Lab 2: INS and Kalman Filter

TTK5: Kalman Filtering and Navigation

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Task 1

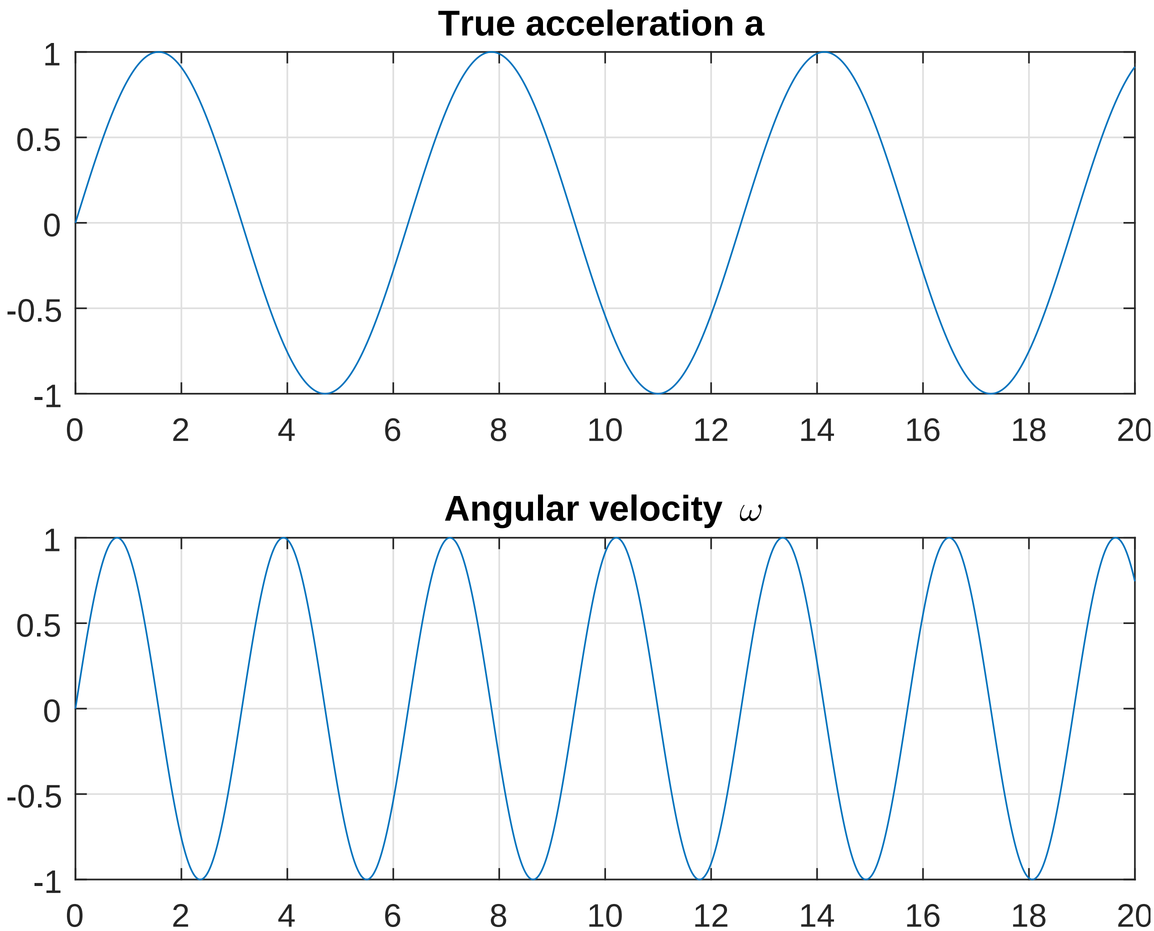

Figure 1: True acceleration and angular velocity.

Task 2

In order to discretize the system, it must first be written as a state space model. The system

$$\dot{x} = v \tag{1a}$$

$$\dot{v} = a \tag{1b}$$

$$\dot{\theta} = \omega \tag{1c}$$

can be written as

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{u} \tag{2a}$$

$$\begin{bmatrix} \dot{x} \\ \dot{v} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ v \\ \theta \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ \omega \end{bmatrix}. \tag{2b}$$

By using forward Euler to discretize the system, it can be written on the form

$$\boldsymbol{x}(t_{k+1}) = (\boldsymbol{I} + h\boldsymbol{A}(t_k))\boldsymbol{x}(t_k) + h\boldsymbol{B}(t_k)\boldsymbol{u}(t_k) \tag{3}$$

where h is the step size. The discretized system then becomes

$$\boldsymbol{x}(t_{k+1}) = \begin{bmatrix} 1 & h & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \boldsymbol{x}(t_k) + \begin{bmatrix} 0 & 0 \\ h & 0 \\ 0 & h \end{bmatrix} \boldsymbol{u}(t_k). \tag{4}$$

Figure 2 shows plots of the states in the discretized system.

Task 3

When white noise is expressed in discrete time it is referred to as a white sequence [1], where the sequence consists of random variables that are uncorrelated [2]. The autocorrelation function for discrete white noise is:

$$R_d(k) = A\delta(k), \;\; \delta(k) = \begin{cases} 1 & k = 0 \\ 0 & k \neq 0 \end{cases}. \tag{5}$$

When using Matlab, white Gaussian noise can be generated by using `wgn()`, which will generate a sequence of uncorrelated random variables, which can be regarded as a white sequence.

The biases b_1 and b_2 can be discretized with forward Euler using equation 3 from task 2. The bias can be written in state space form as

$$\dot{\boldsymbol{x}} = \begin{bmatrix} -\frac{1}{T_1} & 0 \\ 0 & -\frac{1}{T_2} \end{bmatrix} \boldsymbol{x} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \boldsymbol{w} \tag{6}$$

where

$$\boldsymbol{x} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}, \;\; \boldsymbol{w} = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}.$$

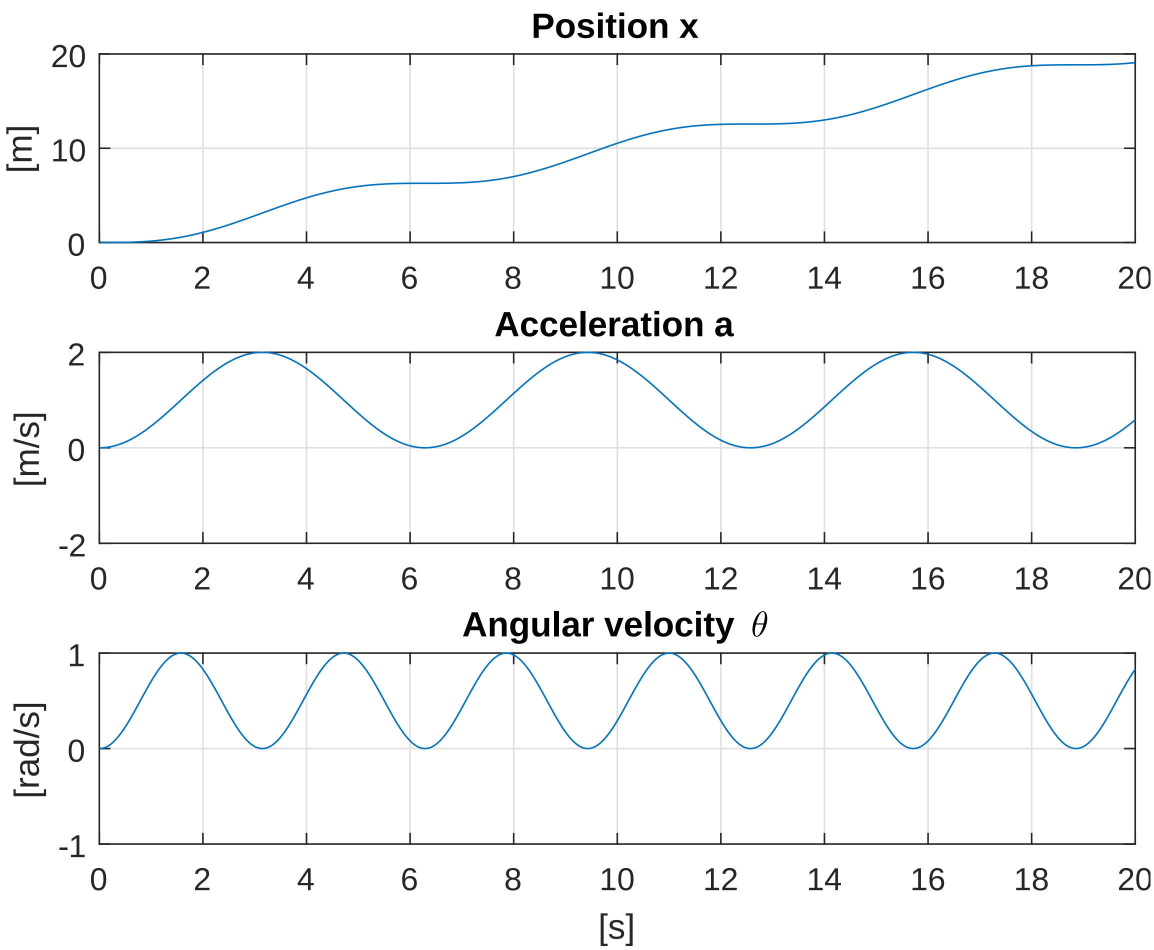

Figure 2: States of the discretized system.

The resulting discretized system is:

$$\boldsymbol{x}(t_{k+1}) = \begin{bmatrix} 1 - \frac{h}{T_1} & 0 \\ 0 & 1 - \frac{h}{T_2} \end{bmatrix} \boldsymbol{x}(t_k) + \begin{bmatrix} h & 0 \\ 0 & h \end{bmatrix} \boldsymbol{w}(t_k). \tag{7}$$

Task 4

The Kalman filter consists of several equations, which can be found in table 4.1 in [1].

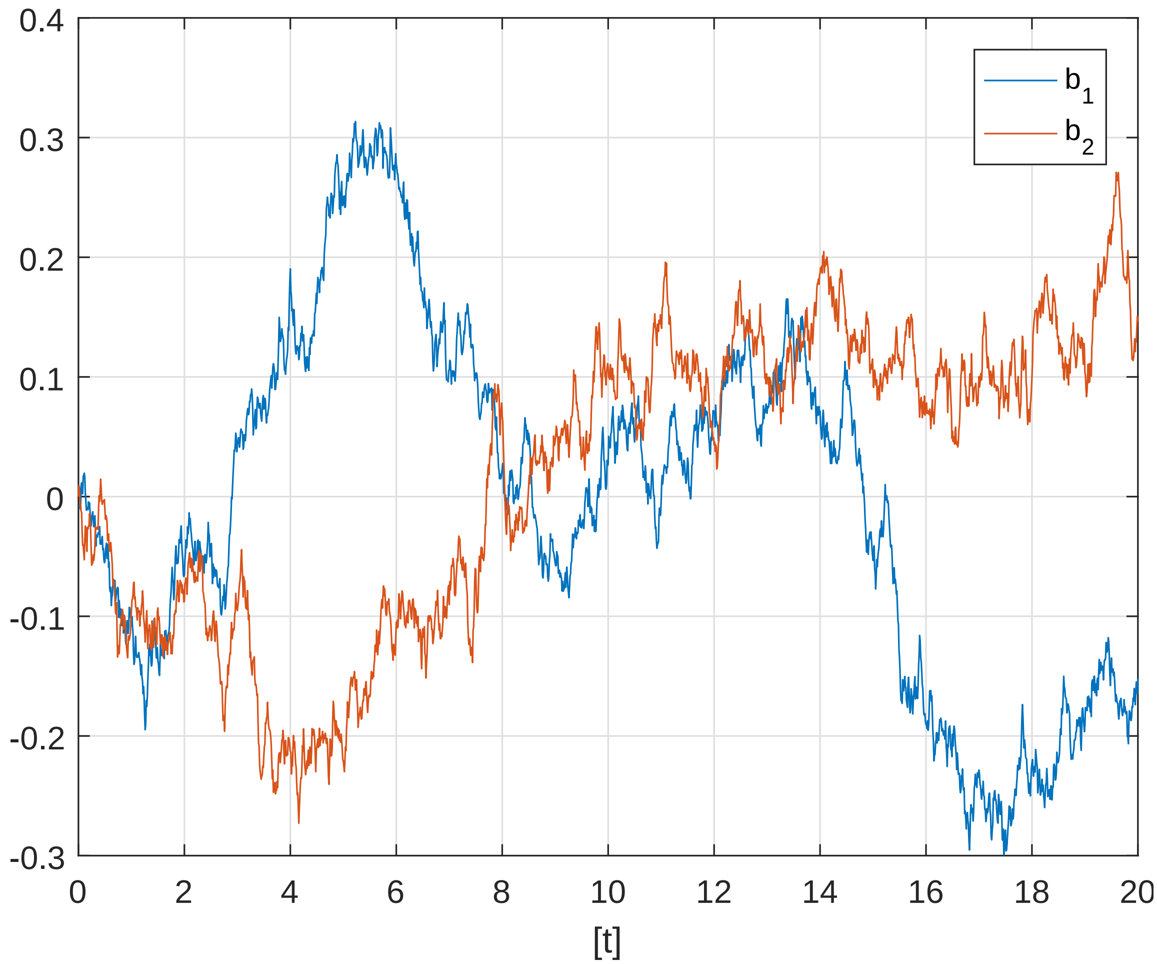

Figure 3: Bias modelled as a Gauss-Markov process.

The design matrices of the Kalman filter, $\boldsymbol{Q}$ and $\boldsymbol{R}$ are chosen based on knowledge of the variance in the system, and must satisfy the condition

$$\boldsymbol{Q}_d(k) = \boldsymbol{Q}_d^T(k) > 0, \quad \boldsymbol{R}_d(k) = \boldsymbol{R}_d(k) > 0. \tag{8}$$

The initial condistions of the filter are chose as

$$\bar{\boldsymbol{x}}(0) = \boldsymbol{x}_0 \tag{9a}$$

$$\bar{\boldsymbol{P}}(0) = E[(\boldsymbol{x}(0) - \hat{\boldsymbol{x}}(0))(\boldsymbol{x}(0) - \hat{\boldsymbol{x}}(0))^T] = \boldsymbol{P}_0. \tag{9b}$$

The Kalman gain matrix is calculated by

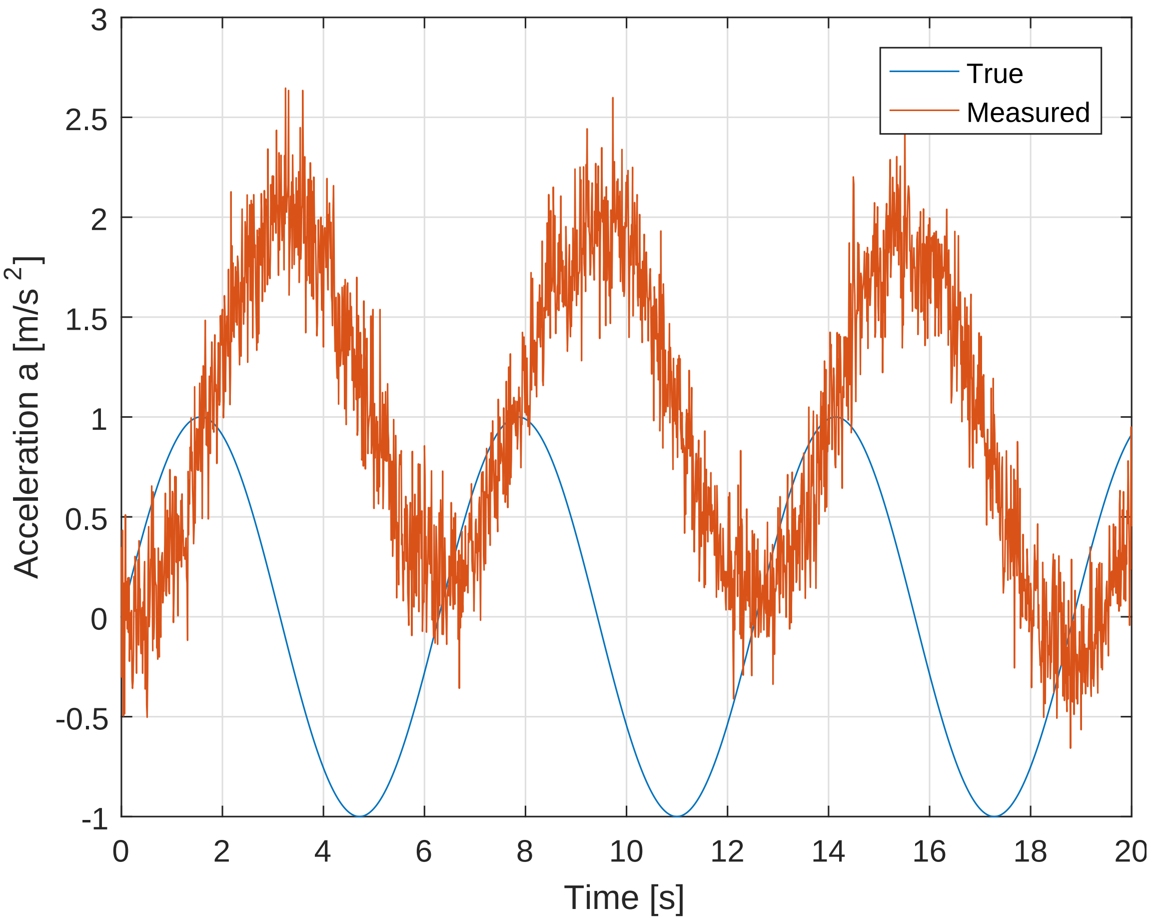

Figure 4: Measured acceleration.

$$\boldsymbol{K}(k) = \bar{\boldsymbol{P}}(k)\boldsymbol{H}^T(k)[\boldsymbol{H}(k)\bar{\boldsymbol{P}}(k)\boldsymbol{H}^T(k) + \boldsymbol{R}_d(k)]^{-1} \tag{10}$$

and the state estimation performed at every timestep is defined as

$$\hat{\boldsymbol{x}}(k) = \bar{\boldsymbol{x}}(k) + \boldsymbol{K}(k)[\boldsymbol{y}(k) - \boldsymbol{H}(k)\bar{\boldsymbol{x}}(k)]. \tag{11}$$

The error covariance update is defined as

$$\hat{\boldsymbol{P}}(k) = [\boldsymbol{I} - \boldsymbol{K}(k)\boldsymbol{H}(k)]\bar{\boldsymbol{P}}(k)[\boldsymbol{I} - \boldsymbol{K}(k)\boldsymbol{H}(k)]^T + \boldsymbol{K}(k)\boldsymbol{R}_d(k)\boldsymbol{K}^T(k). \tag{12}$$

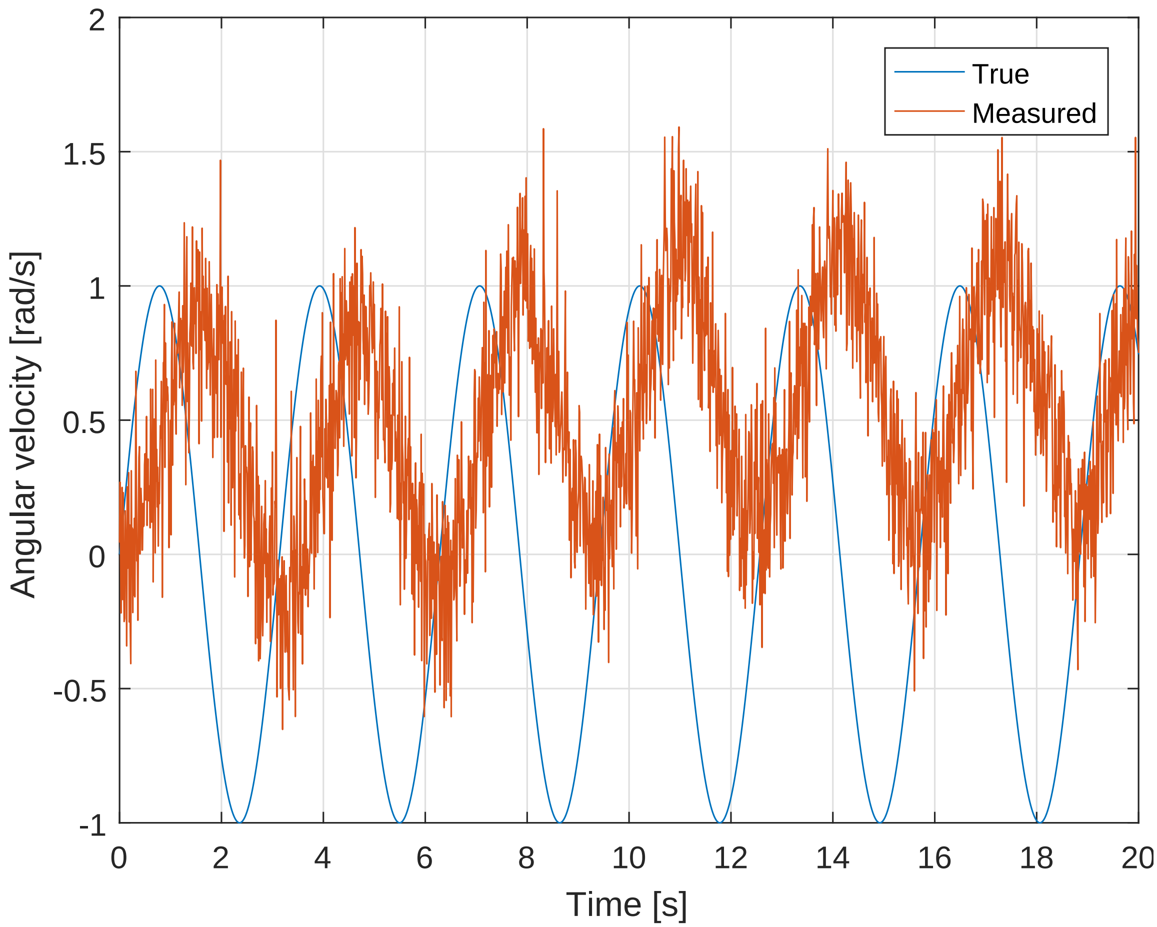

Figure 5: The measure angular velocity.

The propagation of the system is updated in both the state estimation and the error covariance

$$\bar{\boldsymbol{x}}(k+1) = \boldsymbol{\Phi}(k) + \hat{\boldsymbol{x}}(k) + \Delta(k)\boldsymbol{u}(k) \tag{13a}$$

$$\bar{\boldsymbol{P}}(k+1) = \boldsymbol{\Phi}(k)\hat{\boldsymbol{P}}(k)\boldsymbol{\Phi}^T(k) + \boldsymbol{\Gamma}(k)\boldsymbol{Q}_d(k)\boldsymbol{\Gamma}^T(k) \tag{13b}$$

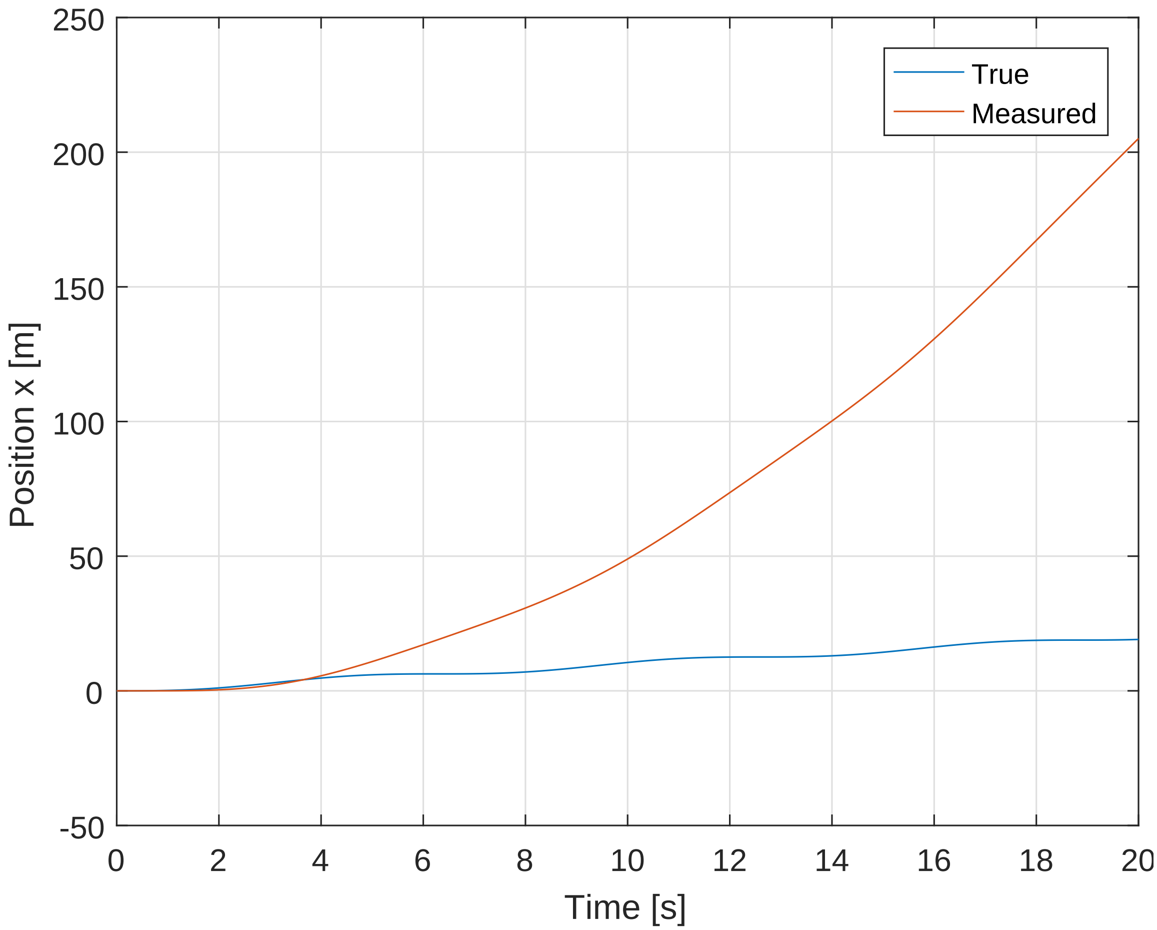

Figure 6: The measured position.

References

[1] Vik, Bjørnar (2014) *"Integrated Satellite and Inertial Navigation Systems"*, Norwegian University of Science and Technology, Department of Engineering Cybernetics, Trondheim

[2] Wikipedia, *https://en.wikipedia.org/wiki/White_noise*, accessed 05.11.2016

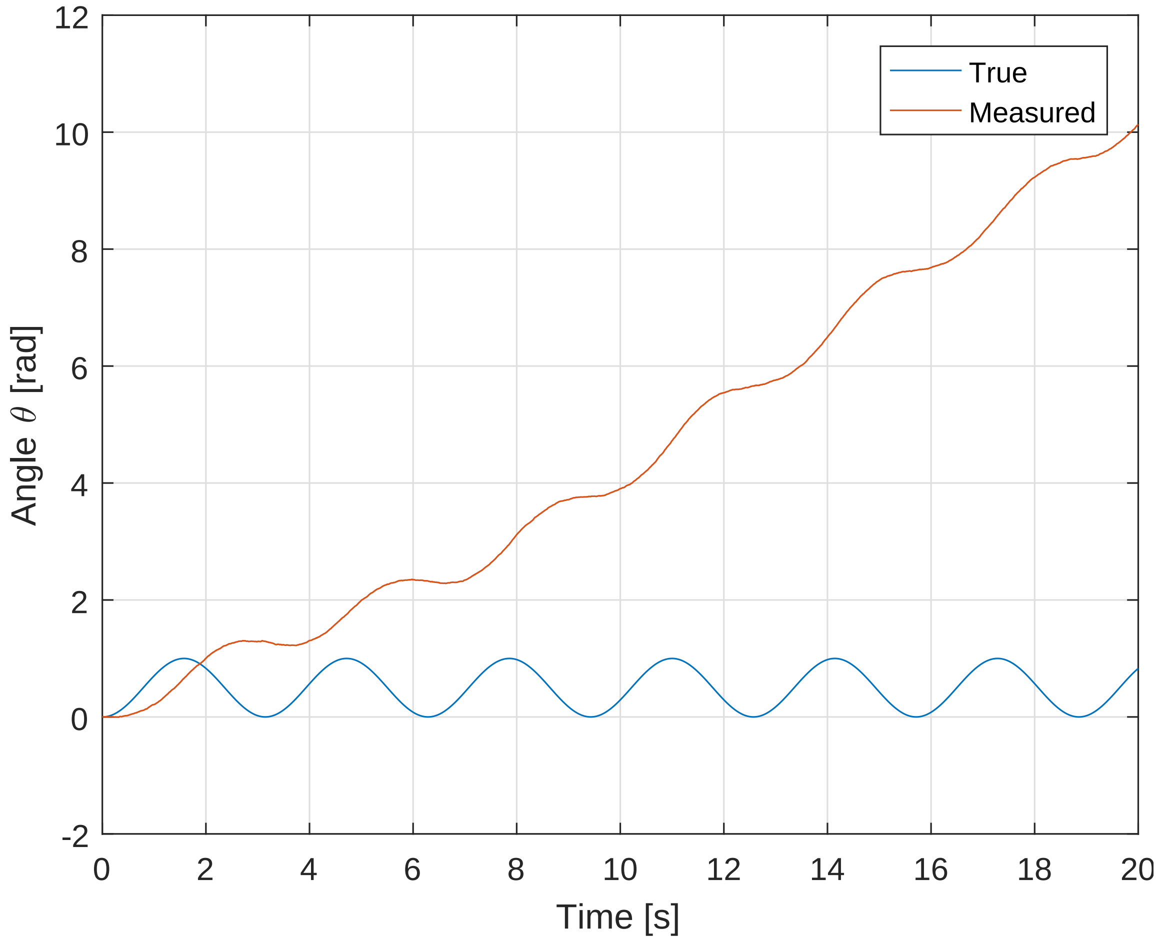

Figure 7: Measured orientation.