Problem set 6, Probability and statistics

1. For which $n > 1$ will be independent

   a) the following events: A: there is at least one head and at least one tail, B: there is at most one tail from n tosses of a coin.

   b) the following events: A: there is at least one head and at least one tail and B: the first result is head, from n flips of a coin.

2. A medical examination result shows condition "A" (e.g. high cholesterol level) with probability 0.05 and condition "B" (e.g. low iron level) with probability 0.03. The probability of joint occurrence is 0.01.

   a) Are the two conditions independent? If not, what would be the probability of joint occurrence in case of independence?

   b) What is the probability that a randomly chosen person has neither of the two conditions?

   c) What is the probability of finding condition "B" among those people with condition "A"? Conversely, what is the probability of finding condition "A" among those people with condition "B"?

3. Let us assume that the number of faults against a given player during a basketball game has Poisson distribution with parameter $\lambda = 2$ and that each of these faults is noticed by the referee with probability 0.5 (independently from the others). Compute the distribution of the noticed faults!

4. Let the density function of X $f(x) = 1 - |x|$ for $-1 < x < 1$ (and 0 otherwise). Compute $E(X)$ and $Var(X)$!

5. Let us assume that the strength of a rope is a normally distributed random variable with expectation 1000 kN. What can the standard deviation be if we know that the probability of a rope to be weaker than 980 kN is 2%?

6. Let X and Y be independent random variables with mean 0 and variance 1 Give a value c such that $\text{cov}(3X - cY, 2X + Y) = 0$.

7. What is shown by the following R-code? What is given by p?

```
temp=rnorm(1000, mean=1, sd=2)
hist(temp, col="orange", main="Normal distribution", xlab="values", ylab="frequencies", freq=F)
curve(dnorm(x, mean=1, sd=2), from=-6, to=8, lwd="3", col="blue", add=T)
p=length(temp[(-1<temp)&(temp<3)])/1000
```

   What is the theoretical value of p?