Notes 4: Finding a General Solution (GS) to a problem

Objective 1: Be able to solve equations for a particular variable.

1. Mathematics is the language used to define the way the world works. Once a problem is understood and defined, the physicist then has an equation that can be analyzed to predict what will happen when one variable, or another, differs in value.

2. It is often necessary to manipulate an equation so that a variable whose value is unknown is expressed in terms of the known quantities. We use algebra to manipulate an equation and determine a GS.

3. For example, consider the equation $v_2 = v_1 + at$, where the values for v_1, v_2, and a are known. You are asked to find t. To determine a GS for t, we begin by subtracting v_1 from both sides, then dividing both sides by a. Finally, we isolate the unknown variable to the left of the equal symbol.

Starting with:	$v_2 = v_1 + at$
Subtract v_1 from both sides:	$v_2 - v_1 = v_1 + at - v_1$
Which simplifies to:	$v_2 - v_1 = at$
Divide each side by *a*:	$\frac{v_2 - v_1}{a} = \frac{at}{a} = \left(\frac{a}{a}\right)(t) = (1)(t) = t$
Isolate *t* on the left side:	$t = \frac{v_2 - v_1}{a}$

4. General rules for solving equations:

   a. Any change made to one side must be made on the other side.
   b. Start by first adding and/or subtracting variables from each side.
   c. Further isolate by multiplying and/or dividing from each side.
   d. ***Square*** or ***take the square of*** each side, as required
   e. Isolate the unknown variable to the left side of the equal sign.
   f. Finally, perform a unit check to verify the algebraic (or general solution).

5. It is possible to check the correctness of the algebraic manipulations performed in solving an equation by performing a ***unit check*** of the ***general solution***. A unit check of the above equation would be done as follows:

Convert $t = \frac{v_2 - v_1}{a}$ into units	$s = \frac{m/s - m/s}{m/s^2}$
Rewritten, this becomes:	$s = \left(\frac{m}{s} - \frac{m}{s}\right)\left(\frac{s^2}{m}\right) = \left(\frac{m}{s}\right)\left(\frac{s^2}{m}\right)$
Simplifying, we get:	*s* = *s* ... which checks!

Objective 2: Be able to solve simultaneous equations.

6. You will often be asked to find an unknown, only to discover that you have two (or more) unknowns in the equation. In these problems, you will be required to find another equation before going further as you can only solve for one unknown if you only have one equation. To solve for two unknowns requires two equations; 3 unknowns — 3 equations; and so on.

7. For example, you are asked to find the kinetic energy of an object with the equation $E_k = \frac{1}{2}mv^2$, but both kinetic energy and velocity are unknown. Using $v = \frac{\Delta x}{\Delta t}$, where you know both Δx and Δt, you can substitute back into the first equation and solve the problem using $E_k = \frac{1}{2}m\left(\frac{\Delta x}{\Delta t}\right)^2$.