$$\boxed{\textit{Theorem: } \text{Conformal metrics } \rho^2 dz \odot d\bar{z} \text{ experience } K_\rho(z) = -\frac{\Delta log(\rho(z))}{\rho^2(z)} \text{ Gauss curvature.}}$$

Proof: Given a two dimensional Riemannian manifold with conformal metric $(\Sigma, e^\phi(dx^1 \otimes dx^1 + dx^2 \otimes dx^2))$, we have $K = -\frac{R_{1212}}{(e^\phi)^2}$ by Gauss' Theorema Egregium. Furthermore, we know that $\Gamma^k_{ij} = \frac{1}{2} g^{kl} \left(\frac{\partial g_{il}}{\partial x^j} + \frac{\partial g_{jl}}{\partial x^i} + \frac{\partial g_{ij}}{\partial x^l} \right)$. Now, observe that at least two of the indices of the Christoffel symbol must agree, yielding:

$$\begin{aligned}
(i = j = k) \quad &\Rightarrow \quad \Gamma^i_{ii} = \frac{1}{2} g^{ii} \left(\frac{\partial g_{ii}}{\partial x^i} \right) = \frac{1}{2} e^{-\phi} \cdot \frac{\partial}{\partial x^i} \left(e^\phi \right) = \frac{1}{2} \frac{\partial}{\partial x^i} (\phi) = \frac{1}{2} \phi_i, \\
(i = j) \neq k \quad &\Rightarrow \quad \Gamma^k_{ii} = \frac{1}{2} g^{kk} \left(-\frac{\partial g_{ii}}{\partial x^k} \right) = \frac{1}{2} e^{-\phi} \cdot \frac{\partial}{\partial x^k} \left(e^\phi \right) = \frac{1}{2} \frac{\partial}{\partial x^k} (\phi) = -\frac{1}{2} \phi_k, \\
(j = k) \neq i \quad &\Rightarrow \quad \Gamma^j_{ij} = \frac{1}{2} g^{jj} \left(\frac{\partial g_{jj}}{\partial x^i} \right) = \frac{1}{2} e^{-\phi} \cdot \frac{\partial}{\partial x^i} \left(e^\phi \right) = \frac{1}{2} \frac{\partial}{\partial x^i} (\phi) = \frac{1}{2} \phi_i, \\
(k = i) \neq j \quad &\Rightarrow \quad \Gamma^i_{ij} = \frac{1}{2} g^{ii} \left(\frac{\partial g_{ii}}{\partial x^j} \right) = \frac{1}{2} e^{-\phi} \cdot \frac{\partial}{\partial x^j} \left(e^\phi \right) = \frac{1}{2} \frac{\partial}{\partial x^j} (\phi) = \frac{1}{2} \phi_j.
\end{aligned}$$

With this in hand, we compute R_{1212} by definition (acquiring a formula along the way):

$$\begin{aligned}
\langle R(\partial_1, \partial_2)\partial_1, \partial_2 \rangle &= \left\langle \nabla_{\partial_1} \nabla_{\partial_2} \partial_1 - \nabla_{\partial_2} \nabla_{\partial_1} \partial_1 - \nabla_{[\partial_1, \partial_2]} \partial_1, \partial_2 \right\rangle \\
&= \left\langle \nabla_{\partial_1} \left(\Gamma^1_{21} \partial_1 + \Gamma^2_{21} \partial_2 \right) - \nabla_{\partial_2} \left(\Gamma^1_{11} \partial_1 + \Gamma^2_{11} \partial_2 \right), \partial_2 \right\rangle \\
&= \left\langle \frac{\partial}{\partial x^1} \Gamma^1_{21} \partial_1 + \Gamma^1_{21} \left(\Gamma^1_{11} \partial_1 + \Gamma^2_{11} \partial_2 \right) + \frac{\partial}{\partial x^1} \Gamma^2_{21} \partial_2 + \Gamma^2_{21} \left(\Gamma^1_{12} \partial_1 + \Gamma^2_{12} \partial_2 \right), \partial_2 \right\rangle \\
&\quad \left\langle - \left[\frac{\partial}{\partial x^2} \Gamma^1_{11} \partial_1 + \Gamma^1_{11} \left(\Gamma^1_{21} \partial_1 + \Gamma^2_{21} \partial_2 \right) + \frac{\partial}{\partial x^2} \Gamma^2_{11} \partial_2 + \Gamma^2_{11} \left(\Gamma^1_{22} \partial_1 + \Gamma^2_{22} \partial_2 \right) \right], \partial_2 \right\rangle \\
&= \left[\Gamma^1_{21} \Gamma^2_{11} + \frac{\partial}{\partial x^1} \Gamma^2_{21} + \Gamma^2_{21} \Gamma^2_{12} - \Gamma^1_{11} \Gamma^2_{21} - \frac{\partial}{\partial x^2} \Gamma^2_{11} - \Gamma^2_{11} \Gamma^2_{22} \right] \cdot \langle \partial_2, \partial_2 \rangle \\
&= e^\phi \cdot \left[\phi_2 \cdot (-\phi_2) + 2 \frac{\partial}{\partial x^1} (\phi_1) + \phi_1 \cdot \phi_1 - \phi_1 \cdot \phi_1 - 2 \frac{\partial}{\partial x^2} (-\phi_2) - (-\phi_2) \cdot \phi_2 \right] \\
&= e^\phi \cdot \frac{1}{2} \Delta \phi
\end{aligned}$$

Therefore, in the metric $e^\phi(dx^1 \otimes dx^1 + dx^2 \otimes dx^2)$, the Gauss curvature K is given by

$$K = -\frac{e^\phi \Delta \phi}{2 \cdot (e^\phi)^2} = -\frac{1}{2} e^{-\phi} \cdot \Delta \phi = -\frac{\Delta \phi}{2 e^\phi}.$$

Since $\rho^2 dz \odot d\bar{z}$ was the form of our metric on Σ, $log(\rho^2(z))$ plays the role of ϕ in the above. Hence,

$$K_\rho(z) = -\frac{\Delta log(\rho^2(z))}{2 e^{log(\rho^2(z))}} = -\frac{\Delta log(\rho(z))}{\rho^2(z)}$$

$\square$