfying the optimality criteria for continuous problems is returned.
- MSK\_SOL\_STA\_INTEGER\_OPTIMAL: An optimal solution satisfying the optimality criteria for integer problems is returned.
- MSK\_SOL\_STA\_PRIM\_FEAS: A solution satisfying the feasibility criteria.
- $\bullet$   $\textit{MSK\_SOL\_STA\_PRIM\_INFEAS\_CER}$  : A primal certificate of infeasibility is returned.
- MSK\_SOL\_STA\_DUAL\_INFEAS\_CER : A dual certificate of infeasibility is returned.

In order to retrieve the primal and dual values of semidefinite variables see  $MSK\_getbarxj$  and  $MSK\_getbarsj$ .

- task (MSKtask\_t) An optimization task. (input)
- whichsol (MSKsoltypee) Selects a solution. (input)
- prosta (MSKprostae by reference) Problem status. (output)
- solsta (MSKsolstae by reference) Solution status. (output)
- skc (MSKstakeye\*) Status keys for the constraints. (output)
- skx (MSKstakeye\*) Status keys for the variables. (output)
- skn (MSKstakeye\*) Status keys for the conic constraints. (output)
- xc (MSKrealt\*) Primal constraint solution. (output)
- xx (MSKrealt\*) Primal variable solution. (output)
- y (MSKrealt\*) Vector of dual variables corresponding to the constraints. (output)
- slc (MSKrealt\*) Dual variables corresponding to the lower bounds on the constraints. (output)
- suc (MSKrealt\*) Dual variables corresponding to the upper bounds on the constraints. (output)
- slx (MSKrealt\*) Dual variables corresponding to the lower bounds on the variables. (output)
- sux (MSKrealt\*) Dual variables corresponding to the upper bounds on the variables. (output)

• snx (MSKrealt\*) – Dual variables corresponding to the conic constraints on the variables. (output)

Return (MSKrescodee) - The function response code.

Groups Solution information, Solution - primal, Solution - dual

 ${\tt MSK\_getsolutioninfo}$ 

```
MSKrescodee (MSKAPI MSK_getsolutioninfo) (
    MSKtask_t task,
    MSKsoltypee whichsol,
    MSKrealt * pobj,
    MSKrealt * pviolcon,
    MSKrealt * pviolvar,
    MSKrealt * pviolbarvar,
    MSKrealt * pviolbarvar,
    MSKrealt * pviolcone,
    MSKrealt * pviolitg,
    MSKrealt * dobj,
    MSKrealt * dviolcon,
    MSKrealt * dviolvar,
    MSKrealt * dviolbarvar,
    MSKrealt * dviolbarvar,
    MSKrealt * dviolcone)
```

Obtains information about a solution.

- task (MSKtask\_t) An optimization task. (input)
- whichsol (MSKsoltypee) Selects a solution. (input)
- pobj (MSKrealt by reference) The primal objective value as computed by MSK\_getprimalobj. (output)
- pviolcon (MSKrealt by reference) Maximal primal violation of the solution associated with the  $x^c$  variables where the violations are computed by  $MSK_qetpviolcon$ . (output)
- pviolvar (MSKrealt by reference) Maximal primal violation of the solution for the x variables where the violations are computed by MSK\_getpviolvar. (output)
- pviolbarvar (MSKrealt by reference) Maximal primal violation of solution for the  $\overline{X}$  variables where the violations are computed by MSK\_getpviolbarvar. (output)
- pviolcone (MSKrealt by reference) Maximal primal violation of solution for the conic constraints where the violations are computed by MSK\_getpviolcones. (output)
- pviolitg (MSKrealt by reference) Maximal violation in the integer constraints. The violation for an integer variable  $x_j$  is given by  $\min(x_j \lfloor x_j \rfloor, \lceil x_j \rceil x_j)$ . This number is always zero for the interior-point and basic solutions. (output)
- dobj (MSKrealt by reference) Dual objective value as computed by MSK\_getdualobj. (output)
- dviolcon (MSKrealt by reference) Maximal violation of the dual solution associated with the  $x^c$  variable as computed by MSK\_qetdviolcon. (output)
- dviolvar (MSKrealt by reference) Maximal violation of the dual solution associated with the x variable as computed by MSK\_getdviolvar. (output)
- dviolbarvar (MSKrealt by reference) Maximal violation of the dual solution associated with the  $\overline{S}$  variable as computed by MSK\_getdviolbarvar. (output)
- dviolcone ( ${\it MSKrealt~by~reference}$ ) Maximal violation of the dual solution associated with the dual conic constraints as computed by  ${\it MSK\_getdviolcones}$ . (output)

```
Return (MSKrescodee) - The function response code.
Groups Solution information
```

MSK\_getsolutionslice

```
MSKrescodee (MSKAPI MSK_getsolutionslice) (
MSKtask_t task,
MSKsoltypee whichsol,
MSKsoliteme solitem,
MSKint32t first,
MSKint32t last,
MSKrealt * values)
```

Obtains a slice of one item from the solution. The format of the solution is exactly as in  $MSK\_getsolution$ . The parameter solitem determines which of the solution vectors should be returned.

### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- whichsol (MSKsoltypee) Selects a solution. (input)
- solitem (MSKsoliteme) Which part of the solution is required. (input)
- first (MSKint32t) First index in the sequence. (input)
- last (MSKint32t) Last index plus 1 in the sequence. (input)
- values (MSKrealt\*) The values in the required sequence are stored sequentially in values. (output)

Return (MSKrescodee) - The function response code.

Groups Solution - primal, Solution - dual, Solution information

MSK\_getsparsesymmat

```
MSKrescodee (MSKAPI MSK_getsparsesymmat) (
   MSKtask_t task,
   MSKint64t idx,
   MSKint64t maxlen,
   MSKint32t * subi,
   MSKint32t * subj,
   MSKrealt * valij)
```

Get a single symmetric matrix from the matrix store.

# Parameters

- task (MSKtask\_t) An optimization task. (input)
- idx (MSKint64t) Index of the matrix to retrieve. (input)
- maxlen  $(\mathit{MSKint64t})$  Length of the output arrays subi, subj and valij. (input)
- subi (MSKint32t\*) Row subscripts of the matrix non-zero elements. (output)
- subj (MSKint32t\*) Column subscripts of the matrix non-zero elements. (output)
- valij (MSKrealt\*) Coefficients of the matrix non-zero elements. (output)

Return (MSKrescodee) - The function response code.

**Groups** Problem data - semidefinite, Inspecting the task

MSK\_getstrparam

```
MSKrescodee (MSKAPI MSK_getstrparam) (
MSKtask_t task,
MSKsparame param,
MSKint32t maxlen,
MSKint32t * len,
char * parvalue)
```

Obtains the value of a string parameter.

## Parameters

- task (MSKtask\_t) An optimization task. (input)
- param (MSKsparame) Which parameter. (input)
- maxlen (MSKint32t) Length of the parvalue buffer. (input)
- len (MSKint32t by reference) The length of the parameter value. (output)
- parvalue (MSKstring\_t) Parameter value. (output)

Return (MSKrescodee) - The function response code.

Groups Names, Parameters

MSK\_getstrparamal

```
MSKrescodee (MSKAPI MSK_getstrparamal) (
    MSKtask_t task,
    MSKsparame param,
    MSKint32t numaddchr,
    MSKstring_t * value)
```

Obtains the value of a string parameter.

### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- param (MSKsparame) Which parameter. (input)
- numaddchr (MSKint32t) Number of additional characters for which room is left in value. (input)
- value (MSKstring\_t by reference) Parameter value. MOSEK will allocate this char buffer of size equal to the actual length of the string parameter plus numaddchr. This memory must be freed by MSK\_freetask. (input/output)

Return (MSKrescodee) - The function response code.

Groups Parameters

MSK\_getstrparamlen

```
MSKrescodee (MSKAPI MSK_getstrparamlen) (
MSKtask_t task,
MSKsparame param,
MSKint32t * len)
```

Obtains the length of a string parameter.

### Parameters

- task (MSKtask\_t) An optimization task. (input)
- param (MSKsparame) Which parameter. (input)
- len (MSKint32t by reference) The length of the parameter value. (output)

Return (MSKrescodee) - The function response code.

Groups Names, Parameters

```
MSKrescodee (MSKAPI MSK_getsuc) (
    MSKtask_t task,
    MSKsoltypee whichsol,
    MSKrealt * suc)
```

Obtains the  $s_u^c$  vector for a solution.

## **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- whichsol (MSKsoltypee) Selects a solution. (input)
- suc (MSKrealt\*) Dual variables corresponding to the upper bounds on the constraints. (output)

Return (MSKrescodee) - The function response code. Groups Solution - dual

MSK\_getsucslice

```
MSKrescodee (MSKAPI MSK_getsucslice) (
MSKtask_t task,
MSKsoltypee whichsol,
MSKint32t first,
MSKint32t last,
MSKrealt * suc)
```

Obtains a slice of the  $s_u^c$  vector for a solution.

# Parameters

- task (MSKtask\_t) An optimization task. (input)
- whichsol (MSKsoltypee) Selects a solution. (input)
- first (MSKint32t) First index in the sequence. (input)
- last (MSKint32t) Last index plus 1 in the sequence. (input)
- suc (MSKrealt\*) Dual variables corresponding to the upper bounds on the constraints. (output)

Return (MSKrescodee) - The function response code. Groups Solution - dual

MSK\_getsux

```
MSKrescodee (MSKAPI MSK_getsux) (
MSKtask_t task,
MSKsoltypee whichsol,
MSKrealt * sux)
```

Obtains the  $s_u^x$  vector for a solution.

### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- whichsol (MSKsoltypee) Selects a solution. (input)
- sux (MSKrealt\*) Dual variables corresponding to the upper bounds on the variables. (output)

Return (MSKrescodee) - The function response code.

Groups Solution - dual

```
MSKrescodee (MSKAPI MSK_getsuxslice) (
MSKtask_t task,
MSKsoltypee whichsol,
MSKint32t first,
MSKint32t last,
MSKrealt * sux)
```

Obtains a slice of the  $s_u^x$  vector for a solution.

## Parameters

- task (MSKtask\_t) An optimization task. (input)
- whichsol (MSKsoltypee) Selects a solution. (input)
- first (MSKint32t) First index in the sequence. (input)
- last (MSKint32t) Last index plus 1 in the sequence. (input)
- sux (MSKrealt\*) Dual variables corresponding to the upper bounds on the variables. (output)

Return (MSKrescodee) - The function response code.

Groups Solution - dual

MSK\_getsymbcon

```
MSKrescodee (MSKAPI MSK_getsymbcon) (
MSKtask_t task,
MSKint32t i,
MSKint32t sizevalue,
char * name,
MSKint32t * value)
```

Obtains the name and corresponding value for the ith symbolic constant.

# Parameters

- task (MSKtask\_t) An optimization task. (input)
- i(MSKint32t) Index. (input)
- sizevalue (MSKint32t) The length of the buffer pointed to by the value argument. (input)
- name (MSKstring\_t) Name of the ith symbolic constant. (output)
- value (MSKint32t by reference) The corresponding value. (output)

Return (MSKrescodee) - The function response code.

Groups Names

MSK\_getsymbcondim

```
MSKrescodee (MSKAPI MSK_getsymbcondim) (
    MSKenv_t env,
    MSKint32t * num,
    size_t * maxlen)
```

Obtains the number of symbolic constants defined by **MOSEK** and the maximum length of the name of any symbolic constant.

- env (MSKenv\_t) The MOSEK environment. (input)
- num (MSKint32t by reference) Number of symbolic constants defined by MOSEK. (output)

• maxlen (size\_t by reference) - Maximum length of the name of any symbolic constants. (output)

Return (MSKrescodee) - The function response code.

Groups Parameters

MSK\_getsymmatinfo

```
MSKrescodee (MSKAPI MSK_getsymmatinfo) (
MSKtask_t task,
MSKint64t idx,
MSKint32t * dim,
MSKint64t * nz,
MSKsymmattypee * type)
```

**MOSEK** maintains a vector denoted by E of symmetric data matrices. This function makes it possible to obtain important information about a single matrix in E.

## **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- idx (MSKint64t) Index of the matrix for which information is requested. (input)
- dim (MSKint32t by reference) Returns the dimension of the requested matrix. (output)
- nz (MSKint64t by reference) Returns the number of non-zeros in the requested matrix. (output)
- type (MSKsymmattypee by reference) Returns the type of the requested matrix. (output)

Return (MSKrescodee) - The function response code.

Groups Problem data - semidefinite, Inspecting the task

MSK\_gettaskname

```
MSKrescodee (MSKAPI MSK_gettaskname) (
MSKtask_t task,
MSKint32t sizetaskname,
char * taskname)
```

Obtains the name assigned to the task.

# Parameters

- task (MSKtask\_t) An optimization task. (input)
- sizetaskname (MSKint32t) Length of the taskname buffer. (input)
- taskname (MSKstring\_t) Returns the task name. (output)

Return (MSKrescodee) - The function response code.

**Groups** Names, Inspecting the task

MSK\_gettasknamelen

```
MSKrescodee (MSKAPI MSK_gettasknamelen) (
MSKtask_t task,
MSKint32t * len)
```

Obtains the length the task name.

### Parameters

• task (MSKtask\_t) - An optimization task. (input)

```
ullet len (MSKint32t by reference) – Returns the length of the task name. (output) Return (MSKrescodee) – The function response code.
```

Groups Names, Inspecting the task

MSK\_getvarbound

```
MSKrescodee (MSKAPI MSK_getvarbound) (
MSKtask_t task,
MSKint32t i,
MSKboundkeye * bk,
MSKrealt * bl,
MSKrealt * bu)
```

Obtains bound information for one variable.

## **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- i (MSKint32t) Index of the variable for which the bound information should be obtained. (input)
- bk (MSKboundkeye by reference) Bound keys. (output)
- bl (MSKrealt by reference) Values for lower bounds. (output)
- bu (MSKrealt by reference) Values for upper bounds. (output)

Return (MSKrescodee) - The function response code.

**Groups** Problem data - linear part, Inspecting the task, Problem data - bounds, Problem data - variables

MSK\_getvarboundslice

```
MSKrescodee (MSKAPI MSK_getvarboundslice) (
MSKtask_t task,
MSKint32t first,
MSKint32t last,
MSKboundkeye * bk,
MSKrealt * bl,
MSKrealt * bu)
```

Obtains bounds information for a slice of the variables.

# Parameters

- task (MSKtask\_t) An optimization task. (input)
- first (MSKint32t) First index in the sequence. (input)
- last (MSKint32t) Last index plus 1 in the sequence. (input)
- bk (MSKboundkeye\*) Bound keys. (output)
- bl (MSKrealt\*) Values for lower bounds. (output)
- bu (MSKrealt\*) Values for upper bounds. (output)

Return (MSKrescodee) - The function response code.

**Groups** Problem data - linear part, Inspecting the task, Problem data - bounds, Problem data - variables

MSK\_getvarname

```
MSKrescodee (MSKAPI MSK_getvarname) (
MSKtask_t task,
MSKint32t j,
MSKint32t sizename,
char * name)
```

Obtains the name of a variable.

### Parameters

- task (MSKtask\_t) An optimization task. (input)
- j (MSKint32t) Index of a variable. (input)
- sizename (MSKint32t) The length of the buffer pointed to by the name argument. (input)
- name (MSKstring\_t) Returns the required name. (output)

Return (MSKrescodee) - The function response code.

**Groups** Names, Problem data - linear part, Problem data - variables, Inspecting the task

MSK\_getvarnameindex

```
MSKrescodee (MSKAPI MSK_getvarnameindex) (
   MSKtask_t task,
   const char * somename,
   MSKint32t * asgn,
   MSKint32t * index)
```

Checks whether the name somename has been assigned to any variable. If so, the index of the variable is reported.

#### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- somename (MSKstring\_t) The name which should be checked. (input)
- asgn (MSKint32t by reference) Is non-zero if the name somename is assigned to a variable. (output)
- index (MSKint32t by reference) If the name somename is assigned to a variable, then index is the index of the variable. (output)

Return (MSKrescodee) - The function response code.

**Groups** Names, Problem data - linear part, Problem data - variables, Inspecting the task

MSK\_getvarnamelen

```
MSKrescodee (MSKAPI MSK_getvarnamelen) (
MSKtask_t task,
MSKint32t i,
MSKint32t * len)
```

Obtains the length of the name of a variable.

### Parameters

- $task (MSKtask_t)$  An optimization task. (input)
- i (MSKint32t) Index of a variable. (input)
- len (MSKint32t by reference) Returns the length of the indicated name. (output)

Return (MSKrescodee) - The function response code.

**Groups** Names, Problem data - linear part, Problem data - variables, Inspecting the

MSK\_getvartype

```
MSKrescodee (MSKAPI MSK_getvartype) (
MSKtask_t task,
MSKint32t j,
MSKvariabletypee * vartype)
```

Gets the variable type of one variable.

### Parameters

- task (MSKtask\_t) An optimization task. (input)
- j (MSKint32t) Index of the variable. (input)
- vartype (MSKvariabletypee by reference) Variable type of the j-th variable. (output)

Return (MSKrescodee) - The function response code.

Groups Inspecting the task, Problem data - variables

# ${\tt MSK\_getvartypelist}$

```
MSKrescodee (MSKAPI MSK_getvartypelist) (
MSKtask_t task,
MSKint32t num,
const MSKint32t * subj,
MSKvariabletypee * vartype)
```

Obtains the variable type of one or more variables. Upon return vartype[k] is the variable type of variable subj[k].

### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- num (MSKint32t) Number of variables for which the variable type should be obtained. (input)
- subj (MSKint32t\*) A list of variable indexes. (input)
- vartype (MSKvariabletypee\*) The variables types corresponding to the variables specified by subj. (output)

Return (MSKrescodee) - The function response code.

Groups Inspecting the task, Problem data - variables

# ${\tt MSK\_getversion}$

```
MSKrescodee (MSKAPI MSK_getversion) (
    MSKint32t * major,
    MSKint32t * minor,
    MSKint32t * revision)
```

Obtains **MOSEK** version information.

## Parameters

- major (MSKint32t by reference) Major version number. (output)
- minor (MSKint32t by reference) Minor version number. (output)
- revision (MSKint32t by reference) Revision number. (output)

Return (MSKrescodee) - The function response code.

Groups Versions

MSK\_getxc

```
MSKrescodee (MSKAPI MSK_getxc) (
   MSKtask_t task,
   MSKsoltypee whichsol,
   MSKrealt * xc)
```

Obtains the  $x^c$  vector for a solution.

### Parameters

- task (MSKtask\_t) An optimization task. (input)
- whichsol (MSKsoltypee) Selects a solution. (input)
- xc (MSKrealt\*) Primal constraint solution. (output)

Return (MSKrescodee) - The function response code.

Groups Solution - primal

MSK\_getxcslice

```
MSKrescodee (MSKAPI MSK_getxcslice) (
MSKtask_t task,
MSKsoltypee whichsol,
MSKint32t first,
MSKint32t last,
MSKrealt * xc)
```

Obtains a slice of the  $x^c$  vector for a solution.

## Parameters

- task (MSKtask\_t) An optimization task. (input)
- whichsol (MSKsoltypee) Selects a solution. (input)
- first (MSKint32t) First index in the sequence. (input)
- last (MSKint32t) Last index plus 1 in the sequence. (input)
- xc (MSKrealt\*) Primal constraint solution. (output)

 ${\bf Return} \ \ ({\it MSKrescodee}\,) - {\bf The} \ {\bf function} \ {\bf response} \ {\bf code}.$ 

Groups Solution - primal

MSK\_getxx

```
MSKrescodee (MSKAPI MSK_getxx) (
MSKtask_t task,
MSKsoltypee whichsol,
MSKrealt * xx)
```

Obtains the  $x^x$  vector for a solution.

# Parameters

- task (MSKtask\_t) An optimization task. (input)
- whichsol (MSKsoltypee) Selects a solution. (input)
- xx (MSKrealt\*) Primal variable solution. (output)

Return (MSKrescodee) - The function response code.

Groups Solution - primal

MSK\_getxxslice

```
MSKrescodee (MSKAPI MSK_getxxslice) (
MSKtask_t task,
MSKsoltypee whichsol,
MSKint32t first,
MSKint32t last,
MSKrealt * xx)
```

Obtains a slice of the  $x^x$  vector for a solution.

### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- whichsol (MSKsoltypee) Selects a solution. (input)
- first (MSKint32t) First index in the sequence. (input)
- last (MSKint32t) Last index plus 1 in the sequence. (input)
- xx (MSKrealt\*) Primal variable solution. (output)

Return (MSKrescodee) - The function response code.

Groups Solution - primal

MSK\_gety

```
MSKrescodee (MSKAPI MSK_gety) (
MSKtask_t task,
MSKsoltypee whichsol,
MSKrealt * y)
```

Obtains the y vector for a solution.

### Parameters

- task (MSKtask\_t) An optimization task. (input)
- whichsol (MSKsoltypee) Selects a solution. (input)
- y (MSKrealt\*) Vector of dual variables corresponding to the constraints. (output)

Return (MSKrescodee) - The function response code.

Groups Solution - dual

MSK\_getyslice

```
MSKrescodee (MSKAPI MSK_getyslice) (
MSKtask_t task,
MSKsoltypee whichsol,
MSKint32t first,
MSKint32t last,
MSKrealt * y)
```

Obtains a slice of the y vector for a solution.

### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- whichsol (MSKsoltypee) Selects a solution. (input)
- first (MSKint32t) First index in the sequence. (input)
- last (MSKint32t) Last index plus 1 in the sequence. (input)
- y (MSKrealt\*) Vector of dual variables corresponding to the constraints. (output)

Return (MSKrescodee) - The function response code.

Groups Solution - dual

```
MSKrescodee (MSKAPI MSK_initbasissolve) (
    MSKtask_t task,
    MSKint32t * basis)
```

Prepare a task for use with the MSK\_solvewithbasis function.

This function should be called

- immediately before the first call to MSK\_solvewithbasis, and
- immediately before any subsequent call to MSK\_solvewithbasis if the task has been modified.

If the basis is singular i.e. not invertible, then the error MSK\_RES\_ERR\_BASIS\_SINGULAR is reported.

### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- basis  $(\mathit{MSKint32t*})$  The array of basis indexes to use. The array is interpreted as follows: If  $\mathtt{basis}[i] \leq numcon 1$ , then  $x^c_{\mathtt{basis}[i]}$  is in the basis at position i, otherwise  $x_{\mathtt{basis}[i]-\mathtt{numcon}}$  is in the basis at position i. (output)

**Return** (MSKrescodee) - The function response code.

**Groups** Solving systems with basis matrix

### MSK\_inputdata

```
MSKrescodee (MSKAPI MSK_inputdata) (
  MSKtask_t task,
 MSKint32t maxnumcon,
 MSKint32t maxnumvar,
  MSKint32t numcon,
  MSKint32t numvar,
  const MSKrealt * c,
 MSKrealt cfix,
  const MSKint32t * aptrb,
  const MSKint32t * aptre,
  const MSKint32t * asub,
  const MSKrealt * aval,
  const MSKboundkeye * bkc,
  const MSKrealt * blc,
  const MSKrealt * buc,
  const MSKboundkeye * bkx,
  const MSKrealt * blx,
  const MSKrealt * bux)
```

Input the linear part of an optimization problem.

The non-zeros of A are inputted column-wise in the format described in Section Column or Row Ordered Sparse Matrix.

For an explained code example see Section Linear Optimization and Section Matrix Formats.

- task (MSKtask\_t) An optimization task. (input)
- maxnumcon (MSKint32t) Number of preallocated constraints in the optimization task. (input)
- maxnumvar (MSKint32t) Number of preallocated variables in the optimization task. (input)
- numcon (MSKint32t) Number of constraints. (input)
- numvar (MSKint32t) Number of variables. (input)

- c (MSKrealt\*) Linear terms of the objective as a dense vector. The length is the number of variables. (input)
- cfix (MSKrealt) Fixed term in the objective. (input)
- aptrb (MSKint32t\*) Row or column start pointers. (input)
- aptre (MSKint32t\*) Row or column end pointers. (input)
- asub (MSKint32t\*) Coefficient subscripts. (input)
- aval (MSKrealt\*) Coefficient values. (input)
- bkc (MSKboundkeye\*) Bound keys for the constraints. (input)
- blc (MSKrealt\*) Lower bounds for the constraints. (input)
- buc (MSKrealt\*) Upper bounds for the constraints. (input)
- bkx (MSKboundkeye\*) Bound keys for the variables. (input)
- blx (MSKrealt\*) Lower bounds for the variables. (input)
- bux (MSKrealt\*) Upper bounds for the variables. (input)

Groups Problem data - linear part, Problem data - bounds, Problem data - constraints

# MSK\_inputdata64

```
MSKrescodee (MSKAPI MSK_inputdata64) (
 MSKtask_t task,
 MSKint32t maxnumcon,
 MSKint32t maxnumvar,
 MSKint32t numcon,
 MSKint32t numvar,
 const MSKrealt * c,
 MSKrealt cfix,
  const MSKint64t * aptrb,
 const MSKint64t * aptre,
 const MSKint32t * asub,
 const MSKrealt * aval,
 const MSKboundkeye * bkc,
 const MSKrealt * blc,
 const MSKrealt * buc,
 const MSKboundkeye * bkx,
  const MSKrealt * blx,
  const MSKrealt * bux)
```

Input the linear part of an optimization problem.

The non-zeros of A are inputted column-wise in the format described in Section Column or Row Ordered Sparse Matrix.

For an explained code example see Section Linear Optimization and Section Matrix Formats.

- task (MSKtask\_t) An optimization task. (input)
- maxnumcon (MSKint32t) Number of preallocated constraints in the optimization task. (input)
- maxnumvar (MSKint32t) Number of preallocated variables in the optimization task. (input)
- numcon (MSKint32t) Number of constraints. (input)
- numvar (MSKint32t) Number of variables. (input)
- c (MSKrealt\*) Linear terms of the objective as a dense vector. The length is the number of variables. (input)
- cfix (MSKrealt) Fixed term in the objective. (input)
- aptrb (MSKint64t\*) Row or column start pointers. (input)

```
• aptre (MSKint64t*) - Row or column end pointers. (input)
```

- asub (MSKint32t\*) Coefficient subscripts. (input)
- aval (MSKrealt\*) Coefficient values. (input)
- bkc (MSKboundkeye\*) Bound keys for the constraints. (input)
- blc (MSKrealt\*) Lower bounds for the constraints. (input)
- buc (MSKrealt\*) Upper bounds for the constraints. (input)
- bkx (MSKboundkeye\*) Bound keys for the variables. (input)
- blx (MSKrealt\*) Lower bounds for the variables. (input)
- bux (MSKrealt\*) Upper bounds for the variables. (input)

Groups Problem data - linear part, Problem data - bounds, Problem data - constraints

## MSK\_iparvaltosymnam

```
MSKrescodee (MSKAPI MSK_iparvaltosymnam) (
MSKenv_t env,
MSKiparame whichparam,
MSKint32t whichvalue,
char * symbolicname)
```

Obtains the symbolic name corresponding to a value that can be assigned to an integer parameter.

### Parameters

- env (MSKenv\_t) The MOSEK environment. (input)
- whichparam (MSKiparame) Which parameter. (input)
- which value (MSKint32t) Which value. (input)
- ullet symbolic name (MSKstring\_t) The symbolic name corresponding to which value. (output)

Return (MSKrescodee) - The function response code.

Groups Parameters, Names

## MSK\_isdouparname

```
MSKrescodee (MSKAPI MSK_isdouparname) (
    MSKtask_t task,
    const char * parname,
    MSKdparame * param)
```

Checks whether parname is a valid double parameter name.

## **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- parname (MSKstring\_t) Parameter name. (input)
- param (MSKdparame by reference) Returns the parameter corresponding to the name, if one exists. (output)

Return (MSKrescodee) - The function response code.

Groups Parameters, Names

## MSK\_isinfinity

```
MSKbooleant (MSKAPI MSK_isinfinity) (
MSKrealt value)
```

Return true if value is considered infinity by MOSEK.

```
Parameters value (MSKrealt) - The value to be checked (input) Return (MSKbooleant) - True if the value represents infinity.
```

MSK\_isintparname

```
MSKrescodee (MSKAPI MSK_isintparname) (
    MSKtask_t task,
    const char * parname,
    MSKiparame * param)
```

Checks whether parname is a valid integer parameter name.

# Parameters

- task (MSKtask\_t) An optimization task. (input)
- parname (MSKstring\_t) Parameter name. (input)
- param (MSKiparame by reference) Returns the parameter corresponding to the name, if one exists. (output)

Return (MSKrescodee) - The function response code.

Groups Parameters, Names

MSK\_isstrparname

```
MSKrescodee (MSKAPI MSK_isstrparname) (
    MSKtask_t task,
    const char * parname,
    MSKsparame * param)
```

Checks whether parname is a valid string parameter name.

# Parameters

- task (MSKtask\_t) An optimization task. (input)
- parname (MSKstring\_t) Parameter name. (input)
- param (MSKsparame by reference) Returns the parameter corresponding to the name, if one exists. (output)

 ${\bf Return} \ \ ({\it MSKrescodee}) - {\bf The} \ {\bf function} \ {\bf response} \ {\bf code}.$ 

Groups Parameters, Names

MSK\_licensecleanup

```
MSKrescodee (MSKAPI MSK_licensecleanup) (void)
```

Stops all threads and deletes all handles used by the license system. If this function is called, it must be called as the last **MOSEK** API call. No other **MOSEK** API calls are valid after this.

```
Return (MSKrescodee) - The function response code.
Groups License system
```

MSK\_linkfiletoenvstream

```
MSKrescodee (MSKAPI MSK_linkfiletoenvstream) (
MSKenv_t env,
MSKstreamtypee whichstream,
const char * filename,
MSKint32t append)
```

Sends all output from the stream defined by whichstream to the file given by filename.

#### **Parameters**

- env (MSKenv\_t) The MOSEK environment. (input)
- whichstream (MSKstreamtypee) Index of the stream. (input)
- filename (MSKstring\_t) A valid file name. (input)
- append (MSKint32t) If this argument is 0 the file will be overwritten, otherwise it will be appended to. (input)

Return (MSKrescodee) - The function response code. Groups Logging

### MSK\_linkfiletotaskstream

```
MSKrescodee (MSKAPI MSK_linkfiletotaskstream) (
MSKtask_t task,
MSKstreamtypee whichstream,
const char * filename,
MSKint32t append)
```

Directs all output from a task stream whichstream to a file filename.

### Parameters

- task (MSKtask\_t) An optimization task. (input)
- whichstream (MSKstreamtypee) Index of the stream. (input)
- filename (MSKstring\_t) A valid file name. (input)
- append (MSKint32t) If this argument is 0 the output file will be overwritten, otherwise it will be appended to. (input)

Return (MSKrescodee) – The function response code.

Groups Logging

### MSK\_linkfunctoenvstream

```
MSKrescodee (MSKAPI MSK_linkfunctoenvstream) (
MSKenv_t env,
MSKstreamtypee whichstream,
MSKuserhandle_t handle,
MSKstreamfunc func)
```

Connects a user-defined function to a stream.

# Parameters

- env (MSKenv\_t) The MOSEK environment. (input)
- whichstream (MSKstreamtypee) Index of the stream. (input)
- handle (MSKuserhandle\_t) A user-defined handle which is passed to the user-defined function func. (input)
- func (MSKstreamfunc) All output to the stream whichstream is passed to func. (input)

Return (MSKrescodee) - The function response code. Groups Logging, Callback

# ${\tt MSK\_linkfunctotaskstream}$

```
MSKrescodee (MSKAPI MSK_linkfunctotaskstream) (
MSKtask_t task,
MSKstreamtypee whichstream,
MSKuserhandle_t handle,
MSKstreamfunc func)
```

Connects a user-defined function to a task stream.

### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- whichstream (MSKstreamtypee) Index of the stream. (input)
- handle (MSKuserhandle\_t) A user-defined handle which is passed to the user-defined function func. (input)
- func (MSKstreamfunc) All output to the stream whichstream is passed to func. (input)

Return (MSKrescodee) - The function response code.

Groups Logging, Callback

 $MSK_makeemptytask$ 

```
MSKrescodee (MSKAPI MSK_makeemptytask) (
MSKenv_t env,
MSKtask_t * task)
```

Creates a new optimization task.

### **Parameters**

- env (MSKenv\_t) The MOSEK environment. (input)
- task (MSKtask\_t by reference) An optimization task. (output)

Return (MSKrescodee) - The function response code.

**Groups** Environment and task management

MSK\_makeenv

```
MSKrescodee (MSKAPI MSK_makeenv) (
   MSKenv_t * env,
   const char * dbgfile)
```

Creates a new MOSEK environment. The environment must be shared among all tasks in a program.

# **Parameters**

- env (MSKenv\_t by reference) The MOSEK environment. (output)
- dbgfile (MSKstring\_t) A user-defined file debug file. (input)

Return (MSKrescodee) - The function response code.

Groups Environment and task management

MSK\_makeenvalloc

```
MSKrescodee (MSKAPI MSK_makeenvalloc) (
   MSKenv_t * env,
   MSKuserhandle_t usrptr,
   MSKmallocfunc usrmalloc,
   MSKcallocfunc usrcalloc,
   MSKreallocfunc usrrealloc,
   MSKreefunc usrfeee,
   const char * dbgfile)
```

Creates a new MOSEK environment with user-defined memory management functions. The environment must be shared among all tasks in a program.

# **Parameters**

- env (MSKenv\_t by reference) The MOSEK environment. (output)
- usrptr (MSKuserhandle\_t) A pointer to a user-defined data structure. The pointer is fed into usrmalloc and usrfree. (input)
- usrmalloc (MSKmallocfunc) A user-defined malloc function or a NULL pointer. (input)
- usrcalloc (MSKcallocfunc) A user-defined calloc function or a NULL pointer. (input)
- usrrealloc (MSKreallocfunc) A user-defined realloc function or a NULL pointer. (input)
- usrfree (MSKfreefunc) A user-defined free function which is used to deallocate space allocated by usrmalloc. This function must be defined if usrmalloc! =null. (input)
- dbgfile (MSKstring\_t) A user-defined file debug file. (input)

Return (MSKrescodee) - The function response code.

Groups Environment and task management

MSK\_maketask

```
MSKrescodee (MSKAPI MSK_maketask) (
   MSKenv_t env,
   MSKint32t maxnumcon,
   MSKint32t maxnumvar,
   MSKtask_t * task)
```

Creates a new task.

#### Parameters

- env (MSKenv\_t) The MOSEK environment. (input)
- maxnumcon (MSKint32t) An optional estimate on the maximum number of constraints in the task. Can be 0 if no such estimate is known. (input)
- maxnumvar (MSKint32t) An optional estimate on the maximum number of variables in the task. Can be 0 if no such estimate is known. (input)
- task (MSKtask\_t by reference) An optimization task. (output)

Return (MSKrescodee) - The function response code.

Groups Environment and task management

MSK\_onesolutionsummary

```
MSKrescodee (MSKAPI MSK_onesolutionsummary) (
MSKtask_t task,
MSKstreamtypee whichstream,
MSKsoltypee whichsol)
```

Prints a short summary of a specified solution.

### Parameters

- task (MSKtask\_t) An optimization task. (input)
- whichstream (MSKstreamtypee) Index of the stream. (input)
- whichsol (MSKsoltypee) Selects a solution. (input)

Return (MSKrescodee) - The function response code.

Groups Logging, Solution information

MSK\_optimize

```
MSKrescodee (MSKAPI MSK_optimize) (
MSKtask_t task)
```

Calls the optimizer. Depending on the problem type and the selected optimizer this will call one of the optimizers in MOSEK. By default the interior point optimizer will be selected for continuous problems. The optimizer may be selected manually by setting the parameter  $MSK\_IPAR\_OPTIMIZER$ .

Response codes come in three categories:

- Errors: Indicate that an error has occurred during the optimization, e.g the optimizer has run out of memory  $(MSK\_RES\_ERR\_SPACE)$ .
- Warnings: Less fatal than errors. E.g MSK\_RES\_WRN\_LARGE\_CJ indicating possibly problematic problem data.
- Termination codes: Relaying information about the conditions under which the optimizer terminated. E.g MSK\_RES\_TRM\_MAX\_ITERATIONS indicates that the optimizer finished because it reached the maximum number of iterations specified by the user.

```
Parameters task (MSKtask_t) - An optimization task. (input) Return (MSKrescodee) - The function response code. Groups Optimization
```

MSK\_optimizermt

```
MSKrescodee (MSKAPI MSK_optimizermt) (
    MSKtask_t task,
    const char * server,
    const char * port,
    MSKrescodee * trmcode)
```

Offload the optimization task to a solver server defined by server:port. The call will block until a result is available or the connection closes.

If the string parameter  $MSK\_SPAR\_REMOTE\_ACCESS\_TOKEN$  is not blank, it will be passed to the server as authentication.

### Parameters

- task (MSKtask\_t) An optimization task. (input)
- server (MSKstring\_t) Name or IP address of the solver server. (input)
- port (MSKstring\_t) Network port of the solver server. (input)
- trmcode (MSKrescodee by reference) Is either MSK\_RES\_OK or a termination response code. (output)

Return (MSKrescodee) - The function response code.

Groups Remote optimization

MSK\_optimizersummary

```
MSKrescodee (MSKAPI MSK_optimizersummary) (
MSKtask_t task,
MSKstreamtypee whichstream)
```

Prints a short summary with optimizer statistics from last optimization.

### Parameters

- task (MSKtask\_t) An optimization task. (input)
- whichstream (MSKstreamtypee) Index of the stream. (input)

**Return** (*MSKrescodee*) – The function response code.

Groups Logging

```
MSKrescodee (MSKAPI MSK_optimizetrm) (
MSKtask_t task,
MSKrescodee * trmcode)
```

Calls the optimizer. Depending on the problem type and the selected optimizer this will call one of the optimizers in MOSEK. By default the interior point optimizer will be selected for continuous problems. The optimizer may be selected manually by setting the parameter  $MSK\_IPAR\_OPTIMIZER$ .

This function is equivalent to MSK\_optimize except for the handling of return values. This function returns errors on the left hand side. Warnings are not returned and termination codes are returned through the separate argument trmcode.

### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- trmcode (MSKrescodee by reference) Is either MSK\_RES\_OK or a termination response code. (output)

Return (MSKrescodee) - The function response code. Groups Optimization

MSK\_potrf

```
MSKrescodee (MSKAPI MSK_potrf) (
   MSKenv_t env,
   MSKuploe uplo,
   MSKint32t n,
   MSKrealt * a)
```

Computes a Cholesky factorization of a real symmetric positive definite dense matrix.

# Parameters

- env (MSKenv\_t) The MOSEK environment. (input)
- uplo (MSKuploe) Indicates whether the upper or lower triangular part of the matrix is stored. (input)
- n (MSKint32t) Dimension of the symmetric matrix. (input)
- a (MSKrealt\*) A symmetric matrix stored in column-major order. Only the lower or the upper triangular part is used, accordingly with the uplo parameter. It will contain the result on exit. (input/output)

Return (MSKrescodee) – The function response code. Groups  $Linear\ algebra$ 

MSK\_primalrepair

```
MSKrescodee (MSKAPI MSK_primalrepair) (
   MSKtask_t task,
   const MSKrealt * wlc,
   const MSKrealt * wuc,
   const MSKrealt * wlx,
   const MSKrealt * wux)
```

The function repairs a primal infeasible optimization problem by adjusting the bounds on the constraints and variables where the adjustment is computed as the minimal weighted sum of relaxations to the bounds on the constraints and variables. Observe the function only repairs the problem but does not solve it. If an optimal solution is required the problem should be optimized after the repair.

The function is applicable to linear and conic problems possibly with integer variables.

Observe that when computing the minimal weighted relaxation the termination tolerance specified by the parameters of the task is employed. For instance the parameter <code>MSK\_IPAR\_MIO\_MODE</code> can be used to make <code>MOSEK</code> ignore the integer constraints during the repair which usually leads to a much faster repair. However, the drawback is of course that the repaired problem may not have an integer feasible solution.

Note the function modifies the task in place. If this is not desired, then apply the function to a cloned task.

#### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- wlc  $(\mathit{MSKrealt*})$   $(w_l^c)_i$  is the weight associated with relaxing the lower bound on constraint i. If the weight is negative, then the lower bound is not relaxed. Moreover, if the argument is NULL, then all the weights are assumed to be 1. (input)
- wuc  $(\mathit{MSKrealt}^*)$   $(w_u^c)_i$  is the weight associated with relaxing the upper bound on constraint i. If the weight is negative, then the upper bound is not relaxed. Moreover, if the argument is NULL, then all the weights are assumed to be 1. (input)
- wlx  $(MSKrealt *) (w_l^x)_j$  is the weight associated with relaxing the lower bound on variable j. If the weight is negative, then the lower bound is not relaxed. Moreover, if the argument is NULL, then all the weights are assumed to be 1. (input)
- wux  $(MSKrealt*) (w_l^x)_i$  is the weight associated with relaxing the upper bound on variable j. If the weight is negative, then the upper bound is not relaxed. Moreover, if the argument is NULL, then all the weights are assumed to be 1. (input)

Return (MSKrescodee) - The function response code. Groups Infeasibility diagnostic

MSK\_primalsensitivity

```
MSKrescodee (MSKAPI MSK_primalsensitivity) (
 MSKtask_t task,
 MSKint32t numi,
  const MSKint32t * subi,
  const MSKmarke * marki,
 MSKint32t numj,
  const MSKint32t * subj,
  const MSKmarke * markj,
 MSKrealt * leftpricei,
 MSKrealt * rightpricei,
 MSKrealt * leftrangei,
 MSKrealt * rightrangei,
 MSKrealt * leftpricej,
 MSKrealt * rightpricej,
 MSKrealt * leftrangej,
 MSKrealt * rightrangej)
```

Calculates sensitivity information for bounds on variables and constraints. For details on sensitivity analysis, the definitions of  $shadow\ price$  and  $linearity\ interval$  and an example see Section  $Sensitivity\ Analysis$ .

The type of sensitivity analysis to be performed (basis or optimal partition) is controlled by the parameter  $MSK\_IPAR\_SENSITIVITY\_TYPE$ .

# Parameters

• task (MSKtask\_t) - An optimization task. (input)

- numi (MSKint32t) Number of bounds on constraints to be analyzed. Length of subi and marki. (input)
- subi (MSKint32t\*) Indexes of constraints to analyze. (input)
- marki (MSKmarke\*) The value of marki[i] indicates for which bound of constraint subi[i] sensitivity analysis is performed. If marki[i] = MSK\_MARK\_UP the upper bound of constraint subi[i] is analyzed, and if marki[i] = MSK\_MARK\_LO the lower bound is analyzed. If subi[i] is an equality constraint, either MSK\_MARK\_LO or MSK\_MARK\_UP can be used to select the constraint for sensitivity analysis. (input)
- numj (MSKint32t) Number of bounds on variables to be analyzed. Length of subj and markj. (input)
- subj (MSKint32t\*) Indexes of variables to analyze. (input)
- markj (MSKmarke\*) The value of markj[j] indicates for which bound of variable subj[j] sensitivity analysis is performed. If markj[j] = MSK\_MARK\_UP the upper bound of variable subj[j] is analyzed, and if markj[j] = MSK\_MARK\_LO the lower bound is analyzed. If subj[j] is a fixed variable, either MSK\_MARK\_LO or MSK\_MARK\_UP can be used to select the bound for sensitivity analysis. (input)
- leftpricei (MSKrealt\*) leftpricei[i] is the left shadow price for the bound marki[i] of constraint subi[i]. (output)
- rightpricei (MSKrealt\*) rightpricei[i] is the right shadow price for the bound marki[i] of constraint subi[i]. (output)
- leftrangei (MSKrealt\*) leftrangei[i] is the left range  $\beta_1$  for the bound marki[i] of constraint subi[i]. (output)
- rightrangei (MSKrealt\*) rightrangei[i] is the right range  $\beta_2$  for the bound marki[i] of constraint subi[i]. (output)
- leftpricej (MSKrealt\*) leftpricej[j] is the left shadow price for the bound markj[j] of variable subj[j]. (output)
- rightpricej (MSKrealt\*) rightpricej[j] is the right shadow price for the bound markj[j] of variable subj[j]. (output)
- leftrangej (MSKrealt\*) leftrangej[j] is the left range  $\beta_1$  for the bound markj[j] of variable subj[j]. (output)
- rightrangej (MSKrealt\*) rightrangej[j] is the right range  $\beta_2$  for the bound markj[j] of variable subj[j]. (output)

 ${\bf Return} \ ({\tt MSKrescodee})$  – The function response code.

Groups Sensitivity analysis

MSK\_printparam

```
MSKrescodee (MSKAPI MSK_printparam) (
MSKtask_t task)
```

Prints the current parameter settings to the message stream.

```
Parameters task (MSKtask_t) - An optimization task. (input) Return (MSKrescodee) - The function response code. Groups Inspecting\ the\ task, Logging
```

MSK\_probtypetostr

```
MSKrescodee (MSKAPI MSK_probtypetostr) (
    MSKtask_t task,
    MSKproblemtypee probtype,
    char * str)
```

Obtains a string containing the name of a given problem type.

### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- probtype (MSKproblemtypee) Problem type. (input)
- str (MSKstring\_t) String corresponding to the problem type key probtype. (output)

Return (MSKrescodee) - The function response code.

Groups Inspecting the task, Names

# MSK\_prostatostr

```
MSKrescodee (MSKAPI MSK_prostatostr) (
MSKtask_t task,
MSKprostae prosta,
char * str)
```

Obtains a string containing the name of a given problem status.

#### Parameters 4 8 1

- $task (MSKtask_t)$  An optimization task. (input)
- prosta (MSKprostae) Problem status. (input)
- str (MSKstring\_t) String corresponding to the status key prosta. (output)

Return (MSKrescodee) - The function response code.

**Groups** Solution information, Names

# MSK\_putacol

```
MSKrescodee (MSKAPI MSK_putacol) (
   MSKtask_t task,
   MSKint32t j,
   MSKint32t nzj,
   const MSKint32t * subj,
   const MSKrealt * valj)
```

Change one column of the linear constraint matrix A. Resets all the elements in column j to zero and then sets

$$a_{\mathtt{subj}[k],j} = \mathtt{valj}[k], \quad k = 0, \dots, \mathtt{nzj} - 1.$$

### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- j (MSKint32t) Index of a column in A. (input)
- nzj (MSKint32t) Number of non-zeros in column j of A. (input)
- subj (MSKint32t\*) Row indexes of non-zero values in column j of A. (input)
- valj (MSKrealt\*) New non-zero values of column j in A. (input)

Return (MSKrescodee) - The function response code.

Groups Problem data - linear part

# MSK\_putacollist

```
MSKrescodee (MSKAPI MSK_putacollist) (
   MSKtask_t task,
   MSKint32t num,
   const MSKint32t * sub,
   const MSKint32t * ptrb,
   const MSKint32t * ptre,
   const MSKint32t * asub,
   const MSKrealt * aval)
```

Change a set of columns in the linear constraint matrix A with data in sparse triplet format. The requested columns are set to zero and then updated with:

$$\label{eq:constraints} \begin{array}{ll} \text{for} & i=0,\dots,num-1\\ & a_{\texttt{asub}[k],\texttt{sub}[i]} = \texttt{aval}[k], \quad k = \texttt{ptrb}[i],\dots,\texttt{ptre}[i]-1. \end{array}$$

### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- num (MSKint32t) Number of columns of A to replace. (input)
- sub (MSKint32t\*) Indexes of columns that should be replaced, no duplicates. (input)
- ptrb (MSKint32t\*) Array of pointers to the first element in each column. (input)
- ptre (MSKint32t\*) Array of pointers to the last element plus one in each column. (input)
- asub (MSKint32t\*) Row indexes of new elements. (input)
- aval (MSKrealt\*) Coefficient values. (input)

Return (MSKrescodee) - The function response code.

Groups Problem data - linear part

# MSK\_putacollist64

```
MSKrescodee (MSKAPI MSK_putacollist64) (
   MSKtask_t task,
   MSKint32t num,
   const MSKint32t * sub,
   const MSKint64t * ptrb,
   const MSKint64t * ptre,
   const MSKint32t * asub,
   const MSKrealt * aval)
```

Change a set of columns in the linear constraint matrix A with data in sparse triplet format. The requested columns are set to zero and then updated with:

$$\label{eq:constraints} \begin{array}{ll} \texttt{for} & i = 0, \dots, num - 1 \\ & a_{\texttt{asub}[k], \texttt{sub}[i]} = \texttt{aval}[k], \quad k = \texttt{ptrb}[i], \dots, \texttt{ptre}[i] - 1. \end{array}$$

# Parameters

- task (MSKtask\_t) An optimization task. (input)
- num (MSKint32t) Number of columns of A to replace. (input)
- sub (MSKint32t\*) Indexes of columns that should be replaced, no duplicates. (input)
- ptrb (MSKint64t\*) Array of pointers to the first element in each column. (input)
- ptre (MSKint64t\*) Array of pointers to the last element plus one in each column. (input)
- asub (MSKint32t\*) Row indexes of new elements. (input)
- aval (MSKrealt\*) Coefficient values. (input)

Return (MSKrescodee) - The function response code.

Groups Problem data - linear part

MSK\_putacolslice

```
MSKrescodee (MSKAPI MSK_putacolslice) (
   MSKtask_t task,
   MSKint32t first,
   MSKint32t last,
   const MSKint32t * ptrb,
   const MSKint32t * ptre,
   const MSKint32t * asub,
   const MSKrealt * aval)
```

Change a slice of columns in the linear constraint matrix A with data in sparse triplet format. The requested columns are set to zero and then updated with:

```
\label{eq:assub} \begin{array}{ll} \texttt{for} & i = \texttt{first}, \dots, \texttt{last} - 1 \\ & a_{\texttt{asub}[k],i} = \texttt{aval}[k], \quad k = \texttt{ptrb}[i], \dots, \texttt{ptre}[i] - 1. \end{array}
```

### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- first (MSKint32t) First column in the slice. (input)
- last (MSKint32t) Last column plus one in the slice. (input)
- ptrb (MSKint32t\*) Array of pointers to the first element in each column. (input)
- ptre (MSKint32t\*) Array of pointers to the last element plus one in each column. (input)
- asub (MSKint32t\*) Row indexes of new elements. (input)
- aval (MSKrealt\*) Coefficient values. (input)

Return (MSKrescodee) - The function response code.

Groups Problem data - linear part

 $MSK_putacolslice64$ 

```
MSKrescodee (MSKAPI MSK_putacolslice64) (
   MSKtask_t task,
   MSKint32t first,
   MSKint32t last,
   const MSKint64t * ptrb,
   const MSKint64t * ptre,
   const MSKint32t * asub,
   const MSKrealt * aval)
```

Change a slice of columns in the linear constraint matrix A with data in sparse triplet format. The requested columns are set to zero and then updated with:

```
\begin{array}{ll} \text{for} & i = \texttt{first}, \dots, \texttt{last} - 1 \\ & a_{\texttt{asub}[k], i} = \texttt{aval}[k], \quad k = \texttt{ptrb}[i], \dots, \texttt{ptre}[i] - 1. \end{array}
```

# Parameters

- task (MSKtask\_t) An optimization task. (input)
- first (MSKint32t) First column in the slice. (input)
- last (MSKint32t) Last column plus one in the slice. (input)
- ptrb (MSKint64t\*) Array of pointers to the first element in each column. (input)
- ptre (MSKint64t\*) Array of pointers to the last element plus one in each column. (input)
- asub (MSKint32t\*) Row indexes of new elements. (input)
- aval (MSKrealt\*) Coefficient values. (input)

Return (MSKrescodee) - The function response code.

# Groups Problem data - linear part

MSK\_putaij

```
MSKrescodee (MSKAPI MSK_putaij) (
   MSKtask_t task,
   MSKint32t i,
   MSKint32t j,
   MSKrealt aij)
```

Changes a coefficient in the linear coefficient matrix A using the method

$$a_{i,j} = aij.$$

### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- i (MSKint32t) Constraint (row) index. (input)
- j (MSKint32t) Variable (column) index. (input)
- $\bullet$  aij (MSKrealt) New coefficient for  $a_{i,j}.$  (input)

Return (MSKrescodee) - The function response code.

Groups Problem data - linear part

MSK\_putaijlist

```
MSKrescodee (MSKAPI MSK_putaijlist) (
   MSKtask_t task,
   MSKint32t num,
   const MSKint32t * subi,
   const MSKint32t * subj,
   const MSKrealt * valij)
```

Changes one or more coefficients in A using the method

$$a_{\texttt{subi}[\texttt{k}],\texttt{subj}[\texttt{k}]} = \texttt{valij}[\texttt{k}], \quad k = 0, \dots, num - 1.$$

Duplicates are not allowed.

# Parameters

- task (MSKtask\_t) An optimization task. (input)
- num (MSKint32t) Number of coefficients that should be changed. (input)
- subi (MSKint32t\*) Constraint (row) indices. (input)
- subj (MSKint32t\*) Variable (column) indices. (input)
- valij (MSKrealt\*) New coefficient values for  $a_{i,j}$ . (input)

Return (MSKrescodee) - The function response code.

Groups Problem data - linear part

MSK\_putaijlist64

```
MSKrescodee (MSKAPI MSK_putaijlist64) (
   MSKtask_t task,
   MSKint64t num,
   const MSKint32t * subi,
   const MSKint32t * subj,
   const MSKrealt * valij)
```

Changes one or more coefficients in A using the method

$$a_{\mathtt{subi}[\mathtt{k}],\mathtt{subj}[\mathtt{k}]} = \mathtt{valij}[\mathtt{k}], \quad k = 0, \dots, num - 1.$$

Duplicates are not allowed.

#### Parameters

- task (MSKtask\_t) An optimization task. (input)
- num (MSKint64t) Number of coefficients that should be changed. (input)
- subi (MSKint32t\*) Constraint (row) indices. (input)
- subj (MSKint32t\*) Variable (column) indices. (input)
- valij (MSKrealt\*) New coefficient values for  $a_{i,j}$ . (input)

Return (MSKrescodee) - The function response code.

Groups Problem data - linear part

### MSK\_putarow

```
MSKrescodee (MSKAPI MSK_putarow) (
   MSKtask_t task,
   MSKint32t i,
   MSKint32t nzi,
   const MSKint32t * subi,
   const MSKrealt * vali)
```

Change one row of the linear constraint matrix A. Resets all the elements in row i to zero and then sets

$$a_{\mathtt{i},\mathtt{subi}[k]} = \mathtt{vali}[k], \quad k = 0, \dots, \mathtt{nzi} - 1.$$

# **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- i (MSKint32t) Index of a row in A. (input)
- nzi (MSKint32t) Number of non-zeros in row i of A. (input)
- subi (MSKint32t\*) Column indexes of non-zero values in row i of A. (input)
- vali (MSKrealt\*) New non-zero values of row i in A. (input)

Return (MSKrescodee) - The function response code.

Groups Problem data - linear part

### MSK\_putarowlist

```
MSKrescodee (MSKAPI MSK_putarowlist) (
   MSKtask_t task,
   MSKint32t num,
   const MSKint32t * sub,
   const MSKint32t * ptrb,
   const MSKint32t * ptre,
   const MSKint32t * asub,
   const MSKrealt * aval)
```

Change a set of rows in the linear constraint matrix A with data in sparse triplet format. The requested rows are set to zero and then updated with:

$$\label{eq:constraints} \begin{array}{ll} \texttt{for} & i = 0, \dots, num - 1 \\ & a_{\mathtt{sub}[i],\mathtt{asub}[k]} = \mathtt{aval}[k], \quad k = \mathtt{ptrb}[i], \dots, \mathtt{ptre}[i] - 1. \end{array}$$

# Parameters

• task (MSKtask\_t) - An optimization task. (input)

- num (MSKint32t) Number of rows of A to replace. (input)
- sub (MSKint32t\*) Indexes of rows that should be replaced, no duplicates. (input)
- ptrb (MSKint32t\*) Array of pointers to the first element in each row. (input)
- ptre (MSKint32t\*) Array of pointers to the last element plus one in each row. (input)
- asub (MSKint32t\*) Column indexes of new elements. (input)
- aval (MSKrealt\*) Coefficient values. (input)

Return (MSKrescodee) - The function response code.

Groups Problem data - linear part

# MSK\_putarowlist64

```
MSKrescodee (MSKAPI MSK_putarowlist64) (
   MSKtask_t task,
   MSKint32t num,
   const MSKint32t * sub,
   const MSKint64t * ptrb,
   const MSKint64t * ptre,
   const MSKint32t * asub,
   const MSKint32t * asub,
   const MSKrealt * aval)
```

Change a set of rows in the linear constraint matrix A with data in sparse triplet format. The requested rows are set to zero and then updated with:

$$\begin{split} \text{for} \quad i = 0, \dots, num - 1 \\ \quad a_{\text{sub}[i], \text{asub}[k]} = \text{aval}[k], \quad k = \text{ptrb}[i], \dots, \text{ptre}[i] - 1. \end{split}$$

### Parameters

- task (MSKtask\_t) An optimization task. (input)
- num (MSKint32t) Number of rows of A to replace. (input)
- $sub\ (MSKint32t*)$  Indexes of rows that should be replaced, no duplicates. (input)
- ptrb (MSKint64t\*) Array of pointers to the first element in each row. (input)
- ptre (MSKint64t\*) Array of pointers to the last element plus one in each row. (input)
- asub (MSKint32t\*) Column indexes of new elements. (input)
- aval (MSKrealt\*) Coefficient values. (input)

Return (MSKrescodee) - The function response code.

Groups Problem data - linear part

# MSK\_putarowslice

```
MSKrescodee (MSKAPI MSK_putarowslice) (
   MSKtask_t task,
   MSKint32t first,
   MSKint32t last,
   const MSKint32t * ptrb,
   const MSKint32t * ptre,
   const MSKint32t * asub,
   const MSKrealt * aval)
```

Change a slice of rows in the linear constraint matrix A with data in sparse triplet format. The requested rows are set to zero and then updated with:

```
\begin{aligned} & \text{for} \quad i = \texttt{first}, \dots, \texttt{last} - 1 \\ & \quad a_{\texttt{sub}[i], \texttt{asub}[k]} = \texttt{aval}[k], \quad k = \texttt{ptrb}[i], \dots, \texttt{ptre}[i] - 1. \end{aligned}
```

### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- first (MSKint32t) First row in the slice. (input)
- last (MSKint32t) Last row plus one in the slice. (input)
- ptrb (MSKint32t\*) Array of pointers to the first element in each row. (input)
- ptre (MSKint32t\*) Array of pointers to the last element plus one in each row. (input)
- asub (MSKint32t\*) Column indexes of new elements. (input)
- aval (MSKrealt\*) Coefficient values. (input)

Return (MSKrescodee) - The function response code.

Groups Problem data - linear part

# MSK\_putarowslice64

```
MSKrescodee (MSKAPI MSK_putarowslice64) (
   MSKtask_t task,
   MSKint32t first,
   MSKint32t last,
   const MSKint64t * ptrb,
   const MSKint64t * ptre,
   const MSKint32t * asub,
   const MSKrealt * aval)
```

Change a slice of rows in the linear constraint matrix A with data in sparse triplet format. The requested rows are set to zero and then updated with:

$$\begin{aligned} & \text{for} \quad i = \texttt{first}, \dots, \texttt{last} - 1 \\ & \quad a_{\texttt{sub}[i], \texttt{asub}[k]} = \texttt{aval}[k], \quad k = \texttt{ptrb}[i], \dots, \texttt{ptre}[i] - 1. \end{aligned}$$

### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- first (MSKint32t) First row in the slice. (input)
- last (MSKint32t) Last row plus one in the slice. (input)
- ptrb (MSKint64t\*) Array of pointers to the first element in each row. (input)
- ptre (MSKint64t\*) Array of pointers to the last element plus one in each row. (input)
- asub (MSKint32t\*) Column indexes of new elements. (input)
- aval (MSKrealt\*) Coefficient values. (input)

Return (MSKrescodee) - The function response code.

Groups Problem data - linear part

# MSK\_putatruncatetol

```
MSKrescodee (MSKAPI MSK_putatruncatetol) (
MSKtask_t task,
MSKrealt tolzero)
```

Truncates (sets to zero) all elements in A that satisfy

$$|a_{i,j}| \leq \text{tolzero}.$$

### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- tolzero (MSKrealt) Truncation tolerance. (input)

Return (MSKrescodee) - The function response code.

# Groups Problem data - linear part

MSK\_putbarablocktriplet

```
MSKrescodee (MSKAPI MSK_putbarablocktriplet) (
   MSKtask_t task,
   MSKint64t num,
   const MSKint32t * subi,
   const MSKint32t * subj,
   const MSKint32t * subk,
   const MSKint32t * subk,
   const MSKint32t * subl,
   const MSKrealt * valijkl)
```

Inputs the  $\overline{A}$  matrix in block triplet form.

### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- num (MSKint64t) Number of elements in the block triplet form. (input)
- subi (MSKint32t\*) Constraint index. (input)
- subj (MSKint32t\*) Symmetric matrix variable index. (input)
- subk (MSKint32t\*) Block row index. (input)
- subl (MSKint32t\*) Block column index. (input)
- valijkl (MSKrealt\*) The numerical value associated with each block triplet. (input)

 $\mathbf{Return} \ (\mathit{MSKrescodee}) - \mathbf{The} \ \mathbf{function} \ \mathbf{response} \ \mathbf{code}.$ 

Groups Problem data - semidefinite

MSK\_putbaraij

```
MSKrescodee (MSKAPI MSK_putbaraij) (
   MSKtask_t task,
   MSKint32t i,
   MSKint32t j,
   MSKint64t num,
   const MSKint64t * sub,
   const MSKrealt * weights)
```

This function sets one element in the  $\overline{A}$  matrix.

Each element in the  $\overline{A}$  matrix is a weighted sum of symmetric matrices from the symmetric matrix storage E, so  $\overline{A}_{ij}$  is a symmetric matrix. By default all elements in  $\overline{A}$  are 0, so only non-zero elements need be added. Setting the same element again will overwrite the earlier entry.

The symmetric matrices from E are defined separately using the function  ${\it MSK\_appendsparsesymmat}$ .

# Parameters

- task (MSKtask\_t) An optimization task. (input)
- i (MSKint32t) Row index of  $\overline{A}$ . (input)
- j (MSKint32t) Column index of  $\overline{A}$ . (input)
- num (MSKint64t) The number of terms in the weighted sum that forms  $\overline{A}_{ij}$ . (input)
- sub (MSKint64t\*) Indices in E of the matrices appearing in the weighted sum for  $\overline{A}_{ij}$ . (input)
- weights (MSKrealt\*) weights [k] is the coefficient of the sub [k]-th element of E in the weighted sum forming  $\overline{A}_{ij}$ . (input)

Return (MSKrescodee) - The function response code.

# Groups Problem data - semidefinite

# MSK\_putbaraijlist

```
MSKrescodee (MSKAPI MSK_putbaraijlist) (
   MSKtask_t task,
   MSKint32t num,
   const MSKint32t * subi,
   const MSKint32t * subj,
   const MSKint64t * alphaptrb,
   const MSKint64t * alphaptre,
   const MSKint64t * matidx,
   const MSKrealt * weights)
```

This function sets a list of elements in the  $\overline{A}$  matrix.

Each element in the  $\overline{A}$  matrix is a weighted sum of symmetric matrices from the symmetric matrix storage E, so  $\overline{A}_{ij}$  is a symmetric matrix. By default all elements in  $\overline{A}$  are 0, so only non-zero elements need be added. Setting the same element again will overwrite the earlier entry.

The symmetric matrices from E are defined separately using the function  $\mathit{MSK\_appendsparsesymmat}$ .

#### Parameters

- task (MSKtask\_t) An optimization task. (input)
- num (MSKint32t) Number of  $\overline{A}$  entries to add. (input)
- subi (MSKint32t\*) Row index of  $\overline{A}$ . (input)
- subj (MSKint32t\*) Column index of  $\overline{A}$ . (input)
- alphaptrb (MSKint64t\*) Start entries for terms in the weighted sum that forms  $\overline{A}_{ij}$ . (input)
- alphaptre (MSKint64t\*) End entries for terms in the weighted sum that forms  $\overline{A}_{ij}$ . (input)
- matidx  $(\underline{MSKint64t}*)$  Indices in E of the matrices appearing in the weighted sum for  $\overline{A}_{ij}$ . (input)
- weights (MSKrealt\*) weights [k] is the coefficient of the sub[k]-th element of E in the weighted sum forming  $\overline{A}_{ij}$ . (input)

Return (MSKrescodee) - The function response code.

Groups Problem data - semidefinite

# MSK\_putbararowlist

```
MSKrescodee (MSKAPI MSK_putbararowlist) (
   MSKtask_t task,
   MSKint32t num,
   const MSKint32t * subi,
   const MSKint64t * ptrb,
   const MSKint64t * ptre,
   const MSKint32t * subj,
   const MSKint64t * nummat,
   const MSKint64t * matidx,
   const MSKrealt * weights)
```

This function replaces a list of rows in the  $\overline{A}$  matrix.

# Parameters

- task (MSKtask\_t) An optimization task. (input)
- num (MSKint32t) Number of  $\overline{A}$  entries to add. (input)
- subi (MSKint32t\*) Row indexes of  $\overline{A}$ . (input)

```
ptrb (MSKint64t*) - Start of rows in A. (input)
ptre (MSKint64t*) - End of rows in A. (input)
subj (MSKint32t*) - Column index of A. (input)
nummat (MSKint64t*) - Number of entries in weighted sum of matrixes. (input)
matidx (MSKint64t*) - Matrix indexes for weighted sum of matrixes. (input)
weights (MSKrealt*) - Weights for weighted sum of matrixes. (input)
Return (MSKrescodee) - The function response code.
```

Groups Problem data - semidefinite

MSK\_putbarcblocktriplet

```
MSKrescodee (MSKAPI MSK_putbarcblocktriplet) (
   MSKtask_t task,
   MSKint64t num,
   const MSKint32t * subj,
   const MSKint32t * subk,
   const MSKint32t * subl,
   const MSKrealt * valjkl)
```

Inputs the  $\overline{C}$  matrix in block triplet form.

### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- num (MSKint64t) Number of elements in the block triplet form. (input)
- subj (MSKint32t\*) Symmetric matrix variable index. (input)
- subk (MSKint32t\*) Block row index. (input)
- subl (MSKint32t\*) Block column index. (input)
- valjkl (MSKrealt\*) The numerical value associated with each block triplet. (input)

Return (MSKrescodee) - The function response code.

Groups Problem data - semidefinite

MSK\_putbarcj

```
MSKrescodee (MSKAPI MSK_putbarcj) (
   MSKtask_t task,
   MSKint32t j,
   MSKint64t num,
   const MSKint64t * sub,
   const MSKrealt * weights)
```

This function sets one entry in the  $\overline{C}$  vector.

Each element in the  $\overline{C}$  vector is a weighted sum of symmetric matrices from the symmetric matrix storage E, so  $\overline{C}_j$  is a symmetric matrix. By default all elements in  $\overline{C}$  are 0, so only non-zero elements need be added. Setting the same element again will overwrite the earlier entry.

The symmetric matrices from E are defined separately using the function  $MSK\_appendsparsesymmat$ .

### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- j (MSKint32t) Index of the element in  $\overline{C}$  that should be changed. (input)
- num (MSKint64t) The number of elements in the weighted sum that forms  $\overline{C}_j$ . (input)
- sub (MSKint64t\*) Indices in E of matrices appearing in the weighted sum for  $\overline{C}_i$  (input)

• weights (MSKrealt\*) - weights [k] is the coefficient of the sub[k]-th element of E in the weighted sum forming  $\overline{C}_j$ . (input)

Return (MSKrescodee) - The function response code.

Groups Problem data - semidefinite, Problem data - objective

MSK\_putbarsj

```
MSKrescodee (MSKAPI MSK_putbarsj) (
MSKtask_t task,
MSKsoltypee whichsol,
MSKint32t j,
const MSKrealt * barsj)
```

Sets the dual solution for a semidefinite variable.

#### Parameters

- task (MSKtask\_t) An optimization task. (input)
- whichsol (MSKsoltypee) Selects a solution. (input)
- j (MSKint32t) Index of the semidefinite variable. (input)
- barsj (MSKrealt\*) Value of  $\overline{S}_j$ . Format as in MSK\_getbarsj. (input)

Return (MSKrescodee) - The function response code.

Groups Solution - semidefinite

MSK\_putbarvarname

```
MSKrescodee (MSKAPI MSK_putbarvarname) (
MSKtask_t task,
MSKint32t j,
const char * name)
```

Sets the name of a semidefinite variable.

# **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- j (MSKint32t) Index of the variable. (input)
- name (MSKstring\_t) The variable name. (input)

Return (MSKrescodee) - The function response code.

Groups Names, Problem data - semidefinite

MSK\_putbarxj

```
MSKrescodee (MSKAPI MSK_putbarxj) (
    MSKtask_t task,
    MSKsoltypee whichsol,
    MSKint32t j,
    const MSKrealt * barxj)
```

Sets the primal solution for a semidefinite variable.

### Parameters

- task (MSKtask\_t) An optimization task. (input)
- whichsol (MSKsoltypee) Selects a solution. (input)
- j (MSKint32t) Index of the semidefinite variable. (input)
- barxj (MSKrealt\*) Value of  $\overline{X}_i$ . Format as in MSK\_getbarxj. (input)

Return (MSKrescodee) - The function response code.

# Groups Solution - semidefinite

MSK\_putcallbackfunc

```
MSKrescodee (MSKAPI MSK_putcallbackfunc) (
MSKtask_t task,
MSKcallbackfunc func,
MSKuserhandle_t handle)
```

Sets a user-defined progress callback function of type MSKcallbackfunc. The callback function is called frequently during the optimization process. See Section Progress and data callback for an example.

### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- func (MSKcallbackfunc) A user-defined function which will be called occasionally from within the MOSEK optimizers. If the argument is a NULL pointer, then a previously defined callback function is removed. The progress function has the type MSKcallbackfunc. (input)
- handle (MSKuserhandle\_t) A pointer to a user-defined data structure. Whenever the function func is called, then handle is passed to the function. (input)

Return (MSKrescodee) - The function response code. Groups Callback

MSK\_putcfix

```
MSKrescodee (MSKAPI MSK_putcfix) (
MSKtask_t task,
MSKrealt cfix)
```

Replaces the fixed term in the objective by a new one.

### Parameters

- task (MSKtask\_t) An optimization task. (input)
- cfix (MSKrealt) Fixed term in the objective. (input)

Return (MSKrescodee) - The function response code.

Groups Problem data - linear part, Problem data - objective

MSK\_putcj

```
MSKrescodee (MSKAPI MSK_putcj) (
MSKtask_t task,
MSKint32t j,
MSKrealt cj)
```

Modifies one coefficient in the linear objective vector c, i.e.

$$c_{j} = c_{j}$$
.

If the absolute value exceeds  $MSK\_DPAR\_DATA\_TOL\_C\_HUGE$  an error is generated. If the absolute value exceeds  $MSK\_DPAR\_DATA\_TOL\_CJ\_LARGE$ , a warning is generated, but the coefficient is inputted as specified.

# Parameters

- task (MSKtask\_t) An optimization task. (input)
- j (MSKint32t) Index of the variable for which c should be changed. (input)
- cj (MSKrealt) New value of  $c_i$ . (input)

```
Return (MSKrescodee) - The function response code.
Groups Problem data - linear part, Problem data - objective
```

MSK\_putclist

```
MSKrescodee (MSKAPI MSK_putclist) (
   MSKtask_t task,
   MSKint32t num,
   const MSKint32t * subj,
   const MSKrealt * val)
```

Modifies the coefficients in the linear term c in the objective using the principle

$$c_{\mathtt{subj[t]}} = \mathtt{val[t]}, \quad t = 0, \dots, num - 1.$$

If a variable index is specified multiple times in subj only the last entry is used. Data checks are performed as in  $MSK_putcj$ .

### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- num (MSKint32t) Number of coefficients that should be changed. (input)
- subj(MSKint32t\*) Indices of variables for which the coefficient in c should be changed. (input)
- val (MSKrealt\*) New numerical values for coefficients in c that should be modified. (input)

Return (MSKrescodee) - The function response code.

Groups Problem data - linear part, Problem data - variables, Problem data - objective

MSK\_putconbound

```
MSKrescodee (MSKAPI MSK_putconbound) (
MSKtask_t task,
MSKint32t i,
MSKboundkeye bkc,
MSKrealt blc,
MSKrealt buc)
```

Changes the bounds for one constraint.

If the bound value specified is numerically larger than  $MSK\_DPAR\_DATA\_TOL\_BOUND\_INF$  it is considered infinite and the bound key is changed accordingly. If a bound value is numerically larger than  $MSK\_DPAR\_DATA\_TOL\_BOUND\_WRN$ , a warning will be displayed, but the bound is inputted as specified.

### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- i (MSKint32t) Index of the constraint. (input)
- bkc (MSKboundkeye) New bound key. (input)
- blc (MSKrealt) New lower bound. (input)
- buc (MSKrealt) New upper bound. (input)

Return (MSKrescodee) - The function response code.

Groups Problem data - linear part, Problem data - constraints, Problem data - bounds

MSK\_putconboundlist

```
MSKrescodee (MSKAPI MSK_putconboundlist) (
   MSKtask_t task,
   MSKint32t num,
   const MSKint32t * sub,
   const MSKboundkeye * bkc,
   const MSKrealt * blc,
   const MSKrealt * buc)
```

Changes the bounds for a list of constraints. If multiple bound changes are specified for a constraint, then only the last change takes effect. Data checks are performed as in MSK\_putconbound.

### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- num (MSKint32t) Number of bounds that should be changed. (input)
- sub (MSKint32t\*) List of constraint indexes. (input)
- bkc (MSKboundkeye\*) Bound keys for the constraints. (input)
- blc (MSKrealt\*) Lower bounds for the constraints. (input)
- buc (MSKrealt\*) Upper bounds for the constraints. (input)

Return (MSKrescodee) - The function response code.

Groups Problem data - linear part, Problem data - constraints, Problem data - bounds

 $MSK_putconboundlistconst$ 

```
MSKrescodee (MSKAPI MSK_putconboundlistconst) (
   MSKtask_t task,
   MSKint32t num,
   const MSKint32t * sub,
   MSKboundkeye bkc,
   MSKrealt blc,
   MSKrealt buc)
```

Changes the bounds for one or more constraints. Data checks are performed as in  $MSK\_putconbound$ .

### Parameters

- task (MSKtask\_t) An optimization task. (input)
- num (MSKint32t) Number of bounds that should be changed. (input)
- sub (MSKint32t\*) List of constraint indexes. (input)
- bkc (MSKboundkeye) New bound key for all constraints in the list. (input)
- blc (MSKrealt) New lower bound for all constraints in the list. (input)
- buc (MSKrealt) New upper bound for all constraints in the list. (input)

Return (MSKrescodee) - The function response code.

Groups Problem data - linear part, Problem data - constraints, Problem data - bounds

MSK\_putconboundslice

```
MSKrescodee (MSKAPI MSK_putconboundslice) (
   MSKtask_t task,
   MSKint32t first,
   MSKint32t last,
   const MSKboundkeye * bkc,
   const MSKrealt * blc,
   const MSKrealt * buc)
```

Changes the bounds for a slice of the constraints. Data checks are performed as in  $MSK\_putconbound$ .

# Parameters

- task (MSKtask\_t) An optimization task. (input)
- first (MSKint32t) First index in the sequence. (input)
- last (MSKint32t) Last index plus 1 in the sequence. (input)
- bkc (MSKboundkeye\*) Bound keys for the constraints. (input)
- blc (MSKrealt\*) Lower bounds for the constraints. (input)
- buc (MSKrealt\*) Upper bounds for the constraints. (input)

Return (MSKrescodee) - The function response code.

Groups Problem data - linear part, Problem data - constraints, Problem data - bounds

MSK\_putconboundsliceconst

```
MSKrescodee (MSKAPI MSK_putconboundsliceconst) (
MSKtask_t task,
MSKint32t first,
MSKint32t last,
MSKboundkeye bkc,
MSKrealt blc,
MSKrealt buc)
```

Changes the bounds for a slice of the constraints. Data checks are performed as in  $MSK\_putconbound$ .

### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- first (MSKint32t) First index in the sequence. (input)
- last (MSKint32t) Last index plus 1 in the sequence. (input)
- bkc (MSKboundkeye) New bound key for all constraints in the slice. (input)
- blc (MSKrealt) New lower bound for all constraints in the slice. (input)
- buc (MSKrealt) New upper bound for all constraints in the slice. (input)

Return (MSKrescodee) - The function response code.

Groups Problem data - linear part, Problem data - constraints, Problem data - bounds

MSK\_putcone

```
MSKrescodee (MSKAPI MSK_putcone) (
   MSKtask_t task,
   MSKint32t k,
   MSKconetypee ct,
   MSKrealt conepar,
   MSKint32t nummem,
   const MSKint32t * submem)
```

Replaces a conic constraint.

# Parameters

- task  $(\textit{MSKtask\_t})$  An optimization task. (input)
- k (MSKint32t) Index of the cone. (input)
- ct (MSKconetypee) Specifies the type of the cone. (input)
- conepar (MSKrealt) For the power cone it denotes the exponent alpha. For other cone types it is unused and can be set to 0. (input)
- nummem (MSKint32t) Number of member variables in the cone. (input)
- submem (MSKint32t\*) Variable subscripts of the members in the cone. (input)

Return (MSKrescodee) - The function response code.

Groups Problem data - cones

```
MSKrescodee (MSKAPI MSK_putconename) (
    MSKtask_t task,
    MSKint32t j,
    const char * name)
```

Sets the name of a cone.

# Parameters

- task (MSKtask\_t) An optimization task. (input)
- j (MSKint32t) Index of the cone. (input)
- name (MSKstring\_t) The name of the cone. (input)

Return (MSKrescodee) - The function response code.

Groups Names, Problem data - cones

MSK\_putconname

```
MSKrescodee (MSKAPI MSK_putconname) (
    MSKtask_t task,
    MSKint32t i,
    const char * name)
```

Sets the name of a constraint.

#### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- i (MSKint32t) Index of the constraint. (input)
- name (MSKstring\_t) The name of the constraint. (input)

Return (MSKrescodee) - The function response code.

Groups Names, Problem data - constraints, Problem data - linear part

MSK\_putconsolutioni

```
MSKrescodee (MSKAPI MSK_putconsolutioni) (
   MSKtask_t task,
   MSKint32t i,
   MSKsoltypee whichsol,
   MSKstakeye sk,
   MSKrealt x,
   MSKrealt sl,
   MSKrealt su)
```

Sets the primal and dual solution information for a single constraint.

# Parameters

- task (MSKtask\_t) An optimization task. (input)
- i (MSKint32t) Index of the constraint. (input)
- whichsol (MSKsoltypee) Selects a solution. (input)
- sk (MSKstakeye) Status key of the constraint. (input)
- x (MSKrealt) Primal solution value of the constraint. (input)
- sl (MSKrealt) Solution value of the dual variable associated with the lower bound. (input)
- $\bullet$  su  $({\it MSKrealt}\,)$  Solution value of the dual variable associated with the upper bound. (input)

```
Return (MSKrescodee) - The function response code.

Groups Solution information, Solution - primal, Solution - dual
```

MSK\_putcslice

```
MSKrescodee (MSKAPI MSK_putcslice) (
MSKtask_t task,
MSKint32t first,
MSKint32t last,
const MSKrealt * slice)
```

Modifies a slice in the linear term c in the objective using the principle

$$c_{j} = slice[j - first], \quad j = first, ..., last - 1$$

Data checks are performed as in MSK\_putcj.

# Parameters

- task (MSKtask\_t) An optimization task. (input)
- first (MSKint32t) First element in the slice of c. (input)
- last (MSKint32t) Last element plus 1 of the slice in c to be changed. (input)
- slice (MSKrealt\*) New numerical values for coefficients in c that should be modified. (input)

 ${\bf Return} \ ({\tt MSKrescodee})$  – The function response code.

Groups Problem data - linear part, Problem data - objective

MSK\_putdouparam

```
MSKrescodee (MSKAPI MSK_putdouparam) (
MSKtask_t task,
MSKdparame param,
MSKrealt parvalue)
```

Sets the value of a double parameter.

# **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- param (MSKdparame) Which parameter. (input)
- parvalue (MSKrealt) Parameter value. (input)

Return (MSKrescodee) - The function response code.

Groups Parameters

MSK\_putexitfunc

```
MSKrescodee (MSKAPI MSK_putexitfunc) (
    MSKenv_t env,
    MSKexitfunc exitfunc,
    MSKuserhandle_t handle)
```

In case **MOSEK** experiences a fatal error, then a user-defined exit function can be called. The exit function should terminate **MOSEK**. In general it is not necessary to define an exit function.

# **Parameters**

- env (MSKenv\_t) The MOSEK environment. (input)
- exitfunc (MSKexitfunc) A user-defined exit function. (input)
- handle (MSKuserhandle\_t) A pointer to a user-defined data structure which is passed to exitfunc when called. (input)

```
Return (MSKrescodee) - The function response code.
Groups Logging, Callback
```

MSK\_putintparam

```
MSKrescodee (MSKAPI MSK_putintparam) (
MSKtask_t task,
MSKiparame param,
MSKint32t parvalue)
```

Sets the value of an integer parameter.

# Parameters

- task (MSKtask\_t) An optimization task. (input)
- param (MSKiparame) Which parameter. (input)
- parvalue (MSKint32t) Parameter value. (input)

 ${\bf Return}~({\it MSKrescodee}\,)$  — The function response code.

Groups Parameters

MSK\_putlicensecode

```
MSKrescodee (MSKAPI MSK_putlicensecode) (
    MSKenv_t env,
    const MSKint32t * code)
```

Input a runtime license code.

### Parameters

- env (MSKenv\_t) The MOSEK environment. (input)
- code (MSKint32t\*) A runtime license code. (input)

Return (MSKrescodee) - The function response code.

Groups License system

MSK\_putlicensedebug

```
MSKrescodee (MSKAPI MSK_putlicensedebug) (
MSKenv_t env,
MSKint32t licdebug)
```

Enables debug information for the license system. If licdebug is non-zero, then MOSEK will print debug info regarding the license checkout.

### **Parameters**

- env (MSKenv\_t) The MOSEK environment. (input)
- licdebug (MSKint32t) Whether license checkout debug info should be printed. (input)

Return (MSKrescodee) - The function response code.

Groups License system

MSK\_putlicensepath

```
MSKrescodee (MSKAPI MSK_putlicensepath) (
MSKenv_t env,
const char * licensepath)
```

Set the path to the license file.

### **Parameters**

- env (MSKenv\_t) The MOSEK environment. (input)
- licensepath (MSKstring\_t) A path specifying where to search for the license. (input)

 ${\bf Return} \ ({\it MSKrescodee}\,)$  — The function response code.

 ${\bf Groups}\ {\it License \ system}$ 

MSK\_putlicensewait

```
MSKrescodee (MSKAPI MSK_putlicensewait) (
MSKenv_t env,
MSKint32t licwait)
```

Control whether **MOSEK** should wait for an available license if no license is available. If licwait is non-zero, then **MOSEK** will wait for licwait-1 milliseconds between each check for an available license.

### Parameters

- env (MSKenv\_t) The MOSEK environment. (input)
- licwait (MSKint32t) Whether MOSEK should wait for a license if no license is available. (input)

Return (MSKrescodee) - The function response code.

Groups License system

MSK\_putmaxnumanz

```
MSKrescodee (MSKAPI MSK_putmaxnumanz) (
MSKtask_t task,
MSKint64t maxnumanz)
```

Sets the number of preallocated non-zero entries in A.

**MOSEK** stores only the non-zero elements in the linear coefficient matrix A and it cannot predict how much storage is required to store A. Using this function it is possible to specify the number of non-zeros to preallocate for storing A.

If the number of non-zeros in the problem is known, it is a good idea to set  $\mathtt{maxnumanz}$  slightly larger than this number, otherwise a rough estimate can be used. In general, if A is inputted in many small chunks, setting this value may speed up the data input phase.

It is not mandatory to call this function, since  $\mathbf{MOSEK}$  will reallocate internal structures whenever it is necessary.

The function call has no effect if both maxnumcon and maxnumvar are zero.

# Parameters

- task (MSKtask\_t) An optimization task. (input)
- maxnumanz (MSKint64t) Number of preallocated non-zeros in A. (input)

Return (MSKrescodee) - The function response code.

Groups Environment and task management, Problem data - semidefinite

 ${\tt MSK\_putmaxnumbarvar}$ 

```
MSKrescodee (MSKAPI MSK_putmaxnumbarvar) (
MSKtask_t task,
MSKint32t maxnumbarvar)
```

Sets the number of preallocated symmetric matrix variables in the optimization task. When this number of variables is reached **MOSEK** will automatically allocate more space for variables.

It is not mandatory to call this function. It only gives a hint about the amount of data to preallocate for efficiency reasons.

Please note that maxnumbarvar must be larger than the current number of symmetric matrix variables in the task.

### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- maxnumbarvar (MSKint32t) Number of preallocated symmetric matrix variables. (input)

 ${\bf Return} \ ({\tt MSKrescodee})$  – The function response code.

Groups Environment and task management, Problem data - semidefinite

### MSK\_putmaxnumcon

```
MSKrescodee (MSKAPI MSK_putmaxnumcon) (
MSKtask_t task,
MSKint32t maxnumcon)
```

Sets the number of preallocated constraints in the optimization task. When this number of constraints is reached **MOSEK** will automatically allocate more space for constraints.

It is never mandatory to call this function, since **MOSEK** will reallocate any internal structures whenever it is required.

Please note that maxnumcon must be larger than the current number of constraints in the task.

#### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- maxnumcon (MSKint32t) Number of preallocated constraints in the optimization task. (input)

Return (MSKrescodee) - The function response code.

Groups Environment and task management, Problem data - constraints

### MSK\_putmaxnumcone

```
MSKrescodee (MSKAPI MSK_putmaxnumcone) (
MSKtask_t task,
MSKint32t maxnumcone)
```

Sets the number of preallocated conic constraints in the optimization task. When this number of conic constraints is reached **MOSEK** will automatically allocate more space for conic constraints.

It is not mandatory to call this function, since **MOSEK** will reallocate any internal structures whenever it is required.

Please note that maxnumcon must be larger than the current number of conic constraints in the task.

### Parameters

- task (MSKtask\_t) An optimization task. (input)
- maxnumcone (MSKint32t) Number of preallocated conic constraints in the optimization task. (input)

**Return** (*MSKrescodee*) – The function response code.

Groups Environment and task management, Problem data - cones

### MSK\_putmaxnumqnz

```
MSKrescodee (MSKAPI MSK_putmaxnumqnz) (
MSKtask_t task,
MSKint64t maxnumqnz)
```

Sets the number of preallocated non-zero entries in quadratic terms.

**MOSEK** stores only the non-zero elements in Q. Therefore, **MOSEK** cannot predict how much storage is required to store Q. Using this function it is possible to specify the number non-zeros to preallocate for storing Q (both objective and constraints).

It may be advantageous to reserve more non-zeros for Q than actually needed since it may improve the internal efficiency of  $\mathbf{MOSEK}$ , however, it is never worthwhile to specify more than the double of the anticipated number of non-zeros in Q.

It is not mandatory to call this function, since **MOSEK** will reallocate internal structures whenever it is necessary.

# Parameters

- task (MSKtask\_t) An optimization task. (input)
- maxnumqnz (MSKint64t) Number of non-zero elements preallocated in quadratic coefficient matrices. (input)

Return (MSKrescodee) - The function response code.

Groups Environment and task management, Problem data - quadratic part

MSK\_putmaxnumvar

```
MSKrescodee (MSKAPI MSK_putmaxnumvar) (
MSKtask_t task,
MSKint32t maxnumvar)
```

Sets the number of preallocated variables in the optimization task. When this number of variables is reached  $\mathbf{MOSEK}$  will automatically allocate more space for variables.

It is not mandatory to call this function. It only gives a hint about the amount of data to preallocate for efficiency reasons.

Please note that maxnumvar must be larger than the current number of variables in the task.

# **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- maxnumvar (MSKint32t) Number of preallocated variables in the optimization task. (input)

Return (MSKrescodee) - The function response code.

Groups Environment and task management, Problem data - variables

MSK\_putnadouparam

```
MSKrescodee (MSKAPI MSK_putnadouparam) (
    MSKtask_t task,
    const char * paramname,
    MSKrealt parvalue)
```

Sets the value of a named double parameter.

# **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- paramname (MSKstring\_t) Name of a parameter. (input)
- parvalue (MSKrealt) Parameter value. (input)

Return (MSKrescodee) - The function response code.

Groups Parameters

```
MSKrescodee (MSKAPI MSK_putnaintparam) (
MSKtask_t task,
const char * paramname,
MSKint32t parvalue)
```

Sets the value of a named integer parameter.

### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- paramname (MSKstring\_t) Name of a parameter. (input)
- parvalue (MSKint32t) Parameter value. (input)

Return (MSKrescodee) - The function response code.

Groups Parameters

MSK\_putnastrparam

```
MSKrescodee (MSKAPI MSK_putnastrparam) (
   MSKtask_t task,
   const char * paramname,
   const char * parvalue)
```

Sets the value of a named string parameter.

### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- paramname (MSKstring\_t) Name of a parameter. (input)
- parvalue (MSKstring\_t) Parameter value. (input)

Return (MSKrescodee) - The function response code.

Groups Parameters

MSK\_putobjname

```
MSKrescodee (MSKAPI MSK_putobjname) (
MSKtask_t task,
const char * objname)
```

Assigns a new name to the objective.

# **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- objname  $(\textit{MSKstring\_t})$  Name of the objective. (input)

Return (MSKrescodee) - The function response code.

Groups Problem data - linear part, Names, Problem data - objective

MSK\_putobjsense

```
MSKrescodee (MSKAPI MSK_putobjsense) (
MSKtask_t task,
MSKobjsensee sense)
```

Sets the objective sense of the task.

# **Parameters**

• task (MSKtask\_t) - An optimization task. (input)

• sense (MSKobjsensee) - The objective sense of the task. The values MSK\_OBJECTIVE\_SENSE\_MAXIMIZE and MSK\_OBJECTIVE\_SENSE\_MINIMIZE mean that the problem is maximized or minimized respectively. (input)

Return (MSKrescodee) - The function response code.

Groups Problem data - linear part, Problem data - objective

MSK\_putoptserverhost

```
MSKrescodee (MSKAPI MSK_putoptserverhost) (
    MSKtask_t task,
    const char * host)
```

Specify an OptServer URL for remote calls. The URL should contain protocol, host and port in the form http://server:port. If the URL is set using this function, all subsequent calls to any MOSEK function that involves synchronous optimization will be sent to the specified OptServer instead of being executed locally. Passing NULL deactivates this redirection.

# Parameters

- task (MSKtask\_t) An optimization task. (input)
- host (MSKstring\_t) A URL specifying the optimization server to be used. (input)

Return (MSKrescodee) - The function response code.

Groups Remote optimization

MSK\_putparam

```
MSKrescodee (MSKAPI MSK_putparam) (
    MSKtask_t task,
    const char * parname,
    const char * parvalue)
```

Checks if parname is valid parameter name. If it is, the parameter is assigned the value specified by parvalue.

# Parameters

- task (MSKtask\_t) An optimization task. (input)
- parname (MSKstring\_t) Parameter name. (input)
- parvalue (MSKstring\_t) Parameter value. (input)

Return (MSKrescodee) - The function response code.

Groups Parameters

MSK\_putqcon

```
MSKrescodee (MSKAPI MSK_putqcon) (
   MSKtask_t task,
   MSKint32t numqcnz,
   const MSKint32t * qcsubk,
   const MSKint32t * qcsubi,
   const MSKint32t * qcsubj,
   const MSKrealt * qcval)
```

Replace all quadratic entries in the constraints. The list of constraints has the form

$$l_k^c \leq \frac{1}{2} \sum_{i=0}^{numvar-1} \sum_{j=0}^{numvar-1} q_{ij}^k x_i x_j + \sum_{j=0}^{numvar-1} a_{kj} x_j \leq u_k^c, \quad k = 0, \dots, m-1.$$

This function sets all the quadratic terms to zero and then performs the update:

$$q_{\mathtt{qcsubi[t]},\mathtt{qcsubj[t]}}^{\mathtt{qcsubk[t]}} = q_{\mathtt{qcsubj[t]},\mathtt{qcsubi[t]}}^{\mathtt{qcsubk[t]}} = q_{\mathtt{qcsubj[t]},\mathtt{qcsubi[t]}}^{\mathtt{qcsubk[t]}} + \mathtt{qcval[t]},$$

for  $t = 0, \dots, numqcnz - 1$ .

Please note that:

- For large problems it is essential for the efficiency that the function <code>MSK\_putmaxnumqnz</code> is employed to pre-allocate space.
- Only the lower triangular parts should be specified because the Q matrices are symmetric. Specifying entries where i < j will result in an error.
- Only non-zero elements should be specified.
- The order in which the non-zero elements are specified is insignificant.
- Duplicate elements are added together as shown above. Hence, it is usually not recommended to specify the same entry multiple times.

For a code example see Section Quadratic Optimization

# Parameters

- task (MSKtask\_t) An optimization task. (input)
- numqcnz (MSKint32t) Number of quadratic terms. (input)
- qcsubk (MSKint32t\*) Constraint subscripts for quadratic coefficients. (input)
- qcsubi (MSKint32t\*) Row subscripts for quadratic constraint matrix. (input)
- qcsubj (MSKint32t\*) Column subscripts for quadratic constraint matrix. (input)
- qcval (MSKrealt\*) Quadratic constraint coefficient values. (input)

Return (MSKrescodee) - The function response code.

Groups Problem data - quadratic part

MSK\_putqconk

```
MSKrescodee (MSKAPI MSK_putqconk) (
   MSKtask_t task,
   MSKint32t k,
   MSKint32t numqcnz,
   const MSKint32t * qcsubi,
   const MSKint32t * qcsubj,
   const MSKrealt * qcval)
```

Replaces all the quadratic entries in one constraint. This function performs the same operations as  $MSK_putqcon$  but only with respect to constraint number k and it does not modify the other constraints. See the description of  $MSK_putqcon$  for definitions and important remarks.

### **Parameters**

- task  $(\textit{MSKtask\_t})$  An optimization task. (input)
- k  $(\mathit{MSKint32t})$  The constraint in which the new Q elements are inserted. (input)
- numqcnz (MSKint32t) Number of quadratic terms. (input)
- qcsubi (MSKint32t\*) Row subscripts for quadratic constraint matrix. (input)
- qcsubj  $(\mathit{MSKint32t}*)$  Column subscripts for quadratic constraint matrix. (input)
- qcval (MSKrealt\*) Quadratic constraint coefficient values. (input)

Return (MSKrescodee) - The function response code.

Groups Problem data - quadratic part

```
MSKrescodee (MSKAPI MSK_putqobj) (
   MSKtask_t task,
   MSKint32t numqonz,
   const MSKint32t * qosubi,
   const MSKint32t * qosubj,
   const MSKrealt * qoval)
```

Replace all quadratic terms in the objective. If the objective has the form

$$\frac{1}{2} \sum_{i=0}^{numvar-1} \sum_{j=0}^{numvar-1} q_{ij}^{o} x_i x_j + \sum_{j=0}^{numvar-1} c_j x_j + c^f$$

then this function sets all the quadratic terms to zero and then performs the update:

$$q^o_{\mathtt{qosubi[t]},\mathtt{qosubj[t]}} = q^o_{\mathtt{qosubj[t]},\mathtt{qosubi[t]}} = q^o_{\mathtt{qosubj[t]},\mathtt{qosubi[t]}} + \mathtt{qoval[t]},$$

for  $t = 0, \dots, numqon z - 1$ .

See the description of MSK\_putqcon for important remarks and example.

### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- numqonz (MSKint32t) Number of non-zero elements in the quadratic objective terms. (input)
- qosubi (MSKint32t\*) Row subscripts for quadratic objective coefficients. (input)
- qosubj (MSKint32t\*) Column subscripts for quadratic objective coefficients.
   (input)
- qoval (MSKrealt\*) Quadratic objective coefficient values. (input)

Return (MSKrescodee) - The function response code.

Groups Problem data - quadratic part, Problem data - objective

MSK\_putqobjij

```
MSKrescodee (MSKAPI MSK_putqobjij) (
MSKtask_t task,
MSKint32t i,
MSKint32t j,
MSKrealt qoij)
```

Replaces one coefficient in the quadratic term in the objective. The function performs the assignment

$$q_{ij}^o = q_{ii}^o = \text{qoij}.$$

Only the elements in the lower triangular part are accepted. Setting  $q_{ij}$  with j > i will cause an error.

Please note that replacing all quadratic elements one by one is more computationally expensive than replacing them all at once. Use  $MSK_putqobj$  instead whenever possible.

### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- i (MSKint32t) Row index for the coefficient to be replaced. (input)
- j (MSKint32t) Column index for the coefficient to be replaced. (input)
- qoij (MSKrealt) The new value for  $q_{ij}^o$ . (input)

```
Return (MSKrescodee) - The function response code.
Groups Problem data - quadratic part, Problem data - objective
```

MSK\_putresponsefunc

```
MSKrescodee (MSKAPI MSK_putresponsefunc) (
MSKtask_t task,
MSKresponsefunc responsefunc,
MSKuserhandle_t handle)
```

Inputs a user-defined error callback which is called when an error or warning occurs.

# Parameters

- task (MSKtask\_t) An optimization task. (input)
- responsefunc (MSKresponsefunc) A user-defined response handling function. (input)
- handle (MSKuserhandle\_t) A user-defined data structure that is passed to the function responsefunc whenever it is called. (input)

 ${\bf Return} \ \ ({\it MSKrescodee}\,) - {\bf The} \ {\bf function} \ {\bf response} \ {\bf code}.$ 

Groups Callback, Logging

MSK\_putskc

```
MSKrescodee (MSKAPI MSK_putskc) (
MSKtask_t task,
MSKsoltypee whichsol,
const MSKstakeye * skc)
```

Sets the status keys for the constraints.

# Parameters

- task (MSKtask\_t) An optimization task. (input)
- whichsol (MSKsoltypee) Selects a solution. (input)
- skc (MSKstakeye\*) Status keys for the constraints. (input)

Return (MSKrescodee) - The function response code.

**Groups** Solution information

MSK\_putskcslice

```
MSKrescodee (MSKAPI MSK_putskcslice) (
MSKtask_t task,
MSKsoltypee whichsol,
MSKint32t first,
MSKint32t last,
const MSKstakeye * skc)
```

Sets the status keys for a slice of the constraints.

# **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- whichsol (MSKsoltypee) Selects a solution. (input)
- first (MSKint32t) First index in the sequence. (input)
- last (MSKint32t) Last index plus 1 in the sequence. (input)
- skc (MSKstakeye\*) Status keys for the constraints. (input)

Return (MSKrescodee) - The function response code.

Groups Solution information

```
MSKrescodee (MSKAPI MSK_putskx) (
   MSKtask_t task,
   MSKsoltypee whichsol,
   const MSKstakeye * skx)
```

Sets the status keys for the scalar variables.

### Parameters

- task (MSKtask\_t) An optimization task. (input)
- whichsol (MSKsoltypee) Selects a solution. (input)
- skx (MSKstakeye\*) Status keys for the variables. (input)

Return (MSKrescodee) - The function response code.

**Groups** Solution information

MSK\_putskxslice

```
MSKrescodee (MSKAPI MSK_putskxslice) (
   MSKtask_t task,
   MSKsoltypee whichsol,
   MSKint32t first,
   MSKint32t last,
   const MSKstakeye * skx)
```

Sets the status keys for a slice of the variables.

### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- whichsol (MSKsoltypee) Selects a solution. (input)
- first (MSKint32t) First index in the sequence. (input)
- last (MSKint32t) Last index plus 1 in the sequence. (input)
- skx (MSKstakeye\*) Status keys for the variables. (input)

Return (MSKrescodee) - The function response code.

**Groups** Solution information

MSK\_putslc

```
MSKrescodee (MSKAPI MSK_putslc) (
    MSKtask_t task,
    MSKsoltypee whichsol,
    const MSKrealt * slc)
```

Sets the  $s_l^c$  vector for a solution.

# **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- whichsol (MSKsoltypee) Selects a solution. (input)
- slc (MSKrealt\*) Dual variables corresponding to the lower bounds on the constraints. (input)

Return (MSKrescodee) - The function response code.

 $\textbf{Groups} \ \ \textit{Solution - dual}$ 

MSK\_putslcslice

```
MSKrescodee (MSKAPI MSK_putslcslice) (
MSKtask_t task,
MSKsoltypee whichsol,
MSKint32t first,
MSKint32t last,
const MSKrealt * slc)
```

Sets a slice of the  $s_l^c$  vector for a solution.

### Parameters

- task (MSKtask\_t) An optimization task. (input)
- whichsol (MSKsoltypee) Selects a solution. (input)
- first (MSKint32t) First index in the sequence. (input)
- last (MSKint32t) Last index plus 1 in the sequence. (input)
- slc(MSKrealt\*) Dual variables corresponding to the lower bounds on the constraints. (input)

Return (MSKrescodee) - The function response code. Groups Solution - dual

MSK\_putslx

```
MSKrescodee (MSKAPI MSK_putslx) (
MSKtask_t task,
MSKsoltypee whichsol,
const MSKrealt * slx)
```

Sets the  $s_l^x$  vector for a solution.

### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- whichsol (MSKsoltypee) Selects a solution. (input)
- $\bullet$  slx (MSKrealt\*) Dual variables corresponding to the lower bounds on the variables. (input)

Return (MSKrescodee) - The function response code.

Groups Solution - dual

MSK\_putslxslice

```
MSKrescodee (MSKAPI MSK_putslxslice) (
MSKtask_t task,
MSKsoltypee whichsol,
MSKint32t first,
MSKint32t last,
const MSKrealt * slx)
```

Sets a slice of the  $s_l^x$  vector for a solution.

### Parameters

- task (MSKtask\_t) An optimization task. (input)
- whichsol (MSKsoltypee) Selects a solution. (input)
- first (MSKint32t) First index in the sequence. (input)
- last (MSKint32t) Last index plus 1 in the sequence. (input)
- slx (MSKrealt\*) Dual variables corresponding to the lower bounds on the variables. (input)

Return (MSKrescodee) - The function response code.

### Groups Solution - dual

MSK\_putsnx

```
MSKrescodee (MSKAPI MSK_putsnx) (
   MSKtask_t task,
   MSKsoltypee whichsol,
   const MSKrealt * sux)
```

Sets the  $s_n^x$  vector for a solution.

#### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- whichsol (MSKsoltypee) Selects a solution. (input)
- sux (MSKrealt\*) Dual variables corresponding to the upper bounds on the variables. (input)

Return (MSKrescodee) – The function response code.

Groups Solution - dual

MSK\_putsnxslice

```
MSKrescodee (MSKAPI MSK_putsnxslice) (
MSKtask_t task,
MSKsoltypee whichsol,
MSKint32t first,
MSKint32t last,
const MSKrealt * snx)
```

Sets a slice of the  $s_n^x$  vector for a solution.

### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- whichsol (MSKsoltypee) Selects a solution. (input)
- first (MSKint32t) First index in the sequence. (input)
- last (MSKint32t) Last index plus 1 in the sequence. (input)
- snx(MSKrealt\*) Dual variables corresponding to the conic constraints on the variables. (input)

Return (MSKrescodee) - The function response code.

Groups Solution - dual

MSK\_putsolution

```
MSKrescodee (MSKAPI MSK_putsolution) (
   MSKtask_t task,
   MSKsoltypee whichsol,
   const MSKstakeye * skc,
   const MSKstakeye * skx,
   const MSKstakeye * skn,
   const MSKrealt * xc,
   const MSKrealt * xx,
   const MSKrealt * y,
   const MSKrealt * slc,
   const MSKrealt * suc,
   const MSKrealt * suc,
   const MSKrealt * slx,
   con
```

Inserts a solution into the task.

#### Parameters

- task (MSKtask\_t) An optimization task. (input)
- whichsol (MSKsoltypee) Selects a solution. (input)
- skc (MSKstakeye\*) Status keys for the constraints. (input)
- skx (MSKstakeye\*) Status keys for the variables. (input)
- skn (MSKstakeye\*) Status keys for the conic constraints. (input)
- xc (MSKrealt\*) Primal constraint solution. (input)
- xx (MSKrealt\*) Primal variable solution. (input)
- y (MSKrealt\*) Vector of dual variables corresponding to the constraints. (input)
- slc (MSKrealt\*) Dual variables corresponding to the lower bounds on the constraints. (input)
- suc (MSKrealt\*) Dual variables corresponding to the upper bounds on the constraints. (input)
- slx (MSKrealt\*) Dual variables corresponding to the lower bounds on the variables. (input)
- sux (MSKrealt\*) Dual variables corresponding to the upper bounds on the variables. (input)
- snx (MSKrealt\*) Dual variables corresponding to the conic constraints on the variables. (input)

Return (MSKrescodee) - The function response code.

Groups Solution information, Solution - primal, Solution - dual

MSK\_putsolutionyi

```
MSKrescodee (MSKAPI MSK_putsolutionyi) (
MSKtask_t task,
MSKint32t i,
MSKsoltypee whichsol,
MSKrealt y)
```

Inputs the dual variable of a solution.

### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- i (MSKint32t) Index of the dual variable. (input)
- whichsol (MSKsoltypee) Selects a solution. (input)
- y (MSKrealt) Solution value of the dual variable. (input)

Return (MSKrescodee) - The function response code.

Groups Solution information, Solution - dual

MSK\_putstrparam

```
MSKrescodee (MSKAPI MSK_putstrparam) (
    MSKtask_t task,
    MSKsparame param,
    const char * parvalue)
```

Sets the value of a string parameter.

- task (MSKtask\_t) An optimization task. (input)
- param (MSKsparame) Which parameter. (input)

```
• parvalue (\textit{MSKstring\_t}) - Parameter value. (input) Return (\textit{MSKrescodee}) - The function response code. Groups Parameters
```

MSK\_putsuc

```
MSKrescodee (MSKAPI MSK_putsuc) (
   MSKtask_t task,
   MSKsoltypee whichsol,
   const MSKrealt * suc)
```

Sets the  $s_u^c$  vector for a solution.

#### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- whichsol (MSKsoltypee) Selects a solution. (input)
- $suc\ (\textit{MSKrealt*})$  Dual variables corresponding to the upper bounds on the constraints. (input)

Return (MSKrescodee) – The function response code.

 ${\bf Groups} \ \ Solution \ - \ dual$ 

MSK\_putsucslice

```
MSKrescodee (MSKAPI MSK_putsucslice) (
   MSKtask_t task,
   MSKsoltypee whichsol,
   MSKint32t first,
   MSKint32t last,
   const MSKrealt * suc)
```

Sets a slice of the  $s_u^c$  vector for a solution.

#### Parameters

- task (MSKtask\_t) An optimization task. (input)
- whichsol (MSKsoltypee) Selects a solution. (input)
- first (MSKint32t) First index in the sequence. (input)
- last (MSKint32t) Last index plus 1 in the sequence. (input)
- suc (MSKrealt\*) Dual variables corresponding to the upper bounds on the constraints. (input)

Return (MSKrescodee) - The function response code.

Groups Solution - dual

MSK\_putsux

```
MSKrescodee (MSKAPI MSK_putsux) (
   MSKtask_t task,
   MSKsoltypee whichsol,
   const MSKrealt * sux)
```

Sets the  $s_u^x$  vector for a solution.

- task (MSKtask\_t) An optimization task. (input)
- whichsol (MSKsoltypee) Selects a solution. (input)
- $\bullet$  sux (MSKrealt\*) Dual variables corresponding to the upper bounds on the variables. (input)

```
Return (MSKrescodee) - The function response code.
Groups Solution - dual
```

MSK\_putsuxslice

```
MSKrescodee (MSKAPI MSK_putsuxslice) (
   MSKtask_t task,
   MSKsoltypee whichsol,
   MSKint32t first,
   MSKint32t last,
   const MSKrealt * sux)
```

Sets a slice of the  $s_u^x$  vector for a solution.

### Parameters

- task (MSKtask\_t) An optimization task. (input)
- whichsol (MSKsoltypee) Selects a solution. (input)
- first (MSKint32t) First index in the sequence. (input)
- last (MSKint32t) Last index plus 1 in the sequence. (input)
- $\bullet$  sux (MSKrealt\*) Dual variables corresponding to the upper bounds on the variables. (input)

 $\textbf{Return} \ \ (\textit{\tt MSKrescodee}) - \textbf{The function response code}.$ 

Groups Solution - dual

MSK\_puttaskname

```
MSKrescodee (MSKAPI MSK_puttaskname) (
MSKtask_t task,
const char * taskname)
```

Assigns a new name to the task.

### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- taskname (MSKstring\_t) Name assigned to the task. (input)

Return (MSKrescodee) - The function response code.

Groups Names, Environment and task management

MSK\_putvarbound

```
MSKrescodee (MSKAPI MSK_putvarbound) (
MSKtask_t task,
MSKint32t j,
MSKboundkeye bkx,
MSKrealt blx,
MSKrealt bux)
```

Changes the bounds for one variable.

If the bound value specified is numerically larger than  $MSK\_DPAR\_DATA\_TOL\_BOUND\_INF$  it is considered infinite and the bound key is changed accordingly. If a bound value is numerically larger than  $MSK\_DPAR\_DATA\_TOL\_BOUND\_WRN$ , a warning will be displayed, but the bound is inputted as specified.

- task (MSKtask\_t) An optimization task. (input)
- j (MSKint32t) Index of the variable. (input)

```
• bkx (MSKboundkeye) - New bound key. (input)
```

- blx (MSKrealt) New lower bound. (input)
- bux (MSKrealt) New upper bound. (input)

Return (MSKrescodee) - The function response code.

**Groups** Problem data - linear part, Problem data - variables, Problem data - bounds

#### MSK\_putvarboundlist

```
MSKrescodee (MSKAPI MSK_putvarboundlist) (
MSKtask_t task,
MSKint32t num,
const MSKint32t * sub,
const MSKboundkeye * bkx,
const MSKrealt * blx,
const MSKrealt * bux)
```

Changes the bounds for one or more variables. If multiple bound changes are specified for a variable, then only the last change takes effect. Data checks are performed as in MSK\_putvarbound.

#### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- num (MSKint32t) Number of bounds that should be changed. (input)
- sub (MSKint32t\*) List of variable indexes. (input)
- bkx (MSKboundkeye\*) Bound keys for the variables. (input)
- blx (MSKrealt\*) Lower bounds for the variables. (input)
- bux (MSKrealt\*) Upper bounds for the variables. (input)

Return (MSKrescodee) - The function response code.

Groups Problem data - linear part, Problem data - variables, Problem data - bounds

# ${\tt MSK\_putvarboundlistconst}$

```
MSKrescodee (MSKAPI MSK_putvarboundlistconst) (
   MSKtask_t task,
   MSKint32t num,
   const MSKint32t * sub,
   MSKboundkeye bkx,
   MSKrealt blx,
   MSKrealt bux)
```

Changes the bounds for one or more variables. Data checks are performed as in MSK\_putvarbound.

#### **Parameters**

- $\bullet$  task ( $\textit{MSKtask\_t}$  ) An optimization task. (input)
- num (MSKint32t) Number of bounds that should be changed. (input)
- sub (MSKint32t\*) List of variable indexes. (input)
- bkx (MSKboundkeye) New bound key for all variables in the list. (input)
- blx (MSKrealt) New lower bound for all variables in the list. (input)
- bux (MSKrealt) New upper bound for all variables in the list. (input)

**Return** (*MSKrescodee*) – The function response code.

Groups Problem data - linear part, Problem data - variables, Problem data - bounds

MSK\_putvarboundslice

```
MSKrescodee (MSKAPI MSK_putvarboundslice) (
MSKtask_t task,
MSKint32t first,
MSKint32t last,
const MSKboundkeye * bkx,
const MSKrealt * blx,
const MSKrealt * bux)
```

Changes the bounds for a slice of the variables. Data checks are performed as in MSK\_putvarbound.

#### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- first (MSKint32t) First index in the sequence. (input)
- last (MSKint32t) Last index plus 1 in the sequence. (input)
- bkx (MSKboundkeye\*) Bound keys for the variables. (input)
- blx (MSKrealt\*) Lower bounds for the variables. (input)
- bux (MSKrealt\*) Upper bounds for the variables. (input)

 ${\bf Return} \ \ ({\it MSKrescodee}\,) - {\bf The} \ {\bf function} \ {\bf response} \ {\bf code}.$ 

Groups Problem data - linear part, Problem data - variables, Problem data - bounds

MSK\_putvarboundsliceconst

```
MSKrescodee (MSKAPI MSK_putvarboundsliceconst) (
MSKtask_t task,
MSKint32t first,
MSKint32t last,
MSKboundkeye bkx,
MSKrealt blx,
MSKrealt bux)
```

Changes the bounds for a slice of the variables. Data checks are performed as in MSK\_putvarbound.

### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- first (MSKint32t) First index in the sequence. (input)
- last (MSKint32t) Last index plus 1 in the sequence. (input)
- bkx (MSKboundkeye) New bound key for all variables in the slice. (input)
- blx (MSKrealt) New lower bound for all variables in the slice. (input)
- bux (MSKrealt) New upper bound for all variables in the slice. (input)

Return (MSKrescodee) - The function response code.

Groups Problem data - linear part, Problem data - variables, Problem data - bounds

MSK\_putvarname

```
MSKrescodee (MSKAPI MSK_putvarname) (
MSKtask_t task,
MSKint32t j,
const char * name)
```

Sets the name of a variable.

- $task (MSKtask_t)$  An optimization task. (input)
- j (MSKint32t) Index of the variable. (input)
- $\bullet$  name (MSKstring\_t) The variable name. (input)

```
Return (MSKrescodee) - The function response code.
Groups Names, Problem data - variables, Problem data - linear part
```

MSK\_putvarsolutionj

```
MSKrescodee (MSKAPI MSK_putvarsolutionj) (
    MSKtask_t task,
    MSKint32t j,
    MSKsoltypee whichsol,
    MSKstakeye sk,
    MSKrealt x,
    MSKrealt sl,
    MSKrealt su,
    MSKrealt su,
```

Sets the primal and dual solution information for a single variable.

#### Parameters

- task (MSKtask\_t) An optimization task. (input)
- j (MSKint32t) Index of the variable. (input)
- whichsol (MSKsoltypee) Selects a solution. (input)
- sk (MSKstakeye) Status key of the variable. (input)
- x (MSKrealt) Primal solution value of the variable. (input)
- $\mathfrak{sl}$  (MSKrealt) Solution value of the dual variable associated with the lower bound. (input)
- su (MSKrealt) Solution value of the dual variable associated with the upper bound. (input)
- sn (MSKrealt) Solution value of the dual variable associated with the conic constraint. (input)

Return (MSKrescodee) - The function response code.

Groups Solution information, Solution - primal, Solution - dual

MSK\_putvartype

```
MSKrescodee (MSKAPI MSK_putvartype) (
    MSKtask_t task,
    MSKint32t j,
    MSKvariabletypee vartype)
```

Sets the variable type of one variable.

#### Parameters

- task (MSKtask\_t) An optimization task. (input)
- j (MSKint32t) Index of the variable. (input)
- vartype (MSKvariabletypee) The new variable type. (input)

Return (MSKrescodee) - The function response code.

Groups Problem data - variables

 ${\tt MSK\_putvartypelist}$ 

```
MSKrescodee (MSKAPI MSK_putvartypelist) (
   MSKtask_t task,
   MSKint32t num,
   const MSKint32t * subj,
   const MSKvariabletypee * vartype)
```

Sets the variable type for one or more variables. If the same index is specified multiple times in subj only the last entry takes effect.

#### Parameters

- task (MSKtask\_t) An optimization task. (input)
- num (MSKint32t) Number of variables for which the variable type should be set. (input)
- subj (MSKint32t\*) A list of variable indexes for which the variable type should be changed. (input)
- vartype (MSKvariabletypee\*) A list of variable types that should be assigned to the variables specified by subj. (input)

 ${\bf Return} \ ({\it MSKrescodee}\,)$  – The function response code.

Groups Problem data - variables

MSK\_putxc

```
MSKrescodee (MSKAPI MSK_putxc) (
MSKtask_t task,
MSKsoltypee whichsol,
MSKrealt * xc)
```

Sets the  $x^c$  vector for a solution.

#### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- whichsol (MSKsoltypee) Selects a solution. (input)
- xc (MSKrealt\*) Primal constraint solution. (output)

Return (MSKrescodee) - The function response code.

Groups Solution - primal

MSK\_putxcslice

```
MSKrescodee (MSKAPI MSK_putxcslice) (
MSKtask_t task,
MSKsoltypee whichsol,
MSKint32t first,
MSKint32t last,
const MSKrealt * xc)
```

Sets a slice of the  $x^c$  vector for a solution.

# **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- whichsol (MSKsoltypee) Selects a solution. (input)
- first (MSKint32t) First index in the sequence. (input)
- last (MSKint32t) Last index plus 1 in the sequence. (input)
- xc (MSKrealt\*) Primal constraint solution. (input)

Return (MSKrescodee) - The function response code.

Groups Solution - primal

MSK\_putxx

```
MSKrescodee (MSKAPI MSK_putxx) (
MSKtask_t task,
MSKsoltypee whichsol,
const MSKrealt * xx)
```

Sets the  $x^x$  vector for a solution.

#### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- whichsol (MSKsoltypee) Selects a solution. (input)
- xx (MSKrealt\*) Primal variable solution. (input)

Return (MSKrescodee) - The function response code.

Groups Solution - primal

MSK\_putxxslice

```
MSKrescodee (MSKAPI MSK_putxxslice) (
   MSKtask_t task,
   MSKsoltypee whichsol,
   MSKint32t first,
   MSKint32t last,
   const MSKrealt * xx)
```

Sets a slice of the  $x^x$  vector for a solution.

#### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- whichsol (MSKsoltypee) Selects a solution. (input)
- first (MSKint32t) First index in the sequence. (input)
- last (MSKint32t) Last index plus 1 in the sequence. (input)
- xx (MSKrealt\*) Primal variable solution. (input)

Return (MSKrescodee) - The function response code.

Groups Solution - primal

MSK\_puty

```
MSKrescodee (MSKAPI MSK_puty) (
MSKtask_t task,
MSKsoltypee whichsol,
const MSKrealt * y)
```

Sets the y vector for a solution.

#### Parameters

- task (MSKtask\_t) An optimization task. (input)
- whichsol (MSKsoltypee) Selects a solution. (input)
- y (MSKrealt\*) Vector of dual variables corresponding to the constraints. (input)

Return (MSKrescodee) - The function response code.

Groups Solution - primal

MSK\_putyslice

```
MSKrescodee (MSKAPI MSK_putyslice) (
MSKtask_t task,
MSKsoltypee whichsol,
MSKint32t first,
MSKint32t last,
const MSKrealt * y)
```

Sets a slice of the y vector for a solution.

#### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- whichsol (MSKsoltypee) Selects a solution. (input)
- first (MSKint32t) First index in the sequence. (input)
- last (MSKint32t) Last index plus 1 in the sequence. (input)
- y (MSKrealt\*) Vector of dual variables corresponding to the constraints. (input)

Return (MSKrescodee) - The function response code. Groups Solution - dual

### MSK\_readdata

```
MSKrescodee (MSKAPI MSK_readdata) (
    MSKtask_t task,
    const char * filename)
```

Reads an optimization problem and associated data from a file. Thex extension of the file name is used to deduce the file format.

For a list of supported file types and their extensions see Supported File Formats.

Data is read from the file filename if it is a nonempty string. Otherwise data is read from the file specified by  $MSK\_SPAR\_DATA\_FILE\_NAME$ .

#### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- filename (MSKstring\_t) A valid file name. (input)

Return (MSKrescodee) - The function response code.

Groups Input/Output

### MSK\_readdataautoformat

```
MSKrescodee (MSKAPI MSK_readdataautoformat) (
    MSKtask_t task,
    const char * filename)
```

Reads an optimization problem and associated data from a file.

### Parameters

- task (MSKtask\_t) An optimization task. (input)
- filename (MSKstring\_t) A valid file name. (input)

Return (MSKrescodee) - The function response code.

Groups Input/Output

### MSK\_readdataformat

```
MSKrescodee (MSKAPI MSK_readdataformat) (
   MSKtask_t task,
   const char * filename,
   int format,
   int compress)
```

Reads an optimization problem and associated data from a file.

- $task (MSKtask_t)$  An optimization task. (input)
- filename  $(MSKstring_t)$  A valid file name. (input)

```
    format (MSKdataformate) - File data format. (input)
    compress (MSKcompresstypee) - File compression type. (input)
    Return (MSKrescodee) - The function response code.
    Groups Input/Output
```

MSK\_readjsonstring

```
MSKrescodee (MSKAPI MSK_readjsonstring) (
MSKtask_t task,
const char * data)
```

Load task data from a JSON string, replacing any data that already exists in the task object. All problem data, parameters and other settings are resorted, but if the string contains solutions, the solution status after loading a file is set to unknown, even if it is optimal or otherwise well-defined.

#### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
   data (MSKstring\_t) Problem data in text format. (input)
   Return (MSKrescodee) The function response code.
   Groups Input/Output
- MSK\_readlpstring

```
MSKrescodee (MSKAPI MSK_readlpstring) (
    MSKtask_t task,
    const char * data)
```

Load task data from a string in LP format, replacing any data that already exists in the task object.

#### **Parameters**

- $\bullet$  task (MSKtask\_t) An optimization task. (input)
- data (MSKstring\_t) Problem data in text format. (input)

Return (MSKrescodee) - The function response code.

Groups Input/Output

MSK\_readopfstring

```
MSKrescodee (MSKAPI MSK_readopfstring) (
MSKtask_t task,
const char * data)
```

Load task data from a string in OPF format, replacing any data that already exists in the task object.

#### Parameters

- task (MSKtask\_t) An optimization task. (input)
- data (MSKstring\_t) Problem data in text format. (input)

Return (MSKrescodee) - The function response code.

Groups Input/Output

MSK\_readparamfile

```
MSKrescodee (MSKAPI MSK_readparamfile) (
MSKtask_t task,
const char * filename)
```

Reads **MOSEK** parameters from a file. Data is read from the file filename if it is a nonempty string. Otherwise data is read from the file specified by MSK\_SPAR\_PARAM\_READ\_FILE\_NAME.

#### Parameters

- task (MSKtask\_t) An optimization task. (input)
- filename (MSKstring\_t) A valid file name. (input)

Return (MSKrescodee) - The function response code.

Groups Input/Output, Parameters

MSK\_readptfstring

```
MSKrescodee (MSKAPI MSK_readptfstring) (
MSKtask_t task,
const char * data)
```

Load task data from a PTF string, replacing any data that already exists in the task object. All problem data, parameters and other settings are resorted, but if the string contains solutions, the solution status after loading a file is set to unknown, even if it is optimal or otherwise well-defined.

#### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- data (MSKstring\_t) Problem data in text format. (input)

Return (MSKrescodee) - The function response code.

Groups Input/Output

MSK\_readsolution

```
MSKrescodee (MSKAPI MSK_readsolution) (
   MSKtask_t task,
   MSKsoltypee whichsol,
   const char * filename)
```

Reads a solution file and inserts it as a specified solution in the task. Data is read from the file filename if it is a nonempty string. Otherwise data is read from one of the files specified by  $MSK\_SPAR\_BAS\_SOL\_FILE\_NAME$ ,  $MSK\_SPAR\_ITR\_SOL\_FILE\_NAME$  or  $MSK\_SPAR\_INT\_SOL\_FILE\_NAME$  depending on which solution is chosen.

#### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- $\bullet \ \ which sol \ (\textit{MSKsoltypee}) Selects \ a \ solution. \ (input)$
- filename  $(MSKstring_t)$  A valid file name. (input)

Return (MSKrescodee) - The function response code.

Groups Input/Output

MSK\_readsummary

```
MSKrescodee (MSKAPI MSK_readsummary) (
MSKtask_t task,
MSKstreamtypee whichstream)
```

Prints a short summary of last file that was read.

#### Parameters

- task (MSKtask\_t) An optimization task. (input)
- whichstream (MSKstreamtypee) Index of the stream. (input)

Return (MSKrescodee) - The function response code.

### **Groups** Input/Output, Inspecting the task

MSK\_readtask

```
MSKrescodee (MSKAPI MSK_readtask) (
    MSKtask_t task,
    const char * filename)
```

Load task data from a file, replacing any data that already exists in the task object. All problem data, parameters and other settings are resorted, but if the file contains solutions, the solution status after loading a file is set to unknown, even if it was optimal or otherwise well-defined when the file was dumped.

See section The Task Format for a description of the Task format.

#### Parameters

- task (MSKtask\_t) An optimization task. (input)
- filename  $(MSKstring_t)$  A valid file name. (input)

Return (MSKrescodee) - The function response code.

Groups Input/Output

MSK\_removebarvars

```
MSKrescodee (MSKAPI MSK_removebarvars) (
MSKtask_t task,
MSKint32t num,
const MSKint32t * subset)
```

The function removes a subset of the symmetric matrices from the optimization task. This implies that the remaining symmetric matrices are renumbered.

#### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- num (MSKint32t) Number of symmetric matrices which should be removed. (input)
- subset (MSKint32t\*) Indexes of symmetric matrices which should be removed. (input)

Return (MSKrescodee) - The function response code.

Groups Problem data - semidefinite

MSK\_removecones

```
MSKrescodee (MSKAPI MSK_removecones) (
MSKtask_t task,
MSKint32t num,
const MSKint32t * subset)
```

Removes a number of conic constraints from the problem. This implies that the remaining conic constraints are renumbered. In general, it is much more efficient to remove a cone with a high index than a low index.

### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- num (MSKint32t) Number of cones which should be removed. (input)
- subset (MSKint32t\*) Indexes of cones which should be removed. (input)

Return (MSKrescodee) - The function response code.

Groups Problem data - cones

```
MSKrescodee (MSKAPI MSK_removecons) (
MSKtask_t task,
MSKint32t num,
const MSKint32t * subset)
```

The function removes a subset of the constraints from the optimization task. This implies that the remaining constraints are renumbered.

### Parameters

- task (MSKtask\_t) An optimization task. (input)
- num (MSKint32t) Number of constraints which should be removed. (input)
- subset (MSKint32t\*) Indexes of constraints which should be removed. (input)

Return (MSKrescodee) - The function response code.

Groups Problem data - constraints, Problem data - linear part

MSK\_removevars

```
MSKrescodee (MSKAPI MSK_removevars) (
    MSKtask_t task,
    MSKint32t num,
    const MSKint32t * subset)
```

The function removes a subset of the variables from the optimization task. This implies that the remaining variables are renumbered.

#### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- num (MSKint32t) Number of variables which should be removed. (input)
- subset (MSKint32t\*) Indexes of variables which should be removed. (input)

Return (MSKrescodee) - The function response code.

Groups Problem data - variables, Problem data - linear part

MSK\_resizetask

```
MSKrescodee (MSKAPI MSK_resizetask) (
MSKtask_t task,
MSKint32t maxnumcon,
MSKint32t maxnumvar,
MSKint32t maxnumcone,
MSKint64t maxnumanz,
MSKint64t maxnumanz)
```

Sets the amount of preallocated space assigned for each type of data in an optimization task.

It is never mandatory to call this function, since it only gives a hint about the amount of data to preallocate for efficiency reasons.

Please note that the procedure is **destructive** in the sense that all existing data stored in the task is destroyed.

- task (MSKtask\_t) An optimization task. (input)
- maxnumcon (MSKint32t) New maximum number of constraints. (input)
- maxnumvar (MSKint32t) New maximum number of variables. (input)
- maxnumcone (MSKint32t) New maximum number of cones. (input)

- maxnumanz (MSKint64t) New maximum number of non-zeros in A. (input)
- maxnumqnz (MSKint64t) New maximum number of non-zeros in all Q matrices. (input)

Return (MSKrescodee) - The function response code.

Groups Environment and task management

 ${\tt MSK\_sensitivityreport}$ 

```
MSKrescodee (MSKAPI MSK_sensitivityreport) (
MSKtask_t task,
MSKstreamtypee whichstream)
```

Reads a sensitivity format file from a location given by <code>MSK\_SPAR\_SENSITIVITY\_FILE\_NAME</code> and writes the result to the stream <code>whichstream</code>. If <code>MSK\_SPAR\_SENSITIVITY\_RES\_FILE\_NAME</code> is set to a non-empty string, then the sensitivity report is also written to a file of this name.

#### Parameters

- task (MSKtask\_t) An optimization task. (input)
- whichstream (MSKstreamtypee) Index of the stream. (input)

Return (MSKrescodee) - The function response code.

Groups Sensitivity analysis

MSK\_setdefaults

```
MSKrescodee (MSKAPI MSK_setdefaults) (
MSKtask_t task)
```

Resets all the parameters to their default values.

```
Parameters task (MSKtask_t) - An optimization task. (input)
Return (MSKrescodee) - The function response code.
Groups Parameters
```

MSK\_setupthreads

```
MSKrescodee (MSKAPI MSK_setupthreads) (
MSKenv_t env,
MSKint32t numthreads)
```

Preallocates a thread pool for the interior-point and conic optimizers in the current process. This function should only be called once per process, before first optimization. Future settings of the parameter  $MSK\_IPAR\_NUM\_THREADS$  will be irrelevant for the conic optimizer.

# Parameters

- env (MSKenv\_t) The MOSEK environment. (input)
- numthreads (MSKint32t) Number of threads. (input)

 ${\bf Return}~({\it MSKrescodee}\,)$  – The function response code.

Groups System, memory and debugging

MSK\_sktostr

```
MSKrescodee (MSKAPI MSK_sktostr) (
MSKtask_t task,
MSKstakeye sk,
char * str)
```

Obtains a status key abbreviation string, one of UN, BS, SB, LL, UL, EQ, \*\*.

#### Parameters

- task (MSKtask\_t) An optimization task. (input)
- sk (MSKstakeye) A valid status key. (input)
- str (MSKstring\_t) Abbreviation string corresponding to the status key sk. (output)

**Return** (MSKrescodee) - The function response code.

Groups Names

MSK\_solstatostr

```
MSKrescodee (MSKAPI MSK_solstatostr) (
MSKtask_t task,
MSKsolstae solsta,
char * str)
```

Obtains an explanatory string corresponding to a solution status.

#### Parameters

- task (MSKtask\_t) An optimization task. (input)
- solsta (MSKsolstae) Solution status. (input)
- str (MSKstring\_t) String corresponding to the solution status solsta. (output)

Return (MSKrescodee) - The function response code.

Groups Names

MSK\_solutiondef

```
MSKrescodee (MSKAPI MSK_solutiondef) (
    MSKtask_t task,
    MSKsoltypee whichsol,
    MSKbooleant * isdef)
```

Checks whether a solution is defined.

#### Parameters

- task (MSKtask\_t) An optimization task. (input)
- whichsol (MSKsoltypee) Selects a solution. (input)
- isdef (MSKbooleant by reference) Is non-zero if the requested solution is defined. (output)

**Return** (*MSKrescodee*) – The function response code.

Groups Solution information

MSK\_solutionsummary

```
MSKrescodee (MSKAPI MSK_solutionsummary) (
MSKtask_t task,
MSKstreamtypee whichstream)
```

Prints a short summary of the current solutions.

#### Parameters

- task (MSKtask\_t) An optimization task. (input)
- whichstream (MSKstreamtypee) Index of the stream. (input)

Return (MSKrescodee) - The function response code.

### Groups Logging, Solution information

MSK\_solvewithbasis

```
MSKrescodee (MSKAPI MSK_solvewithbasis) (
MSKtask_t task,
MSKint32t transp,
MSKint32t * numnz,
MSKint32t * sub,
MSKrealt * val)
```

If a basic solution is available, then exactly numcon basis variables are defined. These numcon basis variables are denoted the basis. Associated with the basis is a basis matrix denoted B. This function solves either the linear equation system

$$B\overline{X} = b \tag{15.3}$$

or the system

$$B^T \overline{X} = b \tag{15.4}$$

for the unknowns  $\overline{X}$ , with b being a user-defined vector. In order to make sense of the solution  $\overline{X}$  it is important to know the ordering of the variables in the basis because the ordering specifies how B is constructed. When calling  $MSK\_initbasissolve$  an ordering of the basis variables is obtained, which can be used to deduce how MOSEK has constructed B. Indeed if the k-th basis variable is variable  $x_i$  it implies that

$$B_{i,k} = A_{i,j}, i = 0, \dots, numcon - 1.$$

Otherwise if the k-th basis variable is variable  $x_i^c$  it implies that

$$B_{i,k} = \begin{cases} -1, & i = j, \\ 0, & i \neq j. \end{cases}$$

The function  $MSK\_initbasissolve$  must be called before a call to this function. Please note that this function exploits the sparsity in the vector b to speed up the computations.

### Parameters

- task (MSKtask\_t) An optimization task. (input)
- transp (MSKint32t) If this argument is zero, then (15.3) is solved, if non-zero then (15.4) is solved. (input)
- numnz (MSKint32t by reference) As input it is the number of non-zeros in b. As output it is the number of non-zeros in  $\overline{X}$ . (input/output)
- sub (MSKint32t\*) As input it contains the positions of non-zeros in b. As output it contains the positions of the non-zeros in  $\overline{X}$ . It must have room for numcon elements. (input/output)
- val  $(\mathit{MSKrealt*})$  As input it is the vector b as a dense vector (although the positions of non-zeros are specified in sub it is required that val[i] = 0 when b[i] = 0). As output val is the vector  $\overline{X}$  as a dense vector. It must have length numcon. (input/output)

Return (MSKrescodee) - The function response code.

Groups Solving systems with basis matrix

MSK\_sparsetriangularsolvedense

```
MSKrescodee (MSKAPI MSK_sparsetriangularsolvedense) (
MSKenv_t env,
MSKtransposee transposed,
MSKint32t n,
const MSKint32t * lnzc,
const MSKint64t * lptrc,
MSKint64t lensubnval,
const MSKint32t * lsubc,
const MSKrealt * lvalc,
MSKrealt * b)
```

The function solves a triangular system of the form

Lx = b

or

$$L^T x = b$$

where L is a sparse lower triangular nonsingular matrix. This implies in particular that diagonals in L are nonzero.

#### Parameters

- env (MSKenv\_t) The MOSEK environment. (input)
- transposed (MSKtransposee) Controls whether to use with L or  $L^T$ . (input)
- n (MSKint32t) Dimension of L. (input)
- lnzc (MSKint32t\*) lnzc[j] is the number of nonzeros in column j. (input)
- lptrc (MSKint64t\*) lptrc[j] is a pointer to the first row index and value in column j. (input)
- lensubnval (MSKint64t) Number of elements in lsubc and lvalc. (input)
- lsubc (MSKint32t\*) Row indexes for each column stored sequentially. Must be stored in increasing order for each column. (input)
- lvalc (MSKrealt\*) The value corresponding to the row index stored in lsubc. (input)
- b (MSKrealt\*) The right-hand side of linear equation system to be solved as a dense vector. (input/output)

Return (MSKrescodee) - The function response code.

Groups Linear algebra

MSK\_strdupdbgtask

```
char * (MSKAPI MSK_strdupdbgtask) (
   MSKtask_t task,
   const char * str,
   const char * file,
   const unsigned line)
```

Make a copy of a string. The string created by this procedure must be freed by MSK\_freetask.

### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- str (MSKstring\_t) String that should be copied. (input)
- file (MSKstring\_t) File from which the function is called. (input)
- line (unsigned) Line in the file from which the function is called. (input)

**Return**  $(MSKstring_t)$  – A copy of the given string.

Groups System, memory and debugging

```
char * (MSKAPI MSK_strduptask) (
   MSKtask_t task,
   const char * str)
```

Make a copy of a string. The string created by this procedure must be freed by MSK\_freetask.

#### Parameters

- task (MSKtask\_t) An optimization task. (input)
- str (MSKstring\_t) String that should be copied. (input)

**Return**  $(MSKstring_t)$  – A copy of the given string.

Groups System, memory and debugging

MSK\_strtoconetype

```
MSKrescodee (MSKAPI MSK_strtoconetype) (
   MSKtask_t task,
   const char * str,
   MSKconetypee * conetype)
```

Obtains cone type code corresponding to a cone type string.

#### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- str (MSKstring\_t) String corresponding to the cone type code conetype. (input)
- conetype (MSKconetypee by reference) The cone type corresponding to the string str. (output)

Return (MSKrescodee) - The function response code.

Groups Names

MSK\_strtosk

```
MSKrescodee (MSKAPI MSK_strtosk) (
MSKtask_t task,
const char * str,
MSKstakeye * sk)
```

Obtains the status key corresponding to an abbreviation string.

# Parameters

- task (MSKtask\_t) An optimization task. (input)
- str (MSKstring\_t) A status key abbreviation string. (input)
- sk (MSKstakeye by reference) Status key corresponding to the string. (output)

Return (MSKrescodee) - The function response code.

Groups Names

MSK\_syeig

```
MSKrescodee (MSKAPI MSK_syeig) (
   MSKenv_t env,
   MSKuploe uplo,
   MSKint32t n,
   const MSKrealt * a,
   MSKrealt * w)
```

Computes all eigenvalues of a real symmetric matrix A. Given a matrix  $A \in \mathbb{R}^{n \times n}$  it returns a vector  $w \in \mathbb{R}^n$  containing the eigenvalues of A.

#### Parameters

- env (MSKenv\_t) The MOSEK environment. (input)
- uplo (MSKuploe) Indicates whether the upper or lower triangular part is used. (input)
- n (MSKint32t) Dimension of the symmetric input matrix. (input)
- a (MSKrealt\*) A symmetric matrix A stored in column-major order. Only the part indicated by uplo is used. (input)
- w (MSKrealt\*) Array of length at least n containing the eigenvalues of A. (output)

Return (MSKrescodee) - The function response code.

Groups Linear algebra

MSK\_syevd

```
MSKrescodee (MSKAPI MSK_syevd) (
   MSKenv_t env,
   MSKuploe uplo,
   MSKint32t n,
   MSKrealt * a,
   MSKrealt * w)
```

Computes all the eigenvalues and eigenvectors a real symmetric matrix. Given the input matrix  $A \in \mathbb{R}^{n \times n}$ , this function returns a vector  $w \in \mathbb{R}^n$  containing the eigenvalues of A and it also computes the eigenvectors of A. Therefore, this function computes the eigenvalue decomposition of A as

$$A = UVU^T$$
.

where  $V = \mathbf{diag}(w)$  and U contains the eigenvectors of A.

Note that the matrix U overwrites the input data A.

### Parameters

- env (MSKenv\_t) The MOSEK environment. (input)
- uplo (MSKuploe) Indicates whether the upper or lower triangular part is used. (input)
- n (MSKint32t) Dimension of the symmetric input matrix. (input)
- a (MSKrealt\*) A symmetric matrix A stored in column-major order. Only the part indicated by uplo is used. On exit it will be overwritten by the matrix U. (input/output)
- w (MSKrealt\*) Array of length at least n containing the eigenvalues of A. (output)

Return (MSKrescodee) - The function response code.

Groups Linear algebra

MSK\_symnamtovalue

```
MSKbooleant (MSKAPI MSK_symnamtovalue) (
  const char * name,
  char * value)
```

Obtains the value corresponding to a symbolic name defined by **MOSEK**.

```
• name (MSKstring_t) - Symbolic name. (input)
```

• value (MSKstring\_t) - The corresponding value. (output)

**Return** (MSKbooleant) - Indicates if the symbolic name has been converted.

Groups Parameters

MSK\_syrk

```
MSKrescodee (MSKAPI MSK_syrk) (
   MSKenv_t env,
   MSKuploe uplo,
   MSKtransposee trans,
   MSKint32t n,
   MSKint32t k,
   MSKrealt alpha,
   const MSKrealt * a,
   MSKrealt beta,
   MSKrealt * c)
```

Performs a symmetric rank-k update for a symmetric matrix.

Given a symmetric matrix  $C \in \mathbb{R}^{n \times n}$ , two scalars  $\alpha, \beta$  and a matrix A of rank  $k \leq n$ , it computes either

$$C := \alpha A A^T + \beta C$$
,

when trans is set to MSK\_TRANSPOSE\_NO and  $A \in \mathbb{R}^{n \times k}$ , or

$$C := \alpha A^T A + \beta C,$$

when trans is set to MSK\_TRANSPOSE\_YES and  $A \in \mathbb{R}^{k \times n}$ .

Only the part of C indicated by uplo is used and only that part is updated with the result. It must not overlap with the other input arrays.

#### **Parameters**

- env (MSKenv\_t) The MOSEK environment. (input)
- uplo (MSKuploe) Indicates whether the upper or lower triangular part of C is used. (input)
- trans (MSKtransposee) Indicates whether the matrix A must be transposed. (input)
- n (MSKint32t) Specifies the order of C. (input)
- k (MSKint32t) Indicates the number of rows or columns of A, depending on whether or not it is transposed, and its rank. (input)
- alpha (MSKrealt) A scalar value multiplying the result of the matrix multiplication. (input)
- a (MSKrealt\*) The pointer to the array storing matrix A in a column-major format. (input)
- beta (MSKrealt) A scalar value that multiplies C. (input)
- c (MSKrealt\*) The pointer to the array storing matrix C in a column-major format. (input/output)

Return (MSKrescodee) - The function response code.

 ${\bf Groups}\ {\it Linear\ algebra}$ 

MSK\_toconic

```
MSKrescodee (MSKAPI MSK_toconic) (
MSKtask_t task)
```

This function tries to reformulate a given Quadratically Constrained Quadratic Optimization problem (QCQP) as a Conic Quadratic Optimization problem (CQO). The first step of the reformulation is to convert the quadratic term of the objective function, if any, into a constraint. Then the following steps are repeated for each quadratic constraint:

- a conic constraint is added along with a suitable number of auxiliary variables and constraints;
- the original quadratic constraint is not removed, but all its coefficients are zeroed out.

Note that the reformulation preserves all the original variables.

The conversion is performed in-place, i.e. the task passed as argument is modified on exit. That also means that if the reformulation fails, i.e. the given QCQP is not representable as a CQO, then the task has an undefined state. In some cases, users may want to clone the task to ensure a clean copy is preserved.

```
Parameters task (\textit{MSKtask\_t}) - An optimization task. (input) Return (\textit{MSKrescodee}) - The function response code. Groups Problem\ data - quadratic\ part
```

MSK\_unlinkfuncfromenvstream

```
MSKrescodee (MSKAPI MSK_unlinkfuncfromenvstream) (
MSKenv_t env,
MSKstreamtypee whichstream)
```

Disconnects a user-defined function from a stream.

### Parameters

- $\bullet$  env ( ${\it MSKenv\_t}$  ) The MOSEK environment. (input)
- whichstream (MSKstreamtypee) Index of the stream. (input)

Return (MSKrescodee) - The function response code.

Groups Logging, Callback

 ${\tt MSK\_unlinkfuncfromtaskstream}$ 

```
MSKrescodee (MSKAPI MSK_unlinkfuncfromtaskstream) (
MSKtask_t task,
MSKstreamtypee whichstream)
```

Disconnects a user-defined function from a task stream.

### Parameters

- task (MSKtask\_t) An optimization task. (input)
- whichstream (MSKstreamtypee) Index of the stream. (input)

Return (MSKrescodee) - The function response code.

Groups Logging, Callback

MSK\_updatesolutioninfo

```
MSKrescodee (MSKAPI MSK_updatesolutioninfo) (
MSKtask_t task,
MSKsoltypee whichsol)
```

Update the information items related to the solution.

- task (MSKtask\_t) An optimization task. (input)
- whichsol (MSKsoltypee) Selects a solution. (input)

```
Return (MSKrescodee) - The function response code.
Groups Information items and statistics
```

MSK\_utf8towchar

```
MSKrescodee (MSKAPI MSK_utf8towchar) (
  const size_t outputlen,
  size_t * len,
  size_t * conv,
  MSKwchart * output,
  const char * input)
```

Converts an UTF8 string to a MSKwchart string.

#### Parameters

- outputlen (size\_t) The length of the output buffer. (input)
- len (size\_t\*) The length of the string contained in the output buffer. (output)
- conv (size\_t\*) Returns the number of characters converted, i.e. input[conv] is the first character which was not converted. If the whole string was converted, then input[conv]=0. (output)
- output (MSKwchart\*) The input string converted to a MSKwchart string. (output)
- input (MSKstring\_t) The UTF8 input string. (input)

Return (MSKrescodee) - The function response code.

**Groups** System, memory and debugging

MSK\_wchartoutf8

```
MSKrescodee (MSKAPI MSK_wchartoutf8) (
  const size_t outputlen,
  size_t * len,
  size_t * conv,
  char * output,
  const MSKwchart * input)
```

Converts a MSKwchart string to an UTF8 string.

#### **Parameters**

- outputlen (size\_t) The length of the output buffer. (input)
- len (size\_t\*) The length of the string contained in the output buffer. (output)
- conv (size\_t\*) Returns the number of characters from converted, i.e. input[conv] is the first char which was not converted. If the whole string was converted, then input[conv]=0. (output)
- output (MSKstring\_t) The input string converted to a UTF8 string. (output)
- input (MSKwchart\*) The MSKwchart input string. (input)

Return (MSKrescodee) - The function response code.

Groups System, memory and debugging

MSK\_whichparam

```
MSKrescodee (MSKAPI MSK_whichparam) (
MSKtask_t task,
const char * parname,
MSKparametertypee * partype,
MSKint32t * param)
```

Checks if parname is a valid parameter name. If yes then partype and param denote the type and the index of the parameter, respectively.

#### **Parameters**

- task  $(\textit{MSKtask\_t})$  An optimization task. (input)
- parname (MSKstring\_t) Parameter name. (input)
- partype (MSKparametertypee by reference) Parameter type. (output)
- param (MSKint32t by reference) Which parameter. (output)

Return (MSKrescodee) - The function response code.

Groups Parameters, Names

MSK\_writedata

```
MSKrescodee (MSKAPI MSK_writedata) (
    MSKtask_t task,
    const char * filename)
```

Writes problem data associated with the optimization task to a file in one of the supported formats. See Section Supported File Formats for the complete list.

The data file format is determined by the file name extension. To write in compressed format append the extension .gz. E.g to write a gzip compressed MPS file use the extension mps.gz.

Please note that MPS, LP and OPF files require all variables to have unique names. If a task contains no names, it is possible to write the file with automatically generated anonymous names by setting the  $MSK\_IPAR\_WRITE\_GENERIC\_NAMES$  parameter to  $MSK\_ON$ .

Data is written to the file filename if it is a nonempty string. Otherwise data is written to the file specified by MSK\_SPAR\_DATA\_FILE\_NAME.

### Parameters

- $task (MSKtask_t)$  An optimization task. (input)
- filename  $(MSKstring_t)$  A valid file name. (input)

Return (MSKrescodee) - The function response code.

Groups Input/Output

MSK\_writejsonsol

```
MSKrescodee (MSKAPI MSK_writejsonsol) (
    MSKtask_t task,
    const char * filename)
```

Saves the current solutions and solver information items in a JSON file.

#### Parameters

- $task(MSKtask_t)$  An optimization task. (input)
- filename  $(MSKstring_t)$  A valid file name. (input)

Return (MSKrescodee) - The function response code.

Groups Input/Output

MSK\_writeparamfile

```
MSKrescodee (MSKAPI MSK_writeparamfile) (
MSKtask_t task,
const char * filename)
```

Writes all the parameters to a parameter file.

```
    task (MSKtask_t) - An optimization task. (input)
    filename (MSKstring_t) - A valid file name. (input)
    Return (MSKrescodee) - The function response code.
```

**Groups** Input/Output, Parameters

MSK\_writesolution

```
MSKrescodee (MSKAPI MSK_writesolution) (
   MSKtask_t task,
   MSKsoltypee whichsol,
   const char * filename)
```

Saves the current basic, interior-point, or integer solution to a file.

#### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- whichsol (MSKsoltypee) Selects a solution. (input)
- filename  $(MSKstring_t)$  A valid file name. (input)

Return (MSKrescodee) - The function response code.

Groups Input/Output

MSK\_writetask

```
MSKrescodee (MSKAPI MSK_writetask) (
MSKtask_t task,
const char * filename)
```

Write a binary dump of the task data. This format saves all problem data, coefficients and parameter settings. See section *The Task Format* for a description of the Task format.

# Parameters

- task (MSKtask\_t) An optimization task. (input)
- filename (MSKstring\_t) A valid file name. (input)

Return (MSKrescodee) - The function response code.

Groups Input/Output

# 15.4 Parameters grouped by topic

### **Analysis**

- MSK\_DPAR\_ANA\_SOL\_INFEAS\_TOL
- MSK\_IPAR\_ANA\_SOL\_BASIS
- MSK\_IPAR\_ANA\_SOL\_PRINT\_VIOLATED
- MSK\_IPAR\_LOG\_ANA\_PRO

### **Basis identification**

- MSK\_DPAR\_SIM\_LU\_TOL\_REL\_PIV
- MSK\_IPAR\_BI\_CLEAN\_OPTIMIZER
- MSK\_IPAR\_BI\_IGNORE\_MAX\_ITER
- MSK\_IPAR\_BI\_IGNORE\_NUM\_ERROR
- MSK\_IPAR\_BI\_MAX\_ITERATIONS
- MSK\_IPAR\_INTPNT\_BASIS
- MSK\_IPAR\_LOG\_BI
- MSK\_IPAR\_LOG\_BI\_FREQ

### Conic interior-point method

- MSK\_DPAR\_INTPNT\_CO\_TOL\_DFEAS
- MSK\_DPAR\_INTPNT\_CO\_TOL\_INFEAS
- MSK\_DPAR\_INTPNT\_CO\_TOL\_MU\_RED
- MSK\_DPAR\_INTPNT\_CO\_TOL\_NEAR\_REL
- MSK\_DPAR\_INTPNT\_CO\_TOL\_PFEAS
- MSK\_DPAR\_INTPNT\_CO\_TOL\_REL\_GAP

### Data check

- MSK\_DPAR\_DATA\_SYM\_MAT\_TOL
- MSK\_DPAR\_DATA\_SYM\_MAT\_TOL\_HUGE
- MSK\_DPAR\_DATA\_SYM\_MAT\_TOL\_LARGE
- MSK\_DPAR\_DATA\_TOL\_AIJ\_HUGE
- MSK\_DPAR\_DATA\_TOL\_AIJ\_LARGE
- MSK\_DPAR\_DATA\_TOL\_BOUND\_INF
- MSK\_DPAR\_DATA\_TOL\_BOUND\_WRN
- MSK\_DPAR\_DATA\_TOL\_C\_HUGE
- MSK\_DPAR\_DATA\_TOL\_CJ\_LARGE
- MSK\_DPAR\_DATA\_TOL\_QIJ
- MSK\_DPAR\_DATA\_TOL\_X
- MSK\_DPAR\_SEMIDEFINITE\_TOL\_APPROX
- MSK\_IPAR\_CHECK\_CONVEXITY
- MSK\_IPAR\_LOG\_CHECK\_CONVEXITY

# Data input/output

- MSK\_IPAR\_INFEAS\_REPORT\_AUTO
- MSK\_IPAR\_LOG\_FILE
- MSK\_IPAR\_OPF\_WRITE\_HEADER
- MSK\_IPAR\_OPF\_WRITE\_HINTS
- MSK\_IPAR\_OPF\_WRITE\_LINE\_LENGTH
- MSK\_IPAR\_OPF\_WRITE\_PARAMETERS
- MSK\_IPAR\_OPF\_WRITE\_PROBLEM
- MSK\_IPAR\_OPF\_WRITE\_SOL\_BAS
- MSK\_IPAR\_OPF\_WRITE\_SOL\_ITG
- MSK\_IPAR\_OPF\_WRITE\_SOL\_ITR
- MSK\_IPAR\_OPF\_WRITE\_SOLUTIONS
- MSK\_IPAR\_PARAM\_READ\_CASE\_NAME
- MSK\_IPAR\_PARAM\_READ\_IGN\_ERROR
- MSK\_IPAR\_PTF\_WRITE\_TRANSFORM
- MSK\_IPAR\_READ\_DEBUG
- MSK\_IPAR\_READ\_KEEP\_FREE\_CON
- MSK\_IPAR\_READ\_LP\_DROP\_NEW\_VARS\_IN\_BOU
- MSK\_IPAR\_READ\_LP\_QUOTED\_NAMES
- MSK\_IPAR\_READ\_MPS\_FORMAT
- MSK\_IPAR\_READ\_MPS\_WIDTH
- MSK\_IPAR\_READ\_TASK\_IGNORE\_PARAM
- MSK\_IPAR\_SOL\_READ\_NAME\_WIDTH
- MSK\_IPAR\_SOL\_READ\_WIDTH
- MSK\_IPAR\_WRITE\_BAS\_CONSTRAINTS
- MSK\_IPAR\_WRITE\_BAS\_HEAD
- MSK\_IPAR\_WRITE\_BAS\_VARIABLES
- MSK\_IPAR\_WRITE\_COMPRESSION
- MSK\_IPAR\_WRITE\_DATA\_PARAM
- MSK\_IPAR\_WRITE\_FREE\_CON
- MSK\_IPAR\_WRITE\_GENERIC\_NAMES
- MSK\_IPAR\_WRITE\_GENERIC\_NAMES\_IO
- MSK\_IPAR\_WRITE\_IGNORE\_INCOMPATIBLE\_ITEMS
- MSK\_IPAR\_WRITE\_INT\_CONSTRAINTS
- MSK\_IPAR\_WRITE\_INT\_HEAD
- MSK\_IPAR\_WRITE\_INT\_VARIABLES

- MSK\_IPAR\_WRITE\_LP\_FULL\_OBJ
- MSK\_IPAR\_WRITE\_LP\_LINE\_WIDTH
- MSK\_IPAR\_WRITE\_LP\_QUOTED\_NAMES
- MSK\_IPAR\_WRITE\_LP\_STRICT\_FORMAT
- MSK\_IPAR\_WRITE\_LP\_TERMS\_PER\_LINE
- MSK\_IPAR\_WRITE\_MPS\_FORMAT
- MSK\_IPAR\_WRITE\_MPS\_INT
- MSK\_IPAR\_WRITE\_PRECISION
- MSK\_IPAR\_WRITE\_SOL\_BARVARIABLES
- MSK\_IPAR\_WRITE\_SOL\_CONSTRAINTS
- MSK\_IPAR\_WRITE\_SOL\_HEAD
- MSK\_IPAR\_WRITE\_SOL\_IGNORE\_INVALID\_NAMES
- MSK\_IPAR\_WRITE\_SOL\_VARIABLES
- MSK\_IPAR\_WRITE\_TASK\_INC\_SOL
- MSK\_IPAR\_WRITE\_XML\_MODE
- MSK\_SPAR\_BAS\_SOL\_FILE\_NAME
- MSK\_SPAR\_DATA\_FILE\_NAME
- MSK\_SPAR\_DEBUG\_FILE\_NAME
- MSK\_SPAR\_INT\_SOL\_FILE\_NAME
- MSK\_SPAR\_ITR\_SOL\_FILE\_NAME
- MSK\_SPAR\_MIO\_DEBUG\_STRING
- MSK\_SPAR\_PARAM\_COMMENT\_SIGN
- MSK\_SPAR\_PARAM\_READ\_FILE\_NAME
- MSK\_SPAR\_PARAM\_WRITE\_FILE\_NAME
- MSK\_SPAR\_READ\_MPS\_BOU\_NAME
- MSK\_SPAR\_READ\_MPS\_OBJ\_NAME
- MSK\_SPAR\_READ\_MPS\_RAN\_NAME
- MSK\_SPAR\_READ\_MPS\_RHS\_NAME
- MSK\_SPAR\_SENSITIVITY\_FILE\_NAME
- MSK\_SPAR\_SENSITIVITY\_RES\_FILE\_NAME
- MSK\_SPAR\_SOL\_FILTER\_XC\_LOW
- MSK\_SPAR\_SOL\_FILTER\_XC\_UPR
- MSK\_SPAR\_SOL\_FILTER\_XX\_LOW
- MSK\_SPAR\_SOL\_FILTER\_XX\_UPR
- MSK\_SPAR\_STAT\_FILE\_NAME
- MSK\_SPAR\_STAT\_KEY
- MSK\_SPAR\_STAT\_NAME
- MSK\_SPAR\_WRITE\_LP\_GEN\_VAR\_NAME

### **Debugging**

• MSK\_IPAR\_AUTO\_SORT\_A\_BEFORE\_OPT

### **Dual simplex**

- MSK\_IPAR\_SIM\_DUAL\_CRASH
- MSK\_IPAR\_SIM\_DUAL\_RESTRICT\_SELECTION
- MSK\_IPAR\_SIM\_DUAL\_SELECTION

# Infeasibility report

- MSK\_IPAR\_INFEAS\_GENERIC\_NAMES
- MSK\_IPAR\_INFEAS\_REPORT\_LEVEL
- MSK\_IPAR\_LOG\_INFEAS\_ANA

### Interior-point method

- MSK\_DPAR\_CHECK\_CONVEXITY\_REL\_TOL
- MSK\_DPAR\_INTPNT\_CO\_TOL\_DFEAS
- $\bullet \ \mathit{MSK\_DPAR\_INTPNT\_CO\_TOL\_INFEAS}$
- MSK\_DPAR\_INTPNT\_CO\_TOL\_MU\_RED
- MSK\_DPAR\_INTPNT\_CO\_TOL\_NEAR\_REL
- MSK\_DPAR\_INTPNT\_CO\_TOL\_PFEAS
- MSK\_DPAR\_INTPNT\_CO\_TOL\_REL\_GAP
- MSK\_DPAR\_INTPNT\_QO\_TOL\_DFEAS
- MSK\_DPAR\_INTPNT\_QO\_TOL\_INFEAS
- MSK\_DPAR\_INTPNT\_QO\_TOL\_MU\_RED
- MSK\_DPAR\_INTPNT\_QO\_TOL\_NEAR\_REL
- MSK\_DPAR\_INTPNT\_QO\_TOL\_PFEAS
- MSK\_DPAR\_INTPNT\_QO\_TOL\_REL\_GAP
- MSK\_DPAR\_INTPNT\_TOL\_DFEAS
- MSK\_DPAR\_INTPNT\_TOL\_DSAFE
- MSK\_DPAR\_INTPNT\_TOL\_INFEAS
- MSK\_DPAR\_INTPNT\_TOL\_MU\_RED
- MSK\_DPAR\_INTPNT\_TOL\_PATH
- MSK\_DPAR\_INTPNT\_TOL\_PFEAS
- MSK\_DPAR\_INTPNT\_TOL\_PSAFE
- MSK\_DPAR\_INTPNT\_TOL\_REL\_GAP
- MSK\_DPAR\_INTPNT\_TOL\_REL\_STEP
- MSK\_DPAR\_INTPNT\_TOL\_STEP\_SIZE

- MSK\_DPAR\_QCQO\_REFORMULATE\_REL\_DROP\_TOL
- MSK\_IPAR\_BI\_IGNORE\_MAX\_ITER
- MSK\_IPAR\_BI\_IGNORE\_NUM\_ERROR
- MSK\_IPAR\_INTPNT\_BASIS
- MSK\_IPAR\_INTPNT\_DIFF\_STEP
- MSK\_IPAR\_INTPNT\_HOTSTART
- MSK\_IPAR\_INTPNT\_MAX\_ITERATIONS
- MSK\_IPAR\_INTPNT\_MAX\_NUM\_COR
- MSK\_IPAR\_INTPNT\_MAX\_NUM\_REFINEMENT\_STEPS
- MSK\_IPAR\_INTPNT\_OFF\_COL\_TRH
- MSK\_IPAR\_INTPNT\_ORDER\_GP\_NUM\_SEEDS
- MSK\_IPAR\_INTPNT\_ORDER\_METHOD
- MSK\_IPAR\_INTPNT\_PURIFY
- MSK\_IPAR\_INTPNT\_REGULARIZATION\_USE
- MSK\_IPAR\_INTPNT\_SCALING
- MSK\_IPAR\_INTPNT\_SOLVE\_FORM
- MSK\_IPAR\_INTPNT\_STARTING\_POINT
- MSK\_IPAR\_LOG\_INTPNT

# License manager

- MSK\_IPAR\_CACHE\_LICENSE
- MSK\_IPAR\_LICENSE\_DEBUG
- MSK\_IPAR\_LICENSE\_PAUSE\_TIME
- MSK\_IPAR\_LICENSE\_SUPPRESS\_EXPIRE\_WRNS
- MSK\_IPAR\_LICENSE\_TRH\_EXPIRY\_WRN
- MSK\_IPAR\_LICENSE\_WAIT

# Logging

- MSK\_IPAR\_LOG
- MSK\_IPAR\_LOG\_ANA\_PRO
- MSK\_IPAR\_LOG\_BI
- MSK\_IPAR\_LOG\_BI\_FREQ
- MSK\_IPAR\_LOG\_CUT\_SECOND\_OPT
- MSK\_IPAR\_LOG\_EXPAND
- MSK\_IPAR\_LOG\_FEAS\_REPAIR
- $\bullet \ \textit{MSK\_IPAR\_LOG\_FILE}$

- MSK\_IPAR\_LOG\_INCLUDE\_SUMMARY
- MSK\_IPAR\_LOG\_INFEAS\_ANA
- MSK\_IPAR\_LOG\_INTPNT
- MSK\_IPAR\_LOG\_LOCAL\_INFO
- MSK\_IPAR\_LOG\_MIO
- MSK\_IPAR\_LOG\_MIO\_FREQ
- MSK\_IPAR\_LOG\_ORDER
- MSK\_IPAR\_LOG\_PRESOLVE
- MSK\_IPAR\_LOG\_RESPONSE
- MSK\_IPAR\_LOG\_SENSITIVITY
- MSK\_IPAR\_LOG\_SENSITIVITY\_OPT
- MSK\_IPAR\_LOG\_SIM
- MSK\_IPAR\_LOG\_SIM\_FREQ
- MSK\_IPAR\_LOG\_STORAGE

# Mixed-integer optimization

- MSK\_DPAR\_MIO\_MAX\_TIME
- MSK\_DPAR\_MIO\_REL\_GAP\_CONST
- MSK\_DPAR\_MIO\_TOL\_ABS\_GAP
- MSK\_DPAR\_MIO\_TOL\_ABS\_RELAX\_INT
- MSK\_DPAR\_MIO\_TOL\_FEAS
- MSK\_DPAR\_MIO\_TOL\_REL\_DUAL\_BOUND\_IMPROVEMENT
- MSK\_DPAR\_MIO\_TOL\_REL\_GAP
- MSK\_IPAR\_LOG\_MIO
- MSK\_IPAR\_LOG\_MIO\_FREQ
- MSK\_IPAR\_MIO\_BRANCH\_DIR
- MSK\_IPAR\_MIO\_CONIC\_OUTER\_APPROXIMATION
- MSK\_IPAR\_MIO\_CUT\_CLIQUE
- MSK\_IPAR\_MIO\_CUT\_CMIR
- MSK\_IPAR\_MIO\_CUT\_GMI
- MSK\_IPAR\_MIO\_CUT\_IMPLIED\_BOUND
- MSK\_IPAR\_MIO\_CUT\_KNAPSACK\_COVER
- MSK\_IPAR\_MIO\_CUT\_SELECTION\_LEVEL
- MSK\_IPAR\_MIO\_FEASPUMP\_LEVEL
- MSK\_IPAR\_MIO\_HEURISTIC\_LEVEL
- MSK\_IPAR\_MIO\_MAX\_NUM\_BRANCHES

- MSK\_IPAR\_MIO\_MAX\_NUM\_RELAXS
- MSK\_IPAR\_MIO\_MAX\_NUM\_ROOT\_CUT\_ROUNDS
- MSK\_IPAR\_MIO\_MAX\_NUM\_SOLUTIONS
- MSK\_IPAR\_MIO\_NODE\_OPTIMIZER
- MSK\_IPAR\_MIO\_NODE\_SELECTION
- MSK\_IPAR\_MIO\_PERSPECTIVE\_REFORMULATE
- MSK\_IPAR\_MIO\_PROBING\_LEVEL
- MSK\_IPAR\_MIO\_PROPAGATE\_OBJECTIVE\_CONSTRAINT
- MSK\_IPAR\_MIO\_RINS\_MAX\_NODES
- MSK\_IPAR\_MIO\_ROOT\_OPTIMIZER
- MSK\_IPAR\_MIO\_ROOT\_REPEAT\_PRESOLVE\_LEVEL
- MSK\_IPAR\_MIO\_SEED
- MSK\_IPAR\_MIO\_VB\_DETECTION\_LEVEL

### **Output information**

- MSK\_IPAR\_INFEAS\_REPORT\_LEVEL
- MSK\_IPAR\_LICENSE\_SUPPRESS\_EXPIRE\_WRNS
- MSK\_IPAR\_LICENSE\_TRH\_EXPIRY\_WRN
- MSK\_IPAR\_LOG
- MSK\_IPAR\_LOG\_BI
- MSK\_IPAR\_LOG\_BI\_FREQ
- MSK\_IPAR\_LOG\_CUT\_SECOND\_OPT
- MSK\_IPAR\_LOG\_EXPAND
- MSK\_IPAR\_LOG\_FEAS\_REPAIR
- MSK\_IPAR\_LOG\_FILE
- MSK\_IPAR\_LOG\_INCLUDE\_SUMMARY
- MSK\_IPAR\_LOG\_INFEAS\_ANA
- MSK\_IPAR\_LOG\_INTPNT
- MSK\_IPAR\_LOG\_LOCAL\_INFO
- MSK\_IPAR\_LOG\_MIO
- MSK\_IPAR\_LOG\_MIO\_FREQ
- MSK\_IPAR\_LOG\_ORDER
- MSK\_IPAR\_LOG\_RESPONSE
- MSK\_IPAR\_LOG\_SENSITIVITY
- MSK\_IPAR\_LOG\_SENSITIVITY\_OPT
- MSK\_IPAR\_LOG\_SIM

- MSK\_IPAR\_LOG\_SIM\_FREQ
- MSK\_IPAR\_LOG\_SIM\_MINOR
- MSK\_IPAR\_LOG\_STORAGE
- MSK\_IPAR\_MAX\_NUM\_WARNINGS

### Overall solver

- MSK\_IPAR\_BI\_CLEAN\_OPTIMIZER
- MSK\_IPAR\_INFEAS\_PREFER\_PRIMAL
- MSK\_IPAR\_LICENSE\_WAIT
- MSK\_IPAR\_MIO\_MODE
- MSK\_IPAR\_OPTIMIZER
- MSK\_IPAR\_PRESOLVE\_LEVEL
- MSK\_IPAR\_PRESOLVE\_MAX\_NUM\_REDUCTIONS
- MSK\_IPAR\_PRESOLVE\_USE
- MSK\_IPAR\_PRIMAL\_REPAIR\_OPTIMIZER
- MSK\_IPAR\_SENSITIVITY\_ALL
- MSK\_IPAR\_SENSITIVITY\_OPTIMIZER
- MSK\_IPAR\_SENSITIVITY\_TYPE
- MSK\_IPAR\_SOLUTION\_CALLBACK

# Overall system

- MSK\_IPAR\_AUTO\_UPDATE\_SOL\_INFO
- MSK\_IPAR\_INTPNT\_MULTI\_THREAD
- MSK\_IPAR\_LICENSE\_WAIT
- MSK\_IPAR\_LOG\_STORAGE
- MSK\_IPAR\_MT\_SPINCOUNT
- MSK\_IPAR\_NUM\_THREADS
- MSK\_IPAR\_REMOVE\_UNUSED\_SOLUTIONS
- MSK\_IPAR\_TIMING\_LEVEL
- MSK\_SPAR\_REMOTE\_ACCESS\_TOKEN

#### **Presolve**

- MSK\_DPAR\_PRESOLVE\_TOL\_ABS\_LINDEP
- MSK\_DPAR\_PRESOLVE\_TOL\_AIJ
- MSK\_DPAR\_PRESOLVE\_TOL\_REL\_LINDEP
- MSK\_DPAR\_PRESOLVE\_TOL\_S
- MSK\_DPAR\_PRESOLVE\_TOL\_X
- MSK\_IPAR\_PRESOLVE\_ELIMINATOR\_MAX\_FILL
- MSK\_IPAR\_PRESOLVE\_ELIMINATOR\_MAX\_NUM\_TRIES
- MSK\_IPAR\_PRESOLVE\_LEVEL
- MSK\_IPAR\_PRESOLVE\_LINDEP\_ABS\_WORK\_TRH
- MSK\_IPAR\_PRESOLVE\_LINDEP\_REL\_WORK\_TRH
- MSK\_IPAR\_PRESOLVE\_LINDEP\_USE
- MSK\_IPAR\_PRESOLVE\_MAX\_NUM\_PASS
- MSK\_IPAR\_PRESOLVE\_MAX\_NUM\_REDUCTIONS
- MSK\_IPAR\_PRESOLVE\_USE

### **Primal simplex**

- MSK\_IPAR\_SIM\_PRIMAL\_CRASH
- MSK\_IPAR\_SIM\_PRIMAL\_RESTRICT\_SELECTION
- MSK\_IPAR\_SIM\_PRIMAL\_SELECTION

# Progress callback

• MSK\_IPAR\_SOLUTION\_CALLBACK

### Simplex optimizer

- MSK\_DPAR\_BASIS\_REL\_TOL\_S
- MSK\_DPAR\_BASIS\_TOL\_S
- MSK\_DPAR\_BASIS\_TOL\_X
- MSK\_DPAR\_SIM\_LU\_TOL\_REL\_PIV
- MSK\_DPAR\_SIMPLEX\_ABS\_TOL\_PIV
- MSK\_IPAR\_BASIS\_SOLVE\_USE\_PLUS\_ONE
- MSK\_IPAR\_LOG\_SIM
- MSK\_IPAR\_LOG\_SIM\_FREQ
- MSK\_IPAR\_LOG\_SIM\_MINOR
- MSK\_IPAR\_SENSITIVITY\_OPTIMIZER
- MSK\_IPAR\_SIM\_BASIS\_FACTOR\_USE
- MSK\_IPAR\_SIM\_DEGEN

- MSK\_IPAR\_SIM\_DUAL\_PHASEONE\_METHOD
- MSK\_IPAR\_SIM\_EXPLOIT\_DUPVEC
- MSK\_IPAR\_SIM\_HOTSTART
- MSK\_IPAR\_SIM\_HOTSTART\_LU
- MSK\_IPAR\_SIM\_MAX\_ITERATIONS
- MSK\_IPAR\_SIM\_MAX\_NUM\_SETBACKS
- MSK\_IPAR\_SIM\_NON\_SINGULAR
- MSK\_IPAR\_SIM\_PRIMAL\_PHASEONE\_METHOD
- MSK\_IPAR\_SIM\_REFACTOR\_FREQ
- MSK\_IPAR\_SIM\_REFORMULATION
- MSK\_IPAR\_SIM\_SAVE\_LU
- MSK\_IPAR\_SIM\_SCALING
- MSK\_IPAR\_SIM\_SCALING\_METHOD
- MSK\_IPAR\_SIM\_SEED
- MSK\_IPAR\_SIM\_SOLVE\_FORM
- MSK\_IPAR\_SIM\_STABILITY\_PRIORITY
- MSK\_IPAR\_SIM\_SWITCH\_OPTIMIZER

### Solution input/output

- MSK\_IPAR\_INFEAS\_REPORT\_AUTO
- MSK\_IPAR\_SOL\_FILTER\_KEEP\_BASIC
- MSK\_IPAR\_SOL\_FILTER\_KEEP\_RANGED
- MSK\_IPAR\_SOL\_READ\_NAME\_WIDTH
- MSK\_IPAR\_SOL\_READ\_WIDTH
- MSK\_IPAR\_WRITE\_BAS\_CONSTRAINTS
- MSK\_IPAR\_WRITE\_BAS\_HEAD
- MSK\_IPAR\_WRITE\_BAS\_VARIABLES
- MSK\_IPAR\_WRITE\_INT\_CONSTRAINTS
- MSK\_IPAR\_WRITE\_INT\_HEAD
- MSK\_IPAR\_WRITE\_INT\_VARIABLES
- MSK\_IPAR\_WRITE\_SOL\_BARVARIABLES
- MSK\_IPAR\_WRITE\_SOL\_CONSTRAINTS
- MSK\_IPAR\_WRITE\_SOL\_HEAD
- MSK\_IPAR\_WRITE\_SOL\_IGNORE\_INVALID\_NAMES
- MSK\_IPAR\_WRITE\_SOL\_VARIABLES
- MSK\_SPAR\_BAS\_SOL\_FILE\_NAME

- MSK\_SPAR\_INT\_SOL\_FILE\_NAME
- MSK\_SPAR\_ITR\_SOL\_FILE\_NAME
- MSK\_SPAR\_SOL\_FILTER\_XC\_LOW
- MSK\_SPAR\_SOL\_FILTER\_XC\_UPR
- MSK\_SPAR\_SOL\_FILTER\_XX\_LOW
- MSK\_SPAR\_SOL\_FILTER\_XX\_UPR

### Termination criteria

- MSK\_DPAR\_BASIS\_REL\_TOL\_S
- MSK\_DPAR\_BASIS\_TOL\_S
- MSK\_DPAR\_BASIS\_TOL\_X
- MSK\_DPAR\_INTPNT\_CO\_TOL\_DFEAS
- MSK\_DPAR\_INTPNT\_CO\_TOL\_INFEAS
- MSK\_DPAR\_INTPNT\_CO\_TOL\_MU\_RED
- MSK\_DPAR\_INTPNT\_CO\_TOL\_NEAR\_REL
- MSK\_DPAR\_INTPNT\_CO\_TOL\_PFEAS
- MSK\_DPAR\_INTPNT\_CO\_TOL\_REL\_GAP
- MSK\_DPAR\_INTPNT\_QO\_TOL\_DFEAS
- MSK\_DPAR\_INTPNT\_QO\_TOL\_INFEAS
- MSK\_DPAR\_INTPNT\_QO\_TOL\_MU\_RED
- MSK\_DPAR\_INTPNT\_QO\_TOL\_NEAR\_REL
- MSK\_DPAR\_INTPNT\_QO\_TOL\_PFEAS
- MSK\_DPAR\_INTPNT\_QO\_TOL\_REL\_GAP
- MSK\_DPAR\_INTPNT\_TOL\_DFEAS
- MSK\_DPAR\_INTPNT\_TOL\_INFEAS
- MSK\_DPAR\_INTPNT\_TOL\_MU\_RED
- MSK\_DPAR\_INTPNT\_TOL\_PFEAS
- MSK\_DPAR\_INTPNT\_TOL\_REL\_GAP
- MSK\_DPAR\_LOWER\_OBJ\_CUT
- MSK\_DPAR\_LOWER\_OBJ\_CUT\_FINITE\_TRH
- MSK\_DPAR\_MIO\_MAX\_TIME
- MSK\_DPAR\_MIO\_REL\_GAP\_CONST
- MSK\_DPAR\_MIO\_TOL\_REL\_GAP
- MSK\_DPAR\_OPTIMIZER\_MAX\_TIME
- MSK\_DPAR\_UPPER\_OBJ\_CUT
- MSK\_DPAR\_UPPER\_OBJ\_CUT\_FINITE\_TRH

- MSK\_IPAR\_BI\_MAX\_ITERATIONS
- MSK\_IPAR\_INTPNT\_MAX\_ITERATIONS
- MSK\_IPAR\_MIO\_MAX\_NUM\_BRANCHES
- MSK\_IPAR\_MIO\_MAX\_NUM\_ROOT\_CUT\_ROUNDS
- MSK\_IPAR\_MIO\_MAX\_NUM\_SOLUTIONS
- MSK\_IPAR\_SIM\_MAX\_ITERATIONS

## Other

• MSK\_IPAR\_COMPRESS\_STATFILE

# 15.5 Parameters (alphabetical list sorted by type)

- Double parameters
- Integer parameters
- String parameters

# 15.5.1 Double parameters

# MSKdparame

The enumeration type containing all double parameters.

# MSK\_DPAR\_ANA\_SOL\_INFEAS\_TOL

If a constraint violates its bound with an amount larger than this value, the constraint name, index and violation will be printed by the solution analyzer.

Default 1e-6

Accepted [0.0; +inf]

Example MSK\_putdouparam(task, MSK\_DPAR\_ANA\_SOL\_INFEAS\_TOL, 1e-6)

 ${\bf Groups} \ \ {\it Analysis}$ 

# MSK\_DPAR\_BASIS\_REL\_TOL\_S

Maximum relative dual bound violation allowed in an optimal basic solution.

 $\textbf{Default} \ \ 1.0\text{e-}12$ 

Accepted [0.0; +inf]

Example MSK\_putdouparam(task, MSK\_DPAR\_BASIS\_REL\_TOL\_S, 1.0e-12)

**Groups** Simplex optimizer, Termination criteria

#### MSK\_DPAR\_BASIS\_TOL\_S

Maximum absolute dual bound violation in an optimal basic solution.

Default 1.0e-6

Accepted [1.0e-9; +inf]

Example MSK\_putdouparam(task, MSK\_DPAR\_BASIS\_TOL\_S, 1.0e-6)

Groups Simplex optimizer, Termination criteria

# MSK\_DPAR\_BASIS\_TOL\_X

Maximum absolute primal bound violation allowed in an optimal basic solution.

Default 1.0e-6

Accepted [1.0e-9; +inf]

Example MSK\_putdouparam(task, MSK\_DPAR\_BASIS\_TOL\_X, 1.0e-6)

Groups Simplex optimizer, Termination criteria

## MSK\_DPAR\_CHECK\_CONVEXITY\_REL\_TOL

This parameter controls when the full convexity check declares a problem to be non-convex. Increasing this tolerance relaxes the criteria for declaring the problem non-convex.

A problem is declared non-convex if negative (positive) pivot elements are detected in the Cholesky factor of a matrix which is required to be PSD (NSD). This parameter controls how much this non-negativity requirement may be violated.

If  $d_i$  is the pivot element for column i, then the matrix Q is considered to not be PSD if:

$$d_i \leq -|Q_{ii}|$$
check convexity rel tol

Default 1e-10

Accepted [0; +inf]

Example MSK\_putdouparam(task, MSK\_DPAR\_CHECK\_CONVEXITY\_REL\_TOL, 1e-10)

Groups Interior-point method

## MSK\_DPAR\_DATA\_SYM\_MAT\_TOL

Absolute zero tolerance for elements in in symmetric matrices. If any value in a symmetric matrix is smaller than this parameter in absolute terms **MOSEK** will treat the values as zero and generate a warning.

**Default** 1.0e-12

**Accepted** [1.0e-16; 1.0e-6]

Example MSK\_putdouparam(task, MSK\_DPAR\_DATA\_SYM\_MAT\_TOL, 1.0e-12)

Groups Data check

#### MSK\_DPAR\_DATA\_SYM\_MAT\_TOL\_HUGE

An element in a symmetric matrix which is larger than this value in absolute size causes an error.

**Default** 1.0e20

Accepted [0.0; +inf]

Example MSK\_putdouparam(task, MSK\_DPAR\_DATA\_SYM\_MAT\_TOL\_HUGE, 1.0e20)

Groups Data check

#### MSK\_DPAR\_DATA\_SYM\_MAT\_TOL\_LARGE

An element in a symmetric matrix which is larger than this value in absolute size causes a warning message to be printed.

**Default** 1.0e10

Accepted [0.0; +inf]

Example MSK\_putdouparam(task, MSK\_DPAR\_DATA\_SYM\_MAT\_TOL\_LARGE, 1.0e10)

Groups Data check

#### MSK\_DPAR\_DATA\_TOL\_AIJ\_HUGE

An element in A which is larger than this value in absolute size causes an error.

**Default** 1.0e20

Accepted [0.0; +inf]

Example MSK\_putdouparam(task, MSK\_DPAR\_DATA\_TOL\_AIJ\_HUGE, 1.0e20)

Groups Data check

# MSK\_DPAR\_DATA\_TOL\_AIJ\_LARGE

An element in A which is larger than this value in absolute size causes a warning message to be printed.

**Default** 1.0e10

Accepted [0.0; +inf]

Example MSK\_putdouparam(task, MSK\_DPAR\_DATA\_TOL\_AIJ\_LARGE, 1.0e10)

Groups Data check

## MSK\_DPAR\_DATA\_TOL\_BOUND\_INF

Any bound which in absolute value is greater than this parameter is considered infinite.

Default 1.0e16

Accepted [0.0; +inf]

Example MSK\_putdouparam(task, MSK\_DPAR\_DATA\_TOL\_BOUND\_INF, 1.0e16)

Groups Data check

### MSK\_DPAR\_DATA\_TOL\_BOUND\_WRN

If a bound value is larger than this value in absolute size, then a warning message is issued.

Default 1.0e8

Accepted [0.0; +inf]

Example MSK\_putdouparam(task, MSK\_DPAR\_DATA\_TOL\_BOUND\_WRN, 1.0e8)

Groups Data check

## MSK\_DPAR\_DATA\_TOL\_C\_HUGE

An element in c which is larger than the value of this parameter in absolute terms is considered to be huge and generates an error.

**Default** 1.0e16

Accepted [0.0; +inf]

Example MSK\_putdouparam(task, MSK\_DPAR\_DATA\_TOL\_C\_HUGE, 1.0e16)

Groups Data check

# MSK\_DPAR\_DATA\_TOL\_CJ\_LARGE

An element in c which is larger than this value in absolute terms causes a warning message to be printed.

Default 1.0e8

Accepted [0.0; +inf]

Example MSK\_putdouparam(task, MSK\_DPAR\_DATA\_TOL\_CJ\_LARGE, 1.0e8)

Groups Data check

## MSK\_DPAR\_DATA\_TOL\_QIJ

Absolute zero tolerance for elements in Q matrices.

**Default** 1.0e-16

Accepted [0.0; +inf]

Example MSK\_putdouparam(task, MSK\_DPAR\_DATA\_TOL\_QIJ, 1.0e-16)

Groups Data check

# MSK\_DPAR\_DATA\_TOL\_X

Zero tolerance for constraints and variables i.e. if the distance between the lower and upper bound is less than this value, then the lower and upper bound is considered identical.

Default 1.0e-8

Accepted [0.0; +inf]

Example MSK\_putdouparam(task, MSK\_DPAR\_DATA\_TOL\_X, 1.0e-8)

Groups Data check

## MSK\_DPAR\_INTPNT\_CO\_TOL\_DFEAS

Dual feasibility tolerance used by the interior-point optimizer for conic problems.

Default 1.0e-8

**Accepted** [0.0; 1.0]

Example MSK\_putdouparam(task, MSK\_DPAR\_INTPNT\_CO\_TOL\_DFEAS, 1.0e-8)

See also MSK\_DPAR\_INTPNT\_CO\_TOL\_NEAR\_REL

Groups Interior-point method, Termination criteria, Conic interior-point method

## MSK\_DPAR\_INTPNT\_CO\_TOL\_INFEAS

Infeasibility tolerance used by the interior-point optimizer for conic problems. Controls when the interior-point optimizer declares the model primal or dual infeasible. A small number means the optimizer gets more conservative about declaring the model infeasible.

**Default** 1.0e-12

**Accepted** [0.0; 1.0]

Example MSK\_putdouparam(task, MSK\_DPAR\_INTPNT\_CO\_TOL\_INFEAS, 1.0e-12)

Groups Interior-point method, Termination criteria, Conic interior-point method

## MSK\_DPAR\_INTPNT\_CO\_TOL\_MU\_RED

Relative complementarity gap tolerance used by the interior-point optimizer for conic problems.

Default 1.0e-8

Accepted [0.0; 1.0]

Example MSK\_putdouparam(task, MSK\_DPAR\_INTPNT\_CO\_TOL\_MU\_RED, 1.0e-8)

Groups Interior-point method, Termination criteria, Conic interior-point method

## MSK\_DPAR\_INTPNT\_CO\_TOL\_NEAR\_REL

Optimality tolerance used by the interior-point optimizer for conic problems. If **MOSEK** cannot compute a solution that has the prescribed accuracy then it will check if the solution found satisfies the termination criteria with all tolerances multiplied by the value of this parameter. If yes, then the solution is also declared optimal.

Default 1000

Accepted [1.0; +inf]

Example MSK\_putdouparam(task, MSK\_DPAR\_INTPNT\_CO\_TOL\_NEAR\_REL, 1000)

Groups Interior-point method, Termination criteria, Conic interior-point method

# MSK\_DPAR\_INTPNT\_CO\_TOL\_PFEAS

Primal feasibility tolerance used by the interior-point optimizer for conic problems.

Default 1.0e-8

Accepted [0.0; 1.0]

Example MSK\_putdouparam(task, MSK\_DPAR\_INTPNT\_CO\_TOL\_PFEAS, 1.0e-8)

See also MSK\_DPAR\_INTPNT\_CO\_TOL\_NEAR\_REL

Groups Interior-point method, Termination criteria, Conic interior-point method

# MSK\_DPAR\_INTPNT\_CO\_TOL\_REL\_GAP

Relative gap termination tolerance used by the interior-point optimizer for conic problems.

Default 1.0e-8

Accepted [0.0; 1.0]

Example MSK\_putdouparam(task, MSK\_DPAR\_INTPNT\_CO\_TOL\_REL\_GAP, 1.0e-8)

See also MSK\_DPAR\_INTPNT\_CO\_TOL\_NEAR\_REL

**Groups** Interior-point method, Termination criteria, Conic interior-point method

#### MSK\_DPAR\_INTPNT\_QO\_TOL\_DFEAS

Dual feasibility tolerance used by the interior-point optimizer for quadratic problems.

Default 1.0e-8

**Accepted** [0.0; 1.0]

Example MSK\_putdouparam(task, MSK\_DPAR\_INTPNT\_QO\_TOL\_DFEAS, 1.0e-8)

See also MSK\_DPAR\_INTPNT\_QO\_TOL\_NEAR\_REL

Groups Interior-point method, Termination criteria

## MSK\_DPAR\_INTPNT\_QO\_TOL\_INFEAS

Infeasibility tolerance used by the interior-point optimizer for quadratic problems. Controls when the interior-point optimizer declares the model primal or dual infeasible. A small number means the optimizer gets more conservative about declaring the model infeasible.

**Default** 1.0e-12

**Accepted** [0.0; 1.0]

Example MSK\_putdouparam(task, MSK\_DPAR\_INTPNT\_QO\_TOL\_INFEAS, 1.0e-12)

Groups Interior-point method, Termination criteria

#### MSK\_DPAR\_INTPNT\_QO\_TOL\_MU\_RED

Relative complementarity gap tolerance used by the interior-point optimizer for quadratic problems.

Default 1.0e-8

**Accepted** [0.0; 1.0]

Example MSK\_putdouparam(task, MSK\_DPAR\_INTPNT\_QO\_TOL\_MU\_RED, 1.0e-8)

Groups Interior-point method, Termination criteria

### MSK\_DPAR\_INTPNT\_QO\_TOL\_NEAR\_REL

Optimality tolerance used by the interior-point optimizer for quadratic problems. If MOSEK cannot compute a solution that has the prescribed accuracy then it will check if the solution found satisfies the termination criteria with all tolerances multiplied by the value of this parameter. If yes, then the solution is also declared optimal.

Default 1000

Accepted [1.0; +inf]

Example MSK\_putdouparam(task, MSK\_DPAR\_INTPNT\_QO\_TOL\_NEAR\_REL, 1000)

Groups Interior-point method, Termination criteria

# MSK\_DPAR\_INTPNT\_QO\_TOL\_PFEAS

Primal feasibility tolerance used by the interior-point optimizer for quadratic problems.

Default 1.0e-8

**Accepted** [0.0; 1.0]

Example MSK\_putdouparam(task, MSK\_DPAR\_INTPNT\_QO\_TOL\_PFEAS, 1.0e-8)

See also MSK\_DPAR\_INTPNT\_QO\_TOL\_NEAR\_REL

Groups Interior-point method, Termination criteria

# MSK\_DPAR\_INTPNT\_QO\_TOL\_REL\_GAP

Relative gap termination tolerance used by the interior-point optimizer for quadratic problems.

Default 1.0e-8

**Accepted** [0.0; 1.0]

Example MSK\_putdouparam(task, MSK\_DPAR\_INTPNT\_QO\_TOL\_REL\_GAP, 1.0e-8)

See also MSK\_DPAR\_INTPNT\_QO\_TOL\_NEAR\_REL

Groups Interior-point method, Termination criteria

# MSK\_DPAR\_INTPNT\_TOL\_DFEAS

Dual feasibility tolerance used by the interior-point optimizer for linear problems.

Default 1.0e-8

**Accepted** [0.0; 1.0]

Example MSK\_putdouparam(task, MSK\_DPAR\_INTPNT\_TOL\_DFEAS, 1.0e-8)

**Groups** Interior-point method, Termination criteria

#### MSK\_DPAR\_INTPNT\_TOL\_DSAFE

Controls the initial dual starting point used by the interior-point optimizer. If the interior-point optimizer converges slowly and/or the constraint or variable bounds are very large, then it might be worthwhile to increase this value.

Default 1.0

Accepted [1.0e-4; +inf]

Example MSK\_putdouparam(task, MSK\_DPAR\_INTPNT\_TOL\_DSAFE, 1.0)

Groups Interior-point method

#### MSK\_DPAR\_INTPNT\_TOL\_INFEAS

Infeasibility tolerance used by the interior-point optimizer for linear problems. Controls when the interior-point optimizer declares the model primal or dual infeasible. A small number means the optimizer gets more conservative about declaring the model infeasible.

**Default** 1.0e-10

**Accepted** [0.0; 1.0]

Example MSK\_putdouparam(task, MSK\_DPAR\_INTPNT\_TOL\_INFEAS, 1.0e-10)

Groups Interior-point method, Termination criteria

#### MSK\_DPAR\_INTPNT\_TOL\_MU\_RED

Relative complementarity gap tolerance used by the interior-point optimizer for linear problems.

**Default** 1.0e-16

Accepted [0.0; 1.0]

Example MSK\_putdouparam(task, MSK\_DPAR\_INTPNT\_TOL\_MU\_RED, 1.0e-16)

Groups Interior-point method, Termination criteria

#### MSK\_DPAR\_INTPNT\_TOL\_PATH

Controls how close the interior-point optimizer follows the central path. A large value of this parameter means the central path is followed very closely. On numerically unstable problems it may be worthwhile to increase this parameter.

Default 1.0e-8

**Accepted** [0.0; 0.9999]

Example MSK\_putdouparam(task, MSK\_DPAR\_INTPNT\_TOL\_PATH, 1.0e-8)

Groups Interior-point method

## MSK\_DPAR\_INTPNT\_TOL\_PFEAS

Primal feasibility tolerance used by the interior-point optimizer for linear problems.

**Default** 1.0e-8

Accepted [0.0; 1.0]

Example MSK\_putdouparam(task, MSK\_DPAR\_INTPNT\_TOL\_PFEAS, 1.0e-8)

Groups Interior-point method, Termination criteria

# MSK\_DPAR\_INTPNT\_TOL\_PSAFE

Controls the initial primal starting point used by the interior-point optimizer. If the interior-point optimizer converges slowly and/or the constraint or variable bounds are very large, then it may be worthwhile to increase this value.

Default 1.0

Accepted [1.0e-4; +inf]

Example MSK\_putdouparam(task, MSK\_DPAR\_INTPNT\_TOL\_PSAFE, 1.0)

Groups Interior-point method

# MSK\_DPAR\_INTPNT\_TOL\_REL\_GAP

Relative gap termination tolerance used by the interior-point optimizer for linear problems.

Default 1.0e-8

Accepted [1.0e-14; +inf]

Example MSK\_putdouparam(task, MSK\_DPAR\_INTPNT\_TOL\_REL\_GAP, 1.0e-8)

Groups Termination criteria, Interior-point method

# MSK\_DPAR\_INTPNT\_TOL\_REL\_STEP

Relative step size to the boundary for linear and quadratic optimization problems.

 $\textbf{Default} \ \ 0.9999$ 

**Accepted** [1.0e-4; 0.999999]

Example MSK\_putdouparam(task, MSK\_DPAR\_INTPNT\_TOL\_REL\_STEP, 0.9999)

# Groups Interior-point method

## MSK\_DPAR\_INTPNT\_TOL\_STEP\_SIZE

Minimal step size tolerance. If the step size falls below the value of this parameter, then the interior-point optimizer assumes that it is stalled. In other words the interior-point optimizer does not make any progress and therefore it is better to stop.

**Default** 1.0e-6 **Accepted** [0.0; 1.0]

Example MSK\_putdouparam(task, MSK\_DPAR\_INTPNT\_TOL\_STEP\_SIZE, 1.0e-6)

Groups Interior-point method

#### MSK\_DPAR\_LOWER\_OBJ\_CUT

If either a primal or dual feasible solution is found proving that the optimal objective value is outside the interval [  $MSK\_DPAR\_LOWER\_OBJ\_CUT$ ,  $MSK\_DPAR\_UPPER\_OBJ\_CUT$ ], then  $\mathbf{MOSEK}$  is terminated.

**Default** -1.0e30

Accepted  $[-\inf; +\inf]$ 

Example MSK\_putdouparam(task, MSK\_DPAR\_LOWER\_OBJ\_CUT, -1.0e30)

See also MSK\_DPAR\_LOWER\_OBJ\_CUT\_FINITE\_TRH

Groups Termination criteria

## MSK\_DPAR\_LOWER\_OBJ\_CUT\_FINITE\_TRH

If the lower objective cut is less than the value of this parameter value, then the lower objective cut i.e.  $MSK\_DPAR\_LOWER\_OBJ\_CUT$  is treated as  $-\infty$ .

**Default** -0.5e30

Accepted  $[-\inf; +\inf]$ 

Example MSK\_putdouparam(task, MSK\_DPAR\_LOWER\_OBJ\_CUT\_FINITE\_TRH, -0.
5e30)

Groups Termination criteria

# MSK\_DPAR\_MIO\_MAX\_TIME

This parameter limits the maximum time spent by the mixed-integer optimizer. A negative number means infinity.

Default -1.0

Accepted  $[-\inf; +\inf]$ 

Example MSK\_putdouparam(task, MSK\_DPAR\_MIO\_MAX\_TIME, -1.0)

 ${\bf Groups}\ {\it Mixed-integer}\ optimization,\ Termination\ criteria$ 

# MSK\_DPAR\_MIO\_REL\_GAP\_CONST

This value is used to compute the relative gap for the solution to an integer optimization problem.

**Default** 1.0e-10

Accepted [1.0e-15; +inf]

Example MSK\_putdouparam(task, MSK\_DPAR\_MIO\_REL\_GAP\_CONST, 1.0e-10)

Groups Mixed-integer optimization, Termination criteria

# MSK\_DPAR\_MIO\_TOL\_ABS\_GAP

Absolute optimality tolerance employed by the mixed-integer optimizer.

Default 0.0

Accepted [0.0; +inf]

Example MSK\_putdouparam(task, MSK\_DPAR\_MIO\_TOL\_ABS\_GAP, 0.0)

**Groups** Mixed-integer optimization

#### MSK\_DPAR\_MIO\_TOL\_ABS\_RELAX\_INT

Absolute integer feasibility tolerance. If the distance to the nearest integer is less than this tolerance then an integer constraint is assumed to be satisfied.

Default 1.0e-5

Accepted [1e-9; +inf]

Example MSK\_putdouparam(task, MSK\_DPAR\_MIO\_TOL\_ABS\_RELAX\_INT, 1.0e-5)

**Groups** Mixed-integer optimization

## MSK\_DPAR\_MIO\_TOL\_FEAS

Feasibility tolerance for mixed integer solver.

Default 1.0e-6

**Accepted** [1e-9; 1e-3]

Example MSK\_putdouparam(task, MSK\_DPAR\_MIO\_TOL\_FEAS, 1.0e-6)

**Groups** Mixed-integer optimization

# MSK\_DPAR\_MIO\_TOL\_REL\_DUAL\_BOUND\_IMPROVEMENT

If the relative improvement of the dual bound is smaller than this value, the solver will terminate the root cut generation. A value of 0.0 means that the value is selected automatically.

Default 0.0

**Accepted** [0.0; 1.0]

**Groups** Mixed-integer optimization

## MSK\_DPAR\_MIO\_TOL\_REL\_GAP

Relative optimality tolerance employed by the mixed-integer optimizer.

Default 1.0e-4

Accepted [0.0; +inf]

Example MSK\_putdouparam(task, MSK\_DPAR\_MIO\_TOL\_REL\_GAP, 1.0e-4)

Groups Mixed-integer optimization, Termination criteria

# MSK\_DPAR\_OPTIMIZER\_MAX\_TIME

Maximum amount of time the optimizer is allowed to spent on the optimization. A negative number means infinity.

**Default** -1.0

Accepted  $[-\inf; +\inf]$ 

Example MSK\_putdouparam(task, MSK\_DPAR\_OPTIMIZER\_MAX\_TIME, -1.0)

Groups Termination criteria

# MSK\_DPAR\_PRESOLVE\_TOL\_ABS\_LINDEP

Absolute tolerance employed by the linear dependency checker.

Default 1.0e-6

Accepted [0.0; +inf]

Example MSK\_putdouparam(task, MSK\_DPAR\_PRESOLVE\_TOL\_ABS\_LINDEP, 1.0e-6)

Groups Presolve

# MSK\_DPAR\_PRESOLVE\_TOL\_AIJ

Absolute zero tolerance employed for  $a_{ij}$  in the presolve.

**Default** 1.0e-12

Accepted [1.0e-15; +inf]

Example MSK\_putdouparam(task, MSK\_DPAR\_PRESOLVE\_TOL\_AIJ, 1.0e-12)

Groups Presolve

```
MSK_DPAR_PRESOLVE_TOL_REL_LINDEP
```

Relative tolerance employed by the linear dependency checker.

**Default** 1.0e-10

Accepted [0.0; +inf]

Example MSK\_putdouparam(task, MSK\_DPAR\_PRESOLVE\_TOL\_REL\_LINDEP, 1.

0e-10)

Groups Presolve

## MSK\_DPAR\_PRESOLVE\_TOL\_S

Absolute zero tolerance employed for  $s_i$  in the presolve.

Default 1.0e-8

Accepted [0.0; +inf]

Example MSK\_putdouparam(task, MSK\_DPAR\_PRESOLVE\_TOL\_S, 1.0e-8)

Groups Presolve

## MSK\_DPAR\_PRESOLVE\_TOL\_X

Absolute zero tolerance employed for  $x_j$  in the presolve.

Default 1.0e-8

Accepted [0.0; +inf]

Example MSK\_putdouparam(task, MSK\_DPAR\_PRESOLVE\_TOL\_X, 1.0e-8)

Groups Presolve

# MSK\_DPAR\_QCQO\_REFORMULATE\_REL\_DROP\_TOL

This parameter determines when columns are dropped in incomplete Cholesky factorization during reformulation of quadratic problems.

Default 1e-15

Accepted [0; +inf]

Groups Interior-point method

# MSK\_DPAR\_SEMIDEFINITE\_TOL\_APPROX

Tolerance to define a matrix to be positive semidefinite.

**Default** 1.0e-10

Accepted [1.0e-15; +inf]

Groups Data check

# MSK\_DPAR\_SIM\_LU\_TOL\_REL\_PIV

Relative pivot tolerance employed when computing the LU factorization of the basis in the simplex optimizers and in the basis identification procedure. A value closer to 1.0 generally improves numerical stability but typically also implies an increase in the computational work.

 $\textbf{Default} \ 0.01$ 

**Accepted** [1.0e-6; 0.999999]

Example MSK\_putdouparam(task, MSK\_DPAR\_SIM\_LU\_TOL\_REL\_PIV, 0.01)

Groups Basis identification, Simplex optimizer

# MSK\_DPAR\_SIMPLEX\_ABS\_TOL\_PIV

Absolute pivot tolerance employed by the simplex optimizers.

Default 1.0e-7

Accepted [1.0e-12; +inf]

Example MSK\_putdouparam(task, MSK\_DPAR\_SIMPLEX\_ABS\_TOL\_PIV, 1.0e-7)

# Groups Simplex optimizer

## MSK\_DPAR\_UPPER\_OBJ\_CUT

If either a primal or dual feasible solution is found proving that the optimal objective value is outside the interval [  $MSK\_DPAR\_LOWER\_OBJ\_CUT$ ,  $MSK\_DPAR\_UPPER\_OBJ\_CUT$ ], then  $\mathbf{MOSEK}$  is terminated.

**Default** 1.0e30

Accepted  $[-\inf; +\inf]$ 

Example MSK\_putdouparam(task, MSK\_DPAR\_UPPER\_OBJ\_CUT, 1.0e30)

See also MSK\_DPAR\_UPPER\_OBJ\_CUT\_FINITE\_TRH

Groups Termination criteria

## MSK\_DPAR\_UPPER\_OBJ\_CUT\_FINITE\_TRH

If the upper objective cut is greater than the value of this parameter, then the upper objective cut  $MSK\_DPAR\_UPPER\_OBJ\_CUT$  is treated as  $\infty$ .

Default 0.5e30

Accepted  $[-\inf; +\inf]$ 

Example MSK\_putdouparam(task, MSK\_DPAR\_UPPER\_OBJ\_CUT\_FINITE\_TRH, 0.
5e30)

Groups Termination criteria

# 15.5.2 Integer parameters

# MSKiparame

The enumeration type containing all integer parameters.

## MSK\_IPAR\_ANA\_SOL\_BASIS

Controls whether the basis matrix is analyzed in solution analyzer.

Default ON

Accepted ON, OFF (see MSKonoffkeye)

Example MSK\_putintparam(task, MSK\_IPAR\_ANA\_SOL\_BASIS, MSK\_ON)

Groups Analysis

## MSK\_IPAR\_ANA\_SOL\_PRINT\_VIOLATED

A parameter of the problem analyzer. Controls whether a list of violated constraints is printed. All constraints violated by more than the value set by the parameter  $MSK\_DPAR\_ANA\_SOL\_INFEAS\_TOL$  will be printed.

Default OFF

Accepted ON, OFF (see MSKonoffkeye)

Example MSK\_putintparam(task, MSK\_IPAR\_ANA\_SOL\_PRINT\_VIOLATED, MSK\_OFF)

Groups Analysis

# MSK\_IPAR\_AUTO\_SORT\_A\_BEFORE\_OPT

Controls whether the elements in each column of A are sorted before an optimization is performed. This is not required but makes the optimization more deterministic.

Default OFF

Accepted ON, OFF (see MSKonoffkeye)

Example MSK\_putintparam(task, MSK\_IPAR\_AUTO\_SORT\_A\_BEFORE\_OPT, MSK\_OFF)

Groups Debugging

# MSK\_IPAR\_AUTO\_UPDATE\_SOL\_INFO

Controls whether the solution information items are automatically updated after an optimization is performed.

Default OFF

```
Accepted ON, OFF (see MSKonoffkeye)

Example MSK_putintparam(task, MSK_IPAR_AUTO_UPDATE_SOL_INFO, MSK_OFF)

Groups Overall system
```

# MSK\_IPAR\_BASIS\_SOLVE\_USE\_PLUS\_ONE

If a slack variable is in the basis, then the corresponding column in the basis is a unit vector with -1 in the right position. However, if this parameter is set to  $MSK_ON$ , -1 is replaced by 1.

This has significance for the results returned by the MSK\_solvewithbasis function.

Default OFF

Accepted ON, OFF (see MSKonoffkeye)

Groups Simplex optimizer

## MSK\_IPAR\_BI\_CLEAN\_OPTIMIZER

Controls which simplex optimizer is used in the clean-up phase. Anything else than  $MSK\_OPTIMIZER\_PRIMAL\_SIMPLEX$  or  $MSK\_OPTIMIZER\_DUAL\_SIMPLEX$  is equivalent to  $MSK\_OPTIMIZER\_FREE\_SIMPLEX$ .

Default FREE

Accepted FREE, INTPNT, CONIC, PRIMAL\_SIMPLEX, DUAL\_SIMPLEX, FREE\_SIMPLEX, MIXED\_INT (see MSKoptimizertypee)

Groups Basis identification, Overall solver

### MSK\_IPAR\_BI\_IGNORE\_MAX\_ITER

If the parameter MSK\_IPAR\_INTPNT\_BASIS has the value MSK\_BI\_NO\_ERROR and the interior-point optimizer has terminated due to maximum number of iterations, then basis identification is performed if this parameter has the value MSK\_ON.

Default OFF

Accepted ON, OFF (see MSKonoffkeye)

Example MSK\_putintparam(task, MSK\_IPAR\_BI\_IGNORE\_MAX\_ITER, MSK\_OFF)

Groups Interior-point method, Basis identification

# MSK\_IPAR\_BI\_IGNORE\_NUM\_ERROR

If the parameter  $MSK\_IPAR\_INTPNT\_BASIS$  has the value  $MSK\_BI\_NO\_ERROR$  and the interior-point optimizer has terminated due to a numerical problem, then basis identification is performed if this parameter has the value  $MSK\_ON$ .

Default OFF

Accepted ON, OFF (see MSKonoffkeye)

Example MSK\_putintparam(task, MSK\_IPAR\_BI\_IGNORE\_NUM\_ERROR, MSK\_OFF)

 ${\bf Groups} \ \ Interior\mbox{-}point \ method, \ Basis \ identification$ 

# MSK\_IPAR\_BI\_MAX\_ITERATIONS

Controls the maximum number of simplex iterations allowed to optimize a basis after the basis identification.

**Default** 1000000

Accepted [0; +inf]

 $\mathbf{Example} \ \mathtt{MSK\_putintparam}(\mathtt{task}, \ \mathtt{MSK\_IPAR\_BI\_MAX\_ITERATIONS}, \ 1000000)$ 

Groups Basis identification, Termination criteria

## MSK\_IPAR\_CACHE\_LICENSE

Specifies if the license is kept checked out for the lifetime of the **MOSEK** environment/model/process  $(MSK\_ON)$  or returned to the server immediately after the optimization  $(MSK\_OFF)$ .

By default the license is checked out for the lifetime of the  $\mathbf{MOSEK}$  environment by the first call to  $\mathit{MSK\_optimize}$ .

Check-in and check-out of licenses have an overhead. Frequent communication with the license server should be avoided.

Default ON

Accepted ON, OFF (see MSKonoffkeye)

Example MSK\_putintparam(task, MSK\_IPAR\_CACHE\_LICENSE, MSK\_ON)

Groups License manager

## MSK\_IPAR\_CHECK\_CONVEXITY

Specify the level of convexity check on quadratic problems.

Default FULL

Accepted NONE, SIMPLE, FULL (see MSKcheckconvexitytypee)

Groups Data check

## MSK\_IPAR\_COMPRESS\_STATFILE

Control compression of stat files.

Default ON

Accepted ON, OFF (see MSKonoffkeye)

Example MSK\_putintparam(task, MSK\_IPAR\_COMPRESS\_STATFILE, MSK\_ON)

# MSK\_IPAR\_INFEAS\_GENERIC\_NAMES

Controls whether generic names are used when an infeasible subproblem is created.

Default OFF

Accepted ON, OFF (see MSKonoffkeye)

Example MSK\_putintparam(task, MSK\_IPAR\_INFEAS\_GENERIC\_NAMES, MSK\_OFF)

Groups Infeasibility report

# ${\tt MSK\_IPAR\_INFEAS\_PREFER\_PRIMAL}$

If both certificates of primal and dual infeasibility are supplied then only the primal is used when this option is turned on.

Default ON

Accepted ON, OFF (see MSKonoffkeye)

Example MSK\_putintparam(task, MSK\_IPAR\_INFEAS\_PREFER\_PRIMAL, MSK\_ON)

Groups Overall solver

# MSK\_IPAR\_INFEAS\_REPORT\_AUTO

Controls whether an infeasibility report is automatically produced after the optimization if the problem is primal or dual infeasible.

Default OFF

Accepted ON, OFF (see MSKonoffkeye)

 $\textbf{Example MSK\_putintparam(task, MSK\_IPAR\_INFEAS\_REPORT\_AUTO, MSK\_OFF)} \\$ 

Groups Data input/output, Solution input/output

# MSK\_IPAR\_INFEAS\_REPORT\_LEVEL

Controls the amount of information presented in an infeasibility report. Higher values imply more information.

#### Default 1

Accepted [0; +inf]

Example MSK\_putintparam(task, MSK\_IPAR\_INFEAS\_REPORT\_LEVEL, 1)

**Groups** Infeasibility report, Output information

## MSK\_IPAR\_INTPNT\_BASIS

Controls whether the interior-point optimizer also computes an optimal basis.

Default ALWAYS

Accepted NEVER, ALWAYS, NO\_ERROR, IF\_FEASIBLE, RESERVERED (see MSKbasindtypee)

Example MSK\_putintparam(task, MSK\_IPAR\_INTPNT\_BASIS, MSK\_BI\_ALWAYS)

See also MSK\_IPAR\_BI\_IGNORE\_MAX\_ITER, MSK\_IPAR\_BI\_IGNORE\_NUM\_ERROR, MSK\_IPAR\_BI\_MAX\_ITERATIONS, MSK\_IPAR\_BI\_CLEAN\_OPTIMIZER

Groups Interior-point method, Basis identification

# MSK\_IPAR\_INTPNT\_DIFF\_STEP

Controls whether different step sizes are allowed in the primal and dual space.

#### Default ON

# Accepted

- *ON*: Different step sizes are allowed.
- *OFF*: Different step sizes are not allowed.

Example MSK\_putintparam(task, MSK\_IPAR\_INTPNT\_DIFF\_STEP, MSK\_ON)

Groups Interior-point method

## MSK\_IPAR\_INTPNT\_HOTSTART

Currently not in use.

Default NONE

Accepted NONE, PRIMAL, DUAL, PRIMAL\_DUAL (see MSKintpnthotstarte)

Groups Interior-point method

# MSK\_IPAR\_INTPNT\_MAX\_ITERATIONS

Controls the maximum number of iterations allowed in the interior-point optimizer.

Default 400

Accepted  $[0; +\inf]$ 

Example MSK\_putintparam(task, MSK\_IPAR\_INTPNT\_MAX\_ITERATIONS, 400)

Groups Interior-point method, Termination criteria

# MSK\_IPAR\_INTPNT\_MAX\_NUM\_COR

Controls the maximum number of correctors allowed by the multiple corrector procedure. A negative value means that **MOSEK** is making the choice.

Default -1

Accepted [-1; +inf]

Example MSK\_putintparam(task, MSK\_IPAR\_INTPNT\_MAX\_NUM\_COR, -1)

 ${\bf Groups} \ \ Interior\mbox{-}point \ method$ 

## MSK\_IPAR\_INTPNT\_MAX\_NUM\_REFINEMENT\_STEPS

Maximum number of steps to be used by the iterative refinement of the search direction. A negative value implies that the optimizer chooses the maximum number of iterative refinement steps.

Default -1

Accepted  $[-\inf; +\inf]$ 

Groups Interior-point method

# MSK\_IPAR\_INTPNT\_MULTI\_THREAD

Controls whether the interior-point optimizers are allowed to employ multiple threads if more threads is available.

Default ON

Accepted ON, OFF (see MSKonoffkeye)

Example MSK\_putintparam(task, MSK\_IPAR\_INTPNT\_MULTI\_THREAD, MSK\_ON)

Groups Overall system

### MSK\_IPAR\_INTPNT\_OFF\_COL\_TRH

Controls how many offending columns are detected in the Jacobian of the constraint matrix.

0	no detection
1	aggressive detection
> 1	higher values mean less aggressive detection

Default 40

Accepted  $[0; +\inf]$ 

Example MSK\_putintparam(task, MSK\_IPAR\_INTPNT\_OFF\_COL\_TRH, 40)

Groups Interior-point method

# MSK\_IPAR\_INTPNT\_ORDER\_GP\_NUM\_SEEDS

The GP ordering is dependent on a random seed. Therefore, trying several random seeds may lead to a better ordering. This parameter controls the number of random seeds tried.

A value of 0 means that MOSEK makes the choice.

Default 0

**Accepted**  $[0; +\inf]$ 

Example MSK\_putintparam(task, MSK\_IPAR\_INTPNT\_ORDER\_GP\_NUM\_SEEDS, 0)

Groups Interior-point method

# MSK\_IPAR\_INTPNT\_ORDER\_METHOD

Controls the ordering strategy used by the interior-point optimizer when factorizing the Newton equation system.

Default FREE

 $\begin{array}{lll} \mathbf{Accepted} & \textit{FREE}, \; \textit{APPMINLOC}, \; \textit{EXPERIMENTAL}, \; \textit{TRY\_GRAPHPAR}, \; \textit{FORCE\_GRAPHPAR}, \; \textit{NONE} \\ & \left( \text{see } \textit{MSKorderingtypee} \right) \end{array}$ 

Example MSK\_putintparam(task, MSK\_IPAR\_INTPNT\_ORDER\_METHOD, MSK\_ORDER\_METHOD\_FREE)

Groups Interior-point method

# MSK\_IPAR\_INTPNT\_PURIFY

Currently not in use.

Default NONE

Accepted NONE, PRIMAL, DUAL, PRIMAL\_DUAL, AUTO (see MSKpurifye)

Example MSK\_putintparam(task, MSK\_IPAR\_INTPNT\_PURIFY, MSK\_PURIFY\_NONE)

Groups Interior-point method

## MSK\_IPAR\_INTPNT\_REGULARIZATION\_USE

Controls whether regularization is allowed.

Default ON

Accepted ON, OFF (see MSKonoffkeye)

Groups Interior-point method

#### MSK\_IPAR\_INTPNT\_SCALING

Controls how the problem is scaled before the interior-point optimizer is used.

Default FREE

Accepted FREE, NONE, MODERATE, AGGRESSIVE (see MSKscalingtypee)

Groups Interior-point method

## MSK\_IPAR\_INTPNT\_SOLVE\_FORM

Controls whether the primal or the dual problem is solved.

Default FREE

Accepted FREE, PRIMAL, DUAL (see MSKsolveforme)

Groups Interior-point method

# MSK\_IPAR\_INTPNT\_STARTING\_POINT

Starting point used by the interior-point optimizer.

Default FREE

Accepted FREE, GUESS, CONSTANT, SATISFY\_BOUNDS (see MSKstartpointtypee)

Groups Interior-point method

# MSK\_IPAR\_LICENSE\_DEBUG

This option is used to turn on debugging of the license manager.

Default OFF

Accepted ON, OFF (see MSKonoffkeye)

Example MSK\_putintparam(task, MSK\_IPAR\_LICENSE\_DEBUG, MSK\_OFF)

Groups License manager

# MSK\_IPAR\_LICENSE\_PAUSE\_TIME

If  $MSK\_IPAR\_LICENSE\_WAIT$  is  $MSK\_ON$  and no license is available, then  $\mathbf{MOSEK}$  sleeps a number of milliseconds between each check of whether a license has become free.

Default 100

**Accepted** [0; 1000000]

Example MSK\_putintparam(task, MSK\_IPAR\_LICENSE\_PAUSE\_TIME, 100)

Groups License manager

# MSK\_IPAR\_LICENSE\_SUPPRESS\_EXPIRE\_WRNS

Controls whether license features expire warnings are suppressed.

Default OFF

Accepted ON, OFF (see MSKonoffkeye)

Groups License manager, Output information

# MSK\_IPAR\_LICENSE\_TRH\_EXPIRY\_WRN

If a license feature expires in a numbers of days less than the value of this parameter then a warning will be issued.

### Default 7

Accepted [0; +inf]

Example MSK\_putintparam(task, MSK\_IPAR\_LICENSE\_TRH\_EXPIRY\_WRN, 7)

Groups License manager, Output information

## MSK\_IPAR\_LICENSE\_WAIT

If all licenses are in use **MOSEK** returns with an error code. However, by turning on this parameter **MOSEK** will wait for an available license.

Default OFF

Accepted ON, OFF (see MSKonoffkeye)

Example MSK\_putintparam(task, MSK\_IPAR\_LICENSE\_WAIT, MSK\_OFF)

Groups Overall solver, Overall system, License manager

## MSK\_IPAR\_LOG

Controls the amount of log information. The value 0 implies that all log information is suppressed. A higher level implies that more information is logged.

Please note that if a task is employed to solve a sequence of optimization problems the value of this parameter is reduced by the value of  $MSK\_IPAR\_LOG\_CUT\_SECOND\_OPT$  for the second and any subsequent optimizations.

## **Default** 10

Accepted [0; +inf]

Example MSK\_putintparam(task, MSK\_IPAR\_LOG, 10)

See also MSK\_IPAR\_LOG\_CUT\_SECOND\_OPT

Groups Output information, Logging

#### MSK\_IPAR\_LOG\_ANA\_PRO

Controls amount of output from the problem analyzer.

Default 1

Accepted [0; +inf]

Example MSK\_putintparam(task, MSK\_IPAR\_LOG\_ANA\_PRO, 1)

Groups Analysis, Logging

# MSK\_IPAR\_LOG\_BI

Controls the amount of output printed by the basis identification procedure. A higher level implies that more information is logged.

# Default 1

Accepted [0; +inf]

Example MSK\_putintparam(task, MSK\_IPAR\_LOG\_BI, 1)

Groups Basis identification, Output information, Logging

# MSK\_IPAR\_LOG\_BI\_FREQ

Controls how frequently the optimizer outputs information about the basis identification and how frequent the user-defined callback function is called.

Default 2500

Accepted [0; +inf]

Example MSK\_putintparam(task, MSK\_IPAR\_LOG\_BI\_FREQ, 2500)

Groups Basis identification, Output information, Logging

### MSK\_IPAR\_LOG\_CHECK\_CONVEXITY

Controls logging in convexity check on quadratic problems. Set to a positive value to turn logging on. If a quadratic coefficient matrix is found to violate the requirement of PSD (NSD) then a list of negative (positive) pivot elements is printed. The absolute value of the pivot elements is also shown.

```
Default 0
```

Accepted [0; +inf]

Example MSK\_putintparam(task, MSK\_IPAR\_LOG\_CHECK\_CONVEXITY, 0)

Groups Data check

## MSK\_IPAR\_LOG\_CUT\_SECOND\_OPT

If a task is employed to solve a sequence of optimization problems, then the value of the log levels is reduced by the value of this parameter. E.g  $MSK\_IPAR\_LOG$  and  $MSK\_IPAR\_LOG\_SIM$  are reduced by the value of this parameter for the second and any subsequent optimizations.

#### Default 1

Accepted  $[0; +\inf]$ 

Example MSK\_putintparam(task, MSK\_IPAR\_LOG\_CUT\_SECOND\_OPT, 1)

See also MSK\_IPAR\_LOG, MSK\_IPAR\_LOG\_INTPNT, MSK\_IPAR\_LOG\_MIO, MSK\_IPAR\_LOG\_SIM

Groups Output information, Logging

#### MSK IPAR LOG EXPAND

Controls the amount of logging when a data item such as the maximum number constrains is expanded.

**Default** 0

**Accepted**  $[0; +\inf]$ 

Example MSK\_putintparam(task, MSK\_IPAR\_LOG\_EXPAND, 0)

Groups Output information, Logging

#### MSK\_IPAR\_LOG\_FEAS\_REPAIR

Controls the amount of output printed when performing feasibility repair. A value higher than one means extensive logging.

# **Default** 1

Accepted [0; +inf]

Example MSK\_putintparam(task, MSK\_IPAR\_LOG\_FEAS\_REPAIR, 1)

Groups Output information, Logging

# MSK\_IPAR\_LOG\_FILE

If turned on, then some log info is printed when a file is written or read.

#### Default 1

Accepted [0; +inf]

Example MSK\_putintparam(task, MSK\_IPAR\_LOG\_FILE, 1)

Groups Data input/output, Output information, Logging

# MSK\_IPAR\_LOG\_INCLUDE\_SUMMARY

If on, then the solution summary will be printed by  $MSK\_optimize$ , so a separate call to  $MSK\_solutionsummary$  is not necessary.

Default OFF

Accepted ON, OFF (see MSKonoffkeye)

Example MSK\_putintparam(task, MSK\_IPAR\_LOG\_INCLUDE\_SUMMARY, MSK\_OFF)

Groups Output information, Logging

# MSK\_IPAR\_LOG\_INFEAS\_ANA

Controls amount of output printed by the infeasibility analyzer procedures. A higher level implies that more information is logged.

# Default 1

Accepted [0; +inf]

Example MSK\_putintparam(task, MSK\_IPAR\_LOG\_INFEAS\_ANA, 1)

# **Groups** Infeasibility report, Output information, Logging

#### MSK\_IPAR\_LOG\_INTPNT

Controls amount of output printed by the interior-point optimizer. A higher level implies that more information is logged.

Default 1

Accepted [0; +inf]

Example MSK\_putintparam(task, MSK\_IPAR\_LOG\_INTPNT, 1)

Groups Interior-point method, Output information, Logging

## MSK\_IPAR\_LOG\_LOCAL\_INFO

Controls whether local identifying information like environment variables, filenames, IP addresses etc. are printed to the log.

Note that this will only affect some functions. Some functions that specifically emit system information will not be affected.

Default ON

Accepted ON, OFF (see MSKonoffkeye)

Example MSK\_putintparam(task, MSK\_IPAR\_LOG\_LOCAL\_INFO, MSK\_ON)

**Groups** Output information, Logging

#### MSK\_IPAR\_LOG\_MIO

Controls the log level for the mixed-integer optimizer. A higher level implies that more information is logged.

Default 4

Accepted [0; +inf]

Example MSK\_putintparam(task, MSK\_IPAR\_LOG\_MIO, 4)

Groups Mixed-integer optimization, Output information, Logging

### MSK\_IPAR\_LOG\_MIO\_FREQ

Controls how frequent the mixed-integer optimizer prints the log line. It will print line every time  $MSK\_IPAR\_LOG\_MIO\_FREQ$  relaxations have been solved.

Default 10

Accepted  $[-\inf; +\inf]$ 

Example MSK\_putintparam(task, MSK\_IPAR\_LOG\_MIO\_FREQ, 10)

Groups Mixed-integer optimization, Output information, Logging

# MSK\_IPAR\_LOG\_ORDER

If turned on, then factor lines are added to the log.

Default 1

Accepted [0; +inf]

Example MSK\_putintparam(task, MSK\_IPAR\_LOG\_ORDER, 1)

Groups Output information, Logging

# MSK\_IPAR\_LOG\_PRESOLVE

Controls amount of output printed by the presolve procedure. A higher level implies that more information is logged.

Default 1

Accepted [0; +inf]

Example MSK\_putintparam(task, MSK\_IPAR\_LOG\_PRESOLVE, 1)

Groups Logging

# MSK\_IPAR\_LOG\_RESPONSE

Controls amount of output printed when response codes are reported. A higher level implies that more information is logged.

```
Default 0
```

Accepted [0; +inf]

Example MSK\_putintparam(task, MSK\_IPAR\_LOG\_RESPONSE, 0)

**Groups** Output information, Logging

#### MSK\_IPAR\_LOG\_SENSITIVITY

Controls the amount of logging during the sensitivity analysis.

- 0. Means no logging information is produced.
- 1. Timing information is printed.
- 2. Sensitivity results are printed.

#### Default 1

Accepted [0; +inf]

Example MSK\_putintparam(task, MSK\_IPAR\_LOG\_SENSITIVITY, 1)

**Groups** Output information, Logging

#### MSK IPAR LOG SENSITIVITY OPT

Controls the amount of logging from the optimizers employed during the sensitivity analysis. 0 means no logging information is produced.

Default 0

Accepted [0; +inf]

Example MSK\_putintparam(task, MSK\_IPAR\_LOG\_SENSITIVITY\_OPT, 0)

Groups Output information, Logging

# MSK\_IPAR\_LOG\_SIM

Controls amount of output printed by the simplex optimizer. A higher level implies that more information is logged.

Default 4

Accepted [0; +inf]

Example MSK\_putintparam(task, MSK\_IPAR\_LOG\_SIM, 4)

**Groups** Simplex optimizer, Output information, Logging

#### MSK\_IPAR\_LOG\_SIM\_FREQ

Controls how frequent the simplex optimizer outputs information about the optimization and how frequent the user-defined callback function is called.

Default 1000

Accepted [0; +inf]

Example MSK\_putintparam(task, MSK\_IPAR\_LOG\_SIM\_FREQ, 1000)

**Groups** Simplex optimizer, Output information, Logging

# MSK\_IPAR\_LOG\_SIM\_MINOR

Currently not in use.

Default 1

Accepted [0; +inf]

Example MSK\_putintparam(task, MSK\_IPAR\_LOG\_SIM\_MINOR, 1)

Groups Simplex optimizer, Output information

### MSK\_IPAR\_LOG\_STORAGE

When turned on, MOSEK prints messages regarding the storage usage and allocation.

**Default** 0

Accepted [0; +inf]

Example MSK\_putintparam(task, MSK\_IPAR\_LOG\_STORAGE, 0)

Groups Output information, Overall system, Logging

#### MSK\_IPAR\_MAX\_NUM\_WARNINGS

Each warning is shown a limited number of times controlled by this parameter. A negative value is identical to infinite number of times.

Default 10

Accepted  $[-\inf; +\inf]$ 

Example MSK\_putintparam(task, MSK\_IPAR\_MAX\_NUM\_WARNINGS, 10)

Groups Output information

#### MSK\_IPAR\_MIO\_BRANCH\_DIR

Controls whether the mixed-integer optimizer is branching up or down by default.

Default FREE

Accepted FREE, UP, DOWN, NEAR, FAR, ROOT\_LP, GUIDED, PSEUDOCOST (see MSKbranchdire)

Example MSK\_putintparam(task, MSK\_IPAR\_MIO\_BRANCH\_DIR, MSK\_BRANCH\_DIR\_FREE)

Groups Mixed-integer optimization

# MSK\_IPAR\_MIO\_CONIC\_OUTER\_APPROXIMATION

If this option is turned on outer approximation is used when solving relaxations of conic problems; otherwise interior point is used.

Default OFF

Accepted ON, OFF (see MSKonoffkeye)

**Groups** Mixed-integer optimization

## MSK\_IPAR\_MIO\_CUT\_CLIQUE

Controls whether clique cuts should be generated.

Default ON

Accepted ON, OFF (see MSKonoffkeye)

Example MSK\_putintparam(task, MSK\_IPAR\_MIO\_CUT\_CLIQUE, MSK\_ON)

 ${\bf Groups}\ \textit{Mixed-integer optimization}$ 

# MSK\_IPAR\_MIO\_CUT\_CMIR

Controls whether mixed integer rounding cuts should be generated.

Default ON

Accepted ON, OFF (see MSKonoffkeye)

Example MSK\_putintparam(task, MSK\_IPAR\_MIO\_CUT\_CMIR, MSK\_ON)

**Groups** Mixed-integer optimization

# MSK\_IPAR\_MIO\_CUT\_GMI

Controls whether GMI cuts should be generated.

Default ON

Accepted ON, OFF (see MSKonoffkeye)

Example MSK\_putintparam(task, MSK\_IPAR\_MIO\_CUT\_GMI, MSK\_ON)

**Groups** Mixed-integer optimization

# MSK\_IPAR\_MIO\_CUT\_IMPLIED\_BOUND

Controls whether implied bound cuts should be generated.

Default OFF

Accepted ON, OFF (see MSKonoffkeye)

Example MSK\_putintparam(task, MSK\_IPAR\_MIO\_CUT\_IMPLIED\_BOUND, MSK\_OFF)

**Groups** Mixed-integer optimization

## MSK\_IPAR\_MIO\_CUT\_KNAPSACK\_COVER

Controls whether knapsack cover cuts should be generated.

Default OFF

Accepted ON, OFF (see MSKonoffkeye)

Example MSK\_putintparam(task, MSK\_IPAR\_MIO\_CUT\_KNAPSACK\_COVER, MSK\_OFF)

**Groups** Mixed-integer optimization

## MSK\_IPAR\_MIO\_CUT\_SELECTION\_LEVEL

Controls how aggressively generated cuts are selected to be included in the relaxation.

- $\bullet$  -1. The optimizer chooses the level of cut selection
- 0. Generated cuts less likely to be added to the relaxation
- 1. Cuts are more aggressively selected to be included in the relaxation

Default -1

Accepted [-1; +1]

Example MSK\_putintparam(task, MSK\_IPAR\_MIO\_CUT\_SELECTION\_LEVEL, -1)

**Groups** Mixed-integer optimization

#### MSK\_IPAR\_MIO\_FEASPUMP\_LEVEL

Controls the way the Feasibility Pump heuristic is employed by the mixed-integer optimizer.

- $\bullet$  -1. The optimizer chooses how the Feasibility Pump is used
- 0. The Feasibility Pump is disabled
- 1. The Feasibility Pump is enabled with an effort to improve solution quality
- 2. The Feasibility Pump is enabled with an effort to reach feasibility early

Default -1

Accepted [-1; 2]

Example MSK\_putintparam(task, MSK\_IPAR\_MIO\_FEASPUMP\_LEVEL, -1)

**Groups** Mixed-integer optimization

# MSK\_IPAR\_MIO\_HEURISTIC\_LEVEL

Controls the heuristic employed by the mixed-integer optimizer to locate an initial good integer feasible solution. A value of zero means the heuristic is not used at all. A larger value than 0 means that a gradually more sophisticated heuristic is used which is computationally more expensive. A negative value implies that the optimizer chooses the heuristic. Normally a value around 3 to 5 should be optimal.

Default -1

Accepted [-inf; +inf]

Example MSK\_putintparam(task, MSK\_IPAR\_MIO\_HEURISTIC\_LEVEL, -1)

**Groups** Mixed-integer optimization

# MSK\_IPAR\_MIO\_MAX\_NUM\_BRANCHES

Maximum number of branches allowed during the branch and bound search. A negative value means infinite.

Default -1

Accepted  $[-\inf; +\inf]$ 

Example MSK\_putintparam(task, MSK\_IPAR\_MIO\_MAX\_NUM\_BRANCHES, -1)

**Groups** Mixed-integer optimization, Termination criteria

# MSK\_IPAR\_MIO\_MAX\_NUM\_RELAXS

Maximum number of relaxations allowed during the branch and bound search. A negative value means infinite.

Default -1

```
Accepted [-\inf; +\inf]
```

Example MSK\_putintparam(task, MSK\_IPAR\_MIO\_MAX\_NUM\_RELAXS, -1)

**Groups** Mixed-integer optimization

## MSK\_IPAR\_MIO\_MAX\_NUM\_ROOT\_CUT\_ROUNDS

Maximum number of cut separation rounds at the root node.

Default 100

Accepted [0; +inf]

Groups Mixed-integer optimization, Termination criteria

## MSK\_IPAR\_MIO\_MAX\_NUM\_SOLUTIONS

The mixed-integer optimizer can be terminated after a certain number of different feasible solutions has been located. If this parameter has the value n > 0, then the mixed-integer optimizer will be terminated when n feasible solutions have been located.

Default -1

Accepted  $[-\inf; +\inf]$ 

Example MSK\_putintparam(task, MSK\_IPAR\_MIO\_MAX\_NUM\_SOLUTIONS, -1)

Groups Mixed-integer optimization, Termination criteria

# MSK\_IPAR\_MIO\_MODE

Controls whether the optimizer includes the integer restrictions when solving a (mixed) integer optimization problem.

Default SATISFIED

Accepted IGNORED, SATISFIED (see MSKmiomodee)

Groups Overall solver

## MSK\_IPAR\_MIO\_NODE\_OPTIMIZER

Controls which optimizer is employed at the non-root nodes in the mixed-integer optimizer.

Default FREE

Accepted FREE, INTPNT, CONIC, PRIMAL\_SIMPLEX, DUAL\_SIMPLEX, FREE\_SIMPLEX, MIXED\_INT (see MSKoptimizertypee)

**Groups** Mixed-integer optimization

# MSK\_IPAR\_MIO\_NODE\_SELECTION

Controls the node selection strategy employed by the mixed-integer optimizer.

Default FREE

Accepted FREE, FIRST, BEST, PSEUDO (see MSKmionodeseltypee)

**Groups** Mixed-integer optimization

# MSK\_IPAR\_MIO\_PERSPECTIVE\_REFORMULATE

Enables or disables perspective reformulation in presolve.

Default ON

Accepted ON, OFF (see MSKonoffkeye)

# **Groups** Mixed-integer optimization

## MSK\_IPAR\_MIO\_PROBING\_LEVEL

Controls the amount of probing employed by the mixed-integer optimizer in presolve.

- $\bullet$  -1. The optimizer chooses the level of probing employed
- 0. Probing is disabled
- 1. A low amount of probing is employed
- 2. A medium amount of probing is employed
- 3. A high amount of probing is employed

```
Default -1
```

Accepted [-1; 3]

Example MSK\_putintparam(task, MSK\_IPAR\_MIO\_PROBING\_LEVEL, -1)

**Groups** Mixed-integer optimization

## MSK\_IPAR\_MIO\_PROPAGATE\_OBJECTIVE\_CONSTRAINT

Use objective domain propagation.

Default OFF

Accepted ON, OFF (see MSKonoffkeye)

**Groups** Mixed-integer optimization

#### MSK\_IPAR\_MIO\_RINS\_MAX\_NODES

Controls the maximum number of nodes allowed in each call to the RINS heuristic. The default value of -1 means that the value is determined automatically. A value of zero turns off the heuristic.

Default -1

Accepted  $[-1; +\inf]$ 

Example MSK\_putintparam(task, MSK\_IPAR\_MIO\_RINS\_MAX\_NODES, -1)

**Groups** Mixed-integer optimization

# MSK\_IPAR\_MIO\_ROOT\_OPTIMIZER

Controls which optimizer is employed at the root node in the mixed-integer optimizer.

Default FREE

Accepted FREE, INTPNT, CONIC, PRIMAL\_SIMPLEX, DUAL\_SIMPLEX, FREE\_SIMPLEX, MIXED\_INT (see MSKoptimizertypee)

**Groups** Mixed-integer optimization

# MSK\_IPAR\_MIO\_ROOT\_REPEAT\_PRESOLVE\_LEVEL

Controls whether presolve can be repeated at root node.

- $\bullet$  -1. The optimizer chooses whether presolve is repeated
- 0. Never repeat presolve
- 1. Always repeat presolve

Default -1

Accepted [-1; 1]

Example MSK\_putintparam(task, MSK\_IPAR\_MIO\_ROOT\_REPEAT\_PRESOLVE\_LEVEL, -1)

**Groups** Mixed-integer optimization

## MSK\_IPAR\_MIO\_SEED

Sets the random seed used for randomization in the mixed integer optimizer. Selecting a different seed can change the path the optimizer takes to the optimal solution.

Default 42

Accepted [0; +inf]

Example MSK\_putintparam(task, MSK\_IPAR\_MIO\_SEED, 42)

**Groups** Mixed-integer optimization

#### MSK\_IPAR\_MIO\_VB\_DETECTION\_LEVEL

Controls how much effort is put into detecting variable bounds.

- $\bullet$  -1. The optimizer chooses
- 0. No variable bounds are detected
- 1. Only detect variable bounds that are directly represented in the problem
- 2. Detect variable bounds in probing

Default -1

Accepted [-1; +2]

Example MSK\_putintparam(task, MSK\_IPAR\_MIO\_VB\_DETECTION\_LEVEL, -1)

**Groups** Mixed-integer optimization

## MSK\_IPAR\_MT\_SPINCOUNT

Set the number of iterations to spin before sleeping.

Default 0

**Accepted** [0; 1000000000]

Example MSK\_putintparam(task, MSK\_IPAR\_MT\_SPINCOUNT, 0)

Groups Overall system

# MSK\_IPAR\_NUM\_THREADS

Controls the number of threads employed by the optimizer. If set to 0 the number of threads used will be equal to the number of cores detected on the machine.

If using the conic optimizer, the value of this parameter set at first optimization remains constant through the lifetime of the process.  $\mathbf{MOSEK}$  will allocate a thread pool of given size, and changing the parameter value later will have no effect. It will, however, remain possible to demand single-threaded execution by setting  $MSK\_IPAR\_INTPNT\_MULTI\_THREAD$ .

For the mixed-integer optimizer and interior-point linear optimizer there is no such restriction.

On the linuxaarch64 platform the settings only applies to the mixed-integer optimizer; the continuous optimizers are single-threaded.

**Default** 0

**Accepted**  $[0; +\inf]$ 

Groups Overall system

# MSK\_IPAR\_OPF\_WRITE\_HEADER

Write a text header with date and MOSEK version in an OPF file.

Default ON

Accepted ON, OFF (see MSKonoffkeye)

Example MSK\_putintparam(task, MSK\_IPAR\_OPF\_WRITE\_HEADER, MSK\_ON)

Groups Data input/output

### MSK\_IPAR\_OPF\_WRITE\_HINTS

Write a hint section with problem dimensions in the beginning of an OPF file.

Default ON

```
Accepted ON, OFF (see MSKonoffkeye)
         Example MSK_putintparam(task, MSK_IPAR_OPF_WRITE_HINTS, MSK_ON)
         Groups Data input/output
MSK_IPAR_OPF_WRITE_LINE_LENGTH
     Aim to keep lines in OPF files not much longer than this.
         Default 80
         Accepted [0; +\inf]
         Example MSK_putintparam(task, MSK_IPAR_OPF_WRITE_LINE_LENGTH, 80)
         Groups Data input/output
MSK_IPAR_OPF_WRITE_PARAMETERS
     Write a parameter section in an OPF file.
         Default OFF
         Accepted ON, OFF (see MSKonoffkeye)
         Example MSK_putintparam(task, MSK_IPAR_OPF_WRITE_PARAMETERS, MSK_OFF)
         Groups Data input/output
MSK_IPAR_OPF_WRITE_PROBLEM
     Write objective, constraints, bounds etc. to an OPF file.
         Default ON
         Accepted ON, OFF (see MSKonoffkeye)
         Example MSK_putintparam(task, MSK_IPAR_OPF_WRITE_PROBLEM, MSK_ON)
         Groups Data input/output
MSK_IPAR_OPF_WRITE_SOL_BAS
     If MSK_IPAR_OPF_WRITE_SOLUTIONS is MSK_ON and a basic solution is defined, include the basic
     solution in OPF files.
         Default ON
         Accepted ON, OFF (see MSKonoffkeye)
         Example MSK_putintparam(task, MSK_IPAR_OPF_WRITE_SOL_BAS, MSK_ON)
         Groups Data input/output
MSK_IPAR_OPF_WRITE_SOL_ITG
     If MSK_IPAR_OPF_WRITE_SOLUTIONS is MSK_ON and an integer solution is defined, write the integer
     solution in OPF files.
         Default ON
         Accepted ON, OFF (see MSKonoffkeye)
         Example MSK_putintparam(task, MSK_IPAR_OPF_WRITE_SOL_ITG, MSK_ON)
         Groups Data input/output
MSK_IPAR_OPF_WRITE_SOL_ITR
     If MSK_IPAR_OPF_WRITE_SOLUTIONS is MSK_ON and an interior solution is defined, write the interior
     solution in OPF files.
         Default ON
         Accepted ON, OFF (see MSKonoffkeye)
         Example MSK_putintparam(task, MSK_IPAR_OPF_WRITE_SOL_ITR, MSK_ON)
         Groups Data input/output
MSK_IPAR_OPF_WRITE_SOLUTIONS
```

Enable inclusion of solutions in the OPF files.

Default OFF

Accepted ON, OFF (see MSKonoffkeye)

```
Example MSK_putintparam(task, MSK_IPAR_OPF_WRITE_SOLUTIONS, MSK_OFF) Groups Data\ input/output
```

#### MSK\_IPAR\_OPTIMIZER

The parameter controls which optimizer is used to optimize the task.

Default FREE

Accepted FREE, INTPNT, CONIC, PRIMAL\_SIMPLEX, DUAL\_SIMPLEX, FREE\_SIMPLEX, MIXED\_INT (see MSKoptimizertypee)

Example MSK\_putintparam(task, MSK\_IPAR\_OPTIMIZER, MSK\_OPTIMIZER\_FREE)

Groups Overall solver

## MSK\_IPAR\_PARAM\_READ\_CASE\_NAME

If turned on, then names in the parameter file are case sensitive.

Default ON

Accepted ON, OFF (see MSKonoffkeye)

Example MSK\_putintparam(task, MSK\_IPAR\_PARAM\_READ\_CASE\_NAME, MSK\_ON)

Groups Data input/output

# MSK\_IPAR\_PARAM\_READ\_IGN\_ERROR

If turned on, then errors in parameter settings is ignored.

Default OFF

Accepted ON, OFF (see MSKonoffkeye)

Example MSK\_putintparam(task, MSK\_IPAR\_PARAM\_READ\_IGN\_ERROR, MSK\_OFF)

Groups Data input/output

## MSK\_IPAR\_PRESOLVE\_ELIMINATOR\_MAX\_FILL

Controls the maximum amount of fill-in that can be created by one pivot in the elimination phase of the presolve. A negative value means the parameter value is selected automatically.

Default -1

Accepted  $[-\inf; +\inf]$ 

Groups Presolve

# MSK\_IPAR\_PRESOLVE\_ELIMINATOR\_MAX\_NUM\_TRIES

Control the maximum number of times the eliminator is tried. A negative value implies  $\mathbf{MOSEK}$  decides.

Default -1

Accepted  $[-\inf; +\inf]$ 

Groups Presolve

# MSK\_IPAR\_PRESOLVE\_LEVEL

Currently not used.

Default -1

Accepted  $[-\inf; +\inf]$ 

Example MSK\_putintparam(task, MSK\_IPAR\_PRESOLVE\_LEVEL, -1)

Groups Overall solver, Presolve

# MSK\_IPAR\_PRESOLVE\_LINDEP\_ABS\_WORK\_TRH

Controls linear dependency check in presolve. The linear dependency check is potentially computationally expensive.

```
Default 100
```

Accepted  $[-\inf; +\inf]$ 

Groups Presolve

## MSK\_IPAR\_PRESOLVE\_LINDEP\_REL\_WORK\_TRH

Controls linear dependency check in presolve. The linear dependency check is potentially computationally expensive.

Default 100

Accepted  $[-\inf; +\inf]$ 

Groups Presolve

# MSK\_IPAR\_PRESOLVE\_LINDEP\_USE

Controls whether the linear constraints are checked for linear dependencies.

Default ON

Accepted ON, OFF (see MSKonoffkeye)

Example MSK\_putintparam(task, MSK\_IPAR\_PRESOLVE\_LINDEP\_USE, MSK\_ON)

Groups Presolve

## MSK\_IPAR\_PRESOLVE\_MAX\_NUM\_PASS

Control the maximum number of times presolve passes over the problem. A negative value implies **MOSEK** decides.

Default -1

Accepted  $[-\inf; +\inf]$ 

Example MSK\_putintparam(task, MSK\_IPAR\_PRESOLVE\_MAX\_NUM\_PASS, -1)

Groups Presolve

# MSK\_IPAR\_PRESOLVE\_MAX\_NUM\_REDUCTIONS

Controls the maximum number of reductions performed by the presolve. The value of the parameter is normally only changed in connection with debugging. A negative value implies that an infinite number of reductions are allowed.

Default -1

Accepted  $[-\inf; +\inf]$ 

Example MSK\_putintparam(task, MSK\_IPAR\_PRESOLVE\_MAX\_NUM\_REDUCTIONS, -1)

Groups Overall solver, Presolve

## MSK\_IPAR\_PRESOLVE\_USE

Controls whether the presolve is applied to a problem before it is optimized.

Default FREE

Accepted OFF, ON, FREE (see MSKpresolvemodee)

Example MSK\_putintparam(task, MSK\_IPAR\_PRESOLVE\_USE,

MSK\_PRESOLVE\_MODE\_FREE)

Groups Overall solver, Presolve

# MSK\_IPAR\_PRIMAL\_REPAIR\_OPTIMIZER

Controls which optimizer that is used to find the optimal repair.

Default FREE

Accepted FREE, INTPNT, CONIC, PRIMAL\_SIMPLEX, DUAL\_SIMPLEX, FREE\_SIMPLEX, MIXED\_INT (see MSKoptimizertypee)

# Groups Overall solver

## MSK\_IPAR\_PTF\_WRITE\_TRANSFORM

If  $MSK\_IPAR\_PTF\_WRITE\_TRANSFORM$  is  $MSK\_ON$ , constraint blocks with identifiable conic slacks are transformed into conic constraints and the slacks are eliminated.

Default ON

Accepted ON, OFF (see MSKonoffkeye)

Example MSK\_putintparam(task, MSK\_IPAR\_PTF\_WRITE\_TRANSFORM, MSK\_ON)

Groups Data input/output

#### MSK\_IPAR\_READ\_DEBUG

Turns on additional debugging information when reading files.

Default OFF

Accepted ON, OFF (see MSKonoffkeye)

Example MSK\_putintparam(task, MSK\_IPAR\_READ\_DEBUG, MSK\_OFF)

Groups Data input/output

#### MSK\_IPAR\_READ\_KEEP\_FREE\_CON

Controls whether the free constraints are included in the problem.

Default OFF

### Accepted

- *ON*: The free constraints are kept.
- OFF: The free constraints are discarded.

Example MSK\_putintparam(task, MSK\_IPAR\_READ\_KEEP\_FREE\_CON, MSK\_OFF)

Groups Data input/output

# MSK\_IPAR\_READ\_LP\_DROP\_NEW\_VARS\_IN\_BOU

If this option is turned on, **MOSEK** will drop variables that are defined for the first time in the bounds section.

Default OFF

Accepted ON, OFF (see MSKonoffkeye)

Groups Data input/output

## MSK\_IPAR\_READ\_LP\_QUOTED\_NAMES

If a name is in quotes when reading an LP file, the quotes will be removed.

Default ON

Accepted ON, OFF (see MSKonoffkeye)

 $\mathbf{Example} \ \mathtt{MSK\_putintparam}(\mathtt{task}, \ \mathtt{MSK\_IPAR\_READ\_LP\_QUOTED\_NAMES}, \ \mathtt{MSK\_ON})$ 

 ${\bf Groups}\ \textit{Data\ input/output}$ 

# MSK\_IPAR\_READ\_MPS\_FORMAT

Controls how strictly the MPS file reader interprets the MPS format.

Default FREE

Accepted STRICT, RELAXED, FREE, CPLEX (see MSKmpsformate)

Example MSK\_putintparam(task, MSK\_IPAR\_READ\_MPS\_FORMAT,

MSK\_MPS\_FORMAT\_FREE)

Groups Data input/output

# MSK\_IPAR\_READ\_MPS\_WIDTH

Controls the maximal number of characters allowed in one line of the MPS file.

Default 1024

```
Accepted [80; +inf]
Example MSK_putintparam(task, MSK_IPAR_READ_MPS_WIDTH, 1024)
Groups Data input/output
```

## MSK\_IPAR\_READ\_TASK\_IGNORE\_PARAM

Controls whether **MOSEK** should ignore the parameter setting defined in the task file and use the default parameter setting instead.

Default OFF
Accepted ON, OFF (see MSKonoffkeye)
Example MSK\_putintparam(task, MSK\_IPAR\_READ\_TASK\_IGNORE\_PARAM, MSK\_OFF)
Groups Data input/output

## MSK\_IPAR\_REMOVE\_UNUSED\_SOLUTIONS

Removes unused solutions before the optimization is performed.

```
Default OFF
Accepted ON, OFF (see MSKonoffkeye)
Example MSK_putintparam(task, MSK_IPAR_REMOVE_UNUSED_SOLUTIONS, MSK_OFF)
Groups Overall system
```

# MSK\_IPAR\_SENSITIVITY\_ALL

If set to  $MSK\_ON$ , then  $MSK\_sensitivityreport$  analyzes all bounds and variables instead of reading a specification from the file.

```
Default OFF

Accepted ON, OFF (see MSKonoffkeye)

Example MSK_putintparam(task, MSK_IPAR_SENSITIVITY_ALL, MSK_OFF)

Groups Overall solver
```

# MSK\_IPAR\_SENSITIVITY\_OPTIMIZER

Controls which optimizer is used for optimal partition sensitivity analysis.

```
Default FREE_SIMPLEX

Accepted FREE, INTPNT, CONIC, PRIMAL_SIMPLEX, DUAL_SIMPLEX, FREE_SIMPLEX,
    MIXED_INT (see MSKoptimizertypee)

Example MSK_putintparam(task, MSK_IPAR_SENSITIVITY_OPTIMIZER,
    MSK_OPTIMIZER_FREE_SIMPLEX)

Groups Overall solver, Simplex optimizer
```

# MSK\_IPAR\_SENSITIVITY\_TYPE

Controls which type of sensitivity analysis is to be performed.

# MSK\_IPAR\_SIM\_BASIS\_FACTOR\_USE

Controls whether an LU factorization of the basis is used in a hot-start. Forcing a refactorization sometimes improves the stability of the simplex optimizers, but in most cases there is a performance penalty.

```
Default ON
Accepted ON, OFF (see MSKonoffkeye)
Example MSK_putintparam(task, MSK_IPAR_SIM_BASIS_FACTOR_USE, MSK_ON)
Groups Simplex optimizer
```

#### MSK\_IPAR\_SIM\_DEGEN

Controls how aggressively degeneration is handled.

Default FREE

Accepted NONE, FREE, AGGRESSIVE, MODERATE, MINIMUM (see MSKsimdegene)

Example MSK\_putintparam(task, MSK\_IPAR\_SIM\_DEGEN, MSK\_SIM\_DEGEN\_FREE)

Groups Simplex optimizer

### MSK\_IPAR\_SIM\_DUAL\_CRASH

Controls whether crashing is performed in the dual simplex optimizer. If this parameter is set to x, then a crash will be performed if a basis consists of more than  $(100 - x) \mod f_v$  entries, where  $f_v$  is the number of fixed variables.

Default 90

Accepted [0; +inf]

Example MSK\_putintparam(task, MSK\_IPAR\_SIM\_DUAL\_CRASH, 90)

Groups Dual simplex

#### MSK\_IPAR\_SIM\_DUAL\_PHASEONE\_METHOD

An experimental feature.

Default 0

**Accepted** [0; 10]

Example MSK\_putintparam(task, MSK\_IPAR\_SIM\_DUAL\_PHASEONE\_METHOD, 0)

Groups Simplex optimizer

# MSK\_IPAR\_SIM\_DUAL\_RESTRICT\_SELECTION

The dual simplex optimizer can use a so-called restricted selection/pricing strategy to choose the outgoing variable. Hence, if restricted selection is applied, then the dual simplex optimizer first choose a subset of all the potential outgoing variables. Next, for some time it will choose the outgoing variable only among the subset. From time to time the subset is redefined. A larger value of this parameter implies that the optimizer will be more aggressive in its restriction strategy, i.e. a value of 0 implies that the restriction strategy is not applied at all.

Default 50

**Accepted** [0; 100]

Example MSK\_putintparam(task, MSK\_IPAR\_SIM\_DUAL\_RESTRICT\_SELECTION, 50)

Groups Dual simplex

### MSK\_IPAR\_SIM\_DUAL\_SELECTION

Controls the choice of the incoming variable, known as the selection strategy, in the dual simplex optimizer.

Default FREE

Accepted FREE, FULL, ASE, DEVEX, SE, PARTIAL (see MSKsimseltypee)

Groups Dual simplex

# MSK\_IPAR\_SIM\_EXPLOIT\_DUPVEC

Controls if the simplex optimizers are allowed to exploit duplicated columns.

Default OFF

Accepted ON, OFF, FREE (see MSKsimdupvece)

Groups Simplex optimizer

# MSK\_IPAR\_SIM\_HOTSTART

Controls the type of hot-start that the simplex optimizer perform.

```
Default FREE
```

Accepted NONE, FREE, STATUS\_KEYS (see MSKsimhotstarte)

Groups Simplex optimizer

#### MSK\_IPAR\_SIM\_HOTSTART\_LU

Determines if the simplex optimizer should exploit the initial factorization.

#### Default ON

# Accepted

- *ON*: Factorization is reused if possible.
- *OFF*: Factorization is recomputed.

Example MSK\_putintparam(task, MSK\_IPAR\_SIM\_HOTSTART\_LU, MSK\_ON)

Groups Simplex optimizer

## MSK\_IPAR\_SIM\_MAX\_ITERATIONS

Maximum number of iterations that can be used by a simplex optimizer.

**Default** 10000000

Accepted [0; +inf]

Example MSK\_putintparam(task, MSK\_IPAR\_SIM\_MAX\_ITERATIONS, 10000000)

Groups Simplex optimizer, Termination criteria

## MSK\_IPAR\_SIM\_MAX\_NUM\_SETBACKS

Controls how many set-backs are allowed within a simplex optimizer. A set-back is an event where the optimizer moves in the wrong direction. This is impossible in theory but may happen due to numerical problems.

Default 250

**Accepted**  $[0; +\inf]$ 

Example MSK\_putintparam(task, MSK\_IPAR\_SIM\_MAX\_NUM\_SETBACKS, 250)

Groups Simplex optimizer

# MSK\_IPAR\_SIM\_NON\_SINGULAR

Controls if the simplex optimizer ensures a non-singular basis, if possible.

Default ON

Accepted ON, OFF (see MSKonoffkeye)

Example MSK\_putintparam(task, MSK\_IPAR\_SIM\_NON\_SINGULAR, MSK\_ON)

Groups Simplex optimizer

# MSK\_IPAR\_SIM\_PRIMAL\_CRASH

Controls whether crashing is performed in the primal simplex optimizer. In general, if a basis consists of more than (100-this parameter value)% fixed variables, then a crash will be performed.

Default 90

Accepted [0; +inf]

Example MSK\_putintparam(task, MSK\_IPAR\_SIM\_PRIMAL\_CRASH, 90)

Groups Primal simplex

# MSK\_IPAR\_SIM\_PRIMAL\_PHASEONE\_METHOD

An experimental feature.

Default 0

**Accepted** [0; 10]

Example MSK\_putintparam(task, MSK\_IPAR\_SIM\_PRIMAL\_PHASEONE\_METHOD, 0)

Groups Simplex optimizer

## MSK\_IPAR\_SIM\_PRIMAL\_RESTRICT\_SELECTION

The primal simplex optimizer can use a so-called restricted selection/pricing strategy to choose the outgoing variable. Hence, if restricted selection is applied, then the primal simplex optimizer first choose a subset of all the potential incoming variables. Next, for some time it will choose the incoming variable only among the subset. From time to time the subset is redefined. A larger value of this parameter implies that the optimizer will be more aggressive in its restriction strategy, i.e. a value of 0 implies that the restriction strategy is not applied at all.

Default 50
Accepted [0; 100]
Example MSK\_putintparam(task, MSK\_IPAR\_SIM\_PRIMAL\_RESTRICT\_SELECTION, 50)
Groups Primal simplex

# MSK\_IPAR\_SIM\_PRIMAL\_SELECTION

Controls the choice of the incoming variable, known as the selection strategy, in the primal simplex optimizer.

## MSK\_IPAR\_SIM\_REFACTOR\_FREQ

Controls how frequent the basis is refactorized. The value 0 means that the optimizer determines the best point of refactorization. It is strongly recommended NOT to change this parameter.

Default 0
Accepted [0; +inf]
Example MSK\_putintparam(task, MSK\_IPAR\_SIM\_REFACTOR\_FREQ, 0)
Groups Simplex optimizer

# MSK\_IPAR\_SIM\_REFORMULATION

Controls if the simplex optimizers are allowed to reformulate the problem.

#### MSK\_IPAR\_SIM\_SAVE\_LU

Controls if the LU factorization stored should be replaced with the LU factorization corresponding to the initial basis.

Default OFF
Accepted ON, OFF (see MSKonoffkeye)
Example MSK\_putintparam(task, MSK\_IPAR\_SIM\_SAVE\_LU, MSK\_OFF)
Groups Simplex optimizer

# MSK\_IPAR\_SIM\_SCALING

Controls how much effort is used in scaling the problem before a simplex optimizer is used.

Default FREE
Accepted FREE, NONE, MODERATE, AGGRESSIVE (see MSKscalingtypee)
Example MSK\_putintparam(task, MSK\_IPAR\_SIM\_SCALING, MSK\_SCALING\_FREE)
Groups Simplex optimizer

#### MSK\_IPAR\_SIM\_SCALING\_METHOD

Controls how the problem is scaled before a simplex optimizer is used.

Default POW2

Accepted POW2, FREE (see MSKscalingmethode)

Groups Simplex optimizer

## MSK\_IPAR\_SIM\_SEED

Sets the random seed used for randomization in the simplex optimizers.

Default 23456

**Accepted** [0; 32749]

Example MSK\_putintparam(task, MSK\_IPAR\_SIM\_SEED, 23456)

Groups Simplex optimizer

#### MSK\_IPAR\_SIM\_SOLVE\_FORM

Controls whether the primal or the dual problem is solved by the primal-/dual-simplex optimizer.

Default FREE

Accepted FREE, PRIMAL, DUAL (see MSKsolveforme)

Example MSK\_putintparam(task, MSK\_IPAR\_SIM\_SOLVE\_FORM, MSK\_SOLVE\_FREE)

Groups Simplex optimizer

## MSK\_IPAR\_SIM\_STABILITY\_PRIORITY

Controls how high priority the numerical stability should be given.

Default 50

**Accepted** [0; 100]

Example MSK\_putintparam(task, MSK\_IPAR\_SIM\_STABILITY\_PRIORITY, 50)

Groups Simplex optimizer

# MSK\_IPAR\_SIM\_SWITCH\_OPTIMIZER

The simplex optimizer sometimes chooses to solve the dual problem instead of the primal problem. This implies that if you have chosen to use the dual simplex optimizer and the problem is dualized, then it actually makes sense to use the primal simplex optimizer instead. If this parameter is on and the problem is dualized and furthermore the simplex optimizer is chosen to be the primal (dual) one, then it is switched to the dual (primal).

Default OFF

Accepted ON, OFF (see MSKonoffkeye)

Example MSK\_putintparam(task, MSK\_IPAR\_SIM\_SWITCH\_OPTIMIZER, MSK\_OFF)

 ${\bf Groups} \ \ Simplex \ optimizer$ 

# MSK\_IPAR\_SOL\_FILTER\_KEEP\_BASIC

If turned on, then basic and super basic constraints and variables are written to the solution file independent of the filter setting.

Default OFF

Accepted ON, OFF (see MSKonoffkeye)

Example MSK\_putintparam(task, MSK\_IPAR\_SOL\_FILTER\_KEEP\_BASIC, MSK\_OFF)

Groups Solution input/output

# MSK\_IPAR\_SOL\_FILTER\_KEEP\_RANGED

If turned on, then ranged constraints and variables are written to the solution file independent of the filter setting.

Default OFF

Accepted ON, OFF (see MSKonoffkeye)

```
Example MSK_putintparam(task, MSK_IPAR_SOL_FILTER_KEEP_RANGED, MSK_OFF)
Groups Solution input/output
```

#### MSK\_IPAR\_SOL\_READ\_NAME\_WIDTH

When a solution is read by **MOSEK** and some constraint, variable or cone names contain blanks, then a maximum name width much be specified. A negative value implies that no name contain blanks

#### Default -1

Accepted  $[-\inf; +\inf]$ 

Example MSK\_putintparam(task, MSK\_IPAR\_SOL\_READ\_NAME\_WIDTH, -1)

Groups Data input/output, Solution input/output

## MSK\_IPAR\_SOL\_READ\_WIDTH

Controls the maximal acceptable width of line in the solutions when read by MOSEK.

Default 1024

Accepted [80; +inf]

Example MSK\_putintparam(task, MSK\_IPAR\_SOL\_READ\_WIDTH, 1024)

Groups Data input/output, Solution input/output

# MSK\_IPAR\_SOLUTION\_CALLBACK

Indicates whether solution callbacks will be performed during the optimization.

Default OFF

Accepted ON, OFF (see MSKonoffkeye)

Example MSK\_putintparam(task, MSK\_IPAR\_SOLUTION\_CALLBACK, MSK\_OFF)

Groups Progress callback, Overall solver

# MSK\_IPAR\_TIMING\_LEVEL

Controls the amount of timing performed inside MOSEK.

#### Default 1

Accepted [0; +inf]

Example MSK\_putintparam(task, MSK\_IPAR\_TIMING\_LEVEL, 1)

Groups Overall system

# MSK\_IPAR\_WRITE\_BAS\_CONSTRAINTS

Controls whether the constraint section is written to the basic solution file.

Default ON

Accepted ON, OFF (see MSKonoffkeye)

Example MSK\_putintparam(task, MSK\_IPAR\_WRITE\_BAS\_CONSTRAINTS, MSK\_ON)

Groups Data input/output, Solution input/output

# MSK\_IPAR\_WRITE\_BAS\_HEAD

Controls whether the header section is written to the basic solution file.

Default ON

Accepted ON, OFF (see MSKonoffkeye)

Example MSK\_putintparam(task, MSK\_IPAR\_WRITE\_BAS\_HEAD, MSK\_ON)

Groups Data input/output, Solution input/output

### MSK\_IPAR\_WRITE\_BAS\_VARIABLES

Controls whether the variables section is written to the basic solution file.

Default ON

Accepted ON, OFF (see MSKonoffkeye)

Example MSK\_putintparam(task, MSK\_IPAR\_WRITE\_BAS\_VARIABLES, MSK\_ON)

Groups Data input/output, Solution input/output

## MSK\_IPAR\_WRITE\_COMPRESSION

Controls whether the data file is compressed while it is written. 0 means no compression while higher values mean more compression.

Default 9

Accepted [0; +inf]

Example MSK\_putintparam(task, MSK\_IPAR\_WRITE\_COMPRESSION, 9)

Groups Data input/output

#### MSK\_IPAR\_WRITE\_DATA\_PARAM

If this option is turned on the parameter settings are written to the data file as parameters.

Default OFF

Accepted ON, OFF (see MSKonoffkeye)

Example MSK\_putintparam(task, MSK\_IPAR\_WRITE\_DATA\_PARAM, MSK\_OFF)

Groups Data input/output

# MSK\_IPAR\_WRITE\_FREE\_CON

Controls whether the free constraints are written to the data file.

Default ON

Accepted ON, OFF (see MSKonoffkeye)

Example MSK\_putintparam(task, MSK\_IPAR\_WRITE\_FREE\_CON, MSK\_ON)

Groups Data input/output

## MSK\_IPAR\_WRITE\_GENERIC\_NAMES

Controls whether generic names should be used instead of user-defined names when writing to the data file.

Default OFF

Accepted ON, OFF (see MSKonoffkeye)

Example MSK\_putintparam(task, MSK\_IPAR\_WRITE\_GENERIC\_NAMES, MSK\_OFF)

Groups Data input/output

# MSK\_IPAR\_WRITE\_GENERIC\_NAMES\_IO

Index origin used in generic names.

Default 1

**Accepted**  $[0; +\inf]$ 

Example MSK\_putintparam(task, MSK\_IPAR\_WRITE\_GENERIC\_NAMES\_IO, 1)

Groups Data input/output

# MSK\_IPAR\_WRITE\_IGNORE\_INCOMPATIBLE\_ITEMS

Controls if the writer ignores incompatible problem items when writing files.

Default OFF

# Accepted

- ON: Ignore items that cannot be written to the current output file format.
- OFF: Produce an error if the problem contains items that cannot the written to the current output file format.

Groups Data input/output

# MSK\_IPAR\_WRITE\_INT\_CONSTRAINTS

Controls whether the constraint section is written to the integer solution file.

Default ON

Accepted ON, OFF (see MSKonoffkeye)

```
Example MSK_putintparam(task, MSK_IPAR_WRITE_INT_CONSTRAINTS, MSK_ON)
          Groups Data input/output, Solution input/output
MSK_IPAR_WRITE_INT_HEAD
     Controls whether the header section is written to the integer solution file.
         Default ON
         Accepted ON, OFF (see MSKonoffkeye)
         Example MSK_putintparam(task, MSK_IPAR_WRITE_INT_HEAD, MSK_ON)
         Groups Data input/output, Solution input/output
MSK_IPAR_WRITE_INT_VARIABLES
     Controls whether the variables section is written to the integer solution file.
         Default ON
          Accepted ON, OFF (see MSKonoffkeye)
         Example MSK_putintparam(task, MSK_IPAR_WRITE_INT_VARIABLES, MSK_ON)
          Groups Data input/output, Solution input/output
MSK_IPAR_WRITE_LP_FULL_OBJ
     Write all variables, including the ones with 0-coefficients, in the objective.
         Default ON
         Accepted ON, OFF (see MSKonoffkeye)
         Example MSK_putintparam(task, MSK_IPAR_WRITE_LP_FULL_OBJ, MSK_ON)
         Groups Data input/output
MSK_IPAR_WRITE_LP_LINE_WIDTH
     Maximum width of line in an LP file written by MOSEK.
         Default 80
          Accepted [40; +inf]
         Example MSK_putintparam(task, MSK_IPAR_WRITE_LP_LINE_WIDTH, 80)
         Groups Data input/output
MSK_IPAR_WRITE_LP_QUOTED_NAMES
     If this option is turned on, then MOSEK will quote invalid LP names when writing an LP file.
         Default ON
          Accepted ON, OFF (see MSKonoffkeye)
         Example MSK_putintparam(task, MSK_IPAR_WRITE_LP_QUOTED_NAMES, MSK_ON)
         Groups Data input/output
MSK_IPAR_WRITE_LP_STRICT_FORMAT
     Controls whether LP output files satisfy the LP format strictly.
         Default OFF
         Accepted ON, OFF (see MSKonoffkeye)
         Example MSK_putintparam(task, MSK_IPAR_WRITE_LP_STRICT_FORMAT, MSK_OFF)
         Groups Data input/output
MSK_IPAR_WRITE_LP_TERMS_PER_LINE
     Maximum number of terms on a single line in an LP file written by MOSEK. 0 means unlimited.
```

```
Default 10
Accepted [0; +\inf]
Example MSK_putintparam(task, MSK_IPAR_WRITE_LP_TERMS_PER_LINE, 10)
Groups Data input/output
```

### MSK\_IPAR\_WRITE\_MPS\_FORMAT

Controls in which format the MPS is written.

Default FREE

Accepted STRICT, RELAXED, FREE, CPLEX (see MSKmpsformate)

 $\textbf{Example} \ \texttt{MSK\_putintparam}(\texttt{task}, \ \texttt{MSK\_IPAR\_WRITE\_MPS\_FORMAT}, \\$ 

MSK\_MPS\_FORMAT\_FREE)

Groups Data input/output

### MSK\_IPAR\_WRITE\_MPS\_INT

Controls if marker records are written to the MPS file to indicate whether variables are integer restricted.

Default ON

Accepted ON, OFF (see MSKonoffkeye)

Example MSK\_putintparam(task, MSK\_IPAR\_WRITE\_MPS\_INT, MSK\_ON)

Groups Data input/output

#### MSK\_IPAR\_WRITE\_PRECISION

Controls the precision with which double numbers are printed in the MPS data file. In general it is not worthwhile to use a value higher than 15.

Default 15

Accepted [0; +inf]

Example MSK\_putintparam(task, MSK\_IPAR\_WRITE\_PRECISION, 15)

Groups Data input/output

#### MSK\_IPAR\_WRITE\_SOL\_BARVARIABLES

Controls whether the symmetric matrix variables section is written to the solution file.

Default ON

Accepted ON, OFF (see MSKonoffkeye)

Example MSK\_putintparam(task, MSK\_IPAR\_WRITE\_SOL\_BARVARIABLES, MSK\_ON)

Groups Data input/output, Solution input/output

## MSK\_IPAR\_WRITE\_SOL\_CONSTRAINTS

Controls whether the constraint section is written to the solution file.

Default ON

Accepted ON, OFF (see MSKonoffkeye)

Example MSK\_putintparam(task, MSK\_IPAR\_WRITE\_SOL\_CONSTRAINTS, MSK\_ON)

Groups Data input/output, Solution input/output

## MSK\_IPAR\_WRITE\_SOL\_HEAD

Controls whether the header section is written to the solution file.

Default ON

Accepted ON, OFF (see MSKonoffkeye)

Example MSK\_putintparam(task, MSK\_IPAR\_WRITE\_SOL\_HEAD, MSK\_ON)

Groups Data input/output, Solution input/output

## MSK\_IPAR\_WRITE\_SOL\_IGNORE\_INVALID\_NAMES

Even if the names are invalid MPS names, then they are employed when writing the solution file.

Default OFF

Accepted ON, OFF (see MSKonoffkeye)

Groups Data input/output, Solution input/output

### MSK\_IPAR\_WRITE\_SOL\_VARIABLES

Controls whether the variables section is written to the solution file.

Default ON

Accepted ON, OFF (see MSKonoffkeye)

Example MSK\_putintparam(task, MSK\_IPAR\_WRITE\_SOL\_VARIABLES, MSK\_ON)

Groups Data input/output, Solution input/output

### MSK\_IPAR\_WRITE\_TASK\_INC\_SOL

Controls whether the solutions are stored in the task file too.

Default ON

Accepted ON, OFF (see MSKonoffkeye)

Example MSK\_putintparam(task, MSK\_IPAR\_WRITE\_TASK\_INC\_SOL, MSK\_ON)

Groups Data input/output

#### MSK\_IPAR\_WRITE\_XML\_MODE

Controls if linear coefficients should be written by row or column when writing in the XML file format.

Default ROW

Accepted ROW, COL (see MSKxmlwriteroutputtypee)

 $\mathbf{Example} \ \mathtt{MSK\_putintparam}(\mathtt{task}, \ \mathtt{MSK\_IPAR\_WRITE\_XML\_MODE},$ 

MSK\_WRITE\_XML\_MODE\_ROW)

Groups Data input/output

# 15.5.3 String parameters

## MSKsparame

The enumeration type containing all string parameters.

MSK\_SPAR\_BAS\_SOL\_FILE\_NAME

Name of the bas solution file.

Accepted Any valid file name.

Example MSK\_putstrparam(task, MSK\_SPAR\_BAS\_SOL\_FILE\_NAME, "somevalue")

Groups Data input/output, Solution input/output

## MSK\_SPAR\_DATA\_FILE\_NAME

Data are read and written to this file.

Accepted Any valid file name.

Example MSK\_putstrparam(task, MSK\_SPAR\_DATA\_FILE\_NAME, "somevalue")

Groups Data input/output

## MSK\_SPAR\_DEBUG\_FILE\_NAME

MOSEK debug file.

Accepted Any valid file name.

Example MSK\_putstrparam(task, MSK\_SPAR\_DEBUG\_FILE\_NAME, "somevalue")

Groups Data input/output

## MSK\_SPAR\_INT\_SOL\_FILE\_NAME

Name of the int solution file.

Accepted Any valid file name.

Example MSK\_putstrparam(task, MSK\_SPAR\_INT\_SOL\_FILE\_NAME, "somevalue")

Groups Data input/output, Solution input/output

## MSK\_SPAR\_ITR\_SOL\_FILE\_NAME

Name of the itr solution file.

Accepted Any valid file name.

Example MSK\_putstrparam(task, MSK\_SPAR\_ITR\_SOL\_FILE\_NAME, "somevalue")

Groups Data input/output, Solution input/output

### MSK\_SPAR\_MIO\_DEBUG\_STRING

For internal debugging purposes.

Accepted Any valid string.

Example MSK\_putstrparam(task, MSK\_SPAR\_MIO\_DEBUG\_STRING, "somevalue")

Groups Data input/output

### MSK\_SPAR\_PARAM\_COMMENT\_SIGN

Only the first character in this string is used. It is considered as a start of comment sign in the **MOSEK** parameter file. Spaces are ignored in the string.

## Default

%%

Accepted Any valid string.

Example MSK\_putstrparam(task, MSK\_SPAR\_PARAM\_COMMENT\_SIGN, "%%")

Groups Data input/output

## MSK\_SPAR\_PARAM\_READ\_FILE\_NAME

Modifications to the parameter database is read from this file.

Accepted Any valid file name.

Example MSK\_putstrparam(task, MSK\_SPAR\_PARAM\_READ\_FILE\_NAME, "somevalue")

Groups Data input/output

## MSK\_SPAR\_PARAM\_WRITE\_FILE\_NAME

The parameter database is written to this file.

Accepted Any valid file name.

Groups Data input/output

## MSK\_SPAR\_READ\_MPS\_BOU\_NAME

Name of the BOUNDS vector used. An empty name means that the first BOUNDS vector is used.

Accepted Any valid MPS name.

Example MSK\_putstrparam(task, MSK\_SPAR\_READ\_MPS\_BOU\_NAME, "somevalue")

Groups Data input/output

# MSK\_SPAR\_READ\_MPS\_OBJ\_NAME

Name of the free constraint used as objective function. An empty name means that the first constraint is used as objective function.

Accepted Any valid MPS name.

Example MSK\_putstrparam(task, MSK\_SPAR\_READ\_MPS\_OBJ\_NAME, "somevalue")

Groups Data input/output

## MSK\_SPAR\_READ\_MPS\_RAN\_NAME

Name of the RANGE vector used. An empty name means that the first RANGE vector is used.

Accepted Any valid MPS name.

Example MSK\_putstrparam(task, MSK\_SPAR\_READ\_MPS\_RAN\_NAME, "somevalue")

Groups Data input/output

## MSK\_SPAR\_READ\_MPS\_RHS\_NAME

Name of the RHS used. An empty name means that the first RHS vector is used.

Accepted Any valid MPS name.

Example MSK\_putstrparam(task, MSK\_SPAR\_READ\_MPS\_RHS\_NAME, "somevalue")

Groups Data input/output

### MSK\_SPAR\_REMOTE\_ACCESS\_TOKEN

An access token used to submit tasks to a remote **MOSEK** server. An access token is a random 32-byte string encoded in base64, i.e. it is a 44 character ASCII string.

Accepted Any valid string.

Example MSK\_putstrparam(task, MSK\_SPAR\_REMOTE\_ACCESS\_TOKEN, "somevalue")

Groups Overall system

### MSK\_SPAR\_SENSITIVITY\_FILE\_NAME

If defined MSK\_sensitivityreport reads this file as a sensitivity analysis data file specifying the type of analysis to be done.

Accepted Any valid string.

Groups Data input/output

## MSK\_SPAR\_SENSITIVITY\_RES\_FILE\_NAME

If this is a nonempty string, then MSK\_sensitivityreport writes results to this file.

**Accepted** Any valid string.

Groups Data input/output

## MSK\_SPAR\_SOL\_FILTER\_XC\_LOW

A filter used to determine which constraints should be listed in the solution file. A value of 0.5 means that all constraints having xc[i]>0.5 should be listed, whereas +0.5 means that all constraints having xc[i]>=blc[i]+0.5 should be listed. An empty filter means that no filter is applied.

Accepted Any valid filter.

 $Example \ \texttt{MSK\_putstrparam}(\texttt{task}, \ \texttt{MSK\_SPAR\_SOL\_FILTER\_XC\_LOW}, \ \texttt{"somevalue"})$ 

Groups Data input/output, Solution input/output

# MSK\_SPAR\_SOL\_FILTER\_XC\_UPR

A filter used to determine which constraints should be listed in the solution file. A value of 0.5 means that all constraints having xc[i]<0.5 should be listed, whereas -0.5 means all constraints having xc[i]<-buc[i]-0.5 should be listed. An empty filter means that no filter is applied.

Accepted Any valid filter.

Example MSK\_putstrparam(task, MSK\_SPAR\_SOL\_FILTER\_XC\_UPR, "somevalue")

Groups Data input/output, Solution input/output

### MSK\_SPAR\_SOL\_FILTER\_XX\_LOW

A filter used to determine which variables should be listed in the solution file. A value of "0.5" means that all constraints having xx[j] >= 0.5 should be listed, whereas "+0.5" means that all constraints having xx[j] >= blx[j] + 0.5 should be listed. An empty filter means no filter is applied.

Accepted Any valid filter.

Example MSK\_putstrparam(task, MSK\_SPAR\_SOL\_FILTER\_XX\_LOW, "somevalue")

Groups Data input/output, Solution input/output

# MSK\_SPAR\_SOL\_FILTER\_XX\_UPR

A filter used to determine which variables should be listed in the solution file. A value of "0.5" means that all constraints having xx[j]<0.5 should be printed, whereas "-0.5" means all constraints having xx[j]<=bux[j]-0.5 should be listed. An empty filter means no filter is applied.

```
Accepted Any valid file name.
```

Example MSK\_putstrparam(task, MSK\_SPAR\_SOL\_FILTER\_XX\_UPR, "somevalue")

Groups Data input/output, Solution input/output

## MSK\_SPAR\_STAT\_FILE\_NAME

Statistics file name.

Accepted Any valid file name.

Example MSK\_putstrparam(task, MSK\_SPAR\_STAT\_FILE\_NAME, "somevalue")

Groups Data input/output

### MSK\_SPAR\_STAT\_KEY

Key used when writing the summary file.

Accepted Any valid string.

Example MSK\_putstrparam(task, MSK\_SPAR\_STAT\_KEY, "somevalue")

Groups Data input/output

### MSK\_SPAR\_STAT\_NAME

Name used when writing the statistics file.

**Accepted** Any valid XML string.

Example MSK\_putstrparam(task, MSK\_SPAR\_STAT\_NAME, "somevalue")

Groups Data input/output

# MSK\_SPAR\_WRITE\_LP\_GEN\_VAR\_NAME

Sometimes when an LP file is written additional variables must be inserted. They will have the prefix denoted by this parameter.

Default xmskgen

Accepted Any valid string.

Example MSK\_putstrparam(task, MSK\_SPAR\_WRITE\_LP\_GEN\_VAR\_NAME, "xmskgen")

Groups Data input/output

# 15.6 Response codes

Response codes include:

- Termination codes
- Warnings
- Errors

The numerical code (in brackets) identifies the response in error messages and in the log output. MSKrescodee

The enumeration type containing all response codes.

# 15.6.1 Termination

MSK\_RES\_OK (0)

No error occurred.

MSK\_RES\_TRM\_MAX\_ITERATIONS (10000)

The optimizer terminated at the maximum number of iterations.

MSK\_RES\_TRM\_MAX\_TIME (10001)

The optimizer terminated at the maximum amount of time.

MSK\_RES\_TRM\_OBJECTIVE\_RANGE (10002)

The optimizer terminated with an objective value outside the objective range.

#### MSK\_RES\_TRM\_MIO\_NUM\_RELAXS (10008)

The mixed-integer optimizer terminated as the maximum number of relaxations was reached.

MSK\_RES\_TRM\_MIO\_NUM\_BRANCHES (10009)

The mixed-integer optimizer terminated as the maximum number of branches was reached.

MSK\_RES\_TRM\_NUM\_MAX\_NUM\_INT\_SOLUTIONS (10015)

The mixed-integer optimizer terminated as the maximum number of feasible solutions was reached.  $MSK_RES_TRM_STALL$  (10006)

The optimizer is terminated due to slow progress.

Stalling means that numerical problems prevent the optimizer from making reasonable progress and that it makes no sense to continue. In many cases this happens if the problem is badly scaled or otherwise ill-conditioned. There is no guarantee that the solution will be feasible or optimal. However, often stalling happens near the optimum, and the returned solution may be of good quality. Therefore, it is recommended to check the status of the solution. If the solution status is optimal the solution is most likely good enough for most practical purposes.

Please note that if a linear optimization problem is solved using the interior-point optimizer with basis identification turned on, the returned basic solution likely to have high accuracy, even though the optimizer stalled.

Some common causes of stalling are a) badly scaled models, b) near feasible or near infeasible problems.

MSK\_RES\_TRM\_USER\_CALLBACK (10007)

The optimizer terminated due to the return of the user-defined callback function.

MSK\_RES\_TRM\_MAX\_NUM\_SETBACKS (10020)

The optimizer terminated as the maximum number of set-backs was reached. This indicates serious numerical problems and a possibly badly formulated problem.

MSK\_RES\_TRM\_NUMERICAL\_PROBLEM (10025)

The optimizer terminated due to numerical problems.

MSK\_RES\_TRM\_INTERNAL (10030)

The optimizer terminated due to some internal reason. Please contact MOSEK support.

MSK\_RES\_TRM\_INTERNAL\_STOP (10031)

The optimizer terminated for internal reasons. Please contact MOSEK support.

# 15.6.2 Warnings

MSK\_RES\_WRN\_OPEN\_PARAM\_FILE (50)

The parameter file could not be opened.

MSK\_RES\_WRN\_LARGE\_BOUND (51)

A numerically large bound value is specified.

MSK\_RES\_WRN\_LARGE\_LO\_BOUND (52)

A numerically large lower bound value is specified.

MSK\_RES\_WRN\_LARGE\_UP\_BOUND (53)

A numerically large upper bound value is specified.

MSK\_RES\_WRN\_LARGE\_CON\_FX (54)

An equality constraint is fixed to a numerically large value. This can cause numerical problems.

MSK\_RES\_WRN\_LARGE\_CJ (57)

A numerically large value is specified for one  $c_j$ .

MSK\_RES\_WRN\_LARGE\_AIJ (62)

A numerically large value is specified for an  $a_{i,j}$  element in A. The parameter  $MSK\_DPAR\_DATA\_TOL\_AIJ\_LARGE$  controls when an  $a_{i,j}$  is considered large.

MSK\_RES\_WRN\_ZERO\_AIJ (63)

One or more zero elements are specified in A.

MSK\_RES\_WRN\_NAME\_MAX\_LEN (65)

A name is longer than the buffer that is supposed to hold it.

MSK\_RES\_WRN\_SPAR\_MAX\_LEN (66)

A value for a string parameter is longer than the buffer that is supposed to hold it.

MSK\_RES\_WRN\_MPS\_SPLIT\_RHS\_VECTOR (70)

An RHS vector is split into several nonadjacent parts in an MPS file.

### MSK\_RES\_WRN\_MPS\_SPLIT\_RAN\_VECTOR (71)

A RANGE vector is split into several nonadjacent parts in an MPS file.

## MSK\_RES\_WRN\_MPS\_SPLIT\_BOU\_VECTOR (72)

A BOUNDS vector is split into several nonadjacent parts in an MPS file.

# MSK\_RES\_WRN\_LP\_OLD\_QUAD\_FORMAT (80)

Missing '/2' after quadratic expressions in bound or objective.

## MSK\_RES\_WRN\_LP\_DROP\_VARIABLE (85)

Ignored a variable because the variable was not previously defined. Usually this implies that a variable appears in the bound section but not in the objective or the constraints.

# MSK\_RES\_WRN\_NZ\_IN\_UPR\_TRI (200)

Non-zero elements specified in the upper triangle of a matrix were ignored.

#### MSK\_RES\_WRN\_DROPPED\_NZ\_QOBJ (201)

One or more non-zero elements were dropped in the Q matrix in the objective.

## MSK\_RES\_WRN\_IGNORE\_INTEGER (250)

Ignored integer constraints.

## MSK\_RES\_WRN\_NO\_GLOBAL\_OPTIMIZER (251)

No global optimizer is available.

### MSK\_RES\_WRN\_MIO\_INFEASIBLE\_FINAL (270)

The final mixed-integer problem with all the integer variables fixed at their optimal values is infeasible.

## MSK\_RES\_WRN\_SOL\_FILTER (300)

Invalid solution filter is specified.

### MSK\_RES\_WRN\_UNDEF\_SOL\_FILE\_NAME (350)

Undefined name occurred in a solution.

## MSK\_RES\_WRN\_SOL\_FILE\_IGNORED\_CON (351)

One or more lines in the constraint section were ignored when reading a solution file.

#### MSK\_RES\_WRN\_SOL\_FILE\_IGNORED\_VAR (352)

One or more lines in the variable section were ignored when reading a solution file.

# MSK\_RES\_WRN\_TOO\_FEW\_BASIS\_VARS (400)

An incomplete basis has been specified. Too few basis variables are specified.

## MSK\_RES\_WRN\_TOO\_MANY\_BASIS\_VARS (405)

A basis with too many variables has been specified.

### MSK\_RES\_WRN\_LICENSE\_EXPIRE (500)

The license expires.

# MSK\_RES\_WRN\_LICENSE\_SERVER (501)

The license server is not responding.

# MSK\_RES\_WRN\_EMPTY\_NAME (502)

A variable or constraint name is empty. The output file may be invalid.

#### MSK\_RES\_WRN\_USING\_GENERIC\_NAMES (503)

Generic names are used because a name is not valid. For instance when writing an LP file the names must not contain blanks or start with a digit.

## MSK\_RES\_WRN\_LICENSE\_FEATURE\_EXPIRE (505)

The license expires.

## MSK\_RES\_WRN\_PARAM\_NAME\_DOU (510)

The parameter name is not recognized as a double parameter.

# MSK\_RES\_WRN\_PARAM\_NAME\_INT (511)

The parameter name is not recognized as a integer parameter.

# MSK\_RES\_WRN\_PARAM\_NAME\_STR (512)

The parameter name is not recognized as a string parameter.

# MSK\_RES\_WRN\_PARAM\_STR\_VALUE (515)

The string is not recognized as a symbolic value for the parameter.

# MSK\_RES\_WRN\_PARAM\_IGNORED\_CMIO (516)

A parameter was ignored by the conic mixed integer optimizer.

# MSK\_RES\_WRN\_ZEROS\_IN\_SPARSE\_ROW (705)

One or more (near) zero elements are specified in a sparse row of a matrix. Since, it is redundant to specify zero elements then it may indicate an error.

#### MSK\_RES\_WRN\_ZEROS\_IN\_SPARSE\_COL (710)

One or more (near) zero elements are specified in a sparse column of a matrix. It is redundant to specify zero elements. Hence, it may indicate an error.

### MSK\_RES\_WRN\_INCOMPLETE\_LINEAR\_DEPENDENCY\_CHECK (800)

The linear dependency check(s) is incomplete. Normally this is not an important warning unless the optimization problem has been formulated with linear dependencies. Linear dependencies may prevent **MOSEK** from solving the problem.

#### MSK\_RES\_WRN\_ELIMINATOR\_SPACE (801)

The eliminator is skipped at least once due to lack of space.

## MSK\_RES\_WRN\_PRESOLVE\_OUTOFSPACE (802)

The presolve is incomplete due to lack of space.

#### MSK\_RES\_WRN\_WRITE\_CHANGED\_NAMES (803)

Some names were changed because they were invalid for the output file format.

## MSK\_RES\_WRN\_WRITE\_DISCARDED\_CFIX (804)

The fixed objective term could not be converted to a variable and was discarded in the output file.

### MSK\_RES\_WRN\_DUPLICATE\_CONSTRAINT\_NAMES (850)

Two constraint names are identical.

### MSK\_RES\_WRN\_DUPLICATE\_VARIABLE\_NAMES (851)

Two variable names are identical.

# MSK\_RES\_WRN\_DUPLICATE\_BARVARIABLE\_NAMES (852)

Two barvariable names are identical.

### MSK\_RES\_WRN\_DUPLICATE\_CONE\_NAMES (853)

Two cone names are identical.

#### MSK\_RES\_WRN\_ANA\_LARGE\_BOUNDS (900)

This warning is issued by the problem analyzer, if one or more constraint or variable bounds are very large. One should consider omitting these bounds entirely by setting them to +inf or -inf.

#### MSK\_RES\_WRN\_ANA\_C\_ZERO (901)

This warning is issued by the problem analyzer, if the coefficients in the linear part of the objective are all zero.

## MSK\_RES\_WRN\_ANA\_EMPTY\_COLS (902)

This warning is issued by the problem analyzer, if columns, in which all coefficients are zero, are found.

### MSK\_RES\_WRN\_ANA\_CLOSE\_BOUNDS (903)

This warning is issued by problem analyzer, if ranged constraints or variables with very close upper and lower bounds are detected. One should consider treating such constraints as equalities and such variables as constants.

# MSK\_RES\_WRN\_ANA\_ALMOST\_INT\_BOUNDS (904)

This warning is issued by the problem analyzer if a constraint is bound nearly integral.

#### MSK\_RES\_WRN\_QUAD\_CONES\_WITH\_ROOT\_FIXED\_AT\_ZERO (930)

For at least one quadratic cone the root is fixed at (nearly) zero. This may cause problems such as a very large dual solution. Therefore, it is recommended to remove such cones before optimizing the problem, or to fix all the variables in the cone to 0.

## MSK\_RES\_WRN\_RQUAD\_CONES\_WITH\_ROOT\_FIXED\_AT\_ZERO (931)

For at least one rotated quadratic cone at least one of the root variables are fixed at (nearly) zero. This may cause problems such as a very large dual solution. Therefore, it is recommended to remove such cones before optimizing the problem, or to fix all the variables in the cone to 0.

## MSK\_RES\_WRN\_EXP\_CONES\_WITH\_VARIABLES\_FIXED\_AT\_ZERO (932)

For at least one exponential cone  $x \ge y \exp(z/y)$  either the variable x or y is fixed at (nearly) zero. This may cause problems such as a very large dual solution. Therefore, it is recommended to remove such cones before optimizing the problem, or to fix all the variables in the cone to 0.

# MSK\_RES\_WRN\_POW\_CONES\_WITH\_ROOT\_FIXED\_AT\_ZERO (933)

For at least one power cone at least one of the root variables are fixed at (nearly) zero. This may cause problems such as a very large dual solution. Therefore, it is recommended to remove such cones before optimizing the problem, or to fix all the variables in the cone to 0.

### MSK\_RES\_WRN\_NO\_DUALIZER (950)

No automatic dualizer is available for the specified problem. The primal problem is solved.

# MSK\_RES\_WRN\_SYM\_MAT\_LARGE (960)

A numerically large value is specified for an  $e_{i,j}$  element in E. The parameter

## 15.6.3 Errors

MSK\_RES\_ERR\_LICENSE (1000)

Invalid license.

MSK\_RES\_ERR\_LICENSE\_EXPIRED (1001)

The license has expired.

MSK\_RES\_ERR\_LICENSE\_VERSION (1002)

The license is valid for another version of **MOSEK**.

MSK\_RES\_ERR\_SIZE\_LICENSE (1005)

The problem is bigger than the license.

MSK\_RES\_ERR\_PROB\_LICENSE (1006)

The software is not licensed to solve the problem.

MSK\_RES\_ERR\_FILE\_LICENSE (1007)

Invalid license file.

MSK\_RES\_ERR\_MISSING\_LICENSE\_FILE (1008)

MOSEK cannot find license file or a token server. See the MOSEK licensing manual for details.

MSK\_RES\_ERR\_SIZE\_LICENSE\_CON (1010)

The problem has too many constraints to be solved with the available license.

MSK\_RES\_ERR\_SIZE\_LICENSE\_VAR (1011)

The problem has too many variables to be solved with the available license.

MSK\_RES\_ERR\_SIZE\_LICENSE\_INTVAR (1012)

The problem contains too many integer variables to be solved with the available license.

MSK\_RES\_ERR\_OPTIMIZER\_LICENSE (1013)

The optimizer required is not licensed.

MSK\_RES\_ERR\_FLEXLM (1014)

The FLEXIm license manager reported an error.

MSK\_RES\_ERR\_LICENSE\_SERVER (1015)

The license server is not responding.

MSK\_RES\_ERR\_LICENSE\_MAX (1016)

Maximum number of licenses is reached.

MSK\_RES\_ERR\_LICENSE\_MOSEKLM\_DAEMON (1017)

The MOSEKLM license manager daemon is not up and running.

MSK\_RES\_ERR\_LICENSE\_FEATURE (1018)

A requested feature is not available in the license file(s). Most likely due to an incorrect license system setup.

MSK\_RES\_ERR\_PLATFORM\_NOT\_LICENSED (1019)

A requested license feature is not available for the required platform.

MSK\_RES\_ERR\_LICENSE\_CANNOT\_ALLOCATE (1020)

The license system cannot allocate the memory required.

MSK\_RES\_ERR\_LICENSE\_CANNOT\_CONNECT (1021)

MOSEK cannot connect to the license server. Most likely the license server is not up and running. MSK\_RES\_ERR\_LICENSE\_INVALID\_HOSTID (1025)

The host ID specified in the license file does not match the host ID of the computer.

MSK\_RES\_ERR\_LICENSE\_SERVER\_VERSION (1026)

The version specified in the checkout request is greater than the highest version number the daemon supports.

MSK\_RES\_ERR\_LICENSE\_NO\_SERVER\_SUPPORT (1027)

The license server does not support the requested feature. Possible reasons for this error include:

- The feature has expired.
- The feature's start date is later than today's date.
- The version requested is higher than feature's the highest supported version.
- A corrupted license file.

Try restarting the license and inspect the license server debug file, usually called lmgrd.log.

#### MSK\_RES\_ERR\_LICENSE\_NO\_SERVER\_LINE (1028)

There is no SERVER line in the license file. All non-zero license count features need at least one SERVER line.

### MSK\_RES\_ERR\_OLDER\_DLL (1035)

The dynamic link library is older than the specified version.

### MSK\_RES\_ERR\_NEWER\_DLL (1036)

The dynamic link library is newer than the specified version.

#### MSK\_RES\_ERR\_LINK\_FILE\_DLL (1040)

A file cannot be linked to a stream in the DLL version.

### MSK\_RES\_ERR\_THREAD\_MUTEX\_INIT (1045)

Could not initialize a mutex.

#### MSK\_RES\_ERR\_THREAD\_MUTEX\_LOCK (1046)

Could not lock a mutex.

## MSK\_RES\_ERR\_THREAD\_MUTEX\_UNLOCK (1047)

Could not unlock a mutex.

### MSK\_RES\_ERR\_THREAD\_CREATE (1048)

Could not create a thread. This error may occur if a large number of environments are created and not deleted again. In any case it is a good practice to minimize the number of environments created.

### MSK\_RES\_ERR\_THREAD\_COND\_INIT (1049)

Could not initialize a condition.

### MSK\_RES\_ERR\_UNKNOWN (1050)

Unknown error.

## MSK\_RES\_ERR\_SPACE (1051)

Out of space.

## MSK\_RES\_ERR\_FILE\_OPEN (1052)

Error while opening a file.

#### MSK\_RES\_ERR\_FILE\_READ (1053)

File read error.

# MSK\_RES\_ERR\_FILE\_WRITE (1054)

File write error.

# MSK\_RES\_ERR\_DATA\_FILE\_EXT (1055)

The data file format cannot be determined from the file name.

#### MSK\_RES\_ERR\_INVALID\_FILE\_NAME (1056)

An invalid file name has been specified.

## MSK\_RES\_ERR\_INVALID\_SOL\_FILE\_NAME (1057)

An invalid file name has been specified.

## MSK\_RES\_ERR\_END\_OF\_FILE (1059)

End of file reached.

## MSK\_RES\_ERR\_NULL\_ENV (1060)

env is a NULL pointer.

### MSK\_RES\_ERR\_NULL\_TASK (1061)

task is a NULL pointer.

## MSK\_RES\_ERR\_INVALID\_STREAM (1062)

An invalid stream is referenced.

# MSK\_RES\_ERR\_NO\_INIT\_ENV (1063)

env is not initialized.

# MSK\_RES\_ERR\_INVALID\_TASK (1064)

The task is invalid.

#### MSK\_RES\_ERR\_NULL\_POINTER (1065)

An argument to a function is unexpectedly a NULL pointer.

## MSK\_RES\_ERR\_LIVING\_TASKS (1066)

All tasks associated with an environment must be deleted before the environment is deleted. There are still some undeleted tasks.

## MSK\_RES\_ERR\_BLANK\_NAME (1070)

An all blank name has been specified.

# MSK\_RES\_ERR\_DUP\_NAME (1071)

The same name was used multiple times for the same problem item type.

MSK\_RES\_ERR\_FORMAT\_STRING (1072)

The name format string is invalid.

MSK\_RES\_ERR\_INVALID\_OBJ\_NAME (1075)

An invalid objective name is specified.

MSK\_RES\_ERR\_INVALID\_CON\_NAME (1076)

An invalid constraint name is used.

MSK\_RES\_ERR\_INVALID\_VAR\_NAME (1077)

An invalid variable name is used.

MSK\_RES\_ERR\_INVALID\_CONE\_NAME (1078)

An invalid cone name is used.

MSK\_RES\_ERR\_INVALID\_BARVAR\_NAME (1079)

An invalid symmetric matrix variable name is used.

MSK\_RES\_ERR\_SPACE\_LEAKING (1080)

MOSEK is leaking memory. This can be due to either an incorrect use of MOSEK or a bug.

MSK\_RES\_ERR\_SPACE\_NO\_INFO (1081)

No available information about the space usage.

MSK\_RES\_ERR\_READ\_FORMAT (1090)

The specified format cannot be read.

MSK\_RES\_ERR\_MPS\_FILE (1100)

An error occurred while reading an MPS file.

MSK\_RES\_ERR\_MPS\_INV\_FIELD (1101)

A field in the MPS file is invalid. Probably it is too wide.

MSK\_RES\_ERR\_MPS\_INV\_MARKER (1102)

An invalid marker has been specified in the MPS file.

MSK\_RES\_ERR\_MPS\_NULL\_CON\_NAME (1103)

An empty constraint name is used in an MPS file.

MSK\_RES\_ERR\_MPS\_NULL\_VAR\_NAME (1104)

An empty variable name is used in an MPS file.

MSK\_RES\_ERR\_MPS\_UNDEF\_CON\_NAME (1105)

An undefined constraint name occurred in an MPS file.

MSK\_RES\_ERR\_MPS\_UNDEF\_VAR\_NAME (1106)

An undefined variable name occurred in an MPS file.

MSK\_RES\_ERR\_MPS\_INV\_CON\_KEY (1107)

An invalid constraint key occurred in an MPS file.

MSK\_RES\_ERR\_MPS\_INV\_BOUND\_KEY (1108)

An invalid bound key occurred in an MPS file.

MSK\_RES\_ERR\_MPS\_INV\_SEC\_NAME (1109)

An invalid section name occurred in an MPS file.

MSK\_RES\_ERR\_MPS\_NO\_OBJECTIVE (1110)

No objective is defined in an MPS file.

MSK\_RES\_ERR\_MPS\_SPLITTED\_VAR (1111)

All elements in a column of the A matrix must be specified consecutively. Hence, it is illegal to specify non-zero elements in A for variable 1, then for variable 2 and then variable 1 again.

MSK\_RES\_ERR\_MPS\_MUL\_CON\_NAME (1112)

A constraint name was specified multiple times in the ROWS section.

MSK\_RES\_ERR\_MPS\_MUL\_QSEC (1113)

Multiple QSECTIONs are specified for a constraint in the MPS data file.

MSK\_RES\_ERR\_MPS\_MUL\_QOBJ (1114)

The Q term in the objective is specified multiple times in the MPS data file.

MSK\_RES\_ERR\_MPS\_INV\_SEC\_ORDER (1115)

The sections in the MPS data file are not in the correct order.

MSK\_RES\_ERR\_MPS\_MUL\_CSEC (1116)

Multiple CSECTIONs are given the same name.

MSK\_RES\_ERR\_MPS\_CONE\_TYPE (1117)

Invalid cone type specified in a CSECTION.

MSK\_RES\_ERR\_MPS\_CONE\_OVERLAP (1118)

A variable is specified to be a member of several cones.

## MSK\_RES\_ERR\_MPS\_CONE\_REPEAT (1119)

A variable is repeated within the CSECTION.

MSK\_RES\_ERR\_MPS\_NON\_SYMMETRIC\_Q (1120)

A non symmetric matrix has been speciefied.

MSK\_RES\_ERR\_MPS\_DUPLICATE\_Q\_ELEMENT (1121)

Duplicate elements is specified in a Q matrix.

MSK\_RES\_ERR\_MPS\_INVALID\_OBJSENSE (1122)

An invalid objective sense is specified.

MSK\_RES\_ERR\_MPS\_TAB\_IN\_FIELD2 (1125)

A tab char occurred in field 2.

MSK\_RES\_ERR\_MPS\_TAB\_IN\_FIELD3 (1126)

A tab char occurred in field 3.

MSK\_RES\_ERR\_MPS\_TAB\_IN\_FIELD5 (1127)

A tab char occurred in field 5.

MSK\_RES\_ERR\_MPS\_INVALID\_OBJ\_NAME (1128)

An invalid objective name is specified.

MSK\_RES\_ERR\_LP\_INCOMPATIBLE (1150)

The problem cannot be written to an LP formatted file.

MSK\_RES\_ERR\_LP\_EMPTY (1151)

The problem cannot be written to an LP formatted file.

MSK\_RES\_ERR\_LP\_DUP\_SLACK\_NAME (1152)

The name of the slack variable added to a ranged constraint already exists.

MSK\_RES\_ERR\_WRITE\_MPS\_INVALID\_NAME (1153)

An invalid name is created while writing an MPS file. Usually this will make the MPS file unreadable.

MSK\_RES\_ERR\_LP\_INVALID\_VAR\_NAME (1154)

A variable name is invalid when used in an LP formatted file.

MSK\_RES\_ERR\_LP\_FREE\_CONSTRAINT (1155)

Free constraints cannot be written in LP file format.

MSK\_RES\_ERR\_WRITE\_OPF\_INVALID\_VAR\_NAME (1156)

Empty variable names cannot be written to OPF files.

MSK\_RES\_ERR\_LP\_FILE\_FORMAT (1157)

Syntax error in an LP file.

MSK\_RES\_ERR\_WRITE\_LP\_FORMAT (1158)

Problem cannot be written as an LP file.

MSK\_RES\_ERR\_READ\_LP\_MISSING\_END\_TAG (1159)

Syntax error in LP file. Possibly missing End tag.

MSK\_RES\_ERR\_LP\_FORMAT (1160)

Syntax error in an LP file.

MSK\_RES\_ERR\_WRITE\_LP\_NON\_UNIQUE\_NAME (1161)

An auto-generated name is not unique.

MSK\_RES\_ERR\_READ\_LP\_NONEXISTING\_NAME (1162)

A variable never occurred in objective or constraints.

MSK\_RES\_ERR\_LP\_WRITE\_CONIC\_PROBLEM (1163)

The problem contains cones that cannot be written to an LP formatted file.

MSK\_RES\_ERR\_LP\_WRITE\_GECO\_PROBLEM (1164)

The problem contains general convex terms that cannot be written to an LP formatted file.

MSK\_RES\_ERR\_WRITING\_FILE (1166)

An error occurred while writing file

MSK\_RES\_ERR\_PTF\_FORMAT (1167)

Syntax error in an PTF file

MSK\_RES\_ERR\_OPF\_FORMAT (1168)

Syntax error in an OPF file

MSK\_RES\_ERR\_OPF\_NEW\_VARIABLE (1169)

Introducing new variables is now allowed. When a [variables] section is present, it is not allowed to introduce new variables later in the problem.

MSK\_RES\_ERR\_INVALID\_NAME\_IN\_SOL\_FILE (1170)

An invalid name occurred in a solution file.

### MSK\_RES\_ERR\_LP\_INVALID\_CON\_NAME (1171)

A constraint name is invalid when used in an LP formatted file.

MSK\_RES\_ERR\_OPF\_PREMATURE\_EOF (1172)

Premature end of file in an OPF file.

MSK\_RES\_ERR\_JSON\_SYNTAX (1175)

Syntax error in an JSON data

MSK\_RES\_ERR\_JSON\_STRING (1176)

Error in JSON string.

MSK\_RES\_ERR\_JSON\_NUMBER\_OVERFLOW (1177)

Invalid number entry - wrong type or value overflow.

MSK\_RES\_ERR\_JSON\_FORMAT (1178)

Error in an JSON Task file

MSK\_RES\_ERR\_JSON\_DATA (1179)

Inconsistent data in JSON Task file

MSK\_RES\_ERR\_JSON\_MISSING\_DATA (1180)

Missing data section in JSON task file.

MSK\_RES\_ERR\_ARGUMENT\_LENNEQ (1197)

Incorrect length of arguments.

MSK\_RES\_ERR\_ARGUMENT\_TYPE (1198)

Incorrect argument type.

MSK\_RES\_ERR\_NUM\_ARGUMENTS (1199)

Incorrect number of function arguments.

MSK\_RES\_ERR\_IN\_ARGUMENT (1200)

A function argument is incorrect.

MSK\_RES\_ERR\_ARGUMENT\_DIMENSION (1201)

A function argument is of incorrect dimension.

MSK\_RES\_ERR\_SHAPE\_IS\_TOO\_LARGE (1202)

The size of the n-dimensional shape is too large.

MSK\_RES\_ERR\_INDEX\_IS\_TOO\_SMALL (1203)

An index in an argument is too small.

MSK\_RES\_ERR\_INDEX\_IS\_TOO\_LARGE (1204)

An index in an argument is too large.

MSK\_RES\_ERR\_PARAM\_NAME (1205)

The parameter name is not correct.

MSK\_RES\_ERR\_PARAM\_NAME\_DOU (1206)

The parameter name is not correct for a double parameter.

MSK\_RES\_ERR\_PARAM\_NAME\_INT (1207)

The parameter name is not correct for an integer parameter.

MSK\_RES\_ERR\_PARAM\_NAME\_STR (1208)

The parameter name is not correct for a string parameter.

MSK\_RES\_ERR\_PARAM\_INDEX (1210)

Parameter index is out of range.

MSK\_RES\_ERR\_PARAM\_IS\_TOO\_LARGE (1215)

The parameter value is too large.

MSK\_RES\_ERR\_PARAM\_IS\_TOO\_SMALL (1216)

The parameter value is too small.

MSK\_RES\_ERR\_PARAM\_VALUE\_STR (1217)

The parameter value string is incorrect.

MSK\_RES\_ERR\_PARAM\_TYPE (1218)

The parameter type is invalid.

MSK\_RES\_ERR\_INF\_DOU\_INDEX (1219)

A double information index is out of range for the specified type.

MSK\_RES\_ERR\_INF\_INT\_INDEX (1220)

An integer information index is out of range for the specified type.

MSK\_RES\_ERR\_INDEX\_ARR\_IS\_TOO\_SMALL (1221)

An index in an array argument is too small.

MSK\_RES\_ERR\_INDEX\_ARR\_IS\_TOO\_LARGE (1222)

An index in an array argument is too large.

#### MSK\_RES\_ERR\_INF\_LINT\_INDEX (1225)

A long integer information index is out of range for the specified type.

### MSK\_RES\_ERR\_ARG\_IS\_TOO\_SMALL (1226)

The value of a argument is too small.

## MSK\_RES\_ERR\_ARG\_IS\_TOO\_LARGE (1227)

The value of a argument is too large.

#### MSK\_RES\_ERR\_INVALID\_WHICHSOL (1228)

whichsol is invalid.

### MSK\_RES\_ERR\_INF\_DOU\_NAME (1230)

A double information name is invalid.

#### MSK\_RES\_ERR\_INF\_INT\_NAME (1231)

An integer information name is invalid.

### MSK\_RES\_ERR\_INF\_TYPE (1232)

The information type is invalid.

## MSK\_RES\_ERR\_INF\_LINT\_NAME (1234)

A long integer information name is invalid.

### MSK\_RES\_ERR\_INDEX (1235)

An index is out of range.

#### MSK\_RES\_ERR\_WHICHSOL (1236)

The solution defined by whichsol does not exists.

## MSK\_RES\_ERR\_SOLITEM (1237)

The solution item number solitem is invalid. Please note that MSK\_SOL\_ITEM\_SNX is invalid for the basic solution.

## MSK\_RES\_ERR\_WHICHITEM\_NOT\_ALLOWED (1238)

whichitem is unacceptable.

### MSK\_RES\_ERR\_MAXNUMCON (1240)

The maximum number of constraints specified is smaller than the number of constraints in the task.

## MSK\_RES\_ERR\_MAXNUMVAR (1241)

The maximum number of variables specified is smaller than the number of variables in the task.

#### MSK\_RES\_ERR\_MAXNUMBARVAR (1242)

The maximum number of semidefinite variables specified is smaller than the number of semidefinite variables in the task.

#### MSK\_RES\_ERR\_MAXNUMQNZ (1243)

The maximum number of non-zeros specified for the Q matrices is smaller than the number of non-zeros in the current Q matrices.

# MSK\_RES\_ERR\_TOO\_SMALL\_MAX\_NUM\_NZ (1245)

The maximum number of non-zeros specified is too small.

### MSK\_RES\_ERR\_INVALID\_IDX (1246)

A specified index is invalid.

## MSK\_RES\_ERR\_INVALID\_MAX\_NUM (1247)

A specified index is invalid.

## MSK\_RES\_ERR\_NUMCONLIM (1250)

Maximum number of constraints limit is exceeded.

#### MSK\_RES\_ERR\_NUMVARLIM (1251)

Maximum number of variables limit is exceeded.

## MSK\_RES\_ERR\_TOO\_SMALL\_MAXNUMANZ (1252)

The maximum number of non-zeros specified for A is smaller than the number of non-zeros in the current A.

#### MSK\_RES\_ERR\_INV\_APTRE (1253)

aptre[j] is strictly smaller than aptrb[j] for some j.

## MSK\_RES\_ERR\_MUL\_A\_ELEMENT (1254)

An element in A is defined multiple times.

# MSK\_RES\_ERR\_INV\_BK (1255)

Invalid bound key.

### MSK\_RES\_ERR\_INV\_BKC (1256)

Invalid bound key is specified for a constraint.

### MSK\_RES\_ERR\_INV\_BKX (1257)

An invalid bound key is specified for a variable.

### MSK\_RES\_ERR\_INV\_VAR\_TYPE (1258)

An invalid variable type is specified for a variable.

## MSK\_RES\_ERR\_SOLVER\_PROBTYPE (1259)

Problem type does not match the chosen optimizer.

### MSK\_RES\_ERR\_OBJECTIVE\_RANGE (1260)

Empty objective range.

### MSK\_RES\_ERR\_UNDEF\_SOLUTION (1265)

**MOSEK** has the following solution types:

- an interior-point solution,
- a basic solution,
- and an integer solution.

Each optimizer may set one or more of these solutions; e.g by default a successful optimization with the interior-point optimizer defines the interior-point solution and, for linear problems, also the basic solution. This error occurs when asking for a solution or for information about a solution that is not defined.

#### MSK\_RES\_ERR\_BASIS (1266)

An invalid basis is specified. Either too many or too few basis variables are specified.

### MSK\_RES\_ERR\_INV\_SKC (1267)

Invalid value in skc.

#### MSK\_RES\_ERR\_INV\_SKX (1268)

Invalid value in skx.

### MSK\_RES\_ERR\_INV\_SKN (1274)

Invalid value in skn.

### MSK\_RES\_ERR\_INV\_SK\_STR (1269)

Invalid status key string encountered.

## MSK\_RES\_ERR\_INV\_SK (1270)

Invalid status key code.

# MSK\_RES\_ERR\_INV\_CONE\_TYPE\_STR (1271)

Invalid cone type string encountered.

## MSK\_RES\_ERR\_INV\_CONE\_TYPE (1272)

Invalid cone type code is encountered.

## MSK\_RES\_ERR\_INVALID\_SURPLUS (1275)

Invalid surplus.

## MSK\_RES\_ERR\_INV\_NAME\_ITEM (1280)

An invalid name item code is used.

## MSK\_RES\_ERR\_PRO\_ITEM (1281)

An invalid problem is used.

## MSK\_RES\_ERR\_INVALID\_FORMAT\_TYPE (1283)

Invalid format type.

## MSK\_RES\_ERR\_FIRSTI (1285)

Invalid firsti.

# MSK\_RES\_ERR\_LASTI (1286)

Invalid lasti.

## MSK\_RES\_ERR\_FIRSTJ (1287)

Invalid firstj.

#### MSK\_RES\_ERR\_LASTJ (1288)

Invalid lastj.

## MSK\_RES\_ERR\_MAX\_LEN\_IS\_TOO\_SMALL (1289)

A maximum length that is too small has been specified.

# MSK\_RES\_ERR\_NONLINEAR\_EQUALITY (1290)

The model contains a nonlinear equality which defines a nonconvex set.

# MSK\_RES\_ERR\_NONCONVEX (1291)

The optimization problem is nonconvex.

#### MSK\_RES\_ERR\_NONLINEAR\_RANGED (1292)

Nonlinear constraints with finite lower and upper bound always define a nonconvex feasible set.

### MSK\_RES\_ERR\_CON\_Q\_NOT\_PSD (1293)

The quadratic constraint matrix is not positive semidefinite as expected for a constraint with finite upper bound. This results in a nonconvex problem. The parameter  $MSK\_DPAR\_CHECK\_CONVEXITY\_REL\_TOL$  can be used to relax the convexity check.

#### MSK\_RES\_ERR\_CON\_Q\_NOT\_NSD (1294)

The quadratic constraint matrix is not negative semidefinite as expected for a constraint with finite lower bound. This results in a nonconvex problem. The parameter  $MSK\_DPAR\_CHECK\_CONVEXITY\_REL\_TOL$  can be used to relax the convexity check.

#### MSK\_RES\_ERR\_OBJ\_Q\_NOT\_PSD (1295)

The quadratic coefficient matrix in the objective is not positive semidefinite as expected for a minimization problem. The parameter MSK\_DPAR\_CHECK\_CONVEXITY\_REL\_TOL can be used to relax the convexity check.

## MSK\_RES\_ERR\_OBJ\_Q\_NOT\_NSD (1296)

The quadratic coefficient matrix in the objective is not negative semidefinite as expected for a maximization problem. The parameter  $MSK\_DPAR\_CHECK\_CONVEXITY\_REL\_TOL$  can be used to relax the convexity check.

## MSK\_RES\_ERR\_ARGUMENT\_PERM\_ARRAY (1299)

An invalid permutation array is specified.

MSK\_RES\_ERR\_CONE\_INDEX (1300)

An index of a non-existing cone has been specified.

MSK\_RES\_ERR\_CONE\_SIZE (1301)

A cone with incorrect number of members is specified.

MSK\_RES\_ERR\_CONE\_OVERLAP (1302)

One or more of the variables in the cone to be added is already member of another cone. Now assume the variable is  $x_j$  then add a new variable say  $x_k$  and the constraint

$$x_j = x_k$$

and then let  $x_k$  be member of the cone to be appended.

MSK\_RES\_ERR\_CONE\_REP\_VAR (1303)

A variable is included multiple times in the cone.

MSK\_RES\_ERR\_MAXNUMCONE (1304)

The value specified for maxnumcone is too small.

MSK\_RES\_ERR\_CONE\_TYPE (1305)

Invalid cone type specified.

MSK\_RES\_ERR\_CONE\_TYPE\_STR (1306)

Invalid cone type specified.

MSK\_RES\_ERR\_CONE\_OVERLAP\_APPEND (1307)

The cone to be appended has one variable which is already member of another cone.

MSK\_RES\_ERR\_REMOVE\_CONE\_VARIABLE (1310)

A variable cannot be removed because it will make a cone invalid.

MSK\_RES\_ERR\_APPENDING\_TOO\_BIG\_CONE (1311)

Trying to append a too big cone.

MSK\_RES\_ERR\_CONE\_PARAMETER (1320)

An invalid cone parameter.

MSK\_RES\_ERR\_SOL\_FILE\_INVALID\_NUMBER (1350)

An invalid number is specified in a solution file.

MSK\_RES\_ERR\_HUGE\_C (1375)

A huge value in absolute size is specified for one  $c_j$ .

MSK\_RES\_ERR\_HUGE\_AIJ (1380)

A numerically huge value is specified for an  $a_{i,j}$  element in A. The parameter  $MSK\_DPAR\_DATA\_TOL\_AIJ\_HUGE$  controls when an  $a_{i,j}$  is considered huge.

MSK\_RES\_ERR\_DUPLICATE\_AIJ (1385)

An element in the A matrix is specified twice.

MSK\_RES\_ERR\_LOWER\_BOUND\_IS\_A\_NAN (1390)

The lower bound specified is not a number (nan).

### MSK\_RES\_ERR\_UPPER\_BOUND\_IS\_A\_NAN (1391)

The upper bound specified is not a number (nan).

MSK\_RES\_ERR\_INFINITE\_BOUND (1400)

A numerically huge bound value is specified.

MSK\_RES\_ERR\_INV\_QOBJ\_SUBI (1401)

Invalid value in qosubi.

MSK\_RES\_ERR\_INV\_QOBJ\_SUBJ (1402)

Invalid value in qosubj.

MSK\_RES\_ERR\_INV\_QOBJ\_VAL (1403)

Invalid value in qoval.

MSK\_RES\_ERR\_INV\_QCON\_SUBK (1404)

Invalid value in qcsubk.

MSK\_RES\_ERR\_INV\_QCON\_SUBI (1405)

Invalid value in qcsubi.

MSK\_RES\_ERR\_INV\_QCON\_SUBJ (1406)

Invalid value in qcsubj.

MSK\_RES\_ERR\_INV\_QCON\_VAL (1407)

Invalid value in qcval.

MSK\_RES\_ERR\_QCON\_SUBI\_TOO\_SMALL (1408)

Invalid value in qcsubi.

MSK\_RES\_ERR\_QCON\_SUBI\_TOO\_LARGE (1409)

Invalid value in qcsubi.

MSK\_RES\_ERR\_QOBJ\_UPPER\_TRIANGLE (1415)

An element in the upper triangle of  $Q^o$  is specified. Only elements in the lower triangle should be specified.

# MSK\_RES\_ERR\_QCON\_UPPER\_TRIANGLE (1417)

An element in the upper triangle of a  $Q^k$  is specified. Only elements in the lower triangle should be specified.

#### MSK\_RES\_ERR\_FIXED\_BOUND\_VALUES (1420)

A fixed constraint/variable has been specified using the bound keys but the numerical value of the lower and upper bound is different.

## MSK\_RES\_ERR\_TOO\_SMALL\_A\_TRUNCATION\_VALUE (1421)

A too small value for the A trucation value is specified.

MSK\_RES\_ERR\_INVALID\_OBJECTIVE\_SENSE (1445)

An invalid objective sense is specified.

MSK\_RES\_ERR\_UNDEFINED\_OBJECTIVE\_SENSE (1446)

The objective sense has not been specified before the optimization.

MSK\_RES\_ERR\_Y\_IS\_UNDEFINED (1449)

The solution item y is undefined.

MSK\_RES\_ERR\_NAN\_IN\_DOUBLE\_DATA (1450)

An invalid floating point value was used in some double data.

MSK\_RES\_ERR\_NAN\_IN\_BLC (1461)

 $l^c$  contains an invalid floating point value, i.e. a NaN.

MSK\_RES\_ERR\_NAN\_IN\_BUC (1462)

 $u^c$  contains an invalid floating point value, i.e. a NaN.

MSK\_RES\_ERR\_NAN\_IN\_C (1470)

c contains an invalid floating point value, i.e. a NaN.

MSK\_RES\_ERR\_NAN\_IN\_BLX (1471)

 $l^x$  contains an invalid floating point value, i.e. a NaN.

MSK\_RES\_ERR\_NAN\_IN\_BUX (1472)

 $u^x$  contains an invalid floating point value, i.e. a NaN.

MSK\_RES\_ERR\_INVALID\_AIJ (1473)

 $a_{i,j}$  contains an invalid floating point value, i.e. a NaN or an infinite value.

MSK\_RES\_ERR\_INVALID\_CJ (1474)

 $c_i$  contains an invalid floating point value, i.e. a NaN or an infinite value.

## MSK\_RES\_ERR\_SYM\_MAT\_INVALID (1480)

A symmetric matrix contains an invalid floating point value, i.e. a NaN or an infinite value.

### MSK\_RES\_ERR\_SYM\_MAT\_HUGE (1482)

A symmetric matrix contains a huge value in absolute size. The parameter  $MSK\_DPAR\_DATA\_SYM\_MAT\_TOL\_HUGE$  controls when an  $e_{i,j}$  is considered huge.

## MSK\_RES\_ERR\_INV\_PROBLEM (1500)

Invalid problem type. Probably a nonconvex problem has been specified.

### MSK\_RES\_ERR\_MIXED\_CONIC\_AND\_NL (1501)

The problem contains nonlinear terms conic constraints. The requested operation cannot be applied to this type of problem.

### MSK\_RES\_ERR\_GLOBAL\_INV\_CONIC\_PROBLEM (1503)

The global optimizer can only be applied to problems without semidefinite variables.

#### MSK\_RES\_ERR\_INV\_OPTIMIZER (1550)

An invalid optimizer has been chosen for the problem.

### MSK\_RES\_ERR\_MIO\_NO\_OPTIMIZER (1551)

No optimizer is available for the current class of integer optimization problems.

# MSK\_RES\_ERR\_NO\_OPTIMIZER\_VAR\_TYPE (1552)

No optimizer is available for this class of optimization problems.

## MSK\_RES\_ERR\_FINAL\_SOLUTION (1560)

An error occurred during the solution finalization.

## MSK\_RES\_ERR\_FIRST (1570)

Invalid first.

### MSK\_RES\_ERR\_LAST (1571)

Invalid index last. A given index was out of expected range.

### MSK\_RES\_ERR\_SLICE\_SIZE (1572)

Invalid slice size specified.

## MSK\_RES\_ERR\_NEGATIVE\_SURPLUS (1573)

Negative surplus.

### MSK\_RES\_ERR\_NEGATIVE\_APPEND (1578)

Cannot append a negative number.

### MSK\_RES\_ERR\_POSTSOLVE (1580)

An error occurred during the postsolve. Please contact MOSEK support.

#### MSK\_RES\_ERR\_OVERFLOW (1590)

A computation produced an overflow i.e. a very large number.

### MSK\_RES\_ERR\_NO\_BASIS\_SOL (1600)

No basic solution is defined.

## MSK\_RES\_ERR\_BASIS\_FACTOR (1610)

The factorization of the basis is invalid.

# MSK\_RES\_ERR\_BASIS\_SINGULAR (1615)

The basis is singular and hence cannot be factored.

#### MSK\_RES\_ERR\_FACTOR (1650)

An error occurred while factorizing a matrix.

## MSK\_RES\_ERR\_FEASREPAIR\_CANNOT\_RELAX (1700)

An optimization problem cannot be relaxed.

## MSK\_RES\_ERR\_FEASREPAIR\_SOLVING\_RELAXED (1701)

The relaxed problem could not be solved to optimality. Please consult the log file for further details.

## MSK\_RES\_ERR\_FEASREPAIR\_INCONSISTENT\_BOUND (1702)

The upper bound is less than the lower bound for a variable or a constraint. Please correct this before running the feasibility repair.

# MSK\_RES\_ERR\_REPAIR\_INVALID\_PROBLEM (1710)

The feasibility repair does not support the specified problem type.

## MSK\_RES\_ERR\_REPAIR\_OPTIMIZATION\_FAILED (1711)

Computation the optimal relaxation failed. The cause may have been numerical problems.

## MSK\_RES\_ERR\_NAME\_MAX\_LEN (1750)

A name is longer than the buffer that is supposed to hold it.

# MSK\_RES\_ERR\_NAME\_IS\_NULL (1760)

The name buffer is a NULL pointer.

## MSK\_RES\_ERR\_INVALID\_COMPRESSION (1800)

Invalid compression type.

#### MSK\_RES\_ERR\_INVALID\_IOMODE (1801)

Invalid io mode.

### MSK\_RES\_ERR\_NO\_PRIMAL\_INFEAS\_CER (2000)

A certificate of primal infeasibility is not available.

## MSK\_RES\_ERR\_NO\_DUAL\_INFEAS\_CER (2001)

A certificate of infeasibility is not available.

#### MSK\_RES\_ERR\_NO\_SOLUTION\_IN\_CALLBACK (2500)

The required solution is not available.

#### MSK\_RES\_ERR\_INV\_MARKI (2501)

Invalid value in marki.

#### MSK\_RES\_ERR\_INV\_MARKJ (2502)

Invalid value in markj.

#### MSK\_RES\_ERR\_INV\_NUMI (2503)

Invalid numi.

## MSK\_RES\_ERR\_INV\_NUMJ (2504)

Invalid numj.

### MSK\_RES\_ERR\_TASK\_INCOMPATIBLE (2560)

The Task file is incompatible with this platform. This results from reading a file on a 32 bit platform generated on a 64 bit platform.

### MSK\_RES\_ERR\_TASK\_INVALID (2561)

The Task file is invalid.

## MSK\_RES\_ERR\_TASK\_WRITE (2562)

Failed to write the task file.

## MSK\_RES\_ERR\_LU\_MAX\_NUM\_TRIES (2800)

Could not compute the LU factors of the matrix within the maximum number of allowed tries.

## MSK\_RES\_ERR\_INVALID\_UTF8 (2900)

An invalid UTF8 string is encountered.

#### MSK\_RES\_ERR\_INVALID\_WCHAR (2901)

An invalid wchar string is encountered.

## MSK\_RES\_ERR\_NO\_DUAL\_FOR\_ITG\_SOL (2950)

No dual information is available for the integer solution.

# MSK\_RES\_ERR\_NO\_SNX\_FOR\_BAS\_SOL (2953)

 $\boldsymbol{s}_n^x$  is not available for the basis solution.

## MSK\_RES\_ERR\_INTERNAL (3000)

An internal error occurred. Please report this problem.

## MSK\_RES\_ERR\_API\_ARRAY\_TOO\_SMALL (3001)

An input array was too short.

## MSK\_RES\_ERR\_API\_CB\_CONNECT (3002)

Failed to connect a callback object.

## MSK\_RES\_ERR\_API\_FATAL\_ERROR (3005)

An internal error occurred in the API. Please report this problem.

#### MSK\_RES\_ERR\_API\_INTERNAL (3999)

An internal fatal error occurred in an interface function.

## MSK\_RES\_ERR\_SEN\_FORMAT (3050)

Syntax error in sensitivity analysis file.

# MSK\_RES\_ERR\_SEN\_UNDEF\_NAME (3051)

An undefined name was encountered in the sensitivity analysis file.

# MSK\_RES\_ERR\_SEN\_INDEX\_RANGE (3052)

Index out of range in the sensitivity analysis file.

## MSK\_RES\_ERR\_SEN\_BOUND\_INVALID\_UP (3053)

Analysis of upper bound requested for an index, where no upper bound exists.

## MSK\_RES\_ERR\_SEN\_BOUND\_INVALID\_LO (3054)

Analysis of lower bound requested for an index, where no lower bound exists.

# MSK\_RES\_ERR\_SEN\_INDEX\_INVALID (3055)

Invalid range given in the sensitivity file.

### MSK\_RES\_ERR\_SEN\_INVALID\_REGEXP (3056)

Syntax error in regexp or regexp longer than 1024.

#### MSK\_RES\_ERR\_SEN\_SOLUTION\_STATUS (3057)

No optimal solution found to the original problem given for sensitivity analysis.

## MSK\_RES\_ERR\_SEN\_NUMERICAL (3058)

Numerical difficulties encountered performing the sensitivity analysis.

## MSK\_RES\_ERR\_SEN\_UNHANDLED\_PROBLEM\_TYPE (3080)

Sensitivity analysis cannot be performed for the specified problem. Sensitivity analysis is only possible for linear problems.

### MSK\_RES\_ERR\_UNB\_STEP\_SIZE (3100)

A step size in an optimizer was unexpectedly unbounded. For instance, if the step-size becomes unbounded in phase 1 of the simplex algorithm then an error occurs. Normally this will happen only if the problem is badly formulated. Please contact **MOSEK** support if this error occurs.

#### MSK\_RES\_ERR\_IDENTICAL\_TASKS (3101)

Some tasks related to this function call were identical. Unique tasks were expected.

## MSK\_RES\_ERR\_AD\_INVALID\_CODELIST (3102)

The code list data was invalid.

## MSK\_RES\_ERR\_INTERNAL\_TEST\_FAILED (3500)

An internal unit test function failed.

## MSK\_RES\_ERR\_XML\_INVALID\_PROBLEM\_TYPE (3600)

The problem type is not supported by the XML format.

### MSK\_RES\_ERR\_INVALID\_AMPL\_STUB (3700)

Invalid AMPL stub.

### MSK\_RES\_ERR\_INT64\_TO\_INT32\_CAST (3800)

A 64 bit integer could not be cast to a 32 bit integer.

### MSK\_RES\_ERR\_SIZE\_LICENSE\_NUMCORES (3900)

The computer contains more cpu cores than the license allows for.

## MSK\_RES\_ERR\_INFEAS\_UNDEFINED (3910)

The requested value is not defined for this solution type.

### MSK\_RES\_ERR\_NO\_BARX\_FOR\_SOLUTION (3915)

There is no  $\overline{X}$  available for the solution specified. In particular note there are no  $\overline{X}$  defined for the basic and integer solutions.

## MSK\_RES\_ERR\_NO\_BARS\_FOR\_SOLUTION (3916)

There is no  $\bar{s}$  available for the solution specified. In particular note there are no  $\bar{s}$  defined for the basic and integer solutions.

# MSK\_RES\_ERR\_BAR\_VAR\_DIM (3920)

The dimension of a symmetric matrix variable has to be greater than 0.

# MSK\_RES\_ERR\_SYM\_MAT\_INVALID\_ROW\_INDEX (3940)

A row index specified for sparse symmetric matrix is invalid.

# MSK\_RES\_ERR\_SYM\_MAT\_INVALID\_COL\_INDEX (3941)

A column index specified for sparse symmetric matrix is invalid.

## MSK\_RES\_ERR\_SYM\_MAT\_NOT\_LOWER\_TRINGULAR (3942)

Only the lower triangular part of sparse symmetric matrix should be specified.

# MSK\_RES\_ERR\_SYM\_MAT\_INVALID\_VALUE (3943)

The numerical value specified in a sparse symmetric matrix is not a floating point value.

# MSK\_RES\_ERR\_SYM\_MAT\_DUPLICATE (3944)

A value in a symmetric matric as been specified more than once.

## MSK\_RES\_ERR\_INVALID\_SYM\_MAT\_DIM (3950)

A sparse symmetric matrix of invalid dimension is specified.

### MSK\_RES\_ERR\_INVALID\_FILE\_FORMAT\_FOR\_SYM\_MAT (4000)

The file format does not support a problem with symmetric matrix variables.

# MSK\_RES\_ERR\_INVALID\_FILE\_FORMAT\_FOR\_CFIX (4001)

The file format does not support a problem with nonzero fixed term in c.

## MSK\_RES\_ERR\_INVALID\_FILE\_FORMAT\_FOR\_RANGED\_CONSTRAINTS (4002)

The file format does not support a problem with ranged constraints.

## MSK\_RES\_ERR\_INVALID\_FILE\_FORMAT\_FOR\_FREE\_CONSTRAINTS (4003)

The file format does not support a problem with free constraints.

## MSK\_RES\_ERR\_INVALID\_FILE\_FORMAT\_FOR\_CONES (4005)

The file format does not support a problem with conic constraints.

### MSK\_RES\_ERR\_INVALID\_FILE\_FORMAT\_FOR\_NONLINEAR (4010)

The file format does not support a problem with nonlinear terms.

MSK\_RES\_ERR\_DUPLICATE\_CONSTRAINT\_NAMES (4500)

Two constraint names are identical.

MSK\_RES\_ERR\_DUPLICATE\_VARIABLE\_NAMES (4501)

Two variable names are identical.

MSK\_RES\_ERR\_DUPLICATE\_BARVARIABLE\_NAMES (4502)

Two barvariable names are identical.

MSK\_RES\_ERR\_DUPLICATE\_CONE\_NAMES (4503)

Two cone names are identical.

MSK\_RES\_ERR\_NON\_UNIQUE\_ARRAY (5000)

An array does not contain unique elements.

MSK\_RES\_ERR\_ARGUMENT\_IS\_TOO\_LARGE (5005)

The value of a function argument is too large.

MSK\_RES\_ERR\_MIO\_INTERNAL (5010)

A fatal error occurred in the mixed integer optimizer. Please contact MOSEK support.

MSK\_RES\_ERR\_INVALID\_PROBLEM\_TYPE (6000)

An invalid problem type.

MSK\_RES\_ERR\_UNHANDLED\_SOLUTION\_STATUS (6010)

Unhandled solution status.

MSK\_RES\_ERR\_UPPER\_TRIANGLE (6020)

An element in the upper triangle of a lower triangular matrix is specified.

MSK\_RES\_ERR\_LAU\_SINGULAR\_MATRIX (7000)

A matrix is singular.

MSK\_RES\_ERR\_LAU\_NOT\_POSITIVE\_DEFINITE (7001)

A matrix is not positive definite.

MSK\_RES\_ERR\_LAU\_INVALID\_LOWER\_TRIANGULAR\_MATRIX (7002)

An invalid lower triangular matrix.

MSK\_RES\_ERR\_LAU\_UNKNOWN (7005)

An unknown error.

MSK\_RES\_ERR\_LAU\_ARG\_M (7010)

Invalid argument m.

MSK\_RES\_ERR\_LAU\_ARG\_N (7011)

Invalid argument n.

MSK\_RES\_ERR\_LAU\_ARG\_K (7012)

Invalid argument k.

MSK\_RES\_ERR\_LAU\_ARG\_TRANSA (7015)

Invalid argument transa.

MSK\_RES\_ERR\_LAU\_ARG\_TRANSB (7016)

Invalid argument transb.

MSK\_RES\_ERR\_LAU\_ARG\_UPLO (7017)

Invalid argument uplo.

MSK\_RES\_ERR\_LAU\_ARG\_TRANS (7018)

Invalid argument trans.

MSK\_RES\_ERR\_LAU\_INVALID\_SPARSE\_SYMMETRIC\_MATRIX (7019)

An invalid sparse symmetric matrix is specified. Note only the lower triangular part with no duplicates is specified.

MSK\_RES\_ERR\_CBF\_PARSE (7100)

An error occurred while parsing an CBF file.

MSK\_RES\_ERR\_CBF\_OBJ\_SENSE (7101)

An invalid objective sense is specified.

MSK\_RES\_ERR\_CBF\_NO\_VARIABLES (7102)

No variables are specified.

MSK\_RES\_ERR\_CBF\_TOO\_MANY\_CONSTRAINTS (7103)

Too many constraints specified.

MSK\_RES\_ERR\_CBF\_TOO\_MANY\_VARIABLES (7104)

Too many variables specified.

MSK\_RES\_ERR\_CBF\_NO\_VERSION\_SPECIFIED (7105)

No version specified.

MSK\_RES\_ERR\_CBF\_SYNTAX (7106)

Invalid syntax.

MSK\_RES\_ERR\_CBF\_DUPLICATE\_OBJ (7107)

Duplicate OBJ keyword.

MSK\_RES\_ERR\_CBF\_DUPLICATE\_CON (7108)

Duplicate CON keyword.

MSK\_RES\_ERR\_CBF\_DUPLICATE\_VAR (7109)

Duplicate VAR keyword.

MSK\_RES\_ERR\_CBF\_DUPLICATE\_INT (7110)

Duplicate INT keyword.

MSK\_RES\_ERR\_CBF\_INVALID\_VAR\_TYPE (7111)

Invalid variable type.

MSK\_RES\_ERR\_CBF\_INVALID\_CON\_TYPE (7112)

Invalid constraint type.

MSK\_RES\_ERR\_CBF\_INVALID\_DOMAIN\_DIMENSION (7113)

Invalid domain dimension.

MSK\_RES\_ERR\_CBF\_DUPLICATE\_OBJACOORD (7114)

Duplicate index in OBJCOORD.

MSK\_RES\_ERR\_CBF\_DUPLICATE\_BCOORD (7115)

Duplicate index in BCOORD.

MSK\_RES\_ERR\_CBF\_DUPLICATE\_ACOORD (7116)

Duplicate index in ACOORD.

MSK\_RES\_ERR\_CBF\_TOO\_FEW\_VARIABLES (7117)

Too few variables defined.

MSK\_RES\_ERR\_CBF\_TOO\_FEW\_CONSTRAINTS (7118)

Too few constraints defined.

MSK\_RES\_ERR\_CBF\_TOO\_FEW\_INTS (7119)

Too few ints are specified.

MSK\_RES\_ERR\_CBF\_TOO\_MANY\_INTS (7120)

Too many ints are specified.

MSK\_RES\_ERR\_CBF\_INVALID\_INT\_INDEX (7121)

Invalid INT index.

MSK\_RES\_ERR\_CBF\_UNSUPPORTED (7122)

Unsupported feature is present.

MSK\_RES\_ERR\_CBF\_DUPLICATE\_PSDVAR (7123)

 $\label{eq:power} \mbox{Duplicate PSDVAR keyword.}$ 

MSK\_RES\_ERR\_CBF\_INVALID\_PSDVAR\_DIMENSION (7124)

Invalid PSDVAR dimension.

MSK\_RES\_ERR\_CBF\_TOO\_FEW\_PSDVAR (7125)

Too few variables defined.

MSK\_RES\_ERR\_CBF\_INVALID\_EXP\_DIMENSION (7126)

Invalid dimension of a exponential cone.

MSK\_RES\_ERR\_CBF\_DUPLICATE\_POW\_CONES (7130)

Multiple POWCONES specified.

MSK\_RES\_ERR\_CBF\_DUPLICATE\_POW\_STAR\_CONES (7131)

Multiple POW\*CONES specified.

MSK\_RES\_ERR\_CBF\_INVALID\_POWER (7132)

Invalid power specified.

MSK\_RES\_ERR\_CBF\_POWER\_CONE\_IS\_TOO\_LONG (7133)

Power cone is too long.

MSK\_RES\_ERR\_CBF\_INVALID\_POWER\_CONE\_INDEX (7134)

Invalid power cone index.

MSK\_RES\_ERR\_CBF\_INVALID\_POWER\_STAR\_CONE\_INDEX (7135)

Invalid power star cone index.

MSK\_RES\_ERR\_CBF\_UNHANDLED\_POWER\_CONE\_TYPE (7136)

An unhandled power cone type.

### MSK\_RES\_ERR\_CBF\_UNHANDLED\_POWER\_STAR\_CONE\_TYPE (7137)

An unhandled power star cone type.

MSK\_RES\_ERR\_CBF\_POWER\_CONE\_MISMATCH (7138)

The power cone does not match with it definition.

MSK\_RES\_ERR\_CBF\_POWER\_STAR\_CONE\_MISMATCH (7139)

The power star cone does not match with it definition.

MSK\_RES\_ERR\_CBF\_INVALID\_NUMBER\_OF\_CONES (7740)

Invalid number of cones.

MSK\_RES\_ERR\_CBF\_INVALID\_DIMENSION\_OF\_CONES (7741)

Invalid dimension of cones.

MSK\_RES\_ERR\_MIO\_INVALID\_ROOT\_OPTIMIZER (7700)

An invalid root optimizer was selected for the problem type.

MSK\_RES\_ERR\_MIO\_INVALID\_NODE\_OPTIMIZER (7701)

An invalid node optimizer was selected for the problem type.

MSK\_RES\_ERR\_TOCONIC\_CONSTR\_Q\_NOT\_PSD (7800)

The matrix defining the quadratric part of constraint is not positive semidefinite.

MSK\_RES\_ERR\_TOCONIC\_CONSTRAINT\_FX (7801)

The quadratic constraint is an equality, thus not convex.

MSK\_RES\_ERR\_TOCONIC\_CONSTRAINT\_RA (7802)

The quadratic constraint has finite lower and upper bound, and therefore it is not convex.

MSK\_RES\_ERR\_TOCONIC\_CONSTR\_NOT\_CONIC (7803)

The constraint is not conic representable.

MSK\_RES\_ERR\_TOCONIC\_OBJECTIVE\_NOT\_PSD (7804)

The matrix defining the quadratric part of the objective function is not positive semidefinite.

MSK\_RES\_ERR\_SERVER\_CONNECT (8000)

Failed to connect to remote solver server. The server string or the port string were invalid, or the server did not accept connection.

MSK\_RES\_ERR\_SERVER\_PROTOCOL (8001)

Unexpected message or data from solver server.

MSK\_RES\_ERR\_SERVER\_STATUS (8002)

Server returned non-ok HTTP status code

MSK\_RES\_ERR\_SERVER\_TOKEN (8003)

The job ID specified is incorrect or invalid

MSK\_RES\_ERR\_SERVER\_PROBLEM\_SIZE (8008)

The size of the problem exceeds the dimensions permitted by the instance of the OptServer where it was run.

# 15.7 Enumerations

# MSKbasindtypee

Basis identification

MSK\_BI\_NEVER (0)

Never do basis identification.

MSK\_BI\_ALWAYS (1)

Basis identification is always performed even if the interior-point optimizer terminates abnormally.

MSK\_BI\_NO\_ERROR (2)

Basis identification is performed if the interior-point optimizer terminates without an error.

MSK\_BI\_IF\_FEASIBLE (3)

Basis identification is not performed if the interior-point optimizer terminates with a problem status saying that the problem is primal or dual infeasible.

MSK\_BI\_RESERVERED (4)

Not currently in use.

# MSKboundkeye

Bound keys

#### MSK\_BK\_LO (0)

The constraint or variable has a finite lower bound and an infinite upper bound.

#### MSK BK UP (1)

The constraint or variable has an infinite lower bound and an finite upper bound.

#### MSK\_BK\_FX (2)

The constraint or variable is fixed.

#### MSK\_BK\_FR (3)

The constraint or variable is free.

## MSK\_BK\_RA (4)

The constraint or variable is ranged.

#### MSKmarke

Mark

## MSK\_MARK\_LO (0)

The lower bound is selected for sensitivity analysis.

#### MSK\_MARK\_UP (1)

The upper bound is selected for sensitivity analysis.

#### MSKsimdegene

Degeneracy strategies

### MSK\_SIM\_DEGEN\_NONE (0)

The simplex optimizer should use no degeneration strategy.

### MSK\_SIM\_DEGEN\_FREE (1)

The simplex optimizer chooses the degeneration strategy.

## MSK\_SIM\_DEGEN\_AGGRESSIVE (2)

The simplex optimizer should use an aggressive degeneration strategy.

### MSK\_SIM\_DEGEN\_MODERATE (3)

The simplex optimizer should use a moderate degeneration strategy.

## MSK\_SIM\_DEGEN\_MINIMUM (4)

The simplex optimizer should use a minimum degeneration strategy.

#### MSKtransposee

Transposed matrix.

# MSK\_TRANSPOSE\_NO (0)

No transpose is applied.

## MSK\_TRANSPOSE\_YES (1)

A transpose is applied.

# MSKuploe

Triangular part of a symmetric matrix.

# MSK\_UPLO\_LO (0)

Lower part.

#### MSK\_UPLO\_UP (1)

Upper part.

# ${\tt MSKsimreforme}$

Problem reformulation.

# MSK\_SIM\_REFORMULATION\_ON (1)

Allow the simplex optimizer to reformulate the problem.

## MSK\_SIM\_REFORMULATION\_OFF (0)

Disallow the simplex optimizer to reformulate the problem.

# MSK\_SIM\_REFORMULATION\_FREE (2)

The simplex optimizer can choose freely.

## MSK\_SIM\_REFORMULATION\_AGGRESSIVE (3)

The simplex optimizer should use an aggressive reformulation strategy.

# MSKsimdupvece

Exploit duplicate columns.

### MSK\_SIM\_EXPLOIT\_DUPVEC\_ON (1)

Allow the simplex optimizer to exploit duplicated columns.

## MSK\_SIM\_EXPLOIT\_DUPVEC\_OFF (0)

Disallow the simplex optimizer to exploit duplicated columns.

#### MSK\_SIM\_EXPLOIT\_DUPVEC\_FREE (2)

The simplex optimizer can choose freely.

### MSKsimhotstarte

Hot-start type employed by the simplex optimizer

### MSK\_SIM\_HOTSTART\_NONE (0)

The simplex optimizer performs a coldstart.

### MSK\_SIM\_HOTSTART\_FREE (1)

The simplex optimize chooses the hot-start type.

### MSK\_SIM\_HOTSTART\_STATUS\_KEYS (2)

Only the status keys of the constraints and variables are used to choose the type of hot-start.

#### MSKintpnthotstarte

Hot-start type employed by the interior-point optimizers.

## MSK\_INTPNT\_HOTSTART\_NONE (0)

The interior-point optimizer performs a coldstart.

## MSK\_INTPNT\_HOTSTART\_PRIMAL (1)

The interior-point optimizer exploits the primal solution only.

#### MSK\_INTPNT\_HOTSTART\_DUAL (2)

The interior-point optimizer exploits the dual solution only.

## MSK\_INTPNT\_HOTSTART\_PRIMAL\_DUAL (3)

The interior-point optimizer exploits both the primal and dual solution.

## MSKpurifye

Solution purification employed optimizer.

# MSK\_PURIFY\_NONE (0)

The optimizer performs no solution purification.

#### MSK\_PURIFY\_PRIMAL (1)

The optimizer purifies the primal solution.

## MSK\_PURIFY\_DUAL (2)

The optimizer purifies the dual solution.

## MSK\_PURIFY\_PRIMAL\_DUAL (3)

The optimizer purifies both the primal and dual solution.

# MSK\_PURIFY\_AUTO (4)

TBD

# MSKcallbackcodee

Progress callback codes

#### MSK\_CALLBACK\_BEGIN\_BI (0)

The basis identification procedure has been started.

## MSK\_CALLBACK\_BEGIN\_CONIC (1)

The callback function is called when the conic optimizer is started.

#### MSK\_CALLBACK\_BEGIN\_DUAL\_BI (2)

The callback function is called from within the basis identification procedure when the dual phase is started.

## MSK\_CALLBACK\_BEGIN\_DUAL\_SENSITIVITY (3)

Dual sensitivity analysis is started.

# MSK\_CALLBACK\_BEGIN\_DUAL\_SETUP\_BI (4)

The callback function is called when the dual BI phase is started.

### MSK\_CALLBACK\_BEGIN\_DUAL\_SIMPLEX (5)

The callback function is called when the dual simplex optimizer started.

#### MSK\_CALLBACK\_BEGIN\_DUAL\_SIMPLEX\_BI (6)

The callback function is called from within the basis identification procedure when the dual simplex clean-up phase is started.

## MSK\_CALLBACK\_BEGIN\_FULL\_CONVEXITY\_CHECK (7)

Begin full convexity check.

### MSK\_CALLBACK\_BEGIN\_INFEAS\_ANA (8)

The callback function is called when the infeasibility analyzer is started.

#### MSK\_CALLBACK\_BEGIN\_INTPNT (9)

The callback function is called when the interior-point optimizer is started.

# MSK\_CALLBACK\_BEGIN\_LICENSE\_WAIT (10)

Begin waiting for license.

### MSK\_CALLBACK\_BEGIN\_MIO (11)

The callback function is called when the mixed-integer optimizer is started.

### MSK\_CALLBACK\_BEGIN\_OPTIMIZER (12)

The callback function is called when the optimizer is started.

### MSK\_CALLBACK\_BEGIN\_PRESOLVE (13)

The callback function is called when the presolve is started.

#### MSK\_CALLBACK\_BEGIN\_PRIMAL\_BI (14)

The callback function is called from within the basis identification procedure when the primal phase is started.

#### MSK\_CALLBACK\_BEGIN\_PRIMAL\_REPAIR (15)

Begin primal feasibility repair.

### MSK\_CALLBACK\_BEGIN\_PRIMAL\_SENSITIVITY (16)

Primal sensitivity analysis is started.

### MSK\_CALLBACK\_BEGIN\_PRIMAL\_SETUP\_BI (17)

The callback function is called when the primal BI setup is started.

## MSK\_CALLBACK\_BEGIN\_PRIMAL\_SIMPLEX (18)

The callback function is called when the primal simplex optimizer is started.

# MSK\_CALLBACK\_BEGIN\_PRIMAL\_SIMPLEX\_BI (19)

The callback function is called from within the basis identification procedure when the primal simplex clean-up phase is started.

## MSK\_CALLBACK\_BEGIN\_QCQO\_REFORMULATE (20)

Begin QCQO reformulation.

#### MSK\_CALLBACK\_BEGIN\_READ (21)

MOSEK has started reading a problem file.

# MSK\_CALLBACK\_BEGIN\_ROOT\_CUTGEN (22)

The callback function is called when root cut generation is started.

#### MSK\_CALLBACK\_BEGIN\_SIMPLEX (23)

The callback function is called when the simplex optimizer is started.

# MSK\_CALLBACK\_BEGIN\_SIMPLEX\_BI (24)

The callback function is called from within the basis identification procedure when the simplex clean-up phase is started.

# MSK\_CALLBACK\_BEGIN\_TO\_CONIC (25)

Begin conic reformulation.

#### MSK\_CALLBACK\_BEGIN\_WRITE (26)

MOSEK has started writing a problem file.

## MSK\_CALLBACK\_CONIC (27)

The callback function is called from within the conic optimizer after the information database has been updated.

## MSK\_CALLBACK\_DUAL\_SIMPLEX (28)

The callback function is called from within the dual simplex optimizer.

#### MSK\_CALLBACK\_END\_BI (29)

The callback function is called when the basis identification procedure is terminated.

### MSK\_CALLBACK\_END\_CONIC (30)

The callback function is called when the conic optimizer is terminated.

#### MSK\_CALLBACK\_END\_DUAL\_BI (31)

The callback function is called from within the basis identification procedure when the dual phase is terminated.

# MSK\_CALLBACK\_END\_DUAL\_SENSITIVITY (32)

Dual sensitivity analysis is terminated.

### MSK\_CALLBACK\_END\_DUAL\_SETUP\_BI (33)

The callback function is called when the dual BI phase is terminated.

### MSK\_CALLBACK\_END\_DUAL\_SIMPLEX (34)

The callback function is called when the dual simplex optimizer is terminated.

### MSK\_CALLBACK\_END\_DUAL\_SIMPLEX\_BI (35)

The callback function is called from within the basis identification procedure when the dual clean-up phase is terminated.

### MSK\_CALLBACK\_END\_FULL\_CONVEXITY\_CHECK (36)

End full convexity check.

#### MSK\_CALLBACK\_END\_INFEAS\_ANA (37)

The callback function is called when the infeasibility analyzer is terminated.

#### MSK\_CALLBACK\_END\_INTPNT (38)

The callback function is called when the interior-point optimizer is terminated.

#### MSK\_CALLBACK\_END\_LICENSE\_WAIT (39)

End waiting for license.

### MSK\_CALLBACK\_END\_MIO (40)

The callback function is called when the mixed-integer optimizer is terminated.

#### MSK\_CALLBACK\_END\_OPTIMIZER (41)

The callback function is called when the optimizer is terminated.

# MSK\_CALLBACK\_END\_PRESOLVE (42)

The callback function is called when the presolve is completed.

### MSK\_CALLBACK\_END\_PRIMAL\_BI (43)

The callback function is called from within the basis identification procedure when the primal phase is terminated.

#### MSK\_CALLBACK\_END\_PRIMAL\_REPAIR (44)

End primal feasibility repair.

## MSK\_CALLBACK\_END\_PRIMAL\_SENSITIVITY (45)

Primal sensitivity analysis is terminated.

## MSK\_CALLBACK\_END\_PRIMAL\_SETUP\_BI (46)

The callback function is called when the primal BI setup is terminated.

# MSK\_CALLBACK\_END\_PRIMAL\_SIMPLEX (47)

The callback function is called when the primal simplex optimizer is terminated.

## MSK\_CALLBACK\_END\_PRIMAL\_SIMPLEX\_BI (48)

The callback function is called from within the basis identification procedure when the primal clean-up phase is terminated.

#### MSK\_CALLBACK\_END\_QCQO\_REFORMULATE (49)

End QCQO reformulation.

## MSK\_CALLBACK\_END\_READ (50)

MOSEK has finished reading a problem file.

### MSK\_CALLBACK\_END\_ROOT\_CUTGEN (51)

The callback function is called when root cut generation is terminated.

#### MSK\_CALLBACK\_END\_SIMPLEX (52)

The callback function is called when the simplex optimizer is terminated.

#### MSK\_CALLBACK\_END\_SIMPLEX\_BI (53)

The callback function is called from within the basis identification procedure when the simplex clean-up phase is terminated.

### MSK\_CALLBACK\_END\_TO\_CONIC (54)

End conic reformulation.

### MSK\_CALLBACK\_END\_WRITE (55)

MOSEK has finished writing a problem file.

#### MSK\_CALLBACK\_IM\_BI (56)

The callback function is called from within the basis identification procedure at an intermediate point.

### MSK\_CALLBACK\_IM\_CONIC (57)

The callback function is called at an intermediate stage within the conic optimizer where the information database has not been updated.

### MSK\_CALLBACK\_IM\_DUAL\_BI (58)

The callback function is called from within the basis identification procedure at an intermediate point in the dual phase.

### MSK\_CALLBACK\_IM\_DUAL\_SENSIVITY (59)

The callback function is called at an intermediate stage of the dual sensitivity analysis.

#### MSK\_CALLBACK\_IM\_DUAL\_SIMPLEX (60)

The callback function is called at an intermediate point in the dual simplex optimizer.

## MSK\_CALLBACK\_IM\_FULL\_CONVEXITY\_CHECK (61)

The callback function is called at an intermediate stage of the full convexity check.

## MSK\_CALLBACK\_IM\_INTPNT (62)

The callback function is called at an intermediate stage within the interior-point optimizer where the information database has not been updated.

# MSK\_CALLBACK\_IM\_LICENSE\_WAIT (63)

MOSEK is waiting for a license.

#### MSK\_CALLBACK\_IM\_LU (64)

The callback function is called from within the LU factorization procedure at an intermediate point.

#### MSK\_CALLBACK\_IM\_MIO (65)

The callback function is called at an intermediate point in the mixed-integer optimizer.

# MSK\_CALLBACK\_IM\_MIO\_DUAL\_SIMPLEX (66)

The callback function is called at an intermediate point in the mixed-integer optimizer while running the dual simplex optimizer.

## MSK\_CALLBACK\_IM\_MIO\_INTPNT (67)

The callback function is called at an intermediate point in the mixed-integer optimizer while running the interior-point optimizer.

## MSK\_CALLBACK\_IM\_MIO\_PRIMAL\_SIMPLEX (68)

The callback function is called at an intermediate point in the mixed-integer optimizer while running the primal simplex optimizer.

## MSK\_CALLBACK\_IM\_ORDER (69)

The callback function is called from within the matrix ordering procedure at an intermediate point.

#### MSK\_CALLBACK\_IM\_PRESOLVE (70)

The callback function is called from within the presolve procedure at an intermediate stage.

## MSK\_CALLBACK\_IM\_PRIMAL\_BI (71)

The callback function is called from within the basis identification procedure at an intermediate point in the primal phase.

#### MSK\_CALLBACK\_IM\_PRIMAL\_SENSIVITY (72)

The callback function is called at an intermediate stage of the primal sensitivity analysis.

#### MSK\_CALLBACK\_IM\_PRIMAL\_SIMPLEX (73)

The callback function is called at an intermediate point in the primal simplex optimizer.

#### MSK\_CALLBACK\_IM\_QO\_REFORMULATE (74)

The callback function is called at an intermediate stage of the conic quadratic reformulation.

### MSK\_CALLBACK\_IM\_READ (75)

Intermediate stage in reading.

## MSK\_CALLBACK\_IM\_ROOT\_CUTGEN (76)

The callback is called from within root cut generation at an intermediate stage.

#### MSK\_CALLBACK\_IM\_SIMPLEX (77)

The callback function is called from within the simplex optimizer at an intermediate point.

### MSK\_CALLBACK\_IM\_SIMPLEX\_BI (78)

The callback function is called from within the basis identification procedure at an intermediate point in the simplex clean-up phase. The frequency of the callbacks is controlled by the  $MSK\_IPAR\_LOG\_SIM\_FREQ$  parameter.

#### MSK\_CALLBACK\_INTPNT (79)

The callback function is called from within the interior-point optimizer after the information database has been updated.

#### MSK\_CALLBACK\_NEW\_INT\_MIO (80)

The callback function is called after a new integer solution has been located by the mixed-integer optimizer.

## MSK\_CALLBACK\_PRIMAL\_SIMPLEX (81)

The callback function is called from within the primal simplex optimizer.

## MSK\_CALLBACK\_READ\_OPF (82)

The callback function is called from the OPF reader.

# MSK\_CALLBACK\_READ\_OPF\_SECTION (83)

A chunk of Q non-zeros has been read from a problem file.

## MSK\_CALLBACK\_SOLVING\_REMOTE (84)

The callback function is called while the task is being solved on a remote server.

#### MSK\_CALLBACK\_UPDATE\_DUAL\_BI (85)

The callback function is called from within the basis identification procedure at an intermediate point in the dual phase.

# MSK\_CALLBACK\_UPDATE\_DUAL\_SIMPLEX (86)

The callback function is called in the dual simplex optimizer.

## MSK\_CALLBACK\_UPDATE\_DUAL\_SIMPLEX\_BI (87)

The callback function is called from within the basis identification procedure at an intermediate point in the dual simplex clean-up phase. The frequency of the callbacks is controlled by the  $MSK\_IPAR\_LOG\_SIM\_FREQ$  parameter.

# MSK\_CALLBACK\_UPDATE\_PRESOLVE (88)

The callback function is called from within the presolve procedure.

## MSK\_CALLBACK\_UPDATE\_PRIMAL\_BI (89)

The callback function is called from within the basis identification procedure at an intermediate point in the primal phase.

#### MSK\_CALLBACK\_UPDATE\_PRIMAL\_SIMPLEX (90)

The callback function is called in the primal simplex optimizer.

## MSK\_CALLBACK\_UPDATE\_PRIMAL\_SIMPLEX\_BI (91)

The callback function is called from within the basis identification procedure at an intermediate point in the primal simplex clean-up phase. The frequency of the callbacks is controlled by the MSK\_IPAR\_LOG\_SIM\_FREQ parameter.

## MSK\_CALLBACK\_WRITE\_OPF (92)

The callback function is called from the OPF writer.

# MSKcheckconvexitytypee

Types of convexity checks.

MSK\_CHECK\_CONVEXITY\_NONE (0)

No convexity check.

MSK\_CHECK\_CONVEXITY\_SIMPLE (1)

Perform simple and fast convexity check.

MSK\_CHECK\_CONVEXITY\_FULL (2)

Perform a full convexity check.

# MSKcompresstypee

Compression types

MSK\_COMPRESS\_NONE (0)

No compression is used.

MSK\_COMPRESS\_FREE (1)

The type of compression used is chosen automatically.

MSK\_COMPRESS\_GZIP (2)

The type of compression used is gzip compatible.

MSK\_COMPRESS\_ZSTD (3)

The type of compression used is zstd compatible.

## MSKconetypee

Cone types

MSK\_CT\_QUAD (0)

The cone is a quadratic cone.

MSK\_CT\_RQUAD (1)

The cone is a rotated quadratic cone.

MSK\_CT\_PEXP (2)

A primal exponential cone.

MSK\_CT\_DEXP (3)

A dual exponential cone.

MSK\_CT\_PPOW (4)

A primal power cone.

MSK\_CT\_DPOW (5)

A dual power cone.

MSK\_CT\_ZERO (6)

The zero cone.

# ${\tt MSKnametypee}$

Name types

MSK\_NAME\_TYPE\_GEN (0)

General names. However, no duplicate and blank names are allowed.

MSK\_NAME\_TYPE\_MPS (1)

MPS type names.

MSK\_NAME\_TYPE\_LP (2)

LP type names.

## MSKscopre

SCopt operator types

MSK\_OPR\_ENT (0)

Entropy

MSK\_OPR\_EXP (1)

Exponential

MSK\_OPR\_LOG (2)

 ${\bf Logarithm}$ 

MSK\_OPR\_POW (3)

Power

#### MSK\_OPR\_SQRT (4)

Square root

### MSKsymmattypee

Cone types

### MSK\_SYMMAT\_TYPE\_SPARSE (0)

Sparse symmetric matrix.

#### MSKdataformate

Data format types

#### MSK\_DATA\_FORMAT\_EXTENSION (0)

The file extension is used to determine the data file format.

#### MSK\_DATA\_FORMAT\_MPS (1)

The data file is MPS formatted.

### MSK\_DATA\_FORMAT\_LP (2)

The data file is LP formatted.

#### MSK\_DATA\_FORMAT\_OP (3)

The data file is an optimization problem formatted file.

## MSK\_DATA\_FORMAT\_FREE\_MPS (4)

The data a free MPS formatted file.

# MSK\_DATA\_FORMAT\_TASK (5)

Generic task dump file.

## MSK\_DATA\_FORMAT\_PTF (6)

(P)retty (T)ext (F)format.

## MSK\_DATA\_FORMAT\_CB (7)

Conic benchmark format,

### MSK\_DATA\_FORMAT\_JSON\_TASK (8)

JSON based task format.

## MSKdinfiteme

Double information items

#### MSK\_DINF\_BI\_CLEAN\_DUAL\_TIME (0)

Time spent within the dual clean-up optimizer of the basis identification procedure since its invocation.

## MSK\_DINF\_BI\_CLEAN\_PRIMAL\_TIME (1)

Time spent within the primal clean-up optimizer of the basis identification procedure since its invocation.

### MSK\_DINF\_BI\_CLEAN\_TIME (2)

Time spent within the clean-up phase of the basis identification procedure since its invocation.

# MSK\_DINF\_BI\_DUAL\_TIME (3)

Time spent within the dual phase basis identification procedure since its invocation.

#### MSK\_DINF\_BI\_PRIMAL\_TIME (4)

Time spent within the primal phase of the basis identification procedure since its invocation.

## MSK\_DINF\_BI\_TIME (5)

Time spent within the basis identification procedure since its invocation.

#### MSK\_DINF\_INTPNT\_DUAL\_FEAS (6)

Dual feasibility measure reported by the interior-point optimizer. (For the interior-point optimizer this measure is not directly related to the original problem because a homogeneous model is employed.)

## MSK\_DINF\_INTPNT\_DUAL\_OBJ (7)

Dual objective value reported by the interior-point optimizer.

## MSK\_DINF\_INTPNT\_FACTOR\_NUM\_FLOPS (8)

An estimate of the number of flops used in the factorization.

#### MSK\_DINF\_INTPNT\_OPT\_STATUS (9)

A measure of optimality of the solution. It should converge to +1 if the problem has a primaldual optimal solution, and converge to -1 if the problem is (strictly) primal or dual infeasible. If the measure converges to another constant, or fails to settle, the problem is usually ill-posed.

### MSK\_DINF\_INTPNT\_ORDER\_TIME (10)

Order time (in seconds).

## MSK\_DINF\_INTPNT\_PRIMAL\_FEAS (11)

Primal feasibility measure reported by the interior-point optimizer. (For the interior-point optimizer this measure is not directly related to the original problem because a homogeneous model is employed).

### MSK\_DINF\_INTPNT\_PRIMAL\_OBJ (12)

Primal objective value reported by the interior-point optimizer.

#### MSK\_DINF\_INTPNT\_TIME (13)

Time spent within the interior-point optimizer since its invocation.

## MSK\_DINF\_MIO\_CLIQUE\_SEPARATION\_TIME (14)

Separation time for clique cuts.

## MSK\_DINF\_MIO\_CMIR\_SEPARATION\_TIME (15)

Separation time for CMIR cuts.

## MSK\_DINF\_MIO\_CONSTRUCT\_SOLUTION\_OBJ (16)

If **MOSEK** has successfully constructed an integer feasible solution, then this item contains the optimal objective value corresponding to the feasible solution.

### MSK\_DINF\_MIO\_DUAL\_BOUND\_AFTER\_PRESOLVE (17)

Value of the dual bound after presolve but before cut generation.

#### MSK\_DINF\_MIO\_GMI\_SEPARATION\_TIME (18)

Separation time for GMI cuts.

## MSK\_DINF\_MIO\_IMPLIED\_BOUND\_TIME (19)

Separation time for implied bound cuts.

# MSK\_DINF\_MIO\_KNAPSACK\_COVER\_SEPARATION\_TIME (20)

Separation time for knapsack cover.

#### MSK\_DINF\_MIO\_OBJ\_ABS\_GAP (21)

Given the mixed-integer optimizer has computed a feasible solution and a bound on the optimal objective value, then this item contains the absolute gap defined by

|(objective value of feasible solution) – (objective bound)|.

Otherwise it has the value -1.0.

## MSK\_DINF\_MIO\_OBJ\_BOUND (22)

The best known bound on the objective function. This value is undefined until at least one relaxation has been solved: To see if this is the case check that  $MSK\_IINF\_MIO\_NUM\_RELAX$  is strictly positive.

### MSK\_DINF\_MIO\_OBJ\_INT (23)

The primal objective value corresponding to the best integer feasible solution. Please note that at least one integer feasible solution must have been located i.e. check  $MSK\_IINF\_MIO\_NUM\_INT\_SOLUTIONS$ .

## MSK\_DINF\_MIO\_OBJ\_REL\_GAP (24)

Given that the mixed-integer optimizer has computed a feasible solution and a bound on the optimal objective value, then this item contains the relative gap defined by

```
\frac{|(\text{objective value of feasible solution}) - (\text{objective bound})|}{\max(\delta, |(\text{objective value of feasible solution})|)}.
```

where  $\delta$  is given by the parameter  $MSK\_DPAR\_MIO\_REL\_GAP\_CONST$ . Otherwise it has the value -1.0.

#### MSK\_DINF\_MIO\_PROBING\_TIME (25)

Total time for probing.

#### MSK\_DINF\_MIO\_ROOT\_CUTGEN\_TIME (26)

Total time for cut generation.

#### MSK\_DINF\_MIO\_ROOT\_OPTIMIZER\_TIME (27)

Time spent in the optimizer while solving the root node relaxation

### MSK\_DINF\_MIO\_ROOT\_PRESOLVE\_TIME (28)

Time spent presolving the problem at the root node.

## MSK\_DINF\_MIO\_TIME (29)

Time spent in the mixed-integer optimizer.

### MSK\_DINF\_MIO\_USER\_OBJ\_CUT (30)

If the objective cut is used, then this information item has the value of the cut.

## MSK\_DINF\_OPTIMIZER\_TIME (31)

Total time spent in the optimizer since it was invoked.

### MSK\_DINF\_PRESOLVE\_ELI\_TIME (32)

Total time spent in the eliminator since the presolve was invoked.

## MSK\_DINF\_PRESOLVE\_LINDEP\_TIME (33)

Total time spent in the linear dependency checker since the presolve was invoked.

#### MSK\_DINF\_PRESOLVE\_TIME (34)

Total time (in seconds) spent in the presolve since it was invoked.

## MSK\_DINF\_PRIMAL\_REPAIR\_PENALTY\_OBJ (35)

The optimal objective value of the penalty function.

### MSK\_DINF\_QCQO\_REFORMULATE\_MAX\_PERTURBATION (36)

Maximum absolute diagonal perturbation occurring during the QCQO reformulation.

# MSK\_DINF\_QCQO\_REFORMULATE\_TIME (37)

Time spent with conic quadratic reformulation.

# MSK\_DINF\_QCQO\_REFORMULATE\_WORST\_CHOLESKY\_COLUMN\_SCALING (38)

Worst Cholesky column scaling.

## MSK\_DINF\_QCQO\_REFORMULATE\_WORST\_CHOLESKY\_DIAG\_SCALING (39)

Worst Cholesky diagonal scaling.

# MSK\_DINF\_RD\_TIME (40)

Time spent reading the data file.

## MSK\_DINF\_SIM\_DUAL\_TIME (41)

Time spent in the dual simplex optimizer since invoking it.

#### MSK\_DINF\_SIM\_FEAS (42)

Feasibility measure reported by the simplex optimizer.

# MSK\_DINF\_SIM\_OBJ (43)

Objective value reported by the simplex optimizer.

### MSK\_DINF\_SIM\_PRIMAL\_TIME (44)

Time spent in the primal simplex optimizer since invoking it.

# MSK\_DINF\_SIM\_TIME (45)

Time spent in the simplex optimizer since invoking it.

## MSK\_DINF\_SOL\_BAS\_DUAL\_OBJ (46)

Dual objective value of the basic solution. Updated if  $MSK\_IPAR\_AUTO\_UPDATE\_SOL\_INFO$  is set or by the method  $MSK\_updatesolutioninfo$ .

### MSK\_DINF\_SOL\_BAS\_DVIOLCON (47)

Maximal dual bound violation for  $x^c$  in the basic solution. Updated if  $\textit{MSK\_IPAR\_AUTO\_UPDATE\_SOL\_INFO}$  is set or by the method  $\textit{MSK\_updatesolutioninfo}$ .

#### MSK\_DINF\_SOL\_BAS\_DVIOLVAR (48)

Maximal dual bound violation for  $x^x$  in the basic solution. Updated if  $\textit{MSK\_IPAR\_AUTO\_UPDATE\_SOL\_INFO}$  is set or by the method  $\textit{MSK\_updatesolutioninfo}$ .

## MSK\_DINF\_SOL\_BAS\_NRM\_BARX (49)

Infinity norm of  $\overline{X}$  in the basic solution.

# MSK\_DINF\_SOL\_BAS\_NRM\_SLC (50)

Infinity norm of  $s_l^c$  in the basic solution.

### MSK\_DINF\_SOL\_BAS\_NRM\_SLX (51)

Infinity norm of  $s_l^x$  in the basic solution.

## MSK\_DINF\_SOL\_BAS\_NRM\_SUC (52)

Infinity norm of  $s_u^c$  in the basic solution.

# MSK\_DINF\_SOL\_BAS\_NRM\_SUX (53)

Infinity norm of  $s_u^X$  in the basic solution.

### MSK\_DINF\_SOL\_BAS\_NRM\_XC (54)

Infinity norm of  $x^c$  in the basic solution.

### MSK\_DINF\_SOL\_BAS\_NRM\_XX (55)

Infinity norm of  $x^x$  in the basic solution.

### MSK\_DINF\_SOL\_BAS\_NRM\_Y (56)

Infinity norm of y in the basic solution.

## MSK\_DINF\_SOL\_BAS\_PRIMAL\_OBJ (57)

Primal objective value of the basic solution. Updated if MSK\_IPAR\_AUTO\_UPDATE\_SOL\_INFO is set or by the method MSK\_updatesolutioninfo.

### MSK\_DINF\_SOL\_BAS\_PVIOLCON (58)

Maximal primal bound violation for  $x^c$  in the basic solution. Updated if  $\textit{MSK\_IPAR\_AUTO\_UPDATE\_SOL\_INFO}$  is set or by the method  $\textit{MSK\_updatesolutioninfo}$ .

#### MSK\_DINF\_SOL\_BAS\_PVIOLVAR (59)

Maximal primal bound violation for  $x^x$  in the basic solution. Updated if  $MSK\_IPAR\_AUTO\_UPDATE\_SOL\_INFO$  is set or by the method  $MSK\_updatesolutioninfo$ .

# MSK\_DINF\_SOL\_ITG\_NRM\_BARX (60)

Infinity norm of  $\overline{X}$  in the integer solution.

# MSK\_DINF\_SOL\_ITG\_NRM\_XC (61)

Infinity norm of  $x^c$  in the integer solution.

## MSK\_DINF\_SOL\_ITG\_NRM\_XX (62)

Infinity norm of  $x^x$  in the integer solution.

## MSK\_DINF\_SOL\_ITG\_PRIMAL\_OBJ (63)

Primal objective value of the integer solution. Updated if  $MSK\_IPAR\_AUTO\_UPDATE\_SOL\_INFO$  is set or by the method  $MSK\_updatesolutioninfo$ .

## MSK\_DINF\_SOL\_ITG\_PVIOLBARVAR (64)

Maximal primal bound violation for  $\overline{X}$  in the integer solution. Updated if  $MSK\_IPAR\_AUTO\_UPDATE\_SOL\_INFO$  is set or by the method  $MSK\_updatesolutioninfo$ .

# MSK\_DINF\_SOL\_ITG\_PVIOLCON (65)

Maximal primal bound violation for  $x^c$  in the integer solution. Updated if  $MSK\_IPAR\_AUTO\_UPDATE\_SOL\_INFO$  is set or by the method  $MSK\_updatesolutioninfo$ .

## MSK\_DINF\_SOL\_ITG\_PVIOLCONES (66)

Maximal primal violation for primal conic constraints in the integer solution. Updated if  $MSK\_IPAR\_AUTO\_UPDATE\_SOL\_INFO$  is set or by the method  $MSK\_updatesolutioninfo$ .

## MSK\_DINF\_SOL\_ITG\_PVIOLITG (67)

Maximal violation for the integer constraints in the integer solution. Updated if  $MSK\_IPAR\_AUTO\_UPDATE\_SOL\_INFO$  is set or by the method  $MSK\_updatesolutioninfo$ .

#### MSK\_DINF\_SOL\_ITG\_PVIOLVAR (68)

Maximal primal bound violation for  $x^x$  in the integer solution. Updated if  $\textit{MSK\_IPAR\_AUTO\_UPDATE\_SOL\_INFO}$  is set or by the method  $\textit{MSK\_updatesolutioninfo}$ .

## MSK\_DINF\_SOL\_ITR\_DUAL\_OBJ (69)

Dual objective value of the interior-point solution. Updated if  $MSK\_IPAR\_AUTO\_UPDATE\_SOL\_INFO$  is set or by the method  $MSK\_updatesolutioninfo$ .

#### MSK\_DINF\_SOL\_ITR\_DVIOLBARVAR (70)

Maximal dual bound violation for  $\overline{X}$  in the interior-point solution. Updated if  $MSK\_IPAR\_AUTO\_UPDATE\_SOL\_INFO$  is set or by the method  $MSK\_updatesolutioninfo$ .

### MSK\_DINF\_SOL\_ITR\_DVIOLCON (71)

Maximal dual bound violation for  $x^c$  in the interior-point solution. Updated if  $\textit{MSK\_IPAR\_AUTO\_UPDATE\_SOL\_INFO}$  is set or by the method  $\textit{MSK\_updatesolutioninfo}$ .

## MSK\_DINF\_SOL\_ITR\_DVIOLCONES (72)

Maximal dual violation for dual conic constraints in the interior-point solution. Updated if  $MSK\_IPAR\_AUTO\_UPDATE\_SOL\_INFO$  is set or by the method  $MSK\_updatesolutioninfo$ .

#### MSK\_DINF\_SOL\_ITR\_DVIOLVAR (73)

Maximal dual bound violation for  $x^x$  in the interior-point solution. Updated if  $MSK\_IPAR\_AUTO\_UPDATE\_SOL\_INFO$  is set or by the method  $MSK\_updatesolutioninfo$ .

## MSK\_DINF\_SOL\_ITR\_NRM\_BARS (74)

Infinity norm of  $\overline{S}$  in the interior-point solution.

## MSK\_DINF\_SOL\_ITR\_NRM\_BARX (75)

Infinity norm of  $\overline{X}$  in the interior-point solution.

# MSK\_DINF\_SOL\_ITR\_NRM\_SLC (76)

Infinity norm of  $s_l^c$  in the interior-point solution.

#### MSK\_DINF\_SOL\_ITR\_NRM\_SLX (77)

Infinity norm of  $s_l^x$  in the interior-point solution.

# MSK\_DINF\_SOL\_ITR\_NRM\_SNX (78)

Infinity norm of  $s_n^x$  in the interior-point solution.

# MSK\_DINF\_SOL\_ITR\_NRM\_SUC (79)

Infinity norm of  $s_u^c$  in the interior-point solution.

# MSK\_DINF\_SOL\_ITR\_NRM\_SUX (80)

Infinity norm of  $s_u^X$  in the interior-point solution.

# MSK\_DINF\_SOL\_ITR\_NRM\_XC (81)

Infinity norm of  $x^c$  in the interior-point solution.

## MSK\_DINF\_SOL\_ITR\_NRM\_XX (82)

Infinity norm of  $x^x$  in the interior-point solution.

### MSK\_DINF\_SOL\_ITR\_NRM\_Y (83)

Infinity norm of y in the interior-point solution.

# MSK\_DINF\_SOL\_ITR\_PRIMAL\_OBJ (84)

Primal objective value of the interior-point solution. Updated if  $MSK\_IPAR\_AUTO\_UPDATE\_SOL\_INFO$  is set or by the method  $MSK\_updatesolutioninfo$ .

## MSK\_DINF\_SOL\_ITR\_PVIOLBARVAR (85)

Maximal primal bound violation for  $\overline{X}$  in the interior-point solution. Updated if  $MSK\_IPAR\_AUTO\_UPDATE\_SOL\_INFO$  is set or by the method  $MSK\_updatesolutioninfo$ .

# MSK\_DINF\_SOL\_ITR\_PVIOLCON (86)

Maximal primal bound violation for  $x^c$  in the interior-point solution. Updated if  $MSK\_IPAR\_AUTO\_UPDATE\_SOL\_INFO$  is set or by the method  $MSK\_updatesolutioninfo$ .

## MSK\_DINF\_SOL\_ITR\_PVIOLCONES (87)

Maximal primal violation for primal conic constraints in the interior-point solution. Updated if MSK\_IPAR\_AUTO\_UPDATE\_SOL\_INFO is set or by the method MSK\_updatesolutioninfo.

# MSK\_DINF\_SOL\_ITR\_PVIOLVAR (88)

Maximal primal bound violation for  $x^x$  in the interior-point solution. Updated if  $MSK\_IPAR\_AUTO\_UPDATE\_SOL\_INFO$  is set or by the method  $MSK\_updatesolutioninfo$ .

## MSK\_DINF\_TO\_CONIC\_TIME (89)

Time spent in the last to conic reformulation.

#### MSKfeaturee

License feature

#### MSK\_FEATURE\_PTS (0)

Base system.

### MSK\_FEATURE\_PTON (1)

Conic extension.

#### MSKliinfiteme

Long integer information items.

# MSK\_LIINF\_BI\_CLEAN\_DUAL\_DEG\_ITER (0)

Number of dual degenerate clean iterations performed in the basis identification.

## MSK\_LIINF\_BI\_CLEAN\_DUAL\_ITER (1)

Number of dual clean iterations performed in the basis identification.

# MSK\_LIINF\_BI\_CLEAN\_PRIMAL\_DEG\_ITER (2)

Number of primal degenerate clean iterations performed in the basis identification.

### MSK\_LIINF\_BI\_CLEAN\_PRIMAL\_ITER (3)

Number of primal clean iterations performed in the basis identification.

#### MSK\_LIINF\_BI\_DUAL\_ITER (4)

Number of dual pivots performed in the basis identification.

### MSK\_LIINF\_BI\_PRIMAL\_ITER (5)

Number of primal pivots performed in the basis identification.

#### MSK\_LIINF\_INTPNT\_FACTOR\_NUM\_NZ (6)

Number of non-zeros in factorization.

#### MSK\_LIINF\_MIO\_ANZ (7)

Number of non-zero entries in the constraint matrix of the probelm to be solved by the mixed-integer optimizer.

### MSK\_LIINF\_MIO\_INTPNT\_ITER (8)

Number of interior-point iterations performed by the mixed-integer optimizer.

### MSK\_LIINF\_MIO\_PRESOLVED\_ANZ (9)

Number of non-zero entries in the constraint matrix of the problem after the mixed-integer optimizer's presolve.

### MSK\_LIINF\_MIO\_SIMPLEX\_ITER (10)

Number of simplex iterations performed by the mixed-integer optimizer.

# MSK\_LIINF\_RD\_NUMANZ (11)

Number of non-zeros in A that is read.

#### MSK\_LIINF\_RD\_NUMQNZ (12)

Number of Q non-zeros.

## MSKiinfiteme

Integer information items.

### MSK\_IINF\_ANA\_PRO\_NUM\_CON (0)

Number of constraints in the problem. This value is set by MSK\_analyzeproblem.

## MSK\_IINF\_ANA\_PRO\_NUM\_CON\_EQ (1)

Number of equality constraints. This value is set by MSK\_analyzeproblem.

## MSK\_IINF\_ANA\_PRO\_NUM\_CON\_FR (2)

Number of unbounded constraints. This value is set by MSK\_analyzeproblem.

## MSK\_IINF\_ANA\_PRO\_NUM\_CON\_LO (3)

Number of constraints with a lower bound and an infinite upper bound. This value is set by MSK\_analyzeproblem.

# MSK\_IINF\_ANA\_PRO\_NUM\_CON\_RA (4)

Number of constraints with finite lower and upper bounds. This value is set by  $MSK\_analyzeproblem$ .

## MSK\_IINF\_ANA\_PRO\_NUM\_CON\_UP (5)

Number of constraints with an upper bound and an infinite lower bound. This value is set by  $MSK\_analyzeproblem$ .

#### MSK\_IINF\_ANA\_PRO\_NUM\_VAR (6)

Number of variables in the problem. This value is set by MSK\_analyzeproblem.

### MSK\_IINF\_ANA\_PRO\_NUM\_VAR\_BIN (7)

Number of binary (0-1) variables. This value is set by MSK\_analyzeproblem.

#### MSK\_IINF\_ANA\_PRO\_NUM\_VAR\_CONT (8)

Number of continuous variables. This value is set by MSK\_analyzeproblem.

#### MSK\_IINF\_ANA\_PRO\_NUM\_VAR\_EQ (9)

Number of fixed variables. This value is set by MSK\_analyzeproblem.

### MSK\_IINF\_ANA\_PRO\_NUM\_VAR\_FR (10)

Number of free variables. This value is set by MSK\_analyzeproblem.

#### MSK\_IINF\_ANA\_PRO\_NUM\_VAR\_INT (11)

Number of general integer variables. This value is set by MSK\_analyzeproblem.

# MSK\_IINF\_ANA\_PRO\_NUM\_VAR\_LO (12)

Number of variables with a lower bound and an infinite upper bound. This value is set by <code>MSK\_analyzeproblem</code>.

#### MSK\_IINF\_ANA\_PRO\_NUM\_VAR\_RA (13)

Number of variables with finite lower and upper bounds. This value is set by  $MSK\_analyzeproblem$ .

#### MSK\_IINF\_ANA\_PRO\_NUM\_VAR\_UP (14)

Number of variables with an upper bound and an infinite lower bound. This value is set by MSK\_analyzeproblem.

# MSK\_IINF\_INTPNT\_FACTOR\_DIM\_DENSE (15)

Dimension of the dense sub system in factorization.

# MSK\_IINF\_INTPNT\_ITER (16)

Number of interior-point iterations since invoking the interior-point optimizer.

# MSK\_IINF\_INTPNT\_NUM\_THREADS (17)

Number of threads that the interior-point optimizer is using.

# MSK\_IINF\_INTPNT\_SOLVE\_DUAL (18)

Non-zero if the interior-point optimizer is solving the dual problem.

### MSK\_IINF\_MIO\_ABSGAP\_SATISFIED (19)

Non-zero if absolute gap is within tolerances.

# MSK\_IINF\_MIO\_CLIQUE\_TABLE\_SIZE (20)

Size of the clique table.

### MSK\_IINF\_MIO\_CONSTRUCT\_SOLUTION (21)

This item informs if MOSEK constructed an initial integer feasible solution.

- -1: tried, but failed,
- 0: no partial solution supplied by the user,
- 1: constructed feasible solution.

### MSK\_IINF\_MIO\_NODE\_DEPTH (22)

Depth of the last node solved.

# MSK\_IINF\_MIO\_NUM\_ACTIVE\_NODES (23)

Number of active branch and bound nodes.

# MSK\_IINF\_MIO\_NUM\_BRANCH (24)

Number of branches performed during the optimization.

# MSK\_IINF\_MIO\_NUM\_CLIQUE\_CUTS (25)

Number of clique cuts.

# MSK\_IINF\_MIO\_NUM\_CMIR\_CUTS (26)

Number of Complemented Mixed Integer Rounding (CMIR) cuts.

### MSK\_IINF\_MIO\_NUM\_GOMORY\_CUTS (27)

Number of Gomory cuts.

### MSK\_IINF\_MIO\_NUM\_IMPLIED\_BOUND\_CUTS (28)

Number of implied bound cuts.

#### MSK\_IINF\_MIO\_NUM\_INT\_SOLUTIONS (29)

Number of integer feasible solutions that have been found.

#### MSK\_IINF\_MIO\_NUM\_KNAPSACK\_COVER\_CUTS (30)

Number of clique cuts.

#### MSK\_IINF\_MIO\_NUM\_RELAX (31)

Number of relaxations solved during the optimization.

# MSK\_IINF\_MIO\_NUM\_REPEATED\_PRESOLVE (32)

Number of times presolve was repeated at root.

### MSK\_IINF\_MIO\_NUMBIN (33)

Number of binary variables in the problem to be solved by the mixed-integer optimizer.

### MSK\_IINF\_MIO\_NUMBINCONEVAR (34)

Number of binary cone variables in the problem to be solved by the mixed-integer optimizer.

#### MSK\_IINF\_MIO\_NUMCON (35)

Number of constraints in the problem to be solved by the mixed-integer optimizer.

# MSK\_IINF\_MIO\_NUMCONE (36)

Number of cones in the problem to be solved by the mixed-integer optimizer.

#### MSK\_IINF\_MIO\_NUMCONEVAR (37)

Number of cone variables in the problem to be solved by the mixed-integer optimizer.

### MSK\_IINF\_MIO\_NUMCONT (38)

Number of continuous variables in the problem to be solved by the mixed-integer optimizer.

# MSK\_IINF\_MIO\_NUMCONTCONEVAR (39)

Number of continuous cone variables in the problem to be solved by the mixed-integer optimizer.

### MSK\_IINF\_MIO\_NUMDEXPCONES (40)

Number of dual exponential cones in the problem to be solved by the mixed-integer optimizer.

# MSK\_IINF\_MIO\_NUMDPOWCONES (41)

Number of dual power cones in the problem to be solved by the mixed-integer optimizer.

# MSK\_IINF\_MIO\_NUMINT (42)

Number of integer variables in the problem to be solved by the mixed-integer optimizer.

# MSK\_IINF\_MIO\_NUMINTCONEVAR (43)

Number of integer cone variables in the problem to be solved by the mixed-integer optimizer.

# MSK\_IINF\_MIO\_NUMPEXPCONES (44)

Number of primal exponential cones in the problem to be solved by the mixed-integer optimizer.

### MSK\_IINF\_MIO\_NUMPPOWCONES (45)

Number of primal power cones in the problem to be solved by the mixed-integer optimizer.

### MSK\_IINF\_MIO\_NUMQCONES (46)

Number of quadratic cones in the problem to be solved by the mixed-integer optimizer.

# MSK\_IINF\_MIO\_NUMRQCONES (47)

Number of rotated quadratic cones in the problem to be solved by the mixed-integer optimizer.

### MSK\_IINF\_MIO\_NUMVAR (48)

Number of variables in the problem to be solved by the mixed-integer optimizer.

# MSK\_IINF\_MIO\_OBJ\_BOUND\_DEFINED (49)

Non-zero if a valid objective bound has been found, otherwise zero.

# MSK\_IINF\_MIO\_PRESOLVED\_NUMBIN (50)

Number of binary variables in the problem after the mixed-integer optimizer's presolve.

### MSK\_IINF\_MIO\_PRESOLVED\_NUMBINCONEVAR (51)

Number of binary cone variables in the problem after the mixed-integer optimizer's presolve.

#### MSK\_IINF\_MIO\_PRESOLVED\_NUMCON (52)

Number of constraints in the problem after the mixed-integer optimizer's presolve.

### MSK\_IINF\_MIO\_PRESOLVED\_NUMCONE (53)

Number of cones in the problem after the mixed-integer optimizer's presolve.

#### MSK\_IINF\_MIO\_PRESOLVED\_NUMCONEVAR (54)

Number of cone variables in the problem after the mixed-integer optimizer's presolve.

### MSK\_IINF\_MIO\_PRESOLVED\_NUMCONT (55)

Number of continuous variables in the problem after the mixed-integer optimizer's presolve.

### MSK\_IINF\_MIO\_PRESOLVED\_NUMCONTCONEVAR (56)

Number of continuous cone variables in the problem after the mixed-integer optimizer's presolve.

### MSK\_IINF\_MIO\_PRESOLVED\_NUMDEXPCONES (57)

Number of dual exponential cones in the problem after the mixed-integer optimizer's presolve.

### MSK\_IINF\_MIO\_PRESOLVED\_NUMDPOWCONES (58)

Number of dual power cones in the problem after the mixed-integer optimizer's presolve.

#### MSK\_IINF\_MIO\_PRESOLVED\_NUMINT (59)

Number of integer variables in the problem after the mixed-integer optimizer's presolve.

### MSK\_IINF\_MIO\_PRESOLVED\_NUMINTCONEVAR (60)

Number of integer cone variables in the problem after the mixed-integer optimizer's presolve.

### MSK\_IINF\_MIO\_PRESOLVED\_NUMPEXPCONES (61)

Number of primal exponential cones in the problem after the mixed-integer optimizer's presolve.

#### MSK\_IINF\_MIO\_PRESOLVED\_NUMPPOWCONES (62)

Number of primal power cones in the problem after the mixed-integer optimizer's presolve.

# MSK\_IINF\_MIO\_PRESOLVED\_NUMQCONES (63)

Number of quadratic cones in the problem after the mixed-integer optimizer's presolve.

### MSK\_IINF\_MIO\_PRESOLVED\_NUMRQCONES (64)

Number of rotated quadratic cones in the problem after the mixed-integer optimizer's presolve.

# MSK\_IINF\_MIO\_PRESOLVED\_NUMVAR (65)

Number of variables in the problem after the mixed-integer optimizer's presolve.

### MSK\_IINF\_MIO\_RELGAP\_SATISFIED (66)

Non-zero if relative gap is within tolerances.

### MSK\_IINF\_MIO\_TOTAL\_NUM\_CUTS (67)

Total number of cuts generated by the mixed-integer optimizer.

### MSK\_IINF\_MIO\_USER\_OBJ\_CUT (68)

If it is non-zero, then the objective cut is used.

### MSK\_IINF\_OPT\_NUMCON (69)

Number of constraints in the problem solved when the optimizer is called.

### MSK\_IINF\_OPT\_NUMVAR (70)

Number of variables in the problem solved when the optimizer is called

### MSK\_IINF\_OPTIMIZE\_RESPONSE (71)

The response code returned by optimize.

### MSK\_IINF\_PURIFY\_DUAL\_SUCCESS (72)

Is nonzero if the dual solution is purified.

# MSK\_IINF\_PURIFY\_PRIMAL\_SUCCESS (73)

Is nonzero if the primal solution is purified.

# MSK\_IINF\_RD\_NUMBARVAR (74)

Number of symmetric variables read.

# MSK\_IINF\_RD\_NUMCON (75)

Number of constraints read.

### MSK\_IINF\_RD\_NUMCONE (76)

Number of conic constraints read.

#### MSK\_IINF\_RD\_NUMINTVAR (77)

Number of integer-constrained variables read.

#### MSK\_IINF\_RD\_NUMQ (78)

Number of nonempty Q matrices read.

### MSK\_IINF\_RD\_NUMVAR (79)

Number of variables read.

### MSK\_IINF\_RD\_PROTYPE (80)

Problem type.

### MSK\_IINF\_SIM\_DUAL\_DEG\_ITER (81)

The number of dual degenerate iterations.

### MSK\_IINF\_SIM\_DUAL\_HOTSTART (82)

If 1 then the dual simplex algorithm is solving from an advanced basis.

### MSK\_IINF\_SIM\_DUAL\_HOTSTART\_LU (83)

If 1 then a valid basis factorization of full rank was located and used by the dual simplex algorithm.

#### MSK\_IINF\_SIM\_DUAL\_INF\_ITER (84)

The number of iterations taken with dual infeasibility.

# MSK\_IINF\_SIM\_DUAL\_ITER (85)

Number of dual simplex iterations during the last optimization.

#### MSK\_IINF\_SIM\_NUMCON (86)

Number of constraints in the problem solved by the simplex optimizer.

### MSK\_IINF\_SIM\_NUMVAR (87)

Number of variables in the problem solved by the simplex optimizer.

### MSK\_IINF\_SIM\_PRIMAL\_DEG\_ITER (88)

The number of primal degenerate iterations.

# MSK\_IINF\_SIM\_PRIMAL\_HOTSTART (89)

If 1 then the primal simplex algorithm is solving from an advanced basis.

# MSK\_IINF\_SIM\_PRIMAL\_HOTSTART\_LU (90)

If 1 then a valid basis factorization of full rank was located and used by the primal simplex algorithm.

# MSK\_IINF\_SIM\_PRIMAL\_INF\_ITER (91)

The number of iterations taken with primal infeasibility.

### MSK\_IINF\_SIM\_PRIMAL\_ITER (92)

Number of primal simplex iterations during the last optimization.

### MSK\_IINF\_SIM\_SOLVE\_DUAL (93)

Is non-zero if dual problem is solved.

### MSK\_IINF\_SOL\_BAS\_PROSTA (94)

Problem status of the basic solution. Updated after each optimization.

### MSK\_IINF\_SOL\_BAS\_SOLSTA (95)

Solution status of the basic solution. Updated after each optimization.

# MSK\_IINF\_SOL\_ITG\_PROSTA (96)

Problem status of the integer solution. Updated after each optimization.

# MSK\_IINF\_SOL\_ITG\_SOLSTA (97)

Solution status of the integer solution. Updated after each optimization.

# MSK\_IINF\_SOL\_ITR\_PROSTA (98)

Problem status of the interior-point solution. Updated after each optimization.

### MSK\_IINF\_SOL\_ITR\_SOLSTA (99)

Solution status of the interior-point solution. Updated after each optimization.

#### MSK\_IINF\_STO\_NUM\_A\_REALLOC (100)

Number of times the storage for storing A has been changed. A large value may indicates that memory fragmentation may occur.

### MSKinftypee

Information item types

### MSK\_INF\_DOU\_TYPE (0)

Is a double information type.

### MSK\_INF\_INT\_TYPE (1)

Is an integer.

### MSK\_INF\_LINT\_TYPE (2)

Is a long integer.

#### MSKiomodee

Input/output modes

### MSK\_IOMODE\_READ (0)

The file is read-only.

### MSK\_IOMODE\_WRITE (1)

The file is write-only. If the file exists then it is truncated when it is opened. Otherwise it is created when it is opened.

### MSK\_IOMODE\_READWRITE (2)

The file is to read and write.

#### MSKbranchdire

Specifies the branching direction.

### MSK\_BRANCH\_DIR\_FREE (0)

The mixed-integer optimizer decides which branch to choose.

#### MSK\_BRANCH\_DIR\_UP (1)

The mixed-integer optimizer always chooses the up branch first.

# MSK\_BRANCH\_DIR\_DOWN (2)

The mixed-integer optimizer always chooses the down branch first.

# MSK\_BRANCH\_DIR\_NEAR (3)

Branch in direction nearest to selected fractional variable.

# MSK\_BRANCH\_DIR\_FAR (4)

Branch in direction farthest from selected fractional variable.

# MSK\_BRANCH\_DIR\_ROOT\_LP (5)

Chose direction based on root lp value of selected variable.

# MSK\_BRANCH\_DIR\_GUIDED (6)

Branch in direction of current incumbent.

# MSK\_BRANCH\_DIR\_PSEUDOCOST (7)

Branch based on the pseudocost of the variable.

#### MSKmiocontsoltypee

Continuous mixed-integer solution type

# MSK\_MIO\_CONT\_SOL\_NONE (0)

No interior-point or basic solution are reported when the mixed-integer optimizer is used.

### MSK\_MIO\_CONT\_SOL\_ROOT (1)

The reported interior-point and basic solutions are a solution to the root node problem when mixed-integer optimizer is used.

#### MSK\_MIO\_CONT\_SOL\_ITG (2)

The reported interior-point and basic solutions are a solution to the problem with all integer variables fixed at the value they have in the integer solution. A solution is only reported in case the problem has a primal feasible solution.

### MSK\_MIO\_CONT\_SOL\_ITG\_REL (3)

In case the problem is primal feasible then the reported interior-point and basic solutions are a solution to the problem with all integer variables fixed at the value they have in the integer solution. If the problem is primal infeasible, then the solution to the root node problem is reported.

#### MSKmiomodee

Integer restrictions

### MSK\_MIO\_MODE\_IGNORED (0)

The integer constraints are ignored and the problem is solved as a continuous problem.

# MSK\_MIO\_MODE\_SATISFIED (1)

Integer restrictions should be satisfied.

#### MSKmionodeseltypee

Mixed-integer node selection types

### MSK\_MIO\_NODE\_SELECTION\_FREE (0)

The optimizer decides the node selection strategy.

### MSK\_MIO\_NODE\_SELECTION\_FIRST (1)

The optimizer employs a depth first node selection strategy.

#### MSK\_MIO\_NODE\_SELECTION\_BEST (2)

The optimizer employs a best bound node selection strategy.

### MSK\_MIO\_NODE\_SELECTION\_PSEUDO (3)

The optimizer employs selects the node based on a pseudo cost estimate.

# MSKmpsformate

MPS file format type

### MSK\_MPS\_FORMAT\_STRICT (0)

It is assumed that the input file satisfies the MPS format strictly.

#### MSK\_MPS\_FORMAT\_RELAXED (1)

It is assumed that the input file satisfies a slightly relaxed version of the MPS format.

### MSK\_MPS\_FORMAT\_FREE (2)

It is assumed that the input file satisfies the free MPS format. This implies that spaces are not allowed in names. Otherwise the format is free.

#### MSK\_MPS\_FORMAT\_CPLEX (3)

The CPLEX compatible version of the MPS format is employed.

# MSKobjsensee

Objective sense types

# MSK\_OBJECTIVE\_SENSE\_MINIMIZE (0)

The problem should be minimized.

# MSK\_OBJECTIVE\_SENSE\_MAXIMIZE (1)

The problem should be maximized.

# MSKonoffkeye

On/off

### MSK\_ON (1)

Switch the option on.

### MSK\_OFF (0)

Switch the option off.

### MSKoptimizertypee

Optimizer types

### MSK\_OPTIMIZER\_CONIC (0)

The optimizer for problems having conic constraints.

# MSK\_OPTIMIZER\_DUAL\_SIMPLEX (1)

The dual simplex optimizer is used.

# MSK\_OPTIMIZER\_FREE (2)

The optimizer is chosen automatically.

### MSK\_OPTIMIZER\_FREE\_SIMPLEX (3)

One of the simplex optimizers is used.

### MSK\_OPTIMIZER\_INTPNT (4)

The interior-point optimizer is used.

# MSK\_OPTIMIZER\_MIXED\_INT (5)

The mixed-integer optimizer.

### MSK\_OPTIMIZER\_PRIMAL\_SIMPLEX (6)

The primal simplex optimizer is used.

# MSKorderingtypee

Ordering strategies

# MSK\_ORDER\_METHOD\_FREE (0)

The ordering method is chosen automatically.

### MSK\_ORDER\_METHOD\_APPMINLOC (1)

Approximate minimum local fill-in ordering is employed.

### MSK\_ORDER\_METHOD\_EXPERIMENTAL (2)

This option should not be used.

### MSK\_ORDER\_METHOD\_TRY\_GRAPHPAR (3)

Always try the graph partitioning based ordering.

### MSK\_ORDER\_METHOD\_FORCE\_GRAPHPAR (4)

Always use the graph partitioning based ordering even if it is worse than the approximate minimum local fill ordering.

### MSK\_ORDER\_METHOD\_NONE (5)

No ordering is used.

#### MSKpresolvemodee

Presolve method.

### MSK\_PRESOLVE\_MODE\_OFF (0)

The problem is not presolved before it is optimized.

# MSK\_PRESOLVE\_MODE\_ON (1)

The problem is presolved before it is optimized.

# MSK\_PRESOLVE\_MODE\_FREE (2)

It is decided automatically whether to presolve before the problem is optimized.

# MSKparametertypee

Parameter type

# MSK\_PAR\_INVALID\_TYPE (0)

Not a valid parameter.

# MSK\_PAR\_DOU\_TYPE (1)

Is a double parameter.

### MSK\_PAR\_INT\_TYPE (2)

Is an integer parameter.

# MSK\_PAR\_STR\_TYPE (3)

Is a string parameter.

# MSKproblemiteme

Problem data items

### MSK\_PI\_VAR (0)

Item is a variable.

### MSK\_PI\_CON (1)

Item is a constraint.

### MSK\_PI\_CONE (2)

Item is a cone.

# MSKproblemtypee

Problem types

# MSK\_PROBTYPE\_LO (0)

The problem is a linear optimization problem.

### MSK\_PROBTYPE\_Q0 (1)

The problem is a quadratic optimization problem.

### MSK\_PROBTYPE\_QCQO (2)

The problem is a quadratically constrained optimization problem.

#### MSK\_PROBTYPE\_CONIC (3)

A conic optimization.

### MSK\_PROBTYPE\_MIXED (4)

General nonlinear constraints and conic constraints. This combination can not be solved by MOSEK.

### MSKprostae

Problem status keys

# MSK\_PRO\_STA\_UNKNOWN (0)

Unknown problem status.

### MSK\_PRO\_STA\_PRIM\_AND\_DUAL\_FEAS (1)

The problem is primal and dual feasible.

# MSK\_PRO\_STA\_PRIM\_FEAS (2)

The problem is primal feasible.

# MSK\_PRO\_STA\_DUAL\_FEAS (3)

The problem is dual feasible.

# MSK\_PRO\_STA\_PRIM\_INFEAS (4)

The problem is primal infeasible.

# MSK\_PRO\_STA\_DUAL\_INFEAS (5)

The problem is dual infeasible.

### MSK\_PRO\_STA\_PRIM\_AND\_DUAL\_INFEAS (6)

The problem is primal and dual infeasible.

# MSK\_PRO\_STA\_ILL\_POSED (7)

The problem is ill-posed. For example, it may be primal and dual feasible but have a positive duality gap.

# MSK\_PRO\_STA\_PRIM\_INFEAS\_OR\_UNBOUNDED (8)

The problem is either primal infeasible or unbounded. This may occur for mixed-integer problems.

# MSKxmlwriteroutputtypee

XML writer output mode

# MSK\_WRITE\_XML\_MODE\_ROW (0)

Write in row order.

### MSK\_WRITE\_XML\_MODE\_COL (1)

Write in column order.

# ${\tt MSKrescodetypee}$

Response code type

### MSK\_RESPONSE\_OK (0)

The response code is OK.

### MSK\_RESPONSE\_WRN (1)

The response code is a warning.

# MSK\_RESPONSE\_TRM (2)

The response code is an optimizer termination status.

#### MSK\_RESPONSE\_ERR (3)

The response code is an error.

# MSK\_RESPONSE\_UNK (4)

The response code does not belong to any class.

#### MSKscalingtypee

Scaling type

#### MSK\_SCALING\_FREE (0)

The optimizer chooses the scaling heuristic.

# MSK\_SCALING\_NONE (1)

No scaling is performed.

#### MSK\_SCALING\_MODERATE (2)

A conservative scaling is performed.

### MSK\_SCALING\_AGGRESSIVE (3)

A very aggressive scaling is performed.

# MSKscalingmethode

Scaling method

### MSK\_SCALING\_METHOD\_POW2 (0)

Scales only with power of 2 leaving the mantissa untouched.

### MSK\_SCALING\_METHOD\_FREE (1)

The optimizer chooses the scaling heuristic.

#### MSKsensitivitytypee

Sensitivity types

### MSK\_SENSITIVITY\_TYPE\_BASIS (0)

Basis sensitivity analysis is performed.

# MSKsimseltypee

Simplex selection strategy

### MSK\_SIM\_SELECTION\_FREE (0)

The optimizer chooses the pricing strategy.

#### MSK\_SIM\_SELECTION\_FULL (1)

The optimizer uses full pricing.

### MSK\_SIM\_SELECTION\_ASE (2)

The optimizer uses approximate steepest-edge pricing.

# MSK\_SIM\_SELECTION\_DEVEX (3)

The optimizer uses devex steepest-edge pricing (or if it is not available an approximate steepedge selection).

# MSK\_SIM\_SELECTION\_SE (4)

The optimizer uses steepest-edge selection (or if it is not available an approximate steep-edge selection).

# MSK\_SIM\_SELECTION\_PARTIAL (5)

The optimizer uses a partial selection approach. The approach is usually beneficial if the number of variables is much larger than the number of constraints.

# ${\tt MSKsoliteme}$

Solution items

# MSK\_SOL\_ITEM\_XC (0)

Solution for the constraints.

# MSK\_SOL\_ITEM\_XX (1)

Variable solution.

### MSK\_SOL\_ITEM\_Y (2)

Lagrange multipliers for equations.

# MSK\_SOL\_ITEM\_SLC (3)

Lagrange multipliers for lower bounds on the constraints.

# MSK\_SOL\_ITEM\_SUC (4)

Lagrange multipliers for upper bounds on the constraints.

# MSK\_SOL\_ITEM\_SLX (5)

Lagrange multipliers for lower bounds on the variables.

### MSK\_SOL\_ITEM\_SUX (6)

Lagrange multipliers for upper bounds on the variables.

#### MSK\_SOL\_ITEM\_SNX (7)

Lagrange multipliers corresponding to the conic constraints on the variables.

#### MSKsolstae

Solution status keys

### MSK\_SOL\_STA\_UNKNOWN (0)

Status of the solution is unknown.

# MSK\_SOL\_STA\_OPTIMAL (1)

The solution is optimal.

### MSK\_SOL\_STA\_PRIM\_FEAS (2)

The solution is primal feasible.

### MSK\_SOL\_STA\_DUAL\_FEAS (3)

The solution is dual feasible.

### MSK\_SOL\_STA\_PRIM\_AND\_DUAL\_FEAS (4)

The solution is both primal and dual feasible.

### MSK\_SOL\_STA\_PRIM\_INFEAS\_CER (5)

The solution is a certificate of primal infeasibility.

# MSK\_SOL\_STA\_DUAL\_INFEAS\_CER (6)

The solution is a certificate of dual infeasibility.

# MSK\_SOL\_STA\_PRIM\_ILLPOSED\_CER (7)

The solution is a certificate that the primal problem is illposed.

### MSK\_SOL\_STA\_DUAL\_ILLPOSED\_CER (8)

The solution is a certificate that the dual problem is illposed.

# MSK\_SOL\_STA\_INTEGER\_OPTIMAL (9)

The primal solution is integer optimal.

#### MSKsoltypee

Solution types

# MSK\_SOL\_BAS (1)

The basic solution.

### MSK\_SOL\_ITR (0)

The interior solution.

# MSK\_SOL\_ITG (2)

The integer solution.

### MSKsolveforme

Solve primal or dual form

# MSK\_SOLVE\_FREE (0)

The optimizer is free to solve either the primal or the dual problem.

# MSK\_SOLVE\_PRIMAL (1)

The optimizer should solve the primal problem.

#### MSK\_SOLVE\_DUAL (2)

The optimizer should solve the dual problem.

# MSKstakeye

Status keys

# MSK\_SK\_UNK (0)

The status for the constraint or variable is unknown.

# MSK\_SK\_BAS (1)

The constraint or variable is in the basis.

# MSK\_SK\_SUPBAS (2)

The constraint or variable is super basic.

### MSK\_SK\_LOW (3)

The constraint or variable is at its lower bound.

#### MSK SK UPR (4)

The constraint or variable is at its upper bound.

#### MSK\_SK\_FIX (5)

The constraint or variable is fixed.

### MSK SK INF (6)

The constraint or variable is infeasible in the bounds.

# MSKstartpointtypee

Starting point types

# MSK\_STARTING\_POINT\_FREE (0)

The starting point is chosen automatically.

### MSK\_STARTING\_POINT\_GUESS (1)

The optimizer guesses a starting point.

### MSK\_STARTING\_POINT\_CONSTANT (2)

The optimizer constructs a starting point by assigning a constant value to all primal and dual variables. This starting point is normally robust.

# MSK\_STARTING\_POINT\_SATISFY\_BOUNDS (3)

The starting point is chosen to satisfy all the simple bounds on nonlinear variables. If this starting point is employed, then more care than usual should employed when choosing the bounds on the nonlinear variables. In particular very tight bounds should be avoided.

# MSKstreamtypee

Stream types

### MSK\_STREAM\_LOG (0)

Log stream. Contains the aggregated contents of all other streams. This means that a message written to any other stream will also be written to this stream.

#### MSK\_STREAM\_MSG (1)

Message stream. Log information relating to performance and progress of the optimization is written to this stream.

### MSK\_STREAM\_ERR (2)

Error stream. Error messages are written to this stream.

### MSK\_STREAM\_WRN (3)

Warning stream. Warning messages are written to this stream.

#### MSKvaluee

Integer values

# MSK\_MAX\_STR\_LEN (1024)

Maximum string length allowed in  $\mathbf{MOSEK}$ .

### MSK\_LICENSE\_BUFFER\_LENGTH (21)

The length of a license key buffer.

# ${\tt MSKvariabletypee}$

Variable types

# MSK\_VAR\_TYPE\_CONT (0)

Is a continuous variable.

# MSK\_VAR\_TYPE\_INT (1)

Is an integer variable.

# 15.8 Data Types

# MSKenv\_t

The **MOSEK** Environment type.

# MSKtask\_t

The **MOSEK** Task type.

# MSKuserhandle\_t

A pointer to a user-defined structure.

### MSKbooleant

A signed integer interpreted as a boolean value.

```
MSKint32t
```

Signed 32bit integer.

MSKint64t

Signed 64bit integer.

MSKwchart

Wide char type. The actual type may differ depending on the platform; it is either a 16 or 32 bits signed or unsigned integer.

MSKrealt

The floating point type used by MOSEK.

MSKstring\_t

The string type used by MOSEK. This is an UTF-8 encoded zero-terminated char string.

# 15.9 Function Types

MSKcallbackfunc

```
MSKint32t (MSKAPI * MSKcallbackfunc) (
    MSKtask_t task,
    MSKuserhandle_t usrptr,
    MSKcallbackcodee caller,
    const MSKrealt * douinf,
    const MSKint32t * intinf,
    const MSKint64t * lintinf)
```

The progress callback function is a user-defined function which will be called by MOSEK occasionally during the optimization process. In particular, the callback function is called at the beginning of each iteration in the interior-point optimizer. For the simplex optimizers  $MSK\_IPAR\_LOG\_SIM\_FREQ$  controls how frequently the callback is called. The callback provides an code denoting the point in the solver from which the call happened, and a set of arrays containing information items related to the current state of the solver. Typically the user-defined callback function displays information about the solution process. The callback function can also be used to terminate the optimization process by returning a non-zero value.

The user *must not* call any **MOSEK** function directly or indirectly from the callback function. The only exception is the possibility to retrieve a current best integer solution from the mixed-integer optimizer, see Section *Progress and data callback*.

#### **Parameters**

- task (MSKtask\_t) An optimization task. (input)
- usrptr (MSKuserhandle\_t) A pointer to a user-defined structure. (input/output)
- caller (MSKcallbackcodee) The caller key indicating the current progress of the solver. (input)
- douinf (MSKrealt\*) An array of double information items. The elements correspond to the definitions in MSKdinfiteme. (input)
- intinf (MSKint32t\*) An array of integer information items. The elements correspond to the definitions in MSKiinfiteme. (input)
- lintinf (MSKint64t\*) An array of long information items. The elements correspond to the definitions in MSKliinfiteme. (input)

**Return** (MSKint32t) – If the return value is non-zero, **MOSEK** terminates whatever it is doing and returns control to the calling application.

MSKcallocfunc

```
void * (MSKAPI * MSKcallocfunc) (
   MSKuserhandle_t usrptr,
   const size_t num,
   const size_t size)
```

A user-defined memory allocation function. The function must be compatible with the C calloc function.

#### **Parameters**

- usrptr (MSKuserhandle\_t) A pointer to a user-defined structure. (input)
- num (size\_t) The number of elements. (input)
- size (size\_t) The size of an element. (input)

**Return** (void\*) - A pointer to the allocated memory.

#### MSKexitfunc

```
void (MSKAPI * MSKexitfunc) (
   MSKuserhandle_t usrptr,
   const char * file,
   MSKint32t line,
   const char * msg)
```

A user-defined exit function which is called in case of fatal errors to handle an error message and terminate the program. The function should never return.

### **Parameters**

- usrptr (MSKuserhandle\_t) A pointer to a user-defined structure. (in-put/output)
- file  $(MSKstring_t)$  The name of the file where the fatal error occurred. (input)
- line (MSKint32t) The line number in the file where the fatal error occurred. (input)
- msg (MSKstring\_t) A message about the error. (input)

Return (void)

# MSKfreefunc

```
void (MSKAPI * MSKfreefunc) (
   MSKuserhandle_t usrptr,
   void * buffer)
```

A user-defined memory freeing function.

### **Parameters**

- usrptr (MSKuserhandle\_t) A pointer to a user-defined structure. (input)
- buffer (void\*) A pointer to the buffer which should be freed. (input/output)

Return (void)

# MSKhreadfunc

```
size_t (MSKAPI * MSKhreadfunc) (
   MSKuserhandle_t handle,
   void * dest,
   const size_t count)
```

Behaves similarly to system read function. Returns the number of bytes read.

#### **Parameters**

- handle (MSKuserhandle\_t) A pointer to a user-defined data structure (or a null pointer). (input/output)
- dest (void\*) Read into this destination (output)
- count (size\_t) Number of bytes to read. (input)

Return (size\_t) - The function response code.

#### MSKhwritefunc

```
size_t (MSKAPI * MSKhwritefunc) (
   MSKuserhandle_t handle,
   void * src,
   const size_t count)
```

Behaves similarly to system read function. Returns the number of bytes written.

### Parameters

- handle (MSKuserhandle\_t) A pointer to a user-defined data structure (or a null pointer). (input/output)
- src (void\*) Write from this location (output)
- count (size\_t) Number of bytes to write. (input)

Return (size\_t) - The function response code.

#### MSKmallocfunc

```
void * (MSKAPI * MSKmallocfunc) (
   MSKuserhandle_t usrptr,
   const size_t size)
```

A user-defined memory allocation function. The function must be compatible with the C malloc function.

### **Parameters**

- usrptr (MSKuserhandle\_t) A pointer to a user-defined structure. (input)
- size (size\_t) The number of characters to allocate. (input)

**Return** (void\*) - A pointer to the allocated memory.

### MSKreallocfunc

```
void * (MSKAPI * MSKreallocfunc) (
   MSKuserhandle_t usrptr,
   void * ptr,
   const size_t size)
```

A user-defined memory reallocation function. The function must be compatible with the C  $\mathtt{realloc}$  function.

#### Parameters

- $\bullet$  usrptr (MSKuserhandle\_t) A pointer to a user-defined structure. (input)
- ptr (void\*) The pointer to the allocated memory. (input/output)
- size (size\_t) Size of the new block. (input)

Return (void\*) - A pointer to the allocated memory.

# MSKresponsefunc

```
MSKrescodee (MSKAPI * MSKresponsefunc) (
   MSKuserhandle_t handle,
   MSKrescodee r,
   const char * msg)
```

Whenever **MOSEK** generates a warning or an error this function is called. The argument  ${\tt r}$  contains the code of the error/warning and the argument  ${\tt msg}$  contains the corresponding error/warning message. This function should always return  ${\tt MSK\_RES\_OK}$ .

### Parameters

- handle (MSKuserhandle\_t) A pointer to a user-defined data structure (or a null pointer). (input/output)
- r (MSKrescodee) The response code corresponding to the exception. (input)
- msg (MSKstring\_t) A string containing the exception message. (input)

Return (MSKrescodee) - The function response code.

#### MSKstreamfunc

```
void (MSKAPI * MSKstreamfunc) (
    MSKuserhandle_t handle,
    const char * str)
```

The message-stream callback function is a user-defined function which can be linked to any of the MOSEK streams. Doing so, the function is called whenever MOSEK sends a message to the stream

The user must not call any MOSEK function directly or indirectly from the callback function.

### Parameters

- handle (MSKuserhandle\_t) A pointer to a user-defined data structure (or a null pointer). (input/output)
- $str(MSKstring_t)$  A string containing a message to a stream. (input) Return (void)

# 15.10 Nonlinear interfaces (obsolete)

**Important:** This is a legacy document for users familiar with SCopt, DGopt, EXPopt, mskenopt, mskscopt and mskgpopt from previous versions of **MOSEK**. These interfaces have now been removed. We assume familiarity with documentation included in version 8. All problems expressible with this interface can (and should) be reformulated using the exponential cone and power cones.

New users should formulate problems involving powers, logarithms and exponentials directly in conic form.

#### Conversion tutorial

We recommend converting all nonlinear problems using SCopt, DGopt, EXPopt, mskenopt, mskscopt and mskgpopt into conic form. Depending on the values of f, g, h either the epigraph or hypograph of a SCopt function if convex, and a bounding variable can be introduced following the basic rules below. We assume all variables are within safe bounds where the SCopt operators are defined and convex. We also assume f > 0.

A more comprehensive modeling guide for these types of problems can be found in the **MOSEK** Modeling Cookbook.

### **Powers**

Consider  $f(x+h)^g$ . This can be reformulated using the power cone.

- If g > 1 then  $t \ge f(x+h)^g$  is equivalent to  $(t/f)^{1/g} \ge |x+h|$ , that is  $(t/f, 1, x+h) \in \mathcal{P}_3^{1/g, 1-1/g}$ .
- If 0 < g < 1 then  $|t| \le f(x+h)^g$  is equivalent to  $(x+h,1,t/f) \in \mathcal{P}_3^{g,1-g}$ .
- If g < 0 then  $t \ge f(x+h)^g$  is equivalent to  $(t/f)(x+h)^{-g} \ge 1$ , that is  $(t/f, x+h, 1) \in \mathcal{P}_3^{1/(1-g), -g/(1-g)}$ .

### Logarithm

The bound  $t \leq f \log(gx + h)$  is equivalent to  $(gx + h, 1, t/f) \in K_{\exp}$ .

#### Entropy

The bound  $t \ge fx \log x$  is equivalent to  $(1, x, -t/f) \in K_{\exp}$ .

# **Exponential**

The bound  $t \ge f \exp(gx + h)$  is equivalent to  $(t/f, 1, gx + h) \in K_{\exp}$ .

# Exponential optimization (EXPopt), Geometric programming (mskgpopt)

For a basic tutorial in geometric programming (GP) see Sec. 6.8.

An exponential optimization problem in standard form consists of constraints of the type:

$$t \ge \log \left( \sum_{i} \exp(a_i^T x + b_i) \right).$$

This log-sum-exp bound is equivalent to

$$\sum_{i} \exp(a_i^T x + b_i - t) \le 1$$

and requires bounding each exponential function as explained above.

# Dual geometric optimization (DGopt)

The objective function of a dual geometric problem involves maximizing expressions of the form

$$x \log \frac{c}{x}$$
 and  $x_i \log \frac{e^T x}{x_i}$ ,

which can be achieved using bounds  $t \leq x \log \frac{y}{x}$ , that is  $(t, x, y) \in K_{\exp}$ .

# Chapter 16

# Supported File Formats

MOSEK supports a range of problem and solution formats listed in Table 16.1 and Table 16.2. The **Task** format is MOSEK's native binary format and it supports all features that MOSEK supports. The **OPF format** is MOSEK's human-readable alternative that supports nearly all features (everything except semidefinite problems). In general, text formats are significantly slower to read, but can be examined and edited directly in any text editor.

### **Problem formats**

Table 16.1: List of supported file formats for optimization problems. The column *Conic* refers to conic problems involving the quadratic, rotated quadratic, power or exponential cone. The last two columns indicate if the format supports solutions and optimizer parameters.

Format Type	Ext.	Binary/Text	LP	QO	Conic	SDP	Sol	Param
LP	lp	plain text	X	X				
MPS	mps	plain text	X	X	X			
OPF	opf	plain text	X	X	X		X	X
PTF	ptf	plain text	X	X	X	X	X	
CBF	cbf	plain text	X		X	X		
Task format	task	binary	X	X	X	X	X	X
Jtask format	jtask	text	X	X	X	X	X	X

### **Solution formats**

Table 16.2: List of supported solution formats.

Format Type	Ext.	Binary/Text	Description
SOL	sol	plain text	Interior Solution
	bas	plain text	Basic Solution
	$_{ m int}$	plain text	Integer
Jsol format	jsol	text	Solution

### Compression

MOSEK supports GZIP and Zstandard compression. Problem files with extension .gz (for GZIP) and .zst (for Zstandard) are assumed to be compressed when read, and are automatically compressed when written. For example, a file called

problem.mps.gz

will be considered as a GZIP compressed MPS file.

# 16.1 The LP File Format

MOSEK supports the LP file format with some extensions. The LP format is not a completely well-defined standard and hence different optimization packages may interpret the same LP file in slightly different ways. MOSEK tries to emulate as closely as possible CPLEX's behavior, but tries to stay backward compatible.

The LP file format can specify problems of the form

$$\begin{array}{lll} \text{minimize/maximize} & & c^Tx + \frac{1}{2}q^o(x) \\ \text{subject to} & l^c & \leq & Ax + \frac{1}{2}q(x) & \leq & u^c, \\ l^x & \leq & x & \leq & u^x, \\ & & & x_{\mathcal{J}} \text{ integer,} \end{array}$$

where

- $x \in \mathbb{R}^n$  is the vector of decision variables.
- $c \in \mathbb{R}^n$  is the linear term in the objective.
- $q^o :\in \mathbb{R}^n \to \mathbb{R}$  is the quadratic term in the objective where

$$q^o(x) = x^T Q^o x$$

and it is assumed that

$$Q^o = (Q^o)^T$$
.

- $A \in \mathbb{R}^{m \times n}$  is the constraint matrix.
- $l^c \in \mathbb{R}^m$  is the lower limit on the activity for the constraints.
- $u^c \in \mathbb{R}^m$  is the upper limit on the activity for the constraints.
- $l^x \in \mathbb{R}^n$  is the lower limit on the activity for the variables.
- $u^x \in \mathbb{R}^n$  is the upper limit on the activity for the variables.
- $q: \mathbb{R}^n \to \mathbb{R}$  is a vector of quadratic functions. Hence,

$$q_i(x) = x^T Q^i x$$

where it is assumed that

$$Q^i = (Q^i)^T$$
.

•  $\mathcal{J} \subseteq \{1, 2, \dots, n\}$  is an index set of the integer constrained variables.

### 16.1.1 File Sections

An LP formatted file contains a number of sections specifying the objective, constraints, variable bounds, and variable types. The section keywords may be any mix of upper and lower case letters.

### **Objective Function**

The first section beginning with one of the keywords

max
maximum
maximize
min
minimum
minimize

defines the objective sense and the objective function, i.e.

$$c^T x + \frac{1}{2} x^T Q^o x.$$

The objective may be given a name by writing

#### myname:

before the expressions. If no name is given, then the objective is named obj.

The objective function contains linear and quadratic terms. The linear terms are written as

```
4 x1 + x2 - 0.1 x3
```

and so forth. The quadratic terms are written in square brackets ([ ]/2) and are either squared or multiplied as in the examples

```
x1^2
```

and

```
x1 * x2
```

There may be zero or more pairs of brackets containing quadratic expressions. An example of an objective section is

```
minimize
myobj: 4 x1 + x2 - 0.1 x3 + [ x1^2 + 2.1 x1 * x2 ]/2
```

Please note that the quadratic expressions are multiplied with  $\frac{1}{2}$ , so that the above expression means

minimize 
$$4x_1 + x_2 - 0.1 \cdot x_3 + \frac{1}{2}(x_1^2 + 2.1 \cdot x_1 \cdot x_2)$$

If the same variable occurs more than once in the linear part, the coefficients are added, so that  $4 \times 1 + 2 \times 1$  is equivalent to  $6 \times 1$ . In the quadratic expressions  $\times 1 \times 2$  is equivalent to  $\times 2 \times 1$  and, as in the linear part, if the same variables multiplied or squared occur several times their coefficients are added.

# Constraints

The second section beginning with one of the keywords

```
subj to
subject to
s.t.
st
```

defines the linear constraint matrix A and the quadratic matrices  $Q^{i}$ .

A constraint contains a name (optional), expressions adhering to the same rules as in the objective and a bound:

```
subject to con1: x1 + x2 + [ x3^2 ]/2 <= 5.1
```

The bound type (here <=) may be any of <, <=, =, >, >= (< and <= mean the same), and the bound may be any number.

In the standard LP format it is not possible to define more than one bound per line, but **MOSEK** supports defining ranged constraints by using double-colon (::) instead of a single-colon (:) after the constraint name, i.e.

$$-5 \le x_1 + x_2 \le 5 \tag{16.1}$$

may be written as

```
con:: -5 < x_1 + x_2 < 5
```

By default MOSEK writes ranged constraints this way.

If the files must adhere to the LP standard, ranged constraints must either be split into upper bounded and lower bounded constraints or be written as an equality with a slack variable. For example the expression (16.1) may be written as

$$x_1 + x_2 - sl_1 = 0, -5 \le sl_1 \le 5.$$

#### **Bounds**

Bounds on the variables can be specified in the bound section beginning with one of the keywords

```
bounds bounds
```

The bounds section is optional but should, if present, follow the **subject to** section. All variables listed in the bounds section must occur in either the objective or a constraint.

The default lower and upper bounds are 0 and  $+\infty$ . A variable may be declared free with the keyword free, which means that the lower bound is  $-\infty$  and the upper bound is  $+\infty$ . Furthermore it may be assigned a finite lower and upper bound. The bound definitions for a given variable may be written in one or two lines, and bounds can be any number or  $\pm\infty$  (written as +inf/-inf/+infinity/-infinity) as in the example

```
bounds

x1 free

x2 <= 5

0.1 <= x2

x3 = 42

2 <= x4 < +inf
```

### Variable Types

The final two sections are optional and must begin with one of the keywords

```
bin
binaries
binary
```

and

```
gen
general
```

Under general all integer variables are listed, and under binary all binary (integer variables with bounds 0 and 1) are listed:

```
general
x1 x2
binary
x3 x4
```

Again, all variables listed in the binary or general sections must occur in either the objective or a constraint.

# **Terminating Section**

Finally, an LP formatted file must be terminated with the keyword

end

# 16.1.2 LP File Examples

# Linear example lo1.lp

```
\ File: lo1.lp
maximize
obj: 3 x1 + x2 + 5 x3 + x4
subject to
c1: 3 x1 + x2 + 2 x3 = 30
c2: 2 x1 + x2 + 3 x3 + x4 >= 15
c3: 2 x2 + 3 x4 <= 25
bounds
0 <= x1 <= +infinity
0 <= x2 <= 10
0 <= x3 <= +infinity
0 <= x4 <= +infinity
end</pre>
```

# Mixed integer example milo1.lp

```
maximize
obj: x1 + 6.4e-01 x2
subject to
c1: 5e+01 x1 + 3.1e+01 x2 <= 2.5e+02
c2: 3e+00 x1 - 2e+00 x2 >= -4e+00
bounds
0 <= x1 <= +infinity
0 <= x2 <= +infinity
general
x1 x2
end
```

# 16.1.3 LP Format peculiarities

#### Comments

Anything on a line after a \ is ignored and is treated as a comment.

#### **Names**

A name for an objective, a constraint or a variable may contain the letters a-z, A-Z, the digits 0-9 and the characters

```
!"#$%&()/,.;?@_'`|~
```

The first character in a name must not be a number, a period or the letter  ${\tt e}$  or  ${\tt E}$ . Keywords must not be used as names.

**MOSEK** accepts any character as valid for names, except **\0**. A name that is not allowed in LP file will be changed and a warning will be issued.

The algorithm for making names LP valid works as follows: The name is interpreted as an utf-8 string. For a Unicode character c:

- If c==\_ (underscore), the output is \_\_ (two underscores).
- If c is a valid LP name character, the output is just c.
- If c is another character in the ASCII range, the output is <code>\_XX</code>, where <code>XX</code> is the hexadecimal code for the character.
- If c is a character in the range 127-65535, the output is \_uXXXX, where XXXX is the hexadecimal code for the character.
- If c is a character above 65535, the output is \_UXXXXXXXX, where XXXXXXXX is the hexadecimal code for the character.

Invalid  $\mathtt{utf-8}$  substrings are escaped as  $\mathtt{XX'}$ , and if a name starts with a period, e or E, that character is escaped as  $\mathtt{XX}$ .

### Variable Bounds

Specifying several upper or lower bounds on one variable is possible but **MOSEK** uses only the tightest bounds. If a variable is fixed (with =), then it is considered the tightest bound.

# MOSEK Extensions to the LP Format

Some optimization software packages employ a more strict definition of the LP format than the one used by **MOSEK**. The limitations imposed by the strict LP format are the following:

- Quadratic terms in the constraints are not allowed.
- Names can be only 16 characters long.
- Lines must not exceed 255 characters in length.

To get around some of the inconveniences converting from other problem formats, **MOSEK** allows lines to contain 1024 characters and names may have any length (shorter than the 1024 characters).

If an LP formatted file created by **MOSEK** should satisfy the strict definition, then the parameter <code>MSK\_IPAR\_WRITE\_LP\_STRICT\_FORMAT</code> should be set; note, however, that some problems cannot be written correctly as a strict LP formatted file. For instance, all names are truncated to 16 characters and hence they may lose their uniqueness and change the problem.

Internally in MOSEK names may contain any (printable) character, many of which cannot be used in LP names. Setting the parameters <code>MSK\_IPAR\_READ\_LP\_QUOTED\_NAMES</code> and <code>MSK\_IPAR\_WRITE\_LP\_QUOTED\_NAMES</code> allows MOSEK to use quoted names. The first parameter tells MOSEK to remove quotes from quoted names e.g, "x1", when reading LP formatted files. The second parameter tells MOSEK to put quotes around any semi-illegal name (names beginning with a number or a period) and fully illegal name (containing illegal characters). As double quote is a legal character in the LP format, quoting semi-illegal names makes them legal in the pure LP format as long as they are still shorter than 16 characters. Fully illegal names are still illegal in a pure LP file.

#### The strict LP format

The LP format is not a formal standard and different vendors have slightly different interpretations of the LP format. To make **MOSEK**'s definition of the LP format more compatible with the definitions of other vendors set the parameter MSK\_IPAR\_WRITE\_LP\_STRICT\_FORMAT to MSK\_ON.

This setting may lead to truncation of some names and hence to an invalid LP file. The simple solution to this problem is to set the parameter MSK\_IPAR\_WRITE\_GENERIC\_NAMES to MSK\_ON which will cause all names to be renamed systematically in the output file.

# Formatting of an LP File

A few parameters control the visual formatting of LP files written by  $\mathbf{MOSEK}$  in order to make it easier to read the files. These parameters are

- MSK\_IPAR\_WRITE\_LP\_LINE\_WIDTH sets the maximum number of characters on a single line. The default value is 80 corresponding roughly to the width of a standard text document.
- MSK\_IPAR\_WRITE\_LP\_TERMS\_PER\_LINE sets the maximum number of terms per line; a term means a sign, a coefficient, and a name (for example + 42 elephants). The default value is 0, meaning that there is no maximum.

#### **Unnamed Constraints**

Reading and writing an LP file with **MOSEK** may change it superficially. If an LP file contains unnamed constraints or objective these are given their generic names when the file is read (however unnamed constraints in **MOSEK** are written without names).

# 16.2 The MPS File Format

**MOSEK** supports the standard MPS format with some extensions. For a detailed description of the MPS format see the book by Nazareth [Naz87].

# 16.2.1 MPS File Structure

The version of the MPS format supported by **MOSEK** allows specification of an optimization problem of the form

maximize/minimize 
$$c^T x + q_0(x)$$
  
 $l^c \leq Ax + q(x) \leq u^c,$   
 $l^x \leq x \leq u^x,$   
 $x \in \mathcal{K},$   
 $x_{\mathcal{T}}$  integer, (16.2)

where

- $x \in \mathbb{R}^n$  is the vector of decision variables.
- $A \in \mathbb{R}^{m \times n}$  is the constraint matrix.
- $l^c \in \mathbb{R}^m$  is the lower limit on the activity for the constraints.
- $u^c \in \mathbb{R}^m$  is the upper limit on the activity for the constraints.
- $l^x \in \mathbb{R}^n$  is the lower limit on the activity for the variables.
- $u^x \in \mathbb{R}^n$  is the upper limit on the activity for the variables.
- $q: \mathbb{R}^n \to \mathbb{R}$  is a vector of quadratic functions. Hence,

$$q_i(x) = \frac{1}{2}x^T Q^i x$$

where it is assumed that  $Q^i = (Q^i)^T$ . Please note the explicit  $\frac{1}{2}$  in the quadratic term and that  $Q^i$  is required to be symmetric. The same applies to  $q_0$ .

- $\mathcal{K}$  is a convex cone.
- $\mathcal{J} \subseteq \{1, 2, \dots, n\}$  is an index set of the integer-constrained variables.
- $\bullet$  c is the vector of objective coefficients.

An MPS file with one row and one column can be illustrated like this:

```
3
                                                     5
                                                                6
*2345678901234567890123456789012345678901234567890
NAME
               [name]
OBJSENSE
    [objsense]
               [objname]
OBJNAME
ROWS
?
    [cname1]
COLUMNS
    [vname1]
               [cname1]
                          [value1]
                                          [cname2]
                                                     [value2]
RHS
               [cname1]
                          [value1]
                                          [cname2]
                                                     [value2]
    [name]
RANGES
    [name]
               [cname1]
                          [value1]
                                          [cname2]
                                                     [value2]
QSECTION
               [cname1]
                                          [vname3]
    [vname1]
               [vname2]
                          [value1]
                                                     [value2]
QMATRIX
    [vname1]
               [vname2]
                          [value1]
QUADOBJ
    [vname1]
               [vname2]
                          [value1]
QCMATRIX
               [cname1]
    [vname1]
               [vname2]
                          [value1]
BOUNDS
?? [name]
               [vname1]
                          [value1]
CSECTION
               [kname1]
                          [value1]
                                          [ktype]
    [vname1]
ENDATA
```

Here the names in capitals are keywords of the MPS format and names in brackets are custom defined names or values. A couple of notes on the structure:

• Fields: All items surrounded by brackets appear in *fields*. The fields named "valueN" are numerical values. Hence, they must have the format

```
[+|-]XXXXXXX.XXXXXX[[e|E][+|-]XXX]
```

where

```
X = [0|1|2|3|4|5|6|7|8|9].
```

- Sections: The MPS file consists of several sections where the names in capitals indicate the beginning of a new section. For example, COLUMNS denotes the beginning of the columns section.
- Comments: Lines starting with an \* are comment lines and are ignored by MOSEK.
- Keys: The question marks represent keys to be specified later.
- Extensions: The sections QSECTION and CSECTION are specific MOSEK extensions of the MPS format. The sections QMATRIX, QUADOBJ and QCMATRIX are included for sake of compatibility with other vendors extensions to the MPS format.
- The standard MPS format is a fixed format, i.e. everything in the MPS file must be within certain fixed positions. **MOSEK** also supports a *free format*. See Sec. 16.2.5 for details.

# Linear example lo1.mps

A concrete example of a MPS file is presented below:

```
* File: lo1.mps
NAME
               lo1
OBJSENSE
    MAX
ROWS
N
   obj
E c1
G
   c2
L
    сЗ
COLUMNS
                          3
               obj
    x1
    x1
               c1
                          3
                          2
    x1
               c2
    x2
                          1
               obj
                          1
    x2
               с1
    x2
               c2
                          1
               сЗ
    x2
                          2
                          5
    хЗ
               obj
                          2
    хЗ
               c1
    xЗ
               c2
                          3
    x4
               obj
                          1
    x4
               c2
                          1
                          3
    x4
               сЗ
RHS
    rhs
               c1
                          30
               c2
                          15
    rhs
    rhs
               сЗ
                          25
RANGES
BOUNDS
UP bound
               x2
                          10
ENDATA
```

Subsequently each individual section in the MPS format is discussed.

# NAME (optional)

In this section a name ([name]) is assigned to the problem.

# OBJSENSE (optional)

This is an optional section that can be used to specify the sense of the objective function. The <code>OBJSENSE</code> section contains one line at most which can be one of the following:

```
MIN
MINIMIZE
MAX
MAXIMIZE
```

It should be obvious what the implication is of each of these four lines.

### OBJNAME (optional)

This is an optional section that can be used to specify the name of the row that is used as objective function. objname should be a valid row name.

#### ROWS

A record in the ROWS section has the form

#### ? [cname1]

where the requirements for the fields are as follows:

Field	Starting Position	Max Width	required	Description
?	2	1	Yes	Constraint key
[cname1]	5	8	Yes	Constraint name

Hence, in this section each constraint is assigned a unique name denoted by [cname1]. Please note that [cname1] starts in position 5 and the field can be at most 8 characters wide. An initial key ? must be present to specify the type of the constraint. The key can have values E, G, L, or N with the following interpretation:

Constraint type	$l_i^c$	$u_i^c$
E (equal)	finite	$=l_i^c$
G (greater)	finite	$\infty$
L (lower)	$-\infty$	finite
N (none)	$-\infty$	$\infty$

In the MPS format the objective vector is not specified explicitly, but one of the constraints having the key N will be used as the objective vector c. In general, if multiple N type constraints are specified, then the first will be used as the objective vector c, unless something else was specified in the section OBJNAME.

### COLUMNS

In this section the elements of A are specified using one or more records having the form:

e1] [cname1] [value1] [cname2] [value2	e2]			
--	-----	--	--	--

where the requirements for each field are as follows:

Field	Starting Position	Max Width	required	Description
[vname1]	5	8	Yes	Variable name
[cname1]	15	8	Yes	Constraint name
[value1]	25	12	Yes	Numerical value
[cname2]	40	8	No	Constraint name
[value2]	50	12	No	Numerical value

Hence, a record specifies one or two elements  $a_{ij}$  of A using the principle that [vname1] and [cname1] determines j and i respectively. Please note that [cname1] must be a constraint name specified in the ROWS section. Finally, [value1] denotes the numerical value of  $a_{ij}$ . Another optional element is specified by [cname2], and [value2] for the variable specified by [vname1]. Some important comments are:

- All elements belonging to one variable must be grouped together.
- Zero elements of A should not be specified.
- At least one element for each variable should be specified.

# RHS (optional)

A record in this section has the format

me] [cna	[cname2]	[value1]	[cname2]
----------	----------	----------	----------

where the requirements for each field are as follows:

Field	Starting Position	Max Width	required	Description
[name]	5	8	Yes	Name of the RHS vector
[cname1]	15	8	Yes	Constraint name
[value1]	25	12	Yes	Numerical value
[cname2]	40	8	No	Constraint name
[value2]	50	12	No	Numerical value

The interpretation of a record is that [name] is the name of the RHS vector to be specified. In general, several vectors can be specified. [cname1] denotes a constraint name previously specified in the ROWS section. Now, assume that this name has been assigned to the i-h constraint and  $v_1$  denotes the value specified by [value1], then the interpretation of  $v_1$  is:

Constraint	$l_i^c$	$u_i^c$
E	$v_1$	$v_1$
G	$v_1$	
L		$v_1$
N		

An optional second element is specified by [cname2] and [value2] and is interpreted in the same way. Please note that it is not necessary to specify zero elements, because elements are assumed to be zero

### RANGES (optional)

A record in this section has the form

.] [cname2] [va	[cname1] [va	[cnam	[value1]	value1] [cname2]	ne1] [value1] [cname2] [value
-----------------	--------------	-------	----------	------------------	-------------------------------

where the requirements for each fields are as follows:

Field	Starting Position	Max Width	required	Description
[name]	5	8	Yes	Name of the RANGE vector
[cname1]	15	8	Yes	Constraint name
[value1]	25	12	Yes	Numerical value
[cname2]	40	8	No	Constraint name
[value2]	50	12	No	Numerical value

The records in this section are used to modify the bound vectors for the constraints, i.e. the values in  $l^c$  and  $u^c$ . A record has the following interpretation: [name] is the name of the RANGE vector and [cname1] is a valid constraint name. Assume that [cname1] is assigned to the *i*-th constraint and let  $v_1$  be the value specified by [value1], then a record has the interpretation:

Constraint type	Sign of $v_1$	$l_i^c$	$u_i^c$
E	_	$u_i^c + v_1$	
E	+		$l_i^c + v_1$
G	- or +		$l_i^c +  v_1 $
L	- or +	$u_i^c -  v_1 $	
N			

Another constraint bound can optionally be modified using [cname2] and [value2] the same way.

# QSECTION (optional)

Within the QSECTION the label [cname1] must be a constraint name previously specified in the ROWS section. The label [cname1] denotes the constraint to which the quadratic terms belong. A record in the QSECTION has the form

[vname3] [va]	name2] [value1]	[vname1]
---------------	-----------------	----------

where the requirements for each field are:

Field	Starting Position	Max Width	required	Description
[vname1]	5	8	Yes	Variable name
[vname2]	15	8	Yes	Variable name
[value1]	25	12	Yes	Numerical value
[vname3]	40	8	No	Variable name
[value2]	50	12	No	Numerical value

A record specifies one or two elements in the lower triangular part of the  $Q^i$  matrix where [cname1] specifies the i. Hence, if the names [vname1] and [vname2] have been assigned to the k-th and j-th variable, then  $Q^i_{kj}$  is assigned the value given by [value1] An optional second element is specified in the same way by the fields [vname1], [vname3], and [value2].

The example

$$\begin{array}{ll} \text{minimize} & -x_2 + \frac{1}{2}(2x_1^2 - 2x_1x_3 + 0.2x_2^2 + 2x_3^2) \\ \text{subject to} & x_1 + x_2 + x_3 & \geq & 1, \\ & x \geq 0 & \end{array}$$

has the following MPS file representation

* F	ile:	qo1.mps	
NAM	Ε	qo1	
ROW	S		
N	obj		
	c1		
COL	UMNS		
	x1	c1	
	x2	obj	
	x2	c1	
	x3	c1	
RHS			
	rhs	c1	
QSE	CTION		
	x1	x1	
	x1	х3	
	x2	x2	
	x3	x3	
END	ATA		

Regarding the QSECTIONs please note that:

- Only one QSECTION is allowed for each constraint.
- The QSECTIONs can appear in an arbitrary order after the COLUMNS section.
- All variable names occurring in the QSECTION must already be specified in the COLUMNS section.
- $\bullet$  All entries specified in a QSECTION are assumed to belong to the lower triangular part of the quadratic term of Q .

### QMATRIX/QUADOBJ (optional)

The QMATRIX and QUADOBJ sections allow to define the quadratic term of the objective function. They differ in how the quadratic term of the objective function is stored:

- QMATRIX stores all the nonzeros coefficients, without taking advantage of the symmetry of the Q
  matrix.
- ullet QUADOBJ stores the upper diagonal nonzero elements of the Q matrix.

A record in both sections has the form:

where the requirements for each field are:

Field	Starting Position	Max Width	required	Description
[vname1]	5	8	Yes	Variable name
[vname2]	15	8	Yes	Variable name
[value1]	25	12	Yes	Numerical value

A record specifies one elements of the Q matrix in the objective function. Hence, if the names [vname1] and [vname2] have been assigned to the k-th and j-th variable, then  $Q_{kj}$  is assigned the value given by [value1]. Note that a line must appear for each off-diagonal coefficient if using a QMATRIX section, while only one entry is required in a QUADOBJ section. The quadratic part of the objective function will be evaluated as  $1/2x^TQx$ .

The example

$$\begin{array}{ll} \text{minimize} & -x_2 + \frac{1}{2}(2x_1^2 - 2x_1x_3 + 0.2x_2^2 + 2x_3^2) \\ \text{subject to} & x_1 + x_2 + x_3 & \geq & 1, \\ & x \geq 0 & \end{array}$$

has the following MPS file representation using QMATRIX

```
* File: qo1_matrix.mps
NAME
               qo1_qmatrix
ROWS
N obj
G c1
COLUMNS
    x1
               c1
                           1.0
                           -1.0
    x2
               obj
                           1.0
    x2
               c1
    xЗ
               c1
                           1.0
RHS
    rhs
                           1.0
               c1
QMATRIX
    x1
               x1
                           2.0
                           -1.0
    x1
               xЗ
    xЗ
                           -1.0
               x1
                          0.2
    x2
               x2
    xЗ
               xЗ
                          2.0
ENDATA
```

or the following using QUADOBJ

```
* File: qo1_quadobj.mps
NAME qo1_quadobj
ROWS
N obj
G c1
```

(continues on next page)

(continued from previous page)

					1 0 /
COLUMNS		·			
x1	c1	1.0			
x2	obj	-1.0			
x2	c1	1.0			
х3	c1	1.0			
RHS					
rhs	c1	1.0			
QUADOBJ					
x1	x1	2.0			
x1	x3	-1.0			
x2	x2	0.2			
x3	x3	2.0			
ENDATA					
1					

Please also note that:

- A QMATRIX/QUADOBJ section can appear in an arbitrary order after the COLUMNS section.
- ullet All variable names occurring in the QMATRIX/QUADOBJ section must already be specified in the COLUMNS section.

# QCMATRIX (optional)

A QCMATRIX section allows to specify the quadratic part of a given constraint. Within the QCMATRIX the label [cname1] must be a constraint name previously specified in the ROWS section. The label [cname1] denotes the constraint to which the quadratic term belongs. A record in the QSECTION has the form

|--|

where the requirements for each field are:

Field	Starting Position	Max Width	required	Description
[vname1]	5	8	Yes	Variable name
[vname2]	15	8	Yes	Variable name
[value1]	25	12	Yes	Numerical value

A record specifies an entry of the  $Q^i$  matrix where [cname1] specifies the i. Hence, if the names [vname1] and [vname2] have been assigned to the k-th and j-th variable, then  $Q^i_{kj}$  is assigned the value given by [value1]. Moreover, the quadratic term is represented as  $1/2x^TQx$ .

The example

$$\begin{array}{lll} \text{minimize} & x_2 \\ \text{subject to} & x_1 + x_2 + x_3 & \geq & 1, \\ & \frac{1}{2} \bigl( -2x_1x_3 + 0.2x_2^2 + 2x_3^2 \bigr) & \leq & 10, \\ & & x \geq 0 \end{array}$$

has the following MPS file representation

* F		qo1.mps qo1			
ROW		401			
N	obj				
G	c1				
L	q1				
COL	UMNS				
	x1	c1	1.0		
	x2	obj	-1.0		
	x2	c1	1.0		
	xЗ	c1	1.0		

(continues on next page)

(continued from previous page)

RHS			 	
rhs	c1	1.0		
rhs	q1	10.0		
QCMATRIX	q1			
x1	x1	2.0		
x1	x3	-1.0		
x3	x1	-1.0		
x2	x2	0.2		
хЗ	x3	2.0		
ENDATA				

Regarding the QCMATRIXs please note that:

- Only one QCMATRIX is allowed for each constraint.
- The QCMATRIXs can appear in an arbitrary order after the COLUMNS section.
- All variable names occurring in the QSECTION must already be specified in the COLUMNS section.
- QCMATRIX does not exploit the symmetry of Q: an off-diagonal entry (i, j) should appear twice.

# BOUNDS (optional)

In the BOUNDS section changes to the default bounds vectors  $l^x$  and  $u^x$  are specified. The default bounds vectors are  $l^x=0$  and  $u^x=\infty$ . Moreover, it is possible to specify several sets of bound vectors. A record in this section has the form

where the requirements for each field are:

Field	Starting Position	Max Width	Required	Description
??	2	2	Yes	Bound key
[name]	5	8	Yes	Name of the BOUNDS vector
[vname1]	15	8	Yes	Variable name
[value1]	25	12	No	Numerical value

Hence, a record in the BOUNDS section has the following interpretation: [name] is the name of the bound vector and [vname1] is the name of the variable for which the bounds are modified by the record. ?? and [value1] are used to modify the bound vectors according to the following table:

??	$l_j^x$	$u_j^x$	Made integer (added to ${\cal J}$ )
FR	$-\infty$	$\infty$	No
FX	$v_1$	$v_1$	No
LO	$v_1$	unchanged	No
MI	$-\infty$	unchanged	No
PL	unchanged	$\infty$	No
UP	unchanged	$v_1$	No
BV	0	1	Yes
LI	$\lceil v_1 \rceil$	unchanged	Yes
UI	unchanged	$\lfloor v_1 \rfloor$	Yes

Here  $v_1$  is the value specified by [value1].

### CSECTION (optional)

The purpose of the CSECTION is to specify the conic constraint

$$x \in \mathcal{K}$$

in (16.2). It is assumed that K satisfies the following requirements. Let

$$x^t \in \mathbb{R}^{n^t}$$
,  $t = 1, \dots, k$ 

be vectors comprised of parts of the decision variables x so that each decision variable is a member of exactly **one** vector  $x^t$ , for example

$$x^1 = \begin{bmatrix} x_1 \\ x_4 \\ x_7 \end{bmatrix}$$
 and  $x^2 = \begin{bmatrix} x_6 \\ x_5 \\ x_3 \\ x_2 \end{bmatrix}$ .

Next define

$$\mathcal{K} := \{ x \in \mathbb{R}^n : x^t \in \mathcal{K}_t, \quad t = 1, \dots, k \}$$

where  $\mathcal{K}_t$  must have one of the following forms:

•  $\mathbb{R}$  set:

$$\mathcal{K}_t = \mathbb{R}^{n^t}$$
.

• Zero cone:

$$\mathcal{K}_t = \{0\} \subseteq \mathbb{R}^{n^t}. \tag{16.3}$$

• Quadratic cone:

$$\mathcal{K}_t = \left\{ x \in \mathbb{R}^{n^t} : x_1 \ge \sqrt{\sum_{j=2}^{n^t} x_j^2} \right\}. \tag{16.4}$$

• Rotated quadratic cone:

$$\mathcal{K}_t = \left\{ x \in \mathbb{R}^{n^t} : 2x_1 x_2 \ge \sum_{j=3}^{n^t} x_j^2, \quad x_1, x_2 \ge 0 \right\}.$$
 (16.5)

• Primal exponential cone:

$$\mathcal{K}_t = \left\{ x \in \mathbb{R}^3 : x_1 \ge x_2 \exp(x_3/x_2), \quad x_1, x_2 \ge 0 \right\}. \tag{16.6}$$

• Primal power cone (with parameter  $0 < \alpha < 1$ ):

$$\mathcal{K}_t = \left\{ x \in \mathbb{R}^{n^t} : x_1^{\alpha} x_2^{1-\alpha} \ge \sqrt{\sum_{j=3}^{n^t} x_j^2}, \quad x_1, x_2 \ge 0 \right\}.$$
 (16.7)

• Dual exponential cone:

$$\mathcal{K}_t = \left\{ x \in \mathbb{R}^3 : x_1 \ge -x_3 e^{-1} \exp(x_2/x_3), \quad x_3 \le 0, x_1 \ge 0 \right\}. \tag{16.8}$$

• Dual power cone (with parameter  $0 < \alpha < 1$ ):

$$\mathcal{K}_t = \left\{ x \in \mathbb{R}^{n^t} : \left(\frac{x_1}{\alpha}\right)^{\alpha} \left(\frac{x_2}{1-\alpha}\right)^{1-\alpha} \ge \sqrt{\sum_{j=3}^{n^t} x_j^2}, \quad x_1, x_2 \ge 0 \right\}.$$
 (16.9)

In general, membership in the  $\mathbb{R}$  set is not specified. If a variable is not a member of any other cone then it is assumed to be a member of the  $\mathbb{R}$  cone.

Next, let us study an example. Assume that the power cone

$$x_4^{1/3} x_5^{2/3} \ge |x_8|$$

and the rotated quadratic cone

$$2x_3x_7 \ge x_1^2 + x_0^2, \quad x_3, x_7 \ge 0,$$

should be specified in the MPS file. One CSECTION is required for each cone and they are specified as follows:

*	1	2	3	4	5	6
*23456	789012	_	_	4567890123456	-	-
CSECTI	ON	konea	3e-1	PPOW		
x4						
x5						
x8						
CSECTI	ON	koneb	0.0	RQUAD		
x7						
x3						
x1						
x0						

In general, a CSECTION header has the format

|--|

where the requirements for each field are as follows:

Field	Starting Position	Max Width	Required	Description
[kname1]	15	8	Yes	Name of the cone
[value1]	25	12	No	Cone parameter
[ktype]	40		Yes	Type of the cone.

The possible cone type keys are:

[ktype]	Members	[value1]	Interpretation.
ZERO	$\geq 0$	unused	Zero cone (16.3).
QUAD	$\geq 1$	unused	Quadratic cone (16.4).
RQUAD	$\geq 2$	unused	Rotated quadratic cone (16.5).
PEXP	3	unused	Primal exponential cone (16.6).
PPOW	$\geq 2$	α	Primal power cone (16.7).
DEXP	3	unused	Dual exponential cone (16.8).
DPOW	$\geq 2$	α	Dual power cone (16.9).

A record in the CSECTION has the format

# [vname1]

where the requirements for each field are

Field	Starting Position	Max Width	required	Description
[vname1]	5	8	Yes	A valid variable name

A variable must occur in at most one CSECTION.

#### **ENDATA**

This keyword denotes the end of the MPS file.

# 16.2.2 Integer Variables

Using special bound keys in the BOUNDS section it is possible to specify that some or all of the variables should be integer-constrained i.e. be members of  $\mathcal{J}$ . However, an alternative method is available. This method is available only for backward compatibility and we recommend that it is not used. This method requires that markers are placed in the COLUMNS section as in the example:

```
COLUMNS
x1
                                                  0.7
           obj
                      -10.0
                                       c1
x1
           c2
                      0.5
                                       сЗ
                                                  1.0
x1
           c4
                      0.1
* Start of integer-constrained variables.
MARK000
                                       'INTORG'
           'MARKER'
x2
           obj
                      -9.0
                                                  1.0
                                       c1
x2
                      0.8333333333
                                                  0.6666667
           c2
                                       с3
x2
           c4
                      0.25
xЗ
                      1.0
                                                  2.0
           obj
                                       c6
MARKO01
           'MARKER'
                                       'INTEND'
* End of integer-constrained variables.
```

Please note that special marker lines are used to indicate the start and the end of the integer variables. Furthermore be aware of the following

- All variables between the markers are assigned a default lower bound of 0 and a default upper bound of 1. This may not be what is intended. If it is not intended, the correct bounds should be defined in the BOUNDS section of the MPS formatted file.
- MOSEK ignores field 1, i.e. MARKO001 and MARKO01, however, other optimization systems require
  them.
- Field 2, i.e. MARKER, must be specified including the single quotes. This implies that no row can be assigned the name MARKER.
- Field 3 is ignored and should be left blank.
- Field 4, i.e. INTORG and INTEND, must be specified.
- It is possible to specify several such integer marker sections within the COLUMNS section.

### 16.2.3 General Limitations

• An MPS file should be an ASCII file.

# 16.2.4 Interpretation of the MPS Format

Several issues related to the MPS format are not well-defined by the industry standard. However, **MOSEK** uses the following interpretation:

- If a matrix element in the COLUMNS section is specified multiple times, then the multiple entries are added together.
- If a matrix element in a QSECTION section is specified multiple times, then the multiple entries are added together.

# 16.2.5 The Free MPS Format

**MOSEK** supports a free format variation of the MPS format. The free format is similar to the MPS file format but less restrictive, e.g. it allows longer names. However, a name must not contain any blanks.

Moreover, by default a line in the MPS file must not contain more than 1024 characters. By modifying the parameter MSK\_IPAR\_READ\_MPS\_WIDTH an arbitrary large line width will be accepted.

The free MPS format is default. To change to the strict and other formats use the parameter  $MSK\_IPAR\_READ\_MPS\_FORMAT$ .

# 16.3 The OPF Format

The Optimization Problem Format (OPF) is an alternative to LP and MPS files for specifying optimization problems. It is row-oriented, inspired by the CPLEX LP format.

Apart from containing objective, constraints, bounds etc. it may contain complete or partial solutions, comments and extra information relevant for solving the problem. It is designed to be easily read and modified by hand and to be forward compatible with possible future extensions.

#### Intended use

The OPF file format is meant to replace several other files:

- The LP file format: Any problem that can be written as an LP file can be written as an OPF file too; furthermore it naturally accommodates ranged constraints and variables as well as arbitrary characters in names, fixed expressions in the objective, empty constraints, and conic constraints.
- Parameter files: It is possible to specify integer, double and string parameters along with the problem (or in a separate OPF file).
- Solution files: It is possible to store a full or a partial solution in an OPF file and later reload it.

### 16.3.1 The File Format

The format uses tags to structure data. A simple example with the basic sections may look like this:

```
[comment]
This is a comment. You may write almost anything here...
[/comment]
# This is a single-line comment.

[objective min 'myobj']
x + 3 y + x^2 + 3 y^2 + z + 1
[/objective]

[constraints]
[con 'con01'] 4 <= x + y [/con]
[/constraints]

[bounds]
[b] -10 <= x,y <= 10 [/b]

[cone quad] x,y,z [/cone]
[/bounds]</pre>
```

A scope is opened by a tag of the form [tag] and closed by a tag of the form [/tag]. An opening tag may accept a list of unnamed and named arguments, for examples:

```
[tag value] tag with one unnamed argument [/tag]
[tag arg=value] tag with one named argument [/tag]
```

Unnamed arguments are identified by their order, while named arguments may appear in any order, but never before an unnamed argument. The value can be a quoted, single-quoted or double-quoted text string, i.e.

```
[tag 'value'] single-quoted value [/tag]
[tag arg='value'] single-quoted value [/tag]
[tag "value"] double-quoted value [/tag]
[tag arg="value"] double-quoted value [/tag]
```

### 16.3.2 Sections

The recognized tags are

### [comment]

A comment section. This can contain *almost* any text: Between single quotes (') or double quotes (") any text may appear. Outside quotes the markup characters ([ and ]) must be prefixed by backslashes. Both single and double quotes may appear alone or inside a pair of quotes if it is prefixed by a backslash.

#### [objective]

The objective function: This accepts one or two parameters, where the first one (in the above example min) is either min or max (regardless of case) and defines the objective sense, and the second one (above myobj), if present, is the objective name. The section may contain linear and quadratic expressions.

If several objectives are specified, all but the last are ignored.

### [constraints]

This does not directly contain any data, but may contain subsections con defining a linear constraint.

# [con]

Defines a single constraint; if an argument is present ([con NAME]) this is used as the name of the constraint, otherwise it is given a null-name. The section contains a constraint definition written as linear and quadratic expressions with a lower bound, an upper bound, with both or with an equality. Examples:

Constraint names are unique. If a constraint is specified which has the same name as a previously defined constraint, the new constraint replaces the existing one.

#### [bounds]

This does not directly contain any data, but may contain subsections **b** (linear bounds on variables) and cone (cones).

#### [b]

Bound definition on one or several variables separated by comma (,). An upper or lower bound on a variable replaces any earlier defined bound on that variable. If only one bound (upper or lower) is given only this bound is replaced. This means that upper and lower bounds can be specified separately. So the OPF bound definition:

results in the bound  $-10 \le x, y \le 10$ .

## [cone]

Specifies a cone. A cone is defined as a sequence of variables which belong to a single unique cone. The supported cone types are:

• quad: a quadratic cone of n variables  $x_1, \ldots, x_n$  defines a constraint of the form

$$x_1^2 \ge \sum_{i=2}^n x_i^2, \quad x_1 \ge 0.$$

• rquad: a rotated quadratic cone of n variables  $x_1, \ldots, x_n$  defines a constraint of the form

$$2x_1x_2 \ge \sum_{i=3}^n x_i^2, \quad x_1, x_2 \ge 0.$$

• pexp: primal exponential cone of 3 variables  $x_1, x_2, x_3$  defines a constraint of the form

$$x_1 \ge x_2 \exp(x_3/x_2), \quad x_1, x_2 \ge 0.$$

• ppow with parameter  $0 < \alpha < 1$ : primal power cone of n variables  $x_1, \ldots, x_n$  defines a constraint of the form

$$x_1^{\alpha} x_2^{1-\alpha} \ge \sqrt{\sum_{j=3}^n x_j^2}, \quad x_1, x_2 \ge 0.$$

• dexp: dual exponential cone of 3 variables  $x_1, x_2, x_3$  defines a constraint of the form

$$x_1 \ge -x_3 e^{-1} \exp(x_2/x_3), \quad x_3 \le 0, x_1 \ge 0.$$

• dpow with parameter  $0 < \alpha < 1$ : dual power cone of n variables  $x_1, \ldots, x_n$  defines a constraint of the form

$$\left(\frac{x_1}{\alpha}\right)^{\alpha} \left(\frac{x_2}{1-\alpha}\right)^{1-\alpha} \ge \sqrt{\sum_{j=3}^{n^t} x_j^2}, \quad x_1, x_2 \ge 0.$$

• zero: zero cone of n variables  $x_1, \ldots, x_n$  defines a constraint of the form

$$x_1 = \dots = x_n = 0$$

A [bounds]-section example:

## 

By default all variables are free.

### [variables]

This defines an ordering of variables as they should appear in the problem. This is simply a space-separated list of variable names.

## [integer]

This contains a space-separated list of variables and defines the constraint that the listed variables must be integer-valued.

### [hints]

This may contain only non-essential data; for example estimates of the number of variables, constraints and non-zeros. Placed before all other sections containing data this may reduce the time spent reading the file.

In the hints section, any subsection which is not recognized by MOSEK is simply ignored. In this section a hint is defined as follows:

```
[hint ITEM] value [/hint]
```

The hints recognized by **MOSEK** are:

- number of variables),
- numcon (number of linear/quadratic constraints),
- numanz (number of linear non-zeros in constraints),
- numqnz (number of quadratic non-zeros in constraints).

#### [solutions]

This section can contain a set of full or partial solutions to a problem. Each solution must be specified using a [solution]-section, i.e.

```
[solutions]
[solution]...[/solution] #solution 1
[solution]...[/solution] #solution 2
#other solutions....
[solution]...[/solution] #solution n
[/solutions]
```

The syntax of a [solution]-section is the following:

```
[solution SOLTYPE status=STATUS]...[/solution]
```

where SOLTYPE is one of the strings

- interior, a non-basic solution,
- basic, a basic solution,
- integer, an integer solution,

and STATUS is one of the strings

- UNKNOWN,
- OPTIMAL,
- INTEGER\_OPTIMAL,
- PRIM\_FEAS,
- DUAL\_FEAS,
- PRIM\_AND\_DUAL\_FEAS,

- NEAR\_OPTIMAL,
- NEAR\_PRIM\_FEAS,
- NEAR\_DUAL\_FEAS,
- NEAR\_PRIM\_AND\_DUAL\_FEAS,
- PRIM\_INFEAS\_CER,
- DUAL\_INFEAS\_CER,
- NEAR\_PRIM\_INFEAS\_CER,
- NEAR\_DUAL\_INFEAS\_CER,
- NEAR\_INTEGER\_OPTIMAL.

Most of these values are irrelevant for input solutions; when constructing a solution for simplex hot-start or an initial solution for a mixed integer problem the safe setting is UNKNOWN.

A [solution]-section contains [con] and [var] sections. Each [con] and [var] section defines solution information for a single variable or constraint, specified as list of KEYWORD/value pairs, in any order, written as

#### KEYWORD=value

Allowed keywords are as follows:

- sk. The status of the item, where the value is one of the following strings:
  - LOW, the item is on its lower bound.
  - UPR, the item is on its upper bound.
  - FIX, it is a fixed item.
  - BAS, the item is in the basis.
  - SUPBAS, the item is super basic.
  - UNK, the status is unknown.
  - INF, the item is outside its bounds (infeasible).
- 1vl Defines the level of the item.
- sl Defines the level of the dual variable associated with its lower bound.
- su Defines the level of the dual variable associated with its upper bound.
- sn Defines the level of the variable associated with its cone.
- y Defines the level of the corresponding dual variable (for constraints only).

A [var] section should always contain the items sk, lvl, sl and su. Items sl and su are not required for integer solutions.

A [con] section should always contain sk, lvl, sl, su and y.

An example of a solution section

```
[solution basic status=UNKNOWN]
[var x0] sk=LOW lvl=5.0 [/var]
[var x1] sk=UPR lvl=10.0 [/var]
[var x2] sk=SUPBAS lvl=2.0 sl=1.5 su=0.0 [/var]

[con c0] sk=LOW lvl=3.0 y=0.0 [/con]
[con c0] sk=UPR lvl=0.0 y=5.0 [/con]
[/solution]
```

• [vendor] This contains solver/vendor specific data. It accepts one argument, which is a vendor ID – for MOSEK the ID is simply mosek – and the section contains the subsection parameters defining solver parameters. When reading a vendor section, any unknown vendor can be safely ignored. This is described later.

Comments using the # may appear anywhere in the file. Between the # and the following line-break any text may be written, including markup characters.

## 16.3.3 Numbers

Numbers, when used for parameter values or coefficients, are written in the usual way by the printf function. That is, they may be prefixed by a sign (+ or -) and may contain an integer part, decimal part and an exponent. The decimal point is always . (a dot). Some examples are

```
1
1.0
.0
1.
1e10
1e+10
1e-10
```

Some invalid examples are

```
e10 # invalid, must contain either integer or decimal part
. # invalid
.e10 # invalid
```

More formally, the following standard regular expression describes numbers as used:

```
[+|-]?([0-9]+[.][0-9]*|[.][0-9]+)([eE][+|-]?[0-9]+)?
```

## 16.3.4 Names

Variable names, constraint names and objective name may contain arbitrary characters, which in some cases must be enclosed by quotes (single or double) that in turn must be preceded by a backslash. Unquoted names must begin with a letter (a-z or A-Z) and contain only the following characters: the letters a-z and A-Z, the digits 0-9, braces ({ and }) and underscore (\_).

Some examples of legal names:

```
an_unquoted_name
another_name{123}
'single quoted name'
"double quoted name"
"name with \\"quote\\" in it"
"name with []s in it"
```

## 16.3.5 Parameters Section

In the vendor section solver parameters are defined inside the parameters subsection. Each parameter is written as

```
[p PARAMETER_NAME] value [/p]
```

where PARAMETER\_NAME is replaced by a **MOSEK** parameter name, usually of the form MSK\_IPAR\_..., MSK\_DPAR\_..., or MSK\_SPAR\_..., and the value is replaced by the value of that parameter; both integer values and named values may be used. Some simple examples are

# 16.3.6 Writing OPF Files from MOSEK

To write an OPF file then make sure the file extension is .opf.

Then modify the following parameters to define what the file should contain:

MSK_IPAR_OPF_WRITE_SOL_BAS	Include basic solution, if defined.	
MSK_IPAR_OPF_WRITE_SOL_ITG	,	
MSK_IPAR_OPF_WRITE_SOL_ITR	Include interior solution, if defined.	
MSK_IPAR_OPF_WRITE_SOLUTIONS Include solutions if they are defined. If this is off, no solutions are		
	included.	
MSK_IPAR_OPF_WRITE_HEADER	Include a small header with comments.	
MSK_IPAR_OPF_WRITE_PROBLEM	Include the problem itself — objective, constraints and bounds.	
MSK_IPAR_OPF_WRITE_PARAMETER\$ nclude all parameter settings.		

# 16.3.7 Examples

MSK\_IPAR\_OPF\_WRITE\_HINTS

This section contains a set of small examples written in OPF and describing how to formulate linear, quadratic and conic problems.

Include hints about the size of the problem.

## Linear Example 101.opf

Consider the example:

having the bounds

In the OPF format the example is displayed as shown in Listing 16.1.

Listing 16.1: Example of an OPF file for a linear problem.

```
[comment]
  The lo1 example in OPF format
[/comment]

[hints]
  [hint NUMVAR] 4 [/hint]
  [hint NUMCON] 3 [/hint]
  [hint NUMANZ] 9 [/hint]
[/hints]
```

```
[variables disallow_new_variables]
    x1 x2 x3 x4
[/variables]

[objective maximize 'obj']
    3 x1 + x2 + 5 x3 + x4
[/objective]

[constraints]
    [con 'c1'] 3 x1 + x2 + 2 x3 = 30 [/con]
    [con 'c2'] 2 x1 + x2 + 3 x3 + x4 >= 15 [/con]
    [con 'c3'] 2 x2 + 3 x4 <= 25 [/con]
[/constraints]

[bounds]
    [b] 0 <= * [/b]
    [b] 0 <= x2 <= 10 [/b]
[/bounds]
```

## Quadratic Example qo1.opf

An example of a quadratic optimization problem is

minimize 
$$x_1^2 + 0.1x_2^2 + x_3^2 - x_1x_3 - x_2$$
 subject to  $1 \le x_1 + x_2 + x_3$ ,  $x > 0$ .

This can be formulated in opf as shown below.

Listing 16.2: Example of an OPF file for a quadratic problem.

```
[comment]
 The qo1 example in OPF format
[/comment]
[hints]
 [hint NUMVAR] 3 [/hint]
  [hint NUMCON] 1 [/hint]
 [hint NUMANZ] 3 [/hint]
 [hint NUMQNZ] 4 [/hint]
[/hints]
[variables disallow_new_variables]
 x1 x2 x3
[/variables]
[objective minimize 'obj']
 # The quadratic terms are often written with a factor of 1/2 as here,
 # but this is not required.
  - x2 + 0.5 ( 2.0 x1 ^2 - 2.0 x3 * x1 + 0.2 x2 ^2 + 2.0 x3 ^2)
[/objective]
[constraints]
 [con 'c1'] 1.0 <= x1 + x2 + x3 [/con]
[/constraints]
```

```
[bounds]
[b] 0 <= * [/b]
[/bounds]
```

## Conic Quadratic Example cqo1.opf

Consider the example:

minimize 
$$x_3 + x_4 + x_5$$
  
subject to  $x_0 + x_1 + 2x_2 = 1$ ,  
 $x_0, x_1, x_2 \ge 0$ ,  
 $x_3 \ge \sqrt{x_0^2 + x_1^2}$ ,  
 $2x_4x_5 \ge x_2^2$ .

Please note that the type of the cones is defined by the parameter to [cone ...]; the content of the cone-section is the names of variables that belong to the cone. The resulting OPF file is in Listing 16.3.

Listing 16.3: Example of an OPF file for a conic quadratic problem.

```
[comment]
 The cqo1 example in OPF format.
[/comment]
[hints]
  [hint NUMVAR] 6 [/hint]
  [hint NUMCON] 1 [/hint]
  [hint NUMANZ] 3 [/hint]
[/hints]
[variables disallow_new_variables]
 x1 x2 x3 x4 x5 x6
[/variables]
[objective minimize 'obj']
  x4 + x5 + x6
[/objective]
[constraints]
  [con 'c1'] x1 + x2 + 2e+00 x3 = 1e+00 [/con]
[/constraints]
[bounds]
 # We let all variables default to the positive orthant
  [b] 0 \ll * [/b]
 # ...and change those that differ from the default
 [b] x4,x5,x6 free [/b]
 # Define quadratic cone: x4 \ge sqrt(x1^2 + x2^2)
 [cone quad 'k1'] x4, x1, x2 [/cone]
 # Define rotated quadratic cone: 2 x5 x6 >= x3^2
  [cone rquad 'k2'] x5, x6, x3 [/cone]
[/bounds]
```

## Mixed Integer Example milo1.opf

Consider the mixed integer problem:

This can be implemented in OPF with the file in Listing 16.4.

Listing 16.4: Example of an OPF file for a mixed-integer linear problem.

```
[comment]
 The milo1 example in OPF format
[/comment]
[hints]
  [hint NUMVAR] 2 [/hint]
  [hint NUMCON] 2 [/hint]
  [hint NUMANZ] 4 [/hint]
[/hints]
[variables disallow_new_variables]
 x1 x2
[/variables]
[objective maximize 'obj']
  x1 + 6.4e-1 x2
[/objective]
[constraints]
  [con 'c1'] 5e+1 x1 + 3.1e+1 x2 \le 2.5e+2 [/con]
  [con 'c2'] -4 \le 3 x1 - 2 x2 [/con]
[/constraints]
[bounds]
 [b] 0 <= * [/b]
[/bounds]
[integer]
 x1 x2
[/integer]
```

## 16.4 The CBF Format

This document constitutes the technical reference manual of the *Conic Benchmark Format* with file extension: .cbf or .CBF. It unifies linear, second-order cone (also known as conic quadratic) and semidefinite optimization with mixed-integer variables. The format has been designed with benchmark libraries in mind, and therefore focuses on compact and easily parsable representations. The problem structure is separated from the problem data, and the format moreover facilitates benchmarking of hotstart capability through sequences of changes.

# 16.4.1 How Instances Are Specified

This section defines the spectrum of conic optimization problems that can be formulated in terms of the keywords of the CBF format.

In the CBF format, conic optimization problems are considered in the following form:

$$\min / \max \quad g^{obj} 
g_i \in \mathcal{K}_i, \quad i \in \mathcal{I}, 
\text{s.t.} \quad G_i \in \mathcal{K}_i, \quad i \in \mathcal{I}^{PSD}, 
x_j \in \mathcal{K}_j, \quad j \in \mathcal{J}, 
\overline{X}_j \in \mathcal{K}_j, \quad j \in \mathcal{J}^{PSD}.$$
(16.10)

- Variables are either scalar variables,  $x_j$  for  $j \in \mathcal{J}$ , or variables,  $\overline{X}_j$  for  $j \in \mathcal{J}^{PSD}$ . Scalar variables can also be declared as integer.
- Constraints are affine expressions of the variables, either scalar-valued  $g_i$  for  $i \in \mathcal{I}$ , or matrix-valued  $G_i$  for  $i \in \mathcal{I}^{PSD}$

$$g_i = \sum_{j \in \mathcal{J}^{PSD}} \langle F_{ij}, X_j \rangle + \sum_{j \in \mathcal{J}} a_{ij} x_j + b_i,$$
$$G_i = \sum_{j \in \mathcal{J}} x_j H_{ij} + D_i.$$

• The **objective function** is a scalar-valued affine expression of the variables, either to be minimized or maximized. We refer to this expression as  $g^{obj}$ 

$$g^{obj} = \sum_{j \in \mathcal{J}^{PSD}} \langle F_j^{obj}, X_j \rangle + \sum_{j \in \mathcal{J}} a_j^{obj} x_j + b^{obj}.$$

CBF format can represent the following cones  $\mathcal{K}$ :

• Free domain - A cone in the linear family defined by

$$\{x \in \mathbb{R}^n\}, \text{ for } n \ge 1.$$

• Positive orthant - A cone in the linear family defined by

$$\{x \in \mathbb{R}^n \mid x_i > 0 \text{ for } j = 1, \dots, n\}, \text{ for } n > 1.$$

• Negative orthant - A cone in the linear family defined by

$$\{x \in \mathbb{R}^n \mid x_i \leq 0 \text{ for } j = 1, \dots, n\}, \text{ for } n \geq 1.$$

• Fixpoint zero - A cone in the linear family defined by

$$\{x \in \mathbb{R}^n \mid x_i = 0 \text{ for } j = 1, \dots, n\}, \text{ for } n \ge 1.$$

• Quadratic cone - A cone in the second-order cone family defined by

$$\left\{ \left( \begin{array}{c} p \\ x \end{array} \right) \in \mathbb{R} \times \mathbb{R}^{n-1}, \ p^2 \ge x^T x, \ p \ge 0 \right\}, \ \text{for } n \ge 2.$$

• Rotated quadratic cone - A cone in the second-order cone family defined by

$$\left\{ \begin{pmatrix} p \\ q \\ x \end{pmatrix} \in \mathbb{R} \times \mathbb{R} \times \mathbb{R}^{n-2}, \ 2pq \ge x^T x, \ p \ge 0, \ q \ge 0 \right\}, \text{ for } n \ge 3.$$

## 16.4.2 The Structure of CBF Files

This section defines how information is written in the CBF format, without being specific about the type of information being communicated.

All information items belong to exactly one of the three groups of information. These information groups, and the order they must appear in, are:

- 1. File format.
- 2. Problem structure.
- 3. Problem data.

The first group, file format, provides information on how to interpret the file. The second group, problem structure, provides the information needed to deduce the type and size of the problem instance. Finally, the third group, problem data, specifies the coefficients and constants of the problem instance.

#### Information items

The format is composed as a list of information items. The first line of an information item is the KEYWORD, revealing the type of information provided. The second line - of some keywords only - is the HEADER, typically revealing the size of information that follows. The remaining lines are the BODY holding the actual information to be specified.



The KEYWORD determines how each line in the HEADER and BODY is structured. Moreover, the number of lines in the BODY follows either from the KEYWORD, the HEADER, or from another information item required to precede it.

## **Embedded hotstart-sequences**

A sequence of problem instances, based on the same problem structure, is within a single file. This is facilitated via the CHANGE within the problem data information group, as a separator between the information items of each instance. The information items following a CHANGE keyword are appending to, or changing (e.g., setting coefficients back to their default value of zero), the problem data of the preceding instance.

The sequence is intended for benchmarking of hotstart capability, where the solvers can reuse their internal state and solution (subject to the achieved accuracy) as warmpoint for the succeeding instance. Whenever this feature is unsupported or undesired, the keyword CHANGE should be interpreted as the end of file.

## File encoding and line width restrictions

The format is based on the US-ASCII printable character set with two extensions as listed below. Note, by definition, that none of these extensions can be misinterpreted as printable US-ASCII characters:

- A line feed marks the end of a line, carriage returns are ignored.
- Comment-lines may contain unicode characters in UTF-8 encoding.

The line width is restricted to 512 bytes, with 3 bytes reserved for the potential carriage return, line feed and null-terminator.

Integers and floating point numbers must follow the ISO C decimal string representation in the standard C locale. The format does not impose restrictions on the magnitude of, or number of significant digits in numeric data, but the use of 64-bit integers and 64-bit IEEE 754 floating point numbers should be sufficient to avoid loss of precision.

### Comment-line and whitespace rules

The format allows single-line comments respecting the following rule:

• Lines having first byte equal to '#' (US-ASCII 35) are comments, and should be ignored. Comments are only allowed between information items.

Given that a line is not a comment-line, whitespace characters should be handled according to the following rules:

- Leading and trailing whitespace characters should be ignored.
  - The seperator between multiple pieces of information on one line, is either one or more whitespace characters.
- Lines containing only whitespace characters are empty, and should be ignored. Empty lines are only allowed between information items.

## 16.4.3 Problem Specification

## The problem structure

The problem structure defines the objective sense, whether it is minimization and maximization. It also defines the index sets,  $\mathcal{J}$ ,  $\mathcal{J}^{PSD}$ ,  $\mathcal{I}$  and  $\mathcal{I}^{PSD}$ , which are all numbered from zero,  $\{0, 1, \ldots\}$ , and empty until explicitly constructed.

• Scalar variables are constructed in vectors restricted to a conic domain, such as  $(x_0, x_1) \in \mathbb{R}^2_+$ ,  $(x_2, x_3, x_4) \in \mathcal{Q}^3$ , etc. In terms of the Cartesian product, this generalizes to

$$x \in \mathcal{K}_1^{n_1} \times \mathcal{K}_2^{n_2} \times \dots \times \mathcal{K}_k^{n_k}$$

which in the CBF format becomes:

```
VAR
n k
K1 n1
K2 n2
...
Kk nk
```

where  $\sum_{i} n_{i} = n$  is the total number of scalar variables. The list of supported cones is found in Table 16.3. Integrality of scalar variables can be specified afterwards.

• **PSD variables** are constructed one-by-one. That is,  $X_j \succeq \mathbf{0}^{n_j \times n_j}$  for  $j \in \mathcal{J}^{PSD}$ , constructs a matrix-valued variable of size  $n_j \times n_j$  restricted to be symmetric positive semidefinite. In the CBF format, this list of constructions becomes:



where N is the total number of PSD variables.

• Scalar constraints are constructed in vectors restricted to a conic domain, such as  $(g_0, g_1) \in \mathbb{R}^2_+$ ,  $(g_2, g_3, g_4) \in \mathcal{Q}^3$ , etc. In terms of the Cartesian product, this generalizes to

$$g \in \mathcal{K}_1^{m_1} \times \mathcal{K}_2^{m_2} \times \cdots \times \mathcal{K}_k^{m_k}$$

which in the CBF format becomes:

```
CON
m k
K1 m1
K2 m2
...
Kk mk
```

where  $\sum_{i} m_{i} = m$  is the total number of scalar constraints. The list of supported cones is found in Table 16.3.

• **PSD constraints** are constructed one-by-one. That is,  $G_i \succeq \mathbf{0}^{m_i \times m_i}$  for  $i \in \mathcal{I}^{PSD}$ , constructs a matrix-valued affine expressions of size  $m_i \times m_i$  restricted to be symmetric positive semidefinite. In the CBF format, this list of constructions becomes

```
PSDCON
M
m1
m2
..
mM
```

where M is the total number of PSD constraints.

With the objective sense, variables (with integer indications) and constraints, the definitions of the many affine expressions follow in problem data.

#### Problem data

The problem data defines the coefficients and constants of the affine expressions of the problem instance. These are considered zero until explicitly defined, implying that instances with no keywords from this information group are, in fact, valid. Duplicating or conflicting information is a failure to comply with the standard. Consequently, two coefficients written to the same position in a matrix (or to transposed positions in a symmetric matrix) is an error.

The affine expressions of the objective,  $g^{obj}$ , of the scalar constraints,  $g_i$ , and of the PSD constraints,  $G_i$ , are defined separately. The following notation uses the standard trace inner product for matrices,  $\langle X, Y \rangle = \sum_{i,j} X_{ij} Y_{ij}$ .

 $\bullet\,$  The affine expression of the objective is defined as

$$g^{obj} = \sum_{j \in \mathcal{J}^{PSD}} \langle F_j^{obj}, X_j \rangle + \sum_{j \in \mathcal{J}} a_j^{obj} x_j + b^{obj},$$

in terms of the symmetric matrices,  $F_j^{obj}$ , and scalars,  $a_j^{obj}$  and  $b^{obj}$ .

• The affine expressions of the scalar constraints are defined, for  $i \in \mathcal{I}$ , as

$$g_i = \sum_{j \in \mathcal{J}^{PSD}} \langle F_{ij}, X_j \rangle + \sum_{j \in \mathcal{J}} a_{ij} x_j + b_i,$$

in terms of the symmetric matrices,  $F_{ij}$ , and scalars,  $a_{ij}$  and  $b_i$ .

• The affine expressions of the PSD constraints are defined, for  $i \in \mathcal{I}^{PSD}$ , as

$$G_i = \sum_{j \in \mathcal{J}} x_j H_{ij} + D_i,$$

in terms of the symmetric matrices,  $H_{ij}$  and  $D_i$ .

### List of cones

The format uses an explicit syntax for symmetric positive semidefinite cones as shown above. For scalar variables and constraints, constructed in vectors, the supported conic domains and their minimum sizes are given as follows.

Table 16.3: Cones available in the CBF format

Name	CBF keyword	Cone family
Free domain	F	linear
Positive orthant	L+	linear
Negative orthant	L-	linear
Fixpoint zero	L=	linear
Quadratic cone	Q	second-order
Rotated quadratic cone	QR	second-order

# 16.4.4 File Format Keywords

#### **VER**

Description: The version of the Conic Benchmark Format used to write the file.

**HEADER:** None

BODY: One line formatted as:

INT

This is the version number.

Must appear exactly once in a file, as the first keyword.

## **OBJSENSE**

Description: Define the objective sense.

**HEADER:** None

BODY: One line formatted as:

STR

having MIN indicates minimize, and MAX indicates maximize. Capital letters are required.

Must appear exactly once in a file.

#### **PSDVAR**

Description: Construct the PSD variables.

**HEADER:** One line formatted as:

INT

This is the number of PSD variables in the problem.

BODY: A list of lines formatted as:

INT

This indicates the number of rows (equal to the number of columns) in the matrix-valued PSD variable. The number of lines should match the number stated in the header.

#### **VAR**

Description: Construct the scalar variables.

**HEADER**: One line formatted as:

INT INT

This is the number of scalar variables, followed by the number of conic domains they are restricted to.

BODY: A list of lines formatted as:

STR INT

This indicates the cone name (see Table 16.3), and the number of scalar variables restricted to this cone. These numbers should add up to the number of scalar variables stated first in the header. The number of lines should match the second number stated in the header.

#### INT

Description: Declare integer requirements on a selected subset of scalar variables.

HEADER: one line formatted as:

INT

This is the number of integer scalar variables in the problem.

BODY: a list of lines formatted as:

INT

This indicates the scalar variable index  $j \in \mathcal{J}$ . The number of lines should match the number stated in the header.

Can only be used after the keyword VAR.

#### **PSDCON**

Description: Construct the PSD constraints.

**HEADER**: One line formatted as:

INT

This is the number of PSD constraints in the problem.

BODY: A list of lines formatted as:

INT

This indicates the number of rows (equal to the number of columns) in the matrix-valued affine expression of the PSD constraint. The number of lines should match the number stated in the header.

Can only be used after these keywords: PSDVAR, VAR.

## **CON**

Description: Construct the scalar constraints.

**HEADER:** One line formatted as:

#### INT INT

This is the number of scalar constraints, followed by the number of conic domains they restrict to. BODY: A list of lines formatted as:

#### STR INT

This indicates the cone name (see Table 16.3), and the number of affine expressions restricted to this cone. These numbers should add up to the number of scalar constraints stated first in the header. The number of lines should match the second number stated in the header.

Can only be used after these keywords: PSDVAR, VAR

## **OBJFCOORD**

Description: Input sparse coordinates (quadruplets) to define the symmetric matrices  $F_j^{obj}$ , as used in the objective.

**HEADER**: One line formatted as:

#### INT

This is the number of coordinates to be specified.

BODY: A list of lines formatted as:

#### INT INT INT REAL

This indicates the PSD variable index  $j \in \mathcal{J}^{PSD}$ , the row index, the column index and the coefficient value. The number of lines should match the number stated in the header.

#### **OBJACOORD**

Description: Input sparse coordinates (pairs) to define the scalars,  $a_j^{obj}$ , as used in the objective. HEADER: One line formatted as:

#### INT

This is the number of coordinates to be specified.

BODY: A list of lines formatted as:

## INT REAL

This indicates the scalar variable index  $j \in \mathcal{J}$  and the coefficient value. The number of lines should match the number stated in the header.

# **OBJBCOORD**

Description: Input the scalar,  $b^{obj}$ , as used in the objective.

HEADER: None.

BODY: One line formatted as:

### REAL

This indicates the coefficient value.

## **FCOORD**

Description: Input sparse coordinates (quintuplets) to define the symmetric matrices,  $F_{ij}$ , as used in the scalar constraints.

**HEADER**: One line formatted as:

INT

This is the number of coordinates to be specified.

BODY: A list of lines formatted as:

#### INT INT INT INT REAL

This indicates the scalar constraint index  $i \in \mathcal{I}$ , the PSD variable index  $j \in \mathcal{J}^{PSD}$ , the row index, the column index and the coefficient value. The number of lines should match the number stated in the header

#### **ACOORD**

Description: Input sparse coordinates (triplets) to define the scalars,  $a_{ij}$ , as used in the scalar constraints. HEADER: One line formatted as:

INT

This is the number of coordinates to be specified.

BODY: A list of lines formatted as:

#### INT INT REAL

This indicates the scalar constraint index  $i \in \mathcal{I}$ , the scalar variable index  $j \in \mathcal{J}$  and the coefficient value. The number of lines should match the number stated in the header.

## **BCOORD**

Description: Input sparse coordinates (pairs) to define the scalars,  $b_i$ , as used in the scalar constraints. HEADER: One line formatted as:

INT

This is the number of coordinates to be specified.

BODY: A list of lines formatted as:

INT REAL

This indicates the scalar constraint index  $i \in \mathcal{I}$  and the coefficient value. The number of lines should match the number stated in the header.

## **HCOORD**

Description: Input sparse coordinates (quintuplets) to define the symmetric matrices,  $H_{ij}$ , as used in the PSD constraints.

**HEADER**: One line formatted as:

INT

This is the number of coordinates to be specified.

BODY: A list of lines formatted as

## INT INT INT INT REAL

This indicates the PSD constraint index  $i \in \mathcal{I}^{PSD}$ , the scalar variable index  $j \in \mathcal{J}$ , the row index, the column index and the coefficient value. The number of lines should match the number stated in the header.

## **DCOORD**

Description: Input sparse coordinates (quadruplets) to define the symmetric matrices,  $D_i$ , as used in the PSD constraints.

**HEADER:** One line formatted as

INT

This is the number of coordinates to be specified.

BODY: A list of lines formatted as:

```
INT INT INT REAL
```

This indicates the PSD constraint index  $i \in \mathcal{I}^{PSD}$ , the row index, the column index and the coefficient value. The number of lines should match the number stated in the header.

#### **CHANGE**

Start of a new instance specification based on changes to the previous. Can be interpreted as the end of file when the hotstart-sequence is unsupported or undesired.

BODY: None Header: None

## 16.4.5 CBF Format Examples

## Minimal Working Example

The conic optimization problem (16.11), has three variables in a quadratic cone - first one is integer - and an affine expression in domain 0 (equality constraint).

minimize 
$$5.1 x_0$$
  
subject to  $6.2 x_1 + 7.3 x_2 - 8.4 \in \{0\}$   
 $x \in \mathcal{Q}^3, x_0 \in \mathbb{Z}.$  (16.11)

Its formulation in the Conic Benchmark Format begins with the version of the CBF format used, to safeguard against later revisions.

VER 1

Next follows the problem structure, consisting of the objective sense, the number and domain of variables, the indices of integer variables, and the number and domain of scalar-valued affine expressions (i.e., the equality constraint).

OBJSENSE
MIN

VAR
3 1
Q 3

INT
1
0

CON
1 1
L= 1

Finally follows the problem data, consisting of the coefficients of the objective, the coefficients of the constraints, and the constant terms of the constraints. All data is specified on a sparse coordinate form.

```
OBJACOORD

1
0 5.1

ACOORD
2
0 1 6.2
0 2 7.3

BCOORD
1
0 -8.4
```

This concludes the example.

## Mixing Linear, Second-order and Semidefinite Cones

The conic optimization problem (16.12), has a semidefinite cone, a quadratic cone over unordered subindices, and two equality constraints.

The equality constraints are easily rewritten to the conic form,  $(g_0, g_1) \in \{0\}^2$ , by moving constants such that the right-hand-side becomes zero. The quadratic cone does not fit under the VAR keyword in this variable permutation. Instead, it takes a scalar constraint  $(g_2, g_3, g_4) = (x_1, x_0, x_2) \in \mathcal{Q}^3$ , with scalar variables constructed as  $(x_0, x_1, x_2) \in \mathbb{R}^3$ . Its formulation in the CBF format is reported in the following list

```
# File written using this version of the Conic Benchmark Format:
      | Version 1.
VER
1
# The sense of the objective is:
      | Minimize.
OBJSENSE
MIN
# One PSD variable of this size:
      | Three times three.
PSDVAR
1
3
# Three scalar variables in this one conic domain:
#
      | Three are free.
VAR
3 1
```

```
F 3
# Five scalar constraints with affine expressions in two conic domains:
      | Two are fixed to zero.
#
      | Three are in conic quadratic domain.
CON
5 2
L= 2
Q3
# Five coordinates in F^{obj}_j coefficients:
      | F^{obj}[0][0,0] = 2.0
      | F^{obj}[0][1,0] = 1.0
      | and more...
OBJFCOORD
0 0 0 2.0
0 1 0 1.0
0 1 1 2.0
0 2 1 1.0
0 2 2 2.0
# One coordinate in a^{obj}_j coefficients:
     | a^{obj}[1] = 1.0
OBJACOORD
1 1.0
# Nine coordinates in F_ij coefficients:
      | F[0,0][0,0] = 1.0
      | F[0,0][1,1] = 1.0
      | and more...
FCOORD
0 0 0 0 1.0
0 0 1 1 1.0
0 0 2 2 1.0
1 0 0 0 1.0
1 0 1 0 1.0
1 0 2 0 1.0
1 0 1 1 1.0
1 0 2 1 1.0
1 0 2 2 1.0
# Six coordinates in a_ij coefficients:
      | a[0,1] = 1.0
      | a[1,0] = 1.0
      | and more...
ACOORD
0 1 1.0
1 0 1.0
1 2 1.0
2 1 1.0
3 0 1.0
4 2 1.0
```

```
# Two coordinates in b_i coefficients:
#   | b[0] = -1.0
#   | b[1] = -0.5
BCOORD
2
0 -1.0
1 -0.5
```

## Mixing Semidefinite Variables and Linear Matrix Inequalities

The standard forms in semidefinite optimization are usually based either on semidefinite variables or linear matrix inequalities. In the CBF format, both forms are supported and can even be mixed as shown in.

minimize 
$$\left\langle \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{bmatrix}, X_1 \right\rangle + x_1 + x_2 + 1$$
  
subject to  $\left\langle \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, X_1 \right\rangle - x_1 - x_2 \qquad \geq 0.0,$   
 $x_1 \begin{bmatrix} 0 & 1 \\ 1 & 3 \end{bmatrix} + x_2 \begin{bmatrix} 3 & 1 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \geq \mathbf{0},$   
 $X_1 \succeq \mathbf{0}.$  (16.13)

Its formulation in the CBF format is written in what follows

```
# File written using this version of the Conic Benchmark Format:
      | Version 1.
VER.
1
# The sense of the objective is:
     | Minimize.
OBJSENSE
MIN
# One PSD variable of this size:
# | Two times two.
PSDVAR
1
2
# Two scalar variables in this one conic domain:
      | Two are free.
VAR
2 1
F 2
# One PSD constraint of this size:
# | Two times two.
PSDCON
1
2
# One scalar constraint with an affine expression in this one conic domain:
#
      | One is greater than or equal to zero.
CON
1 1
```

```
L+ 1
# Two coordinates in F^{obj}_j coefficients:
# | F^{obj}[0][0,0] = 1.0
# | F^{obj}[0][1.1] = 1.0
      | F^{obj}[0][1,1] = 1.0
OBJFCOORD
0 0 0 1.0
0 1 1 1.0
# Two coordinates in a^{obj}_j coefficients:
# | a^{obj}[0] = 1.0
      | a^{obj}[1] = 1.0
OBJACOORD
0 1.0
1 1.0
# One coordinate in b^{obj} coefficient:
# | b^{obj} = 1.0
OBJBCOORD
1.0
\# One coordinate in F_ij coefficients:
# | F[0,0][1,0] = 1.0
FCOORD
0 0 1 0 1.0
# Two coordinates in a_ij coefficients:
     | a[0,0] = -1.0
      | a[0,1] = -1.0
ACOORD
0 0 -1.0
0 1 -1.0
# Four coordinates in H_ij coefficients:
# | H[0,0][1,0] = 1.0
     | H[0,0][1,1] = 3.0
# | and more...
HCOORD
0 0 1 0 1.0
0 0 1 1 3.0
0 1 0 0 3.0
0 1 1 0 1.0
# Two coordinates in D_i coefficients:
# D[0][0,0] = -1.0
     | D[0][1,1] = -1.0
#
DCOORD
0 0 0 -1.0
0 1 1 -1.0
```

## Optimization Over a Sequence of Objectives

The linear optimization problem (16.14), is defined for a sequence of objectives such that hotstarting from one to the next might be advantages.

$$\begin{array}{llll} \text{maximize}_k & g_k^{obj} \\ \text{subject to} & 50 \, x_0 + 31 & \leq & 250 \, , \\ & 3 \, x_0 - 2 x_1 & \geq & -4 \, , \\ & & x \in \mathbb{R}_+^2, \end{array} \tag{16.14}$$

given,

```
1. g_0^{obj} = x_0 + 0.64x_1.

2. g_1^{obj} = 1.11x_0 + 0.76x_1.

3. g_2^{obj} = 1.11x_0 + 0.85x_1.
```

Its formulation in the CBF format is reported in Listing 16.5.

Listing 16.5: Problem (16.14) in CBF format.

```
# File written using this version of the Conic Benchmark Format:
#
      | Version 1.
VER
# The sense of the objective is:
      | Maximize.
OBJSENSE
MAX
# Two scalar variables in this one conic domain:
      | Two are nonnegative.
VAR
2 1
L+ 2
# Two scalar constraints with affine expressions in these two conic domains:
      | One is in the nonpositive domain.
      | One is in the nonnegative domain.
CON
2 2
L- 1
L+ 1
# Two coordinates in a^{obj}_j coefficients:
      | a^{obj}[0] = 1.0
      | a^{obj}[1] = 0.64
OBJACOORD
0 1.0
1 0.64
# Four coordinates in a_ij coefficients:
      | a[0,0] = 50.0
      | a[1,0] = 3.0
#
      | and more...
ACOORD
```

```
0 0 50.0
1 0 3.0
0 1 31.0
1 1 -2.0
# Two coordinates in b_i coefficients:
      | b[0] = -250.0
      | b[1] = 4.0
BCOORD
0 -250.0
1 4.0
# New problem instance defined in terms of changes.
# Two coordinate changes in a^{obj}_j coefficients. Now it is:
      | a^{obj}[0] = 1.11
      | a^{obj}[1] = 0.76
OBJACOORD
2
0 1.11
1 0.76
# New problem instance defined in terms of changes.
# One coordinate change in a\{obj\}_j coefficients. Now it is:
      | a^{obj}[0] = 1.11
      | a^{obj}[1] = 0.85
OBJACOORD
1 0.85
```

# 16.5 The PTF Format

The PTF format is a new human-readable, natural text format. Its features and structure are similar to the *OPF* format, with the difference that the PTF format **does** support semidefinite terms.

## 16.5.1 The overall format

The format is indentation based, where each section is started by a head line and followed by a section body with deeper indentation that the head line. For example:

```
Header line
Body line 1
Body line 1
Body line 1
```

Section can also be nested:

```
Header line A

Body line in A

Header line A.1

Body line in A.1
```

```
Body line in A.1
Body line in A
```

The indentation of blank lines is ignored, so a subsection can contain a blank line with no indentation. The character # defines a line comment and anything between the # character and the end of the line is ignored.

In a PTF file, the first section must be a Task section. The order of the remaining section is arbitrary, and sections may occur multiple times or not at all.

MOSEK will ignore any top-level section it does not recognize.

#### **Names**

In the description of the format we use following definitions for name strings:

```
NAME: PLAIN_NAME | QUOTED_NAME

PLAIN_NAME: [a-zA-Z_] [a-zA-Z0-9_-.!|]

QUOTED_NAME: "'" ( [^'\\r\n] | "\\" ( [\\rn] | "x" [0-9a-fA-F] [0-9a-fA-F] ) )* "'"
```

### **Expressions**

An expression is a sum of terms. A term is either a linear term (a coefficient and a variable name, where the coefficient can be left out if it is 1.0), or a matrix inner product.

An expression:

```
EXPR: EMPTY | [+-]? TERM ( [+-] TERM )*
TERM: LINEAR_TERM | MATRIX_TERM
```

A linear term

```
LINEAR_TERM: FLOAT? NAME
```

A matrix term

```
MATRIX_TERM: "<" FLOAT? NAME ( [+-] FLOAT? NAME)* ";" NAME ">"
```

Here the right-hand name is the name of a (semidefinite) matrix variable, and the left-hand side is a sum of symmetric matrixes. The actual matrixes are defined in a separate section.

Expressions can span multiple lines by giving subsequent lines a deeper indentation.

For example following two section are equivalent:

## 16.5.2 Task section

The first section of the file must be a Task. The text in this section is not used and may contain comments, or meta-information from the writer or about the content.

Format:

```
Task NAME
Anything goes here...
```

NAME is a the task name.

# 16.5.3 Objective section

The Objective section defines the objective name, sense and function. The format:

```
"Objective" NAME?
( "Minimize" | "Maximize" ) EXPR
```

For example:

```
Objective 'obj'
Minimize x1 + 0.2 x2 + < M1; X1 >
```

## 16.5.4 Constraints section

The constraints section defines a series of constraints. A constraint defines a term  $A \cdot x + b \in K$ . For linear constraints A is just one row, while for conic constraints it can be multiple rows. If a constraint spans multiple rows these can either be written inline separated by semi-colons, or each expression in a separete sub-section.

Simple linear constraints:

```
"Constraints"
NAME? "[" [-+] (FLOAT | "Inf") (";" [-+] (FLOAT | "Inf") )? "]" EXPR
```

If the brackets contain two values, they are used as upper and lower bounds. It they contain one value the constraint is an equality.

For example:

```
Constraints
'c1' [0;10] x1 + x2 + x3
[0] x1 + x2 + x3
```

Constraint blocks put the expression either in a subsection or inline. The cone type (domain) is written in the brackets, and MOSEK currently supports following types:

- SOC(N) Second order cone of dimension N
- RSOC(N) Rotated second order cone of dimension N
- PSD(N) Symmetric positive semidefinite cone of dimension N. This contains N\*(N+1)/2 elements.
- PEXP Primal exponential cone of dimension 3
- DEXP Dual exponential cone of dimension 3
- PPOW(N,P) Primal power cone of dimension N with parameter P
- DPOW(N,P) Dual power cone of dimension N with parameter P
- ZERO(N) The zero-cone of dimension N.

```
"Constraints"
NAME? "[" DOMAIN "]" EXPR_LIST
```

For example:

```
Constraints

'K1' [SOC(3)] x1 + x2 ; x2 + x3 ; x3 + x1

'K2' [RSOC(3)]

x1 + x2

x2 + x3

x3 + x1
```

## 16.5.5 Variables section

Any variable used in an expression must be defined in a variable section. The variable section defines each variable domain.

```
"Variables"

NAME "[" [-+] (FLOAT | "Inf") (";" [-+] (FLOAT | "Inf") )? "]"

NAME "[" DOMAIN "]" NAMES

For example, a linear variable
```

```
Variables x1 [0;Inf]
```

As with constraints, members of a conic domain can be listed either inline or in a subsection:

```
Variables
k1 [SOC(3)] x1; x2; x3
k2 [RSOC(3)]
x1
x2
x3
```

## 16.5.6 Integer section

This section contains a list of variables that are integral. For example:

```
Integer x1 x2 x3
```

## 16.5.7 SymmetricMatrixes section

This section defines the symmetric matrixes used for matrix coefficients in matrix inner product terms. The section lists named matrixes, each with a size and a number of non-zeros. Only non-zeros in the lower triangular part should be defined.

```
"SymmetricMatrixes"

NAME "SYMMAT" "(" INT ")" ( "(" INT "," INT "," FLOAT ")" )*

...
```

For example:

```
SymmetricMatrixes
M1 SYMMAT(3) (0,0,1.0) (1,1,2.0) (2,1,0.5)
M2 SYMMAT(3)
(0,0,1.0)
(1,1,2.0)
(2,1,0.5)
```

## 16.5.8 Solutions section

Each subsection defines a solution. A solution defines for each constraint and for each variable exactly one primal value and either one (for conic domains) or two (for linear domains) dual values. The values follow the same logic as in the **MOSEK** C API. A primal and a dual solution status defines the meaning of the values primal and dual (solution, certificate, unknown, etc.)

The format is this:

```
"Solutions"
  "Solution" WHICHSOL
      "ProblemStatus" PROSTA PROSTA?
  "SolutionStatus" SOLSTA SOLSTA?
  "Objective" FLOAT FLOAT
  "Variables"
      # Linear variable status: level, slx, sux
     NAME "[" STATUS "]" FLOAT (FLOAT FLOAT)?
      # Conic variable status: level, snx
      NAME.
         "[" STATUS "]" FLOAT FLOAT?
         . . .
   "Constraints"
      # Linear variable status: level, slx, sux
      NAME "[" STATUS "]" FLOAT (FLOAT FLOAT)?
      # Conic variable status: level, snx
      NAME
         "[" STATUS "]" FLOAT FLOAT?
```

Following values for WHICHSOL are supported:

- interior Interior solution, the result of an interior-point solver.
- basic Basic solution, as produced by a simplex solver.
- integer Integer solution, the solution to a mixed-integer problem. This does not define a dual solution.

Following values for PROSTA are supported:

- unknown The problem status is unknown
- feasible The problem has been proven feasible
- infeasible The problem has been proven infeasible
- illposed The problem has been proved to be ill posed
- infeasible\_or\_unbounded The problem is infeasible or unbounded

Following values for SOLSTA are supported:

- unknown The solution status is unknown
- feasible The solution is feasible
- optimal The solution is optimal
- infeas\_cert The solution is a certificate of infeasibility
- illposed\_cert The solution is a certificate of illposedness

Following values for STATUS are supported:

- unknown The value is unknown
- super\_basic The value is super basic

- at\_lower The value is basic and at its lower bound
- at\_upper The value is basic and at its upper bound
- fixed The value is basic fixed
- infinite The value is at infinity

# 16.6 The Task Format

The Task format is MOSEK's native binary format. It contains a complete image of a MOSEK task, i.e.

- Problem data: Linear, conic, semidefinite and quadratic data
- Problem item names: Variable names, constraints names, cone names etc.
- Parameter settings
- Solutions

There are a few things to be aware of:

- Status of a solution read from a file will always be unknown.
- Parameter settings in a task file *always override* any parameters set on the command line or in a parameter file.

The format is based on the TAR (USTar) file format. This means that the individual pieces of data in a .task file can be examined by unpacking it as a TAR file. Please note that the inverse may not work: Creating a file using TAR will most probably not create a valid **MOSEK** Task file since the order of the entries is important.

## 16.7 The JSON Format

**MOSEK** provides the possibility to read/write problems in valid JSON format.

JSON (JavaScript Object Notation) is a lightweight data-interchange format. It is easy for humans to read and write. It is easy for machines to parse and generate. It is based on a subset of the JavaScript Programming Language, Standard ECMA-262 3rd Edition - December 1999. JSON is a text format that is completely language independent but uses conventions that are familiar to programmers of the C-family of languages, including C, C++, C#, Java, JavaScript, Perl, Python, and many others. These properties make JSON an ideal data-interchange language.

The official JSON website http://www.json.org provides plenty of information along with the format definition.

**MOSEK** defines two JSON-like formats:

- jtask
- jsol

Despite being text-based human-readable formats, jtask and jsol files will include no indentation and no new-lines, in order to keep the files as compact as possible. We therefore strongly advise to use JSON viewer tools to inspect jtask and jsol files.

# 16.7.1 jtask format

It stores a problem instance. The *jtask* format contains the same information as a *task format*. Even though a *jtask* file is human-readable, we do not recommend users to create it by hand, but to rely on **MOSEK**.

You can read and write jtask files using  $MSK\_readdata$  and  $MSK\_writedata$  specifying the extension .jtask.

# 16.7.2 isol format

It stores a problem solution. The *jsol* format contains all solutions and information items. You can write a *jsol* file using MSK\_writejsonsol. You can not read a *jsol* file into MOSEK.

# 16.7.3 A jtask example

In Listing 16.6 we present a file in the *jtask* format that corresponds to the sample problem from lo1.lp. The listing has been formatted for readability.

Listing 16.6: A formatted *jtask* file for the lol.lp example.

```
{
    "$schema": "http://mosek.com/json/schema#",
    "Task/INFO":{
        "taskname": "lo1",
        "numvar":4,
        "numcon":3,
        "numcone":0,
         "numbarvar":0,
        "numanz":9,
        "numsymmat":0,
        "mosekver":[
             8,
             0,
             0,
             9
        ]
    },
    "Task/data":{
        "var":{
             "name":[
                 "x1",
                 "x2",
                 "x3"
                 "x4"
             ],
             "bk":[
                 "lo",
                 "ra",
                 "lo",
                 "lo"
             ],
             "bl":[
                 0.0,
                 0.0,
                 0.0.
                 0.0
             ],
             "bu":[
```

```
1e+30,
        1e+1,
        1e+30,
        1e+30
    ],
    "type":[
        "cont",
        "cont",
        "cont",
        "cont"
    ]
},
"con":{
    "name":[
        "c1",
        "c2",
        "c3"
    ],
    "bk":[
        "fx",
        "lo",
        "up"
    ],
    "bl":[
        3e+1,
        1.5e+1,
            -1e+30
    ],
    "bu":[
        3e+1,
        1e+30,
        2.5e+1
    ]
},
"objective":{
    "sense":"max",
    "name":"obj",
    "c":{
        "subj":[
            0,
            1,
            2,
            3
        ],
        "val":[
            3e+0,
            1e+0,
            5e+0,
            1e+0
        ]
    },
    "cfix":0.0
},
"A":{
    "subi":[
        Ο,
```

```
Ο,
            0,
            1,
            1,
            1,
            1,
            2,
            2
        ],
        "subj":[
            0,
            1,
            2,
            0,
            1,
            2,
            3,
            1,
            3
        ],
        "val":[
            3e+0,
            1e+0,
            2e+0,
            2e+0,
            1e+0,
            3e+0.
            1e+0,
            2e+0,
            3e+0
        ]
    }
"Task/parameters":{
    "iparam":{
        "ANA_SOL_BASIS":"ON",
        "ANA_SOL_PRINT_VIOLATED": "OFF",
        "AUTO_SORT_A_BEFORE_OPT": "OFF",
        "AUTO_UPDATE_SOL_INFO":"OFF",
        "BASIS_SOLVE_USE_PLUS_ONE":"OFF",
        "BI_CLEAN_OPTIMIZER": "OPTIMIZER_FREE",
        "BI_IGNORE_MAX_ITER": "OFF",
        "BI_IGNORE_NUM_ERROR":"OFF",
        "BI_MAX_ITERATIONS":1000000,
        "CACHE_LICENSE": "ON",
        "CHECK_CONVEXITY": "CHECK_CONVEXITY_FULL",
        "COMPRESS_STATFILE":"ON",
        "CONCURRENT_NUM_OPTIMIZERS":2,
        "CONCURRENT_PRIORITY_DUAL_SIMPLEX":2,
        "CONCURRENT_PRIORITY_FREE_SIMPLEX":3,
        "CONCURRENT_PRIORITY_INTPNT":4,
        "CONCURRENT_PRIORITY_PRIMAL_SIMPLEX":1,
        "FEASREPAIR_OPTIMIZE": "FEASREPAIR_OPTIMIZE_NONE",
        "INFEAS_GENERIC_NAMES":"OFF",
        "INFEAS_PREFER_PRIMAL": "ON",
        "INFEAS_REPORT_AUTO": "OFF",
```

```
"INFEAS_REPORT_LEVEL":1,
"INTPNT_BASIS": "BI_ALWAYS",
"INTPNT_DIFF_STEP": "ON",
"INTPNT_FACTOR_DEBUG_LVL":0,
"INTPNT_FACTOR_METHOD":0,
"INTPNT_HOTSTART": "INTPNT_HOTSTART_NONE",
"INTPNT_MAX_ITERATIONS":400,
"INTPNT_MAX_NUM_COR":-1,
"INTPNT_MAX_NUM_REFINEMENT_STEPS":-1,
"INTPNT_OFF_COL_TRH":40,
"INTPNT_ORDER_METHOD": "ORDER_METHOD_FREE",
"INTPNT_REGULARIZATION_USE": "ON",
"INTPNT_SCALING": "SCALING_FREE",
"INTPNT_SOLVE_FORM": "SOLVE_FREE",
"INTPNT_STARTING_POINT": "STARTING_POINT_FREE",
"LIC_TRH_EXPIRY_WRN":7,
"LICENSE_DEBUG": "OFF",
"LICENSE_PAUSE_TIME":0,
"LICENSE_SUPPRESS_EXPIRE_WRNS": "OFF",
"LICENSE_WAIT": "OFF",
"LOG":10,
"LOG_ANA_PRO":1,
"LOG_BI":4,
"LOG_BI_FREQ":2500,
"LOG_CHECK_CONVEXITY":0,
"LOG_CONCURRENT":1,
"LOG_CUT_SECOND_OPT":1,
"LOG_EXPAND":0,
"LOG_FACTOR":1,
"LOG_FEAS_REPAIR":1,
"LOG_FILE":1,
"LOG_HEAD":1,
"LOG_INFEAS_ANA":1,
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"LOG MIO":4.
"LOG_MIO_FREQ":1000,
"LOG_OPTIMIZER":1,
"LOG_ORDER":1,
"LOG_PRESOLVE":1,
"LOG_RESPONSE":0,
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"LOG_SENSITIVITY_OPT":0,
"LOG_SIM":4,
"LOG_SIM_FREQ":1000,
"LOG_SIM_MINOR":1,
"LOG_STORAGE":1,
"MAX_NUM_WARNINGS":10,
"MIO_BRANCH_DIR": "BRANCH_DIR_FREE",
"MIO_CONSTRUCT_SOL": "OFF",
"MIO_CUT_CLIQUE": "ON",
"MIO_CUT_CMIR": "ON",
"MIO_CUT_GMI": "ON",
"MIO_CUT_KNAPSACK_COVER": "OFF",
"MIO_HEURISTIC_LEVEL":-1,
"MIO_MAX_NUM_BRANCHES":-1,
"MIO_MAX_NUM_RELAXS":-1,
```

```
"MIO_MAX_NUM_SOLUTIONS":-1,
"MIO_MODE": "MIO_MODE_SATISFIED",
"MIO_MT_USER_CB": "ON",
"MIO_NODE_OPTIMIZER":"OPTIMIZER_FREE",
"MIO_NODE_SELECTION": "MIO_NODE_SELECTION_FREE",
"MIO_PERSPECTIVE_REFORMULATE": "ON",
"MIO_PROBING_LEVEL":-1,
"MIO_RINS_MAX_NODES":-1,
"MIO_ROOT_OPTIMIZER":"OPTIMIZER_FREE",
"MIO_ROOT_REPEAT_PRESOLVE_LEVEL":-1,
"MT_SPINCOUNT":0,
"NUM_THREADS":0,
"OPF_MAX_TERMS_PER_LINE":5,
"OPF_WRITE_HEADER": "ON",
"OPF_WRITE_HINTS":"ON",
"OPF_WRITE_PARAMETERS": "OFF",
"OPF_WRITE_PROBLEM": "ON",
"OPF_WRITE_SOL_BAS": "ON"
"OPF_WRITE_SOL_ITG":"ON"
"OPF_WRITE_SOL_ITR":"ON",
"OPF_WRITE_SOLUTIONS": "OFF";
"OPTIMIZER": "OPTIMIZER_FREE",
"PARAM_READ_CASE_NAME": "ON",
"PARAM_READ_IGN_ERROR": "OFF"
"PRESOLVE_ELIMINATOR_MAX_FILL": -1;
"PRESOLVE_ELIMINATOR_MAX_NUM_TRIES":-1,
"PRESOLVE_LEVEL":-1,
"PRESOLVE_LINDEP_ABS_WORK_TRH":100,
"PRESOLVE_LINDEP_REL_WORK_TRH": 100,
"PRESOLVE_LINDEP_USE": "ON",
"PRESOLVE_MAX_NUM_REDUCTIONS":-1,
"PRESOLVE_USE": "PRESOLVE_MODE_FREE",
"PRIMAL_REPAIR_OPTIMIZER": "OPTIMIZER_FREE",
"QO_SEPARABLE_REFORMULATION": "OFF",
"READ_DATA_COMPRESSED": "COMPRESS_FREE",
"READ_DATA_FORMAT": "DATA_FORMAT_EXTENSION",
"READ_DEBUG": "OFF",
"READ_KEEP_FREE_CON": "OFF",
"READ_LP_DROP_NEW_VARS_IN_BOU": "OFF",
"READ_LP_QUOTED_NAMES": "ON",
"READ_MPS_FORMAT": "MPS_FORMAT_FREE",
"READ_MPS_WIDTH": 1024,
"READ_TASK_IGNORE_PARAM":"OFF",
"SENSITIVITY_ALL": "OFF",
"SENSITIVITY_OPTIMIZER": "OPTIMIZER_FREE_SIMPLEX",
"SENSITIVITY_TYPE": "SENSITIVITY_TYPE_BASIS",
"SIM_BASIS_FACTOR_USE": "ON",
"SIM_DEGEN": "SIM_DEGEN_FREE";
"SIM_DUAL_CRASH":90,
"SIM_DUAL_PHASEONE_METHOD":0,
"SIM_DUAL_RESTRICT_SELECTION":50,
"SIM_DUAL_SELECTION": "SIM_SELECTION_FREE",
"SIM_EXPLOIT_DUPVEC": "SIM_EXPLOIT_DUPVEC_OFF",
"SIM_HOTSTART": "SIM_HOTSTART_FREE",
"SIM_HOTSTART_LU":"ON",
"SIM_INTEGER":0,
```

```
"SIM_MAX_ITERATIONS":10000000,
    "SIM_MAX_NUM_SETBACKS":250,
    "SIM_NON_SINGULAR": "ON",
    "SIM_PRIMAL_CRASH":90,
    "SIM_PRIMAL_PHASEONE_METHOD":0,
    "SIM_PRIMAL_RESTRICT_SELECTION":50,
    "SIM_PRIMAL_SELECTION": "SIM_SELECTION_FREE",
    "SIM_REFACTOR_FREQ":0,
    "SIM_REFORMULATION": "SIM_REFORMULATION_OFF",
    "SIM_SAVE_LU":"OFF",
    "SIM_SCALING": "SCALING_FREE",
    "SIM_SCALING_METHOD": "SCALING_METHOD_POW2",
    "SIM_SOLVE_FORM": "SOLVE_FREE",
    "SIM_STABILITY_PRIORITY":50,
    "SIM_SWITCH_OPTIMIZER":"OFF".
    "SOL_FILTER_KEEP_BASIC": "OFF",
    "SOL_FILTER_KEEP_RANGED": "OFF",
    "SOL_READ_NAME_WIDTH":-1,
    "SOL_READ_WIDTH": 1024,
    "SOLUTION_CALLBACK": "OFF",
    "TIMING_LEVEL":1,
    "WRITE_BAS_CONSTRAINTS": "ON",
    "WRITE_BAS_HEAD": "ON",
    "WRITE_BAS_VARIABLES": "ON",
    "WRITE_DATA_COMPRESSED":0,
    "WRITE_DATA_FORMAT": "DATA_FORMAT_EXTENSION",
    "WRITE_DATA_PARAM": "OFF",
    "WRITE_FREE_CON": "OFF",
    "WRITE_GENERIC_NAMES": "OFF",
    "WRITE_GENERIC_NAMES_IO":1,
    "WRITE_IGNORE_INCOMPATIBLE_CONIC_ITEMS": "OFF",
    "WRITE_IGNORE_INCOMPATIBLE_ITEMS":"OFF",
    "WRITE_IGNORE_INCOMPATIBLE_NL_ITEMS":"OFF"
    "WRITE_IGNORE_INCOMPATIBLE_PSD_ITEMS": "OFF",
    "WRITE_INT_CONSTRAINTS": "ON",
    "WRITE_INT_HEAD": "ON",
    "WRITE_INT_VARIABLES": "ON",
    "WRITE_LP_FULL_OBJ": "ON",
    "WRITE_LP_LINE_WIDTH":80,
    "WRITE_LP_QUOTED_NAMES": "ON",
    "WRITE_LP_STRICT_FORMAT": "OFF",
    "WRITE_LP_TERMS_PER_LINE":10,
    "WRITE_MPS_FORMAT": "MPS_FORMAT_FREE",
    "WRITE_MPS_INT":"ON",
    "WRITE_PRECISION":15,
    "WRITE_SOL_BARVARIABLES": "ON",
    "WRITE_SOL_CONSTRAINTS": "ON",
    "WRITE_SOL_HEAD": "ON",
    "WRITE_SOL_IGNORE_INVALID_NAMES": "OFF",
    "WRITE_SOL_VARIABLES": "ON",
    "WRITE_TASK_INC_SOL":"ON",
    "WRITE_XML_MODE": "WRITE_XML_MODE_ROW"
},
"dparam":{
    "ANA_SOL_INFEAS_TOL":1e-6,
    "BASIS_REL_TOL_S":1e-12,
```

```
"BASIS_TOL_S":1e-6,
"BASIS_TOL_X":1e-6,
"CHECK_CONVEXITY_REL_TOL":1e-10,
"DATA_TOL_AIJ":1e-12,
"DATA_TOL_AIJ_HUGE":1e+20,
"DATA_TOL_AIJ_LARGE":1e+10,
"DATA_TOL_BOUND_INF": 1e+16,
"DATA_TOL_BOUND_WRN":1e+8,
"DATA_TOL_C_HUGE":1e+16,
"DATA_TOL_CJ_LARGE":1e+8,
"DATA_TOL_QIJ":1e-16,
"DATA_TOL_X":1e-8,
"FEASREPAIR_TOL":1e-10,
"INTPNT_CO_TOL_DFEAS":1e-8,
"INTPNT_CO_TOL_INFEAS":1e-10,
"INTPNT_CO_TOL_MU_RED":1e-8,
"INTPNT_CO_TOL_NEAR_REL":1e+3,
"INTPNT_CO_TOL_PFEAS":1e-8,
"INTPNT_CO_TOL_REL_GAP":1e-7,
"INTPNT_NL_MERIT_BAL":1e-4,
"INTPNT_NL_TOL_DFEAS":1e-8,
"INTPNT_NL_TOL_MU_RED":1e-12,
"INTPNT_NL_TOL_NEAR_REL":1e+3,
"INTPNT_NL_TOL_PFEAS":1e-8,
"INTPNT_NL_TOL_REL_GAP":1e-6,
"INTPNT_NL_TOL_REL_STEP":9.95e-1,
"INTPNT_QO_TOL_DFEAS":1e-8,
"INTPNT_QO_TOL_INFEAS":1e-10,
"INTPNT_QO_TOL_MU_RED":1e-8,
"INTPNT_QO_TOL_NEAR_REL":1e+3,
"INTPNT_QO_TOL_PFEAS":1e-8,
"INTPNT_QO_TOL_REL_GAP":1e-8,
"INTPNT_TOL_DFEAS":1e-8,
"INTPNT_TOL_DSAFE":1e+0,
"INTPNT_TOL_INFEAS":1e-10,
"INTPNT_TOL_MU_RED":1e-16,
"INTPNT_TOL_PATH":1e-8,
"INTPNT_TOL_PFEAS": 1e-8,
"INTPNT_TOL_PSAFE":1e+0,
"INTPNT_TOL_REL_GAP":1e-8,
"INTPNT_TOL_REL_STEP":9.999e-1,
"INTPNT_TOL_STEP_SIZE":1e-6,
"LOWER_OBJ_CUT":-1e+30,
"LOWER_OBJ_CUT_FINITE_TRH":-5e+29,
"MIO_DISABLE_TERM_TIME":-1e+0,
"MIO_MAX_TIME":-1e+0,
"MIO_MAX_TIME_APRX_OPT":6e+1,
"MIO_NEAR_TOL_ABS_GAP":0.0,
"MIO_NEAR_TOL_REL_GAP":1e-3,
"MIO_REL_GAP_CONST":1e-10,
"MIO_TOL_ABS_GAP":0.0,
"MIO_TOL_ABS_RELAX_INT": 1e-5,
"MIO_TOL_FEAS":1e-6,
"MIO_TOL_REL_DUAL_BOUND_IMPROVEMENT":0.0,
"MIO_TOL_REL_GAP":1e-4,
"MIO_TOL_X":1e-6,
```

```
"OPTIMIZER_MAX_TIME":-1e+0,
            "PRESOLVE_TOL_ABS_LINDEP":1e-6,
            "PRESOLVE_TOL_AIJ":1e-12,
            "PRESOLVE_TOL_REL_LINDEP":1e-10,
            "PRESOLVE_TOL_S":1e-8,
            "PRESOLVE_TOL_X":1e-8,
            "QCQO_REFORMULATE_REL_DROP_TOL":1e-15,
            "SEMIDEFINITE_TOL_APPROX":1e-10,
            "SIM_LU_TOL_REL_PIV":1e-2,
            "SIMPLEX_ABS_TOL_PIV":1e-7,
            "UPPER_OBJ_CUT":1e+30,
            "UPPER_OBJ_CUT_FINITE_TRH":5e+29
        },
        "sparam":{
            "BAS_SOL_FILE_NAME":"",
            "DATA_FILE_NAME": "examples/tools/data/lo1.mps",
            "DEBUG_FILE_NAME":"",
            "INT_SOL_FILE_NAME":""
            "ITR_SOL_FILE_NAME":""
            "MIO_DEBUG_STRING":"",
            "PARAM_COMMENT_SIGN":"%%",
            "PARAM_READ_FILE_NAME":""
            "PARAM_WRITE_FILE_NAME":""
            "READ_MPS_BOU_NAME":"",
            "READ_MPS_OBJ_NAME":"",
            "READ_MPS_RAN_NAME":"",
            "READ_MPS_RHS_NAME":"",
            "SENSITIVITY_FILE_NAME":""
            "SENSITIVITY_RES_FILE_NAME":"",
            "SOL_FILTER_XC_LOW":"",
            "SOL_FILTER_XC_UPR":"",
            "SOL_FILTER_XX_LOW":"",
            "SOL_FILTER_XX_UPR":"",
            "STAT_FILE_NAME":"",
            "STAT_KEY":"",
            "STAT_NAME":""
            "WRITE_LP_GEN_VAR_NAME": "XMSKGEN"
        }
    }
}
```

## 16.8 The Solution File Format

MOSEK provides several solution files depending on the problem type and the optimizer used:

- basis solution file (extension .bas) if the problem is optimized using the simplex optimizer or basis identification is performed,
- interior solution file (extension .sol) if a problem is optimized using the interior-point optimizer and no basis identification is required,
- integer solution file (extension .int) if the problem contains integer constrained variables.

All solution files have the format:

(continued from previous page)

SOLUTION STATUS : <status of the solution> OBJECTIVE NAME : <name of the objective function> PRIMAL OBJECTIVE : <pri> <pri> corresponding to the solution> DUAL OBJECTIVE : <dual objective value corresponding to the solution> CONSTRAINTS AT ACTIVITY LOWER LIMIT UPPER LIMIT DUAL LOWER DUAL UPPER INDEX NAME ?? <a value> <a value> <a value> <a value> <a value> <name> VARIABLES AT ACTIVITY INDEX NAME LOWER LIMIT UPPER LIMIT DUAL LOWER DUAL UPPER →CONIC DUAL <name> ?? <a value> <a value> <a value> <a value> <a value> →<a value>

In the example the fields ? and <> will be filled with problem and solution specific information. As can be observed a solution report consists of three sections, i.e.

- HEADER In this section, first the name of the problem is listed and afterwards the problem and solution status are shown. Next the primal and dual objective values are displayed.
- CONSTRAINTS For each constraint i of the form

$$l_i^c \le \sum_{j=1}^n a_{ij} x_j \le u_i^c, \tag{16.15}$$

the following information is listed:

- INDEX: A sequential index assigned to the constraint by MOSEK
- NAME: The name of the constraint assigned by the user.
- AT: The status of the constraint. In Table 16.4 the possible values of the status keys and their interpretation are shown.

Status key	Interpretation
UN	Unknown status
BS	Is basic
SB	Is superbasic
LL	Is at the lower limit (bound)
UL	Is at the upper limit (bound)
EQ	Lower limit is identical to upper limit
**	Is infeasible i.e. the lower limit is greater than the upper limit.

Table 16.4: Status keys.

- ACTIVITY: the quantity  $\sum_{j=1}^{n} a_{ij}x_{j}^{*}$ , where  $x^{*}$  is the value of the primal solution.
- LOWER LIMIT: the quantity  $l_i^c$  (see (16.15).)
- UPPER LIMIT: the quantity  $u_i^c$  (see (16.15).)
- DUAL LOWER: the dual multiplier corresponding to the lower limit on the constraint.
- DUAL UPPER: the dual multiplier corresponding to the upper limit on the constraint.
- VARIABLES The last section of the solution report lists information about the variables. This information has a similar interpretation as for the constraints. However, the column with the header CONIC DUAL is included for problems having one or more conic constraints. This column shows the dual variables corresponding to the conic constraints.

### Example: lo1.sol

In Listing 16.7 we show the solution file for the lol.opf problem.

Listing 16.7: An example of  $\,$  .sol file.

NAME				
	: PRIMAL_AND_DUAL_FEASIBLE			
SOLUTION STATUS				
OBJECTIVE NAME				
	: 8.33333333e+01			
	: 8.33333332e+01			
CONSTRAINTS				
INDEX NAME	AT ACTIVITY	LOWER LIMIT	UPPER LIMI	Τ 👝
→ DUAL LOWER	DUAL UPPER			
0 c1	EQ 3.000000000000e+01	3.00000000e+01	3.	
→00000000e+01	-0.0000000000000e+00 -2.4999	9999741653e+00		
1 c2	SB 5.33333333049187e+01	1.50000000e+01	NONE	Ш
	-0.00000000000000000000000000000000000	+00		
2 <b>c3</b>	UL 2.4999999842049e+01 -0.000000000000000e+00 -3.3333	NONE	2.	
→50000000e+01	-0.0000000000000e+00 -3.3333	3332895108e-01		
VARIABLES	AT ACTIVITY	LOUIDD LIMITE	IIDDED I TMT	
		LUWEK LIMII	OPPER LIMI	I 🗆
→ DUAL LUWER	DUAL UPPER LL 1.67020427038537e-09	0 00000000-100	MONE	
	9528054e+00 -0.0000000000000000		NUNE	П
	LL 2.93510446211883e-09		1.	
	-2.16666666494915e+00 6.20868		Ι.	
	SB 1.49999999899424e+01		NONE	
	7245553e-10 -0.0000000000000000		1101112	Ш
	SB 8.3333332273115e+00		NONE	
	78848200e-09 -0.000000000000000000000000000000000		14014	П
-1.0313031	-0.000000000000000			

## Chapter 17

# List of examples

List of examples shipped in the distribution of Optimizer API for C:

Table 17.1: List of distributed examples

Eil	Table 17.1. List of distributed examples	
File	Description	
blas_lapack.c	Demonstrates the <b>MOSEK</b> interface to BLAS/LAPACK linear algebra routines	
callback.c	An example of data/progress callback	
ceo1.c	A simple conic exponential problem	
concurrent1.cc	Implementation of a concurrent optimizer for linear and mixed-integer problems	
cqo1.c	A simple conic quadratic problem	
errorreporting.	Demonstrates how error reporting can be customized	
С		
feasrepairex1.	A simple example of how to repair an infeasible problem	
С		
gp1.c	A simple geometric program (GP) in conic form	
lo1.c	A simple linear problem	
lo2.c	A simple linear problem	
logistic.c	Implements logistic regression and simple log-sum-exp (CEO)	
mico1.c	A simple mixed-integer conic problem	
milo1.c	A simple mixed-integer linear problem	
mioinitsol.c	A simple mixed-integer linear problem with an initial guess	
modelLib.c	Library of implementations of basic functions	
opt_server_async. Uses MOSEK OptServer to solve an optimization problem asynchronously		
С	-	
opt_server_sync.	Uses MOSEK OptServer to solve an optimization problem synchronously	
С		
parallel.cc	Demonstrates parallel optimization	
parameters.c	Shows how to set optimizer parameters and read information items	
portfolio_1_basi	c Portfolio optimization - basic Markowitz model	
c	•	
portfolio_2_fron	t Pertfolio optimization - efficient frontier	
c	•	
portfolio_3_impa	ctPortfolio optimization - market impact costs	
c	-	
portfolio_4_transdastfolio optimization - transaction costs		
c		
portfolio_5_card	. Portfolio optimization - cardinality constraints	
C	·	
pow1.c	A simple power cone problem	
qcqo1.c	A simple quadratically constrained quadratic problem	
qo1.c	A simple quadratic problem	
1	L L L L	

continues on next page

Table 17.1 – continued from previous page

File	Description
reoptimization.	Demonstrate how to modify and re-optimize a linear problem
С	
response.c	Demonstrates proper response handling
sdo1.c	A simple semidefinite problem with one matrix variable and a quadratic cone
sdo2.c	A simple semidefinite problem with two matrix variables
sensitivity.c	Sensitivity analysis performed on a small linear problem
simple.c	A simple I/O example: read problem from a file, solve and write solutions
solutionquality.	Demonstrates how to examine the quality of a solution
С	
solvebasis.c	Demonstrates solving a linear system with the basis matrix
solvelinear.c	Demonstrates solving a general linear system
sparsecholesky.	Shows how to find a Cholesky factorization of a sparse matrix
С	
unicode.c	Demonstrates string conversion to Unicode

Additional examples can be found on the  $\mathbf{MOSEK}$  website and in other  $\mathbf{MOSEK}$  publications.

## Chapter 18

## Interface changes

The section shows interface-specific changes to the **MOSEK** Optimizer API for C in version 9.3 compared to version 8. See the release notes for general changes and new features of the **MOSEK** Optimization Suite.

### 18.1 Backwards compatibility

- Parameters. Users who set parameters to tune the performance and numerical properties of the solver (termination criteria, tolerances, solving primal or dual, presolve etc.) are recommended to reevaluate such tuning. It may be that other, or default, parameter settings will be more beneficial in the current version. The hints in Sec. 8 may be useful for some cases.
- All functions using the enum MSKaccmodee were removed. Use corresponding separate functions for manipulating variables and constraints. For example, instead of

```
MSK_putbound(task, MSK_ACC_VAR, ...);
MSK_putbound(task, MSK_ACC_CON, ...);
```

use

```
MSK_putvarbound(task, ...);
MSK_putconbound(task, ...);
```

and so on.

- Removed all Near problem and solution statuses i.e. MSK\_SOL\_STA\_NEAR\_OPTIMAL, MSK\_SOL\_STA\_NEAR\_PRIM\_INFEAS\_CER, etc. See Sec. 13.3.3.
- All functions related to the general nonlinear optimizer and Scopt have been removed. See Sec. 15.10.

#### 18.2 Functions

#### **Added**

- MSK\_setupthreads
- MSK\_appendsparsesymmatlist
- MSK\_generateconenames
- MSK\_generateconnames
- MSK\_generatevarnames
- MSK\_getacolslice

- $\bullet \ \textit{MSK\_getacolslicenumnz}$
- MSK\_getarowslice
- MSK\_getarowslicenumnz
- MSK\_getatruncatetol
- MSK\_getbarsslice
- MSK\_getbarxslice
- MSK\_getclist
- $\bullet$  MSK\_getskn
- MSK\_putatruncatetol
- MSK\_putbaraijlist
- $\bullet \ \textit{MSK\_putbararowlist}$
- $\bullet \ \textit{MSK\_putconboundlistconst}$
- $\bullet \ \textit{MSK\_putconboundsliceconst}$
- $\bullet$  MSK\_putconsolutioni
- $\bullet$  MSK\_putoptserverhost
- $\bullet \ \textit{MSK\_putvarboundlistconst}$
- $\bullet \ \textit{MSK\_putvarboundsliceconst}$
- MSK\_putvarsolutionj
- MSK\_readjsonstring
- MSK\_readlpstring
- MSK\_readopfstring
- $\bullet \ \textit{MSK\_readptfstring}$

#### Removed

- MSK\_checkconvexity
- MSK\_chgbound
- MSK\_getaslice
- MSK\_getaslicenumnz
- MSK\_getbound
- MSK\_getboundslice
- MSK\_getnlfunc
- MSK\_getsolutioni
- MSK\_printdata
- MSK\_putbound
- MSK\_putboundlist
- MSK\_putboundslice
- MSK\_putnlfunc
- MSK\_putsolutioni

#### 18.3 Parameters

#### **Added**

- MSK\_IPAR\_INTPNT\_ORDER\_GP\_NUM\_SEEDS
- MSK\_IPAR\_INTPNT\_PURIFY
- MSK\_IPAR\_LOG\_INCLUDE\_SUMMARY
- MSK\_IPAR\_LOG\_LOCAL\_INFO
- MSK\_IPAR\_MIO\_CONIC\_OUTER\_APPROXIMATION
- MSK\_IPAR\_MIO\_FEASPUMP\_LEVEL
- MSK\_IPAR\_MIO\_MAX\_NUM\_ROOT\_CUT\_ROUNDS
- MSK\_IPAR\_MIO\_PROPAGATE\_OBJECTIVE\_CONSTRAINT
- MSK\_IPAR\_MIO\_SEED
- MSK\_IPAR\_OPF\_WRITE\_LINE\_LENGTH
- MSK\_IPAR\_PRESOLVE\_MAX\_NUM\_PASS
- MSK\_IPAR\_PTF\_WRITE\_TRANSFORM
- MSK\_IPAR\_SIM\_SEED
- MSK\_IPAR\_WRITE\_COMPRESSION

#### Removed

- MSK\_DPAR\_DATA\_TOL\_AIJ
- MSK\_DPAR\_INTPNT\_NL\_MERIT\_BAL
- MSK\_DPAR\_INTPNT\_NL\_TOL\_DFEAS
- MSK\_DPAR\_INTPNT\_NL\_TOL\_MU\_RED
- MSK\_DPAR\_INTPNT\_NL\_TOL\_NEAR\_REL
- MSK\_DPAR\_INTPNT\_NL\_TOL\_PFEAS
- MSK\_DPAR\_INTPNT\_NL\_TOL\_REL\_GAP
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- MSK\_IPAR\_MIO\_MT\_USER\_CB
- MSK\_IPAR\_OPF\_MAX\_TERMS\_PER\_LINE
- MSK\_IPAR\_READ\_DATA\_COMPRESSED
- MSK\_IPAR\_READ\_DATA\_FORMAT
- MSK\_IPAR\_WRITE\_DATA\_COMPRESSED
- MSK\_IPAR\_WRITE\_DATA\_FORMAT

#### 18.4 Constants

#### **Added**

- MSK\_COMPRESS\_ZSTD
- MSK\_CT\_DEXP
- MSK\_CT\_DPOW
- MSK\_CT\_PEXP
- MSK\_CT\_PPOW
- MSK\_CT\_ZERO
- MSK\_DATA\_FORMAT\_PTF
- MSK\_IINF\_MIO\_NUMBIN
- MSK\_IINF\_MIO\_NUMBINCONEVAR
- MSK\_IINF\_MIO\_NUMCONE
- MSK\_IINF\_MIO\_NUMCONEVAR
- MSK\_IINF\_MIO\_NUMCONT
- MSK\_IINF\_MIO\_NUMCONTCONEVAR
- MSK\_IINF\_MIO\_NUMDEXPCONES
- MSK\_IINF\_MIO\_NUMDPOWCONES
- MSK\_IINF\_MIO\_NUMINTCONEVAR
- MSK\_IINF\_MIO\_NUMPEXPCONES
- MSK\_IINF\_MIO\_NUMPPOWCONES
- MSK\_IINF\_MIO\_NUMQCONES
- MSK\_IINF\_MIO\_NUMRQCONES
- MSK\_IINF\_MIO\_PRESOLVED\_NUMBINCONEVAR
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- MSK\_IINF\_MIO\_PRESOLVED\_NUMDPOWCONES
- MSK\_IINF\_MIO\_PRESOLVED\_NUMINTCONEVAR
- MSK\_IINF\_MIO\_PRESOLVED\_NUMPEXPCONES
- MSK\_IINF\_MIO\_PRESOLVED\_NUMPPOWCONES
- MSK\_IINF\_MIO\_PRESOLVED\_NUMQCONES
- MSK\_IINF\_MIO\_PRESOLVED\_NUMRQCONES
- MSK\_IINF\_PURIFY\_DUAL\_SUCCESS
- MSK\_IINF\_PURIFY\_PRIMAL\_SUCCESS
- MSK\_LIINF\_MIO\_ANZ

#### Removed

- MSK\_DATAFORMAT\_XML
- MSK\_DINFITEM\_MIO\_HEURISTIC\_TIME
- MSK\_DINFITEM\_MIO\_OPTIMIZER\_TIME
- MSK\_IINFITEM\_MIO\_CONSTRUCT\_NUM\_ROUNDINGS
- MSK\_IINFITEM\_MIO\_INITIAL\_SOLUTION
- MSK\_IINFITEM\_MIO\_NEAR\_ABSGAP\_SATISFIED
- MSK\_IINFITEM\_MIO\_NEAR\_RELGAP\_SATISFIED
- MSK\_LIINFITEM\_MIO\_SIM\_MAXITER\_SETBACKS
- MSK\_MIONODESELTYPE\_HYBRID
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- MSK\_PROBLEMTYPE\_GECO
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- MSK\_PROSTA\_NEAR\_PRIM\_AND\_DUAL\_FEAS
- MSK\_PROSTA\_NEAR\_PRIM\_FEAS
- MSK\_SENSITIVITYTYPE\_OPTIMAL\_PARTITION
- MSK\_SOLSTA\_NEAR\_DUAL\_FEAS
- MSK\_SOLSTA\_NEAR\_DUAL\_INFEAS\_CER
- MSK\_SOLSTA\_NEAR\_INTEGER\_OPTIMAL
- MSK\_SOLSTA\_NEAR\_OPTIMAL
- MSK\_SOLSTA\_NEAR\_PRIM\_AND\_DUAL\_FEAS
- MSK\_SOLSTA\_NEAR\_PRIM\_FEAS
- MSK\_SOLSTA\_NEAR\_PRIM\_INFEAS\_CER

### 18.5 Response Codes

#### Added

- MSK\_RES\_ERR\_APPENDING\_TOO\_BIG\_CONE
- MSK\_RES\_ERR\_CBF\_DUPLICATE\_POW\_CONES
- MSK\_RES\_ERR\_CBF\_DUPLICATE\_POW\_STAR\_CONES
- MSK\_RES\_ERR\_CBF\_INVALID\_DIMENSION\_OF\_CONES
- MSK\_RES\_ERR\_CBF\_INVALID\_EXP\_DIMENSION
- MSK\_RES\_ERR\_CBF\_INVALID\_NUMBER\_OF\_CONES
- MSK\_RES\_ERR\_CBF\_INVALID\_POWER
- MSK\_RES\_ERR\_CBF\_INVALID\_POWER\_CONE\_INDEX

- MSK\_RES\_ERR\_CBF\_INVALID\_POWER\_STAR\_CONE\_INDEX
- MSK\_RES\_ERR\_CBF\_POWER\_CONE\_IS\_TOO\_LONG
- MSK\_RES\_ERR\_CBF\_POWER\_CONE\_MISMATCH
- MSK\_RES\_ERR\_CBF\_POWER\_STAR\_CONE\_MISMATCH
- MSK\_RES\_ERR\_CBF\_UNHANDLED\_POWER\_CONE\_TYPE
- MSK\_RES\_ERR\_CBF\_UNHANDLED\_POWER\_STAR\_CONE\_TYPE
- MSK\_RES\_ERR\_CONE\_PARAMETER
- MSK\_RES\_ERR\_FORMAT\_STRING
- MSK\_RES\_ERR\_INVALID\_CJ
- MSK\_RES\_ERR\_INVALID\_FILE\_FORMAT\_FOR\_CFIX
- MSK\_RES\_ERR\_INVALID\_FILE\_FORMAT\_FOR\_FREE\_CONSTRAINTS
- MSK\_RES\_ERR\_INVALID\_FILE\_FORMAT\_FOR\_NONLINEAR
- $\bullet \ \textit{MSK}\_\textit{RES}\_\textit{ERR}\_\textit{INVALID}\_\textit{FILE}\_\textit{FORMAT}\_\textit{FOR}\_\textit{RANGED}\_\textit{CONSTRAINTS}$
- MSK\_RES\_ERR\_NUM\_ARGUMENTS
- MSK\_RES\_ERR\_PTF\_FORMAT
- MSK\_RES\_ERR\_SERVER\_PROBLEM\_SIZE
- MSK\_RES\_ERR\_SHAPE\_IS\_TOO\_LARGE
- MSK\_RES\_ERR\_SLICE\_SIZE
- MSK\_RES\_ERR\_TOO\_SMALL\_A\_TRUNCATION\_VALUE
- MSK\_RES\_WRN\_EXP\_CONES\_WITH\_VARIABLES\_FIXED\_AT\_ZERO
- MSK\_RES\_WRN\_POW\_CONES\_WITH\_ROOT\_FIXED\_AT\_ZERO

#### Removed

- MSK\_RES\_ERR\_CANNOT\_CLONE\_NL
- MSK\_RES\_ERR\_CANNOT\_HANDLE\_NL
- MSK\_RES\_ERR\_INVALID\_ACCMODE
- MSK\_RES\_ERR\_INVALID\_FILE\_FORMAT\_FOR\_GENERAL\_NL
- MSK\_RES\_ERR\_NONLINEAR\_FUNCTIONS\_NOT\_ALLOWED
- MSK\_RES\_ERR\_NR\_ARGUMENTS
- MSK\_RES\_ERR\_OPEN\_DL
- MSK\_RES\_ERR\_USER\_FUNC\_RET
- MSK\_RES\_ERR\_USER\_FUNC\_RET\_DATA
- MSK\_RES\_ERR\_USER\_NLO\_EVAL
- MSK\_RES\_ERR\_USER\_NLO\_EVAL\_HESSUBI
- MSK\_RES\_ERR\_USER\_NLO\_EVAL\_HESSUBJ

- MSK\_RES\_ERR\_USER\_NLO\_FUNC
- MSK\_RES\_TRM\_MIO\_NEAR\_ABS\_GAP
- MSK\_RES\_TRM\_MIO\_NEAR\_REL\_GAP
- MSK\_RES\_WRN\_CONSTRUCT\_INVALID\_SOL\_ITG
- MSK\_RES\_WRN\_CONSTRUCT\_NO\_SOL\_ITG
- MSK\_RES\_WRN\_CONSTRUCT\_SOLUTION\_INFEAS
- MSK\_RES\_WRN\_NO\_NONLINEAR\_FUNCTION\_WRITE

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