Butterfly scan

The algorithms and programs described here are in the repository on GitHub:

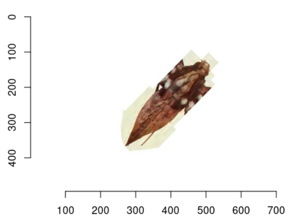
https://github.com/andy-aa/butterfly_scan

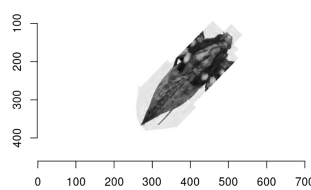
The algorithms are implemented in the R programming language[1].

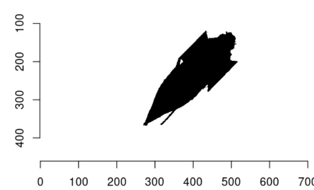
Calculation of the body volume of a butterfly

The algorithm for estimating the volume of the body of a butterfly:

1. Converting a color image to black and white. Three-channel RGB image → Single-channel grayscale image → Single-channel black and white images. When converting an image to black and white, it is necessary to use an empirical coefficient that is in the range from 0 to 1. The empirical coefficient is a color code used to separate all colors in an image into two groups, black and white.







2. Search for the axis of rotation of an arbitrarily oriented body of a butterfly. The axis of rotation is a line connecting the two most distant points from each other. To reduce the search time, the convex hull of the figure is first constructed.

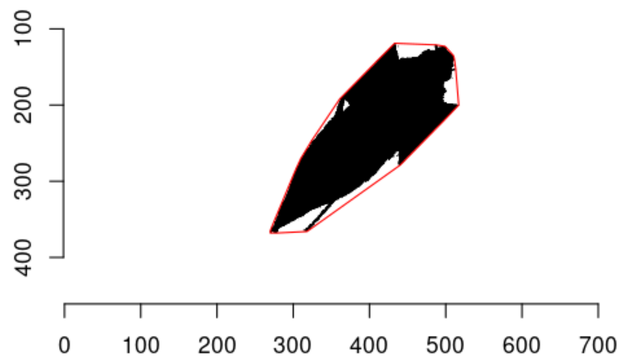


Figure 1: Convex hull

3. In the set of points of the convex hull, we need to find two points that have a maximum distance function: $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

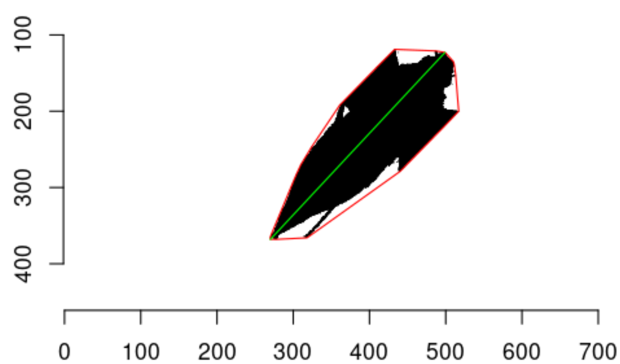
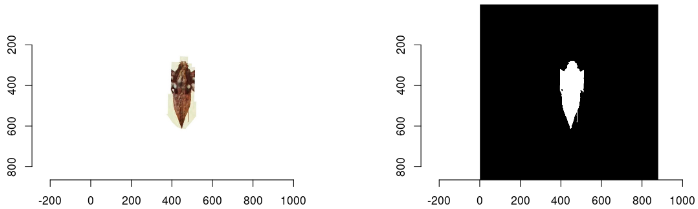


Figure 2: Axis of rotation

4. Finding the angle between the rotation axis and the x-axis. Rotate the image so that the rotation axis is perpendicular to the x-axis and parallel to the y-axis.

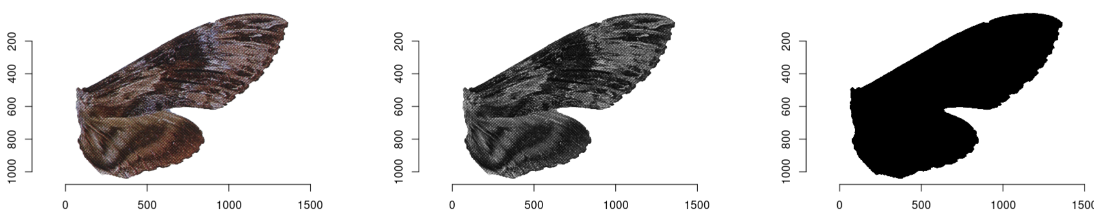


5. Dividing the silhouette (contour) into layers parallel to the x-axis. Finding the width of each layer. The width (w_i) of a layer is equal to the number of pixels in that layer multiplied by the physical size of a pixel. The layer height (h) is equal to the physical pixel size. Layer volume is: $v_i = (w_i/2)^2 * pi * h$. The total volume (V) is the sum of the volumes of its layers $V = \sum_{i=1}^{n} v_i$.

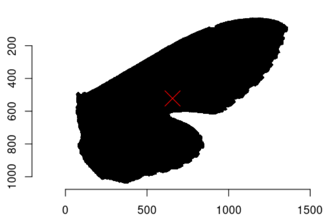
Calculation of the moment of inertia of a butterfly wing

The algorithm for estimating the moment of inertia of a butterfly wing:

1. Converting a color image to black and white.



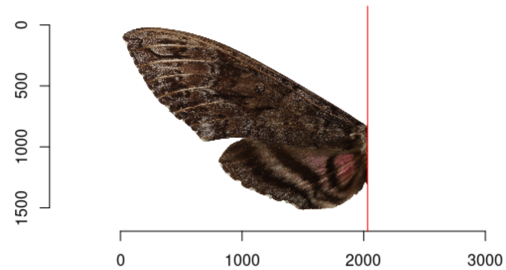
2. Calculation of the coordinates of the conditional center of mass. The classic formula for calculating the center of mass is: $\vec{r}_c = \frac{\sum\limits_i m_i \vec{r}_i}{\sum\limits_i m_i}$. In the case of a two-dimensional object, the formula takes the form: $x_c = \frac{\sum_{i=1}^{n} m_i x_i}{\sum_{i=1}^{n} m_i}$ and $y_c = \frac{\sum_{i=1}^{n} m_i y_i}{\sum_{i=1}^{n} m_i}$; If we take the mass of each point equal to 1, the formulas are noticeably simplified: $x_c = \bar{x}$ and $y_c = \bar{y}$.



3. The classical formula for calculating the ***moment of inertia*** is: $J_p = \sum_{i=1}^{n} m_i r_i^2$, where m_i is the mass of i-th point; r_i is the distance from the i-th point to the axis. If we take the mass of each point equal to 1, and for the point of the axis we take the center of mass, the formula for ***polar moment of inertia*** will take the form: $J_p = \sum_{i=1}^{n} r_i^2 = \sum_{i=1}^{n}((x_i - x_c)^2 + (y_i - y_c)^2)$

4. In our case ***moment of inertia about the axis*** will be calculated relative to the wing attachment line. This is due to the way the wing is attached to the body and how it moves during flight [2]. The formula

for calculating the moment of inertia about the axis will take the form:: $J_a = \sum_{i=1}^{n} r_i^2 = \sum_{i=1}^{n} (x_i - x_a)^2$, where x_a is the distance to the axis of rotation.



Geometric classification and measurement of wing parameters

In order to highlight the characteristic features of the butterfly wings, it is of interest to approximate the butterfly wing using polygons[3]. A simplified representation of a wings shape allows you to highlight the main geometric properties.

Left wing of Hemaris diffinis

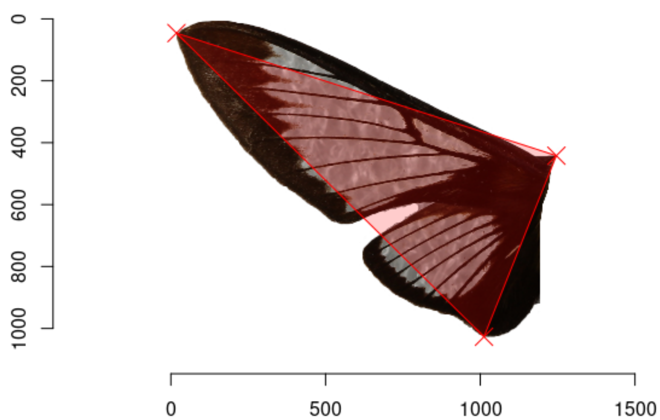


Figure 3: Threangle

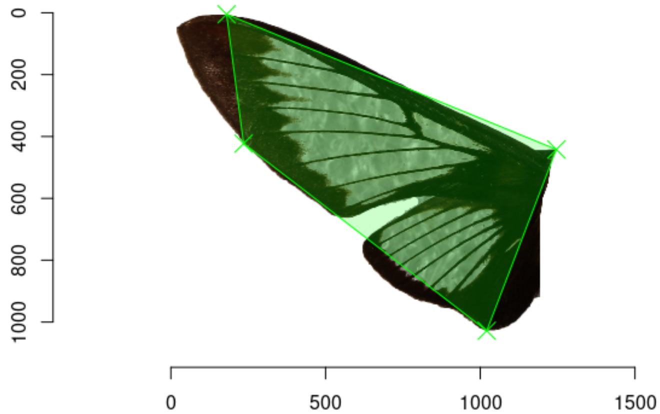


Figure 4: Quadrangle

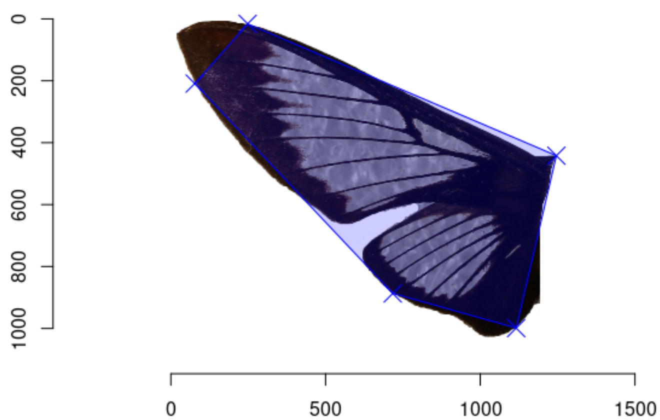


Figure 5: Pentagon

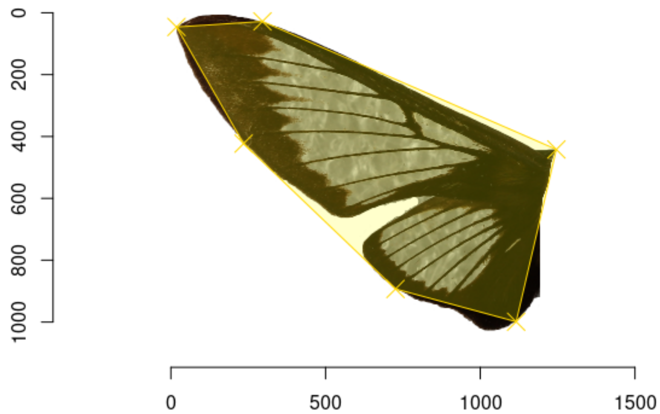


Figure 6: Hexagon

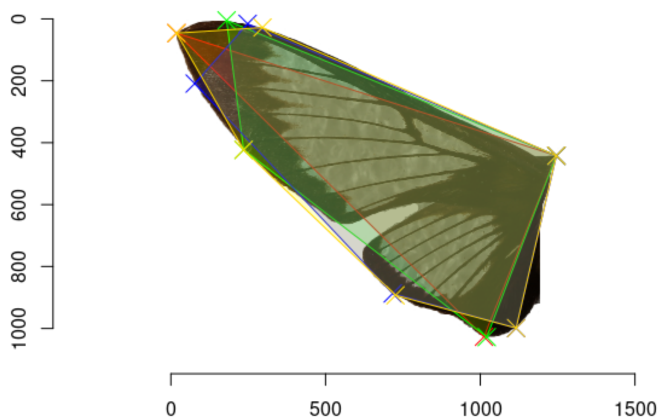


Figure 7: Superposition of polygons

Left wing of Manduca rustica

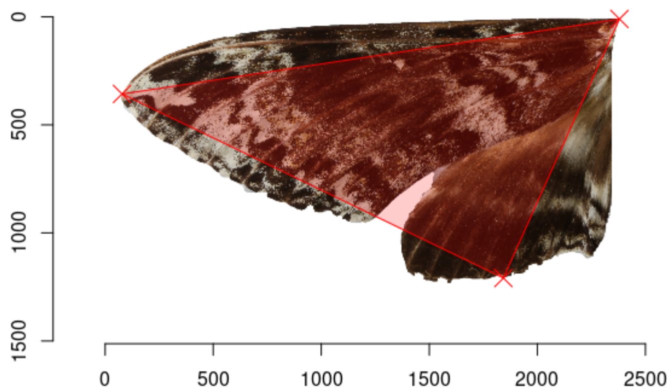


Figure 8: Threangle

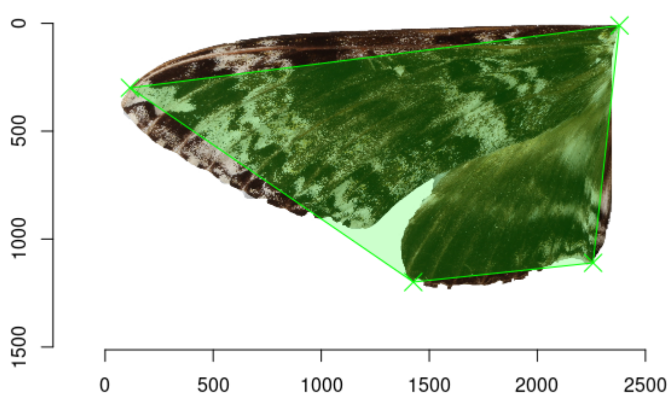


Figure 9: Quadrangle

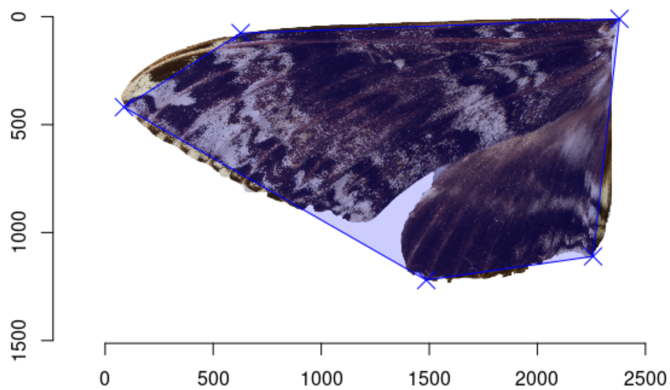


Figure 10: Pentagon

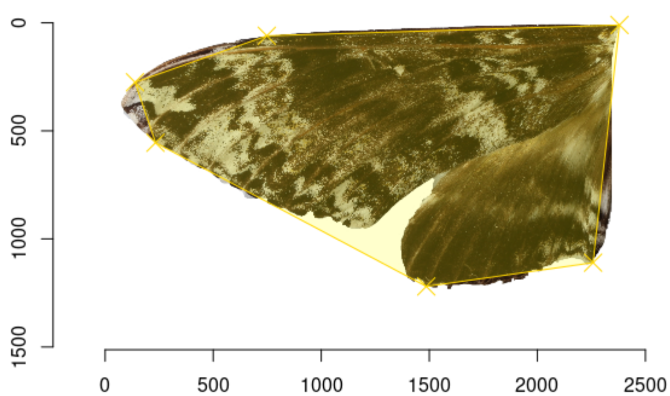


Figure 11: Hexagon

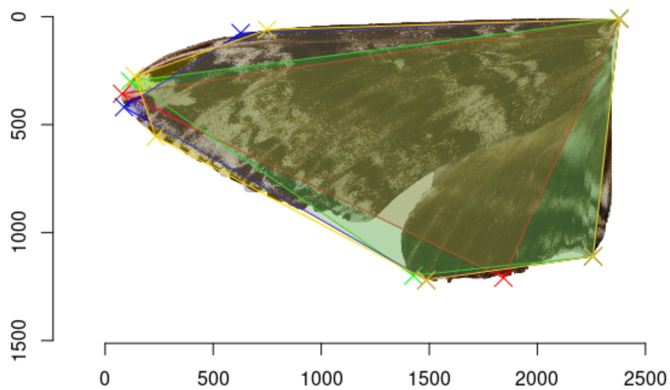


Figure 12: Superposition of polygons

Butterfly wing boundary definition

To find the boundaries of objects in images, one of the most common algorithms is the Canny algorithm [4].

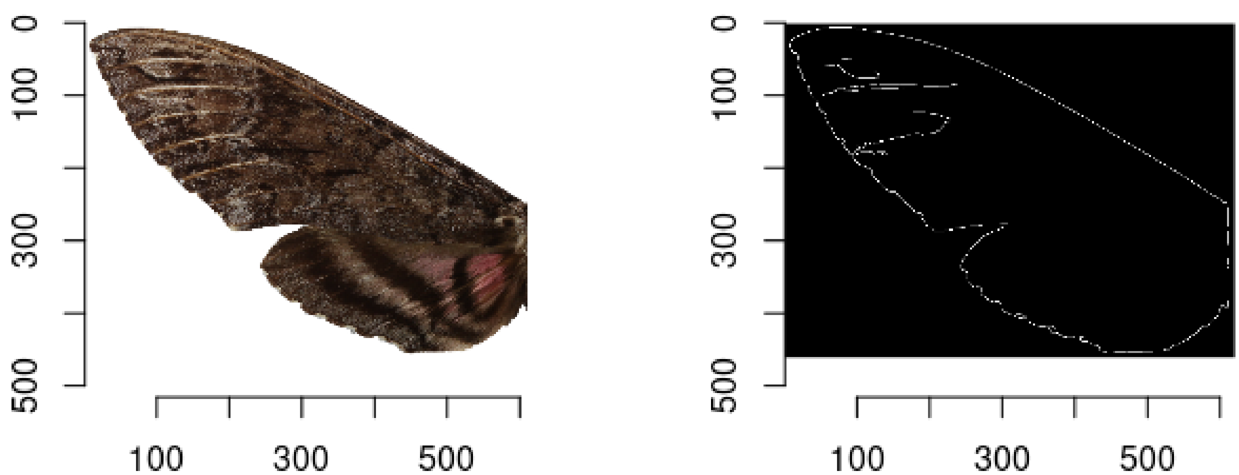


Figure 13: Finding edges in a color image

When using this algorithm, the boundaries will be found not only outside the object but also inside. To avoid this, we can first convert the image to black and white. The disadvantage of using this algorithm is that at the output we have an unordered point cloud, which is the edge of the object.

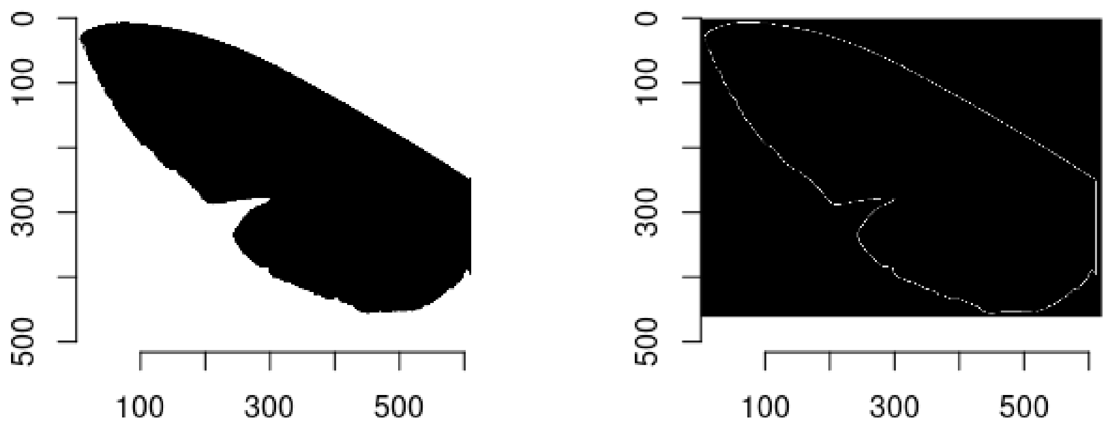


Figure 14: Finding edges in a contrasting black and white image

The boundary of the butterfly wing object is actually a concave hull. One of the algorithms for finding the edges of a concave hull is the Alpha shape algorithm [5].

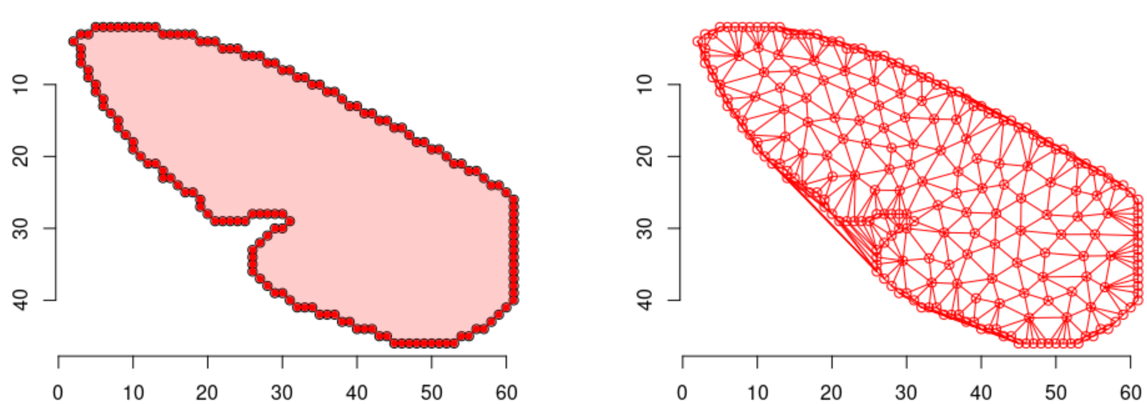


Figure 15: Concave Hull of a wing object

As a result of the Alpha shape algorithm, we can get a polygon that does not intersect itself and has no holes. To find the area of such polygon, we can use the shoelace Gauss algorithm. [6]

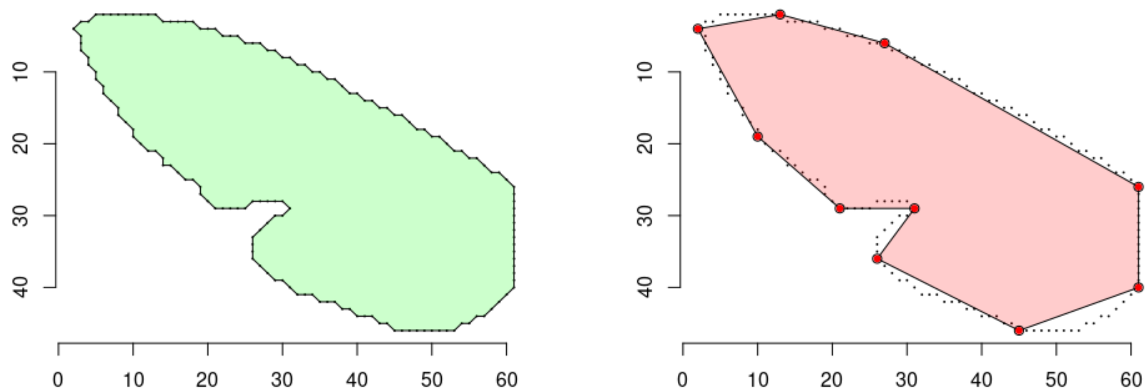
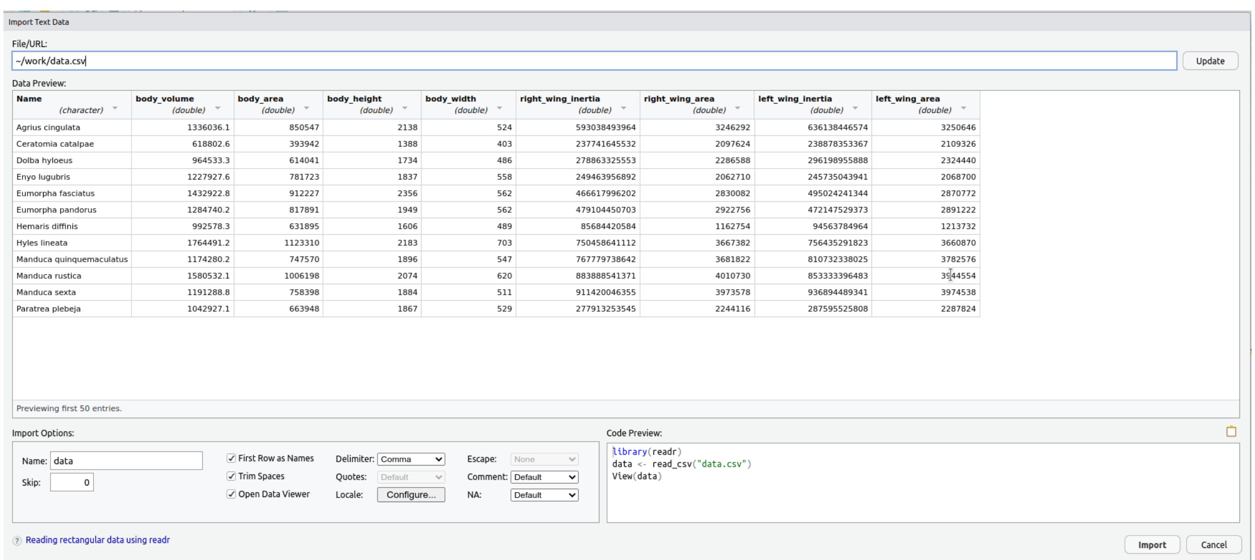


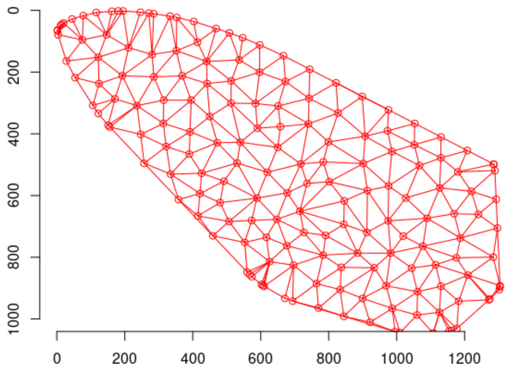
Figure 16: Concave hull after minimizing the number of points

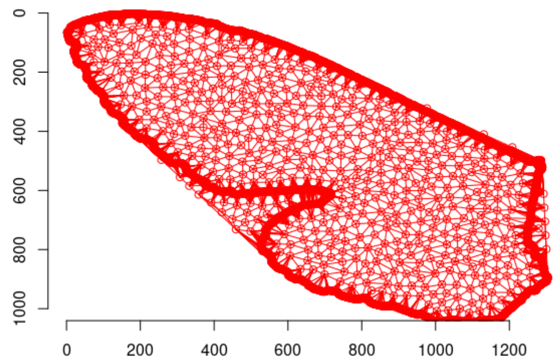
To be continued. . .



Name (character)	body_volume (double)	body_area (double)	body_height (double)	body_width (double)	right_wing_inertia (double)	right_wing_area (double)	left_wing_inertia (double)	left_wing_area (double)
Agrius cingulata	1336036.1	850547	2138	524	593038493964	3246292	636138446574	3250646
Ceratomia catalpae	618802.6	393942	1388	403	237741645532	2097624	238878353367	2109326
Dolba hyloeus	964533.3	614041	1734	486	278863325553	2286588	296198955888	2324440
Enyo lugubris	1227927.6	781723	1837	558	249463956892	2062710	245735043941	2068700
Eumorpha fasciatus	1432922.8	912227	2356	562	466617996202	2830082	495024241344	2870772
Eumorpha pandorus	1284740.2	817891	1949	562	479104450703	2922756	472147529373	2891222
Hemaris diffinis	992578.3	631895	1606	489	85684420584	1162754	94563784964	1213732
Hyles lineata	1764491.2	1123310	2183	703	750458641112	3667382	756435291823	3660870
Manduca quinquemaculatus	1174280.2	747570	1896	547	767779738642	3681822	810732338025	3782576
Manduca rustica	1580532.1	1006198	2074	620	883888541371	4010730	853333396483	3944554
Manduca sexta	1191288.8	758398	1884	511	911420046355	3973578	936894489341	3974538
Paratrea plebeja	1042927.1	663948	1867	529	277913253545	2244116	287595525808	2287824

Figure 17: At the moment, the results of the calculations look like this





References

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3. Bronstein EM (2008) Approximation of convex sets by polytopes. *J Math Sci* 153: pages727–762.
4. Wikipedia.org (2022) Canny edge detector.
5. Wikipedia.org (2022) Alpha shape.
6. Wikipedia.org (2022) Gauss's area formula.