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Crude oil price forecasting based on a novel hybrid long memory GARCH-M and wavelet analysis model

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Abstract: This paper proposes a novel hybrid forecast model to forecast crude oil price on considering the long memory, asymmetric, heavy-tail distribution, nonlinear and non-stationary characteristics of crude oil price. First, we use a signal de-noising method to reduce excessive noise significantly in the crude oil price. Then we employ empirical mode decomposition to transform the de-noised price into different intrinsic mode functions (IMFs). Finally, some complex long memory GARCH-M models are used to forecast different IMFs and a residual. Empirical results show that the proposed hybrid forecasting model WPD-EMD-ARMA-FIGARCH-M achieves significant effect during periods of extreme incidents. The robustness test shows that this hybrid model is superior to traditional models.

Key words: Crude oil price forecasting; Structural break period; Wavelet de-noising; Empirical mode decomposition; Complex long memory GARCH-M models

1. Introduction

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Recently, crude prices are on all the news wires for the rapid development of industrial economy and the consumer indispensable resource in daily life. And the government also make full use of international crude resources and establish national strategies crude reserves to ensure the country a sustainable and steady development. Besides, As a commodity in the futures market for sale, crude oil usually entails risks so that oil price jumps or plummets frequently (Tang and Wu et al., 2010; Masih and Peters et al., 2011). Furthermore, the geopolitical conflict and natural disasters and many other uncertainty events can also affect the volatility of crude oil price. Therefore, volatilities of crude oil price may exert a huge influence on countries stability, the global growth, the macro economy and social security (Sadorsky, 1999; Wu and Zhang, 2014). Thus, it is essential to develop a reliable model for crude oil price forecasting. However, since changes of crude oil price usually possess nonlinear, non-stationary, long memory and asymmetry characteristics, the oil price forecasting becomes more complicated and challenging. In other words, it is difficult to forecast and any results obtained uncertain. Therefore, crude price forecasting remains a huge challenge.

In recent literatures, there exists two types of forecasting model which can be classified into econometric forecasting models and soft scientific computing models. Some researchers have forecast short-term crude oil futures prices or oil price movement using econometric models (Morana, 2001; Abosedra and Baghestani, 2004; Moshiri and Foroutan, 2006; Coppola, 2008; Ye et al., 2009; Baumeister and Kilian, 2012). Other researchers have forecast spot and future crude oil prices by soft scientific computing models (Fan et al., 2008; Yu et al., 2008; Ghaffari and Zare, 2009; He et al. 2012; Jammazi et al., 2012; Zhang et al., 2015). Among them, (Karanasos, 2001; Mohammadi and Su, 2010) suggest that ARMA-GARCH type models or GARCH in mean type models seem to better deal with oil price forecasting. These models can also prove effective in forecasting other prices (Hickey et al., 2012; Liu et al., 2013). Cheong (2009) shows that the complex characteristics, like the asymmetry, heavy-tail distribution and leverage effect often occurs in the WTI crude oil market (Aloui and Mabrouk, 2010; Cheng and Hung, 2011), which makes crude oil price forecasting more complicated. Therefore, the traditional ARMA-GARCH type models or GARCH in mean type models that further consider the long memory and asymmetry behavior in them may well capture the complex crude oil behaviors and accurately analysis the trend of oil price and then increase the accuracy of crude oil forecasting. What is more, recent sudden events, like the global financial crisis, the European debt crisis and uncertain politics, have seen crude oil price become more irregular. This leads to characteristics of complex nonlinearity, dynamic variation and high irregularity in crude oil price (Watkins and Plourde (1994)). Thus, single and linear ARMA-GARCH type models or GARCH in mean type models may not deal with crude oil price forecasting accurately, especially during periods of extreme incidents. Therefore, nonlinear and non-stationary behavior should also consider into the crude oil forecasting. In other words, we should deal with these behaviors (nonlinear and non-stationary) before we can make a good forecasting. And then, we can make data processing before prediction. And the process can make crude oil price become smooth and it can be benefit for the following GARCH type models mentioned in the paper used to do the forecasting. In addition, in order to judge the effective of our crude oil price forecasting, we should distinguish the large volatilities and relatively gentle volatilities of crude oil forecasting interval. Besides, in order to illustrate the prediction accuracy, the different frequencies (daily or weekly) of crude oil price should also be considered in the forecasting. More importantly, the recent relatively volatility forecasting models should be considered in order to finally test the robust results.

For these reasons, this paper proposes a new hybrid oil price forecast model based upon wavelet analysis (soft scientific computing model) and complex long memory GARCH-M (econometric model). Firstly, in order to test our hybrid forecast model's ability in periods of extreme incidents, we use Markov Switching Dynamic Regression (MS-DR) model to determine two high volatility periods under daily and weekly crude oil prices. Secondly, we compare our hybrid crude oil price forecasting models under these period intervals with different (daily and weekly) frequencies. Thirdly, we compare other volatility models used in the crude oil price forecasting. In detail, a four-layer wavelet packet de-noise method can eliminate excessive noise in the high frequency of crude oil price, then empirical mode decomposition can divide the de-noised price into numbers of IMFs and a residual. Thus, this signal of de-noised oil price tends to be more stable and better for us to make forecasts. Next, complex long memory GARCH-M models enable us to make comparatively precise forecasts respectively under the consistent principle of stability, because these forecasting models are able to achieve the asymmetry and long memory features of crude oil price. Especially long memory GARCH-M models have allowed the volatility of crude oil price into oil price forecasting. Finally, in order to conduct a robust test, we compare our relatively superior hybrid forecasting model with other traditional forecasting models during random periods.

Our contributions are as follows: Firstly, we propose a hybrid crude oil price forecasting model combining wavelet analysis (consider the nonlinear and non-stationary behavior in the crude oil price) and complex long memory and asymmetry GARCH-M (consider the long memory and asymmetry behavior in the crude oil price). Secondly, we use the MS-DR model to determine the high volatility and gentle volatility of different frequencies (daily and weekly) of crude oil price forecasting interval. And we do the forecasting and compare the hybrid forecasting models in the paper under these intervals. Thirdly, in order to test our accurate predict results and examine the robustness of our hybrid forecast model's results, we compare the relatively superior hybrid forecast model with other models during random periods. The results show our hybrid forecasting model can better deal with the complex forecasting work both during periods of extreme incidents and during random periods.

This paper is structured as follows. Section 2 reviews the related literature. Section 3 describes the data used. Section 4 analyses the proposed empirical methods and presents the empirical results. Finally, Section 5 presents our summary and outlook.

2. Literature review

As the main driving force in the energy market, crude oil plays an important role in the development of the world economy. However, changes of crude oil price lead to fluctuations of stock markets, which may have an impact on the stable operations of economies (Narayan and Sharma (2014)). Thus, forecasting crude oil price not only benefits investors, but also affords advantages to policy makers. Within the current forecasting literature, there exists two types of forecasting model which are broadly classified into econometric forecasting models and soft scientific computing models. That is, GARCH family auto regressive augmented type models and neural network or wavelet analysis type models.

GARCH family models and augmented GARCH family models have been commonly used in recent forecasting literature, because classes of GARCH models are better able to characterize mean or variance of financial time series. In the short run, it is more appropriate for us to use univariate GARCH-type model to make forecasts. In the long run, GARCH classes of model which possess long memory and asymmetry properties may prove superior to other forecasting models, as the perturbation term of self-correlation coefficient slowly decays with hyperbolic rate. A summary of GARCH-type models in the recent literature is shown in Table 1. However, crude oil price is susceptible to the macro-economy and policy adjustments owing to the rapid development of the oil market. Thus, crude oil price sharply fluctuates in these periods, which may increase the residual error term simultaneously. Therefore, hybrid forecast GARCH models have been increasingly used in recent forecasting literature, including augmented GARCH-M type models, RS-GARCH type models, wavelet analysis methods or neural-network models, which can be combined with GARCH-type models, etc.

Table 1 Related forecasting literature using GARCH-type models

Related literature	Forecasting models	Main conclusion
Kang et al. (2009)	CGARCH and FIGARCH models	CGARCH and FIGARCH models are useful for modelling and forecasting persistence in the volatility of crude oil prices.
Wei et al. (2010）	GARCH-class models	Nonlinear GARCH-class models are capable of capturing long-memory and asymmetric volatility, which exhibit greater forecasting accuracy than linear ones, especially when forecasting over longer time horizons.
Aloui and Mabrouk (2010)	Three ARCH/GARCH-type models	The FIAPARCH model outperforms other models for VaR prediction.
Wang and Wu (2012)	Univariate and multivariate GARCH-class models	Univariate models allowing for asymmetric effects display the greatest accuracy.
Hou and Suardi (2012)	Parametric and nonparametric GARCH models	GARCH models yield superior performance relative to an extensive class of parametric GARCH models.
Arouri et al. (2012)	A variety of GARCH-type models	The FIGARCH model better fit the study data, but the degree of volatility persistence diminished significantly after adjusting for structural breaks.
Martos et al. (2013)	Univariate and multivariate GARCH models	Multivariate models improve prediction accuracy.
Chkili et al. (2014)	Linear and nonlinear GARCH-type models	The FIAPARCH model was found to be the best model for estimating VaR forecasts for

		both short and long trading positions.
Bentes (2015)	Three GARCH family models	FIGARCH (1, d, 1) model was superior compared to other models.
Klein and Walther (2016)	Mixture memory GARCH model	MMGARCH's forecasting of volatility and Value-at-Risk outperforms all other models.
Pan et al. (2017)	Nonlinear GARCH-MIDAS model	The nonlinear GARCH-MIDAS model performs significantly in the forecasting results.
Hung and Thach (2018)	GARCH type models	EGARCH(1,1) model with Student's t-distribution provides the most accurate forecast.
Herrera et al. (2018)	Linear and nonlinear GARCH type models	EGARCH(1,1) yields the most accurate forecasts at medium horizons.

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Wavelet analysis methods have become more commonly used for energy forecasting in recent years. In general, most time series contain noisy peaks and mutations. Besides, these noisy signals are not always white noise. So wavelet de-noising method can effectively distinguish these abrupt changes and noise within the signal, thus obtaining better results. Original sequence decomposition may improve forecasting accuracy by using methods of wavelet filtering (Joo and Kim (2015)). A summary of wavelet analysis literature is shown in Table 2. However, single wavelet models are limited by their data frequency and decomposition level, which may have some impacts on forecasting errors. Therefore, we hypothesize that hybrid forecasting models are conducive to reducing forecasting errors.

Table 2 Related forecasting literature using the wavelet analysis type models

Related literature	Forecasting models	Main conclusion
Yousef et al. (2005)	Discrete wavelet transform (DWT)	The wavelet-based forecasting procedure works best for large sample sizes, approximately 100 and larger.
He et al. (2012)	Novel wavelet decomposed ensemble algorithm	Superior performance of the proposed algorithm against the benchmark models in terms of both level and directional predictive accuracy.
Haven et al. (2012)	Wavelet de-noising and decomposition	Out-of-sample price forecasting is significantly improved after noise is removed using the wavelet method.
Tang et al. (2015)	CEEMD based on EELM	A novel CEEMD-based EELM ensemble model statistically outperforms all listed benchmarks in prediction accuracy.

Hua and Jiang (2015)	Improved empirical model decomposition (EMD) method	Prediction results after decomposition approximate real results and suggest more accurate price trends, which can help investors to choose better strategies in the big data era.
Pany and Ghoshal (2015)	Local linear wavelet neural network model	The results presented in this paper demonstrate the potential of the proposed approach and show its effectiveness for electricity price forecasting.
Yu et al. (2015)	A decomposition–ensemble model with data-characteristic-driven reconstruction	Empirical study statistically verifies the effectiveness of the proposed model.
Yu et al. (2016)	A novel decomposition ensemble model with extended extreme learning machine	The model is shown superior in terms of high accuracy, time saving and robustness.
Yu et al. (2016)	Grid-GA-based LSSVR model	It can be utilized as one effective forecasting tool for crude oil price with high volatility and irregularity
Yu et al. (2017)	A novel ensemble forecasting approach	The superiority of the SR - based FNN method over some other popular forecasting models and similar ensemble models (with other decomposition tools).
Yu et al. (2017)	An LSSVR ensemble learning paradigm with uncertain parameters	The crude oil price prediction verifies effectiveness of the model.
Zhao et al. (2017)	A deep learning ensemble approach	Some competing approaches and shows superior forecasting ability that is statistically proved by three tests.
Ding (2018)	A novel decompose-ensemble methodology with AIC-ANN	This approach outperforms some other benchmark models by means of forecasting accuracy and hypothesis tests.
Sun et al. (2018)	Interval decomposition ensemble approach	This approach outperforms some other benchmark models by means of forecasting accuracy and hypothesis tests.
Tang et al. (2018)	A randomized-algorithm-based decomposition-ensemble learning methodology	The proposed method outperforms popular single methods and ensemble variants in accuracy.
Tang et al. (2018)	A non-iterative decomposition-ensemble learning paradigm using RVFL network	It outperforms popular methods and the respective ensemble variants in accuracy.

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Dong et al. (2019)	EMD-based methods	The proposed OEMD - based methods considering multiscale complexity significantly outperform the benchmarks based on typical EMDs in prediction accuracy.
Wu et al. (2019)	Improved EEMD and LSTM networks	This hybrid model is promising for forecasting crude oil price.

Finally, financial instability or policy uncertainty may impact on energy prices, which usually presents as irregular fluctuations. Thus, it is relatively difficult to achieve precise forecasting, especially when dealing with samples with different frequencies and periods of extreme incidents. Generally, single forecasting model precision is likely to be reduced. As a consequence, hybrid forecasting models have become more common in recent years. Hybrid forecasting models are generally superior to single conventional forecasting models. Typically, these hybrid models have a more promising forecasting prospect when compared to single models. A brief review of the literature on hybrid forecasting models is shown in Table 3.

Table 3 Related literature using hybrid forecasting models

Related literature	Forecasting models	Main conclusion
E Silva et al. (2010)	Wavelet analysis and HMM model	The proposed methodology might be a useful decision support tool for agents participating in the crude oil market.
Dong et al. (2011)	EMD and ARIMA model	The proposed model may improve prediction accuracy noticeably.
Liu et al. (2011)	Five different GARCH approaches	ARMA–GARCH (-M) approaches may effectively catch trend changes in the mean and volatility.
Islam et al. (2012)	MEMD and ARMA model	MEMD-ARMA-based methods perform better than other methods.
Chang (2012)	Flexible regime-switching EGARCH model	Regime switches and asymmetric basis effects play decisive roles in forecasting returns, volatility and tail distributions.
Karthikeyan and Kumar (2013)	Wavelet and EMD based time series models	A wavelet-based time series algorithm can be used to model events such as droughts with reasonable accuracy.
Liu and Shi (2013)	ARMA–GARCH models	ARMA–GARCH-M models are in general an effective tool for modelling and forecasting the mean and volatility of electricity prices, while ARMA–SGARCH-M models are simple and robust and the ARMA–GJRGARCH-M model is very competitive.

Kang and Yoon (2013)	ARFIMA–GARCH-class models	The ARFIMA–FIGARCH model better captures long-memory properties of returns and volatility. The out-of-sample analysis indicates no unique model for all three types of petroleum futures contracts.
Mensi et al. (2014)	ARFIMA–FIGARCH model	Out-of-sample forecasts provide mixed results and indicate that none of the specifications of the volatility model are appropriate for analyzing LM dynamics in the Saudi Arabian exchange market.
Aloui and Hamida (2014)	ARFIMA-FIGARCH and ARFIMA-FIAPARCH models	The forecasting ability analysis showed that the FIAPARCH model under skewed a Student-*t* distribution substantially improved VaR and the ES forecasts.
Sun et al. (2015)	ARMA–GARCH models	ARMA–GARCH-M models make the mean radiation equations more sufficient.
Li et al. (2017)	Econometric model and AI techniques	The results statistically support the powerful predictive effectiveness.
Yu et al. (2017)	AI techniques combine with Markov-switching ARFIMA model	It is a fairly good candidate for crude oil price forecasting in terms of either one-step prediction or multi-step prediction.
Yu et al. (2019)	Econometric model and AI techniques	The proposed online big-data-driven forecasting work with Google trends improves on the traditional techniques.

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Overall, continuous sharp rises or drops of crude oil price drives a non-linear and non-stationary trend. Thus, multi-step forecasting has improved simultaneously. Data pre-processing is an important step in the process of energy forecasting in existing forecasting literature. However, crude oil price forecasting still faces many uncertainties, and the robustness is not high across different samples. Improper handling of the data under different frequencies may lead to high deviation in forecasting results. Moreover, financial crises or uncertainty incidents may lead oil prices to fluctuate excessively, so the series often presents high volatility intervals and some abnormal trends. Thus, forecasting in this interval may often fail.

In summary, this paper tries to use hybrid forecasting and multi-step forecasting methods on the WTI crude oil price. In detail, firstly, this paper uses a MS-DR method to determine two forecasting intervals; secondly, we conduct a multi-step forecast with a hybrid forecasting model based upon wavelet analysis and a complex long-term memory GARCH-M model under skewed t distribution. The detailed prediction step is shown in Figure 5.

3. Data and descriptive statistics

3.1 Data description

In this paper, we use daily and weekly WTI crude oil prices between 2000 and 2015, and we select preferable hybrid forecasting models as mentioned above. Next, daily, weekly and monthly crude oil prices are selected as random forecasting samples. Crude oil prices in the paper all convert into returns. The formula is as follows, $R_t = \log\left(P_t / P_{t-1}\right)$. P_t indicates the price of crude oil at a time point.

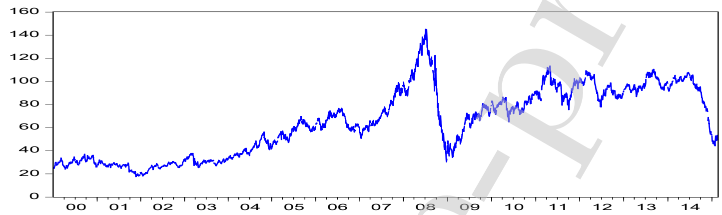


Fig. 1. Fluctuation in WTI crude oil (daily) price

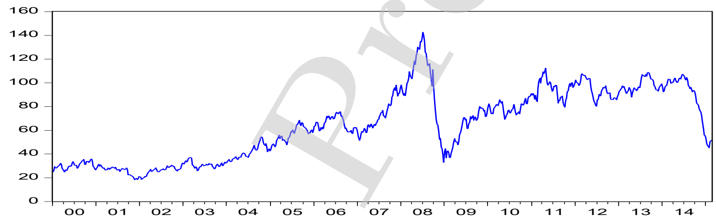
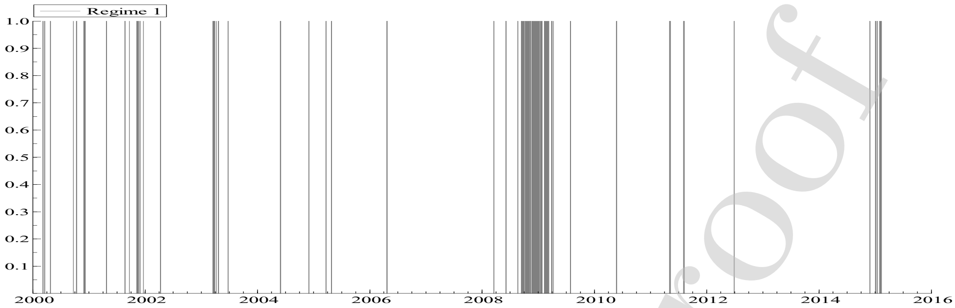


Fig. 2. Fluctuation in WTI crude oil (weekly) price

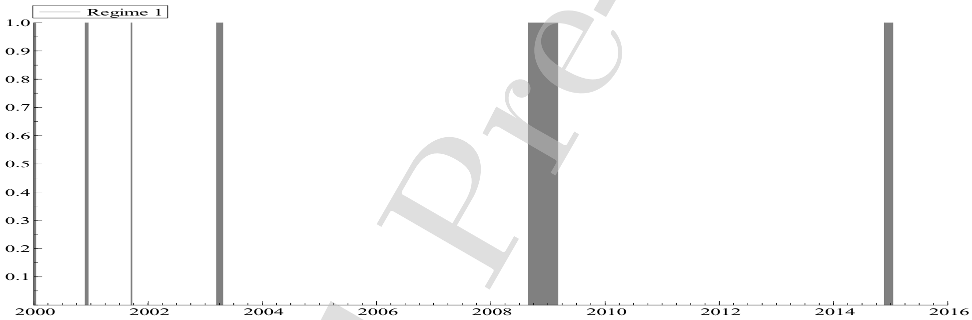
As is shown in Figure 1 and 2, influenced by global financial turmoil, slow global economic growth, unbalanced political structures in districts and imperfect energy markets, the crude oil price continuously fluctuates. More specifically, all of the products were affected by the Asian financial crisis in 1998 and the subsequent impact of the 911 terrorist attacks in 2001. In 2003, the crude oil price rose slightly as a result of general strikes in crude oil exporting countries such as Venezuela. In 2004 and 2005, against a background of increasing demand for crude oil worldwide, OPEC declared that it will reduce its output of crude oil, which drove the oil price up continuously until it hit the peak before the financial crisis. Besides, there was a sharp increase in demand for crude oil as a result of regional politics and the economic development of emerging countries over the same period. However, affected by the global financial crisis in 2008, the crude oil price dropped dramatically and then fell to its lowest point. Afterwards, with the recovery of the world economy in 2009 and

2010, the price of crude oil increased rapidly and maintained steady growth until the first half of 2015. After that, crude oil price fell again and reached its second lowest point since the financial crisis in 2008 due to regional economic and political incidents.



WTI

Fig.3. High volatility of crude oil (daily) price based upon MS-DR method



WTI

Fig.4. High volatility of crude oil (weekly) price based upon MS-DR method

We divided the high volatility of forecasting interval (periods of extreme incidents) according to the MS-DR model, which is the extension of the Markov Switching autoregressive model and the Markov Switching Regression (Hamilton, 1989). The grey-shaded regions in Figures 3 and 4 indicate excessive oil price fluctuations. In detail, we use January 4, 2000 to June 5, 2008 as the sample observation interval and June 6, 2008 to July 29, 2009 as the daily oil price forecasting interval under financial crisis period. We use January 7, 2000 to September 12, 2008 as the observation interval and September 19, 2008 to November 6, 2009 as the weekly oil price forecasting interval under financial crisis period. We use January 4, 2000 to October 20, 2014 as the observation interval and October 21, 2014 to February 20, 2015 as the daily oil price forecasting interval under uncertainty policy incident period. Finally, we use January 7, 2000 to December 26, 2014 as the observation interval and January 2, 2015 to February 13, 2015 as the weekly oil price forecasting interval under uncertainty policy incident period.

3.2 Data statistics analysis

As can be seem from Table 4, the means of daily and weekly crude oil prices are all positive, but the overall level is relatively low. That means crude oil price is usually subject to the impact of external factors; the rapid changes of oil price entails large risk so that the level of its return is comparatively low in this period. Besides, standard deviation shows that weekly crude oil price fluctuates significantly in comparison to daily oil price. Skewness of the crude oil prices are all negative, which indicates daily and weekly oil prices are in a bad situation most of the time. The Jarque-Bera test, Ljung-Box test and ARCH test are demonstrate the fat-tail distribution. The ADF and KPSS unit root test all show the stability of the crude oil price.

Table 4 Data description and unit root for crude oil price

	WTI crude oil (Daily）	WTI crude oil (weekly）
Data description		
Mean	0.0002	0.0009
Standard deviation	0.0245	0.0427
Skewness	-0.2536	-0.43819
Kurtosis	4.8712	3.483
Q(20)	48.6051***	64.698**
$Q^2(20)$	1915.94***	463.117**
Unit root test		
ADF	-35.0766***	-13.8907***
KPSS	0.167	0.2003

Notes: The table mainly reports crude oil price descriptive statistics. J-B test, Q (20), Q^2 (20) represents J-B test for normal distribution, L-B test for oil prices and square prices autocorrelation under 20th order respectively. ADF and KPSS test are used for unit root test for crude oil returns. *,**,*** represent rejection of the null hypothesis at the 10%, 5% and 1% level, respectively.

4. Empirical methodology

4.1. WPD and EMD

Affected by the economy and other factors, the crude oil price is often mixed with noise. Therefore, a de-noising process is a very important step before attempting an accurate forecast. Generally speaking, helpful information for forecasting often exists in low frequency or stationary signals in the crude oil price, while the noise usually exists in high frequency signals. The wavelet packet de-noising method can divide signals into multiple levels, and then decompose part of the high-frequency which cannot be subdivided by multi-resolution analysis. The method can choose corresponding frequency bands to analysis signal character, so it can match the signal spectrum and improve the time and frequency resolution. The specific steps are as follows:

In this paper, we let $\psi(t)\in L^2(R)$, $\psi(t)$ be a basic wavelet or mother wavelet in multi-scale resolution space, $L^2(R)=\underset{j\in Z}{\oplus}W_j$ at different scale factors j decomposed $L^2(R)$ into the sum of the orthogonal in all subspaces $W_j(j\in Z)$. Among them, W_j is the closure of wavelet function $\psi(t)$. We let $U_j^0=V_j, U_j^1=W_j, j\in Z$, U_j^n represent the wavelet subspace and V_j represent the scale space. The subspace U_j^n is defined as a function of closure space U_n^t. U_n^t is the closure space of functions U_{2n}^t. Let U_n^t satisfy the two scale equation:

$$U_{2n}(t)=\sqrt{2}\sum_{k\in Z}h(k)u_n(2t-k) \tag{1}$$

$$U_{2n+1}(t)=\sqrt{2}\sum_{k\in Z}g(k)u_n(2t-k) \tag{2}$$

Among them, $g(k)=(-1)^k h(1-k)$ is satisfied in the above equation, that is, $h(k)$ shows an orthogonal relation with $g(k)$. When $n\in Z_+$ (non-integer) is established, we obtain:

$$U_{j+1}^n=U_j^n\oplus U_j^{2n+1}, j\in Z, n\in Z_+ \tag{3}$$

When $n=0$ is established, $u_0(t)$ and $u_1(t)$ are degenerated to scale function $\varphi(t)$ and wavelet basis function $\psi(t)$. That is, the equations are as follows:

$$\varphi(t)=\sum_{k\in Z}h_k\varphi(2t-k),\{h_k\}_{k\in Z}\in l^2 \tag{4}$$

$$\psi(t)=\sum_{k\in Z}g_k\varphi(2t-k),\{g_k\}_{k\in Z}\in l^2 \tag{5}$$

As mentioned above, sequence $\{u_n(t)\}$ constructed by (4) and (5) is called an orthogonal wavelet packet, which is based on basis function $u_0(t)=\varphi(t)$. The wavelet packet is called as follows:

$$\psi_{j,k,n}(t)=2^{-j/2}\psi_n\left(2^{-j}t-k\right) \tag{6}$$

In the above mentioned, j represents the scale index; k represents the location index; and n represents the frequency index.

In this paper, we use the wavelet packet de-noising method to decompose the crude oil price into high frequency and low frequency signals, then we handle part of the high frequency and low frequency signal simultaneously in the form of a multilayer quantization method with a threshold value, and finally reconstruct the signal. The detailed process is shown in Figure 5.

The Hilbert Huang transform time frequency analysis is a new signal analytical method proposed by E. Huang Norden et al. (1998). The model is specialized in analysis of non-linear and non-stationary data under different times and frequencies. Its greatest advantage over previous methods is that it can make non-stationary signals stationary by way of empirical mode decomposition. Overall, detailed steps must be satisfied with the following conditions:

a. The function of the number of local extreme points and number of zero crossings must be equal or at most one across the whole-time range.
b. The mean value of envelope (the upper envelope) and envelope of the local minimum (the lower envelope) must be zero at any time.

Generally speaking, intrinsic mode function is shown as a simple oscillation model. Contrary to simple harmonic function, the crude oil price can be decomposed by the following procedures:

c. Distinguish local extreme value of crude oil price, including local maxima and local minima.
d. Upper and lower envelopes which aggregate all of extreme points are generated by the spline.
e. Corresponding envelope mean value is calculated by upper and lower envelopes according to the following formula:

$$m_t=\left(x_{up,t}+x_{low,t}\right)/2 \tag{7}$$

f. Extracting envelope mean from the crude oil price, the difference is then defined between envelope and mean of envelope, which is calculated as follows:

$$c_t = x_t - m_t \tag{8}$$

g. If c_t meets the above two conditions, then intrinsic mode function is extracted and replaced with the residual error $r_t = x_t - c_t$ with x_t; if c_t is not intrinsic mode function, using c_t replace with x_t.

h. The above steps are repeated until all standards are met. Through the above process, the crude oil price can ultimately be described as a module function and residual error, which is as follows:

$$x_t = \sum_{j=1}^{n} c_{j,t} + r_{n,t} \tag{9}$$

In the above equation, n represents the number of intrinsic mode functions; $r_{n,t}$ represents residual error; and $c_{j,t}$ represents the j th intrinsic mode function. All of the intrinsic mode functions are orthogonal to each other and have zero mean value. Thus, the crude oil price can be decomposed into n intrinsic mode functions and one residual error. Each frequency of intrinsic mode function is different and varies with the changes in the crude oil price x_t; $r_{n,t}$ represents the concentration trend.

R1Q2

Although the method mentioned above (empirical mode decomposition) still has some defects. For example, some partially decomposed signal can be confused within one IMF. Besides, some partially decomposed signal corresponding to the events can be integrated into different IMFs. But we still use this approach to do our crude oil forecasting. The detail reasons are as follows: Firstly, we started with the WPD and because the this method can adaptively select the best basis function according to the characteristics of the analyzed signal, so as to match it with the signal, and then improve the analysis ability of the signal firstly. And the application of the WPD and EMD can more accurately decompose the de-noised crude oil price signal and then effectively avoid the single EMD method defects. Secondly, wavelet analysis is used to de-noise the excessive noise in the high frequency of crude oil price, and an EMD method further stabilizes the de-noised signal. Meanwhile, these methods do not change the statistical properties of the crude oil price.

4.2. Complex long memory GARCH-M models

Because the crude oil price has fluctuated significantly and frequently in recent years, complex classes of GARCH-M model can improve forecast accuracy with data pre-processing. In accordance with the GARCH standard model proposed by Bollerslev (1986) and the ARCH-M model proposed by Engle et al. (1987), we obtain the GARCH-M model as follows:

$$y_t = \mu + \gamma h_t + \varepsilon_t \tag{10}$$

In the above equation (10), y_t represents conditional crude oil return (the difference of the log oil price); h_t represents conditional crude oil variance. ψ_{t-1} represents crude oil information at time t; $h_t = E\left[y_t \middle| \psi_{t-1}\right]$ and $\varepsilon_t = y_t - E\left(y_t \middle| \psi_{t-1}\right)$ is satisfied in the above equation (10).

Granger and Joyeux (1980) and Hosking (1981) mention that ARFIMA-type models can be used for measuring long memory according to established parameters. From the point of the conditional mean, $I(d)$ is considered as the fractional integration process, which is also the conditional mean process. ARFIMA(p,d,q) is as follows:

$$\varepsilon_t = u_t \sqrt{h_t} \tag{11}$$

$$\Psi(L)(1-L)^d (y_t - \mu) = \Theta(L)\varepsilon_t \tag{12}$$

In the above equation (12), L represents lag operator and the equations like $\Psi(L) = 1 - \Psi_1 L - \Psi_2 L^2 - \cdots - \Psi_p L^p$ and $\Theta(L) = 1 + \theta_1 L + \theta_2 L^2 + \cdots + \theta_q L^q$ must conform to the above equations (11) and (12), in which the root of all polynomials of autoregressive and moving average is located outside the unit circle.

Baillie et al. (1996) propose the FIGARCH model, which is distinguished by the slow decay of long memory. The model can distinguish long memory and short memory with conditional variance. The FIGARCH (1, d, 1) model is often defined as follows:

$$h_t = \omega + \beta h_{t-1} + \left[1 - \left(1 - \beta L^{-1}\right)\left(1 - \lambda L\right)\left(1 - L\right)^d\right]\varepsilon_t^2 \tag{13}$$

In the above equation (13), parameters are met in the following conditions: $\omega > 0$, $\beta < 1$, $\lambda < 1$. Fractional integral parameter d refers to the degree of long memory, that is, whether or not there are persistent shocks in conditional variance of crude oil price. Fractional integral parameter d must satisfy the following requirements: $0 \le d \le 1$. If condition $0 < d < 1$ is met in the model, it means there is a moderate impact and the shocks on crude oil decline at the speed of the decayed hyperbolic rate. If condition $d = 0$ is met in the model, it means the model can decline to the model of GARCH (1,1) and possesses short memory. If condition $d = 1$ is met in the model, it means the model can decline to the model of IGARCH (1,1), so the model and conditional variance process no longer show the mean reversion.

Tse (1998) proposed the FIAPARCH model and explained the long memory and asymmetry properties of conditional volatility simultaneously. The model is as follows:

$$\left(\sqrt{h_t}\right)^{\delta} = \omega\left(1-\beta L\right)^{-1} + \left[1-\left(1-\beta L\right)^{-1}\left(1-\lambda L\right)\left(1-L\right)^{d}\right]\left(\left|\varepsilon_t\right| - \gamma\varepsilon_t\right)^{\delta} \tag{14}$$

In the above equation (14), parameters are often met in the following conditions, $\omega > 0$, $\delta > 0$, $\beta < 1$, $\lambda < 1$. Among them, γ represents an asymmetry property and must satisfy the condition $-1 < \gamma < 1$. If $\gamma > 0$ is satisfied in equation (14), it indicates the volatility of the crude oil price may suffer a greater negative impact. If $0 \le d \le 1$ is satisfied in equation (14), it indicates there must have been long memory in the conditional variance process; the situation is the same as the fractional integral parameter d in the FIGARCH model.

These models mentioned above are all under skewed t distributions, that is $Z_t \sim Skst\left(0,1,k,v\right)$. The skewed t distributions with maximum likelihood values are as follows:

$$\begin{aligned} L_{Skst} = {} & T\left\{In\Gamma\left(\frac{\nu+1}{2}\right) - In\Gamma\left(\frac{\nu}{2}\right) - \frac{1}{2}In\left[\pi\left(\nu-2\right)\right] + In\left(\frac{2}{k+\left(1/k\right)}\right) + In\left(s\right)\right\} \\ & -\frac{1}{2}\sum_{t=1}^{T}\left[In\left(\sigma_t^2\right) + \left(1+\nu\right)In\left[1+\left(1+\frac{\left(SZ_t+m\right)^2}{\left(\nu-2\right)}\right)k^{-2I_t}\right]\right] \end{aligned} \tag{15}$$

In the above equation (15), $\Gamma\left(\cdot\right)$ represents a gamma function and $2 < \nu \le \infty$ must also satisfy the condition. If parameter $Z_t \ge m/s$ or $Z_t < m/s$ is met in the above equation (15), then $I_t = -1$. Contrary to normal distribution, parameter ν represents the degrees of freedom under skewed t distribution, and the parameter is used to measure the density of heavy-tail distribution.

4.3. Hybrid forecasting based upon complex long memory GARCH-M models and wavelet analysis

As the crude oil price possesses non-linear and unstable characteristics, hybrid forecasting based upon complex long memory GARCH-M models and wavelet analysis may achieve a better effect. We found superior results of these models in our paper. In detail, signal de-noising methods can reduce relative noise during periods of excessive fluctuation. The EMD method can make time and frequency decompositions, thus de-noising the WTI crude oil price. Due to their characteristics of long memory, heavy-tail distributions and asymmetry as described above, complex long memory GARCH-M models are used to make forecasts, and then corresponding indicators are used to evaluate the hybrid forecasting models. The detailed forecasting steps are as follows:

Step 1. As is shown in figure 5, we employ signal de-noising methods with four layer decomposition, and then we decompose the upper level of the low frequency and high frequency bands simultaneously. After this transformation, multiple resolutions are well-suited for our wavelet de-noising method. We are then able to better capture the non-stationary characteristics of the signal, including spikes and break points.

Step 2. Following Step 1, the EMD method is used to allow different time and frequency decomposition, that is $X_t = \sum_{j=1}^{n} C_{j,t} + R_{n,t}$, n represents number of intrinsic mode functions; $R_{n,t}$ represents residual and $C_{j,t}$ represents the j th intrinsic mode function.

Step 3. Hybrid forecasting models with long memory, asymmetry and skewed t distributions are used to make subsection forecasts, and then generate the final forecasting results. Evaluation indicators ME, MAE and MSE are used to test the model.

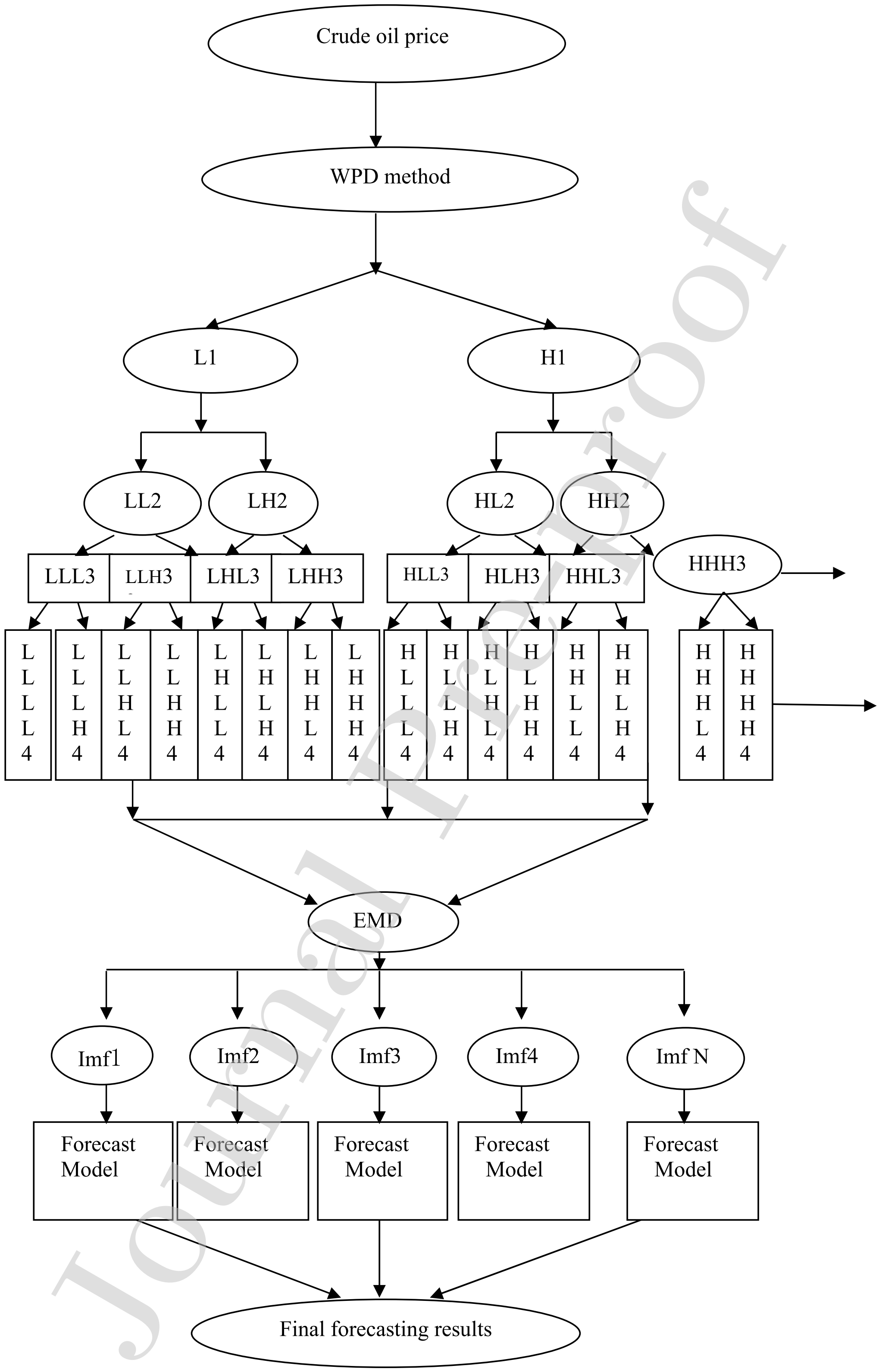


Fig. 5. Hybrid forecasting model for WTI crude oil price structure diagram

Notes: L represents the low frequency of WTI crude oil price; H represents the high frequency of WTI crude oil price; M represents the ARMA-FIGARCH-M, ARMA-FIAPARCH-M, ARFIMA-FIGARCH-M, ARFIMA-FIAPARCH-M forecasting models.

4.4. Forecasting evaluating indicators

In this paper, ME, MSE, MAE are used to evaluate the forecasting model. These indicators are as follows:

$$ME = \frac{1}{T}\sum_{t=1}^{T} e_t \tag{16}$$

$$MSE = \frac{1}{T}\sum_{t=1}^{T} e^2 \tag{17}$$

$$MAE = \frac{1}{T}\sum_{t=1}^{T} |e_t| \tag{18}$$

The indicator is calculated by $e_t = y_t - \hat{y}_t$. Among them, y_t represents the crude oil price; $\hat{y}_t$ represents their forecast price. T represents forecasting time interval.

5. Empirical results analyses

5.1 Results for crude oil price parameter estimation

In order to examine the long memory, asymmetry and fat tail distribution characteristics in the daily and weekly crude oil price, we choose six complex long memory GARCH-M models with skew t distribution in this paper.

Table 2 demonstrates that almost all models are significant at the level of 1%. This means these models are well established. So these models can effectively analyze the rapid changes of the daily crude oil price. Among them, AR coefficients are all negative, which means past crude oil price can carry negative information on current oil price. Parameter d shows that the long memory character can also exists in the daily crude oil price. This is mainly caused by the extreme incidents between 2000 and 2015. The ARCH coefficients are all positive, which indicates oil price easily subject to positive impacts from its own shocks. The GARCH coefficients show that the daily crude oil price fluctuates frequently. Besides, the APARCH coefficients demonstrate that negative impacts are larger than positive impacts on the crude oil price. That is, the leverage effect can also exist in oil price, which is the asymmetric character. The fat tail indicates skew t distribution is useful in our established models.

The result of the parameter estimation for the weekly crude oil price in Table 3 is similar to Table 2. Most of the parameters are almost significant at the 1% level, and there seems to be long memory, asymmetry and fat tail distribution in the weekly crude oil price. In all, the daily crude oil price and weekly crude oil price possess these characteristics mentioned above, so it is beneficial for us to have data pre-processing and then have a relatively accurate forecast.

Table 5 Parameter estimation for daily crude oil price

	ARMA-FIGARCH-M	ARMA-FIAPARCH-M	ARFIMA-FIGARCH-M	ARFIMA-FIAPARCH-M
Const(m)	0.0004	-0.0003	0.0004	-0.0004
	(-0.0005)	(-0.0006)	(-0.0005)	(-0.0007)
AR(1)	-0.1295	-0.1547	-0.1031	-0.0154
	(-0.2689)	(-0.1941)	(-0.2903)	(-0.2184)
MA(1)	0.089	0.1183	0.0594	-0.0434
	(-0.2692)	(-0.1933)	(-0.3038)	(-0.2337)
d-FIGARCH	0.5588	0.4044	0.5582	0.3975
	(-0.0677)***	(0.0554)***	(0.0681)***	(0.0621)***
d-ARFIMA			0.0033	0.0235
			(-0.0256)	(-0.0279)
ARCH	0.3454	0.384	0.3461	0.3844
	(0.0537)***	(0.0563)***	(0.0547)***	(0.0559)***
GARCH	0.7832	0.6821	0.7829	0.6772
	(0.0571)***	(0.0678)***	(0.0573)***	(0.0709)***
APARCH(γ)		0.4737		0.5135
		(0.1531)***		(0.1954)***
APARCH(δ)		1.3142		1.2951
		(0.1545)***		(0.1834)***
Asymmetry	-0.0788	-0.09	-0.0786	-0.0908
	(0.0242)***	(0.0234)***	(-0.0244)***	(0.0236)***
Tail	6.6694	7.5768	6.6607	7.4987
	(0.6928)***	(0.8971)***	(0.6985)***	(0.8879)***
ARCH-in-mean	0.1164	1.4292	0.1112	1.7852
	(-0.9483)	(-1.237)	-0.9727	-1.6153

Notes: This table mainly reports the daily crude oil price parameter estimation with six complex GARCH-M models. Standard errors are all presented in parentheses. *,**,*** indicate the rejection of null hypothesis under the significant level of 10%, 5% and 1%, respectively.

Table 6 Parameter estimation for weekly crude oil price

	ARMA-FIGARCH-M	ARMA-FIAPARCH-M	ARFIMA-FIGARCH-M	ARFIMA-FIAPARCH-M

Const(m)	-0.0019	-0.0012	-0.001	0.0003
	(-0.0025)	(-0.003)	(-0.0026)	(-0.0031)
AR(1)	-0.2533	-0.1614	-0.2462	-0.2708
	(-0.1536)	(-0.2778)	(-0.2146)	(-0.1809)
MA(1)	0.4554	0.373	0.4518	0.4612
	(0.1483)***	(-0.2713)	(0.1743)***	(0.1499)***
d-FIGARCH	0.5868	0.3774	0.6794	0.6981
	(0.1180)***	(0.1188)***	(0.1058)***	(0.1365)***
d-ARFIMA			-0.0067	0.0222
			(-0.0731)	(-0.0718)
ARCH	0.1722	0.282	0.1513	0.1494
	(0.0735)**	(0.1147)**	(0.0677)**	(0.0900)*
GARCH	0.7075	0.5958	0.783	0.7972
	(0.0922)***	(0.1567)***	(0.0655)***	(0.0826)***
APARCH(γ)		0.5872		0.2593
		(0.2742)**		(-0.2813)
APARCH(δ)		1.1768		1.9931
		(0.3059)***		(0.3522)***
Asymmetry	-0.1941	-0.203	-0.1821	-0.204
	(0.0627)***	(0.0630)***	(0.0551)***	(0.0604)***
Tail	7.4873	9.3679	9.1294	8.5073
	(1.8261)***	(2.7528)***	(2.2440)***	(2.2963)***
ARCH-in-mean	1.8971	1.4644	1.8765	0.2563
	(-1.3544)	(-2.3043)	-1.4531	-1.7832

Notes: This table mainly reports the weekly crude oil price parameter estimation with six complex GARCH-M models. Standard errors are all presented in parentheses. *, **, *** indicate the rejection of null hypothesis under the significant level of 10%, 5% and 1%, respectively.

5.2. Results for crude oil price under hybrid forecasting models

(1) Crude oil daily price forecasting results

The global financial crisis caused enormous fluctuations in the crude oil price, which represents a large swath of volatility between 2008 and 2009. Therefore, it is essential for investors or policy makers to develop a proper forecasting method, which is readily available and able to accurately predict the crude oil price over the longer term. Before the global financial crisis, the "911" attack, the war in Iraq, the Venezuelan workers strike, OPEC policy and other factors led to fluctuations in the crude oil price, which may have long memory on the time-period between 2008 and 2009. Thus, long memory and asymmetric GARCH-M models may benefit the analysis of large fluctuations in the crude oil price. The WPD (wavelet packet de-noise) method can effectively eliminate noise in the changes of oil price, stabilizing it. The de-noised oil price can be further subjected to EMD wavelet decomposition, which is conducive to subsection forecasting and reduces forecast error. ARMA auto-regression models are more helpful for short-term forecasting, but the financial crisis lasted for a long time and its impact was persistent. Thus, a simple ARMA process is not suitable for its forecasting; our complex long memory GARCH-M model is effective in crude oil price forecasting over longer periods. This means risk in mean GARCH models combined with long memory and asymmetric effect may be more reliable and more accurate in forecasting. Thus, these models are suitable to sudden event periods of forecasting.

Daily forecasting results for the WTI crude oil price in Table 7 show that the WPD-EMD-ARMA-FIGARCH-M model is superior to other hybrid forecasting models proposed in this paper for the ME, MAE and MSE evaluation indicators. To sum up, wavelet analysis can render the crude oil price smooth and the FIGARCH-M model can accurately model the long-term sustainability of persistent fluctuations caused by historical events, such as the high volatility presented in Figure 3 and Figure 4. Together, these advancements make the forecasting performance more advanced as a whole. In addition to hybrid forecasts using complex FIGARCH-M models and wavelet analysis methods, forecast accuracy is also relatively high for hybrid forecasting models which combine the ARMA- FIAPARCH-M model with wavelet analysis, which means parameters in this model can roughly capture the volatility characteristics of the crude oil price over the study period. The negative impacts are far greater than the positive impacts for daily crude oil prices, and our results also show that the crude oil price continually experiences sustained and strong fluctuations during global financial crises, therefore, these models which possess long memory and asymmetric features can also better forecast them. Finally, our forecasting results show that hybrid forecasting models with simple long memory GARCH-M are better than models with dual long memory GARCH-M during high volatility periods of daily crude oil prices.

R1Q3

The detail explanations are as follows: Firstly, The global financial crisis cause the huge impact on crude oil market, and the negative influence are far more than the positive influence at that period. Therefore, long memory behavior in GARCH model can accurately capture the trend of volatility in crude oil market. Secondly, the de-noised signal combined with the wavelet decomposed method decreases the originated asymmetry behavior in the crude oil prices. Thus, the forecasting accuracy of the long memory and asymmetry behavior in GARCH model is a little inferior than that of the model that only considered long memory. Thirdly, the wavelet method can deal with the nonlinear and non-stationary crude oil price behaviors. And then retain the statistical characteristics of crude oil prices. Thus, long memory can be easily analysis the essential fluctuations of crude oil price. Therefore, government should establish the economic and financial risk alert mechanism and aware of the dynamic risk transmission between crude oil market and other markets. Besides, the managers and policy makers should figure out the extent of the influence of financial crisis on crude oil market and the duration of the impact that last.

Table 7 Errors in daily crude oil price forecast based upon hybrid forecasting models during a period of global financial crisis

2008.6.6—2009.7.29 (WTI crude oil)

Hybrid forecasting model	ME	MAE	MSE
WPD-EMD-ARMA-FIGARCH-M	**-0.0029**	**0.0333**	**0.002**
WPD-EMD-ARMA-FIAPARCH-M	-0.0137	0.0355	0.0022
WPD-EMD-ARFIMA-FIGARCH-M	-0.0216	0.0383	0.0025
WPD-EMD-ARFIMA-FIAPARCH-M	-0.0435	0.0522	0.004

Note：This form reports errors in daily crude oil price forecasting based upon hybrid models during financial crises. Forecast indicators are ME, MAE and MSE. Black/Bold forecast evaluation indicators represent a relatively better forecasting model.

Because the crude oil market is imperfect, oil price is easily influenced by external factors, such as macro-economic regulation, regional conflicts, economic and political events, etc. Therefore, crude oil was unbalanced in demand and supply between 2014 and 2015. Thus, the crude oil price a demonstrated high volatility during that period. However, the impact is less than during the global financial crisis. Crude oil price during this period also experienced a relative greater degree of short-term fluctuation. However, the global financial crisis, the European debt crisis, regional politics, and other factors have some direct and indirect effects on the crude oil price. This can further bring some fluctuations to the forecasting period between 2014 and 2015. To eliminate noise due to excessive fluctuations caused by these several uncertain factors, this paper uses a four-layer wavelet de-noising method to render these prices smooth. Then, EMD is further employed for de-noised prices under time and frequency decomposition, so complex long memory GARCH-M models can be used to better estimate parameters and make accurate forecasts. That is, long memory GARCH-M models consider the risk in mean, which can capture the unforeseen volatility in crude oil price forecasting.

The daily spot forecasting results for WTI crude oil in Table 8 show that the WPD-EMD-ARMA-FIGARCH-M hybrid forecasting model is superior to all other forecasting models. As mentioned above, the forecasting interval is relatively small and also conducive to the ARMA process for short-term forecasting. However, financial crises or other incidents still play an uncertain role on the current crude oil price. Therefore, hybrid forecasting models with long memory can predict the current price more accurately. Consistent with daily crude oil forecasts during financial crises, the FIAPARCH model without long memory in the mean may result in a relatively good performance in short-term price forecasting under huge fluctuations in crude oil prices during uncertain policy incidents. Furthermore, the performance of our hybrid forecasting model with simple long memory is relatively stable compared to other models.

R1Q3

The detailed reasons are as follows: Firstly, the large fluctuations of crude oil price in that period can be attributed to the irregular events. And these numerous irregular events are recurring within the sample analysis interval of this paper. Therefore, long memory behavior in the GARCH model can be easily identified once they occurred. Secondly, the asymmetry behavior is not strong in the irregular events and the time period is not last long. Hence, long memory and asymmetry GARCH model used in the crude oil price forecasting at that period may not perform well when compare with the only long memory GARCH model. Thirdly, according to the data processing by using of wavelet decomposition method, the long memory behavior can be significantly easier track the trend of irregular volatilities of crude oil prices. Hence, investors and managers should distinguish the irregular events and the extremely events from the volatilities of crude oil market. And then tests the long memory behavior in each component. Therefore, the prediction model based on long memory can be beneficial to do the crude oil market forecasting in the short term. Besides, government should establish and put forward the emergency measures to prevent irregular events to affect the crude oil market.

Table 8 Errors in daily crude oil price forecast based upon hybrid forecasting models in periods of uncertain policy incidents

2014.10.21—2015.2.20 （WTI crude oil）

Hybrid forecasting model	ME	MAE	MSE
WPD-EMD-ARMA-FIGARCH-M	**-0.0042**	**0.0251**	**0.001**
WPD-EMD-ARMA-FIAPARCH-M	-0.007	0.0257	0.0011
WPD-EMD-ARFIMA-FIGARCH-M	-0.0126	0.0271	0.0012
WPD-EMD-ARFIMA-FIAPARCH-M	-0.0147	0.0276	0.0013

Note：This form reports errors of daily crude oil price forecasting based on hybrid models in periods of uncertain policy incidents. Forecast indicators are ME, MAE and MSE. Black/Bold forecast evaluation indicator represents a relatively better forecasting model.

(2) Crude oil weekly price forecasting results

Because frequency of weekly data is relatively lower than the daily data, the extreme incident interval in weekly data is a little shorter than daily data. Thus, the degree of change in the weekly crude oil price is less than in the daily crude oil price. Therefore, our weekly spot crude oil price tends to be smoother under the same four-layer de-noising method and can better obtain frequency of the real signal. Then the EMD method can make the de-noised price more stable. That is to say, the excessive noise caused by the 911 terrorist attacks, the Iraq war, the OPEC policy incidents and other factors may reduce significantly. Thus, it is more suitable for complex long memory GARCH-M models under the principle of stability, because these models can better track the trends of the crude oil price because of its long memory and asymmetric features. In other words, it seems that hybrid forecasting models may achieve a relatively accurate result under lower frequency samples, particularly when forecasting in a sudden events period.

The weekly spot return forecasting results in Table 9 show that the WPD-EMD-ARMA-FIGARCH-M model in a period of financial crisis is also better than other models by using the forecasting evaluation indicators MAE and MSE. Thus, performance of this model is robust and the forecasting accuracy is superior to other models under different data frequencies during a period of financial crisis. In detail, the range of the weekly crude oil high volatility interval is smaller than the daily crude oil price high volatility interval. The ARMA process and FIGARCH-M model combined with wavelet analysis is relatively significant in forecasting. Forecasting effects are also relatively significant for our hybrid forecasting model based upon ARMA- FIAPARCH-M with wavelet analysis under weekly data during financial crises, which means historical volatility can also have an effect on the recent situation and the asymmetric effect may be a good variable in GARCH-M forecasting models.

R1Q3

The explanations for the weekly of crude oil price forecasting at the time period of financial crisis is almost the same for the daily data of crude oil price prediction. That is, the wavelet decompose method can effective denoise and decompose the original signal and make the processed data more stable. Therefore, the fluctuations in different intrinsic mode functions of crude oil price are not perform intensely. And the long memory GARCH model can be more suitable for the forecasting. Similarly, managers and investors should predict the downward trend of crude oil price and be able to determine in advance whether there is danger ahead of time response. Thus, minimize the losses due to the sharp fluctuations of crude oil market to the whole economy and the stability of the country.

Table 9 Errors in weekly crude oil price forecast based upon hybrid forecasting models during a period of global financial crisis

2008.9.19—2009.11.6 (WTI crude oil)

Hybrid forecasting model	ME	MAE	MSE
WPD-EMD-ARMA-FIGARCH-M	**0.0237**	**0.0635**	**0.0067**
WPD-EMD-ARMA-FIAPARCH-M	0.0385	0.0717	0.0079
WPD-EMD-ARFIMA-FIGARCH-M	0.0587	0.0844	0.01
WPD-EMD-ARFIMA-FIAPARCH-M	0.1097	0.1279	0.0207

Note：This form reports errors of weekly crude oil price forecasting based on hybrid models during global financial crises. Forecast indicators are ME, MAE and MSE. Black/Bold forecast evaluation indicator represents a relatively better forecasting model.

Figure 4 indicates that the crude oil price has high volatility during the global financial crisis between 2008 and 2009 and the uncertain policy incidents between 2014 and 2015, under weekly data. This indicates that weekly crude oil price fluctuations are almost the same as that of the daily oil price. This phenomenon also reflects that the crude oil price is influence by government policies and economic incidents. Besides, the high volatility interval is relatively shorter for the weekly crude oil price. So it is of benefit for us to use the ARMA process have a short forecast, while the global financial crisis and the European debt crisis may have had a large impact on trends of future crude oil price. So, complex long memory GARCH-M models can provide accurate oil price forecasts when effectively combined with the wavelet analysis method.

The results for the weekly crude oil price forecasting in Table 10 also show that the WPD-EMD-ARFIMA-FIGARCH-M model is superior to other models during periods of uncertain policy incidents. As in the above conclusions, ARMA-FIAPARCH-M combined with wavelet analysis is also significant. Thus, the long memory feature combined with the wavelet type method seems to provide suitable and effective forecasting both under different periods (extreme incident periods) and different frequencies. Besides, the asymmetric feature combined with the wavelet type method can also take effect in that situation.

R1Q3

The explanations for the weekly of crude oil price forecasting at the time period of uncertain policy incidents is the same as the daily data of crude oil forecasting. For the weekly data time period is shorter. And it is more adaptable for the long memory behavior to do the forecasting when combine with the wavelet decompose method. Analogously, managers should predict the trend of crude oil price from the irregular events and the extent by which the crude oil market could fall. And then, the effective defence system for the irregular events to the crude oil market can be better improved in the long term.

Table 10 Errors in weekly crude oil price forecast based on hybrid forecasting models in periods of uncertain policy incidents

2015.1.2—2015.2.13 (WTI)

Hybrid forecasting model	ME	MAE	MSE
WPD-EMD-ARMA-FIGARCH-M	**-0.0056**	**0.0366**	**0.0026**
WPD-EMD-ARMA-FIAPARCH-M	0.0084	0.0381	0.003
WPD-EMD-ARFIMA-FIGARCH-M	0.0126	0.0381	0.0031
WPD-EMD-ARFIMA-FIAPARCH-M	0.0131	0.0384	0.0031

Note：This form reports errors of weekly crude oil price forecasting based on hybrid models during uncertain policy incidents. Forecast indicators are ME, MAE and MSE. Black/Bold forecast evaluation indicator represents a relatively better forecasting model.

5.3 Comparing the hybrid forecasting model with traditional models

Frequent fluctuations in the crude oil price usually results in a lot of noise, which is an increasing challenge in crude oil price forecasting. In order to test the robustness of the hybrid forecasting model, we use different frequencies of crude oil price and compare the relatively superior hybrid forecasting model with other traditional models. Firstly, we use the daily data between 2008 and 2010 as the random forecasting interval, because this period covers the whole global financial crisis process. So, the crude oil price fluctuated sharply and then fell, followed by relatively stable fluctuations. Secondly, we choose the period between 2011 and 2015 as the weekly random forecasting period. During this period, European debt crisis, uncertain policy incidents may have caused the relatively sharp fluctuations between the smooth fluctuations. Lastly, monthly data between 2013 and 2014 was also used for the random testing. Because monthly volatility of the crude oil price is relatively slight, it is of benefit for us to track the oil price trend and then make forecasts. In all, we compare the hybrid forecasting model based upon complex FIGARCH-M and a wavelet analysis method to existing energy forecasting models in the recent literature.

Table 11 shows that the WPD-EMD-ARMA-FIGARCH-M model is also better than other models in daily crude oil price forecasting. While the forecasting indicators MSE and MAE are almost the same as each other, the ME indicator in our hybrid forecasting model is superior to the others. For this period, the oil price experienced the extreme global financial crisis incident, so most of the models may have lost their original forecast functions. Thus, as a commodity in the financial markets, added to the complexity of investor moods, the crude oil price tends towards irregularity and nonlinearity. Thus, our hybrid forecasting model may well established to have a forecast in this period after compare with other models. And this is almost the same with the high volatility interval forecasting results mentioned above.

R1Q3

The detail explanations are as follows: the financial crisis in the time period 2008 and 2009 has caused a tremendous impact on crude oil market, and then the followed European debt crisis continued this effect on crude oil market. Thus, the long memory behavior can be more intensely exists in crude oil market. Added with the wavelet decomposition method, the data processing can make the decomposed signal smoother and then more suitable for long memory GARCH model to forecasting. Consequently, policy makers should precisely judge the impact of crisis events on crude oil market. And then the long memory behavior can be used to measure the impact and further do the trend of crude oil price forecasting.

Table 12 shows that the WPD-EMD-ARMA-FIGARCH-M model is also superior to other models in weekly crude oil price forecasting. During this period, the financial global crisis may have induced the secondary fluctuations from the following European debt crisis and uncertain policy incidents, along with other incidents, like terrorist attacks, regional distributes and increasing demands of emerging markets. The crude oil price fluctuates continuously, so a wavelet method combined with a long memory model can accurately capture the effective information before this period and then provide relatively accurate forecasts. Besides, an asymmetric character model can also be useful for negative impacts, which are often larger than positive impacts on the crude oil price.

R1Q3

The detail explanations are as follows: At this time period, the global economy tends to recovery and runs to stable. And then the crude oil market experience a relatively stable fluctuations at that sample period. In other words, the irregular events gathered and the small volatility caused by these events can persist have impact on crude oil market. Besides, the irregular events are almost occur in the high frequency component of intrinsic mode function of crude oil price, and wavelet decompose method can easily identify these features. Therefore, it is advantage for the long memory behavior to play an important role in crude oil price forecasting. Thus, investors and policy makers should effectively identified the period of stable fluctuations of crude oil market. And then the fluctuations of crude oil market caused by the irregular events can be well predicted.

Table 13 further shows that the robustness of WPD-EMD-ARMA-FIGARCH-M model forecasting for the crude oil price. Because monthly data can better demonstrate the trend of oil price, our hybrid model can easily provide forecasts under this random period. In all, hybrid forecasting based on wavelet analysis and complex long memory GARCH-M models play an important role in both the high volatility periods and random periods under different frequencies.

R1Q3

The detail explanations are as follows: After experienced the large fluctuations caused by the extremely events. The crude oil markets showed a slight fluctuations. However, the extremely events can also influence the trend of crude oil price for a period of time. Thus, the long memory can better solve this problem combine with the wavelet decompose method. Therefore, although there is a period of steady fluctuation in the crude oil market. The crude oil price volatility can still have some influence on the future oil price fluctuations. And the investors and policy makers still can use the long memory GARCH model to do the forecasting.

Table 11 Errors in daily crude oil price forecast based upon hybrid forecasting models using random period

2008.1.4—2010.12 .31 (WTI crude oil)

Hybrid forecasting model	ME	MAE	MSE
WPD-EMD-ARMA-FIGARCH-M	**-0.0001**	**0.0222**	**0.0010**
ARMA-GARCH-M	-0.0015	0.0222	0.0010

FIGARCH-M	-0.0012	0.0222	0.0010
ARFIMA-FIGARCH	-0.0012	0.0222	0.0010
ARFIMA-FIAPARCH	-0.0010	0.0222	0.0010

Note：This table reports errors in daily crude oil price forecasting based upon hybrid models under random periods. Forecast indicators are ME, MAE and MSE. Black/Bold forecast evaluation indicators represent relatively better forecasting models.

Table 12 Errors in weekly crude oil price forecast based upon hybrid forecasting models using random period

2011.1.7—2015.2 .13 (WTI crude oil)

Hybrid forecasting model	ME	MAE	MSE
WPD-EMD-ARMA-FIGARCH-M	**-0.0001**	**0.0226**	**0.0009**
ARMA-GARCH-M	-0.0059	0.0233	0.0010
FIGARCH-M	-0.0050	0.0231	0.0010
ARFIMA-FIGARCH	-0.0053	0.0231	0.0010
ARFIMA-FIAPARCH	-0.0045	0.0230	0.0010

Note：This table reports errors in weekly crude oil price forecasting based upon hybrid models under random period. Forecast indicators are ME, MAE and MSE. Black/Bold forecast evaluation indicators represent relatively better forecasting models.

Table 13 Errors in monthly crude oil price forecast based upon hybrid forecasting models using random period

2013.1—2014.12 (WTI crude oil)

Hybrid forecasting model	ME	MAE	MSE
WPD-EMD-ARMA-FIGARCH-M	**-0.0377**	**0.057**	**0.0076**
ARMA-GARCH-M	-0.0391	0.0573	0.0076
FIGARCH-M	-0.0390	0.0571	0.0076
ARFIMA-FIGARCH	-0.0446	0.0596	0.0076
ARFIMA-FIAPARCH	-0.0401	0.0579	0.0111

Note：This table reports errors in monthly crude oil price forecasting based upon hybrid models under random period. Forecast indicators are ME, MAE and MSE. Black/Bold forecast evaluation indicators represents relatively better forecasting models.

6. Conclusion

Because of long memory, asymmetries, and the nonlinear and non-stable characteristics of the crude oil price, in this paper, we propose a hybrid forecasting model combining complex long memory GARCH-M models with wavelet analysis, and then forecast the crude oil price during periods of extreme incidents. Energy prices are more sensitive to sample objects, sample frequencies and high volatility intervals. In order to model large fluctuations within the forecasting interval accurately (extreme incidents interval), we employ am MS-DR method and a period of dramatic crude oil volatility as the forecasting interval in this paper. We try to eliminate noise under circumstances of excessive violent fluctuations and obtain more accurate forecasting results for multiple timescales and frequencies. First, wavelets and EMD methods are used to de-noise the crude oil price. Secondly, we construct complex long memory GARCH-M models and do subsection prediction under high volatility intervals at different frequencies. Finally, we compare the model to forecasting models in the current literature under random periods and different frequencies.

Empirical results firstly show that signal de-noising method can render crude oil prices smooth. Then, signals from oil prices can be extracted by eliminating high-frequency noise. Wavelet empirical mode decomposition then decomposes different times and frequencies of the intrinsic mode function, which makes the non-stationary signals stationary and so improves forecasting accuracy. Secondly, we try to introduce complex long memory GARCH-M models under skewed t distribution and predict oil prices across different timescales and frequencies. These models achieve characteristics of long memory, asymmetry, persistence volatility, strenuous fluctuation and heavy tailed distribution, according to empirical results in the paper. Thirdly, our forecasting results with different frequencies during high volatility interval show that the WPD-EMD-ARMA-FIGARCH-M is better than other models in oil price forecasting. Moreover, in comparison with traditionally forecasting models from the literature for random periods, we find that our WPD-EMD-ARMA-FIGARCH-M model is also superior to other forecasting models through the evaluation of ME, MAE and MSE. This means the model possesses relatively robust forecasting performance under different frequencies, especially under two high volatility intervals. That is, a long memory GARCH-M model combined with wavelet analysis is reliable in forecasting crude oil prices. In addition, our complex long memory GARCH-M models, such as the ARMA-FIAPARCH-M model combined with a wavelet analysis method, also have relatively high prediction rates.

Overall, the WTI crude oil price shows properties of long memory and asymmetry during high volatility intervals represented by financial crises and uncertain policy incidents. The model proposed in this paper is effectively established and can capture beneficial information for the process of forecasting. The paper use skewed t distribution which can better capture leptokurtic features of oil price volatility, which can help investors determine general trends in oil price and avoid significant market risks. These forecasting models can also aid in the forming of effective policies. In a word, no matter the irregular events or the extremely events that occur in the crude oil market, policy makers should be carefully identified and should be aware of the influence on crude oil market. Therefore, long memory behavior can play an important role in measure this process and then accurately predict the trend of the crude oil price in the short and long term.

R1Q3

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Declaration of interests

☒ The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

☐The authors declare the following financial interests/personal relationships which may be considered as potential competing interests:

Highlights:

- We propose a novel hybrid WPD-EMD-ARMA-FIGARCH-M model to forecast crude oil price.
- Long memory, asymmetric, heavy-tail, nonlinear and non-stationary are included.
- Empirical results show that the proposed model achieves significant effects.
- The robustness test shows that this hybrid model is superior to traditional models.