Math Review

PA 393K/393G

Andrew Waxman
Fall 2020

This Week's Class

1. Graphing
2. Equations
3. Practice Problems

Some useful stuff we'll learn about today

1. Slope
2. Equations for Lines
3. Tangency
4. Solving SOEs

Math Review

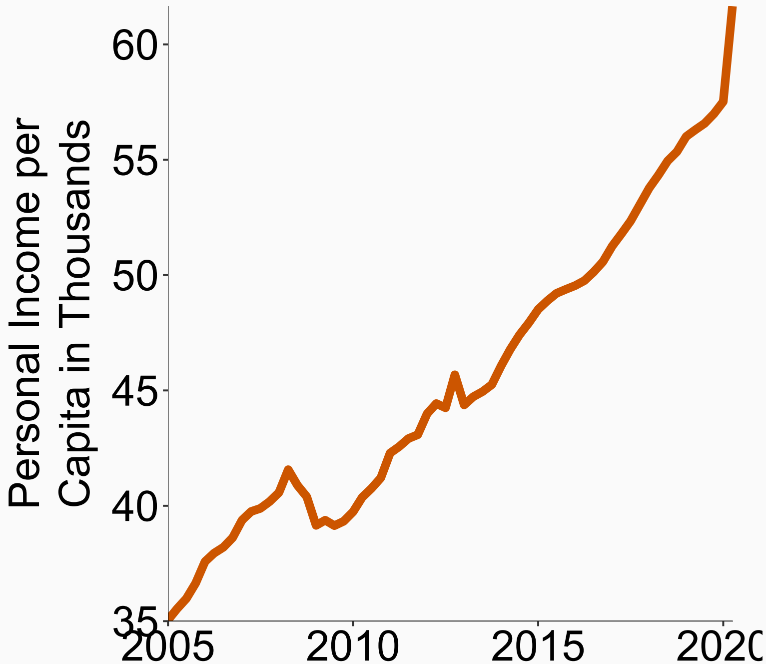
Graphing - Basic Definitions

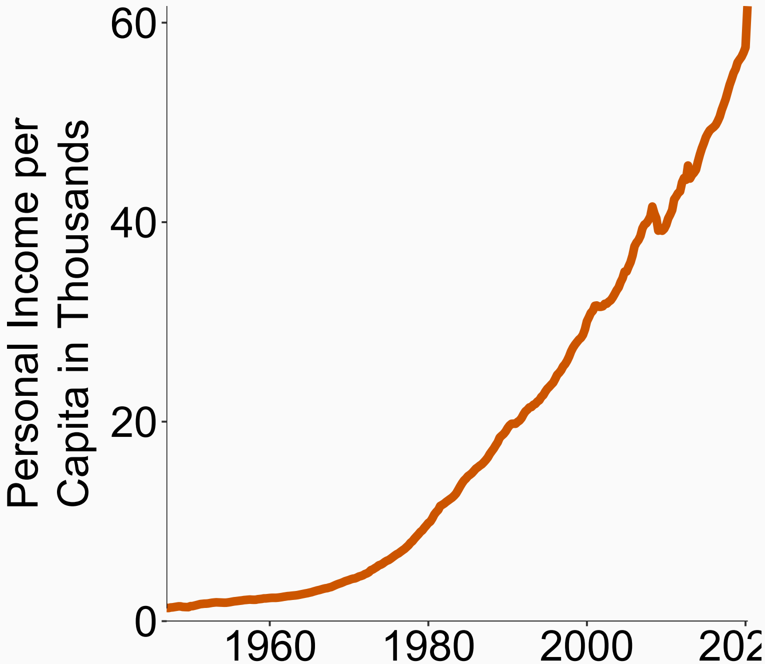
- **Variable:** quantity that can take on more than one value (hence "variable")
  - Example: **income** for one person may be $100K, another $25K
- **Units:** what is being measured. **Very important** to help identify what we are seeing.
  - Example: **annual income per capita in $1,000's** (*Source:* FRED)

Things that matter

- Labels: tell us what is what
- Units: tell us how things are measured
- Scale: Affects what we see and therefore what we infer from th epicture

Scale


Personal Income per Capita in Thousands
60
55
50
45
40
35
2005
2010
2015
2020


Personal Income per Capita in Thousands
60
40
20
0
1960
1980
2000
202

Graphing - Basic Definitions

Variables:

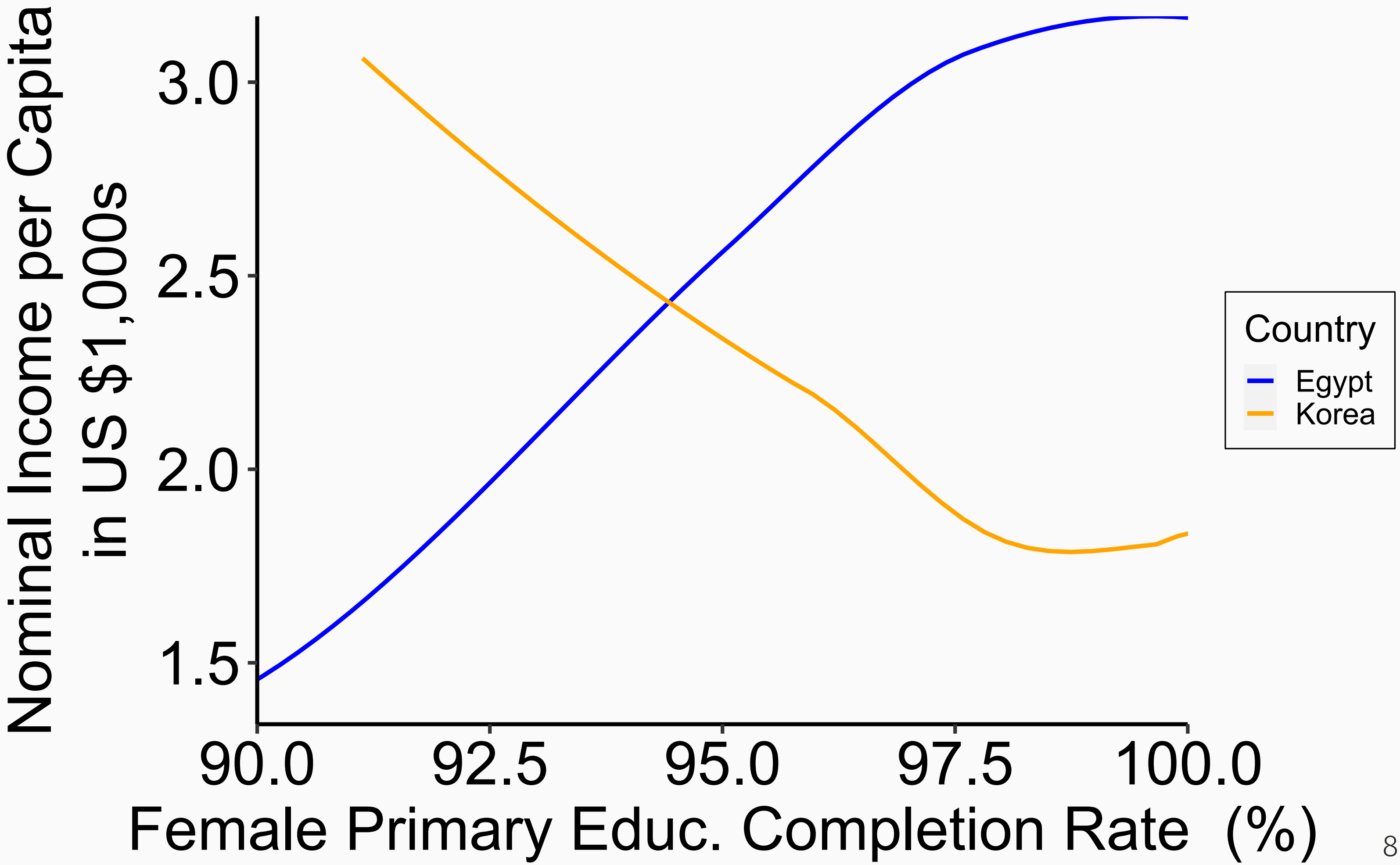
- We just showed you income. What if we are interested in the relationship between income and years of schooling?

Two Variables

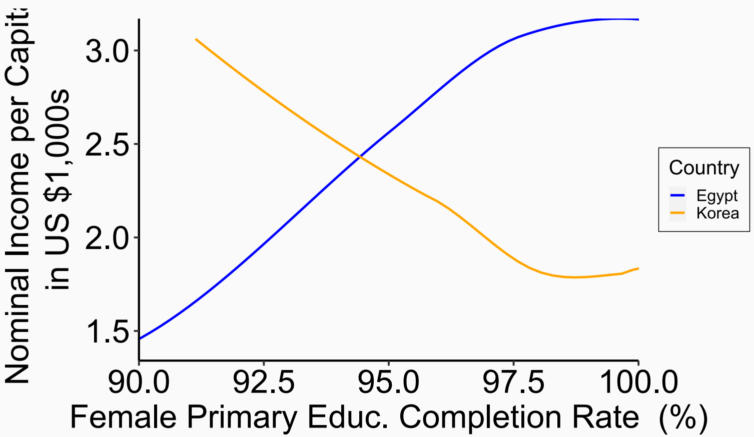
Common convention (i.e., no specific meaning to it) in math:

- Call one variable the x-variable
  - Put this one on the horizontal axis
- Call the other the y-variable -This one on the vertical axis

Example: Two Y-variables, One X-



An aside...



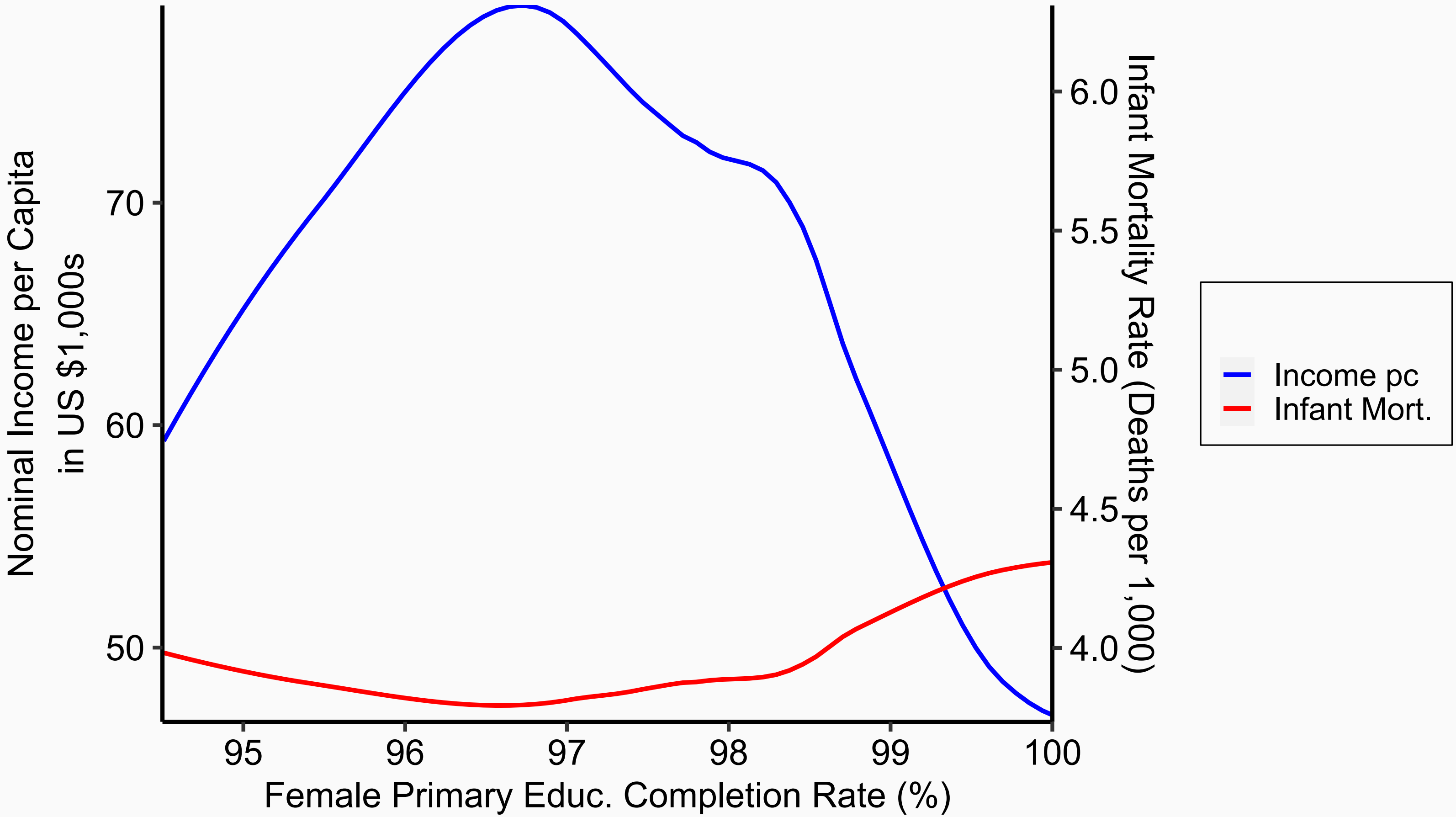
- inflation - rate of growth of prices in economy
  - higher inflation rate -> faster cost of living rises

- We may be interested in adjusting variables based on this relative cost of living: called defating the variable

- real $'s (income, prices, etc.) - deflated to account for inflation

- nominal $'s (income, prices, etc.) - undeflated -> like the actual value at the time

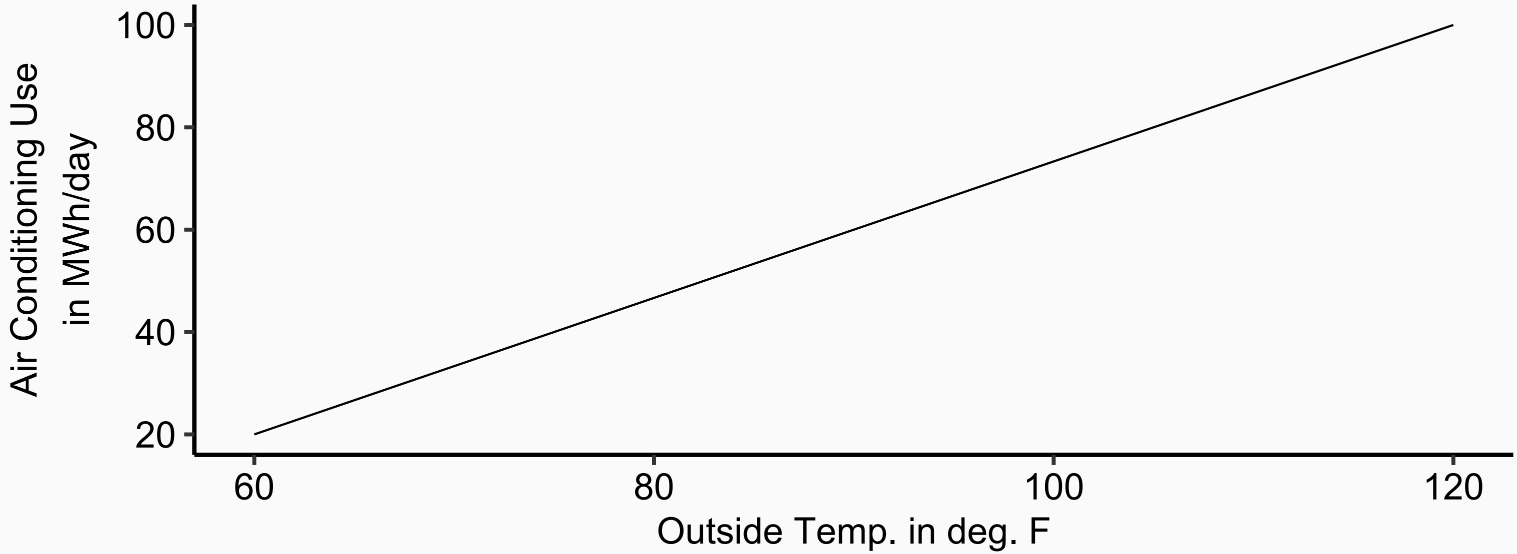
Two Axis Graph



Types of Curves

curve - any line connecting points on a graph

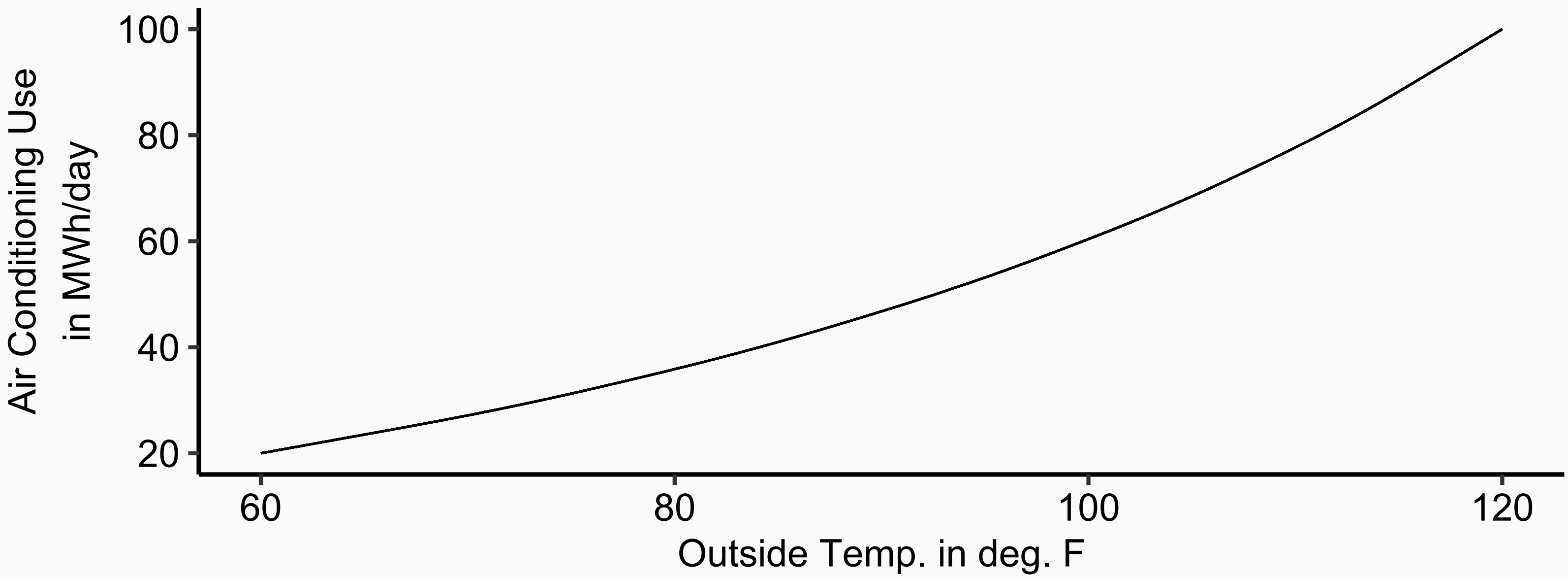
- (doesn't have to be straight, but maybe)
- straight: linear relationship



Types of Curves

curve - any line connecting points on a graph

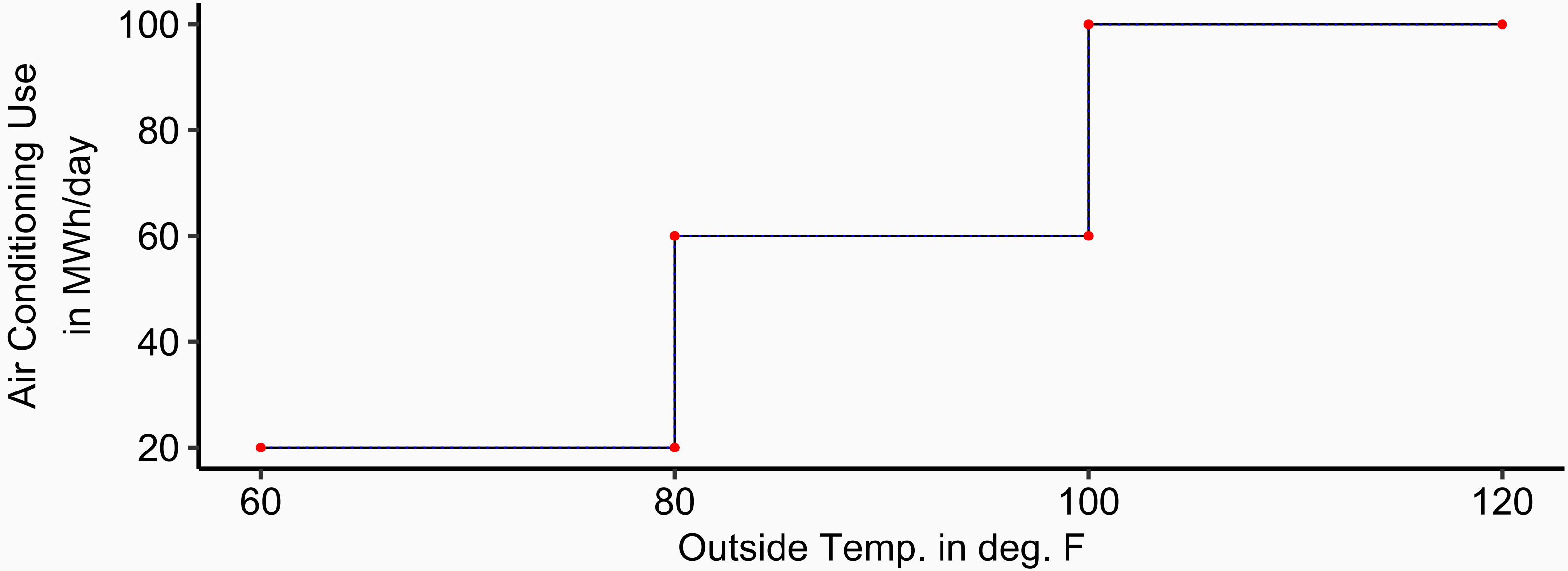
- (doesn't have to be straight, but maybe)
- straight: linear relationship
- curved: non-linear relationsip



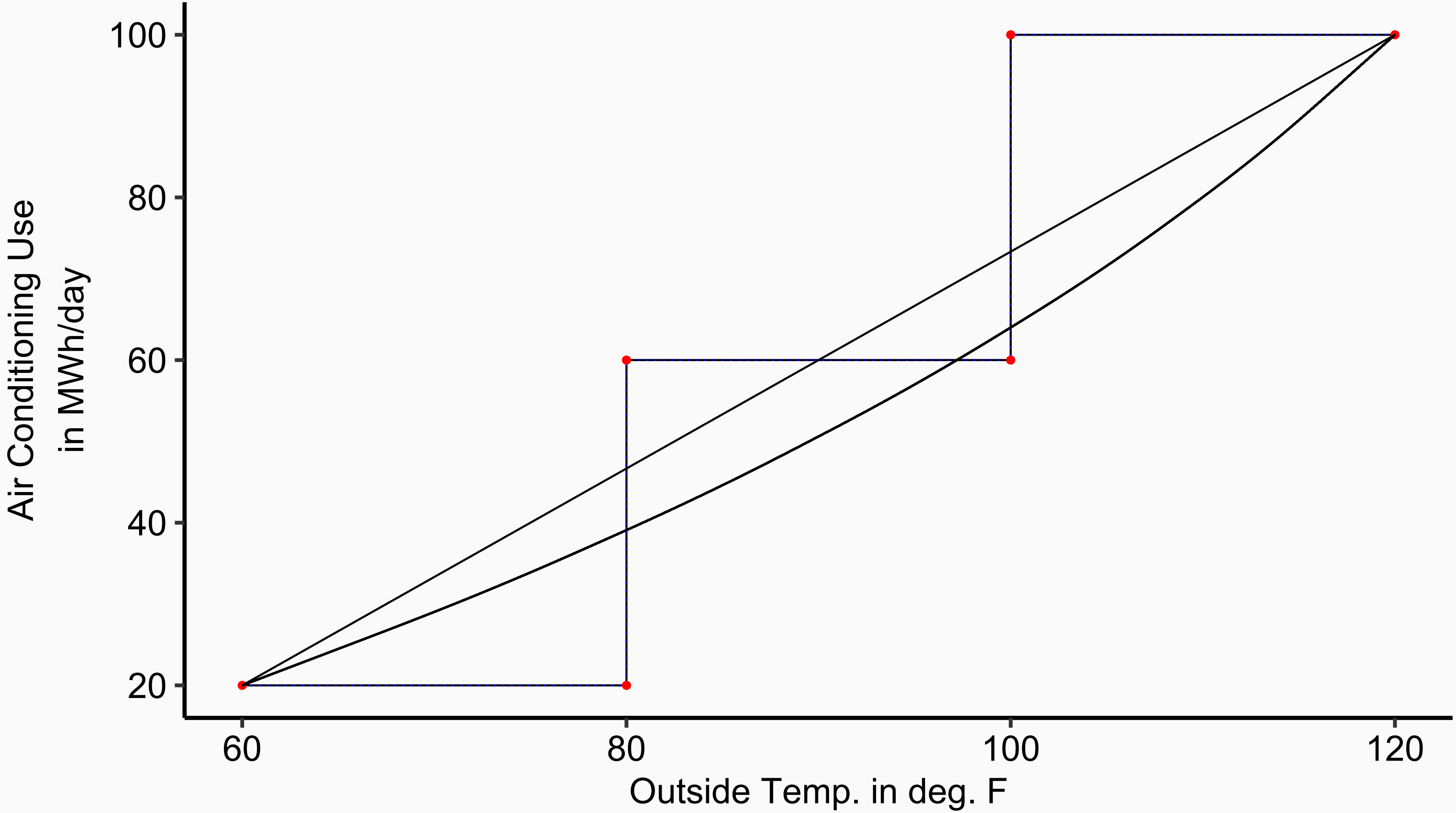
Types of Curves

<u>curve</u> - any line connecting points on a graph

- (doesn't have to be straight, but maybe)
- straight: <u>linear relationship</u>
- curved: <u>non-linear relationsip</u>
- something else (here piece-wise linear)



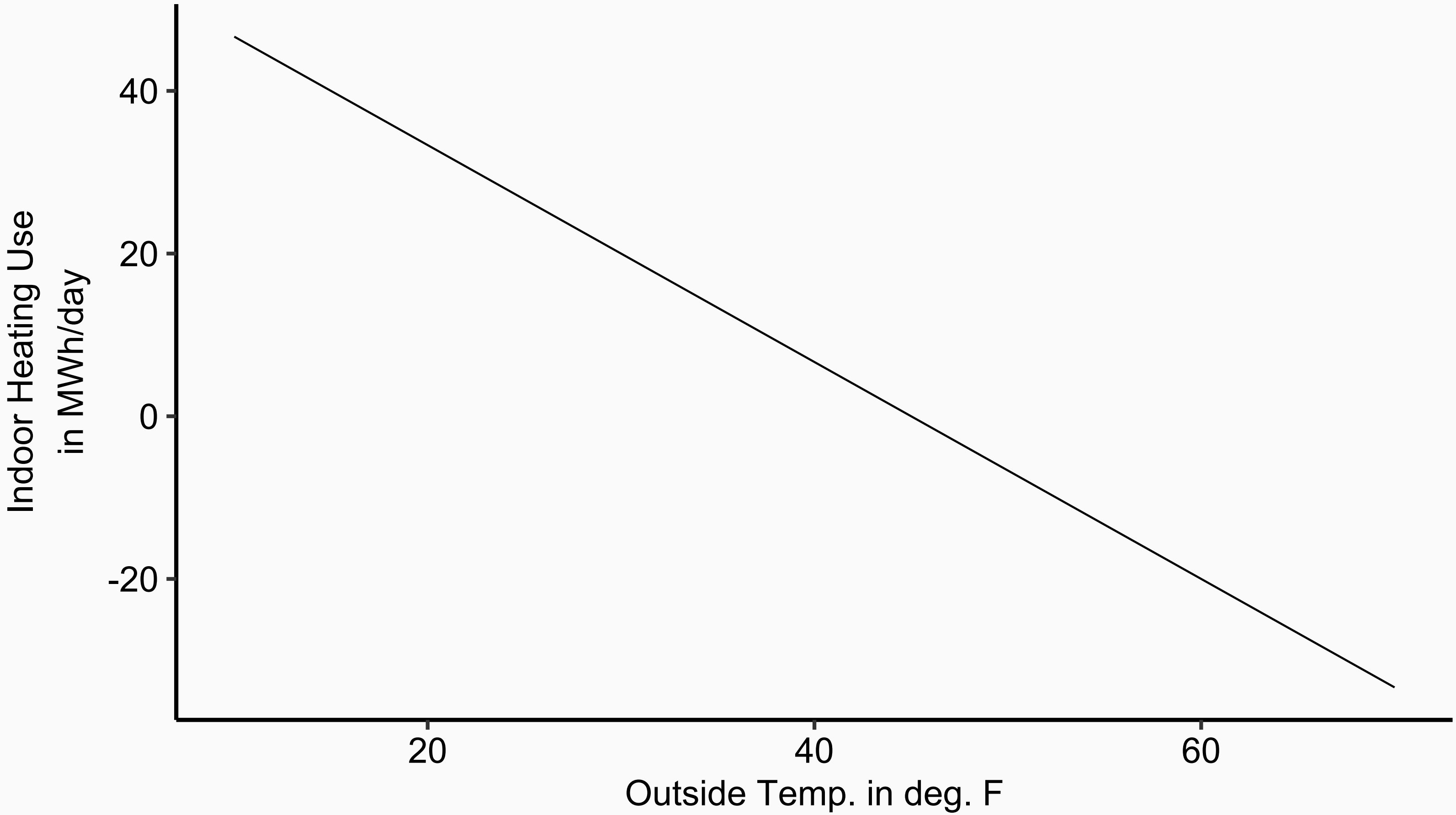
Relationships between Variables



- These are **all** positive relationships: as the temps increase, so too does A/C use

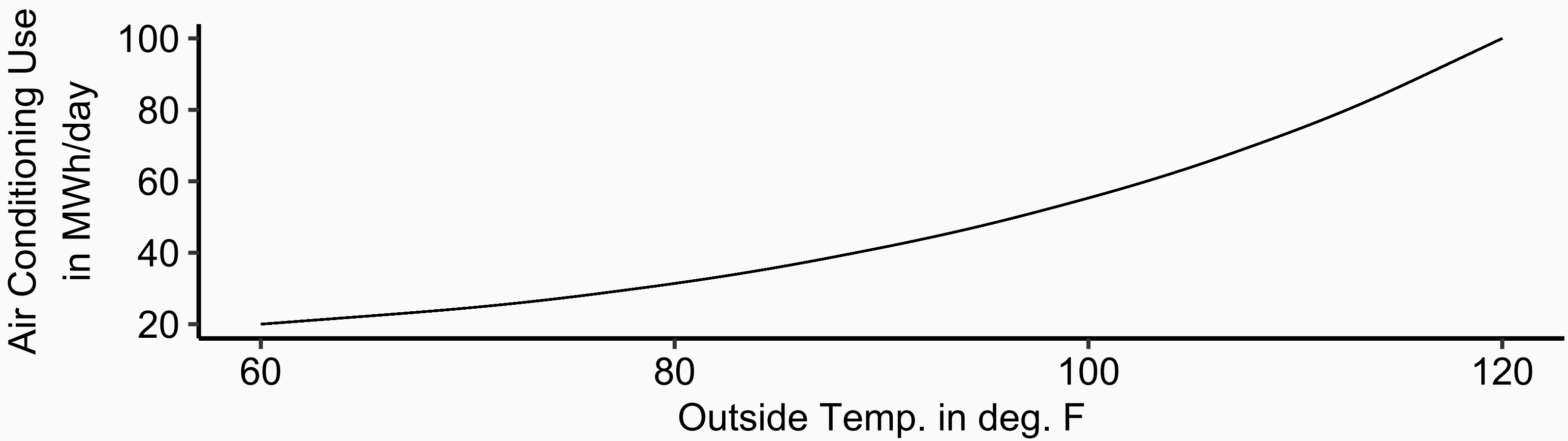
Negative Relationships

- as the temps increase, indoor heating use **goes down**

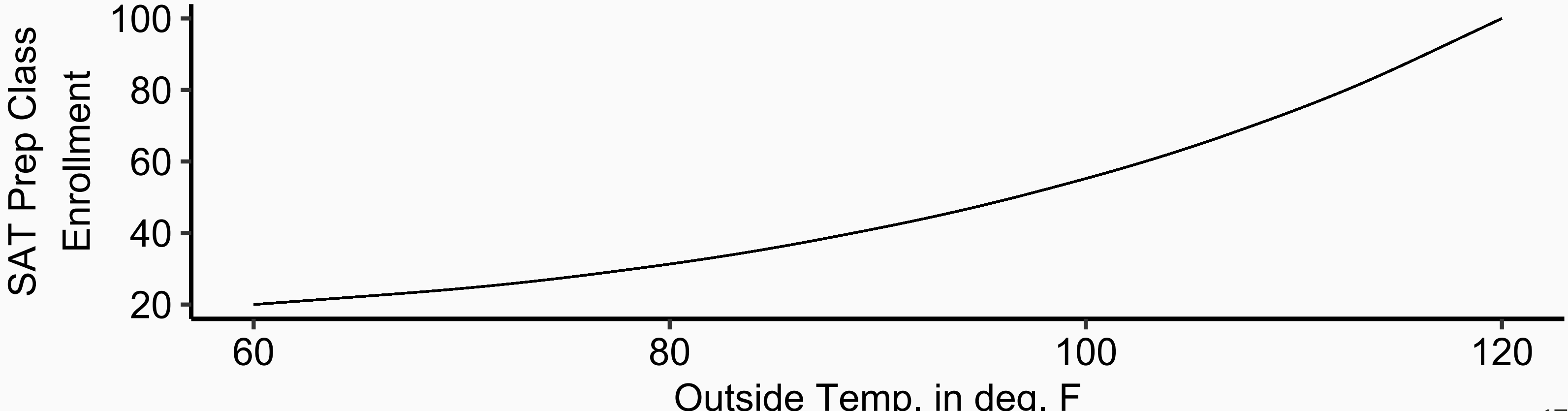


Useful thing to consider...

Causal Relationship



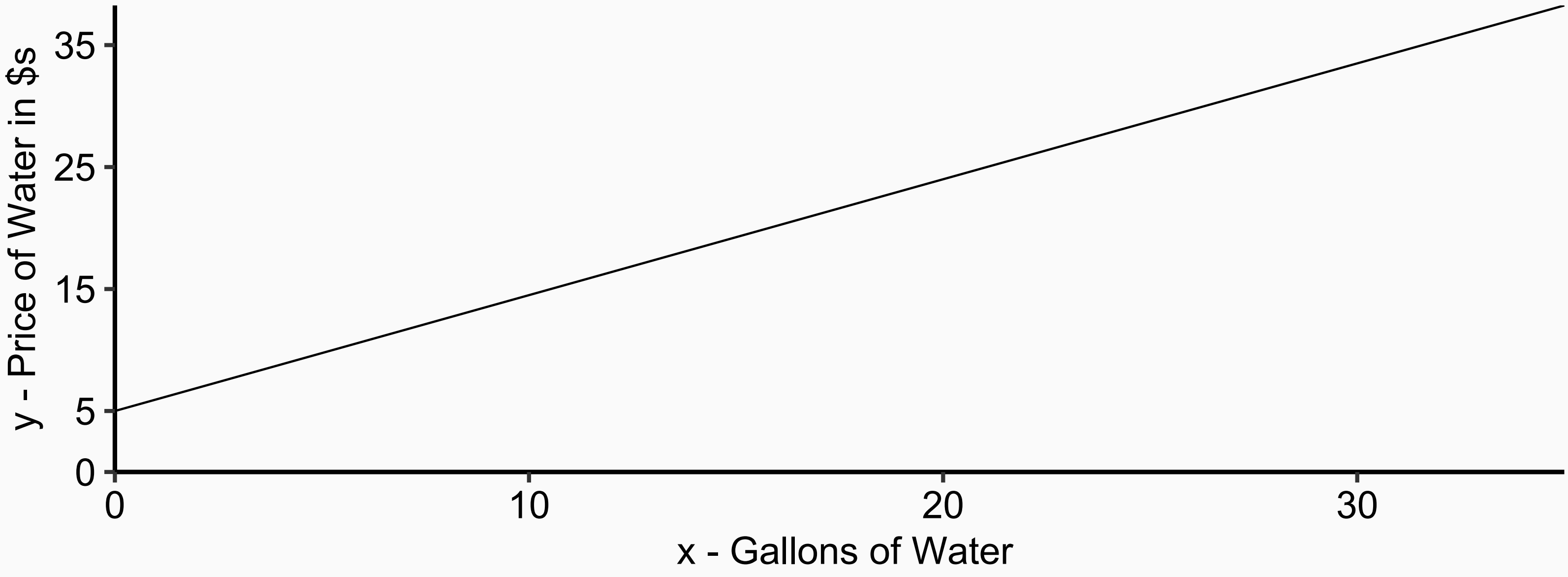
Non-Causal Relationship



Important Characteristics

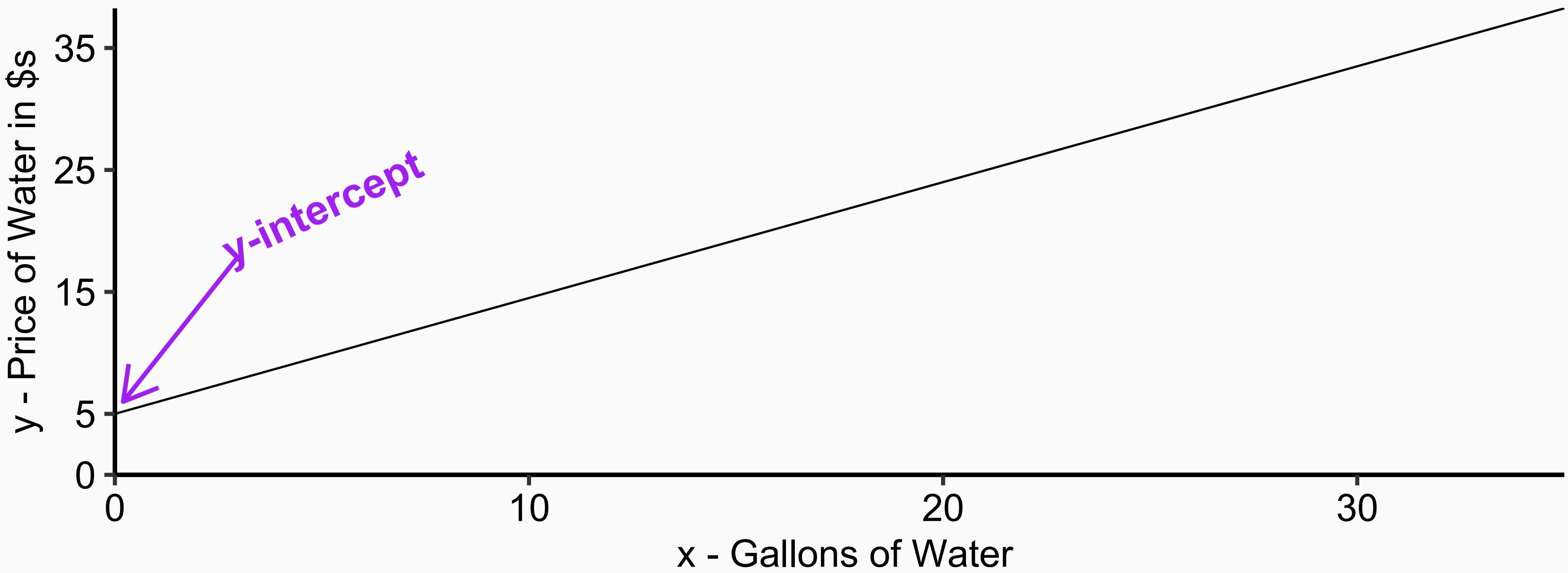
- Sometimes useful to know where graph intersects axes (things with the numbers on them)

- **Common** to call the horizontal axis "x-axis" & vertical axis "y-axis"



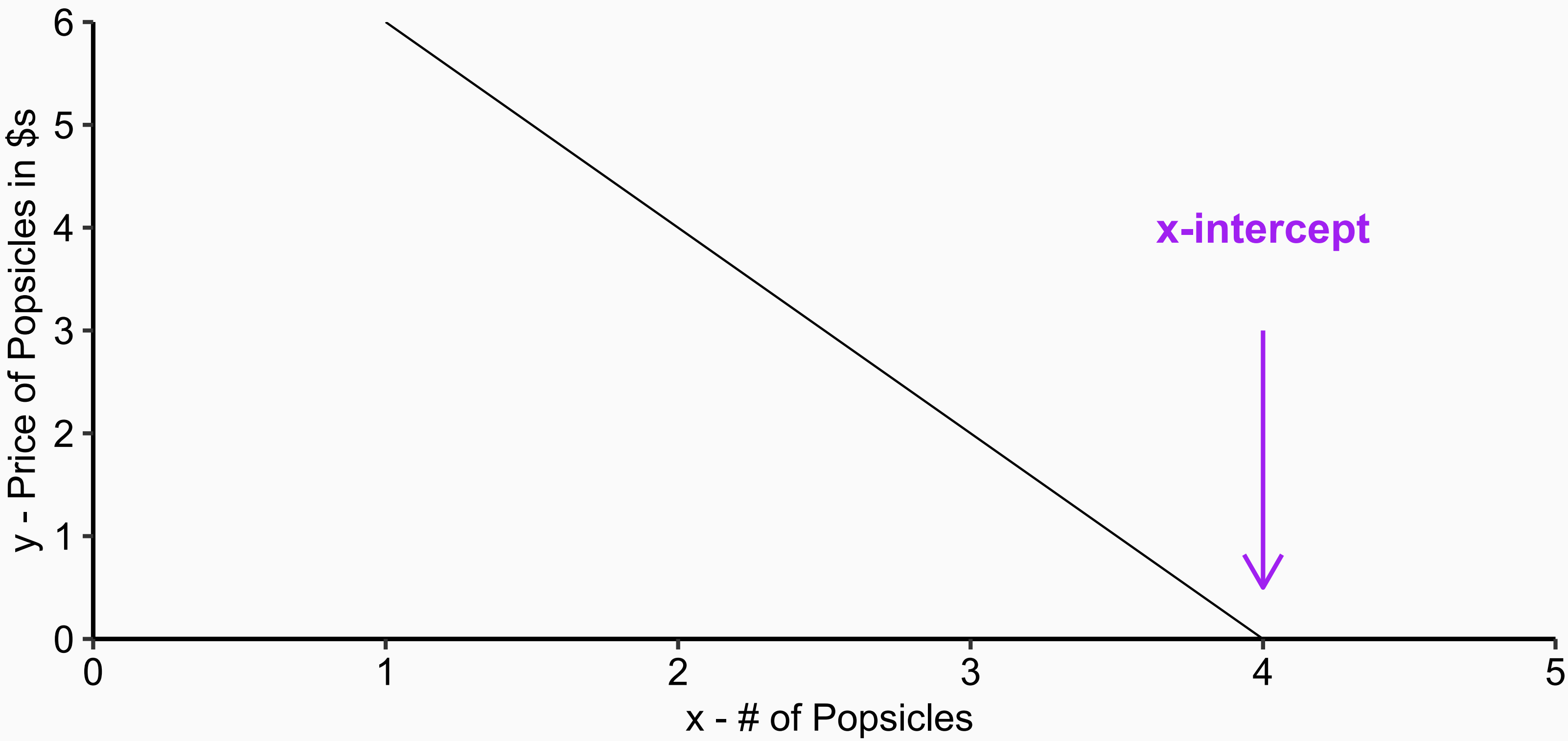
Y-intercept

- vertical intercept (aka y-intercept) at \$5 per gallon



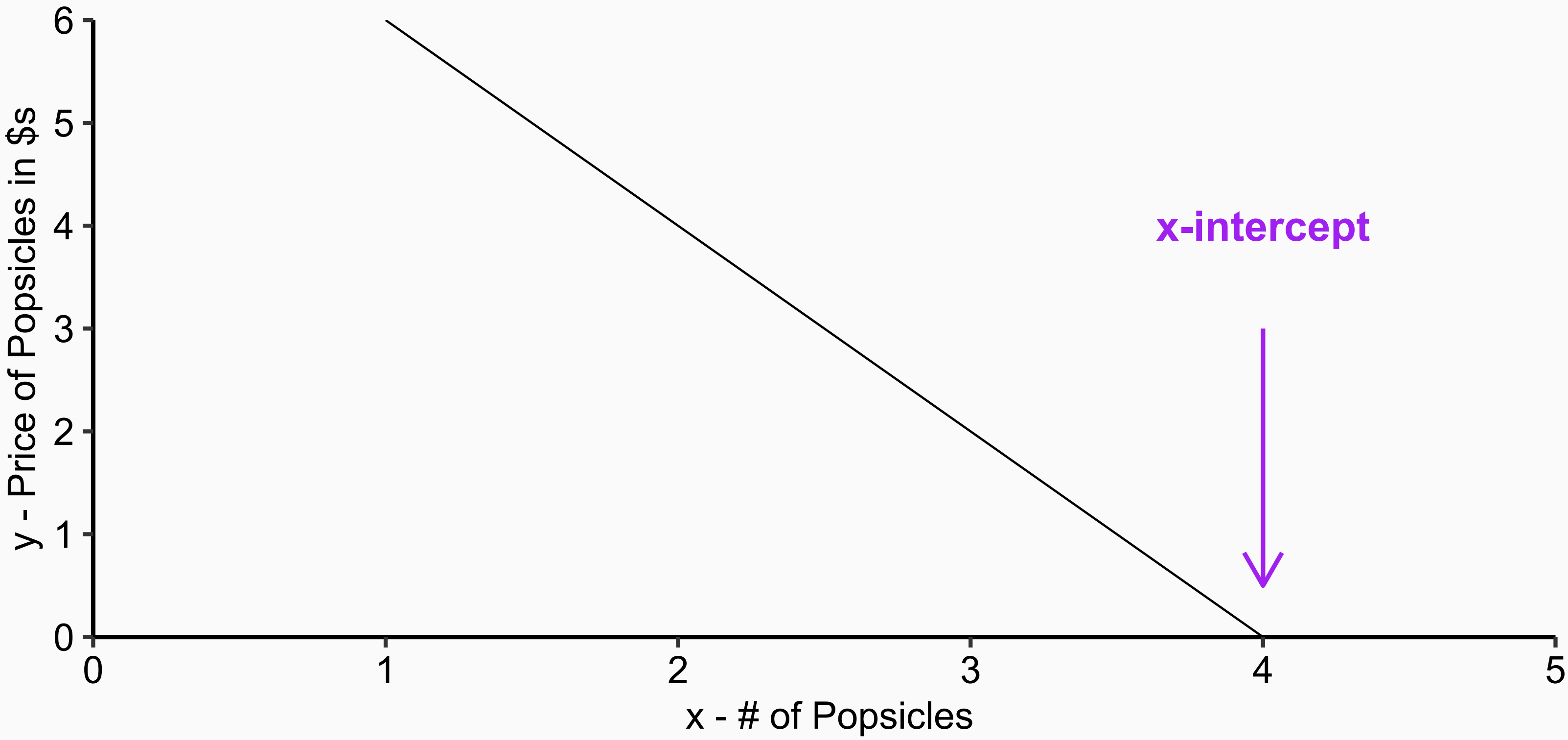
- This tells us the price of the **first** gallon of water

X-intercept


x-intercept
y - Price of Popsicles in $s
0
1
2
3
4
5
6
0
1
2
3
4
5
x - # of Popsicles

X-intercept

- horizontal intercept (aka y-intercept) at 4 popsicles



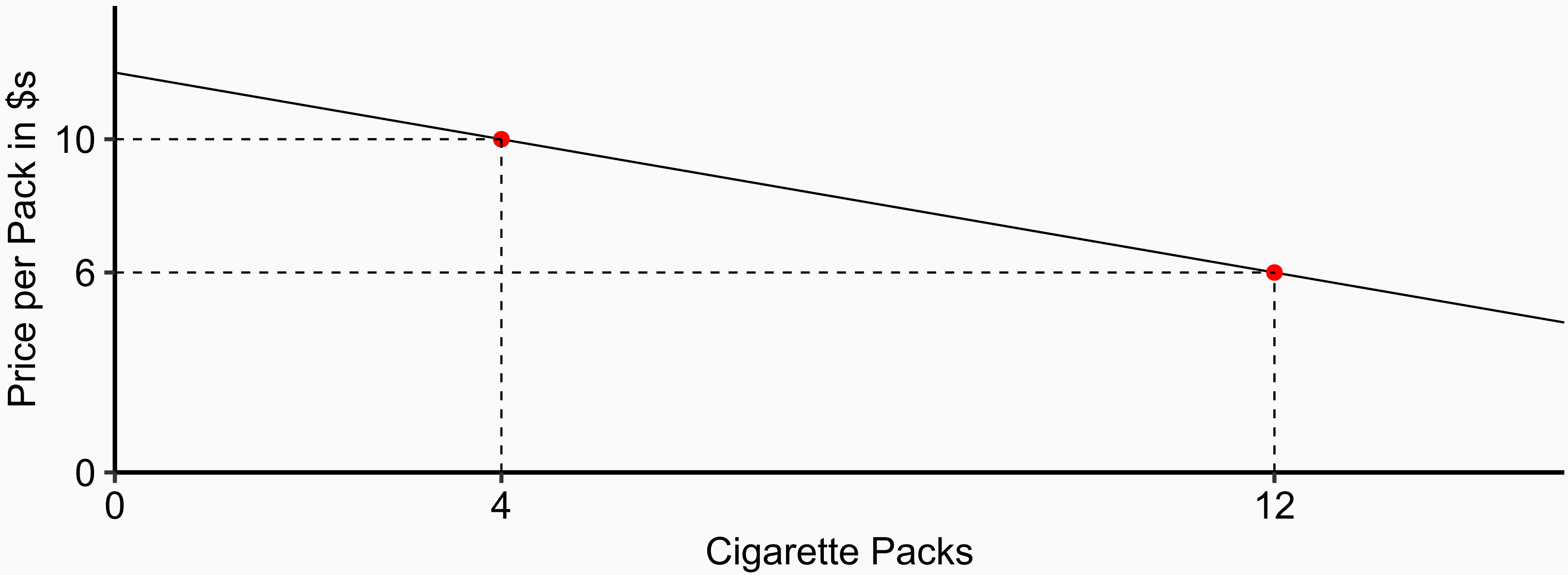
- I won't be adding "x" and "y" to the axes from here on, just remember which is whiche

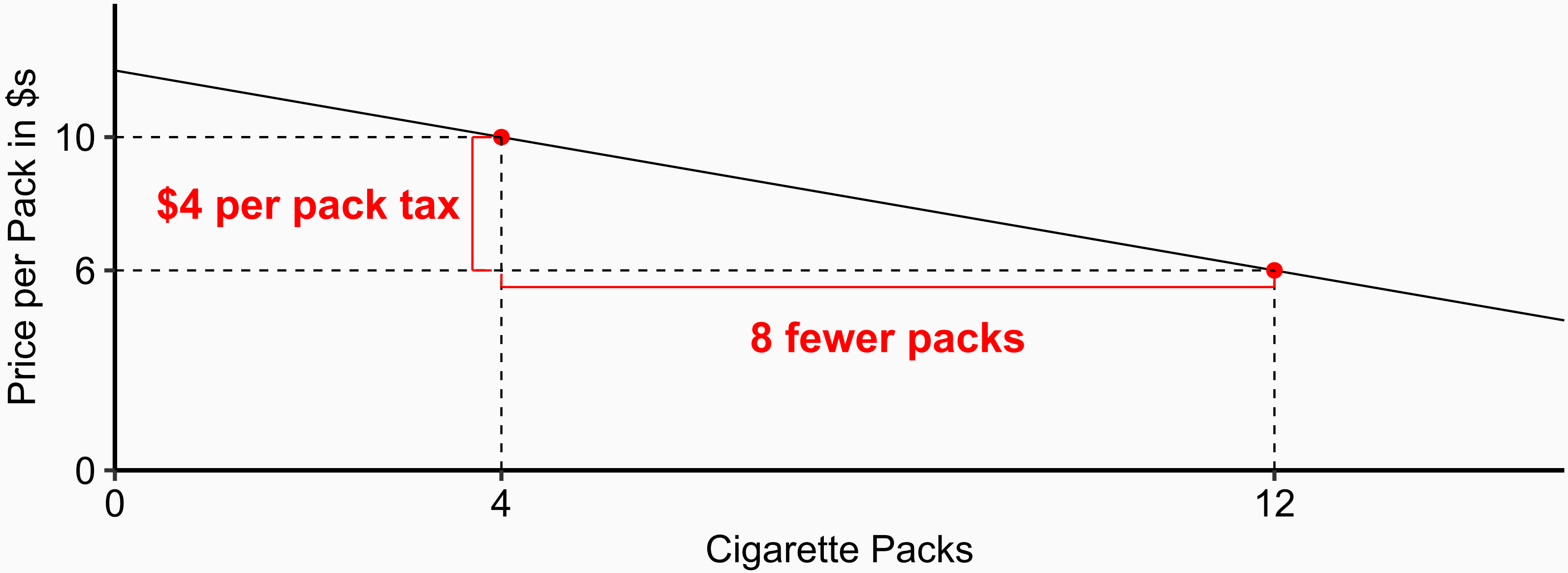
Slope

- How does the y-variable change as I change the x variable?
- Sounds boring, but one of the most used characteristics in this class
    - The slope often represents some policy relevant change
    - How will cigarette consumption change if we tax them by $4 per pack?
    - Let's look at data of how cigarette consumption changes with price...

Cigarette consumption vs. Price

- Say price is now $6 and 12 packs are consumed
- If we tax cigarettes by $4 per pack, then the price per pack raises to $10
- The line shows only 4 packs are consumed under this higher price

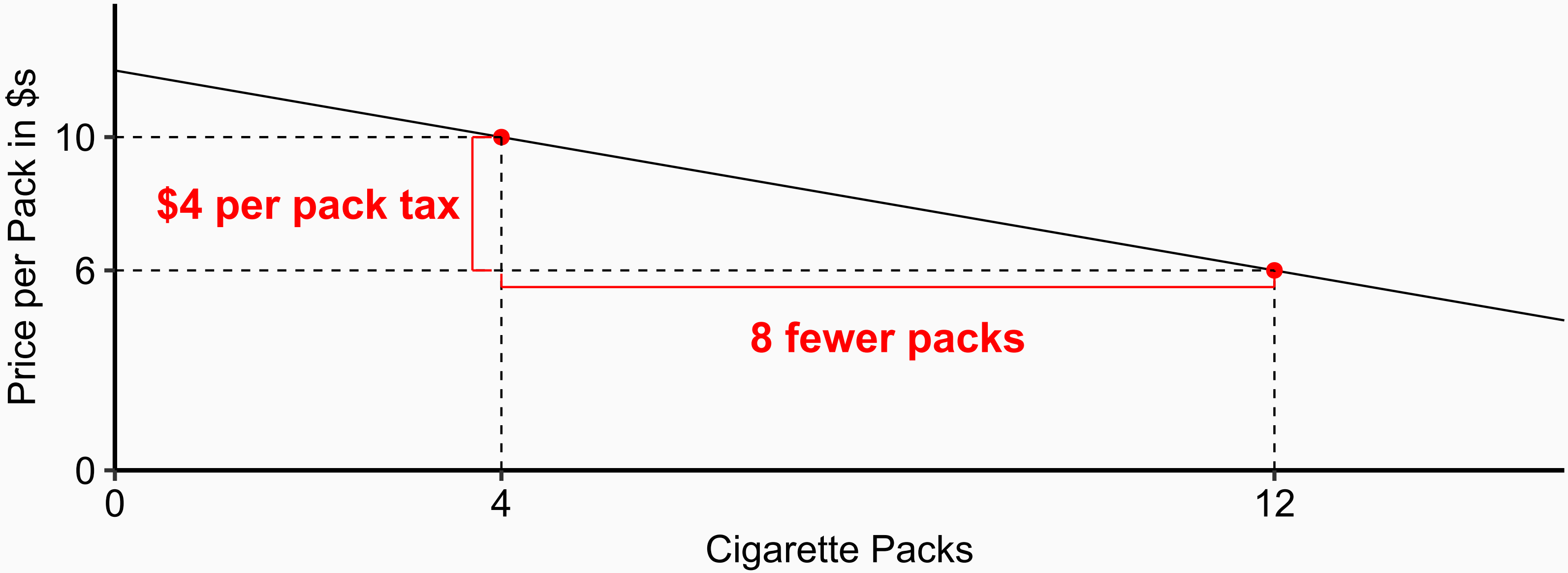




- Slope - change in y variable divided by change in x variable

  in Math:

$$m = \frac{\Delta Y}{\Delta X} = \frac{Y_2 - Y_1}{X_2 - X_1},$$



Slope - change in y variable divided by change in x variable

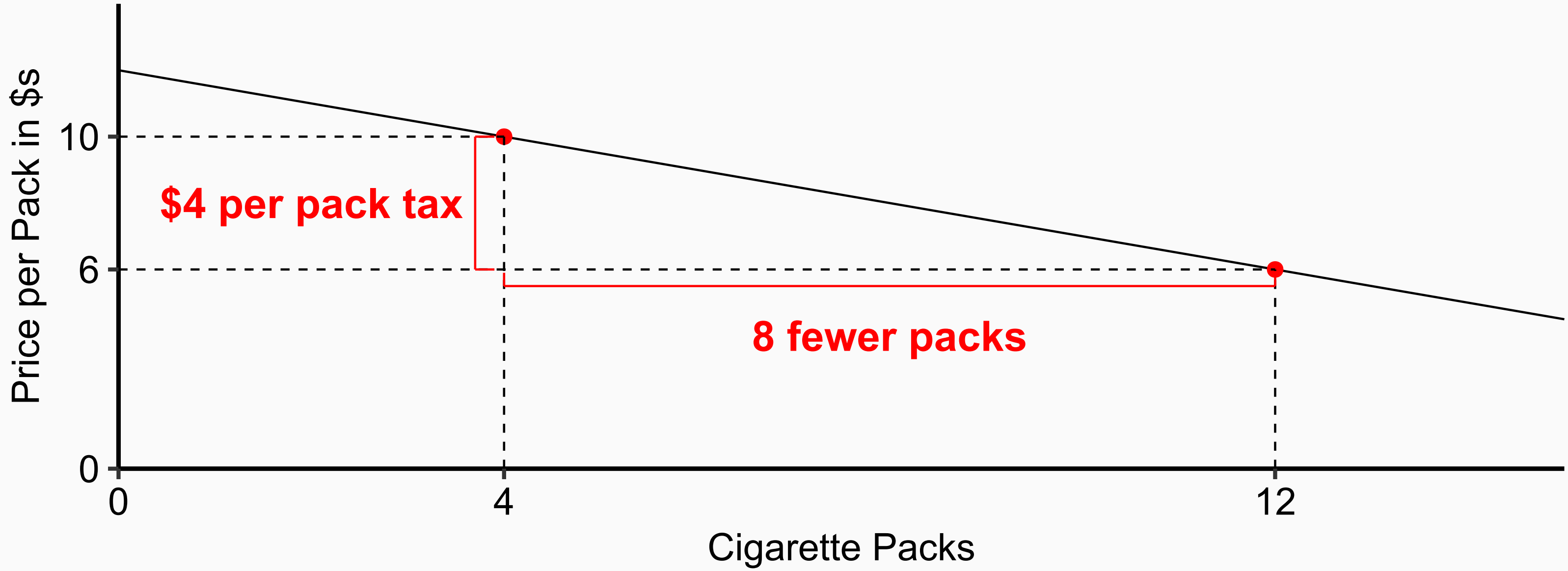
in Math:

$$m = \frac{\Delta Y}{\Delta X} = \frac{Y_2 - Y_1}{X_2 - X_1},$$

for this graph:

$$Slope = \frac{10 - 6}{4 - 12} = \frac{-4}{8} = -.5$$

Careful: Need to be consistent about order of x's and y's



Slope - change in y variable divided by change in x variable

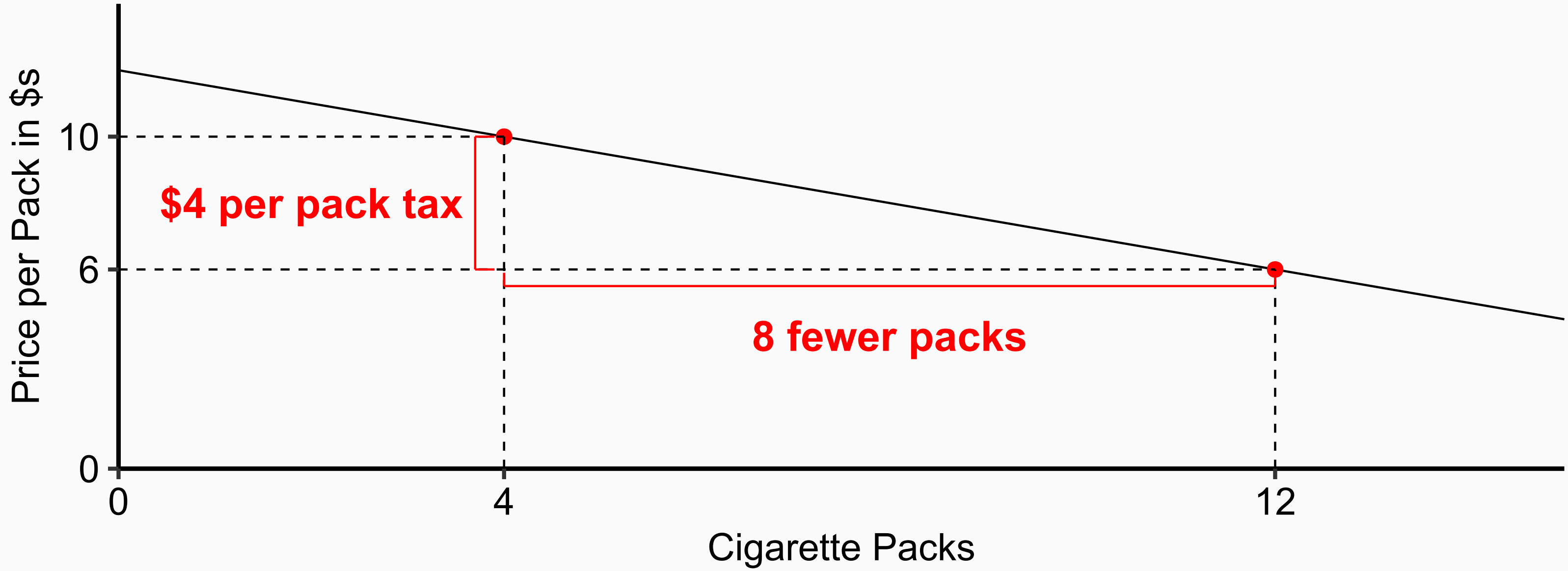
in Math:

$$m = \frac{\Delta Y}{\Delta X} = \frac{Y_2 - Y_1}{X_2 - X_1},$$

for this graph:

$$Slope = \frac{10-6}{4-12} = \frac{-4}{8} = -.5$$

Note: often we denote slope by the letter **m**



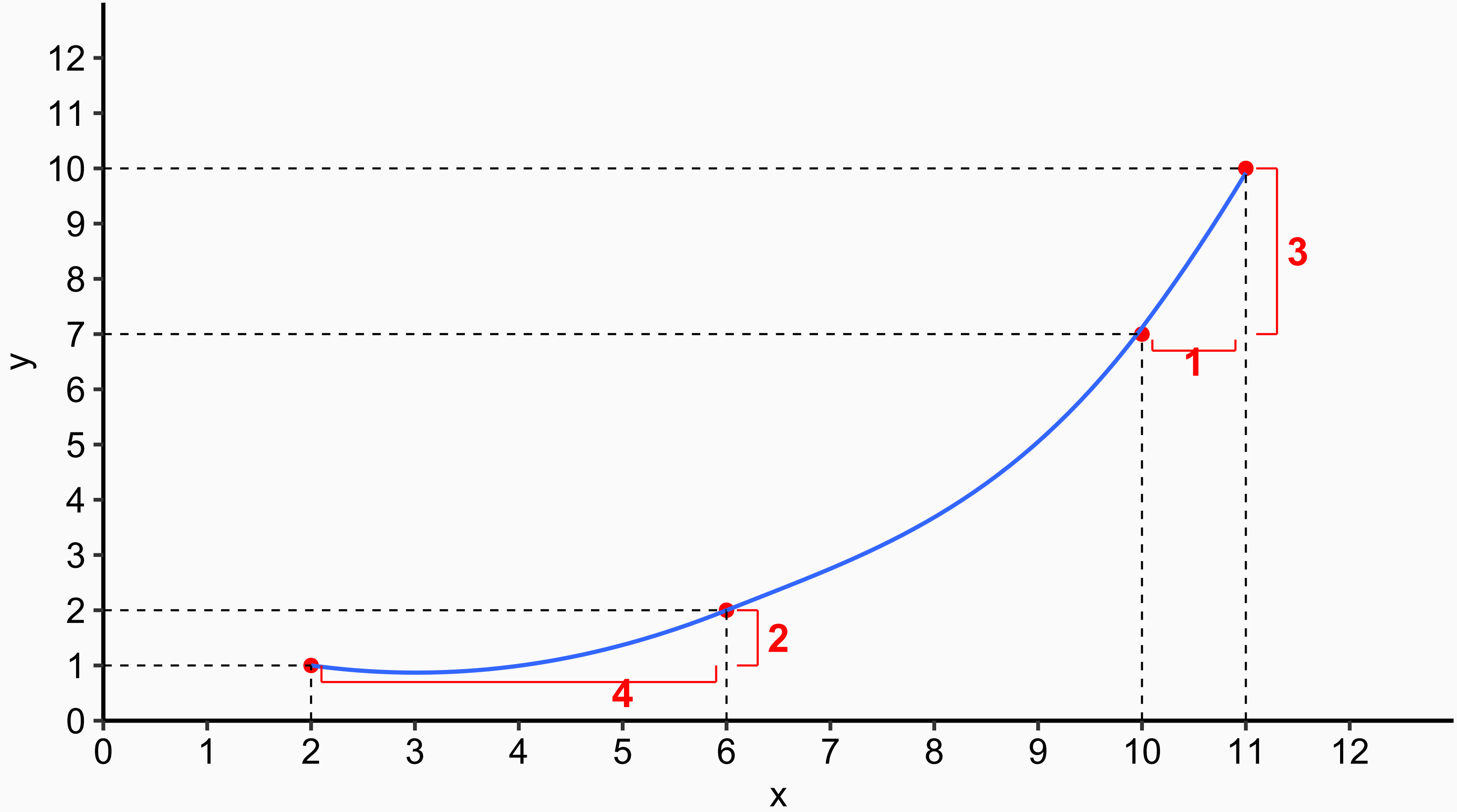
for this graph:

$$Slope = \frac{10-6}{4-12} = \frac{-4}{8} = -.5$$

Interpretation: Every $1 increase in price, demand for cigs falls by $\frac{1}{2}$

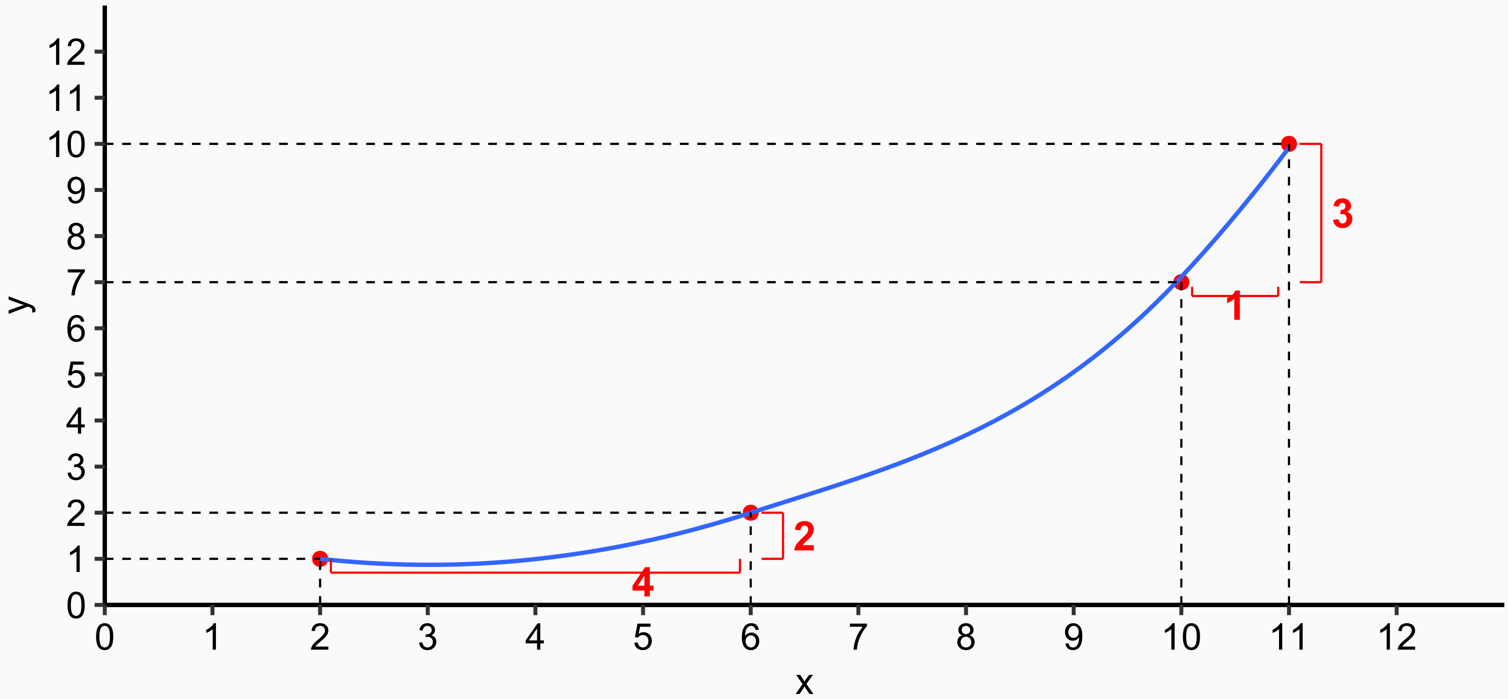
Note: Along a straight line, the slope is the same at every point!

Along a non-straight curve, the slope may vary from point to point

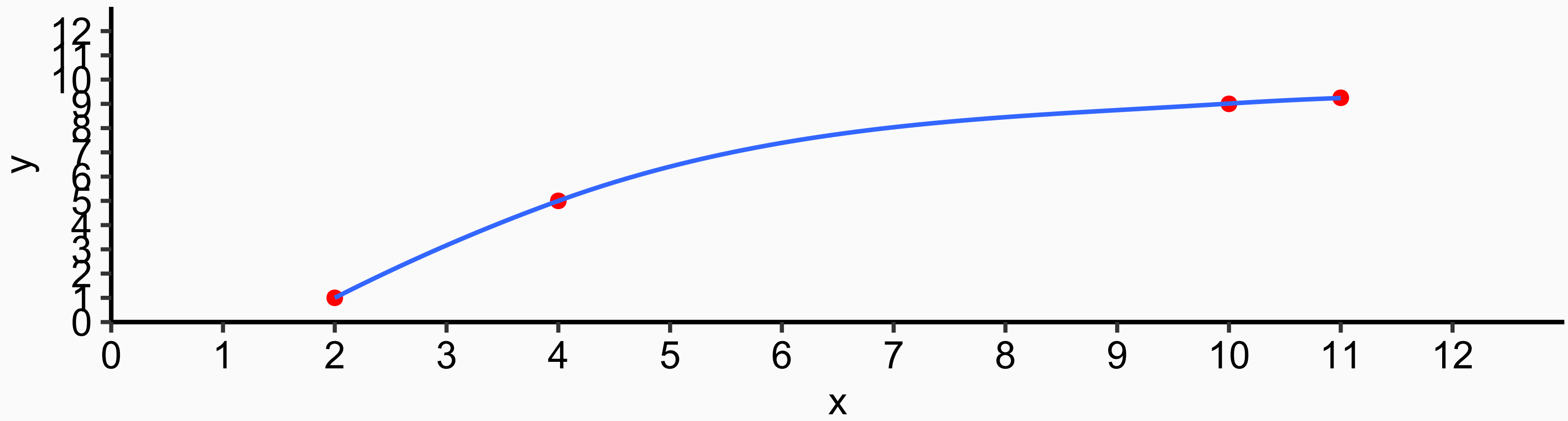


The slope...

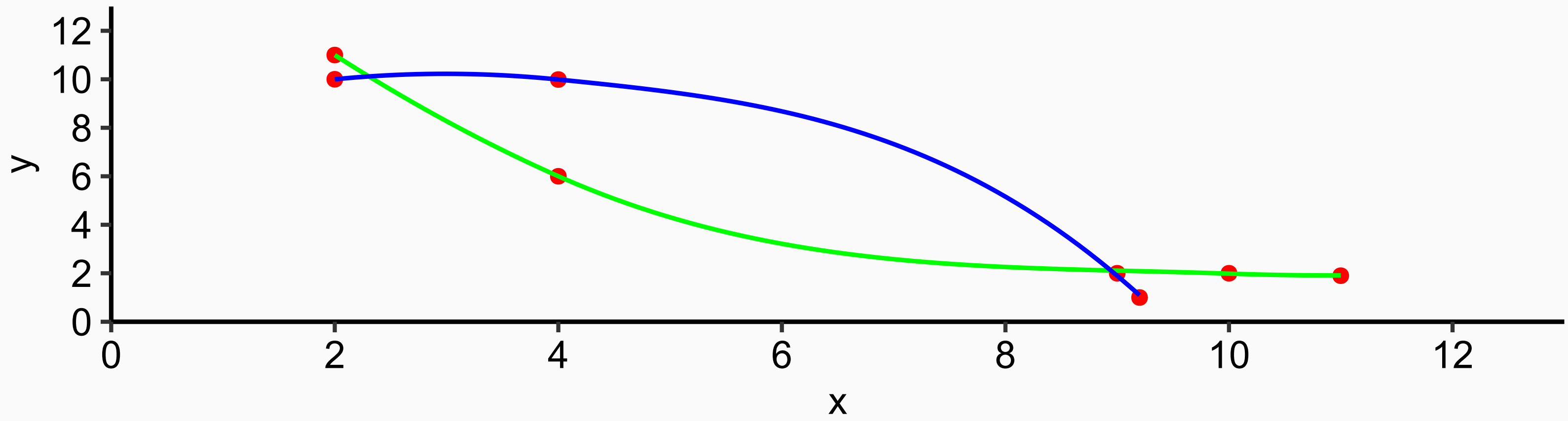
1. Not the same at every point
2. Gets larger *(in this case)*
3. Always positive *(in this case)*



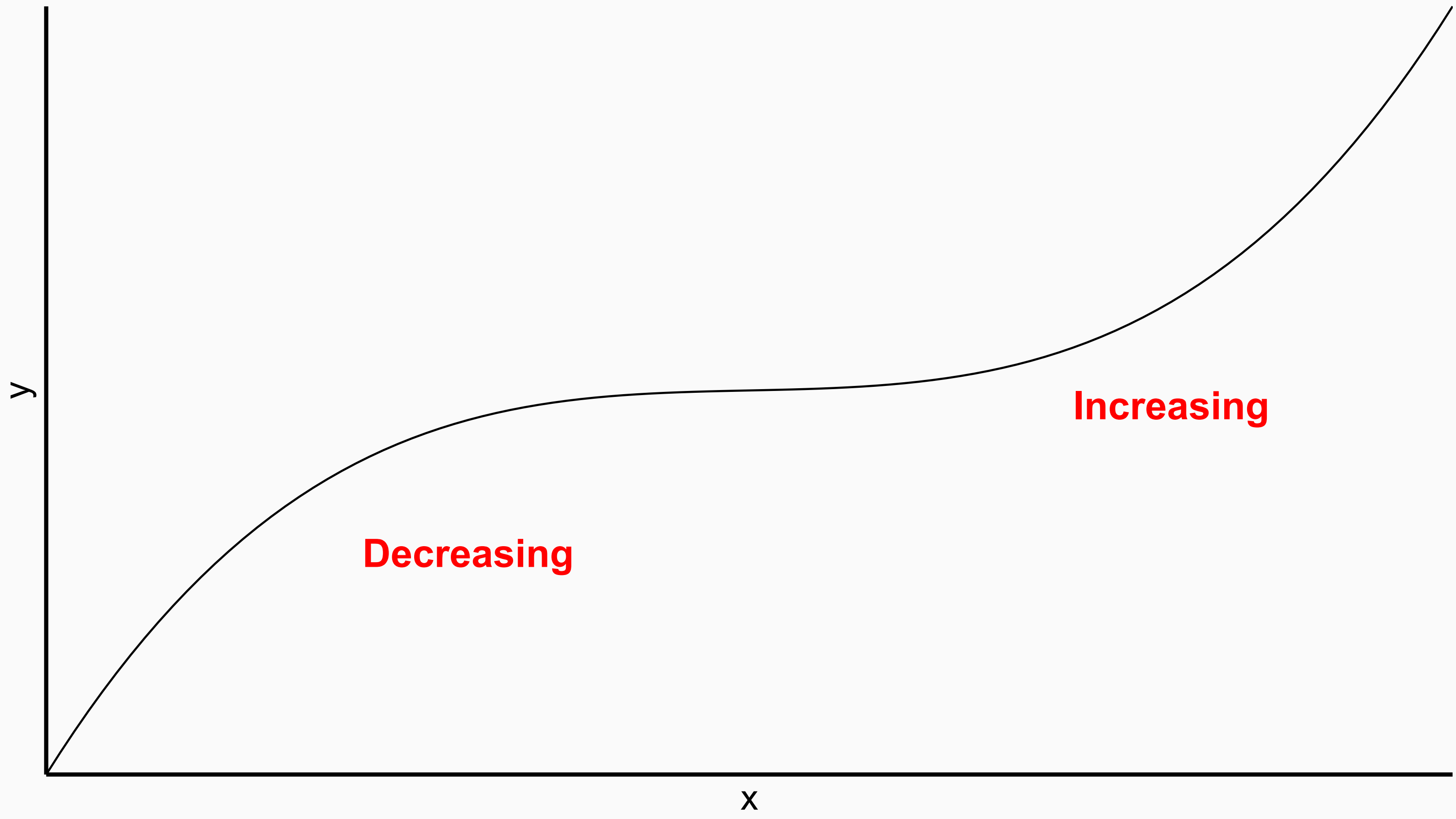
Can draw graphs with positive slope where slope gets smaller



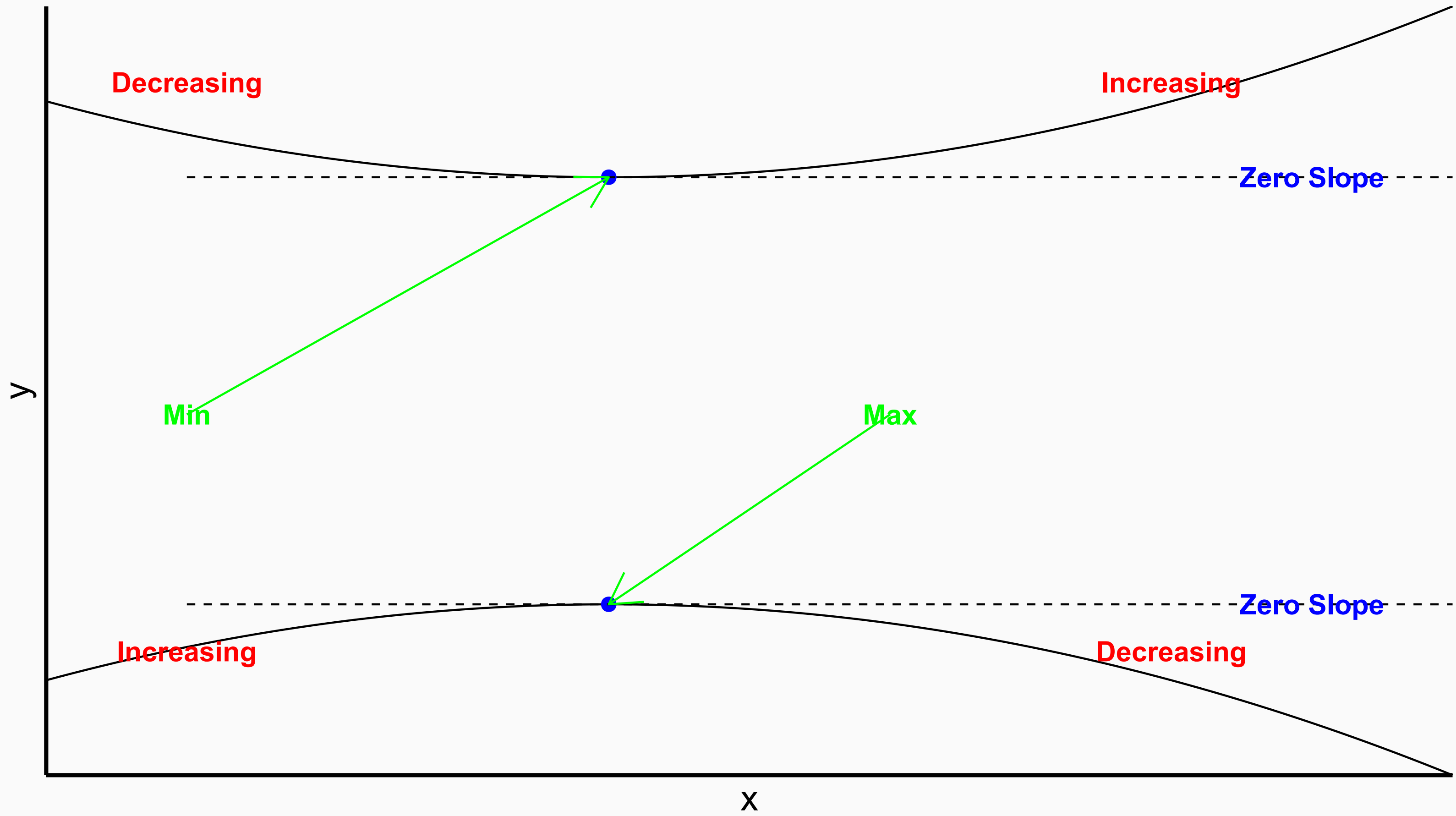
Can draw graphs with negative slope where slope gets bigger/smaller



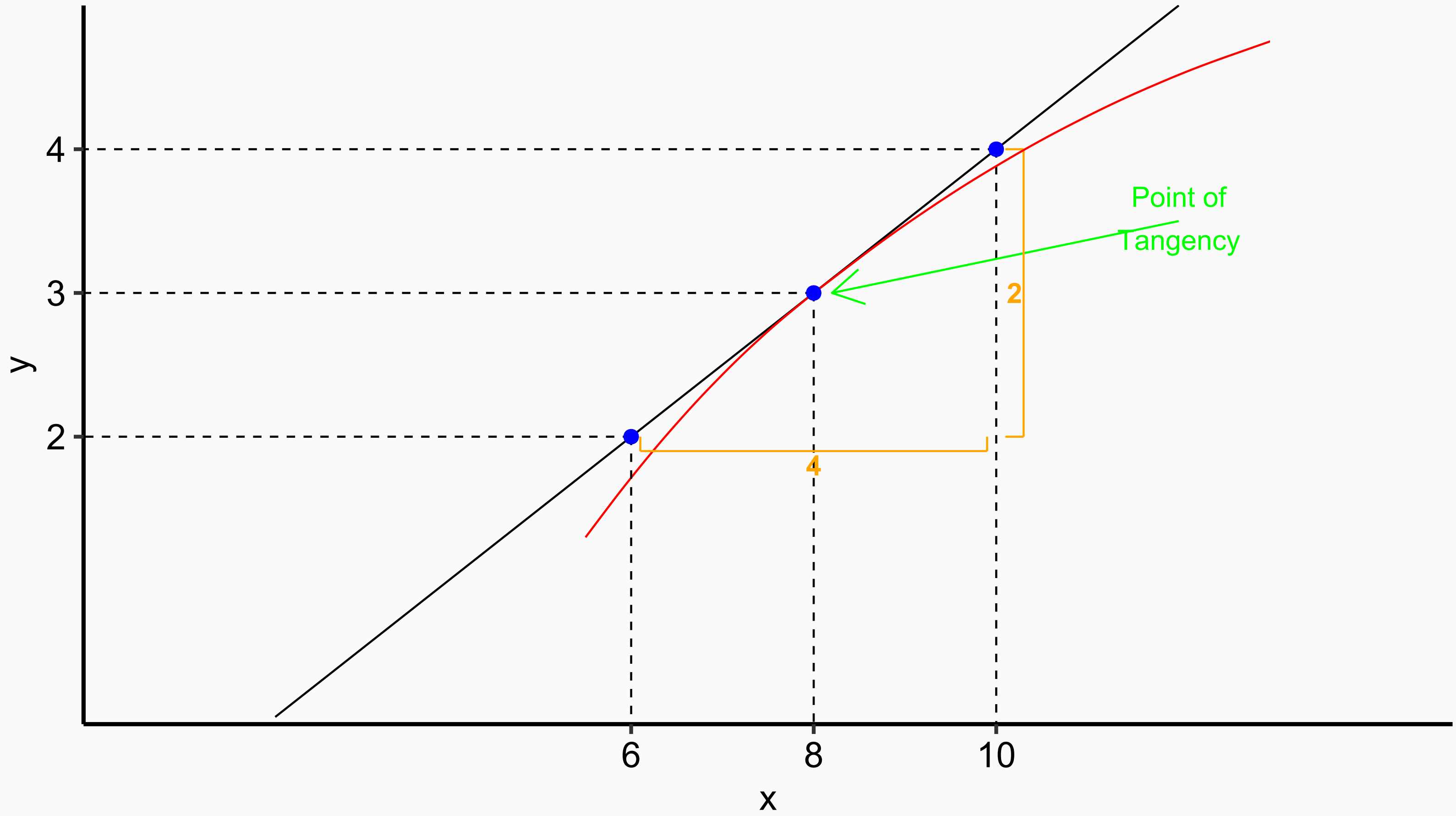
Can draw graphs where slope is both increasing and decreasing



Can draw graphs where slope is both positive and negative

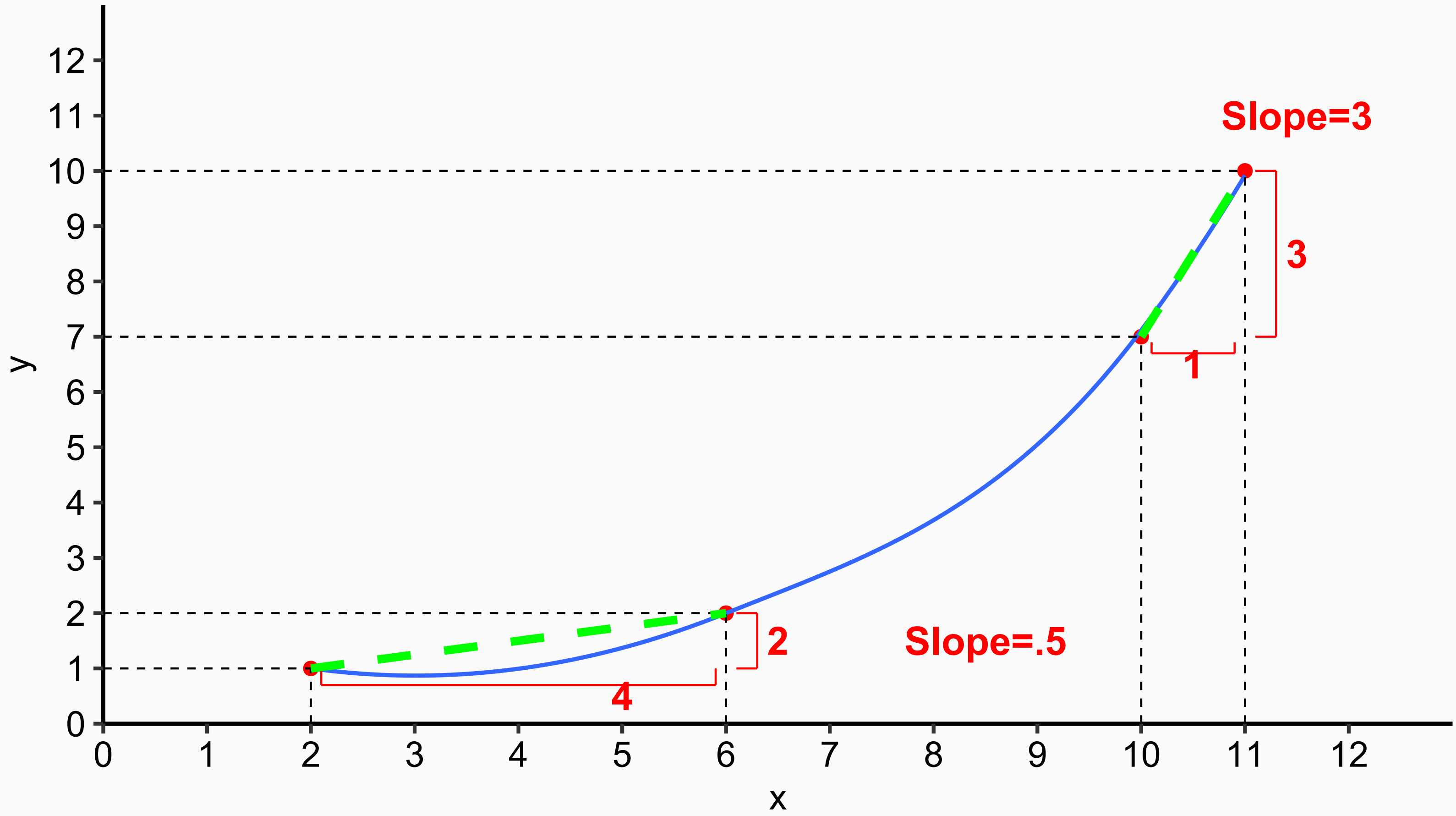


Calculating Slope of a Curve at a Point

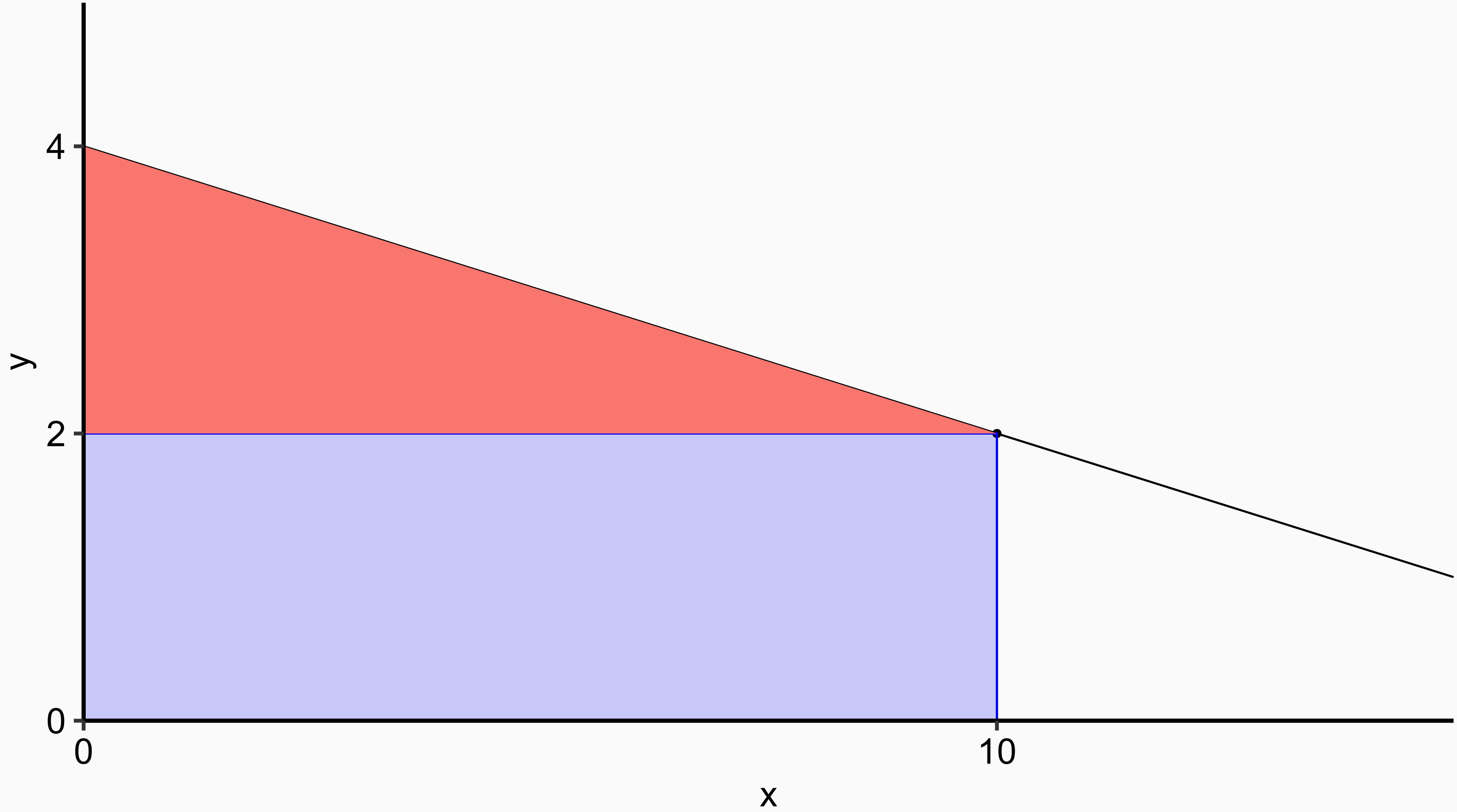


Slope is $\frac{2}{4} = \frac{1}{2}$

Arc Method of Slope Calculation - calculate the slope of line connecting two points on curve

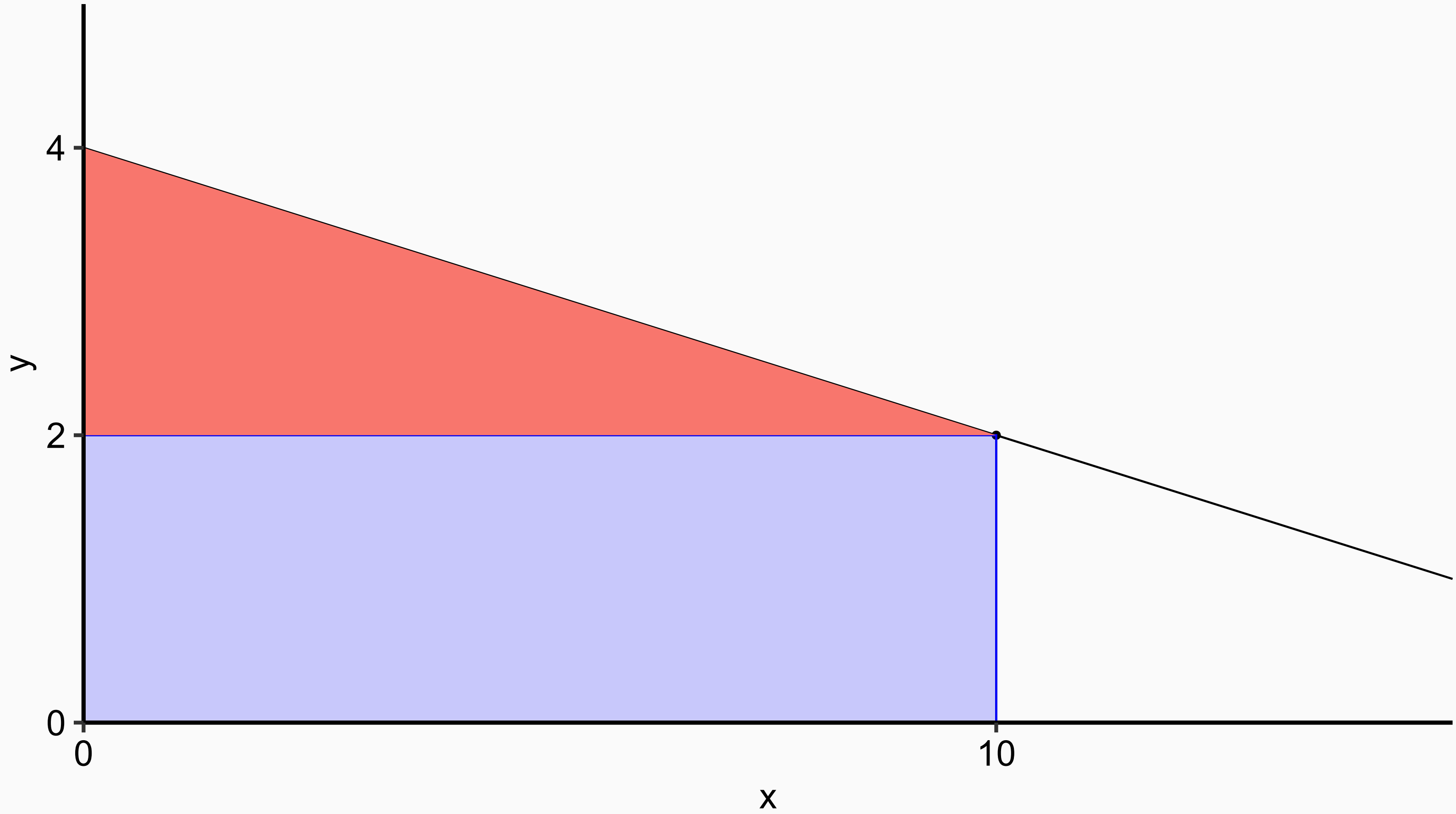


Area Under a Line



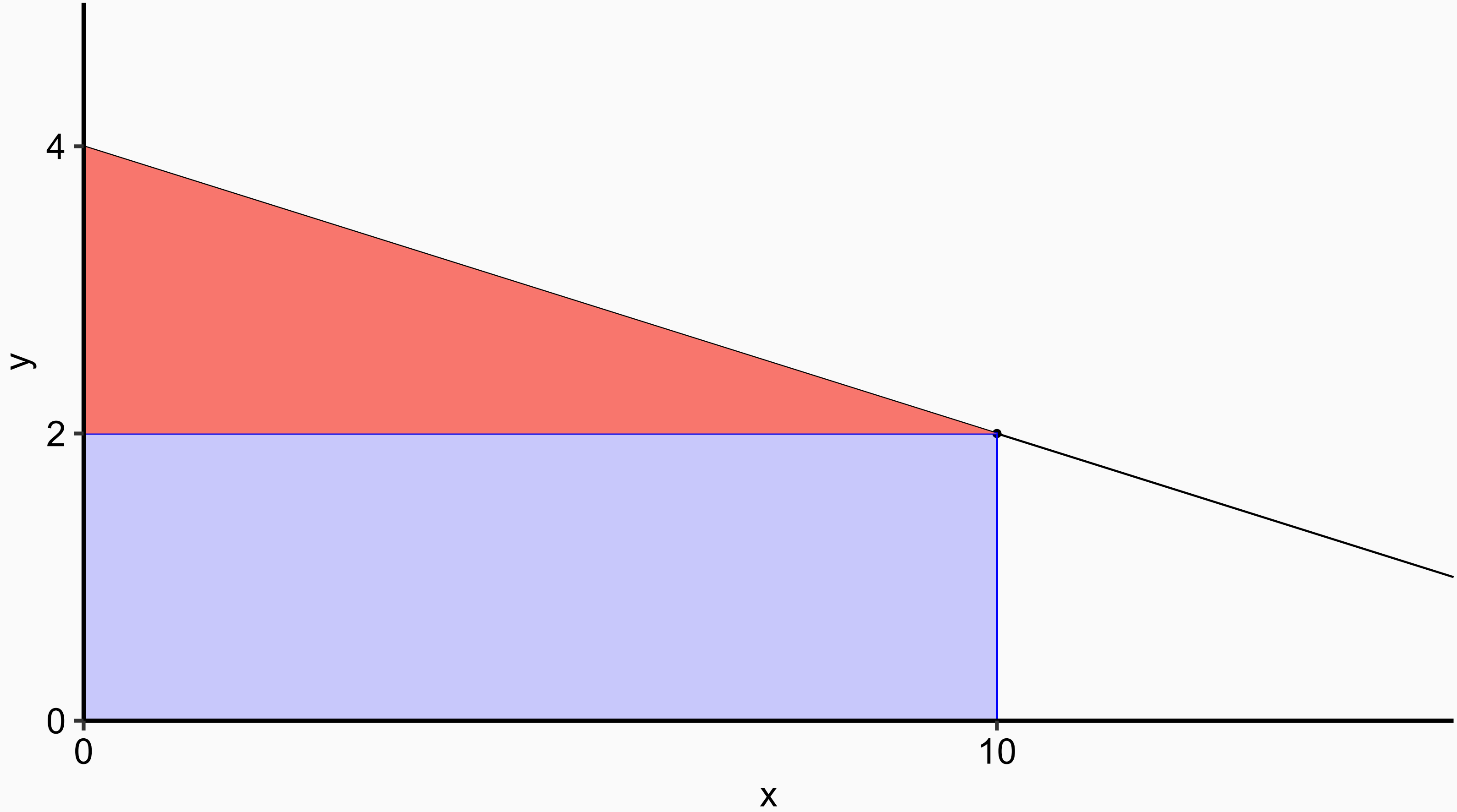
Area of the curve is ▲ + □

Area Under a Line



Area of ▲ = $\frac{1}{2} \times base \times height = .5 \times 10 \times (4-2) = 10$

Area Under a Line

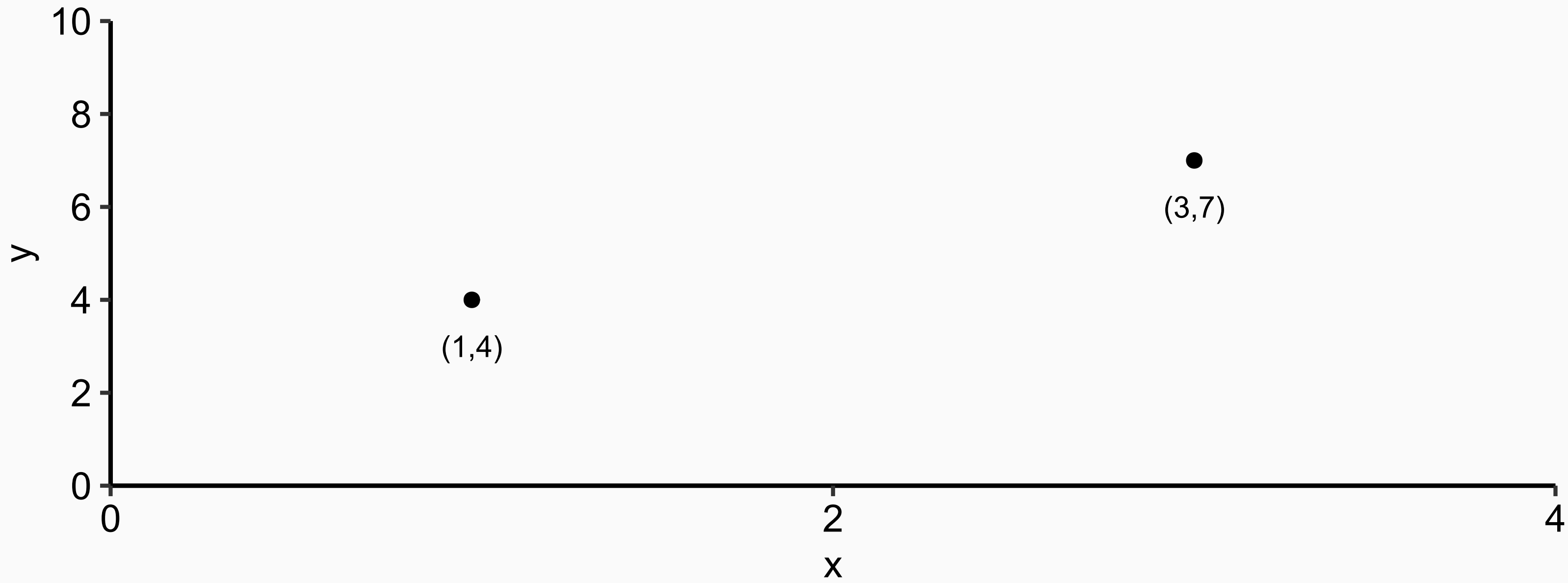


Area of $\square$: $base \times height = 10 \times 2 = 20$

$\blacktriangle$ + $\square$ = 20 + 10 = 30

Cartesian Coordinate System

Express location of points on a graph in parentheses: (x-value, y-value)



If we have two points, we may want to determine the line that connects them

HOW?

Equations, Expressions & Operators

- **Variable:** something that can take more than one value: $x = 1, 3, 5, 6$
- **Operator:** performs a mathematical operation
  - Recall order of operations: **P-E-M-D-A-S**
  - (), x^2, $x \times y$, $\frac{\pi}{2}$, 🥕 $+$ 🎂, $x - 3$
- **Expression:** any combination of mathematical symbols, variables, operators
  - Ex.: $3 + 4$, x^3, 16, $y + 2$
- **Equation:** Relates a pair of expression using an equals sign (=) to state equality
  - $2 + 2 = 4$, $x - 3 = y$
  - Can express non-equality: $2 + 3 \neq 4$
  - Can express inequality: $2 + 3 > 4$, $1 + 6 \geq x$

Representing lines

Slope-Intercept Form

$$\boldsymbol{y = 3x + 2}$$

y-var. = slope * x + y-intercept (AKA `(y=mx+b)`)

- m = 3 is the slope
- b = 2 is the y-intercept

Point-Slope Form

$$\boldsymbol{y - y_1 = m \times (x - x_1)}$$

$$\boldsymbol{\Rightarrow y = m(x - x_1) + y_1}$$

$$\boldsymbol{= mx - mx_1 + y_1}$$

$$\boldsymbol{\Rightarrow \textbf{y-intercept} = y_1 - m \times x_1}$$

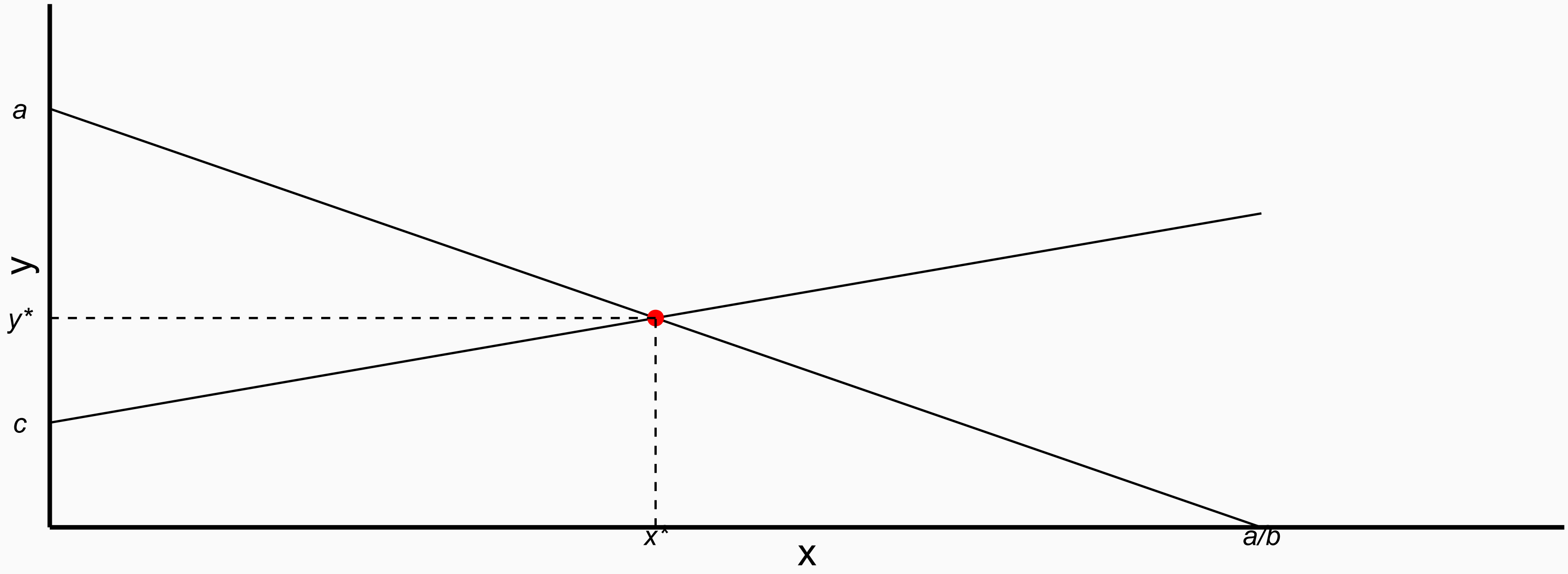
A very common problem...

Two equations:

$$y = a - bx$$

$$y = c + dx$$

where a,b,c,d are parameters (any constant number)



Two easy ways to solve for (x^*, y^*)

1. Subtract equations

$$\begin{aligned} & y = a - bx \\ - \quad & y = c + dx \end{aligned}$$

$$\Rightarrow 0 = a - c - bx - dx$$

$$\Rightarrow (b + d)x = a - c$$

$$x^* = \frac{a - c}{b + d}$$

If we plug back into one of the original equations to find y^*:

$$y* = a - b\left(\frac{a - c}{b + d}\right)$$

Two easy ways to solve for (x^*, y^*)

1. Substitute one equation into the other

$$y = y$$

$$a - bx = c + dx$$

$$x^* = \frac{a-c}{b+d}$$

$$y* = a - b\left(\frac{a-c}{b+d}\right)$$

Example problem:

$$y = 27 - 3x$$

$$y = 9x - 9$$

$$\Rightarrow 27 - 3x = 9x - 9$$

$$\Rightarrow 36 = 12x$$

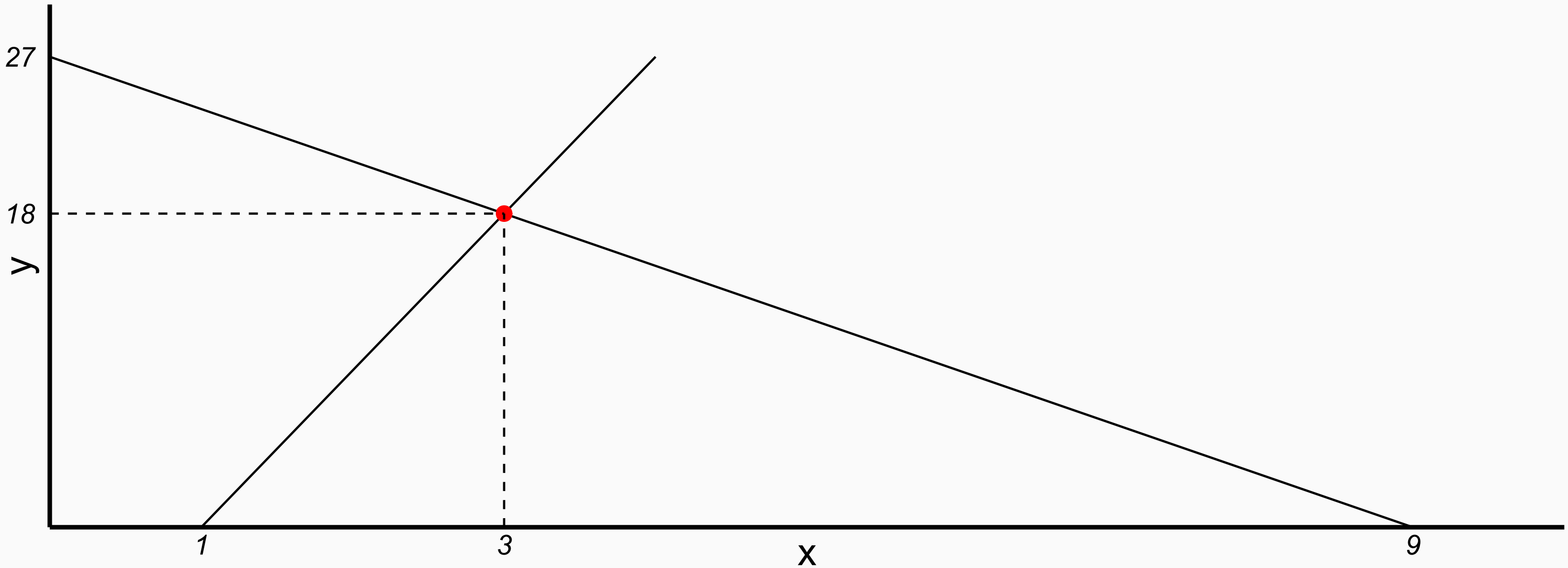
$$x^* = 3$$

$$y^* = 9x^* - 9 = 9 * 3 - 9 = 27 - 9 = 18$$

$$y = 27 - 3x$$

$$y = 9x - 9$$

solution: $(x^*, y^*) = (3, 18)$



Charts from Data

Example: City of Buda - High School Exit Test Scores vs. Student to Teach ratio

Year	Stu/Teach	Exit Test Score
2000	20	80
2002	26	65
2004	28	60
2006	32	50
2008	30	55
2010	22	75
2012	24	79

Sort it by independent variable: student-teacher ratio

Year	Stu/Teach	Exit Test Score
2000	20	80
2002	26	65
2004	28	60
2006	32	50
2008	30	55
2010	22	75
2012	24	79

Stu/Teach	Exit Test Score
20	80
22	75
24	70
26	65
28	60
30	55
32	50

Stu/Teach	Exit Test Score
20	80
22	75
24	70
26	65
28	60
30	55
32	50

- What is slope?
  - Note that for every increase of 2 students to 1 teacher, test scores go down by 5
  - Slope = -5/2 = -2.5

Stu/Teach	Exit Test Score
20	80
22	75
24	70
26	65
28	60
30	55
32	50

- What is y-intercept?: each step in Stu/Teach ratio is 2: 10 steps -> scores increase by 5 for each step: -> 50 points
  - 10*5 + 80 = 130
- equation for line: TestScore = 130 - 2.5 * StudentTeacherRatio

Practice Problem

Austin Integrated School District Enrollment in 2018: 82,520 Enrollment in 2019: 80,495 Expenditures in 2018: $1.6 billion Expenditures in 2019: $2.082 billion

1. What is average spending per student over 2018-19?
2. Calculate percentage change in expenditures 2018-9?
3. What is the slope of the line that connects 2018-9, where enrollment is x and expenditures is y?

Inflation vs. PPP: **Big Mac Index**

Visualization is recreated by using Tableau Desktop which inspired by http://www.economist.com/content/big-mac-index where it only in US dollar as base currency and the index in its original, raw form

The Big Mac Index is an informal way for measure the purchasing power parity (PPP) between two currencies.

currency over time. Allowing you to hover over / tap the line chart and see detailed indicators for your selection over time. To unfreeze the map, click on the highlighted country again.

Source: McDonald's; Thomson Reuters; IMF; *The Economist*

The panel at the bottom displays a scatter chart plotting the local price of a Big Mac (expressed in the current base currency) against GDP per person in that country. Hover over individual points for details.