December 15, 2023 Group/Teacher: _____

Test 2

1. Evaluate the following integrals: *(5+6 points)*

   (a) $\displaystyle\int \left(\frac{1}{x^2} - \sqrt{1+\frac{1}{x}}\right)^2 dx \quad (x \in (0,+\infty))$;

   (b) $\displaystyle\int (1-2x)\cdot \arctan x\, dx \quad (x \in \mathbb{R})$.

2. Find the area of the bounded and closed region enclosed by the following curves: *(6 points)*

$$y = x^2 + 2x,\ y = 4 - x^2.$$

3. Find the volume of the solid obtained by rotating the graph of the following function around x axes: *(8 points)*

$$f(x) := \frac{1}{x}\cdot\sqrt{\frac{x+1}{x-1}} \quad (x \in [2,3]).$$

4. Compute the local extrema of the following function: *(7 points)*

$$f(x,y) = \frac{x^3}{3} - 3x^2 - 4xy + y^2 \quad ((x,y) \in \mathbb{R}^2).$$

5. Consider the function:

$$f(x,y) = \frac{\sin^2 x}{y^2} \quad ((x,y) \in \mathbb{R}\times(0,+\infty)).$$

Evaluate the double integral of f over the following domain: *(8 points)*

$$D := \{(x,y) \in \mathbb{R}^2 : \pi/4 \le x \le \pi/2,\ \sin x \le y \le 1\}$$