



Research Article

Multifractal Detrended Cross-Correlation Analysis of the Return-Volume Relationship of Bitcoin Market

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We investigate the cross-correlations of return-volume relationship of the Bitcoin market. In particular, we select eight exchange rates whose trading volume accounts for more than 98% market shares to synthesize Bitcoin indexes. The empirical results based on multifractal detrended cross-correlation analysis (MF-DCCA) reveal that (1) the nonlinear dependencies and power-law cross-correlations in return-volume relationship are found; (2) all cross-correlations are multifractal, and there are antipersistent behaviors of cross-correlation for $q = 2$; (3) the price of small fluctuations is more persistent than that of the volume, while the volume of larger fluctuations is more antipersistent; and (4) the rolling window method shows that the cross-correlations of return-volume are antipersistent in the entire sample period.

1. Introduction

Since 2016, the market value and amount of cryptocurrencies based on blockchain technology have risen rapidly. At the same time, the risen price is accompanied by very violent fluctuations. For example, Bitcoin, the leading cryptocurrency, fell down more than 40% to less than \$12,000 in a month from more than \$20,000 on December 17, 2017. Despite the recent market crash, some people still attributed the rapid rise of cryptocurrencies to the bright prospect of the application of blockchain technology (<https://www.cnn.com/eran-eyal-bitcoin-will-rise-above-100000/> (assessed date: February 1, 2018)). The article mentions that some blockchain backers believe Bitcoin can exceed \$100,000, while others think that the cryptocurrencies are products of speculation rather than the mediums of exchange [1, 2] and Bitcoin is in a bubble period [3]. In order to increase the price discovery function of Bitcoin in the financial market and meet the needs of investors in indirect cryptocurrencies, US financial firms CME and CBOE have launched Bitcoin futures, respectively. In addition, Bitcoin, to a certain extent, is an option for asset allocation and

hedging [4] due to the distinctly different returns, volatility, and correlation characteristics from other assets [5]. As a form of cryptocurrency with a long history and the largest market capitalization, Bitcoin has a limit on the total amount at the beginning of the design. Besides, the correlation between price and volume is an important indicator for investigating market dynamics in classical finance. The price change denotes the market reaction to new information, while the volume change reflects investors' different understanding of new information. Studying the cross-correlations between price and volume can reveal the speed, spread, and absorption of market information flows and the extent to which prices reflect market information. This issue has been explored in the stock market [6–9], commodity market [10, 11], and so on. We believe it is necessary to conduct this study in the Bitcoin market. Therefore, we fill the current research gap by investigating the return-volume relationship of the Bitcoin market.

In recent years, economic literature on Bitcoin has shown that the returns of Bitcoin run counter to the efficient market hypothesis (EMH) [12, 13]. Urquhart [14] uses several tests to test the inefficiency of Bitcoin and finds that while Bitcoin

is not efficient, it is in the process of becoming effective. Bariviera [15] employs a dynamic detrended fluctuation analysis approach to support Urquhart's conclusion and finds that the volatility of Bitcoin has long-term memory. Besides, there is a literature on Bitcoin from the perspective of stylized facts [16–19] and relationships with other assets [20, 21]. In line with the researches of the relationship between investor attention and asset performance [22–24], Urquhart [25] and Zhang et al. [26] explore the relation with retailed investor attention measured by Google Trends. All of these studies show that Bitcoin is different from traditional assets. Based on the scholars' findings, we study the return-volume relationship of the Bitcoin market by employing an econophysics method.

The fractal theory proposed by Mandelbrot [27] can explain the economic phenomena that cannot be explained by the traditional efficient market hypothesis. Early fractal studies focus on rescaled range (R/S) analysis proposed by Hurst [28, 29] in the field of hydrology. However, Lo [30] finds that the R/S Hurst method is sensitive to short-term autocorrelation, which may lead to bias error of nonstationary time series. To overcome this shortcoming, Peng et al. [31] propose a widely used detrended fluctuation analysis that uses a long-range power law to avoid significant long-range autocorrelation false detection [32]. As a multifractal extension of DFA approach, Kantelhardt et al. [33] introduce the MF-DFA method widely used in financial markets. Podobnik and Stanley [34] extend the DFA by introducing a detrended cross-correlation analysis (DCCA) approach that can be used to study long-range cross-correlation between two nonstationary time series. On this basis, Zhou [35] proposes a new method multifractal detrended cross-correlation analysis (MF-DCCA, MF-DXA, or MF-X-DFA), which is a combination of MF-DFA and DCCA. Then, a series of improvement methods have been proposed, such as the method of MF-PX-DFA and MF-PX-DMA [36], MF-X-DMA [37], MF-HXA [38], and MF-X-PF [39]. These increase the efficiency of some applications of the MF-DCCA method. The MF-DCCA method has been widely applied to describe the multifractal characteristics of two cross-correlated nonstationary time series in a financial field such as the foreign exchange market [40, 41], the stock market [42–44], the crude oil market [45–47], carbon market [48, 49], and the commodity market [50, 51]. Zhang et al. also employ MF-DCCA to examine the relationship between mass media and new media [52] and to quantify the cross-correlation between investor sentiment proxies [53], that is, FEARS [54] and Twitter happiness sentiment [55].

Previously, the relationships between the volume and price of the stock market [6–9], agricultural commodity future markets [11], and the gold market [10] have been studied, using the multifractal detrended cross-correlation analysis method. Therefore, we conduct the MF-DCCA to investigate the cross-correlations between the return and volume of the Bitcoin market.

This paper is structured as follows. Section 2 describes the data and method of processing data. We introduce the method of MF-DCCA in Section 3. We provide empirical results and conclusions in Section 4 and Section 5, respectively.

TABLE 1: Daily volume of different exchange rates. The volume is the average daily trading volume in 2017. Mean volume represents the average volume of the exchange rate for each market. The bold is the top 8 exchange rates which account for more than 98.35% market share.

Currency	Volume	Mkt. number	Mean vol
ARS	4.470354	1	4.470354
AUD	461.0146	2	230.5073
BRL	914.7376	3	304.9125
CAD	107.9026	2	53.95132
CHF	2.974501	1	2.974501
CLP	1.101236	1	1.101236
CZK	3.483695	2	1.741847
DKK	1.672666	1	1.672666
EUR	18751.29	13	1442.407
GBP	2357.178	4	589.2945
HKD	12.3385	1	12.3385
IDR	715.4489	2	357.7245
ILS	34.51753	2	17.25877
INR	29.57731	1	29.57731
JPY	72848.44	6	12141.41
KRW	5889.262	1	5889.262
MXN	8.559472	1	8.559472
MYR	190.0112	1	190.0112
NGN	70.77995	1	70.77995
NOK	17.85582	1	17.85582
NZD	21.38022	1	21.38022
PKR	14.49338	1	14.49338
PLN	1422.03	5	284.406
RUB	2046.448	4	511.6121
SEK	29.7208	2	14.8604
SGD	76.45168	3	25.48389
SLL	51.89407	1	51.89407
THB	24.94223	1	24.94223
USD	47526.69	16	2970.418
VEF	64.25689	2	32.12844
VND	12.52992	2	6.264962
ZAR	583.2546	2	291.6273

2. Data Description

The data in this paper is from <https://coincharts.com/>, a website that provides Bitcoin financial and technical data. The Coincharts website collects market data from various Bitcoin markets around the world and divides them into active and inactive markets. The symbol provided by the website, such as bitstampUSD, consists of two parts; the lowercase part is the trading market and the uppercase part is the exchange rate. In the active section, the website has 90 pairs (market and exchange rate), of which bcEUR, bitmarketEUR, and bitmarketPLN are not provided with complete data (<https://coincharts.com/markets/list/>). Therefore, we choose 87 pairs out of 90 pairs.

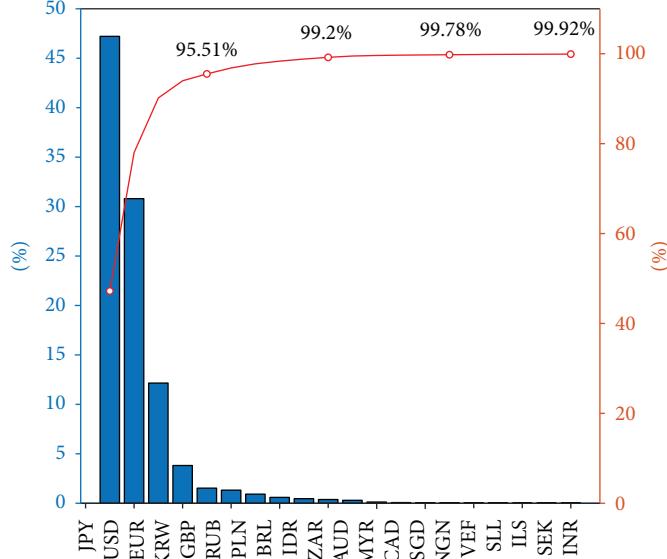


FIGURE 1: Share of each exchange rate and accumulative share.

We rank the daily trading volume of 2017 in different exchange rates and find that the top 8 exchange rates account for more than 98%. The top 8 exchange rates' symbols BRL, EUR, GBP, JPY, KRW, PLN, RUB, and USD denote Brazilian real, euro, British pound, Japanese yen, Korean won, Polish zloty and US dollars. Table 1 reports the volume of each currency in 2017 and the average volume of each market. And the proportion of each exchange rate and cumulative proportions are shown in Figure 1. From Table 2, we can see that the oldest market in each exchange rate is not the largest volume. Likewise, the market with the largest volume does not have the longest span of time. And since different markets in the first eight exchange rates have different starting dates for trading and the average daily trading volume is different, we use the raw data for each market to calculate the volume-weighted average composite price index in order to avoid market bias. In the synthesis procedure, we get the weighted sum of the closing price that is denominated in different exchange rates to get the index price: $\text{price}_t = \sum_{i=1}^n \text{cp}_{i,t} * \text{volume}_{i,t} / \text{total volume}_t$, where $\text{cp}_{i,t}$ and $\text{volume}_{i,t}$ denote the closing price and trading volume of the i th symbol at t day, and $\text{total volume}_t = \sum_{i=1}^n \text{volume}_{i,t}$. The volume of the composite index is the sum of symbols denominated in the same currency. Table 2 shows the basic information for each of the first eight exchange rates, including the start date and the average daily volume for 2017. And Table 3 shows the start date and end date of the synthetic composite Bitcoin price index. USD, EUR, and BRL have the longest span of time, with more than 2200 data points.

The daily returns and volume change are defined:

$$\begin{aligned} r_t &= \ln \left(\frac{\text{price}_t}{\text{price}_{t-1}} \right), \\ v_t &= \ln \left(\frac{\text{volume}_t}{\text{volume}_{t-1}} \right), \end{aligned} \quad (1)$$

where price_t and volume_t are the closing price and trading volume on trading day t , respectively. The daily return data for the entire period of the Bitcoin market is given in Figure 2, indicating the presence of volatility clustering. However, even though the eight charts in Figure 2 are all about Bitcoin returns, the difference between different exchange rates is significant. For BRL, EUR, and USD that have similar time spans, extrema and degree of volatility clustering significantly vary. Similarly, fluctuations in the trading volume of BRL, RUB, PLN, and USD show a declining trend, while other exchange rates are relatively stable, and GBP may even rise (see Figure 3). This also proves that the global Bitcoin market is not fully synchronized, so it is necessary to study the different markets of Bitcoin. Table 4 shows the statistical properties of the Bitcoin market. In panel A, we can see that the BRL and RUB indicators are significantly different from other exchange rates, with greater standard deviation, kurtosis, and J-B statistics, which means the BRL and RUB markets are more volatile. The Jarque-Bera test rejects the null hypothesis that time series obeys normal distribution at a significance level of 1%. The kurtosis of returns and volume change is greater than 3, indicating that the time series is leptokurtic. Both the return and volume change series are not normal distribution.

3. Methodology

Zhou [35] introduces the MF-DCCA method to analyze the cross-correlation of two nonstationary series. Let us consider two time series $\{x_i\}$ and $\{y_i\}$ of the same length N , where $i = 1, 2, \dots, N$. Then, we determine the profile: $X_i = \sum_{k=1}^i (x_k - \bar{x})$, $Y_i = \sum_{k=1}^i (y_k - \bar{y})$, $i = 1, 2, \dots, N$, where \bar{x} and \bar{y} are the mean values of the time series $\{x_i\}$ and $\{y_i\}$.

The integrated time series X_i and Y_i have eliminated the trend. Then, we divide the two profiles into $N_s \equiv \text{int}(N/s)$ nonoverlapping segments of equal length s , respectively. If length N cannot be divisible by scale s , then a short part will

TABLE 2: Basic information for each market of top 8 exchange rates.

Currency	Market	Start date	Daily volume
BRL	Foxbit	20141211	532.92
BRL	Localbtc	20130318	15.67
BRL	Mrcd	20110728	366.15
EUR	Abucoins	20170921	10.02
EUR	Bitbay	20150317	22.66
EUR	Btcde	20110826	845.45
EUR	Cex	20140926	152.75
EUR	Coinbase	20150423	1961.44
EUR	Coinfalcon	20171013	157.68
EUR	Coinsbank	20170703	1563.57
EUR	Itbit	20131117	142.06
EUR	Kraken	20140108	11132.42
EUR	Localbtc	20130312	95.46
EUR	Rock	20111109	267.31
EUR	Wex	20171017	158.55
EUR	Zyado	20140424	2241.92
GBP	Coinbase	20150421	339.00
GBP	Coinfloor	20140329	790.40
GBP	Coinsbank	20170703	951.48
GBP	Localbtc	20130311	276.29
JPY	Bitflyer	20170704	18769.10
JPY	Btcbox	20140409	4251.26
JPY	Coincheck	20141031	32811.68
JPY	Fisco	20171018	9707.05
JPY	Kraken	20170611	69.24
JPY	Zaif	20170627	7240.11
KRW	Korbit	20130903	5889.26
PLN	Abucoins	20170918	60.35
PLN	Bitbay	20140329	1306.06
PLN	Bitmaszyna	20170804	7.56
PLN	Localbtc	20130624	2.60
PLN	Nevbit	20170412	45.45
RUB	Cex	20150806	2.71
RUB	Coinsbank	20170629	1331.30
RUB	Localbtc	20130328	497.04
RUB	Wex	20171017	215.39
USD	Abucoins	20170921	9.66
USD	Bitbay	20140516	14.43
USD	Bitkonan	20130702	2.18
USD	Bitstamp	20110913	12873.29
USD	Btcalpha	20161101	147.61
USD	Btcc	20161102	87.78
USD	Cex	20140718	1170.66
USD	Coinbase	20141201	15057.84
USD	Coinsbank	20170703	1701.17
USD	Getbtc	20170616	851.68
USD	Itbit	20130825	2785.99
USD	Kraken	20140107	5048.12

TABLE 2: Continued.

Currency	Market	Start date	Daily volume
USD	Lake	20140301	4872.08
USD	Localbtc	20130311	529.29
USD	Okcoin	20170807	559.40
USD	Wex	20170919	1815.51

TABLE 3: Start date and end date of top 8 exchange rates.

Currency	Start date	End date	Obs.
BRL	20110728	20180118	2242
EUR	20110826	20180118	2336
GBP	20130311	20180118	1775
JPY	20140409	20180118	1380
KRW	20130903	20180118	1590
PLN	20130624	20180118	1565
RUB	20130328	20180118	1578
USD	20110913	20180118	2302

be kept at the end of the profile. In order not to neglect the short part, we repeat the same procedure starting from the end of the profile, so $2N_s$ segments are obtained.

The local trend of the series $\tilde{X}_v(i)$ and $\tilde{Y}_v(i)$ is determined for each segment v ; $\tilde{X}_v(i) = \tilde{a}_k i^m + \dots + \tilde{a}_1 i + \tilde{a}_0$ and $\tilde{Y}_v(i) = \tilde{b}_k i^m + \dots + \tilde{b}_1 i + \tilde{b}_0$, where $i = 1, 2, \dots, s$, $v = 1, 2, \dots, 2N_s$, and $m = 1, 2, \dots$. Then, we calculate the polynomial fit of each segment v and obtain the detrended covariance

$$F^2(s, v) = \frac{1}{s} \sum_{j=1}^s [X((v-1)s+j) - \tilde{X}_v(i)] \cdot [Y((v-1)s+j) - \tilde{Y}_v(i)], \quad (2)$$

for $v = 1, 2, \dots, N_s$, and

$$F^2(s, v) = \frac{1}{s} \sum_{j=1}^s [X(N - (v - N_s)s + j) - \tilde{X}_v(i)] \cdot [Y(N - (v - N_s)s + j) - \tilde{Y}_v(i)], \quad (3)$$

for $v = N_s + 1, N_s + 2, \dots, 2N_s$, where $\tilde{X}_v(i)$ and $\tilde{Y}_v(i)$ are the local trends.

The q -order fluctuation function is obtained by averaging all the segments

$$F_q(s) = \left\{ \frac{1}{2N_s} \sum_{\lambda=1}^{2N_s} F^2(s, v)^{q/2} \right\}^{1/q}. \quad (4)$$

And when $q = 0$, the equation is $F_0(s) = \exp \{1/4N_s \sum_{\lambda=1}^{2N_s} \ln[F^2(s, v)]\}$.

The scale behavior of the fluctuation function is analyzed by observing the logarithmic plots of $F_q(s)$ versus s . If the two

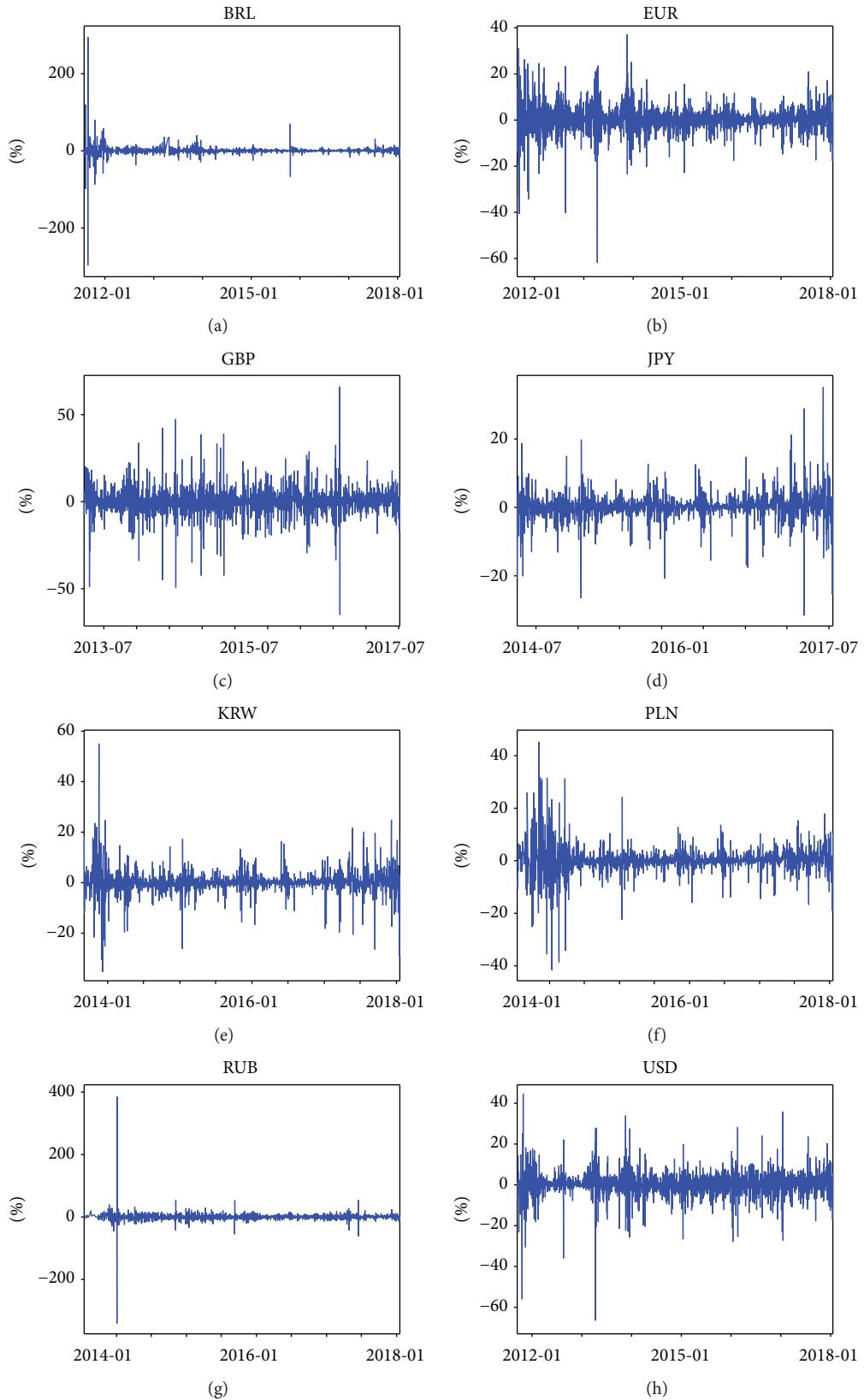


FIGURE 2: Returns of the Bitcoin market.

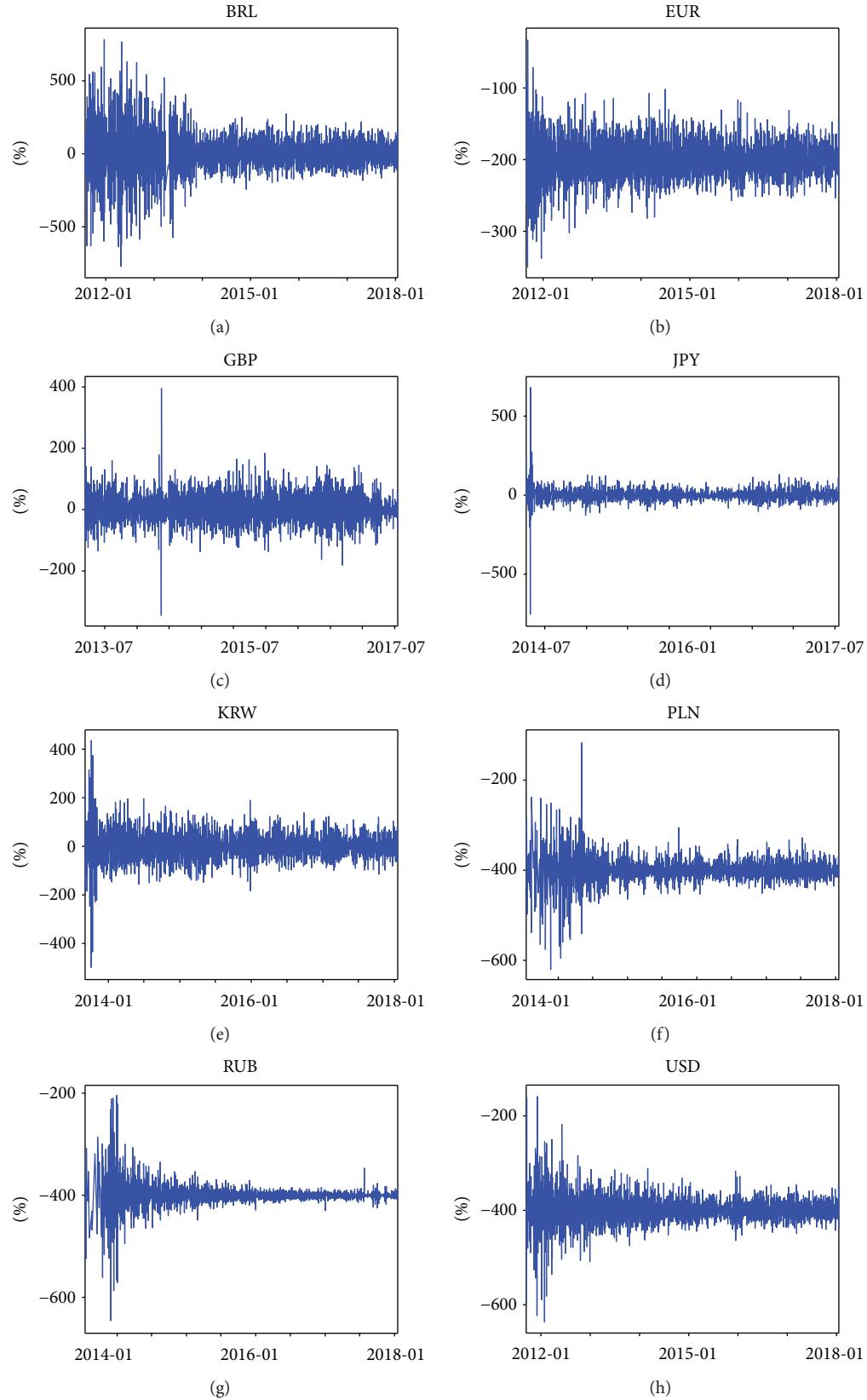


FIGURE 3: Volume change of the Bitcoin market.

TABLE 4: Descriptive statistics for the Bitcoin market time series.

(a) Statistical properties of price change									
Currency	Mean	Median	Std	Max.	Min.	Skewness	Kurtosis	Jarque-Bera	Obs.
BRL	0.0033	0.0012	0.1172	2.9429	-2.9673	-0.0678	373.9244	12846991.37***	2241
EUR	0.0031	0.0029	0.0565	0.3709	-0.6175	-0.7343	15.9916	16630.86***	2335
GBP	0.0031	0.0039	0.0804	0.6603	-0.6496	-0.2600	13.7927	8629.91***	1774
JPY	0.0024	0.0020	0.0426	0.3514	-0.3154	-0.2271	15.0605	8369.46***	1379
KRW	0.0029	0.0012	0.0492	0.5494	-0.3532	0.3396	21.1442	21827.20***	1589
PLN	0.0030	0.0021	0.0535	0.4523	-0.4154	0.1621	18.3915	15434.84***	1563
RUB	0.0036	0.0010	0.1558	3.8493	-3.4028	2.8241	382.8340	9482091.38***	1577
USD	0.0033	0.0038	0.0591	0.4455	-0.6636	-1.0286	19.8176	27522.38***	2301

(b) Statistical properties of volume change									
Currency	Mean	Median	Std	Max.	Min.	Skewness	Kurtosis	Jarque-Bera	Obs.
BRL	0.0027	-0.0297	1.3674	7.8313	-7.7223	0.0621	8.1312	2459.95***	2241
EUR	0.0035	-0.0080	0.5457	3.3349	-2.9957	0.0697	5.3577	542.69***	2335
GBP	0.0026	-0.0277	0.4949	3.9505	-3.4478	0.4587	7.3116	1436.31***	1774
JPY	0.0088	-0.0122	0.4832	6.8152	-7.5347	-0.4213	74.9053	297122.05***	1379
KRW	0.0059	-0.0164	0.6397	4.3582	-4.9870	0.0017	9.3632	2680.84***	1589
PLN	0.0057	-0.0289	0.8054	7.0761	-5.5133	-0.0655	13.8337	7644.75***	1563
RUB	0.0048	0.0060	0.6393	4.8952	-6.1465	0.0260	24.8634	31409.28***	1577
USD	0.0030	-0.0288	0.7592	6.0276	-5.9090	0.1337	13.5211	10619.66***	2301

***Denotes statistical significance at 1% level.

time series are cross-correlated, we get the following power-law relationship:

$$F_q(s) \propto s^{H_{xy}(q)}. \quad (5)$$

Equation (5) can be rewritten as

$$\log F_q(s) = H_{xy}(q) \cdot \log s + \log C. \quad (6)$$

In the empirical section, we will draw logarithmic graphs according to (6). The cross-correlation exponent $H_{xy}(q)$ in (5) can describe the power-law relationship between two nonstationary time series. In particular, MF-DCCA is identical to MF-DFA if time series x is the same as y . If $q = 2$, the cross-correlation exponent $H_{xy}(q)$ is equivalent to the generalized Hurst exponent.

According to Shadkho and Jafari [56], the similarity between the classical multifractal exponent $\tau_{xy}(q)$ and q can be given by

$$\tau_{xy}(q) = qH_{xy}(q) - 1. \quad (7)$$

If $\tau_{xy}(q)$ is a linear function of q , the cross-correlation of the two correlated series is monofractal; otherwise, it is

multiperiodic. Through Legendre conversion, we can get the following relationship:

$$\alpha_{xy}(q) = H_{xy}(q) + H'_{xy}(q), \quad (8)$$

$$f_{xy}(\alpha) = q[\alpha_{xy} - H_{xy}(q)] + 1. \quad (9)$$

Multifractal intensity can be estimated by the width of the multifractal spectrum, which is given by the following equation:

$$\Delta\alpha = \alpha_{\max} - \alpha_{\min}. \quad (10)$$

Based on Podobnik and Stanley [34], when $q=2$, for two fractionally autoregressive integral moving average (ARFIMA) processes that share the same random noise, the cross-correlation exponent is approximately identical to the mean value of the individual Hurst exponents. Zhou [35] also observes that for the two time series constructed by the binomial measure of the p -model, there is the following equation:

$$H_{xy}(q) = \frac{[H_{xx}(q) + H_{yy}(q)]}{2}. \quad (11)$$

4. Empirical Results

4.1. Cross-Correlation Test. In order to study the correlation between the volume and price of Bitcoin markets, we employ

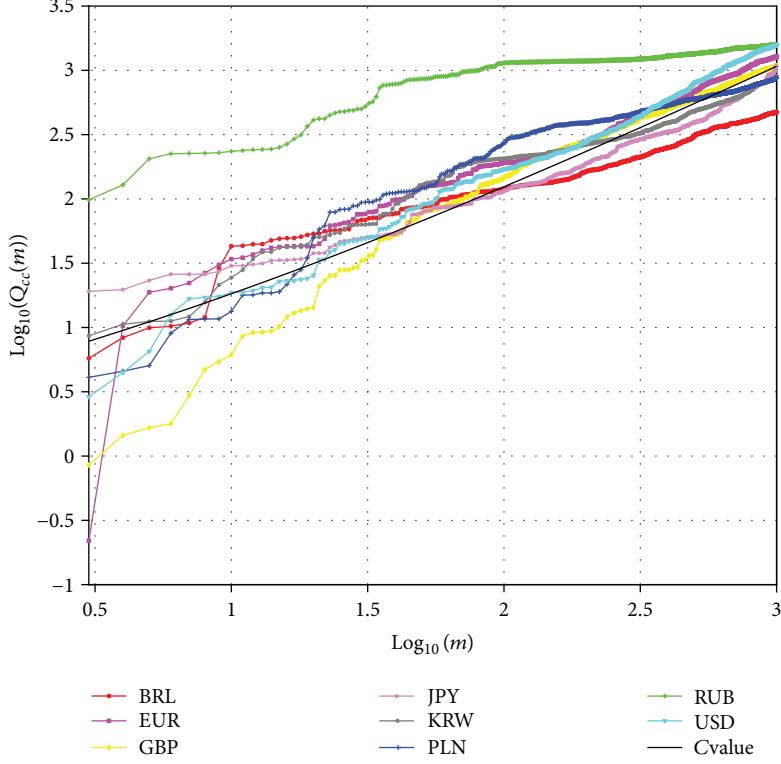


FIGURE 4: The cross-correlation statistics between price and volume.

the cross-correlation test developed by Podobnik et al. [57]. This is a qualitative way of testing the level of significance of cross-correlation between any two nonstationary time series.

For the two time series $\{x_i\}$ and $\{y_i\}$ with the equal length N , the test statistic is defined as

$$Q_{cc}(m) = N^2 \sum_{i=1}^m \frac{X_i^2}{N-i}, \quad (12)$$

where X_i^2 is the cross-correlation function

$$X_i = \frac{\sum_{k=i+1}^N x_k y_{k-i}}{\sqrt{\sum_{k=1}^N x_k^2 \sum_{k=1}^N y_k^2}}. \quad (13)$$

The test statistic $Q_{cc}(m)$ is approximately $\chi^2(m)$ distributed with m degrees of freedom. We use Q_{cc} statistics to test the null hypothesis that the first m has a nonzero correlation coefficient. Therefore, if the value of Q_{cc} is larger than the critical value of $\chi^2(m)$, the cross-correlation between the two time series is significant.

In Figure 4, the black line is the critical value of $\chi^2(m)$ at 5% significant level, and the degrees of freedom vary from 1 to 1000. The cross-correlation statistics show that the value of $Q_{cc}(m)$ deviates from the critical value $\chi^2(m)$ at 5% significant level, indicating that the two time series have nonlinear cross-correlation regardless of m changes. Thus, we can reject the null hypothesis of no cross-correlations.

Besides, we can clearly see that the green line representing the RUB deviates from the other lines. In combination with Table 2 and Figures 2 and 3, the RUB does show different characteristics. We speculate that this is related to the proposed banning of Bitcoin by the Russian Ministry of Finance in August 2014 [58].

4.2. Multifractal Detrended Cross-Correlation Analysis. The cross-correlation test in Section 4.1 has qualitatively described the existence of nonlinear cross-correlations between the time series of price and volume. To quantify the cross-correlation, we employ the MF-DCCA method to quantitatively estimate the cross-correlation exponent in this section. And the scale q is set from -10 to 10 in this paper.

According to (2), (3), (4), (5), and (6), the log-log plots of fluctuation function $F_{xyq}(s)$ versus time scale s of the return-volume in the Bitcoin market are shown in Figure 5. We find that, although the increasing rates of the fluctuation function $F_{xyq}(s)$ of eight exchange rates vary, they all increase linearly with the scale s . Therefore, there exist power-law behavior and long-range cross-correlation of price and volume time series in the Bitcoin market.

The legends $H_{xx} - p$, $H_{yy} - p$, and $H_{xy} - pv$ in Figure 6 denote the Hurst exponent of price, volume, and bivariate. The average scaling exponents are the black lines with asterisk in Figure 6. The general Hurst exponents $H_{xx}(q)$ and $H_{yy}(q)$ are calculated by using MF-DFA. MF-DCCA is identical to DCCA for $q=2$. And the bivariate Hurst

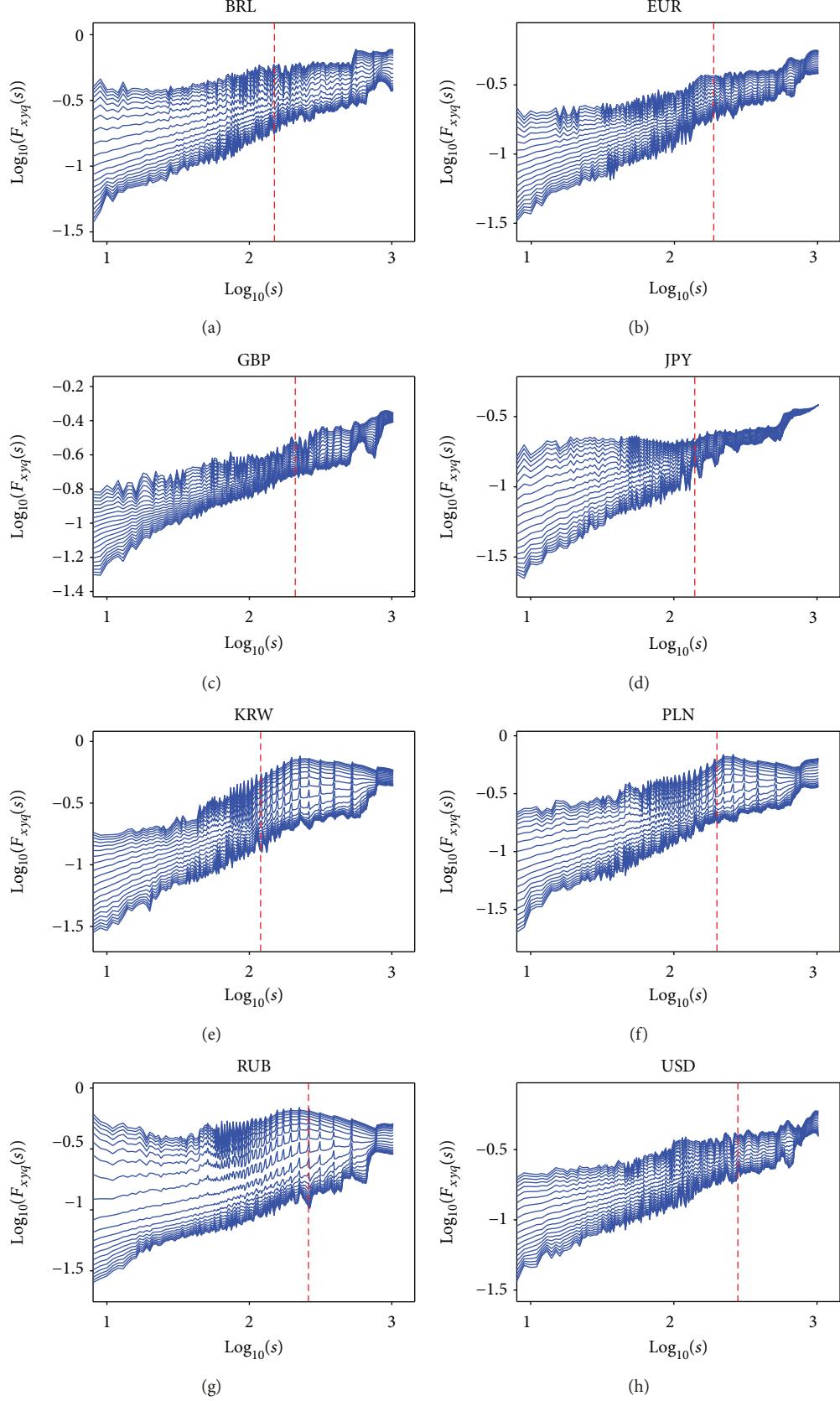


FIGURE 5: The log-log plots of fluctuation function $F_{xyq}(q)$ versus s for price-volume series when the polynomial order $m = 1$.

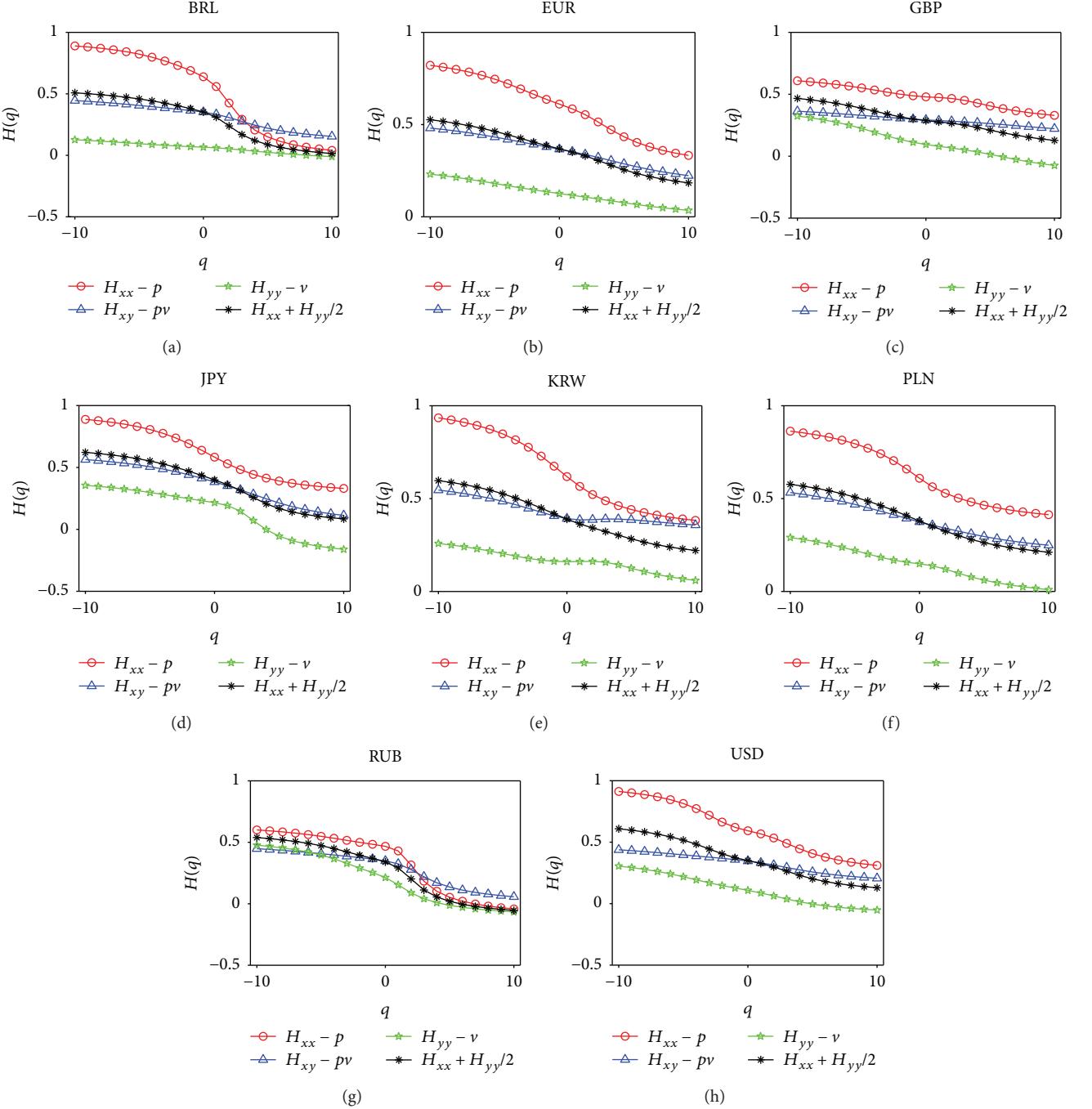
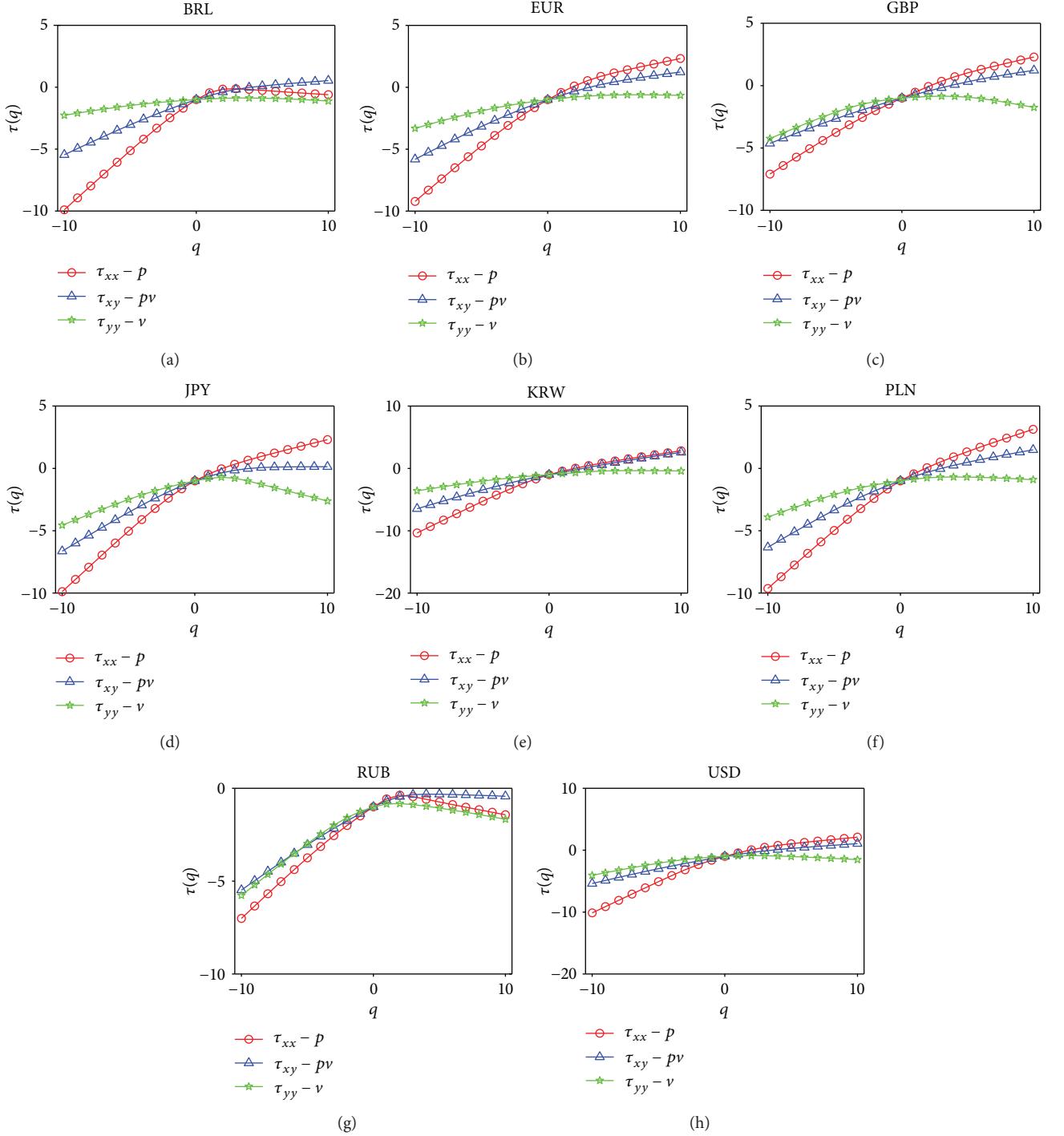


FIGURE 6: The generalized Hurst exponent values of H_{xx} , H_{yy} , H_{xy} , and the value $(H_{xx} + H_{yy})/2$ for different q values.

exponent H_{xy} has similar properties and interpretation as the univariate Hurst exponent. Figure 6 shows the relationship between the generalized Hurst exponents H_{xx} , H_{yy} , and H_{xy} . In our findings, for the Bitcoin market, the cross-correlation exponent is less than the average generalized Hurst exponent for $q > 0$. This result is consistent with the conclusions of the agricultural future market [59] and the gold market [10].

If H is a constant, the time series is monofractal; otherwise, it is multifractal. As we can see in Figure 6, the cross-

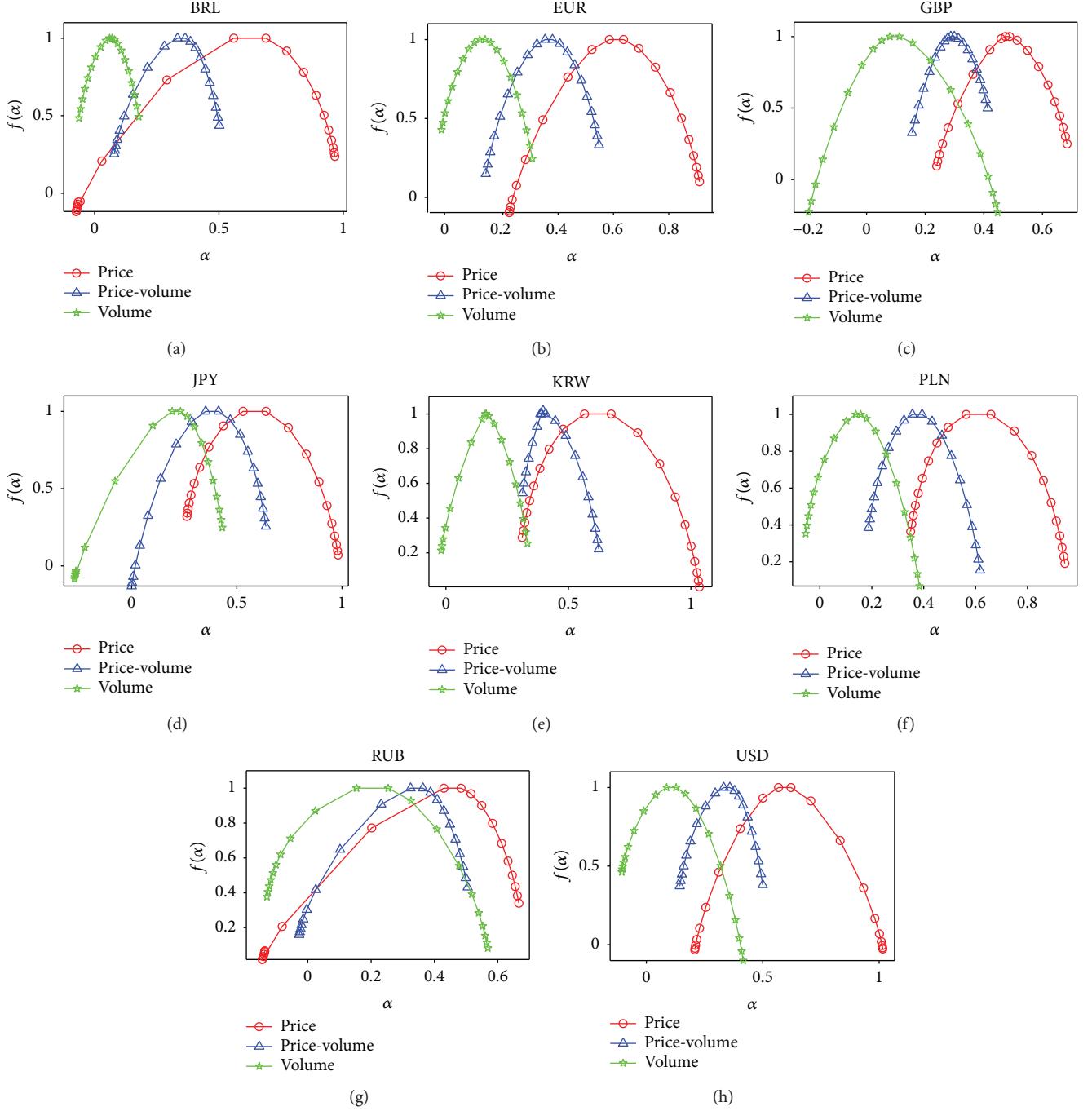
correlation of return-volume has been declining, indicating that all the eight exchange rates have multifractal features. In particular, the Hurst exponent $H_{xx}(q)$ is greater than 0.5, $H_{xy}(q)$ and $H_{yy}(q)$ for $q < 0$, which means that the price behavior of small fluctuations is more persistent than the volume and cross-correlation behavior. In contrast, the anti-persistence of the volume is stronger for $q > 0$. Because the cross-correlation Hurst exponent $H_{xy}(2) < 0.5$, we can see that there is an antipersistent cross-correlated behavior of return-volume in the Bitcoin market.

FIGURE 7: The mass exponent values of $\tau(q)$ for different q values.

According to (7), we calculate the Renyi exponent $\tau_{xy}(q)$ that characterizes the multifractality. The legends $\tau_{xx} - p$, $\tau_{yy} - p$, and $\tau_{xy} - p\nu$ in Figure 7 denote the Renyi exponent of price, volume, and bivariate. According to Figure 7, $\tau_{xy}(q)$ is the nonlinear curves of the eight exchange rates dependent on q .

Then, we employ (8) and (9) to calculate the multifractal spectrum of the eight exchange rates. The $\Delta\alpha$ in (10) means the degree of multifractality. The larger $\Delta\alpha$ implies stronger

multipfractality and lower $\Delta\alpha$ indicates weaker multifractal behavior. If $\Delta\alpha$ is close to zero, it is monofractal. According to Figure 8, the multifractal spectra of the eight exchange rates are not a point; there exist the multifractal characteristics of return-volume. The width of the multifractal spectrum of BRL and RUB is obviously different, and the price spectrum width is larger than the sum of the volume spectrum width and price-volume spectrum width. This may be related to the Russian ban on Bitcoin. As early as January 2014,

FIGURE 8: The multifractal spectrum $f(\alpha)$ as a function of α .

Russian officials issued a statement that it was illegal to issue agency currency in Russia. In the ensuing period, various stakeholders have been wrestling for many times. The policy on the issuance and circulation of cryptocurrencies has also changed several times (“A Concise History of the Bitcoin Ban in Russia”; please refer to: <http://www.coinfox.info/news/reviews/5982-a-concise-history-of-the-bitcoin-ban-in-russia>). In the early 2018, Russia finalized the federal law on cryptocurrency regulation (<http://Bitcion.com/> reported this news: <https://news.bitcoin.com/russia-finalizes-federal-law-cryptocurrency-regulation>).

4.3. Long-Term and Short-Term Analysis. We define “crossover” S^* as a turning point when the linear tendency of the curve changes fundamentally. Short-term market behavior is very vulnerable to external market factors, while long-term behavior is mainly determined by the internal factors of assets. With time going by, the short-term impact will gradually decay; the long-term market supply and demand mechanisms will play a role. When $s < S^*$, the scale exponent means short-term-related behavior. And when $s > S^*$, the scale exponent reflects long-term-related behavior. According to Figure 4, we find that the “crossover” of the top eight

TABLE 5: Long-term and short-term scaling exponents for each exchange rate. Short denotes the term $s < S^*$, long is the term $s > S^*$. Here, we define $\Delta H_q = \max(H_q) - \min(H_q)$, and the larger the ΔH_q , the higher the degree of multifractality.

q	BRL	EUR	GBP	JPY	KRW	PLN	RUB	USD
	Short	Long	Short	Long	Short	Long	Short	Long
-10	0.44118	0.4020	0.4954	0.4302	0.4004	0.4417	0.5909	0.4811
-9	0.4370	0.3979	0.4886	0.4283	0.3913	0.4418	0.5635	0.4751
-8	0.4311	0.3937	0.4805	0.4267	0.3811	0.4421	0.5737	0.4695
-7	0.4242	0.3896	0.4711	0.4254	0.3698	0.4425	0.5629	0.4630
-6	0.4163	0.3858	0.4599	0.4245	0.3574	0.4430	0.5502	0.4558
-5	0.4077	0.3825	0.4468	0.4240	0.3441	0.4433	0.5352	0.4480
-4	0.3985	0.3801	0.4316	0.4236	0.3303	0.4433	0.5173	0.4394
-3	0.3882	0.3786	0.4145	0.4228	0.3163	0.4424	0.4961	0.4299
-2	0.3757	0.3774	0.3958	0.4206	0.3026	0.4400	0.4711	0.4195
-1	0.3660	0.3744	0.3762	0.4156	0.2895	0.4350	0.4421	0.4082
0	0.3409	0.3649	0.3563	0.4060	0.2773	0.4264	0.4093	0.3961
1	0.3175	0.3449	0.3365	0.3910	0.2662	0.4134	0.3726	0.3834
2	0.2885	0.3160	0.3165	0.3716	0.2562	0.3960	0.3310	0.3705
3	0.2562	0.2851	0.2961	0.3499	0.2472	0.3755	0.2844	0.3576
4	0.2264	0.2581	0.2753	0.3288	0.2389	0.3540	0.2359	0.3452
5	0.2022	0.2364	0.2549	0.3101	0.2310	0.3333	0.1907	0.3335
6	0.1834	0.2193	0.2362	0.2944	0.2232	0.3147	0.1526	0.3227
7	0.1687	0.2056	0.2198	0.2816	0.2156	0.2986	0.1218	0.3129
8	0.1571	0.1945	0.2059	0.2711	0.2083	0.2849	0.0975	0.3041
9	0.1476	0.1854	0.1941	0.2626	0.2013	0.2734	0.0782	0.2962
10	0.1397	0.1777	0.1843	0.2555	0.1948	0.2637	0.0627	0.2892
ΔH_q	0.3021	0.2243	0.3112	0.1747	0.2055	0.1796	0.5282	0.1919

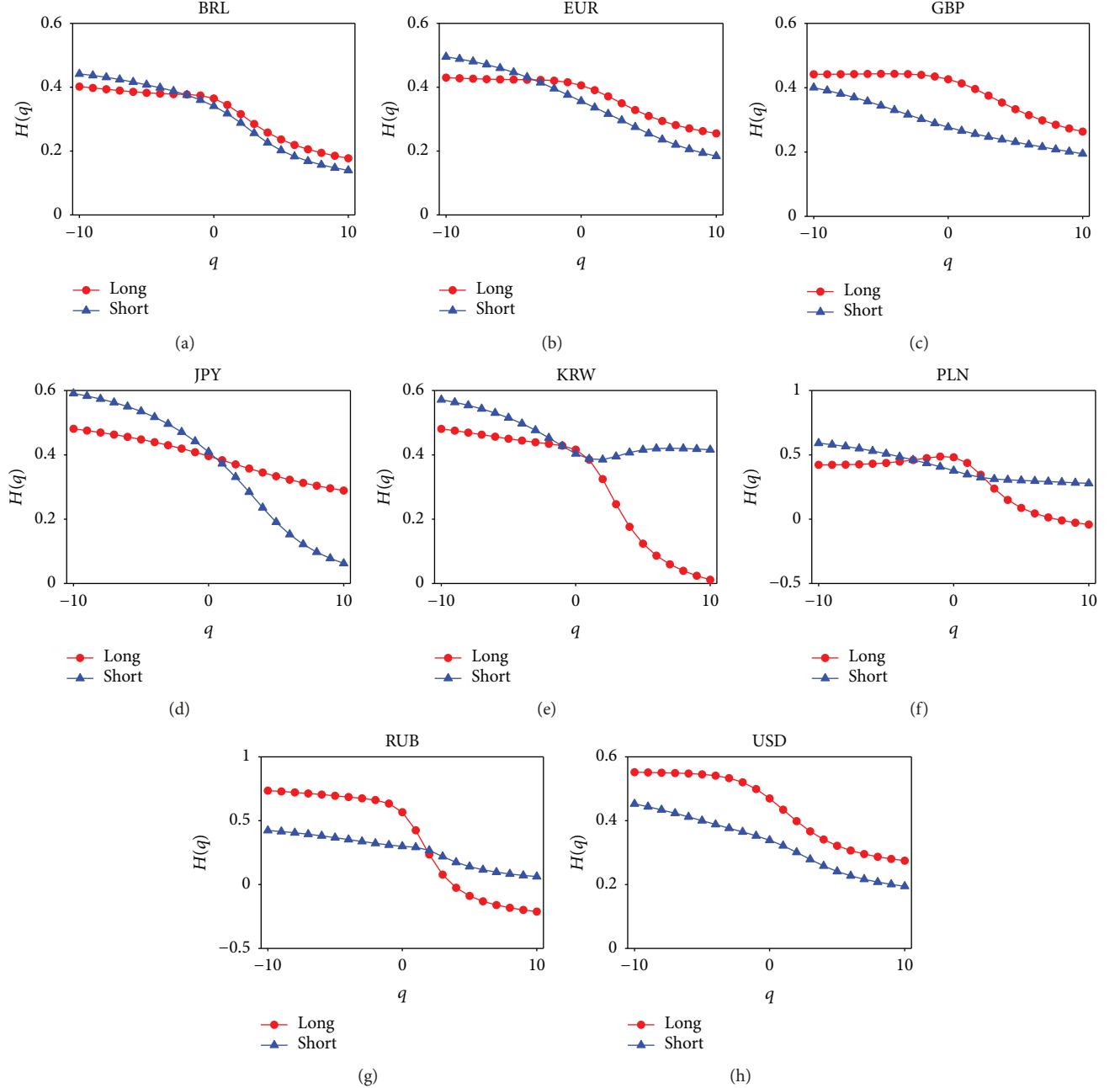


FIGURE 9: The long-term and short-term generalized Hurst exponent values H_{xy} .

exchange rates (BRL, EUR, GBP, JPY, KRW, PLN, RUB, and USD) is located at approximately $\log(S^* = 2.1761)$ (150 days, about 5 months), $\log(S^* = 2.2788)$ (190 days, about 6.3333 months), $\log(S^* = 2.3222)$ (210 days, about 7 months), $\log(S^* = 2.1461)$ (140 days, about 4.6667 months), $\log(S^* = 2.0792)$ (120 days, about 4 months), $\log(S^* = 2.301)$ (200 days, about 6.6667 months), $\log(S^* = 2.415)$ (260 days, about 8.6667 months), and $\log(S^* = 2.4472)$ (280 days, about 9.3333 months), respectively.

To study the different long-term and short-term changes in the cross-correlation, we calculate the cross-correlation scaling exponents for q from -10 to 10. Table 5 reports the long-term and short-term scaling exponents. ΔH_q

denotes the degree of multifractality. KRW, PLN, RUB, and USD are strongly multifractal in the long term than the short term, while the rest is the opposite. From Figure 9, for the highest degree of fractal exchange rate, RUB, long-term scaling exponent is greater than 0.5 when $q < 0$, indicating that the cross-correlation of small fluctuations in the long-term persists (positive), while almost all the exponents of the other exchange rates are less than 0.5 in the long term. Long-term and short-term exponents of BRL, EUR, and GBP are less than 0.5 with q varying from -10 to 10. For EUR and GBP, the long-term value of $H_{xy}(q)$ is weakly dependent on q , indicating that long-term cross-correlation is weakly multifractal. Long-term

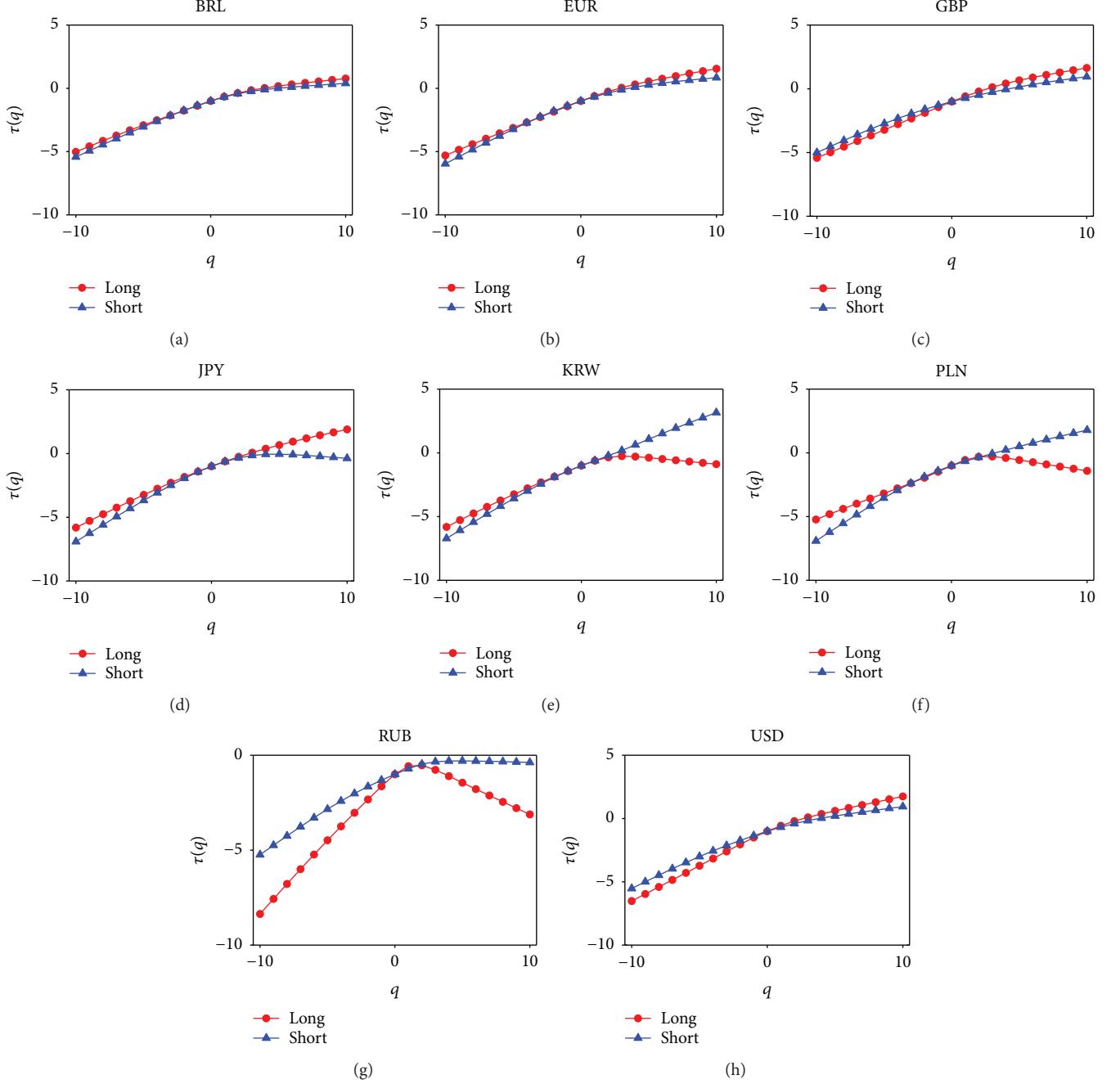


FIGURE 10: The long-term and short-term mass exponent values of $\tau_{xy}(q)$.

and short-term scaling exponents $H_{xy}(2)$ of all exchange rates are all less than 0.5, indicating antipersistent cross-correlations. This conclusion is consistent with the findings in Section 4.2. Besides, the particularity of RUB may require further analysis.

From Figure 10, we find the $\tau_{xy}(q)$ of KRW, PLN, and RUB are more curved in the long term, which indicates that the three exchange rates are more multifractal in the long term. The rest exchange rates are the opposite. This evidence supports the findings of Table 5.

In Figure 11, although some points are clustered together, the $\Delta\alpha$ is not zero. As can be seen from Figure 11, the

multiplicative strength of the spectrum of RUB in the long term is the largest, which means the multiplicative degree of RUB is stronger than others. The multiplicities of KRW, PLN, RUB, and USD are stronger in the long term, while the spectra of the rest are just the opposite.

4.4. Rolling Window Analysis. The method of rolling windows is very popular in the financial field due to its ability to analyze the dynamics of time series. The result of the rolling method depends on the window length. In particular, if the length is set too short, short-term fluctuations may be very dramatic; we may not be able to accurately determine

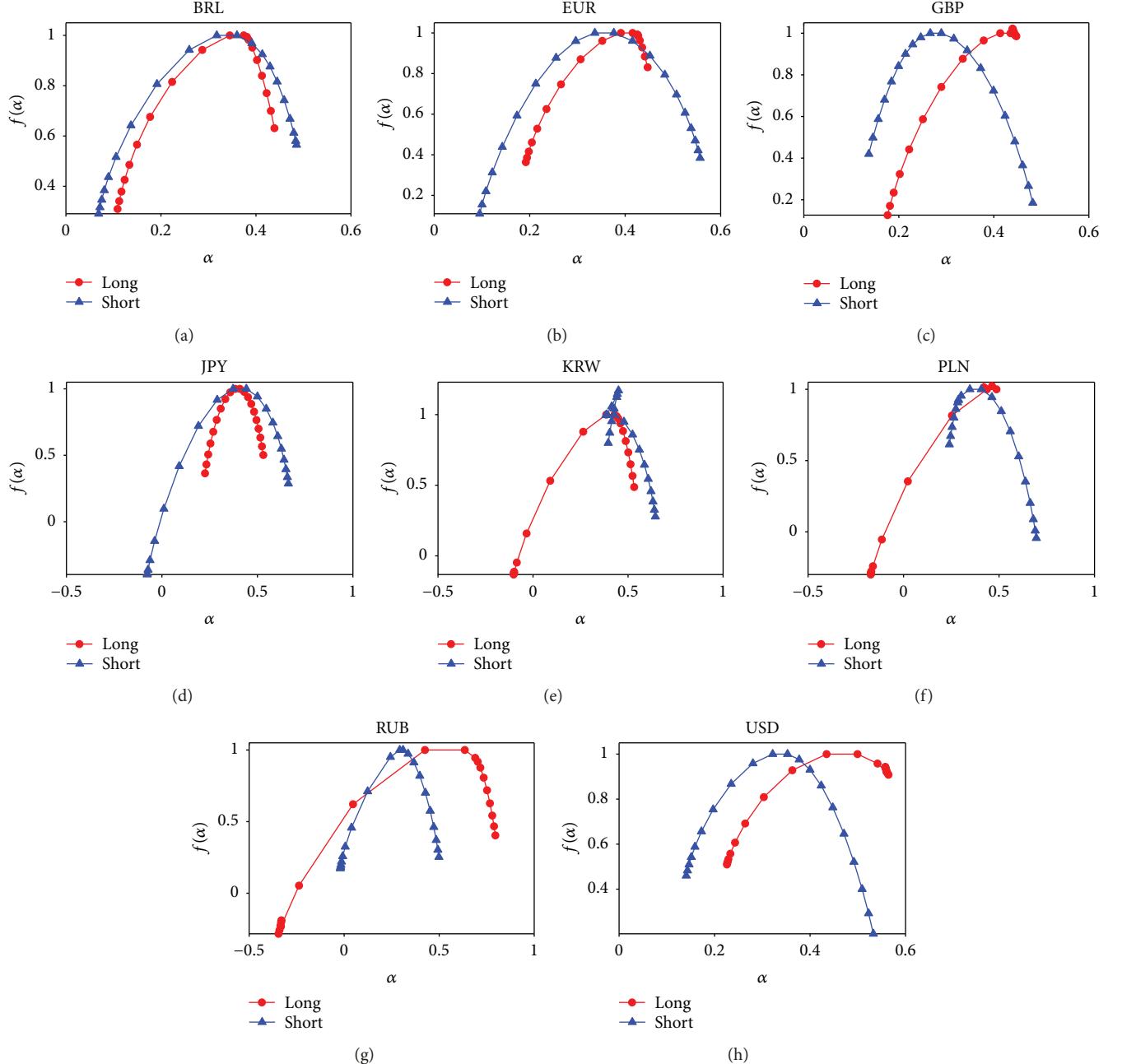


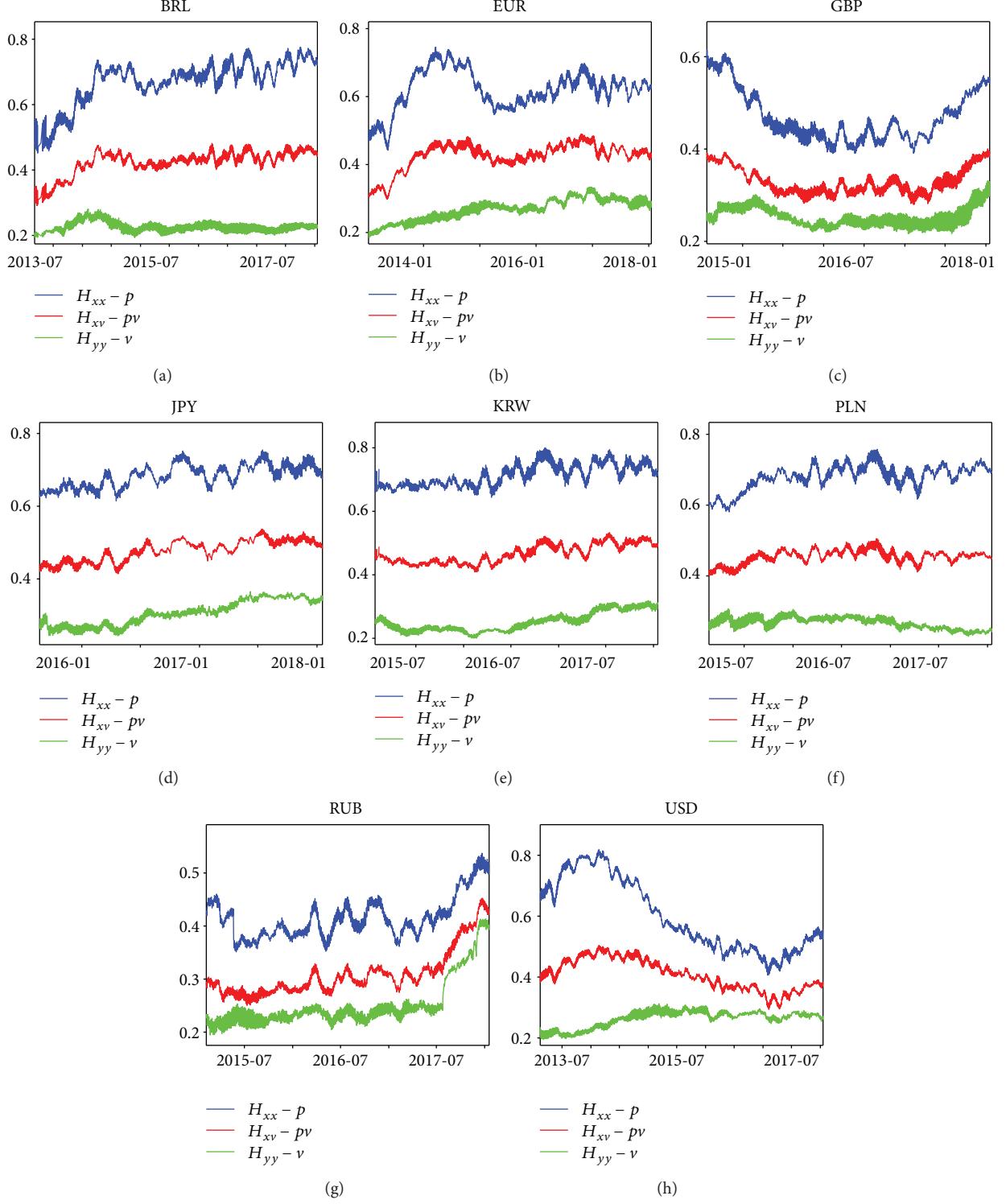
FIGURE 11: The long-term and short-term multifractal spectrum $f_{xy}(\alpha)$.

the long-term trend. If the length is set too long, we cannot observe the local detail changes.

According to the previous study of Bitcoin using the rolling window approach [15, 16], in this paper, we use a 500-day sliding window to estimate the scaling exponents. Specifically, we use the overlay window to move forward the one-day data point.

The legends $H_{xx} - p$, $H_{yy} - v$, and $H_{xy} - pv$ in Figure 12 denote price, volume, and bivariate. The blue line and green line are the Hurst exponents of price, respectively, by employing DFA. And the red line is the exponent for the cross-correlation for return-volume. The green lines in the

eight charts of Figure 12 are less than 0.5 over the entire sample period, which means the volume change is antipersistent. The blue lines of BRL, EUR, JPY, KRW, and PLN are greater than 0.5, which means Bitcoin is not efficient in those markets. The GBP price exponent trend is first to decline and then rise. In the chart of USD, the Hurst exponent of price is greater than 0.5 before 2014 and wanders around 0.5 after 2014, which means USD-denominated Bitcoin is in the process of moving efficiently. This evidence supports the conclusion of Urquhart [14] and Bariviera [15]. The price Hurst exponent of RUB is less than 0.5. And this is another proof of the special nature of the RUB. All the scaling

FIGURE 12: Time varying scaling exponents when $q = 2$.

exponents of the cross-correlation are less than 0.5, indicating that returns and price are not persistently cross-correlated. In particular, the exponent of the RUB is lower and wanders around 0.3, which indicates the cross-correlation of RUB is more antipersistently cross-correlated (under the circumstance that H_{xy} is less than 0.5 which

means antipersistently cross-correlated, if H_{xy} of RUB is smaller than that of other currencies when they are both less than 0.5, we say RUB is more antipersistently cross-correlated). The RUB rise of the scaling exponents of the cross-correlation after July 2017 may be related to the madness of the global Bitcoin market.

5. Concluding Remarks

We selected the most actively traded markets in the world and synthesize the composite index of the top eight exchange rates with the highest volume on the basis of the volume-weighted composite index. In particular, we conduct the cross-correlation test with multifractal detrended cross-correlation analysis (MF-DCCA), and the empirical results show that first, the global Bitcoin markets are not fully synchronized. The statistics and multifractal features of Bitcoin denominated in different exchange rates are very different. Among them, the RUB is most evident. Second, we use the cross-correlation test statistic Q_{cc} to find the nonlinear dependencies and power-law cross-correlations in return-volume relationship in all of the Bitcoin markets. Third, all cross-correlations of the eight exchange rates are multifractal. The cross-correlation Hurst exponent $H_{xy}(2) < 0.5$, which means antipersistent behavior for the return-volume in the Bitcoin markets. The cross-correlation exponent H_{xy} is less than the average generalized Hurst exponent for $q > 0$. And the price behavior of small fluctuations is more persistent than the volume and cross-correlation behavior, while the volume of larger fluctuations is more antipersistent. Fourth, we find that the bivariate cross-correlation exponent is less than 0.5 by employing a rolling window method. This indicates that the cross-correlation of return-volume is antipersistent with time going by. It would be interesting to investigate the impacts of the policy changes on the return-volume relationship of the Bitcoin market. We leave this for future research.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this article.

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