Optimization Equation

The optimization problem will seek to minimize the distance between the centre of the camera footprint and the path that is to be observed. The states of the UAV $\mathbf{x}$ and inputs $\mathbf{u}$ are defined as

$$\mathbf{x}_j = \begin{bmatrix} p_{n\,j} \\ p_{e\,j} \\ p_{d\,j} \\ u_j \\ v_j \\ w_j \\ \phi_j \\ \theta_j \\ \psi_j \\ p_j \\ q_j \\ r_j \\ q1_j \\ q2_j \\ q3_j \\ q4_j \end{bmatrix}, \quad \mathbf{u}_j = \begin{bmatrix} \delta_{e\,j} \\ \delta_{a\,j} \\ \delta_{r\,j} \\ \delta_{t\,j} \end{bmatrix}. \tag{1}$$

The length of the optimization is given by K timesteps. The optimization vector $\mathbf{z}$ will consist of UAV states $\mathbf{x}_j$ and inputs $\mathbf{u}_j$ for each of the K timesteps. $\mathbf{x}_0$ and $\mathbf{u}_0$ equals the trimmed state of the UAV.

$$\mathbf{z} = \begin{bmatrix} \mathbf{x}_1 & \mathbf{x}_2 & \dots & \mathbf{x}_K & \mathbf{u}_1 & \mathbf{u}_2 & \dots & \mathbf{u}_K \end{bmatrix}^T. \tag{2}$$

Problem Definition

$$\begin{array}{rrcl} \min\limits_{\mathbf{z}} & J_k = \frac{1}{2}\sum\limits_{i=k}^{k+L}[\mathbf{e}(\mathbf{z}_i)^\intercal \mathbf{Q}\mathbf{e}(\mathbf{z}_i)] + \frac{1}{2}\sum\limits_{i=k}^{k+L}[\mathbf{u}_i^\intercal \mathbf{R}\mathbf{u}_i] & & \\ \text{s.t} & \mathbf{x}^{low} \leq & \mathbf{x}_i & \leq \mathbf{x}^{high} \\ & \mathbf{u}^{low} \leq & \mathbf{u}_i & \leq \mathbf{u}^{high} \\ & \mathbf{x}_{k+1} & = & \mathbf{A}_d\mathbf{x}_k + \mathbf{B}_d\mathbf{u}_k \end{array} \tag{3}$$

where the inequality constraints represent the limits put on both the UAV states and inputs, and the equaltiy constraints represent the discrete model of the UAV. The MPC horizon is $[k, k+L]$.

Cost Function

The objective of the cost function is to minimize the cross track error e between the path and the centre of the camera footprint of the UAVs current and future position. The position of the camera centre is expressed as (c_x^n, c_y^n), calculated by:

$$\begin{aligned} \mathbf{c}^n &= \mathbf{p} + \mathbf{c}_b^b \\ &= \begin{bmatrix} p_n \\ p_e \end{bmatrix} + \mathbf{R}_{z,\psi} \begin{bmatrix} p_d tan(\theta) \\ p_d tan(\phi) \end{bmatrix}. \end{aligned} \tag{4}$$

The parametrized path will be expressed by a path variable t so that points on the path that is to be observed $\mathbf{c}_d^n$ can be written as $(x_d(t), y_d(t), z_d(t))$. z_d represents the aircraft altitude, and not the position of the path that is to be observed. The cross-track error e_i can be written as

$$\begin{aligned} \mathbf{e}(\mathbf{z}_i) &= ||\mathbf{c}^n - \mathbf{c}_d^n|| \\ &= \sqrt{(c_x^n - x_d(t))^2 + (c_y^n - y_d(t))^2 + (p_d - z_d(t))^2} \end{aligned} \tag{5}$$