
Lab 2: INS and Kalman Filter

TTK5: Kalman Filtering and Navigation

By:

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Task 1

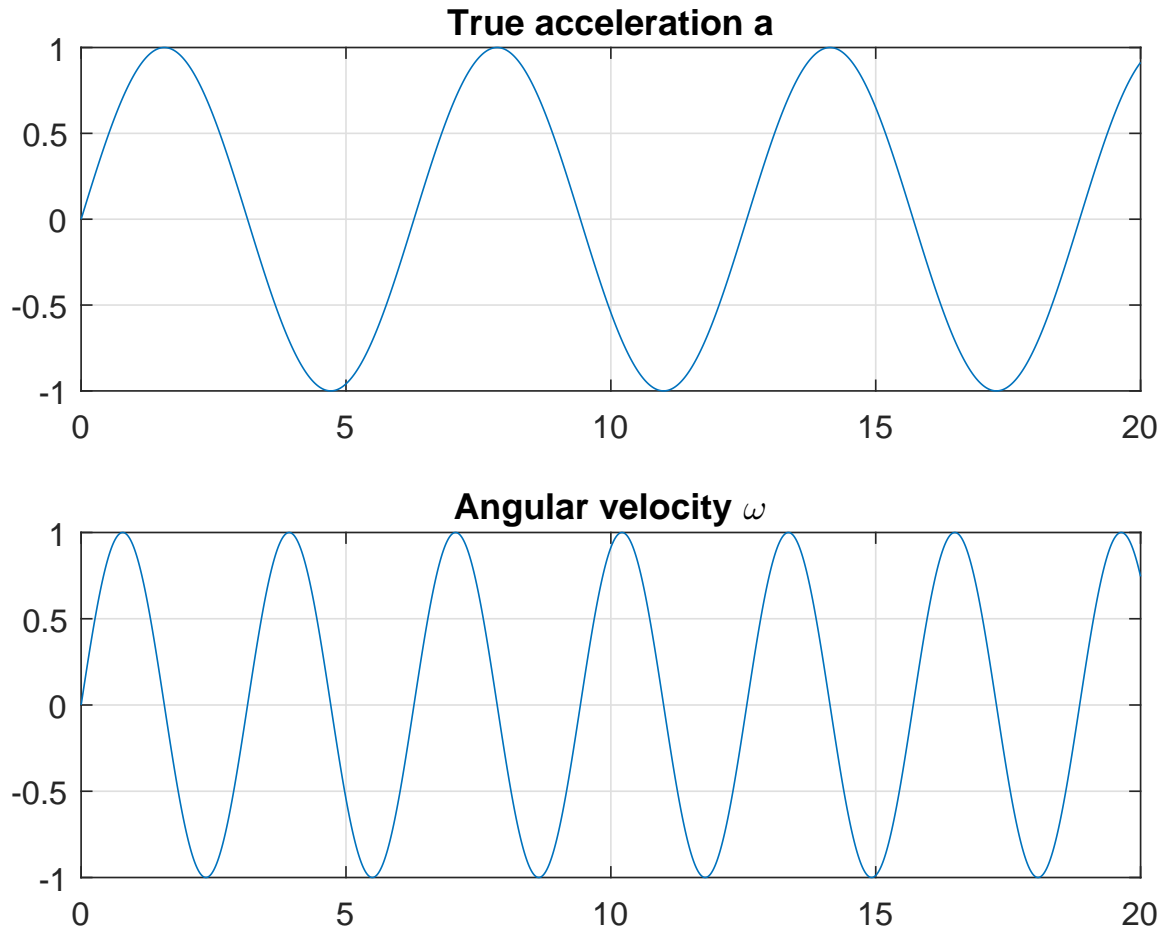


Figure 1: True acceleration and angular velocity.

Task 2

In order to discretize the system, it must first be written as a state space model.
The system

$$\dot{x} = v \quad (1a)$$

$$\dot{v} = a \quad (1b)$$

$$\dot{\theta} = \omega \quad (1c)$$

can be written as

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} \quad (2a)$$

$$\begin{bmatrix} \dot{x} \\ \dot{v} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ v \\ \theta \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ \omega \end{bmatrix}. \quad (2b)$$

By using forward Euler to discretize the system, it can be written on the form

$$\mathbf{x}(t_{k+1}) = (\mathbf{I} + h\mathbf{A}(t_k))\mathbf{x}(t_k) + h\mathbf{B}(t_k)\mathbf{u}(t_k) \quad (3)$$

where h is the step size. The discretized system then becomes

$$\mathbf{x}(t_{k+1}) = \begin{bmatrix} 1 & h & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \mathbf{x}(t_k) + \begin{bmatrix} 0 & 0 \\ h & 0 \\ 0 & h \end{bmatrix} \mathbf{u}(t_k). \quad (4)$$

Figure 2 shows plots of the states in the discretized system.

Task 3

When white noise is expressed in discrete time it is referred to as a white sequence [1], where the sequence consists of random variables that are uncorrelated [2]. The autocorrelation function for discrete white noise is:

$$R_d(k) = A\delta(k), \quad \delta(k) = \begin{cases} 1 & k = 0 \\ 0 & k \neq 0 \end{cases}. \quad (5)$$

When using Matlab, white Gaussian noise can be generated by using `wgn()`, which will generate a sequence of uncorrelated random variables, which can be regarded as a white sequence.

The biases b_1 and b_2 can be discretized with forward Euler using equation 3 from task 2. The bias can be written in state space form as

$$\dot{\mathbf{x}} = \begin{bmatrix} -\frac{1}{T_1} & 0 \\ 0 & -\frac{1}{T_2} \end{bmatrix} \mathbf{x} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \mathbf{w} \quad (6)$$

where

$$\mathbf{x} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}.$$

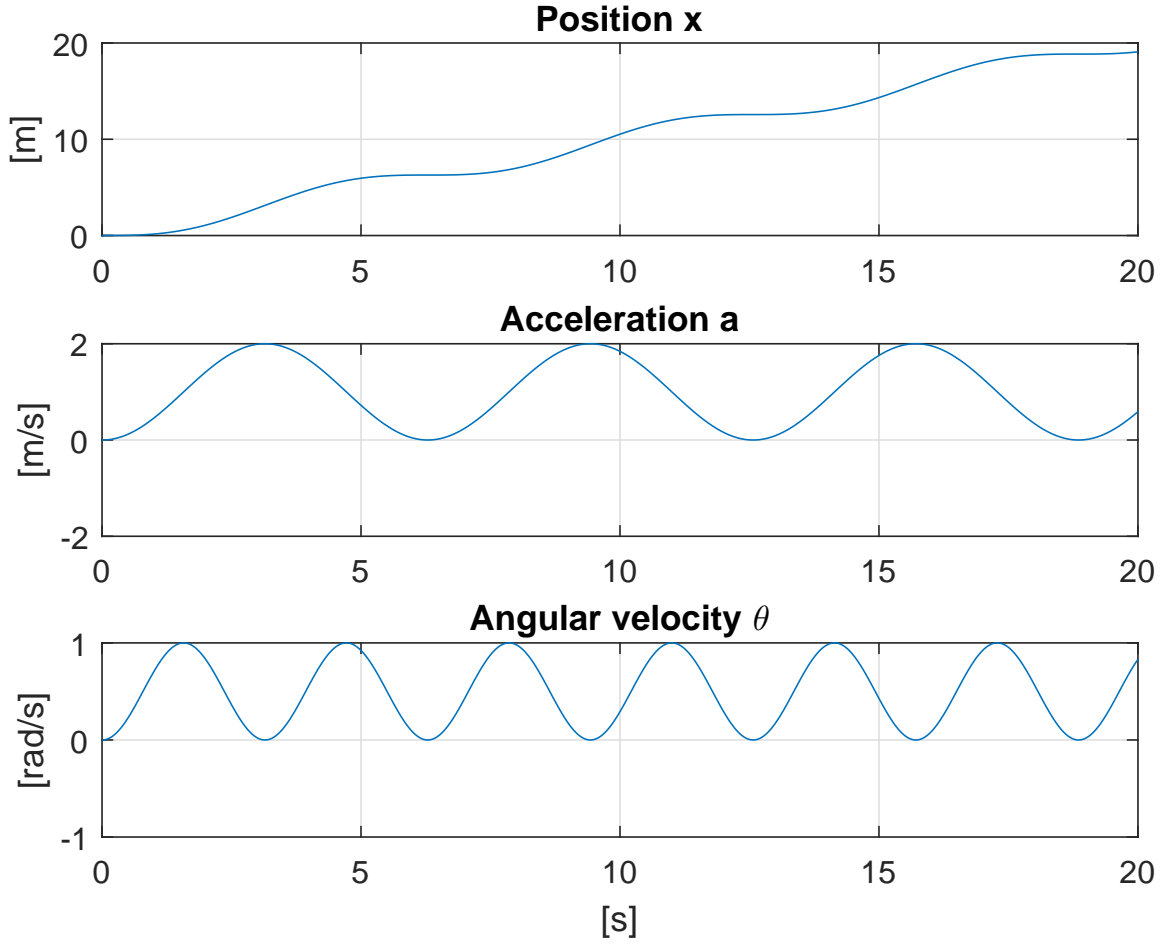


Figure 2: States of the discretized system.

The resulting discretized system is:

$$\mathbf{x}(t_{k+1}) = \begin{bmatrix} 1 - \frac{h}{T_1} & 0 \\ 0 & 1 - \frac{h}{T_2} \end{bmatrix} \mathbf{x}(t_k) + \begin{bmatrix} h & 0 \\ 0 & h \end{bmatrix} \mathbf{w}(t_k). \quad (7)$$

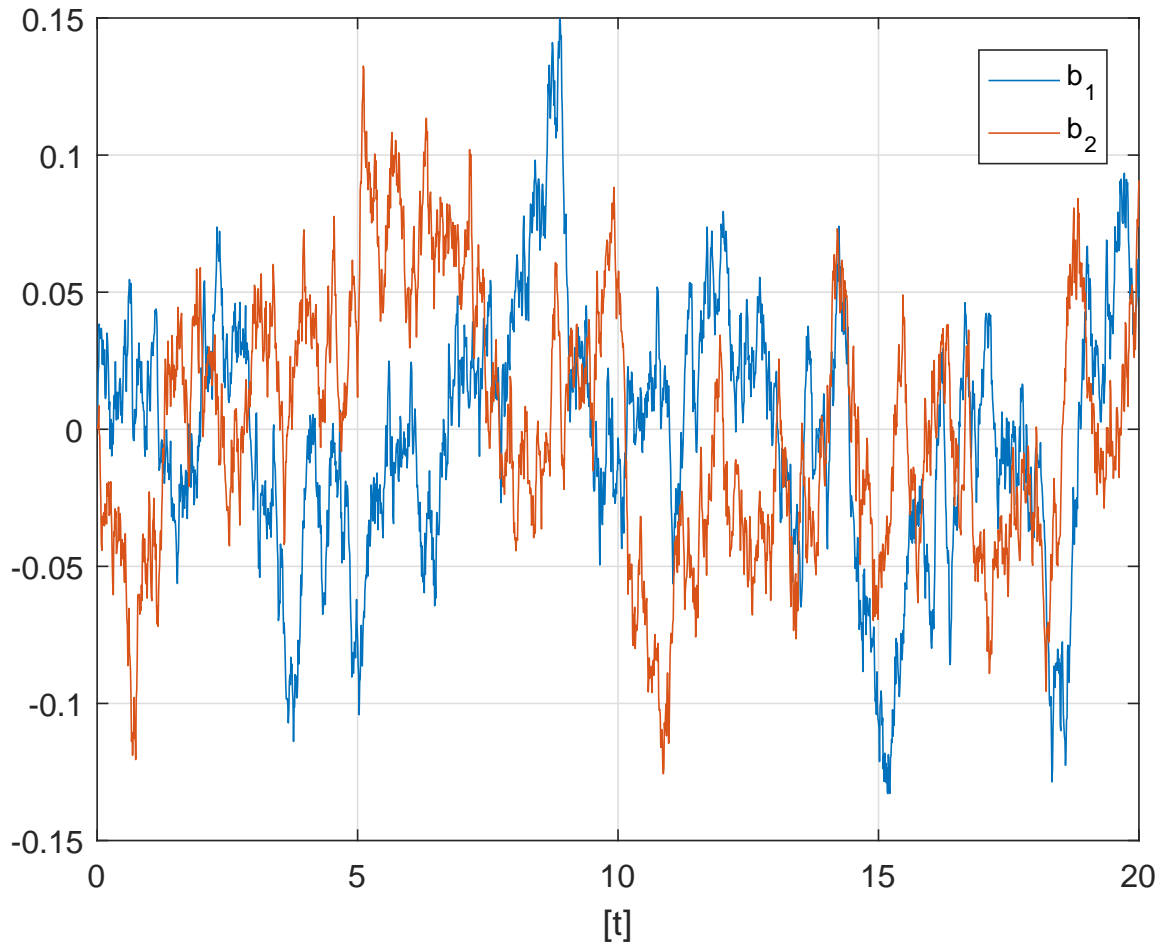


Figure 3: Bias modelled as a Gauss-Markov process.

References

- [1] Vik, Bjørnar (2014) *"Integrated Satellite and Inertial Navigation Systems"*, Norwegian University of Science and Technology, Department of Engineering Cybernetics, Trondheim
- [2] Wikipedia, https://en.wikipedia.org/wiki/White_noise, accessed 05.11.2016

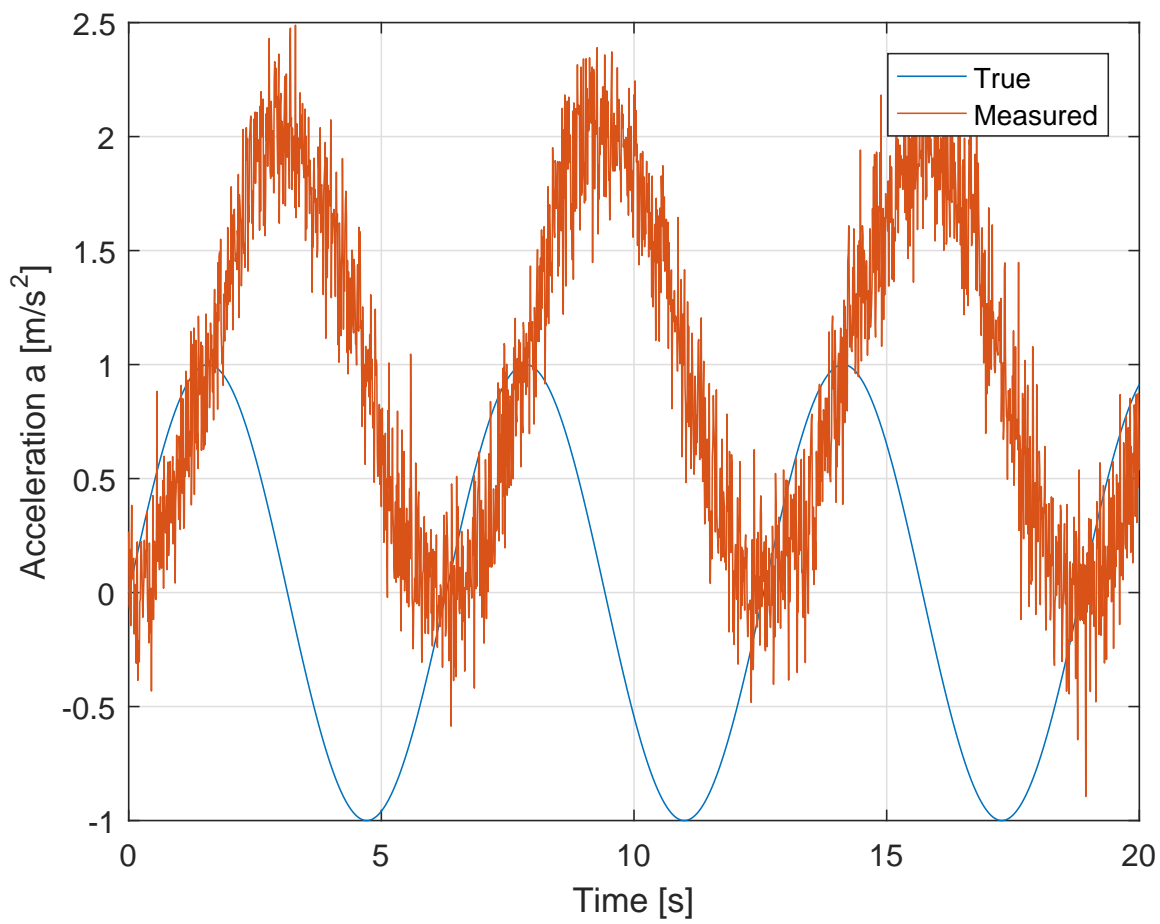


Figure 4: Measured acceleration.

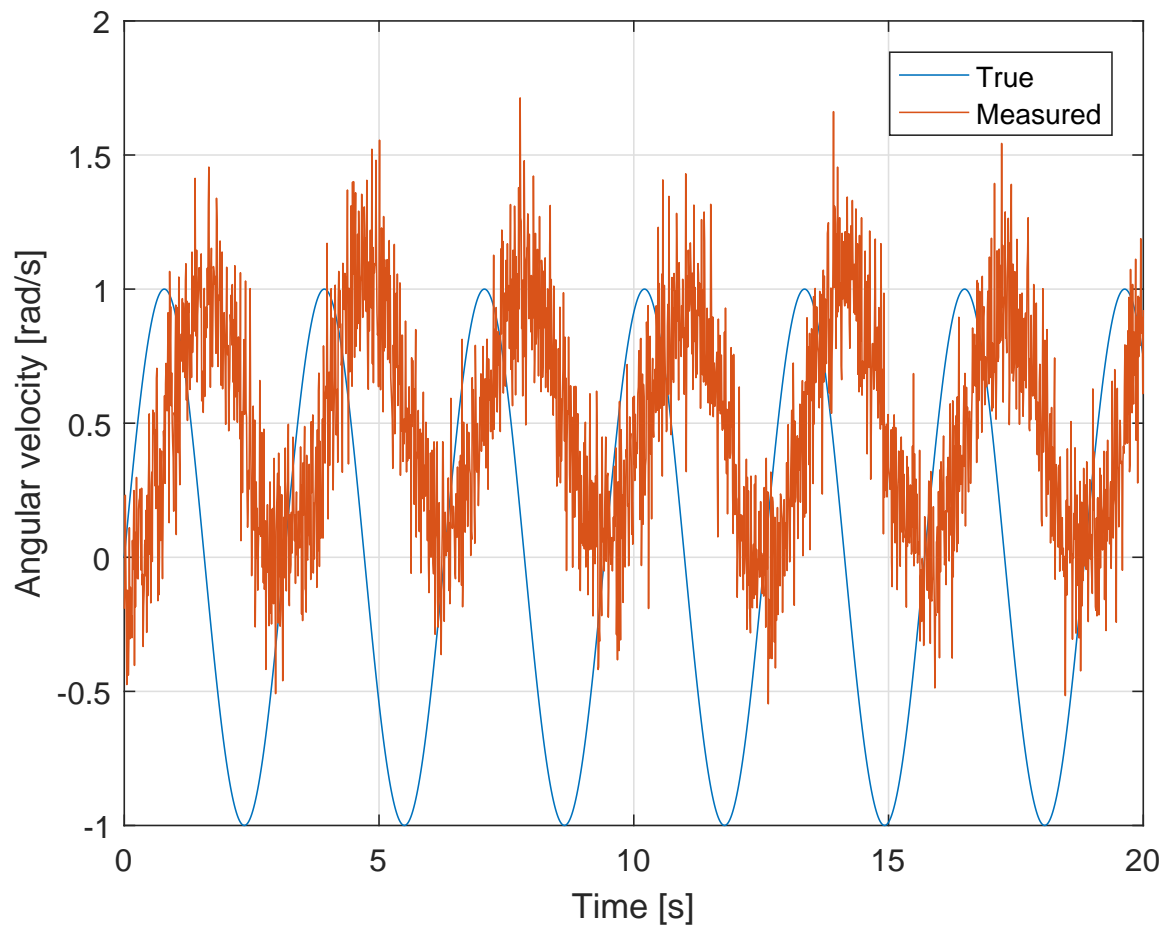


Figure 5: The measure angular velocity.

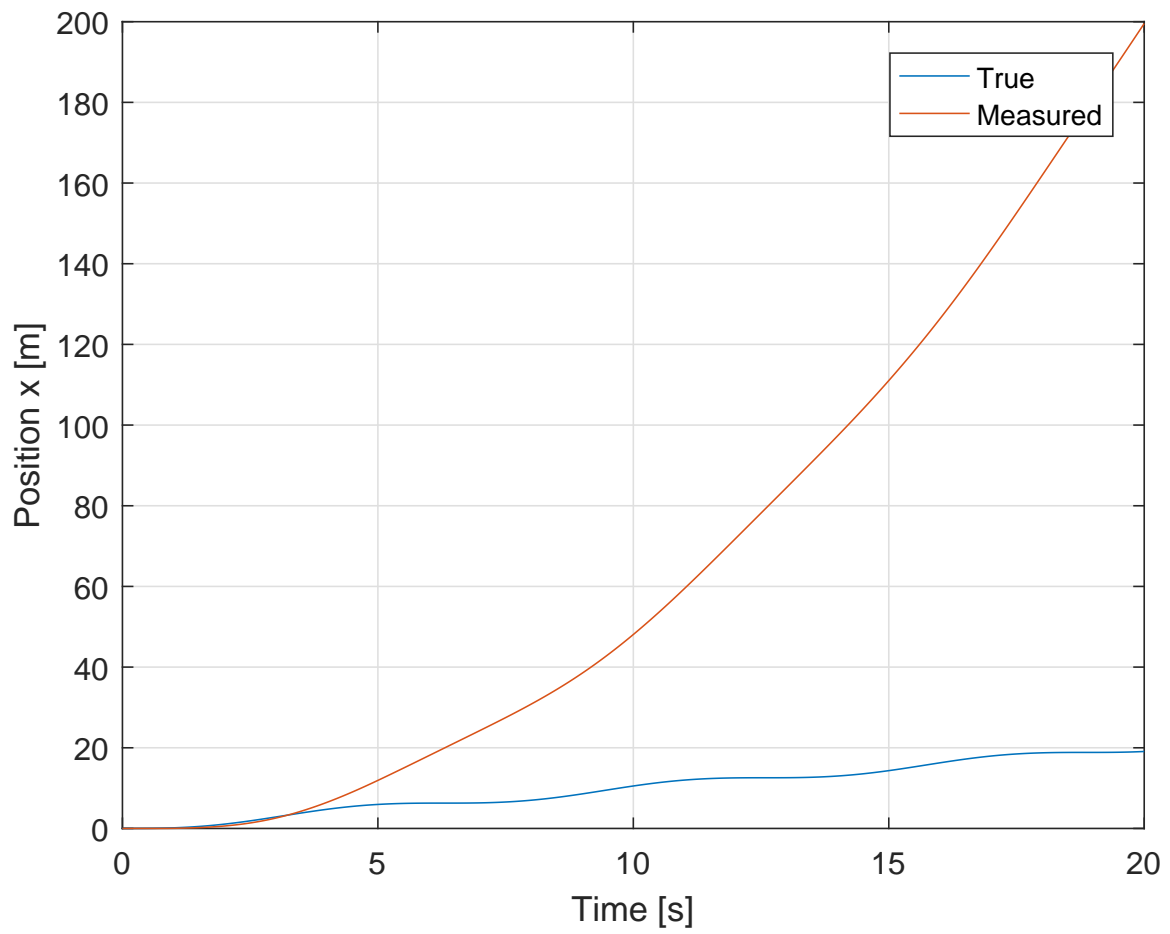


Figure 6: The measured position.

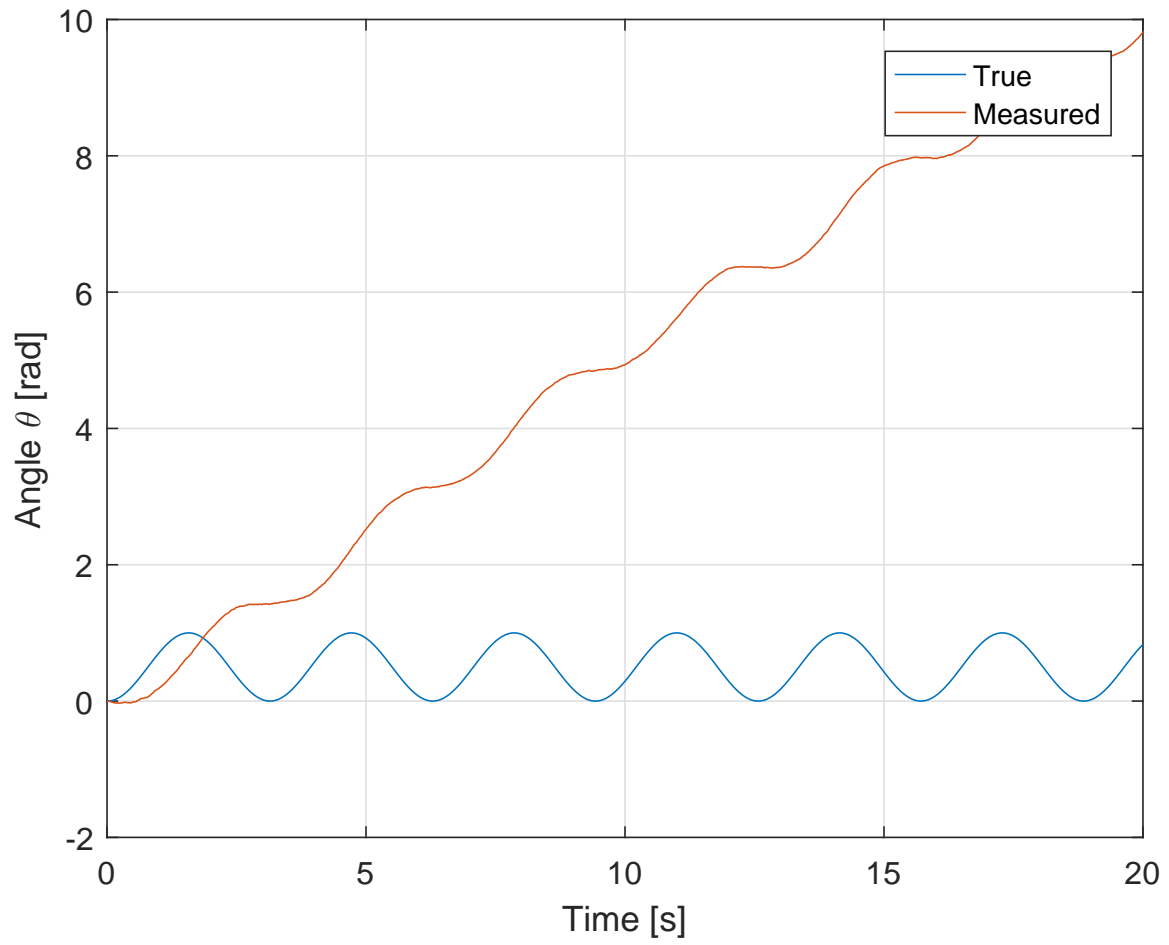


Figure 7: Measured orientation.