UAV Kinematics

Position of the UAV will be given in reference frame {n} using NED. This gives the states:

$$\boldsymbol{p}_{b/n}^{n} = \begin{bmatrix} N \\ E \\ D \end{bmatrix} = \begin{bmatrix} x_n \\ y_n \\ z_n \end{bmatrix} \tag{1}$$

Attitude will be given as Euler-angles:

$$\boldsymbol{\Theta}_{nb} = \begin{bmatrix} \phi \\ \theta \\ \psi \end{bmatrix} \tag{2}$$

Together these make up the position and orientation vector $\boldsymbol{\eta}$:

$$\boldsymbol{\eta} = \begin{bmatrix} \boldsymbol{p}_{b/n}^{n} \\ \boldsymbol{\Theta}_{nb} \end{bmatrix} \tag{3}$$

Wind

Wind will introduce what is called crab angle χ_c in the horizontal plane and the angle of attack γ_a in the vertical plane. This will change the UAVs actual heading ψ and pitch θ to:

$$\chi = \psi + \chi_c \tag{4a}$$

$$\gamma = \theta + \gamma_a \tag{4b}$$

where χ is called the course and γ is maybe called something???? These angles will only affect the navigational part, and not what the camera is pointed at.

Camera position

When assuming flat earth, the centre point of the camera on the ground can be expressed in the body frame {b} using the attitude $\boldsymbol{\Theta}_{nb}$ of the UAV and the height z_n:

$$\boldsymbol{c}_b^b = \begin{bmatrix} c_{x/b}^b \\ c_{y/b}^b \end{bmatrix} = \begin{bmatrix} z_n sin(\theta) \\ z_n sin(\phi) \end{bmatrix} \tag{5}$$

The distance from the UAV to the camera centre point can be expressed in {n} by using the rotational matrix $\boldsymbol{R}_{z,\psi}$:

$$\boldsymbol{c}_b^n = \begin{bmatrix} c_{x/b}^n \\ c_{y/b}^n \end{bmatrix} = \boldsymbol{R}_{z,\psi} \boldsymbol{c}_b^b \tag{6}$$

In order to translate this to position in NED, it needs to be added to the UAV's NED position:

$$\boldsymbol{c}^n = \begin{bmatrix} c_x^n \\ c_y^n \end{bmatrix} = \begin{bmatrix} x_n + c_{x/b}^n \\ y_n + c_{y/b}^n \end{bmatrix} \tag{7}$$

Camera Angle of View

Since the camera isn't only focusing on one specific point, it can be useful describing the camera point of focus as two extremities instead of one center point. Equation (5) can easily be changed to do this. Assuming the camera as an angle of view σ, the equation now becomes

$$\boldsymbol{e}_b^b = \begin{bmatrix} e_{x_1/b}^b \\ e_{y_1/b}^b \\ e_{x_2/b}^b \\ e_{y_2/b}^b \end{bmatrix} = \begin{bmatrix} z_n sin(\theta + \sigma) \\ z_n sin(\theta - \sigma) \\ z_n sin(\phi + \sigma) \\ z_n sin(\phi - \sigma) \end{bmatrix} \tag{8}$$

The steps for translating the points to the NED frame are the same as in (6) and (7):

$$\boldsymbol{e}_b^n = \begin{bmatrix} e_{x_1/b}^n \\ e_{y_1/b}^n \\ e_{x_2/b}^n \\ e_{y_2/b}^n \end{bmatrix} = \boldsymbol{R}_{z,\psi} \boldsymbol{e}_b^b \tag{9}$$

$$\boldsymbol{e}^n = \begin{bmatrix} e_{x_1}^n \\ e_{y_1}^n \\ e_{x_2}^n \\ e_{y_2}^n \end{bmatrix} = \begin{bmatrix} x_n + e_{x_1/b}^n \\ y_n + e_{y_1/b}^n \\ x_n + e_{x_2/b}^n \\ y_n + e_{y_2/b}^n \end{bmatrix} \tag{10}$$

ADD TRANSLATIONAL MATRIX FOR POSITION OF CAMERA