Math Review

PA 393K/393L

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Fall 2020

This Week's Class

A. Logistics

1. Schedule
2. Motivation for this Course

B. Math Review

1. Graphing
2. Equations
3. Practice Problems

Some useful stuff we'll learn about today

1. Slope
2. Equations for Lines
3. Tangency
4. Solving SOEs

Logistics

name: logistics

Schedule

Today

- Welcome, check in, admin, and survey
- Research basics: *Why are we here? Baumol*: "Hard Heads, Soft Hearts"
- Our class: *What are we doing?*, *Why are we doing it?*
- Today's class: *How are we doing it?* - Basic analytical mathematics

Upcoming

- Learn application to microeconomic issues
- Learn the basic theoretical models.
- Build momentum.

Long run

Motivation

Why are we here?

- **Policy:** Understand an economic basis human, social, and/or economic behaviors.

- **Master's Program:** Learn methods, tools, skills, and intution required for applying economic concepts to poly.

- **Microeconomic theory:** Build a toolbox of *theoretical & empirical* methods, tools, and skills to that combine data and statistical insights to test and/or measure theories and policies.

- **You:** You should be thinking about this question throughout your program/work/life. **Self awareness and mental health are important.**

Motivation

This class

For those without bachelor's in quantitative topics, **this course marks a big shift** in how school works.

- Readings, problem solving, problem sets, exams.
- Like sudoku, the mathematics in this class is easy if you have been practicing, but seemingly impossible otherwise.
- Online/remote learning makes this even more difficult.

While the academic contents may be forgotten by January 1st, the intution and concepts ought to be pivotal for **a lot** of what you will do in your future careers.

Motivation

Take responsibility for your education and career.

- Get your calendar out today and plan the time each week when you expect to be working for this course.
  - 9 course hours = 3 hours lecture + 6 hours self-study per week
- Be proactive and curious.
- Ask questions.
- *Respectfully* challenge.
- Apply.

Math Review

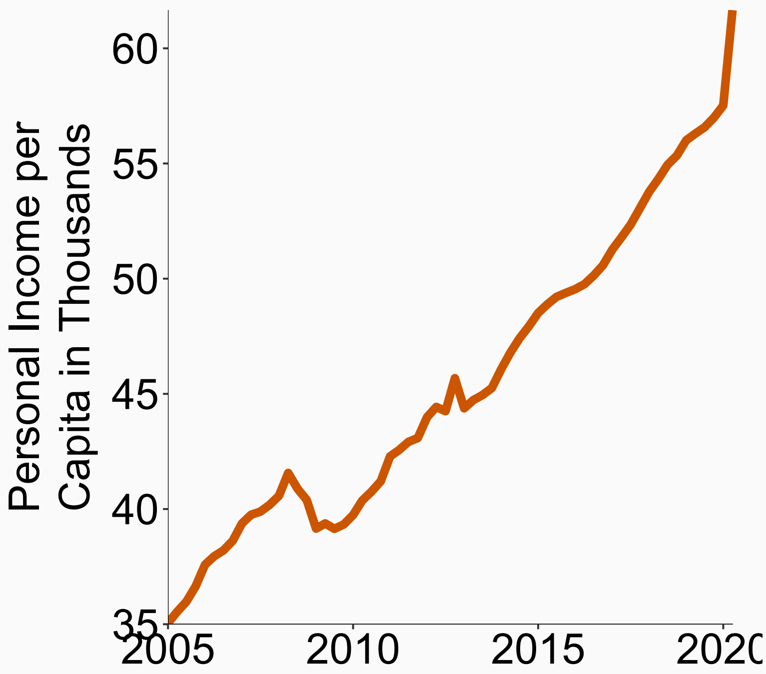
Graphing - Basic Definitions

- **Variable:** quantity that can take on more than one value (hence "variable")
    - Example: **income** for one person may be $100K, another $25K
- **Units:** what is being measured. **Very important** to help identify what we are seeing.
    - Example: **annual income per capita in $1,000's** (*Source:* FRED)

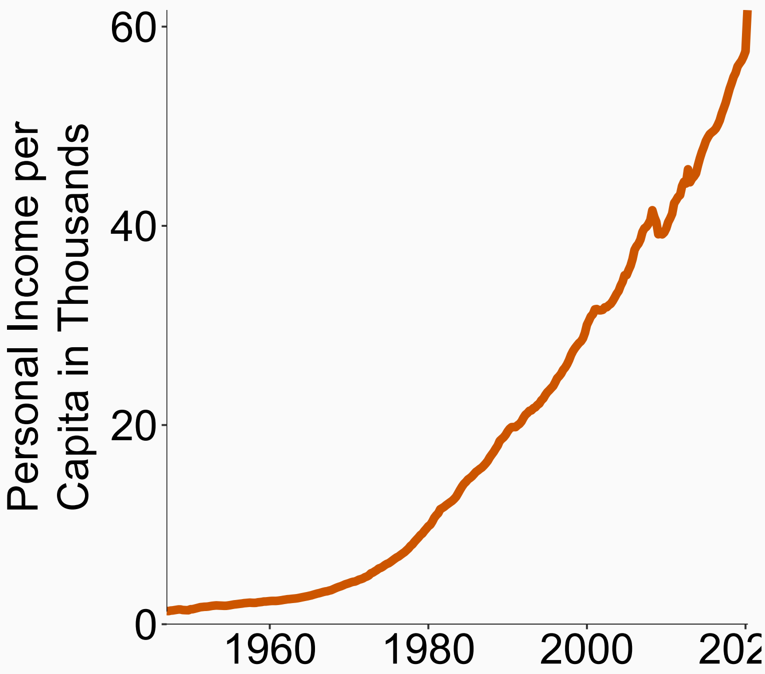
Things that matter

- Labels: tell us what is what
- Units: tell us how things are measured
- Scale: Affects what we see and therefore what we infer from th epicture

Scale



Personal Income per Capita in Thousands
60
55
50
45
40
35
2005
2010
2015



Personal Income per Capita in Thousands
60
40
20
0
1960
1980
2000

Graphing - Basic Definitions

Variables:

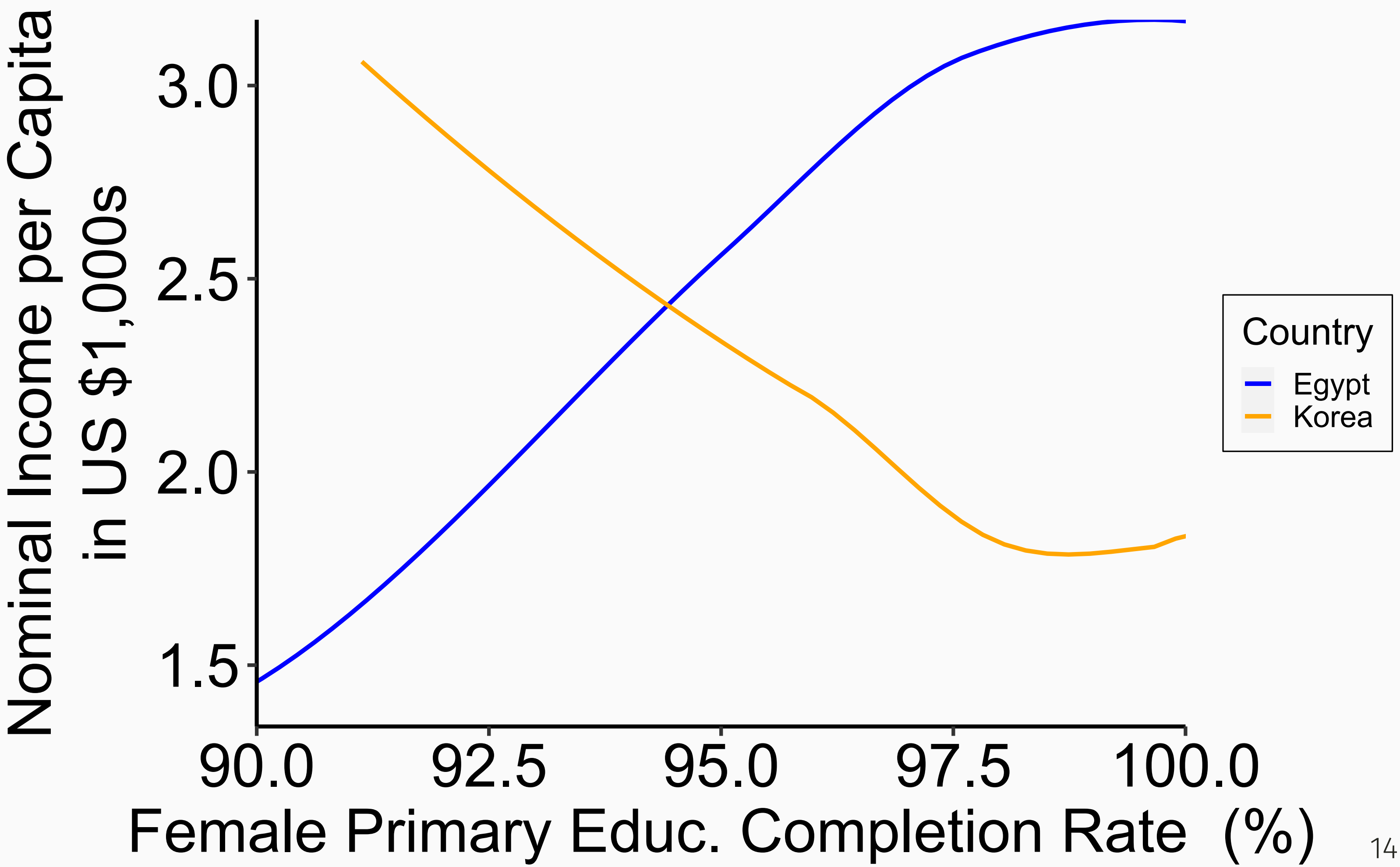
- We just showed you income. What if we are interested in the relationship between income and years of schooling?

Two Variables

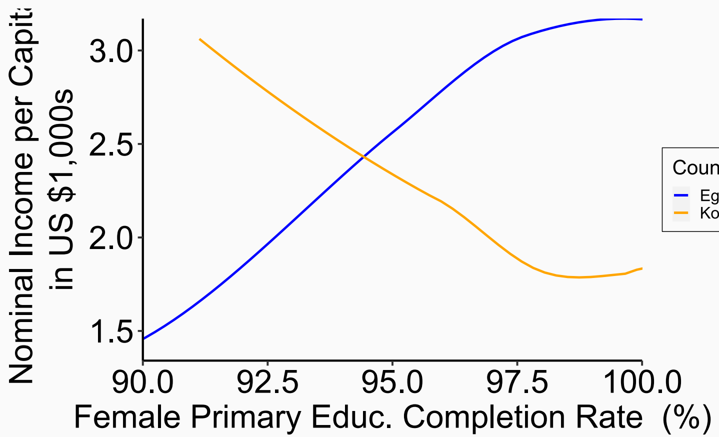
Common convention (i.e., no specific meaning to it) in math:

- Call one variable the x-variable
  - Put this one on the horizontal axis
- Call the other the y-variable -This one on the vertical axis

Example: Two Y-variables, One X-



An aside...



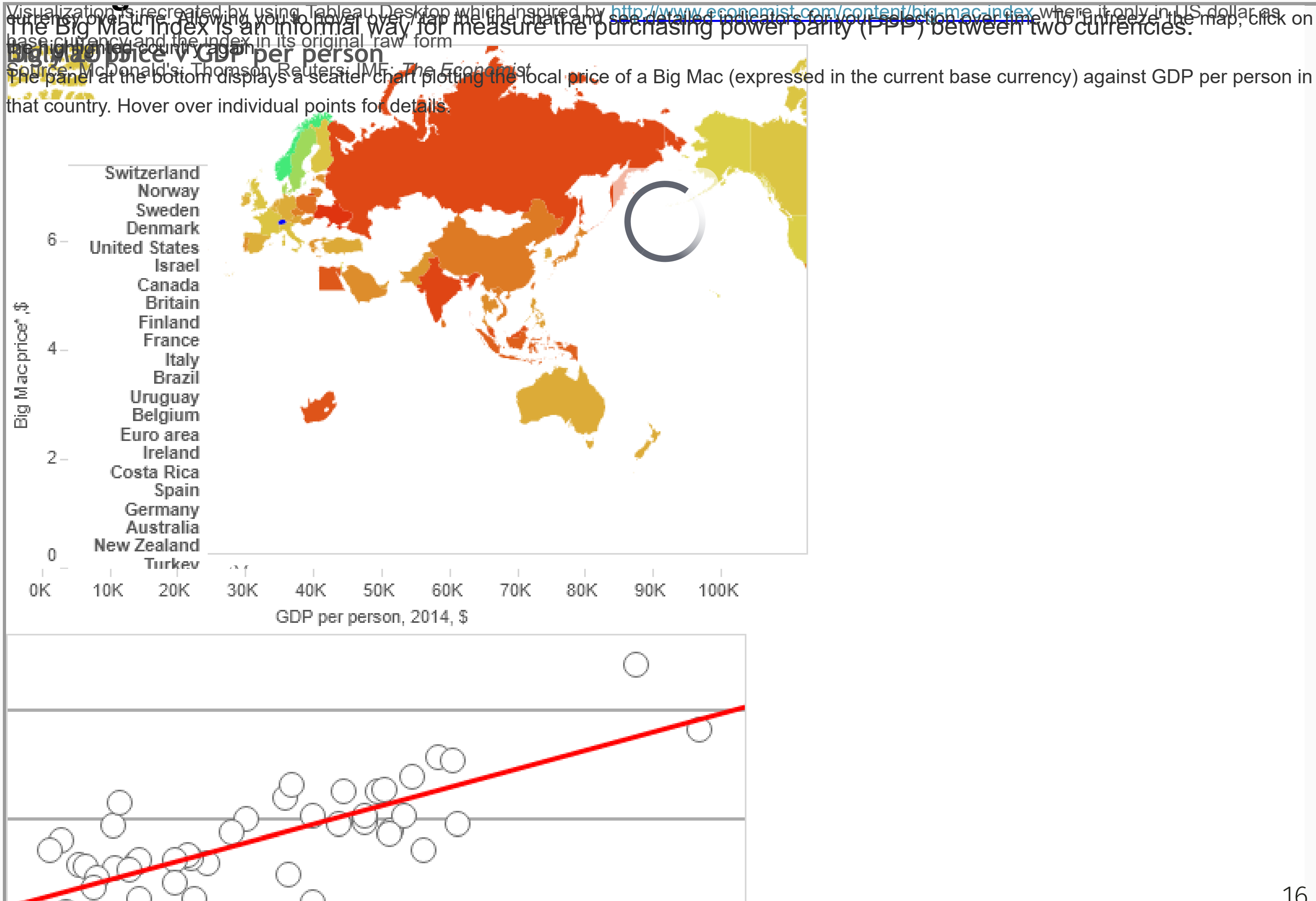
- inflation - rate of growth of prices in economy
  - higher inflation rate -> faster cost of living rises

- We may be interested in adjusting variables based on this relative cost of living: called defating the variable

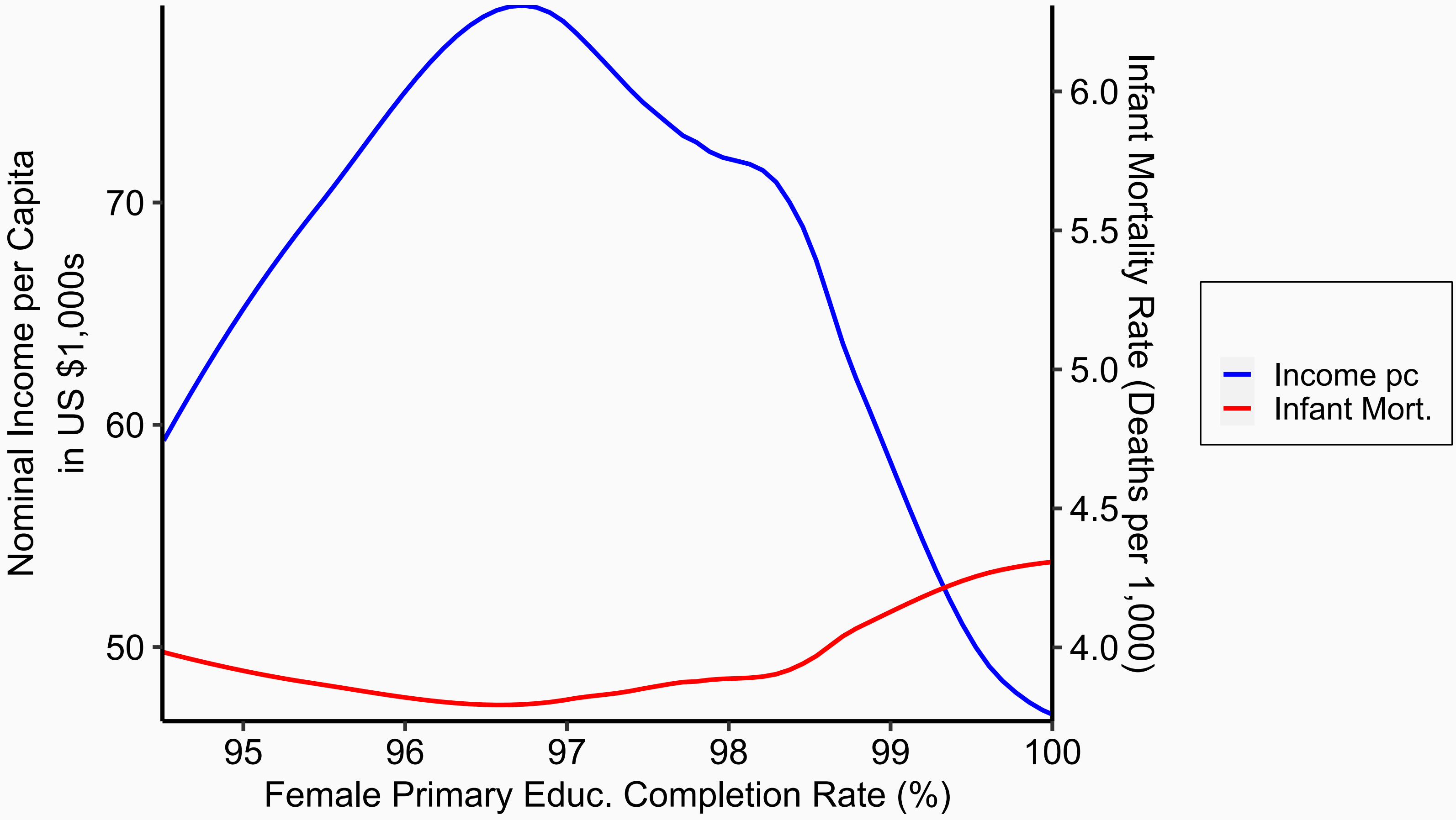
- real $'s (income, prices, etc.) - deflated to account for inflation

- nominal $'s (income, prices, etc.) - undeflated -> like the actual value at the time

Inflation vs. PPP: **Big Mac Index** 🍔 💵



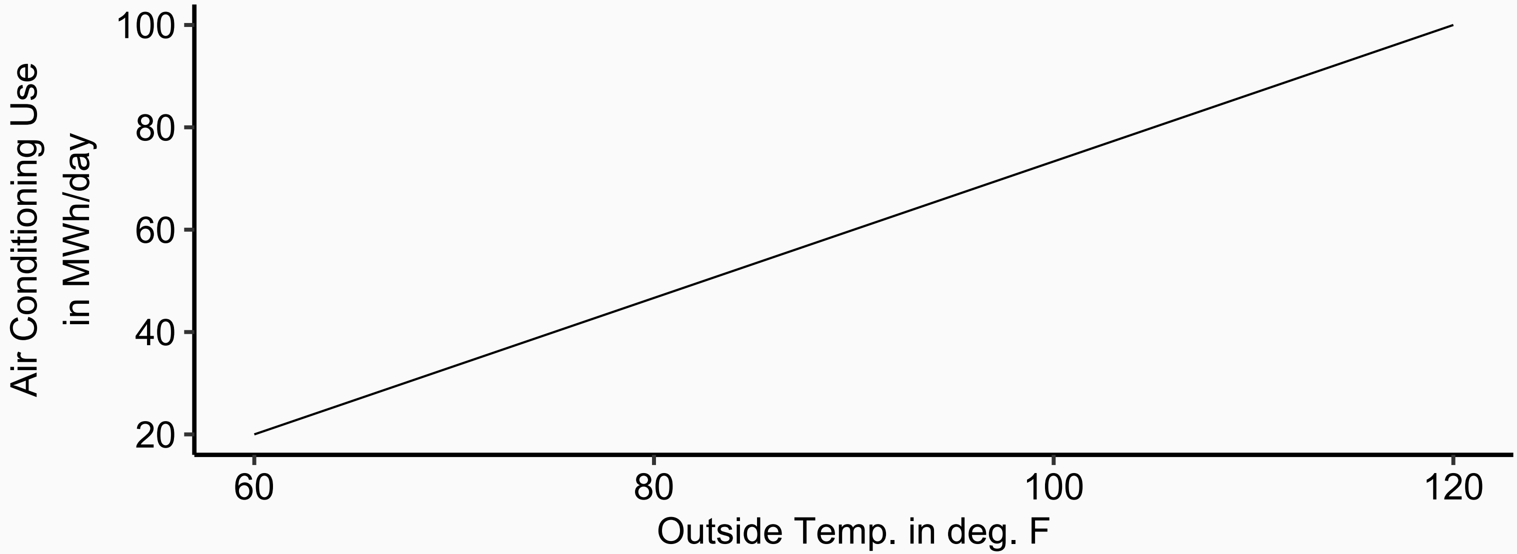
Two Axis Graph



Types of Curves

curve - any line connecting points on a graph

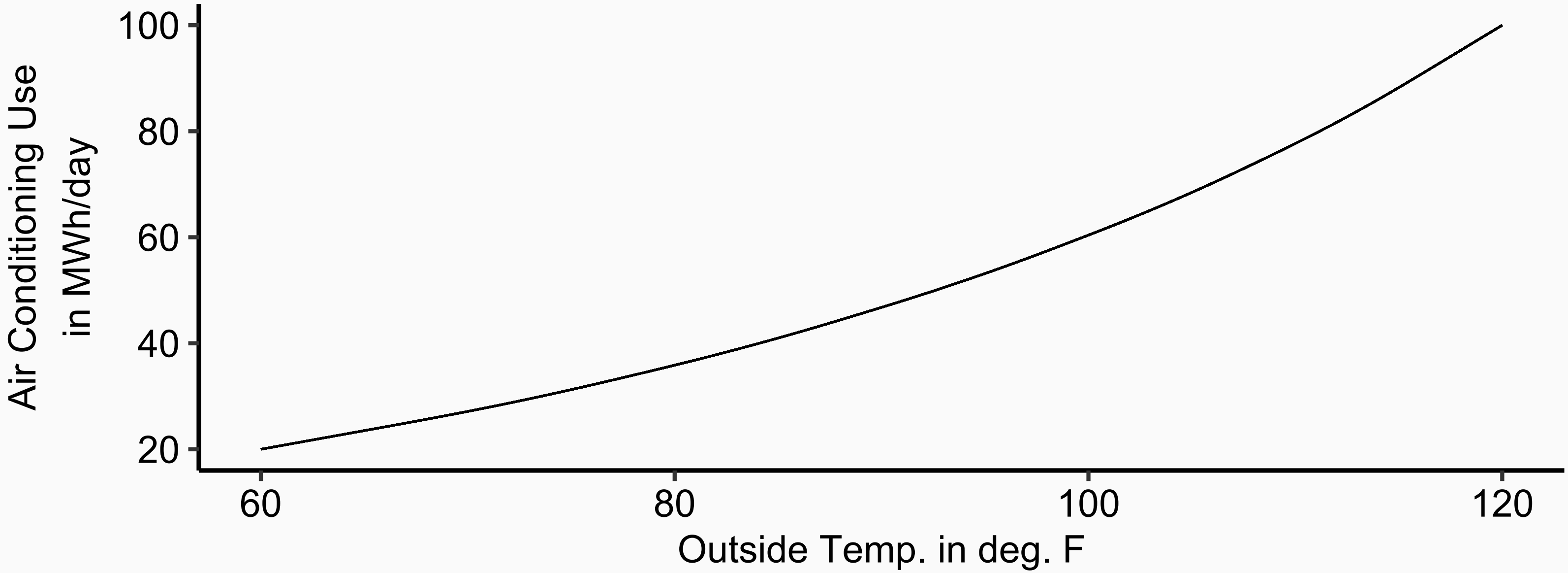
- (doesn't have to be straight, but maybe)
- straight: linear relationship



Types of Curves

<u>curve</u> - any line connecting points on a graph

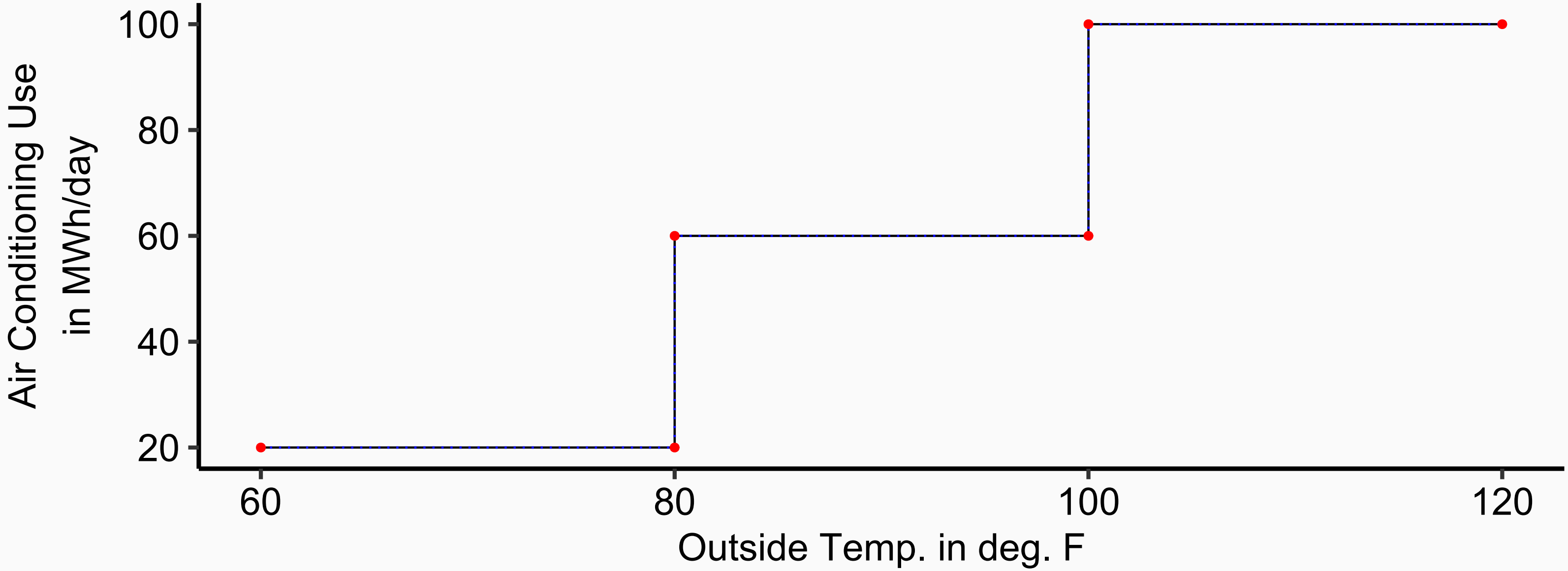
- (doesn't have to be straight, but maybe)
- straight: <u>linear relationship</u>
- curved: <u>non-linear relationsip</u>



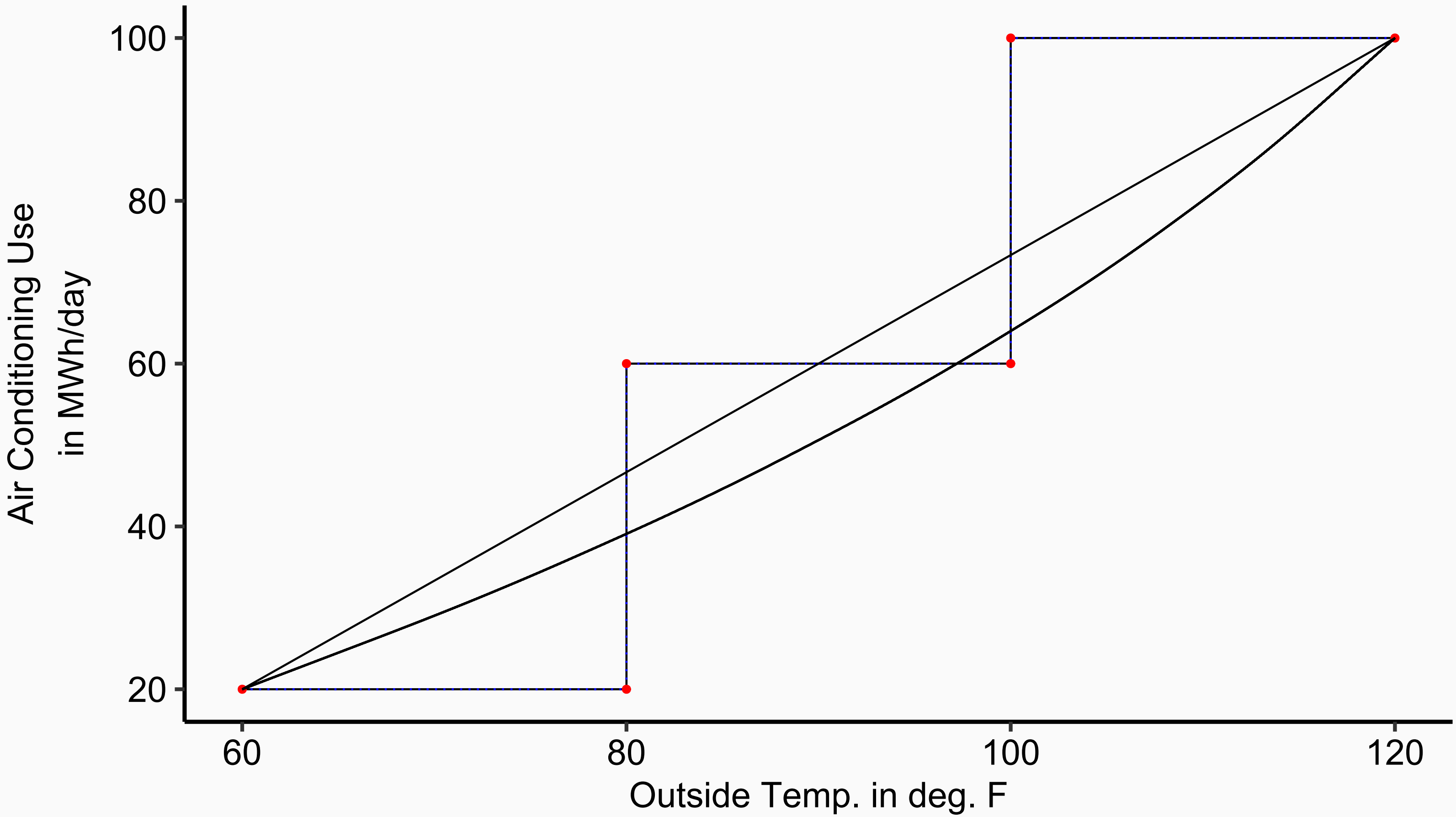
Types of Curves

curve - any line connecting points on a graph

- (doesn't have to be straight, but maybe)
- straight: linear relationship
- curved: non-linear relationsip
- something else (here piece-wise linear)



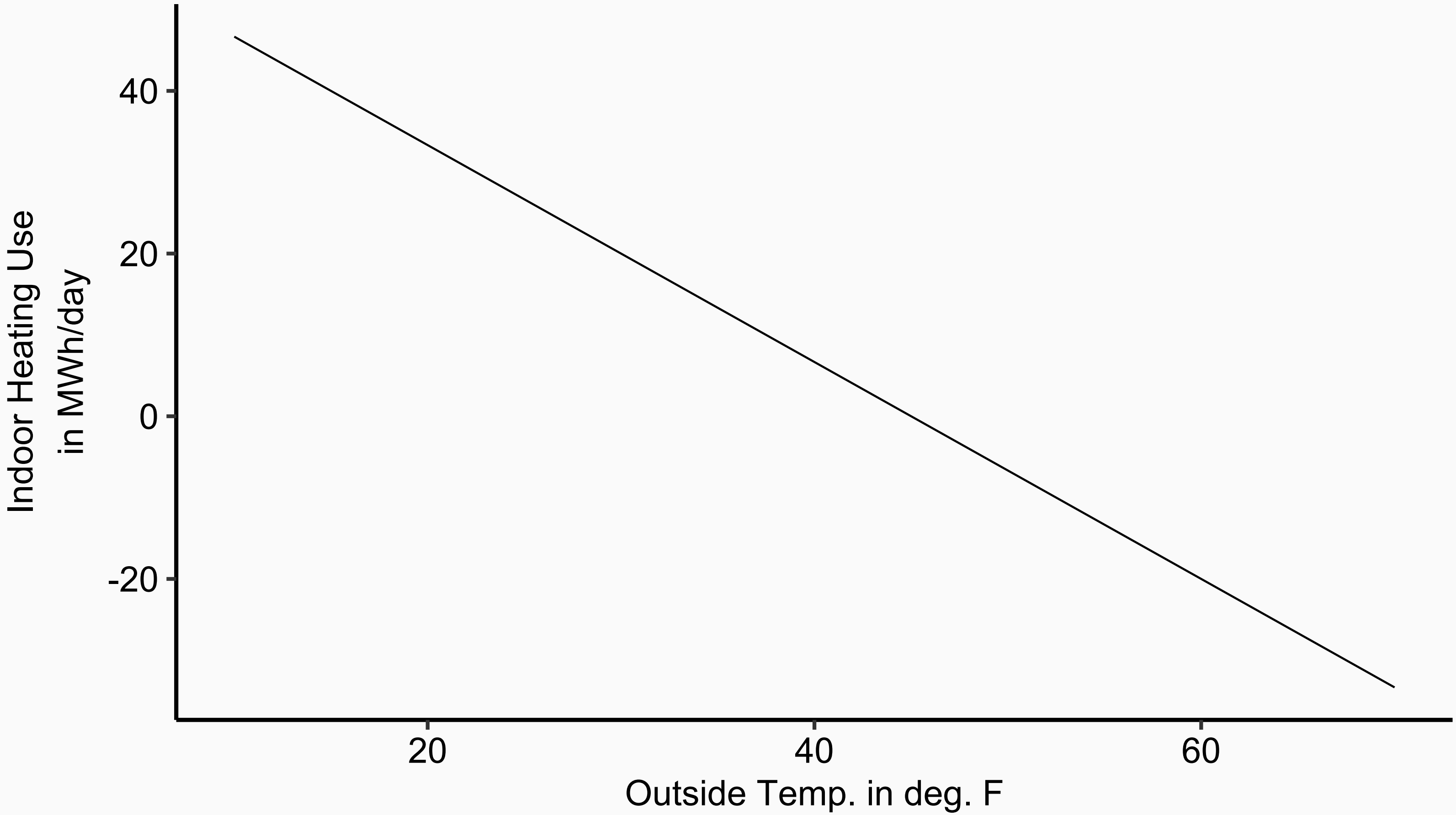
Relationships between Variables



- These are **all** positive relationships: as the temps increase, so too does A/C use

Negative Relationships

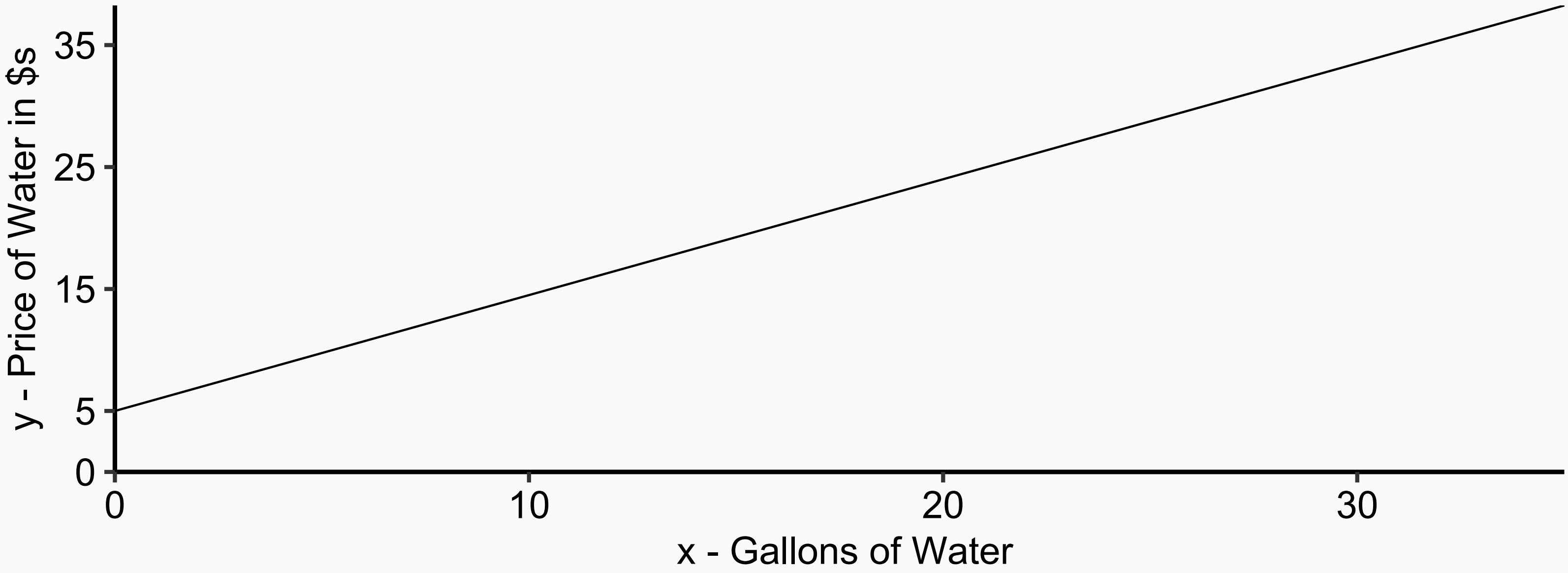
- as the temps increase, indoor heating use **goes down**



Important Characteristics

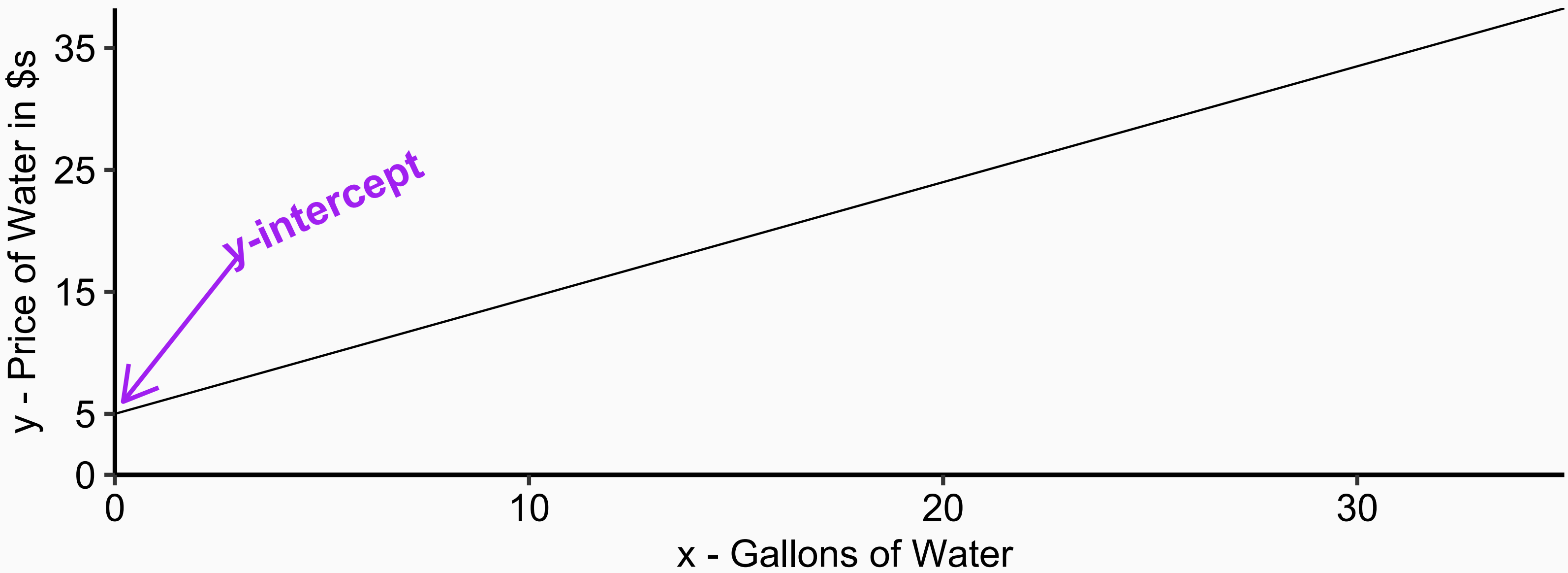
- Sometimes useful to know where graph intersects axes (things with the numbers on them)

- **Common** to call the horizontal axis "x-axis" & vertical axis "y-axis"



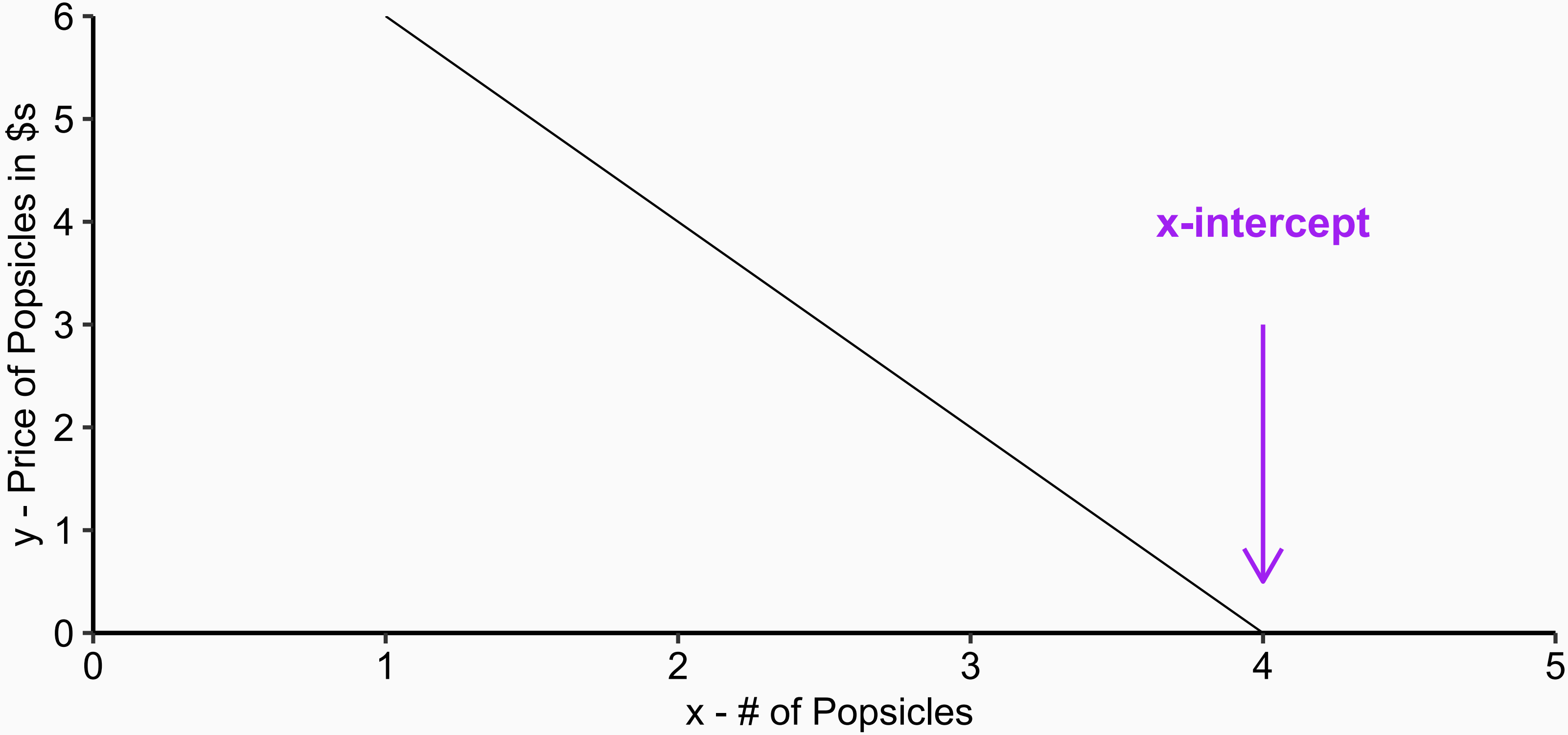
Y-intercept

- vertical intercept (aka y-intercept) at $5 per gallon



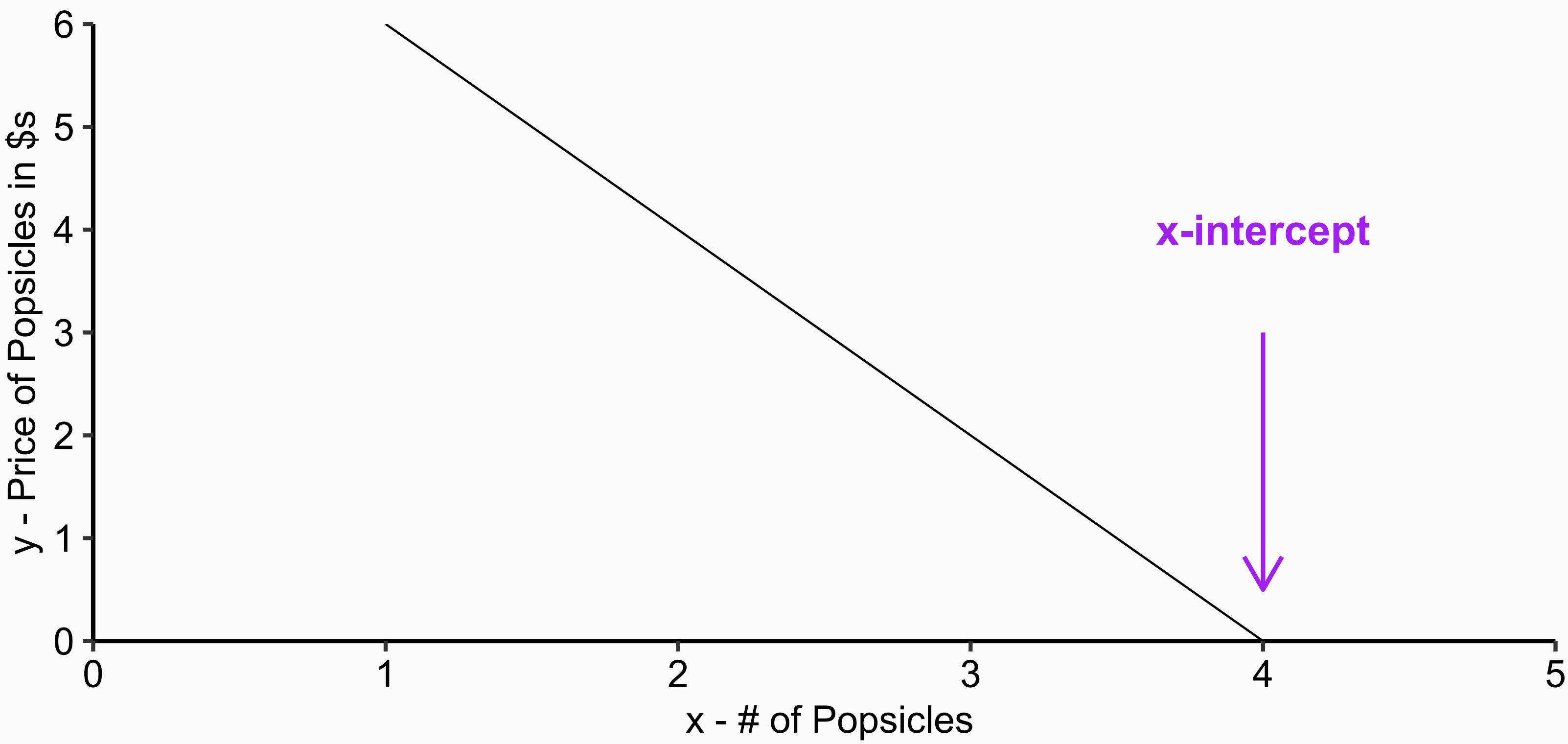
- This tells us the price of the **first** gallon of water

X-intercept



X-intercept

- horizontal intercept (aka y-intercept) at 4 popsicles



- I won't be adding "x" and "y" to the axes from here on, just remember which is whiche

Slope

name: slope

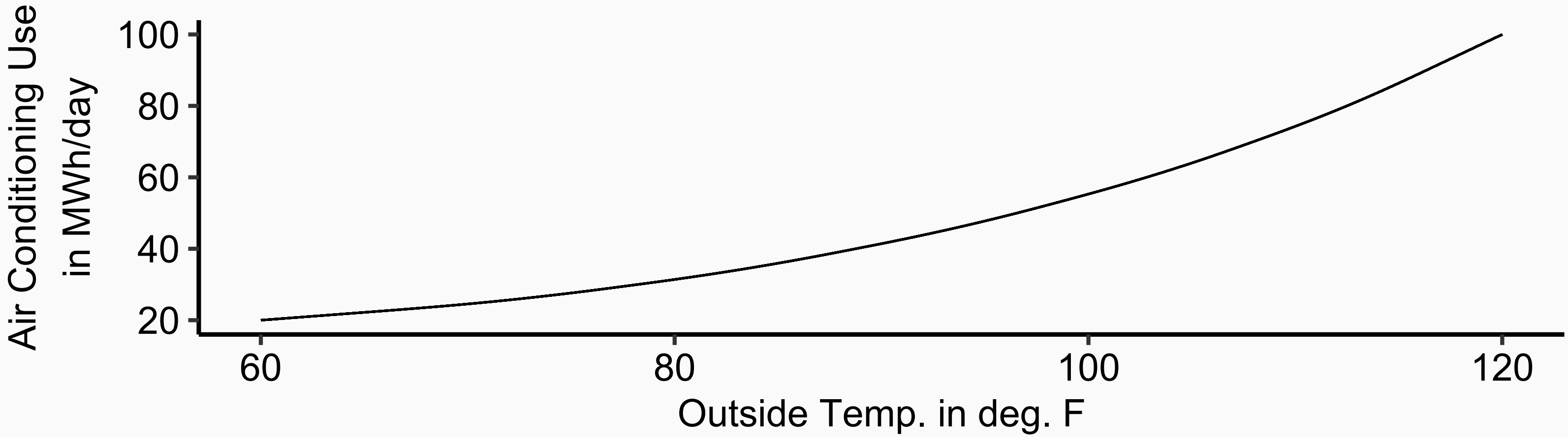
- How does the y-variable change as I change the x variable?

- Sounds boring, but one of the most used characteristics in this class

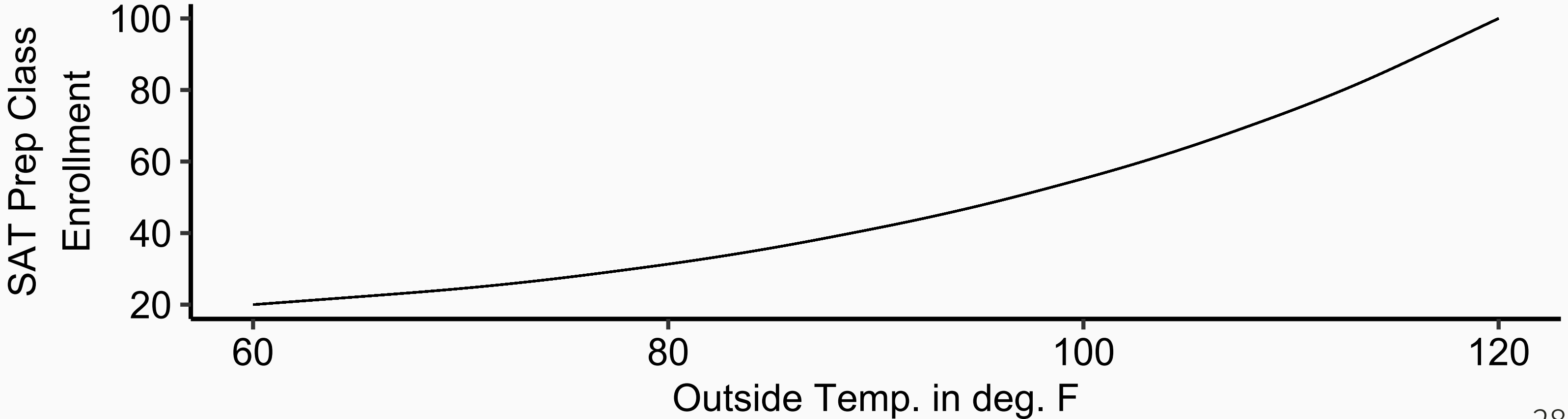
    - The slope often represents some policy relevant change
    - How will cigarette consumption change if we tax them by $4 per pack?
    - Let's look at data of how cigarette consumption changes with price...

Useful thing to consider...

Causal Relationship

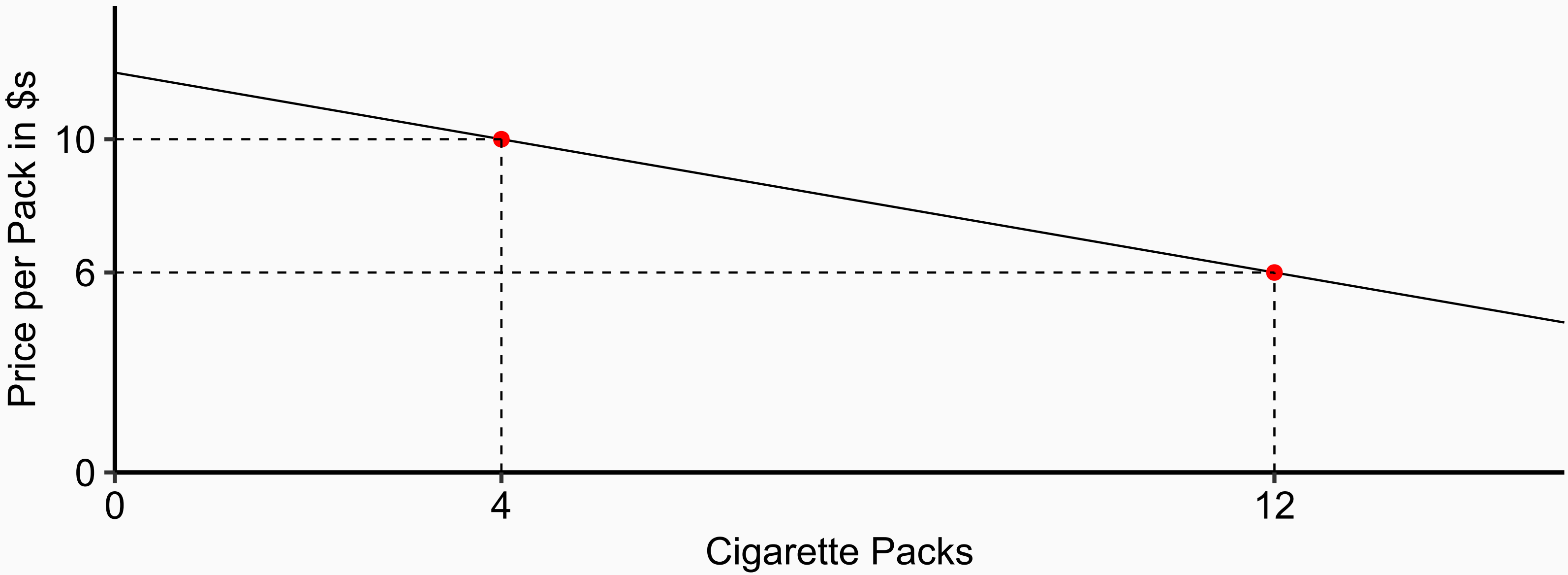


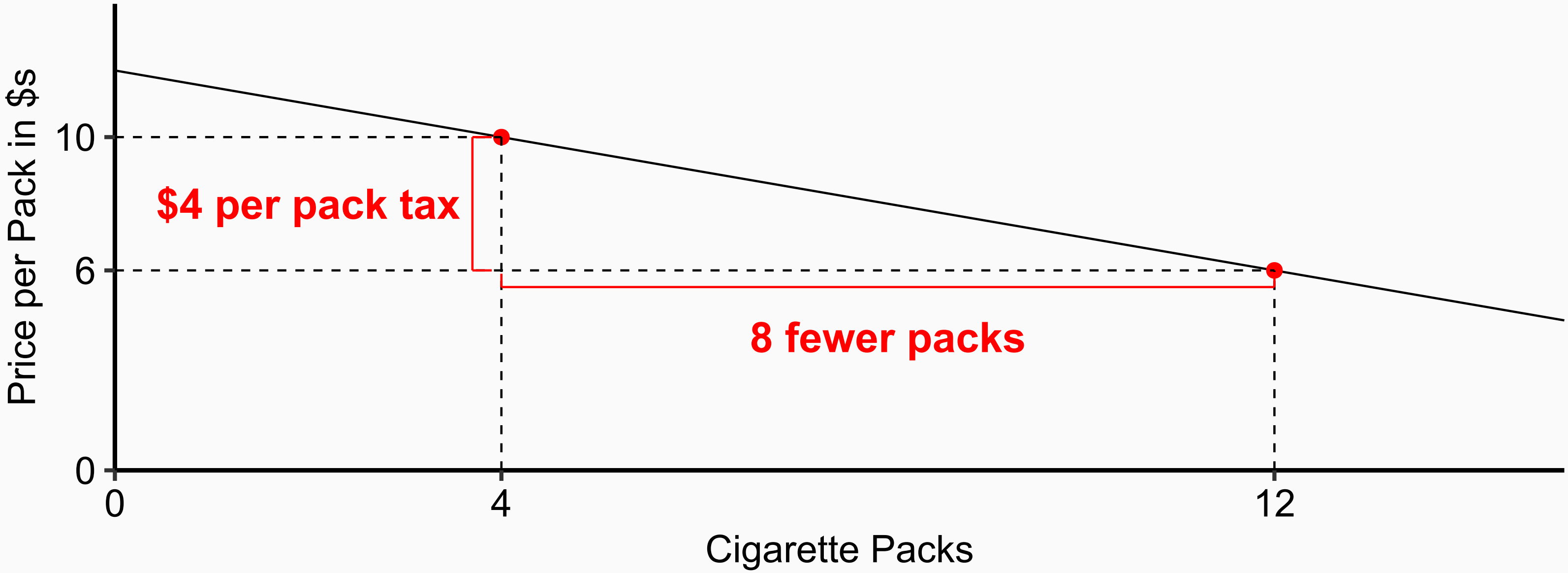
Non-Causal Relationship



Cigarette consumption vs. Price

- Say price is now \$6 and 12 packs are consumed
- If we tax cigarettes by \$4 per pack, then the price per pack raises to \$10
- The line shows only 4 packs are consumed under this higher price

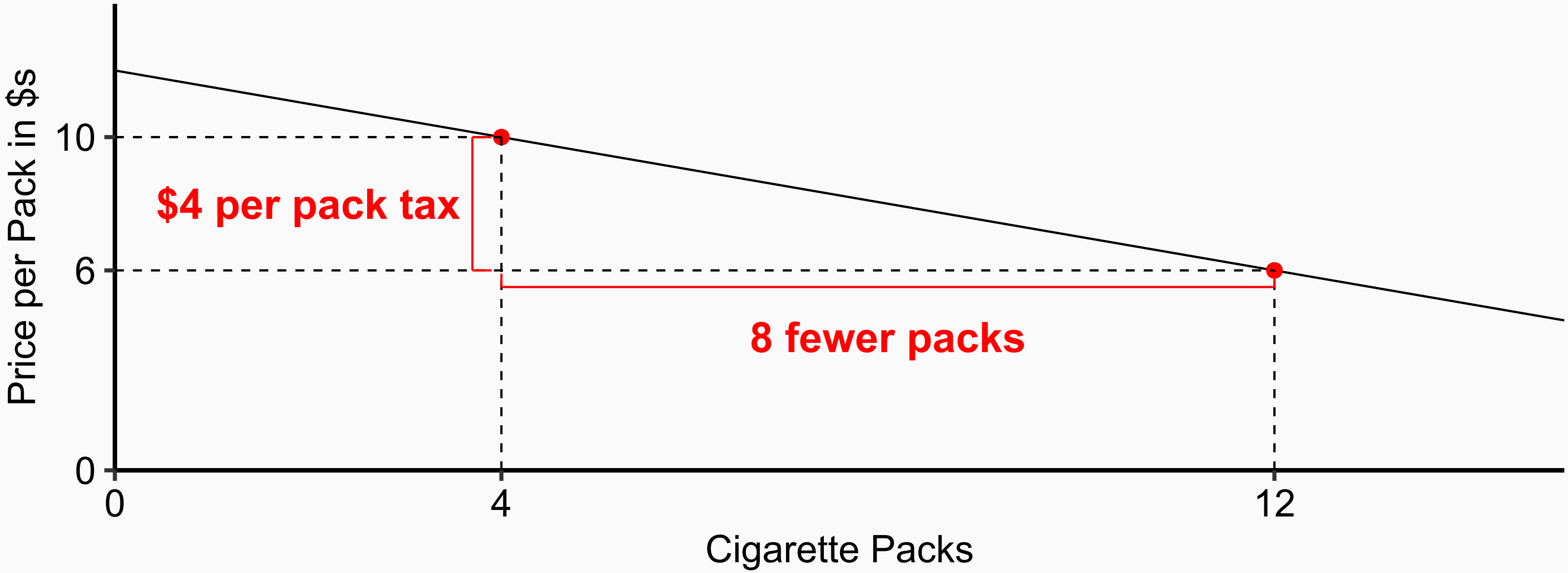




- Slope - change in y variable divided by change in x variable

  in Math:

$$m = \frac{\Delta Y}{\Delta X} = \frac{Y_2 - Y_1}{X_2 - X_1},$$



Slope - change in y variable divided by change in x variable

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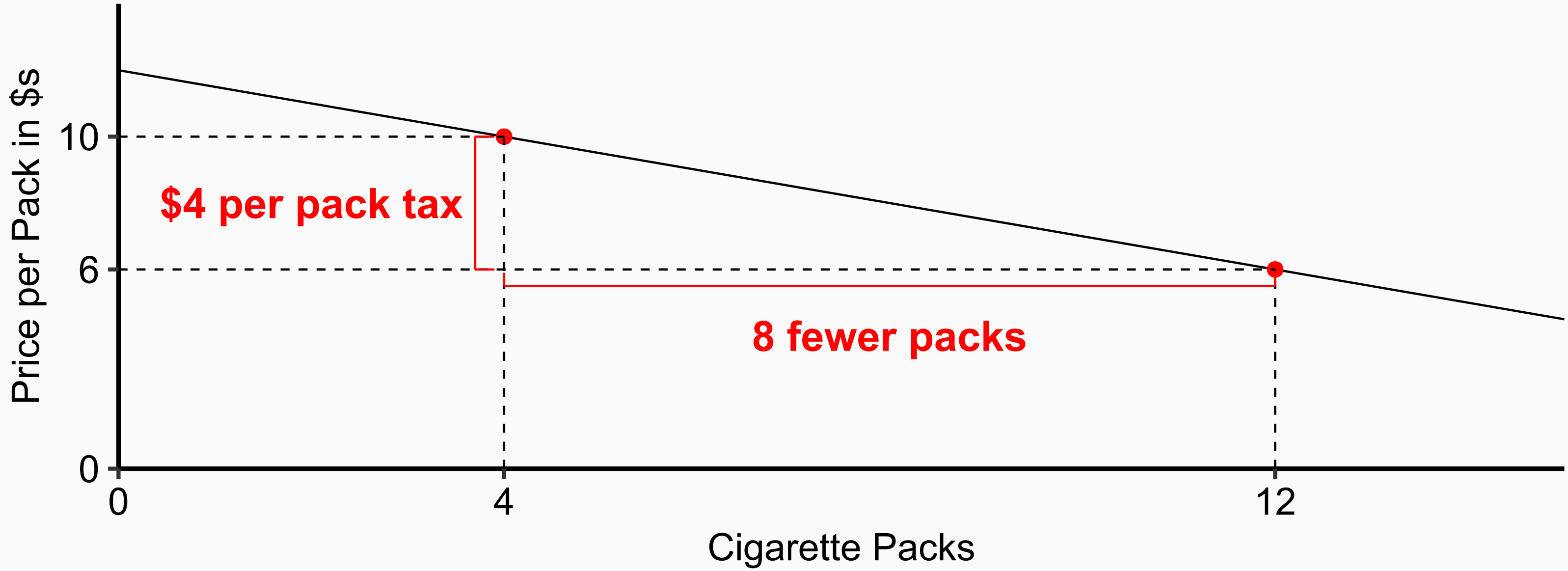
for this graph:

$$Slope = \frac{6 - 10}{12 - 4} = \frac{-4}{8} = -.5$$

Careful: Need to be consistent about order of x's and y's



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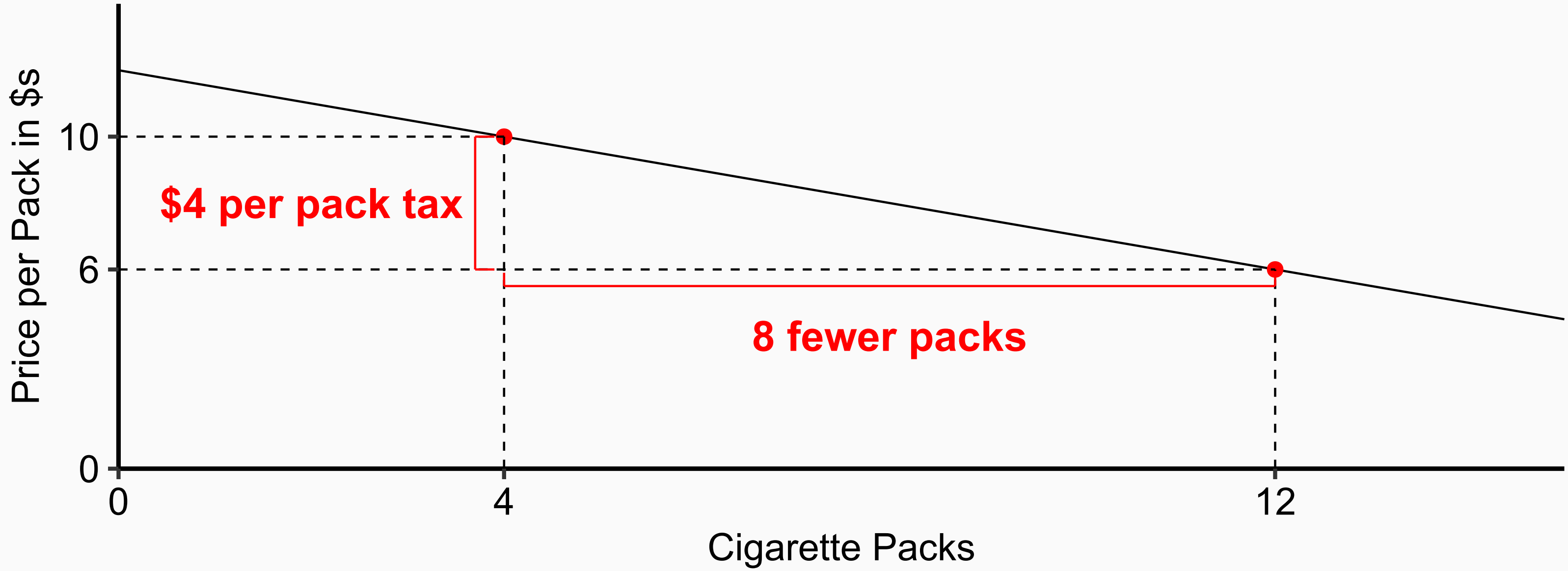
Note: often we denote slope by the letter **m**

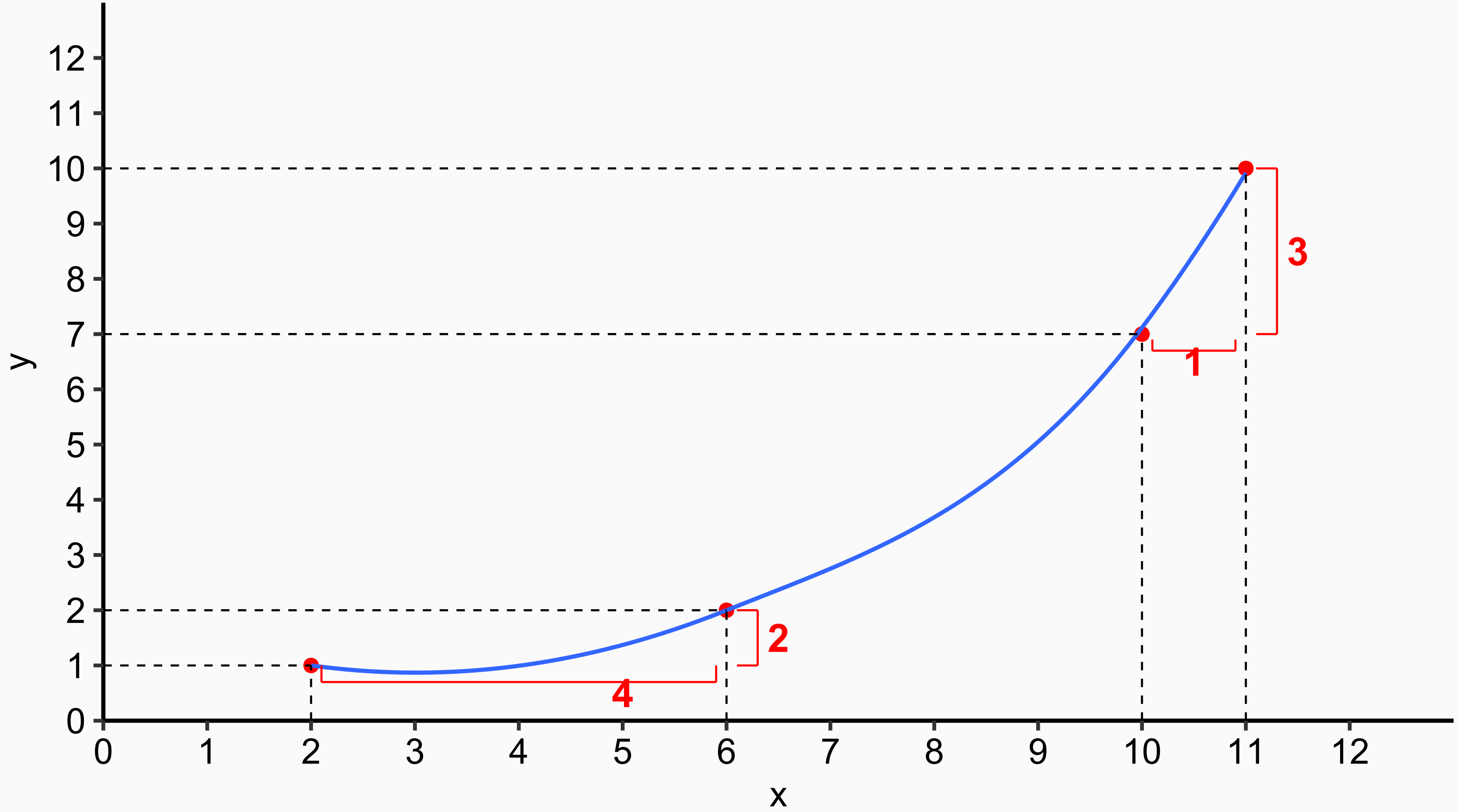


$$Slope = \frac{6-10}{12-4} = \frac{-4}{8} = -.5$$

Interpretation: Every \$1 increase in price, demand for cigs falls by $\frac{1}{2}$

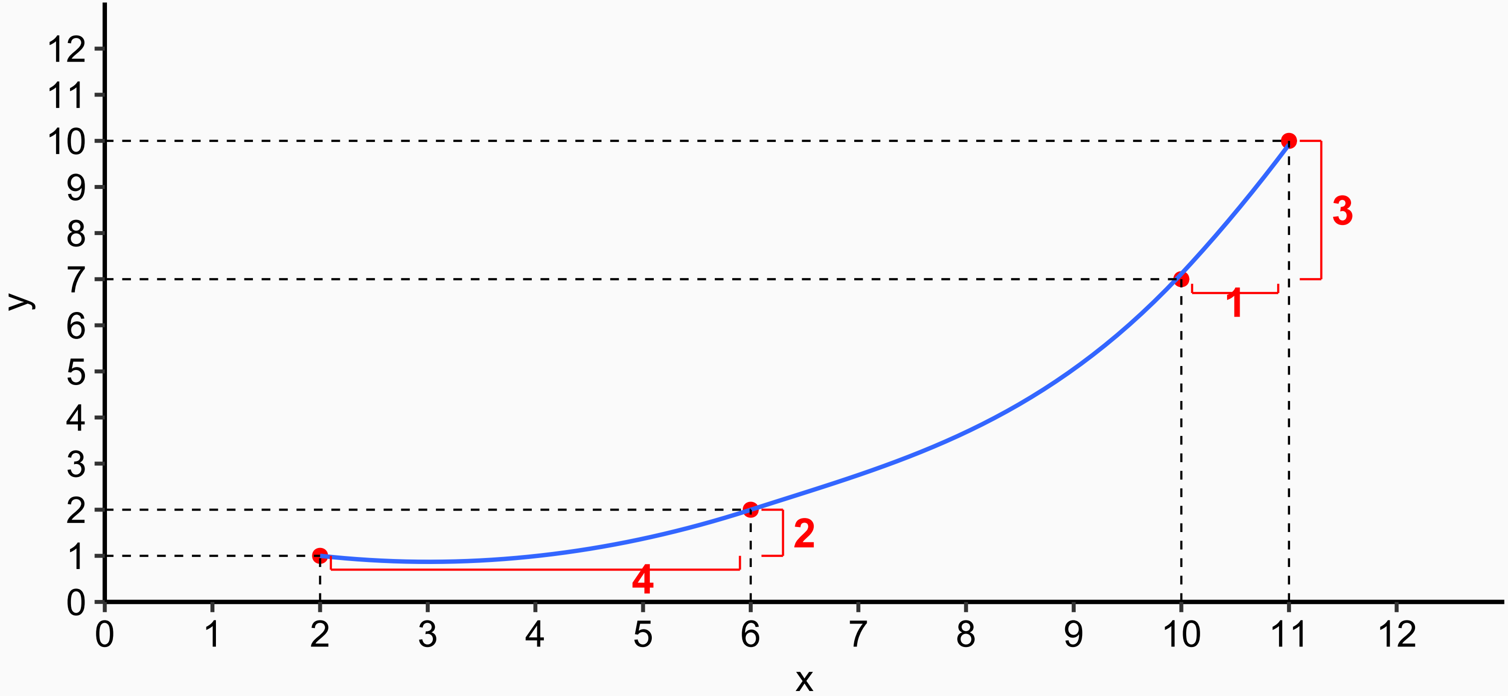
Note: Along a straight line, the slope is the same at every point!

Along a non-straight curve, the slope may vary from point to point

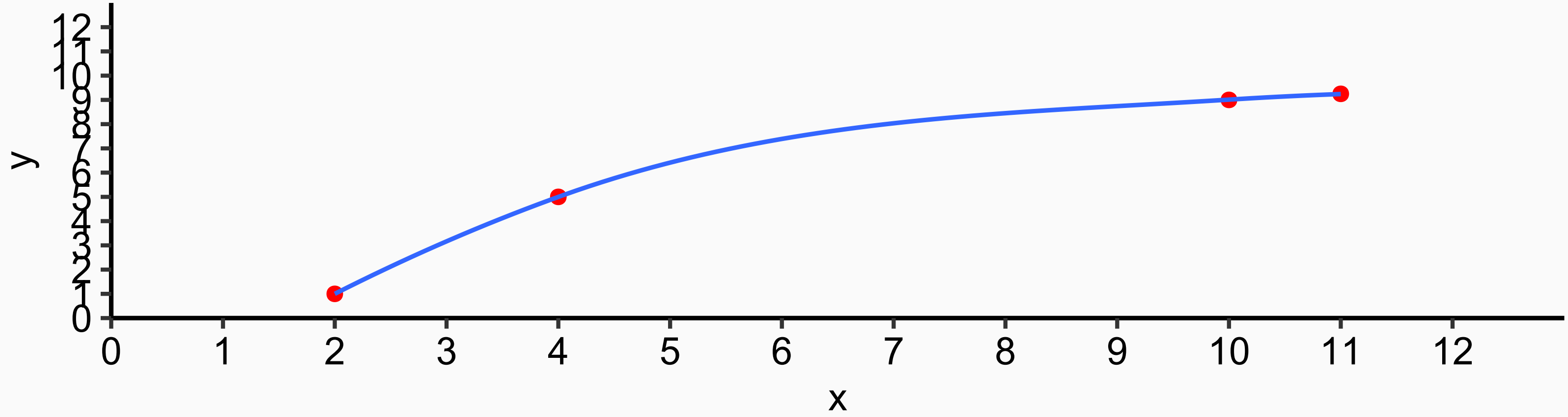


The slope...

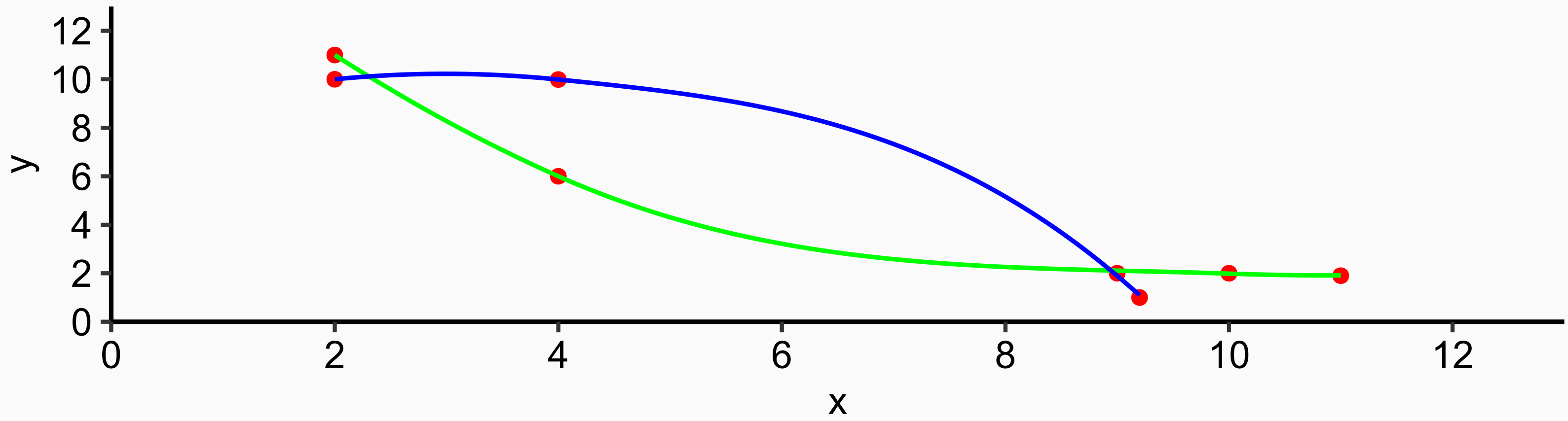
1. Not the same at every point
2. Gets larger *(in this case)*
3. Always positive *(in this case)*



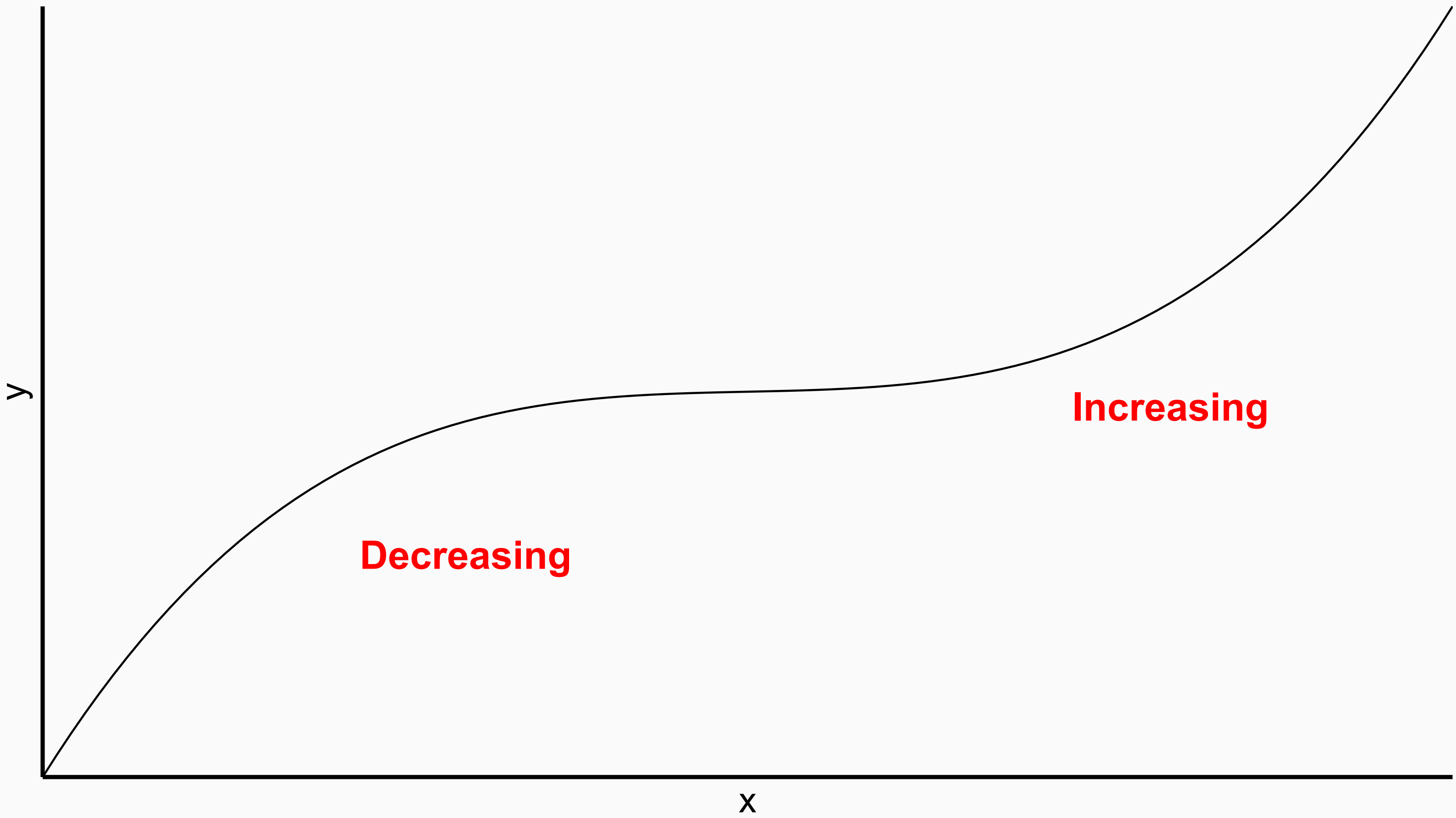
Can draw graphs with positive slope where slope gets smaller



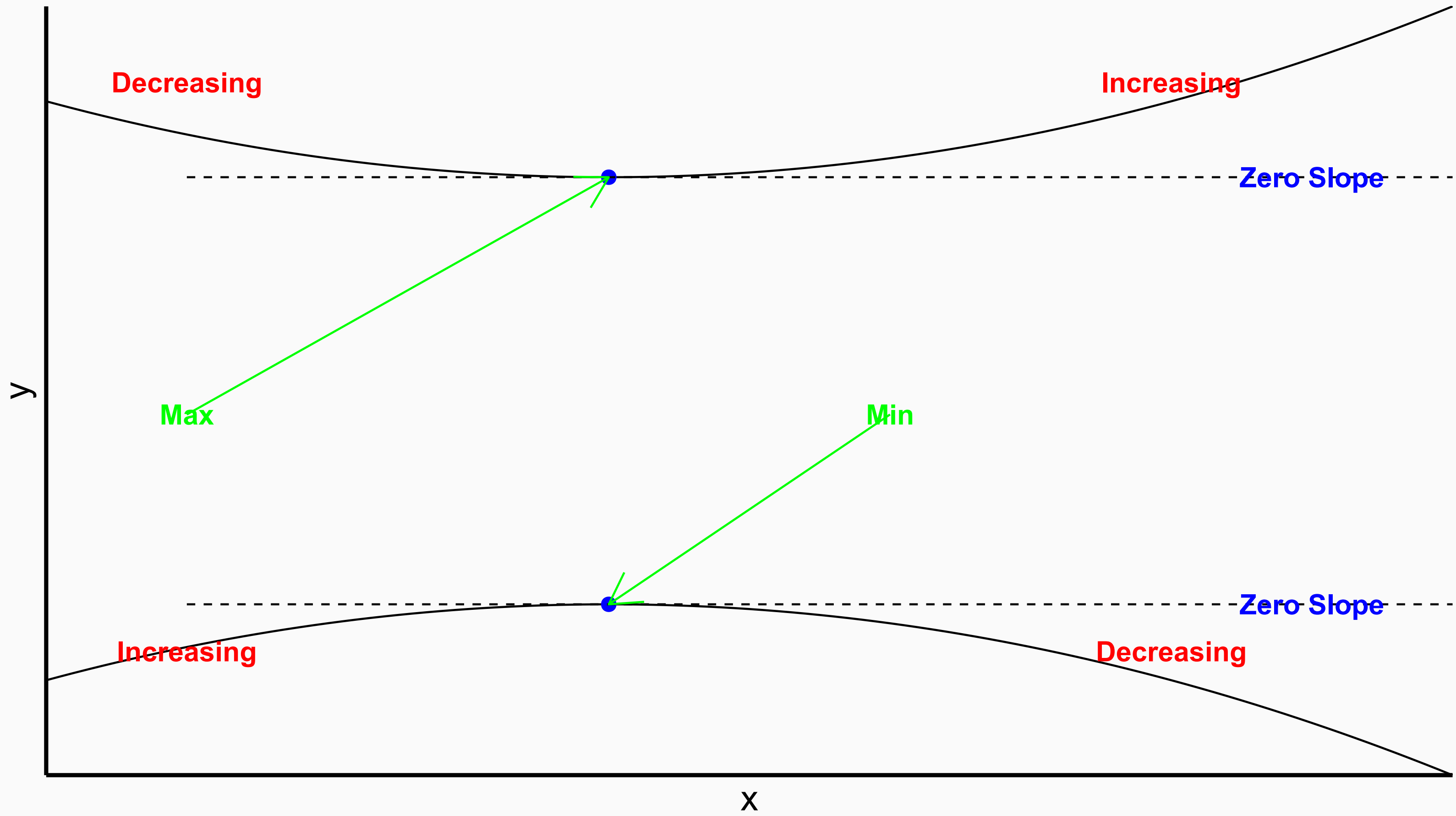
Can draw graphs with negative slope where slope gets bigger/smaller



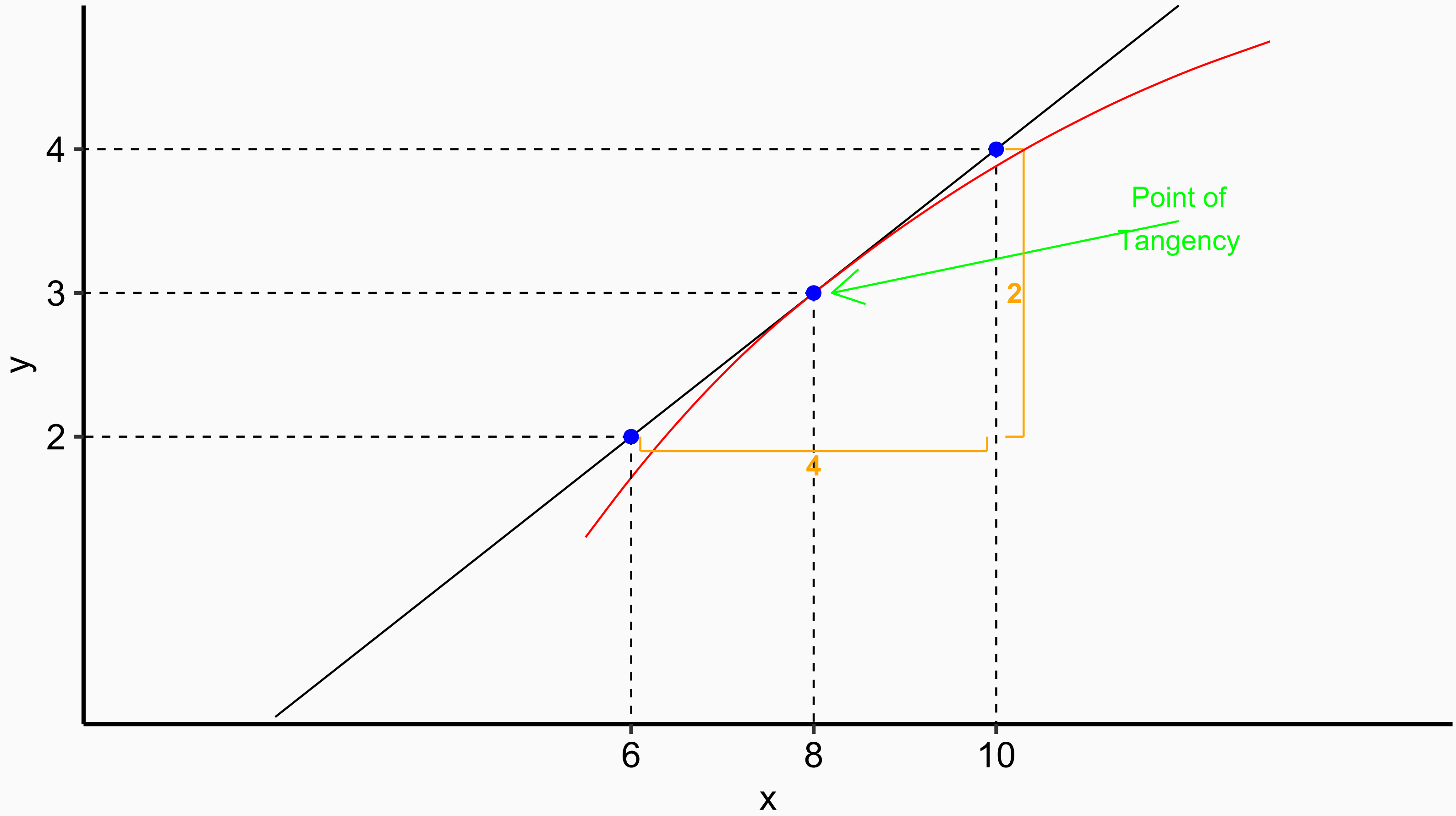
Can draw graphs where slope is both increasing and decreasing



Can draw graphs where slope is both positive and negative

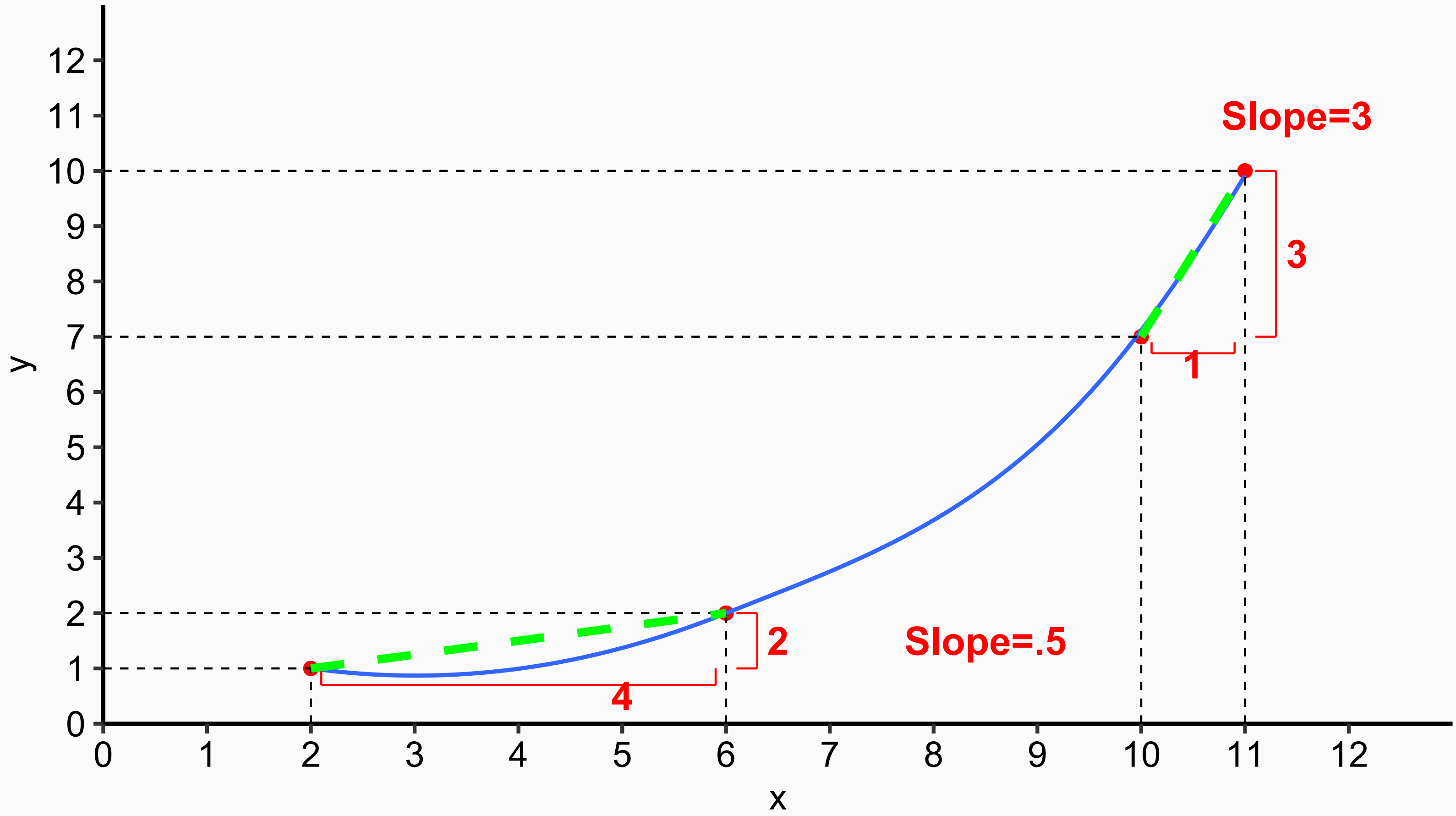


Calculating Slope of a Curve at a Point

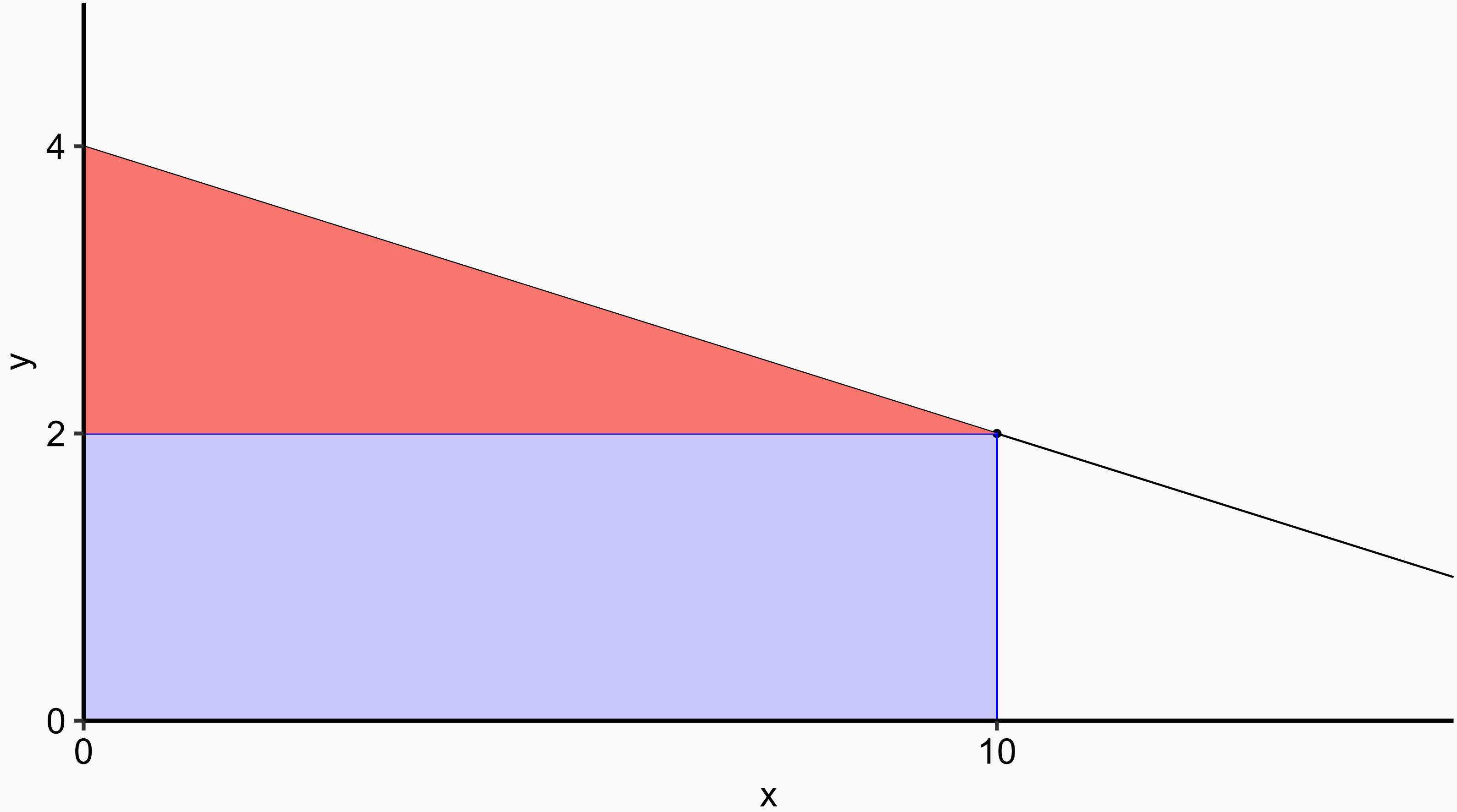


Slope is $\frac{2}{4} = \frac{1}{2}$

Arc Method of Slope Calculation - calculate the slope of line connecting two points on curve

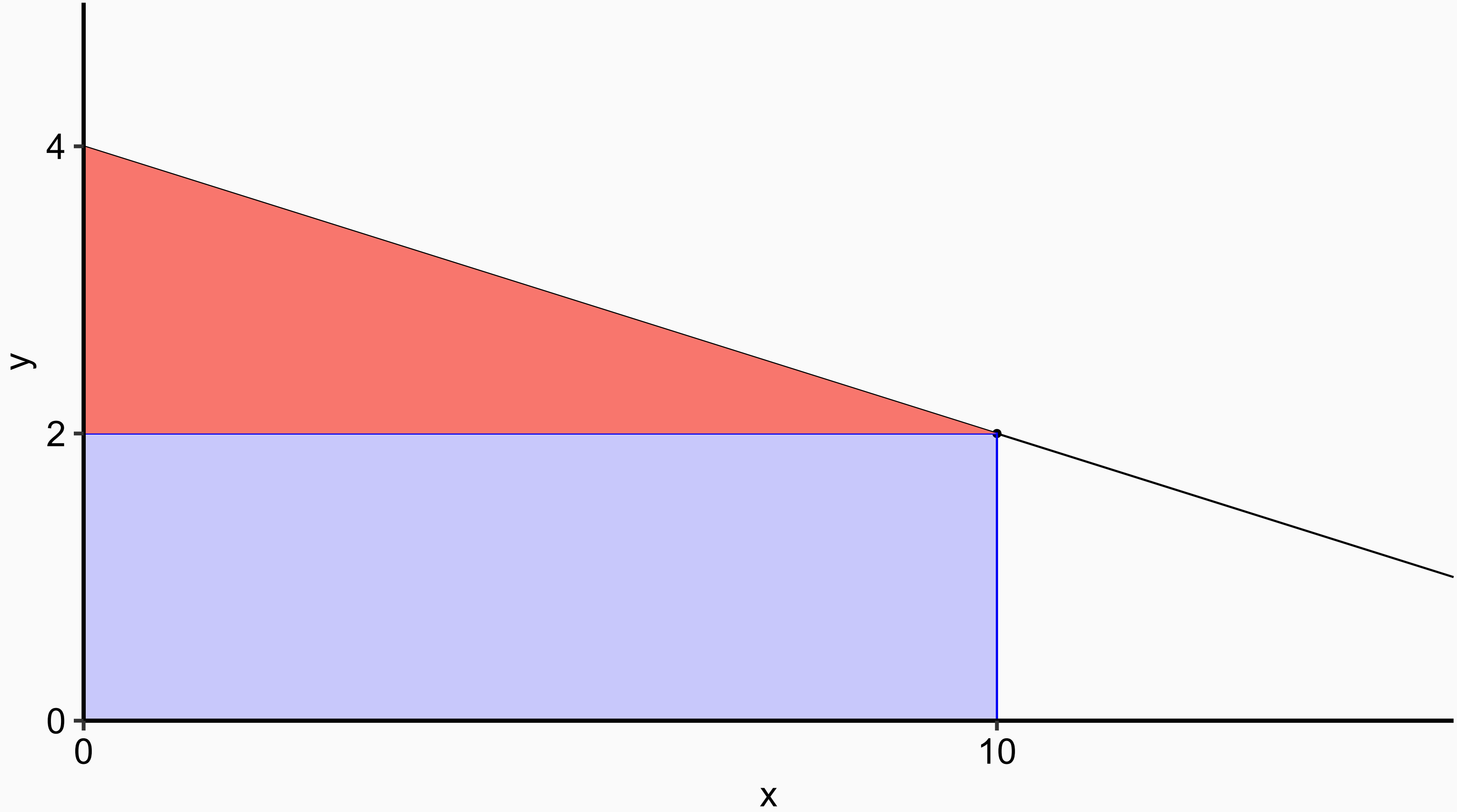


Area Under a Line



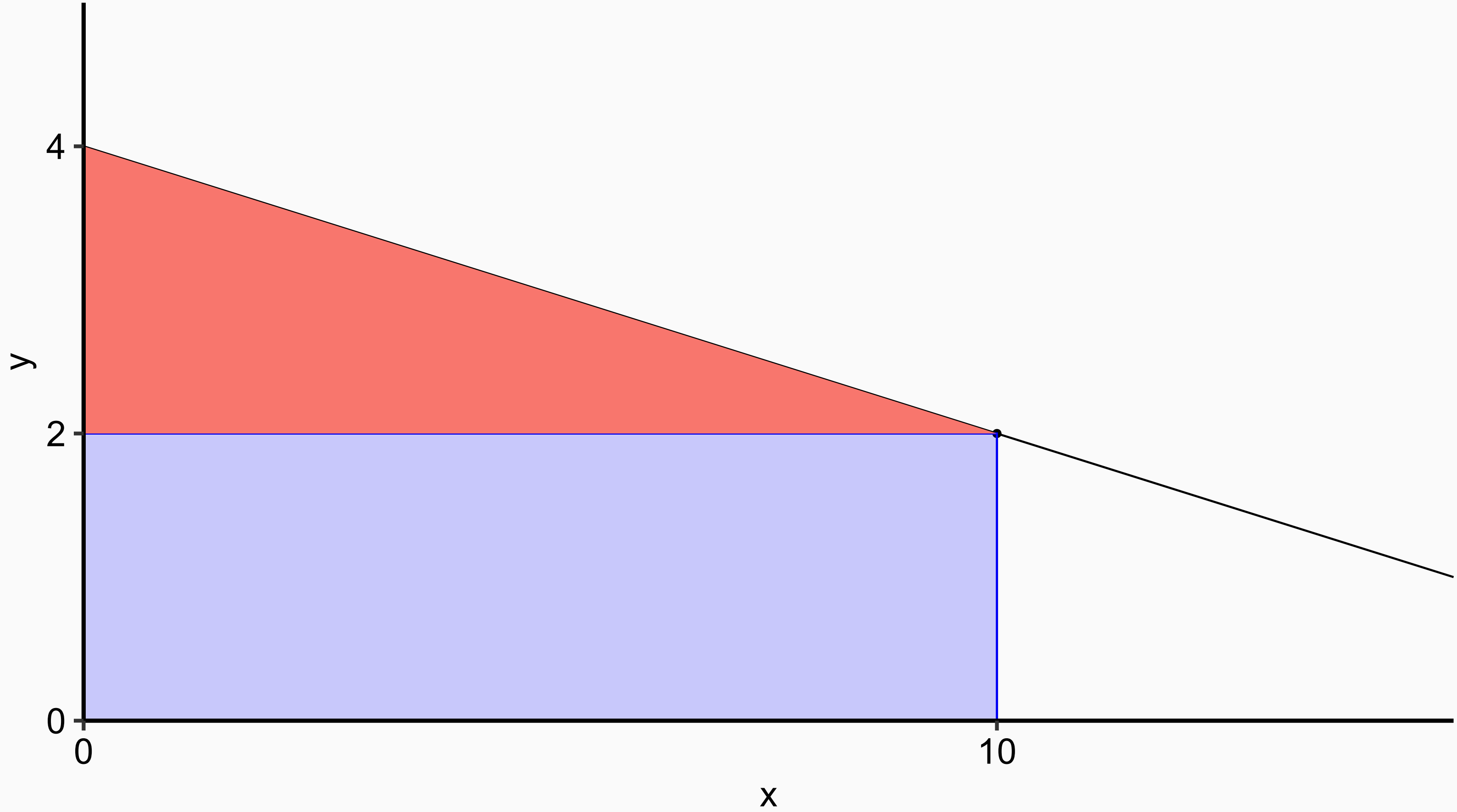
Area of the curve is ▲ + □

Area Under a Line



Area of ▲ = $\frac{1}{2} \times base \times height = .5 \times 10 \times (4-2) = 10$

Area Under a Line

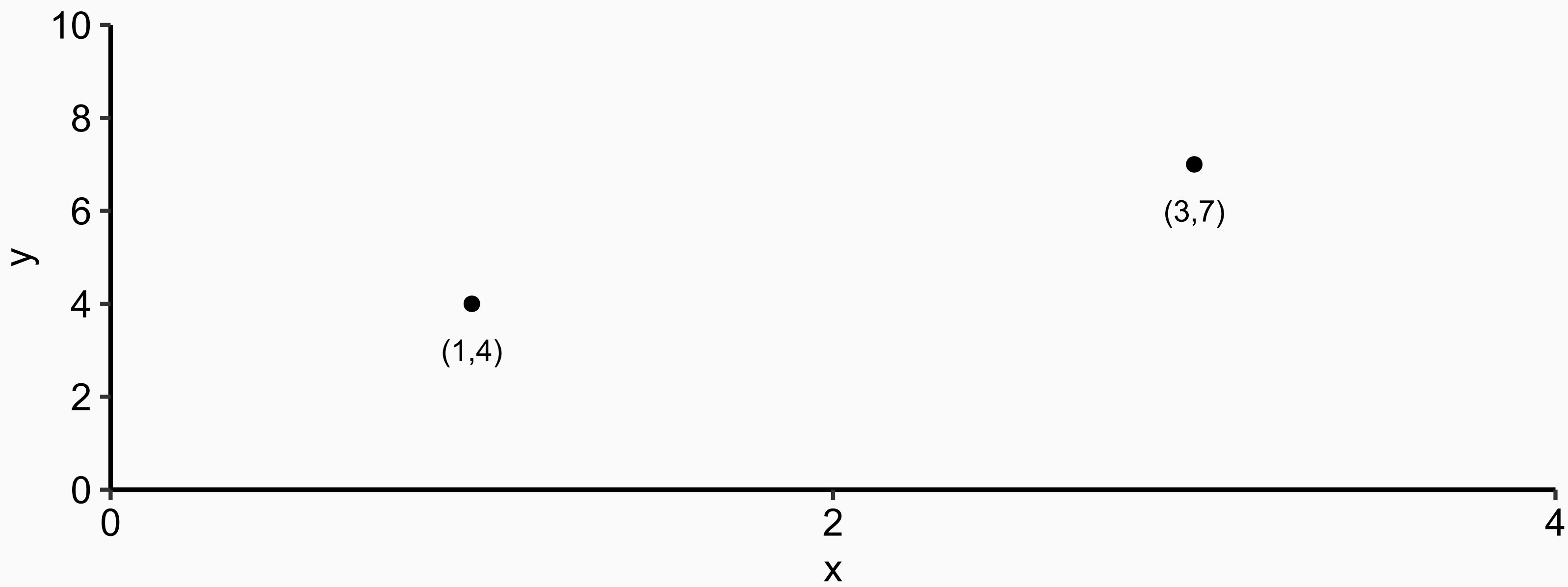


Area of $\square$: $\boldsymbol{base \times height = 10 \times 2 = 20}$

▲ + $\square$ = 20 + 10 = 30

Cartesian Coordinate System

Express location of points on a graph in parentheses: (x-value, y-value)



If we have two points, we may want to determine the line that connects them

HOW?

Equations, Expressions & Operators

- **Variable:** something that can take more than one value: $x = 1, 3, 5, 6$
- **Operator:** performs a mathematical operation
  - Recall order of operations: **P-E-M-D-A-S**
  - (), x^2, $x \times y$, $\frac{\pi}{2}$, 🥕 $+$ 🎂, $x - 3$
- **Expression:** any combination of mathematical symbols, variables, operators
  - Ex.: $3 + 4$, x^3, 16, $y + 2$
- **Equation:** Relates a pair of expression using an equals sign ($=$) to state equality
  - $2 + 2 = 4$, $x - 3 = y$
  - Can express non-equality: $2 + 3 \neq 4$
  - Can express inequality: $2 + 3 > 4$, $1 + 6 \geq x$

Representing lines

Slope-Intercept Form

$$y = \mathbf{3}x + \mathbf{2}$$

y-var. = slope * x + y-intercept (AKA `(y=mx+b)`)

- m = 3 is the slope
- b = 2 is the y-intercept

Point-Slope Form

$$y - y_1 = m \times (x - x_1)$$

$$\Rightarrow y = m(x - x_1) + y_1$$

$$= mx - mx_1 + y_1$$

$$\Rightarrow \textbf{y-intercept} = y_1 - m \times x_1$$

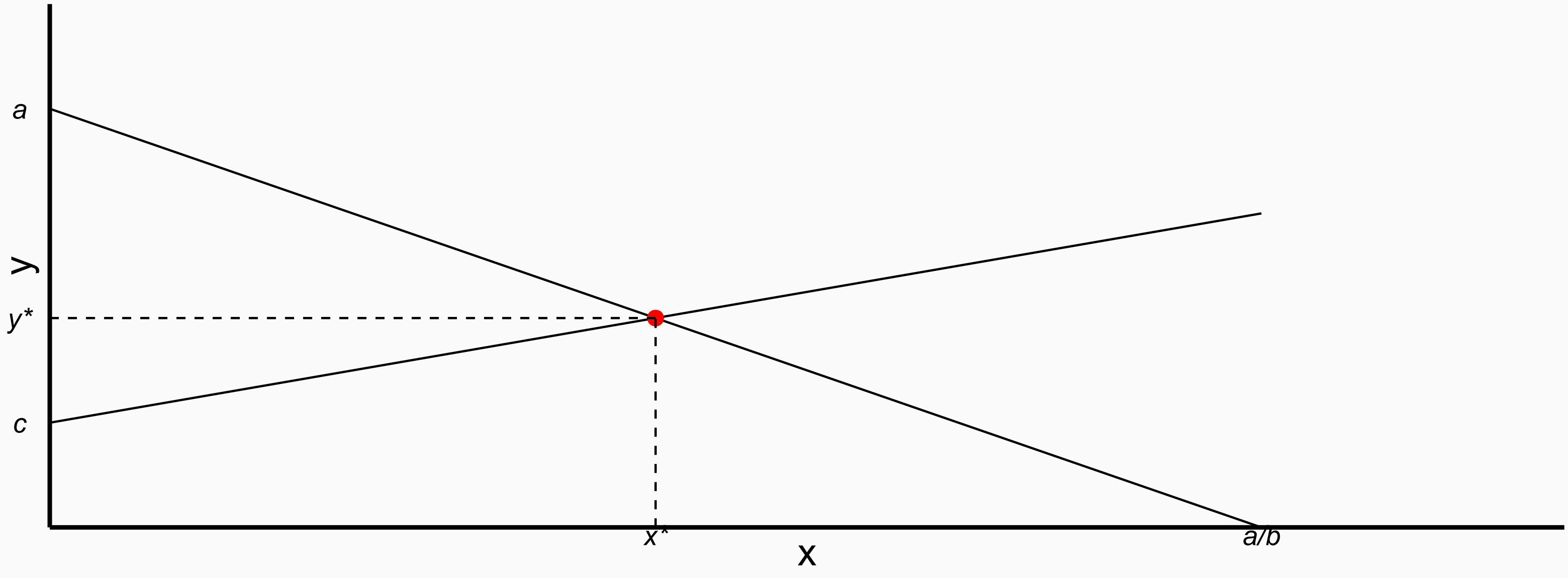
A very common problem...

Two equations:

$$y = a - bx$$

$$y = c + dx$$

where a,b,c,d are parameters (any constant number)



Two easy ways to solve for (x^*, y^*)

1. Subtract equations

$$\begin{aligned} y &= a - bx \\ - \quad y &= c + dx \end{aligned}$$

$$\Rightarrow 0 = a - c - bx - dx$$

$$\Rightarrow a - bx = c + dx$$

$$\Rightarrow (b + x)x = a - c$$

$$x^* = \frac{a - c}{b + d}$$

If we plug back into one of the original equations to find y^*:

$$y* = a - b\left(\frac{a - c}{b + d}\right)$$

Two easy ways to solve for (x^*, y^*)

1. Substitute one equation into the other

$$y = y$$

$$a - bx = c + dx$$

$$x^* = \frac{a-c}{b+d}$$

$$y* = a - b\left(\frac{a-c}{b+d}\right)$$

Example problem:

$$y = 27 - 3x$$

$$y = 9x - 9$$

$$\Rightarrow 27 - 3x = 9x - 9$$

$$\Rightarrow 36 = 12x$$

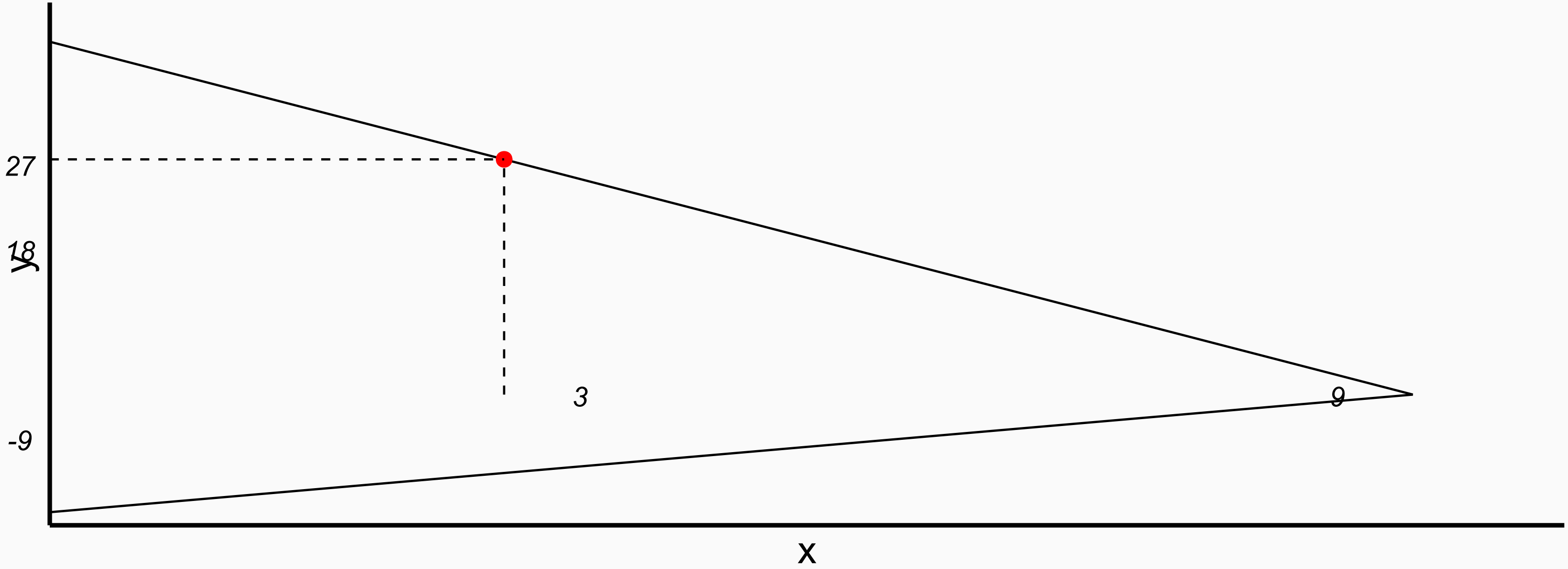
$$x^* = 3$$

$$y^* = 9x^* - 9 = 9 * 3 - 9 = 27 - 9 = 18$$

$$y = 27 - 3x$$

$$y = 9x - 9$$

solution: `(\left(x^,y^\right)=\left(3,18\right))`



Charts from Data

Example: City of Buda - High School Exit Test Scores vs. Student to Teach ratio

Year | Stu/Teach | Exit Test Score

2000 | 20 | 80 2002 | 26 | 65 2004 | 28 | 60 2006 | 32 | 50 2008 | 30 | 55 2010 | 22 | 75 2012 | 24 | 79

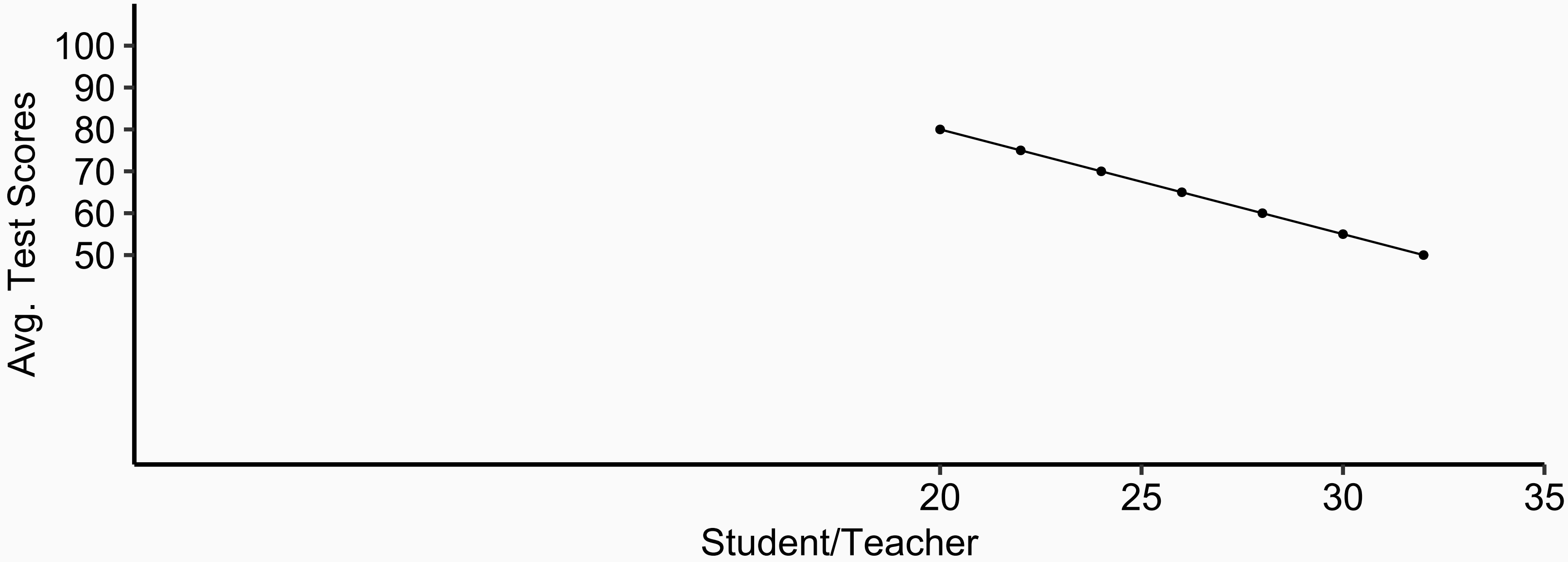
Sort it by independent variable: student-teacher ratio

Year | Stu/Teach | Exit Test Score

2000 | 20 | 80 2002 | 26 | 65 2004 | 28 | 60 2006 | 32 | 50 2008 | 30 | 55 2010 | 22 | 75 2012 | 24 | 79

Stu/Teach | Exit Test Score

20 | 80 22 | 75 24 | 70 26 | 65 28 | 60 30 |55 32 |50



- What is slope?
- Note that for every increase of 2 students to 1 teacher, test scores go down by 5
- Slope = -5/2 = -2.5
- What is y-intercept?: each step in Stu/Teach ratio is 2: 10 steps -> scores increase by 5 for each step: -> 50 points
    - *105 + 80 = 130 -We could write an equation for this line as: TestScore = 130 - 2.5* StudentTeacherRatio

Practice Problem

Austin Integrated School District Enrollment in 2018: 82,520 Enrollment in 2019: 80,495 Expenditures in 2018: $1.6 billion Expenditures in 2019: $2.082 billion

1. What is spending per student?
2. Calculate percentage change in each variable 2018-9
3. What is the slope of the line that connects 2018, where enrollment is x and expenditures is y?