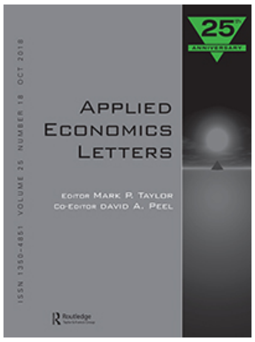

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ARTICLE

Multifractal detrended cross-correlations between WTI crude oil price fluctuations and investor fear gauges

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ABSTRACT
This article investigates the cross-correlations between WTI crude oil prices and fear gauges using cross-correlation statistic test and multifractal detrended cross-correlation analysis. The results show that the cross-correlations between crude oil prices and three different kinds of fear gauges are multifractal. By finding the 'crossover', we separate the three pairs of time series into the short term and long term, and find that cross-correlations of small fluctuations are persistent in the short and long terms, cross-correlations of large fluctuations are strongly anti-persistent in the short and long terms. The relationship is useful to profit in future markets.

KEYWORDS
Cross-correlations; MF-DCCA; WTI; fear gauges

JEL CLASSIFICATION
C32; C61

I. Introduction

Fractal theory is the frontier of nonlinear science theory, and is widely used in examining the relationship between two financial time series. Building on fractal theory, Kantelhardt et al. (2002) propose multifractal detrended fluctuation analysis (MF-DFA), while Podobnik and Stanley (2008) present detrended cross-correlation analysis (DCCA). Based on these two methods, Zhou (2008) proposes multifractal detrended cross-correlation analysis (MF-DCCA or MF-DXA) to detect the multifractal nature of the cross-correlations between two financial series. Pal, Rao, and Manimarana (2014) research the nonlinear relationship between gold and crude oil markets using MF-DCCA. Dutta (2016) compare results of autocorrelations (cross-correlations) using MF-DFA and MF-DCCA, and show that foreign exchange and SENSEX fluctuations in India are multifractal cross-correlated, the degree of multifractality can be effective in predicting fluctuations of SENSEX and FX rate. Auer (2016) provide evidence that Hurst coefficients in fractal theory are valuable tools for development of profitable trading rules in precious metals markets.

Maghyereh, Awartani, and Bouri (2016) investigate the directional connectedness between oil and stock prices' volatility, show the bi-directional information spillovers between oil and equity markets. Bouri et al. (2017) exam causality dynamics between gold and stock markets of China and India, and find a bi-directional causality between gold and the Chinese and Indian stock markets for short- and long-run horizons. Sarwar (2017) shows that increases in the stock market volatility (VIX) lead to contemporaneous and delayed increase in the volatilities of T-note, gold and silver prices. Though most studies examine the linkages between two financial time series, few study uses three kinds of volatilities of the stock to examine correlations with fluctuations of WTI oil futures. Then, previous studies examine the relationships often uses a qualitative way which cannot measure relevance degree, multifractal theory is not only a qualitative way but also a quantitative analysis to investigate relevance degree which is useful to profit in future markets.

This article focuses on the cross-correlations between crude oil prices and three kinds of market anxiety gauges. For the crude oil price, we choose the West Texas Intermediate (WTI) spot price, which is the most active crude oil in four major global crude oil futures market. Because the VIX,[1]

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[1]The VIX is a crucial gauge of the market's expectations of near-term (30-day) volatility which is widely considered to be the world's premier barometer of investor sentiment and market volatility. VIX values greater than 30 are generally associated with a large amount of volatility due to investor fear or uncertainty, while values below 20 generally correspond to less stress in the markets.

VXD[2] and VXN[3] are often referred to as ‘investor fear gauges’, we choose them to represent market anxiety and emotions. Our study contributes to understand the relationship between WTI crude oil price returns and expected volatilities of stocks (three kinds of fear gauges) using both qualitative and quantitative analyses. Through the MF-DCCA, we found the multifractality characteristics for the cross-correlated degrees between WTI crude oil price returns and fear gauges. It is important to estimate one financial time series’ trading volumes from the other according to the cross-correlated degrees.

II. Methodology

MF-DCCA proposed by Zhou (2008) can be expressed as follows.

Step 1. Supposed two time serijes $x(i)$ and $y(i)$ (i = 1, 2,..., N), where N is the equal length of these two series. Then, the ‘profile’ of each series is determined as follows:

$$X_i = \sum_{k=1}^{i}(x_k - \bar{x}),\ Y_i = \sum_{k=1}^{i}(y_k - \bar{y}),\ i = 1, 2, \ldots, N, \tag{1}$$

Where $(\bar{x})$ and $(\bar{y})$ describe the average returns of the two time series $x(i)$ and $y(i)$.

Step 2. The two time series $x(i)$ and $y(i)$ are divided into N_s = $[N/s]$ non-overlapping segments of the same length s. Since the length N of the series is not always a multiple of the considered time scale s, a short part of the profile (1) may remain. To ensure the completeness of the information is not disregarded in the time series, the same procedure is repeated starting with the opposite of the two series $x(i)$ and $y(i)$. Thus, $2N_s$ segments are obtained together.

Step 3. Define the local trends from an mth-order polynomial fit.

$$X_\lambda(j) = \alpha_k j^m + \alpha_{k-1} j^{m-1} + \ldots + \alpha_1 j + \alpha_0 \tag{2}$$

$$Y_\lambda(j) = \beta_k j^m + \beta_{k-1} j^{m-1} + \ldots + \beta_1 j + \beta_0. \tag{3}$$

Where $j = 1, 2, \ldots, s$, λ = 1, 2, ...,$2N_s$, m = 1,2,...

Step 4. Calculate the local trends for each $2N_s$ segment by an mth-order polynomial fit. The detrended covariance is determined by

$$F^2(s,\lambda) = \frac{1}{s}\sum_{j=1}^{s}\left|X_{(\lambda-1)s+j}(j) - X_\lambda(j)\right|\left|Y_{(\lambda-1)s+j}(j) - Y_\lambda(j)\right| \tag{4}$$

For each segment λ, λ = 1, 2, ..., Ns and

$$F^2(s,\lambda) = \frac{1}{s}\sum_{j=1}^{s}\left|X_{N-(\lambda-N_s)s+j}(j) - X_\lambda(j)\right|\left|Y_{N-(\lambda-N_s)s+j}(j) - Y_\lambda(j)\right| \tag{5}$$

For each segment $\lambda, \lambda = N_s + 1, N_s + 2, \ \ldots, \ 2N_s$. $X_\lambda(j)$ and $Y_\lambda(j)$ are the fitting polynomial of each profile with order m in segment λ, which is also referred to as MF-DCCA-m.

Step 5. Obtain the qth order fluctuation function by averaging all segments λ,

$$F_q(s) = \left\{\frac{1}{2N_s}\sum_{\lambda=1}^{2N_s} F^2(s,\ \lambda)^{q/2}\right\}^{1/q}. \tag{6}$$

While q = 0, Equation 4 can be defined as follows:

$$F_0(s) = \exp\left\{\frac{1}{4N_s}\sum_{\lambda=1}^{2N_s} \ln\left[F^2(s,\ \lambda)\right]\right\}. \tag{7}$$

Step 6. Analyze the scaling behaviour of the fluctuation function by observing the log–log plots $F_q(s)$ for each value of q. If the two series $x(i)$ and $y(i)$ are long-range cross-correlated, we can derive that $F_q(s)$ have large values of s. Thus, a power–law relationship can be expressed as follows:

$$F_q(s) \sim s^{H_{xy}(q)}. \tag{8}$$

Equation (8) can be written as follows:

$$\log F_q(s) = H_{xy}(q)\log(s) + \log(C). \tag{9}$$

Where C is the constant. The scaling exponent $H_{xy}(q)$, known as the generalized cross-correlation exponent, can be obtained from observing the slope of the log–log plots of $F_q(s)$ versus s by the method of ordinary least squares (OLS) for each value of q. If $H_{xy}(q) > 0.5$, the cross-correlations between the two time series related to q are persistent, expressing an increase of the series is statistically likely to be followed by an increase of the other series. If $H_{xy}(q) < 0.5$, the cross-correlations

[2]The VXD (CBOE DJIA (Dow Jones Industrial Average) Volatility Index) is based on real-time prices of options on the DJIA (DJIA, with an options ticker of DJX).
[3]The VXN index is based on the Nasdaq-100 index.

between the two time series related to q are anti-persistent, expressing an increase of the series is statistically likely to be followed by a decrease of the other series. If $H_{xy}(q) = 0.5$, the two series are not cross-correlated with each other, which means the alterations in one series do not affect the behaviour of the other. Zunino et al. (2008) proposed further measure the degree of multifractality ΔH, ΔH is descried as follows:

$$\Delta H = H_{max}(q) - H_{\min}(q). \tag{10}$$

Where the larger value of ΔH, the greater degree of multifractality and vice versa.

III. Data

We use daily WTI crude oil closing prices and three kinds of 'investor fear gauge', the VIX, the VXD and the VXN. As the VIX expanded from using options based on the volatility of S&P 100 to the volatility of the S&P 500 stock index option prices in 2 January 2004, the VXD expanded in 7 October 1997, and the VXN expanded in 2 February 2001. We choose sample data covering the period from 2 January 2004 to 19 June 2017, deleting missing data; each price series contain 3,389 observations. The original data are derived from the Chicago Board Options Exchange (CBOE) website and Wind financial database. The daily returns of WTI crude oil are defined as the logarithmic difference of the daily closing price P_t.

$$r_t = \log(P_t) - \log(P_{t\text{-}1}). \tag{11}$$

Since VIX, VXD and VXN are quoted as percentages, we used daily changes in these three fear gauges as follows:

$$\Delta V_t = V_t - V_{t-1}. \tag{12}$$

Where V denotes VIX, VXD or VXN respectively. r_t, ΔV_t observations are illustrated in Figure 1.

Table 1 shows the descriptive statistics for daily VIX changes, daily VXD changes, daily VXN changes, and daily returns of WTI crude oil. Each index of the mean value is close to zero, while each standard deviation is larger than zero. Each skewness is non-zero and kurtosis are all larger than 15, indicating strong deviations from normality. The Jarque-Bera test statistics show the rejection of the null hypothesis of normality at the 5% significance level. The ADF test shows the stationarity of daily VIX changes, daily VXD changes, daily VXN changes and daily WTI crude oil price returns.

IV. Empirical results

Cross-correlation test

For two time series, $\{x_i,\ i = 1, \ldots N\}$ and $\{y_i,\ i = 1, \ldots N\}$, the test statistic $Q_{cc}(m)$ proposed by Podobnik and Stanley (2008) expresses as follows:

$$Q_{cc}(m) = N^2 \sum_{i=1}^{m} \frac{{c_i}^2}{N - i}. \tag{13}$$

Where their cross-correlation function is:

$$c_i = \frac{\sum_{k=i+1}^{N} x_k y_{k-i}}{\sqrt{\sum_{k=1}^{N} x_k^2 \sum_{k=1}^{N} y_k^2}}. \tag{14}$$

The cross-correlation statistic $Q_{cc}(m)$ is approximately $\lambda\chi^2(m)$ distributed with m degrees of freedom. We select any degree of freedom, ranging from 10° t o $10^{3,}$ using $\lambda\chi^2$ (m) distribution. If there is no cross-correlation between the two time series, the $Q_{cc}(m)$ is well aligned with the $\chi^2(m)$ distribution. If the value of $Q_{cc}(m)$ strongly deviate from the value of χ^2 (m) distribution at the 5% level of significance, nonlinear cross-correlations exit between the two time series.

Figure 2 shows the cross-correlation statistic for daily changes in each 'fear gauge' and daily returns of WTI crude oil price compared with the critical values for the $\chi^2(m)$ distribution at the 5% level of significance, with the degree of freedom m ranging from 1 to 1,000. We can see that a nonlinear cross-correlated relationship existed between the two time series; thus, the MF-DCCA method is used to investigate the cross-correlations between the WTI crude oil and fear gauges further.

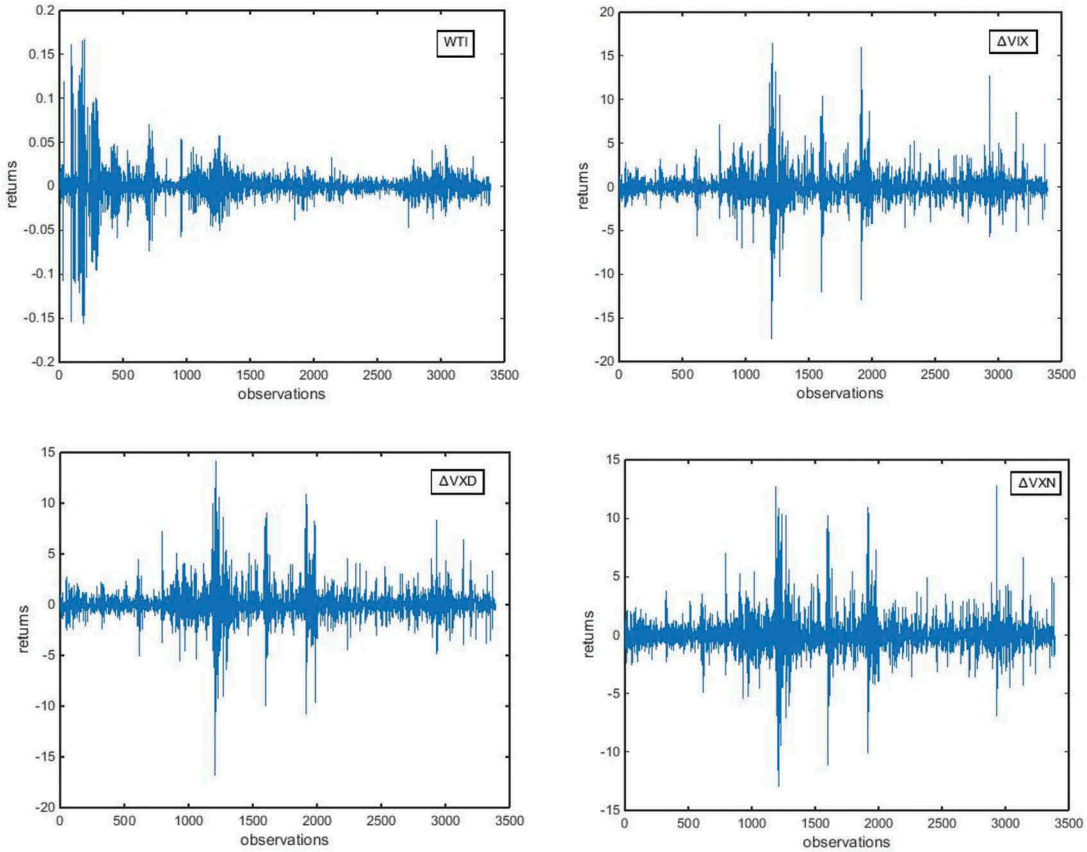

Figure 1. Returns of WTI crude oil and daily changes of three fear gauges (ΔVIX, ΔVXD, ΔVXN), respectively.

Table 1. Descriptive statistic for daily returns of WTI and daily VIX, daily VXD, daily VXN changes.

Variable	Mean (%)	Min	Max	SD	Variance	Skewness	Kurtosis	JB	ADF	No.
WTI	.0039	.1672	−.1566	.0188	.0004	.3726	28.65	92,976*	−88.32*	3388
ΔVIX	−.2317	16.54	−17.36	1.7475	3.0539	.6878	22.18	52,190*	−66.04*	3388
ΔVXD	−.1995	14.16	−16.82	1.5675	2.4571	.3261	20.95	45,554*	−69.13*	3388
ΔVXN	−.2925	12.75	−12.96	1.5817	2.5019	.6053	16.54	26,094*	−61.31*	3388

Note: *The rejection of the null hypothesis at the significance level of 5%.

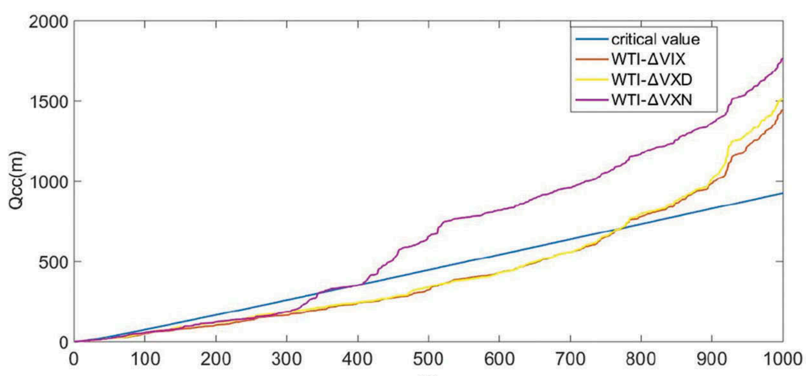

Figure 2. (Colors online) The cross-correlation statistic $Q_{cc}(m)$ for daily returns of WTI prices and daily fear gauges changes. The blue line denotes the critical value of the cross-correlation statistic; the red curve denotes the cross-correlation statistic for daily returns of WTI prices and daily fear gauge VIX changes; the yellow curve denotes the $Q_{cc}(m)$ for WTI-ΔVXD; and the purple curve denotes the $Q_{cc}(m)$ for WTI-ΔVXN. Each curve is derived from critical values of $\chi^2_{0.95}\ (m)$.

Multifractal detrended cross-correlation analysis (MF-DCCA)

To observe the nonlinear relationship between WTI crude oil price and fear gauges further, we adopt MF-DCCA to model the scaling behaviour of fluctuations and the multifractality of cross-correlations between different time series in a quantitative way.

Per Equations (1–9), we set $-10 \leq q \leq 10$, $8 \leq s \leq N/4$, polynomial order $m = 1$ and log–log plots of the fluctuation function $F_q(s)$ versus time scale s, depict for cross-correlations between daily returns of WTI crude oil price and daily changes of three different fear gauges from Figure 3.

Figure 3 shows there is a turning point in each pair of series called the 'crossover' per Podobnik et al. The crossover s^* divides the time span into the short term and long term. Short term financial market behaviour ($s < s^*$) is influenced by external market factors and long term behaviour ($s > s^*$) by internal factors; external shocks gradually decay and internal effects take place over a long period.

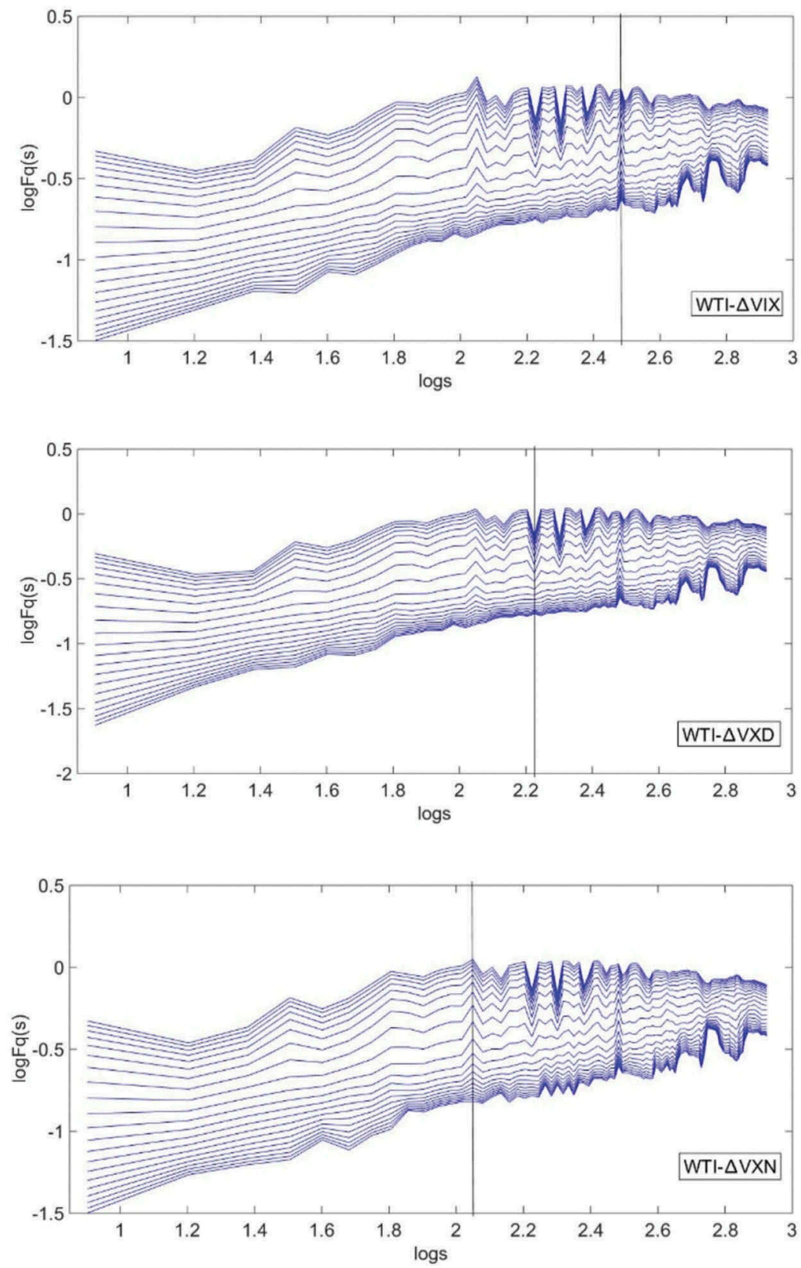

Figure 3. Log–log plots of the fluctuation function $F_q(s)$ versus time scale s for WTI-ΔVIX, WTI-ΔVXD and WTI-ΔVXN, respectively.

According to the log–log plots figures, the specific turning point for the s^* of the WTI returns and VIX changes occur at about $\log(s^*) = 2.4829$ (304days, or approximately 10 months); the crossover of WTI-ΔVXD at about $\log(s^*) = 2.2253$ (168 days, or approximately 5.5 months), and the crossover of WTI-ΔVXN at about $\log(s^*) = 2.0792$ (120 days, or approximately 4 months). To further explore the different cross-correlation exponents in the short term and long term, we calculate the cross-correlation exponent $H_{xy}(q)$ from Equation (9) using OLS versus different values of q for $s < s^*$ and $s > s^*$ (see Table 2).

From Figure 3 and Table 2, we can see $H_{xy}(q)$ changes versus different values of q, meaning the value of $H_{xy}(q)$ is dependent on q, and the cross-correlations between WTI price returns and three kinds of fear gauges are multifractal. For $s < s^*$ in the short term, except for $q = -10$, the exponent H_{xy} for WTI-ΔVIX is larger than 0.5, and except for $q \leq -6$, the cross-correlation exponents for WTI-ΔVXD, WTI-ΔVXN are larger than 0.5, while other values of exponent H_{xy} are all smaller than 0.5. This indicates that the cross-correlated behaviour of small fluctuations are persistent but the cross-correlated behaviour of large fluctuations are anti-persistent in the short term for the three pairs of time series. For $s > s^*$ in the long term, except for $q \leq -2$, the exponent H_{xy} for WTI-ΔVIX is larger than 0.5; except for $q \leq -4$, the cross-correlation exponent for WTI-ΔVXD is larger than 0.5; and except for $q \leq -6$, the cross-correlation exponent for WTI-ΔVXN is also larger than 0.5. Other values of exponent H_{xy} are all smaller than 0.5, showing that the cross-correlated behaviour of small fluctuations are persistent but the cross-correlated behaviour of large fluctuations are anti-persistent in the long term for these three pairs of time series.

To investigate the degree of multifractality for each pair of time series, we can calculate the value of $\Delta H(q)$ based on Equation (10). In Table 2, we can see the greatest degree of multifractality is WTI-ΔVIX in the long term, while the lowest degree of multifractality is WTI-ΔVIX in the short term. These suggest that the linkage effect of WTI crude oil and VIX changes is the strongest in the long term, and the lowest in the short term.

Our analysis highlights the important of using MF-DCCA in studying the dynamic relationship between WTI crude oil and implied volatility. This relationship between crude oil futures and stock market can help investors to find capital risk and chances. For instance, the linkage effect of crude oil and implied volatility implies that oil price uncertainty cannot be ignored, contemporaneously, such as hedge funds, it is possible to have chances to

Table 2. Cross-correlation exponents $H_{xy}(q)$ for WTI-ΔVIX, WTI-ΔVXD and WTI-ΔVXN, respectively.

	WTI-ΔVIX		WTI-ΔVXD		WTI-ΔVXN	
	$s^* = 304$		$s^* = 168$		$s^* = 120$	
Q	$s < s^*$	$s > s^*$	$s < s^*$	$s > s^*$	$s < s^*$	$s > s^*$
−10	0.5041	0.7307	0.6139	0.5522	0.5961	0.5302
−9	0.4937	0.7185	0.5984	0.5466	0.5815	0.5249
−8	0.4815	0.7046	0.5798	0.5406	0.5642	0.5191
−7	0.4673	0.6884	0.5575	0.5341	0.5440	0.5128
−6	0.4510	0.6696	0.5306	0.5269	0.5208	0.5060
−5	0.4330	0.6470	0.4996	0.5186	0.4951	0.4985
−4	0.4144	0.6194	0.4662	0.5087	0.4678	0.4898
−3	0.3970	0.5846	0.4342	0.4956	0.4402	0.4791
−2	0.3835	0.5395	0.4072	0.4768	0.4145	0.4642
−1	0.3769	0.4799	0.3881	0.4479	0.3937	0.4410
0	0.3795	0.3998	0.3805	0.4001	0.3823	0.4018
1	0.3900	0.2938	0.3894	0.3224	0.3865	0.3362
2	0.3985	0.1730	0.4117	0.2181	0.4094	0.2448
3	0.3929	0.0646	0.4295	0.1150	0.4376	0.1489
4	0.3760	−0.0180	0.4321	0.0339	0.4538	0.0697
5	0.3564	−0.0774	0.4251	−0.0247	0.4573	0.0110
6	0.3384	−0.1203	0.4150	−0.0668	0.4537	−0.0318
7	0.3231	−0.1523	0.4050	−0.0979	0.4474	−0.0635
8	0.3103	−0.1767	0.3959	−0.1216	0.4403	−0.0877
9	0.2996	−0.1959	0.3881	−0.1402	0.4334	−0.1066
10	0.2906	−0.2115	0.3814	−0.1552	0.4270	−0.1218
MF-degree	0.2135	0.9422	0.2334	0.7074	0.2138	0.6520

Note: 'MF-degree' is the degree of multifractality ΔH .

obtain abnormal returns by using the strong linkages between crude oil futures and stock market, and quantitative analysis suggests that linkages of WTI-ΔVIX in the long term should be applied first.

V. Conclusions

Previous studies show that the prediction of stock markets' implied volatility can play a role in hedging in futures trading. This article add the quantitative relations between three kinds of implied volatility and WTI crude oil price returns using economy physics theory MF-DCCA. It is meaningful to determine the amount of arbitrage transactions. Conclusions are obtained as follows.

Qualitative and quantitative analyses all confirmed that the cross-correlations between daily WTI crude oil price returns and changes in three kinds of daily fear gauges changes are multifractal. The strongest degree of multifractality is WTI-ΔVIX in the long term, and the lowest degree of multifractality is WTI-ΔVIX in the short term. Then, for three pairs of two time series (WTI-ΔVIX, WTI-ΔVXD and WTI-ΔVXN), the empirical results show that the cross-correlations of small fluctuations are persistent in the short term and long term, while the cross-correlations of large fluctuations are anti-persistent in the short term and long term.

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Disclosure statement

No potential conflict of interest was reported by the authors.

References

Auer, B. R. 2016. "On the Performance of Simple Trading Rules Derived from the Fractal Dynamics of Gold and Silver Price Fluctuations." *Finance Research Letters* 16: 255–267. doi:10.1016/j.frl.2015.12.009.

Bouri, E., D. Roubaud, R. Jammazi, and A. Assaf. 2017. "Uncovering Frequency Domain Causality between Gold and the Stock Markets of China and India: Evidence from Implied Volatility Indices." *Finance Research Letters* 23: 23–30. doi:10.1016/j.frl.2017.06.010.

Kantelhardt, J. W., S. A. Zschiegner, E. Koscielny-Bunde, S. Havlin, A. Bunde, and E. H. Stanley. 2002. "Multifractal Detrended Fluctuation Analysis of Nonstationary Time Series." *Physica A: Statistical Mechanics and Its Applications* 316: 87–114. doi:10.1016/S0378-4371(02)01383-3.

Maghyereh, A. I., B. Awartani, and E. Bouri. 2016. "The Directional Volatility Connectedness between Crude Oil and Equity Markets: New Evidence from Implied Volatility Indexes." *Energy Economics* 57: 78–93. doi:10.1016/j.eneco2016.04.010.

Pal, M., P. M. Rao, and P. Manimarana. 2014. "Multifractal Detrended Cross-Correlation Analysis on Gold, Crude Oil and Foreign Exchange Rate Time Series." *Physica A: Statistical Mechanics and Its Applications* 416: 452–460. doi:10.1016/j.physa.2014.09.004.

Podobnik, B., and H. E. Stanley. 2008. "Detrended Cross-Correlation Analysis: A New Method for Analyzing Two Non-Stationary Time Series." *Physical Review Letters* 100: 084102. doi:10.1103/physrevlett.100.084102.

Sarwar, G. 2017. "Examining the Flight-To-Safety with the Implied Volatilities." *Finance Research Letters* 20: 118–124. doi:10.1016/j.frl.2016.09.015.

Zhou, W. X. 2008. "Multifractal Detrended Cross-Correlation Analysis for Two Nonstationary Signals." *Pyhsical Review E* 77: 066211. doi:10.1103/Physreve.77.066211.

Zunino, L., B. M. Tabak, A. Figliola, D. G. Prez, M. Garavaglia, and O. A. Rosso. 2008. "A Multifractal Approach for Stock Market Inefficiency." *Physica A: Statistical Mechanics and Its Applications* 387: 6558–6566. doi:10.1016/j.physa.2008.08.028.