6.867 Homework 1

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I. Implementing Gradient Descent

II. Linear Basis Function Regression

A. Closed-form Solution with Polynomial Basis

Consider the basis function

$$\Phi_M(x) = [\phi_0(x), \ldots, \phi_M(x)],$$

where $\phi_k(x) = x^k$. Applying Φ_M to $\mathbf{X}$ gives the desired basis change, so we have the generalized linear model

$$\mathbf{Y} = \Phi_M(\mathbf{X}) \cdot \boldsymbol{\beta} + \boldsymbol{\epsilon},$$

which has the close-form solution

$$\hat{\boldsymbol{\beta}} = (\Phi_M(\mathbf{X})^T \Phi_M(\mathbf{X}))^{-1} \Phi_M(\mathbf{X})^T \mathbf{Y},$$

where $\hat{\boldsymbol{\beta}}$ is the maximum-likelihood estimator of the regression coefficients.

We ran regressions on the data using a few different degrees $(M = 0, 1, 3, 10)$.[1] Below are plots of the resulting polynomial functions, compared to the given data and the true function:

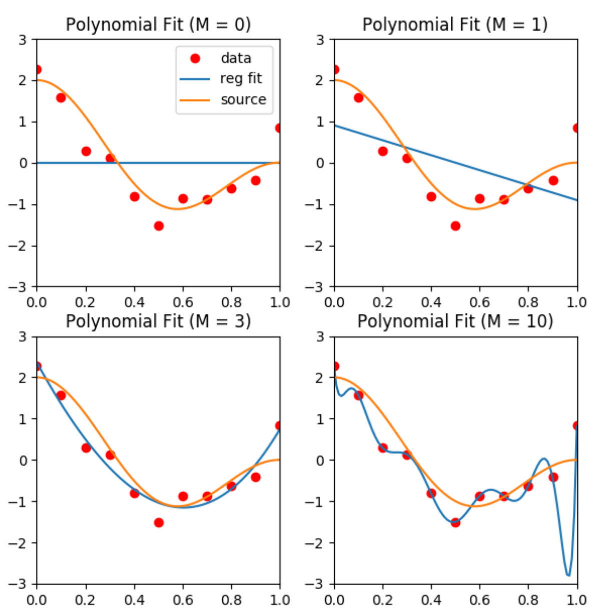

Fig. 1: M-Degree Polynomial Fit (M = 0, 1, 3, 10)

[1]See the Appendix for the numerical values of the weights.

B. Gradient Descent Solution with Polynomial Basis

C. Closed-form Solution with Cosine Function Basis

III. Ridge Regression

IV. Sparsity and Lasso

V. Appendix