Optimization Equation

The states included in the optimization problem:

$$\mathbf{x} = \begin{bmatrix} p_n \\ p_e \\ p_d \\ u \\ v \\ w \\ \phi \\ \theta \\ \psi \\ p \\ q \\ r \end{bmatrix}. \tag{1}$$

Problem Definition

$$\begin{aligned} \min_{\mathbf{x}} \quad & \mathbf{\Phi} = ||\mathbf{c}_d^n - \mathbf{c}^n|| \\ \text{s.t} \quad & \mathbf{A}\mathbf{x} \leq \mathbf{b} \\ & \dot{\mathbf{x}} = f(\mathbf{x}, \mathbf{u}) \end{aligned} \tag{2}$$

where $f(\mathbf{x}, \mathbf{u})$ represents the aircraft dynamics and $\mathbf{A}\mathbf{x} \leq \mathbf{b}$ represents the constraints put on the UAV states. $\mathbf{\Phi}$ is the distance between the center point of the camera $\mathbf{c}^n$ and the ground path that is to be observed $\mathbf{c}_d^n$.

Cost Function

The center point of the camera $\mathbf{c}^n$ is expressed as a function of the position of the UAV and its attitude states

$$\begin{aligned} \mathbf{c}^n &= \mathbf{p} + \mathbf{c}_b^b \\ &= \begin{bmatrix} p_n \\ p_e \end{bmatrix} + \mathbf{R}_{z,\psi} \begin{bmatrix} p_d tan(\theta) \\ p_d tan(\phi) \end{bmatrix}. \end{aligned} \tag{3}$$

The objective of the cost function is to minimize the distance from camera center point $\mathbf{c}^n$ to the ground observation path $\mathbf{c}_d^n$.

Constraints

The equality constraints $\dot{\mathbf{x}} = f(\mathbf{x}, \mathbf{u})$ represents the full UAV model.