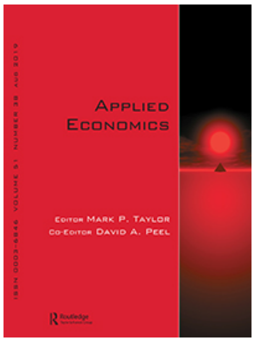

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Dynamic relationship between stock market trading volumes and investor fear gauges movements

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ABSTRACT
This article applies multifractal detrended fluctuation analysis (MF-DFA) and multifractal detrended cross-correlation analysis (MF-DCCA) to investigate cross-correlation behaviours between two kinds of stock markets trading volumes and investor fear gauges covering the data of U.S. stock markets from 2 January 2004 to 31 July 2018. The empirical results show that the dynamic relationship between stock markets trading volume fluctuations and different kinds of investor fear gauges are multifractal and find that the dynamic relationship is strongly anti-persistent. Moreover, financial crisis in 2008 has a significant impact on the cross-correlated behaviour, suggesting that stock market trading volume fluctuations and investor fear gauges are more susceptible to each other during the financial crisis period. Through the rolling windows analysis, we also find that the stock markets trading volume fluctuations and different kinds of investor fear gauges are anti-persistent dynamic cross-correlated.

KEYWORDS
Dynamic; MF-DFA; MF-DCCA; stock market trading volumes; investor fear gauges

JEL CLASSIFICATION
C32; C61

I. Introduction

Financial market trading volumes, especially stock market trading volumes play a significant role in capital markets. Trading volume reflects not only the efficiency of stock market but also market's sentiment. As we know, if market sentiment is down, the trading volume of a financial market may be correspondingly poor. Contrarily, if market sentiment is up, the trading volume may be corresponding improve. The existence of dynamic relationship between stock market trading volumes and market sentiment is crucial for businesses, investors and capital market. Previous studies about stock market trading volumes or market sentiment are based on the Efficient Market Hypothesis (Karpoff 1987; Bessembinder and Seguin 1993; Statman, Thorley, and Vorkink 2006). The limitation of Efficient Market Hypothesis lies in its inability to explain the real market situation, such as insider information, transaction cost, and momentum effect (Jegadeesh and Titman 1993).

Fractal Market Hypothesis (FMH) proposed by Mandelbrot (1982) is the frontier of nonlinear theory while financial market cannot be adequately addressed by using the traditional Efficient Market Hypothesis. Since then, a number of financial physics methods have proposed in order to test the dynamic relationship between two financial time series. Peng et al. (1994) proposed detrended fluctuation analysis (DFA), Kantelhardt et al. (2002) improved on it by proposing the multifractal form of DFA (MF-DFA). Subsequently, Alessio et al. (2002) proposed the detrending moving average (DMA), Gu and Zhou (2010) then developed the multifractal detrending moving average (MF-DMA). These two methods are both based on the moving average that was first researched by Vandewalle and Ausloos (1998). Podobnik and Stanley (2008) proposed detrended cross-correlation analysis (DCCA) to investigate power-law cross-correlations between nonstationary time series. Zhou (2008) then integrated DCCA into MF-DFA to derive multifractal detrended cross-correlation analysis (MF-DCCA).

The MF-DFA and MF-DCCA methods have been widely used to detect the cross-correlations between two financial series. Fleming and Kirby (2011) documented the relationship characteristics of volume and volatility, and show that volume and volatility both display long memory. Sukpitak and Hengpunya (2016) investigated the cross-

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correlation between market efficiency and trading volume, and show weak cross-correlation between market efficiency and trading volume. Lu, Sun, and Ge (2017) investigated the dynamic relationship between Japanese Yen exchange rates and the market anxiety gauge VIX, concluding that the cross-correlations between the two sets of time series are multifractal and susceptible to economic uncertainties and risks. Cai and Ren (2018) investigated the cross-correlations between crude oil price and investor fear gauges, showing cross-correlations of large fluctuations are strongly anti-persistent in both short- and long-term.

In 2004, the Chicago Board Options Exchange (CBOE) established its Volatility Index (the VIX) expanded from S&P 100 index to the volatility of options on the S&P 500 index. The VIX is a crucial gauge of the market's expectations of near-term (30-day) volatility, which is widely considered to be the world's premier barometer of investor's sentiment and market volatility. VIX values greater than 30 are generally associated with a large amount of volatility due to investors' fear or uncertainty, while values below 20 generally correspond to less stressful, even complacent periods in the market. The CBOE Dow Jones Industrial Average (DJIA) Volatility Index (VXD) is based on real-time prices of options on the DJIA (such options represented by the ticker symbol DJX) and is designed to reflect investors' consensus view of future (30-day) expected stock market volatility. The VIX and the VXD are all widely used measures of market sentiment, and often referred to 'investor fear gauges.' Stock market trading volume fluctuations which measured by Podobnik et al. (2009) explored the possible relationships between price changes and volume changes, and find power-law cross-correlations between absolute of logarithmic volume change and logarithmic price change of S&P500 Index. As the VIX is based on the S&P 500 index after 2004, and the VXD is based on DJIA, we focus on the relationship between investor fear gauges and trading volume fluctuations of each dependent index. We will test the cross-correlation features of the VIX, the VXD changes, S&P500 and DJIA trading volume fluctuations using MF-DFA and MF-DCCA methods, respectively.

This article focuses on investigating the cross-correlation between stock markets trading volume fluctuations and two kinds of investors fear gauges which represent market sentiment gauges. For the trading volume, we chose the trading volume logarithmic fluctuations of S&P500 and DJIA. Our study examines the relationship between trading volume logarithmic fluctuations and two kinds of investor fear gauges using both qualitative and quantitative analyses. By applying the MF-DFA and MF-DCCA method, we find the characteristic of multifractality for the cross-correlated degrees of stock market trading volume fluctuations and investor fear gauges. Through the rolling-windows method, the features of multifractality for the cross-correlations between stock market trading volume fluctuations and two kinds of investor fear gauges are re-examined. And cross-correlated characteristics are pay attention under 2008 financial crisis background. Ftiti, Jawadi, and Louhichi (2017) investigate the relationship between realized volatility and trading volume of energy and stock markets, which supporting the mixture distribution hypothesis. Guo et al. (2017) find that the greater uncertainty leads to lower trading volume, higher price fluctuations on subsequent months by panel VAR and causality analysis. Gupta et al. (2018) show the linkage between market returns and trading volume of two major emerging stock market-China and India using a wavelet-based vector autoregression approach. Though most studies examine the linkages between trading volume and other financial markets, few studies investigate market sentiment to examine correlations with trading volume fluctuations of investor fear gauges' dependent stock market using MF-DFA and MF-DCCA methods based on FMH. Then, previous studies examine the relationship often uses a qualitative way which cannot measure relevance degree, multifractal theory is not only a qualitative way but also a quantitative analysis to show relevance degree which is helpful for investors and policymakers.

The main contribution of this article is to fill the gap in the literature by thoroughly finding the multifractality and dynamic relationship between trading volume fluctuations and two kinds of investors fear gauges with the use of MF-DFA, MF-DCCA and rolling windows methods. And we find the strongest multifractality during the financial crisis period, which means the capital market has a stronger non-linear structure while it is affected by economic shocks.

The rest of the article is organized as follows: Section II introduces the methodology. Section III describes the data to be used. Section IV reports the empirical results. Section V concludes the article.

II. Methodology

To explore the cross-correlations between stock market trading volume fluctuations and investor fear gauges, MF-DFA, DCCA and MF-DCCA are proposed in this article. These approaches can be expressed as follows:

Step 1. Imagine two time series $x(t)$ and $y(t)$(t= 1, 2,. . ., N), where N is the equal length of these two series. The 'profile' of each series is then determined as follows:

$$X_t = \sum_{k=1}^{t} (x_k - \bar{x}) Y_t = \sum_{k=1}^{t} (y_k - \bar{y}) \quad t = 1, 2, \ldots, N. \tag{1}$$

where $(\bar{x})$ and$(\bar{y})$ describe the average returns of the two time series $x(t)$ and $y(t)$.

Step 2. The two series $x(t)$ and $y(t)$ are divided into N_s= $[N/s]$ non-overlapping segments of the same length s. Since the length N of the series is not always a multiple of the considered timescale s, a short part of the profile (1) may remain. To ensure that the complete information is not disregarded in the time series, the same procedure is repeated starting from the opposite of the two series $x(t)$ and $y(t)$. Thus, $2N_s$ segments are obtained together.

Step 3. Define the local trends from an m^{th}-order polynomial fit:

$$X_\lambda(j) = \alpha_k j^m + \alpha_{k-1} j^{m-1} + \ldots + \alpha_1 j + \alpha_0 \tag{2}$$

$$Y_\lambda(j) = \beta_k j^m + \beta_{k-1} j^{m-1} + \ldots + \beta_1 j + \beta_0 \tag{3}$$

Where $j = 1, 2, \ldots, s$, $\lambda = 1,\ 2,\ \ldots,\ 2N_s$, m= 1, 2, ...

Step 4. Calculate the local trends for each $2N_s$ segment by an m^{th}-order polynomial fit. The detrended covariance is determined by:

$$F^2(s,\lambda) = \frac{1}{s} \sum_{j=1}^{s} \left| X_{(\lambda-1)s+j}(j) - X_\lambda(j) \right| \left| Y_{(\lambda-1)s+j}(j) - Y_\lambda(j) \right| \tag{4}$$

For each segment λ, $\lambda = 1,\ 2,\ \ldots,\ N_s$ and

$$F^2(s,\lambda) = \frac{1}{s} \sum_{j=1}^{s} \left| X_{N-(\lambda-N_s)s+j}(j) - X_\lambda(j) \right| \left| Y_{N-(\lambda-N_s)s+j}(j) - Y_\lambda(j) \right| \tag{5}$$

For each segment λ, $\lambda = N_s + 1, N_s + 2,\ \ldots,\ 2N_s$. $X_\lambda(j)$ and $Y_\lambda(j)$ are the fitting polynomial of each profile with order m in segment λ, which is also referred to as MF-DCCA-m.

Step 5. Obtain the q^{th} order fluctuation function from averaging all segments λ.

$$F_q(s) = \left\{ \frac{1}{2N_s} \sum_{\lambda=1}^{2N_s} F^2(s,\ \lambda)^{q/2} \right\}^{1/q} \tag{6}$$

When q= 0, Equation (6) can be re-expressed as follows:

$$F_0(s) = exp\left\{ \frac{1}{4N_s} \sum_{\lambda=1}^{2N_s} ln\left[F^2(s,\ \lambda)\right] \right\} \tag{7}$$

Step 6. Analyze the scaling behaviour of the fluctuation function by observing the log-log plot $F_q(s)$ against each value of q. If the two series $x(t)$ and $y(t)$ are long-range cross-correlated, we can derive that $F_q(s)$ has large values of s. Thus, a power–law relationship can be expressed as follows:

$$F_q(s) \sim s^{H_{xy}(q)} \tag{8}$$

Equation (8) can be rewritten as follows:

$$\log F_q(s) = H_{xy}(q) \log(s) + \log(C) \tag{9}$$

where C in Equation (9) is the constant. The scaling exponent $H_{xy}(q)$, which is known as the generalized cross-correlation exponent, can be obtained by observing the slope of the log-log plot of $F_q(s)$ versus s using the method of ordinary least squares for each value of q. If $H_{xy}(q) > 0.5$, the cross-correlations between the two time series related to q are persistent, which demonstrates that an increase in one series is statistically likely to be followed by an increase in the other series. If $H_{xy}(q) < 0.5$, the cross-correlations between the two time series related to q are anti-persistent, which demonstrates that an increase in one series is statistically likely to be followed by a decrease in the other series. If $H_{xy}(q) = 0.5$, the two series are not cross-correlated with each other, which means that alterations in one series do not affect the behaviour of the other series. When q= 2 in Equation (8), the MF-DCCA method reduces to DCCA.

The scaling exponent $H_{xy}(q)$ changes to $H_{xy}(2)$, which is identical to the well-known Hurst exponent H. If $x(t)$ and $y(t)(t = 1, 2, \ldots, N)$ are the same series, the MF-DCCA method reduces to MF-DFA. Where $H_{xy}(q)$ is independent of q, the cross-correlation between the two series is monofractal; otherwise, it is multifractal.

Step 7. The Renyi exponent $\tau_{xy}(q)$ adopts a multifractal nature, so the exponent $\tau_{xy}(q)$ can be expressed as follows:

$$\tau_{xy}(q) = qH_{xy}(q) - 1 \tag{10}$$

If the scaling exponent function $\tau_{xy}(q)$ is linear with q, the cross-correlation between the two series is monofractal; otherwise, it is multifractal.

Step 8. Through the Legendre transformation, the singularity content of the time series can be deduced from the multifractal spectrum $f_{xy}(\alpha)$.

$$\alpha_{xy}(q) = \tau'_{xy}(q) = H_{xy}(q) + qH'_{xy}(q) \tag{11}$$

$$\begin{aligned} f_{xy}(\alpha) &= q\alpha_{xy} - \tau_{xy}(q) \\ &= q\left[\alpha_{xy} - H_{xy}(q)\right] + 1 \end{aligned} \tag{12}$$

where $H'_{xy}(q)$ is the derivative of $H_{xy}(q)$ with respect to q, $\tau'_{xy}(q)$ is the derivative of $\tau_{xy}(q)$ with respect to q, and α is the Holder exponent or singularity strength that expresses the singularity and monofractality in time series. The width of the spectrum determines the strength of multifractality, obtained by $\Delta\alpha_{xy} = \max(\alpha_{xy}) - \min(\alpha_{xy})$. The broader the width of the spectrum, the greater the strength of the multifractality, and vice versa. If a multifractal spectrum appears as a point, it is monofractal. The width of the spectrum α_{xy} can be fitted by the following function:

$$\alpha_{xy} = -\frac{1}{ln2} \times \frac{a^q lna + b^q lnb}{a^q + b^q}. \tag{13}$$

Where $\alpha_{xy}(-\infty) = -lna/ln2$ denotes the weakest singularity α_{min}, $\alpha_{xy}(+\infty) = -lnb/ln2$ reflects the strongest singularity α_{max}. Thus, the $\Delta\alpha_{xy}$ can be estimated by the parameters a, b.

Step 9. To further measure the degree of multifractality, ΔH is described as follows:

$$\Delta H = H_{max}(q) - H_{min}(q) \tag{14}$$

where the larger ΔH is, the greater the degree of multifractality, and vice versa.

III. Data

In this article, we use daily S&P500、DJIA trading volume fluctuations and two different kinds of investor fear gauges, the VIX and VXD. The sample data covered the period from 2 January 2004 to 31 July 2018, and each series contains 3656 observations. The original data were derived from the CBOE website and the iFinD financial database. Based on trading volume logarithmic changes measured by Podobnik et al. (2009), we set the trading volume fluctuations of S&P500 index (SP500) and the trading volume fluctuations of DJIA index (DJIA) as follows:

$$\Delta VOL_t = log(VOL_t) - log(VOL_{t-1}). \tag{15}$$

Where VOL_t is the daily trading volume of each stock market.

Since VIX and VXD are quoted as percentages, we use daily changes in these two kinds of investor fear gauges as follows:

$$\Delta V_t = V_t - V_{t-1} \tag{16}$$

Where V denotes VIX or VXD, respectively. The results for VOL_t, V_t, ΔVOL_t and ΔV_t are illustrated in Figure 1 and Table 1.

As shown in Figure 1, trading volume fluctuations and investor fear gauges changes demonstrate the clusters of small and large fluctuations. During the global financial crisis, the investor fear gauges experienced more drastic fluctuations and substantial increase, but not the significant same scenario appeared in the trading volumes fluctuations. After the global financial crisis, trading volumes of stock markets decreased slowly over the time and raised since 2017.

Table 1 shows the descriptive statistics of daily trading volume fluctuations of S&P500, DJIA and daily VIX changes, daily VXD changes. Each index of the mean value is close to zero, each standard deviation is larger than zero. The skewness is nonzero, and kurtosis is larger than 20, indicating significant deviations from normality. The Jarque–Bera statistical test shows the rejection of the null hypothesis of normality at the 5%

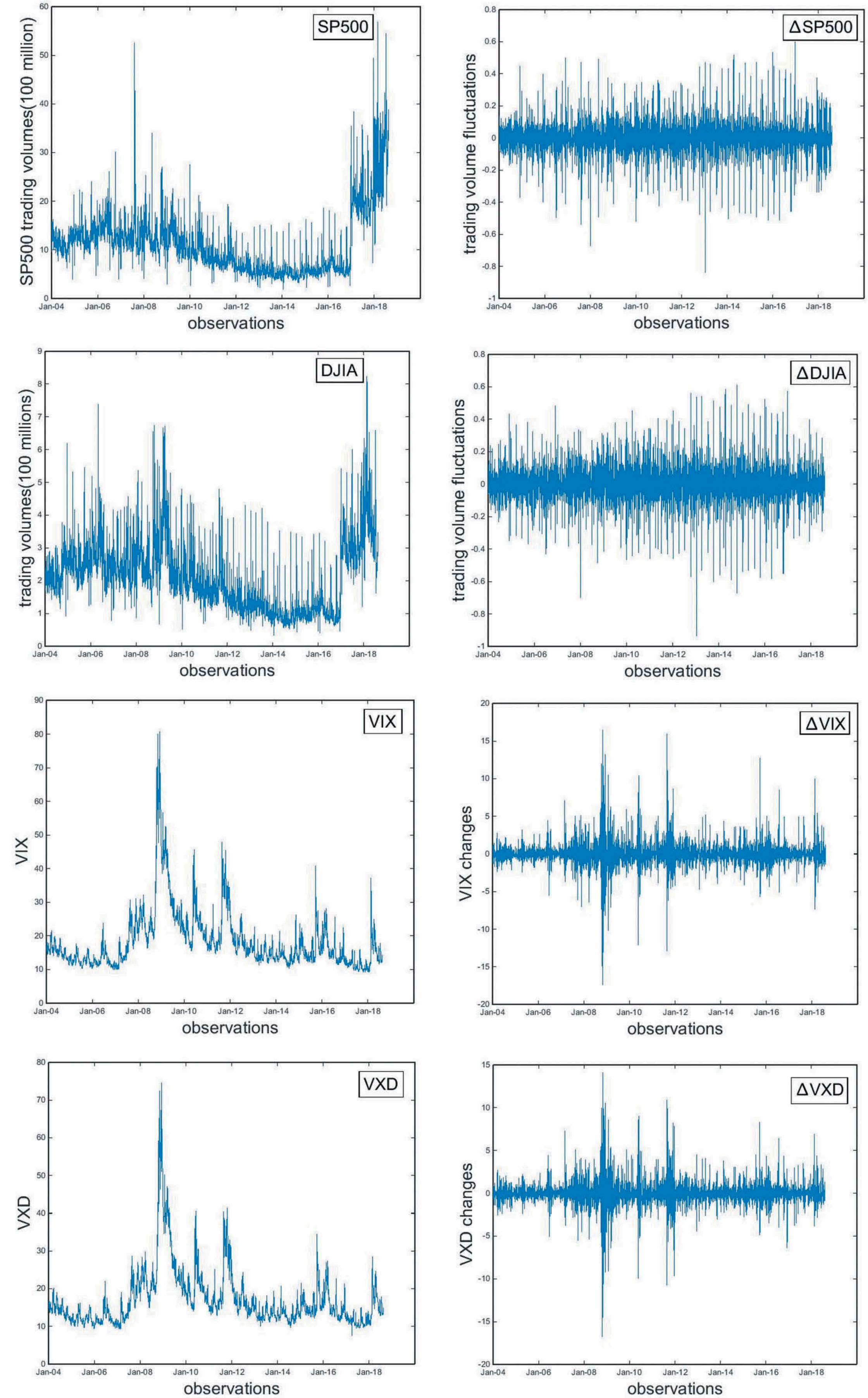


Figure 1. Trading volume (fluctuations) of S&P500 trading volume (fluctuations) of DJIA and daily (changes) in two different kinds of fear gauges, VIX:VIX ,VXD:VXDrespectively.

Table 1. Descriptive statistic for daily trading volume fluctuations of S&P500, DJIA, VIX and VXD changes.

	Mean(%)	Min	Max	S.D.	Skew	Kurtosis	JBtest	ADF	Observation
ΔSP500	0.0166	−1.9856	2.3942	0.1267	0.7164	89.7478	1146300*	−94.6326*	3655
ΔDJIA	0.0086	−1.9897	2.1003	0.1371	0.1193	53.7770	392660*	−94.1681*	3655
ΔVIX	0.0400	−17.36	20.01	1. 7520	1.0387	25.2525	76068*	−68.6508*	3655
ΔVXD	−0.0744	−16.82	14.16	1. 5457	0.3510	20.8627	48668*	−71.5630*	3655

'Min', 'Max', 'SD', 'Skew', 'Kurt' denote Maximum, Minimum, Standard Deviation, Skewness and Kurtosis, respectively. JBtest denotes Jarque–Bera test. ADF denotes Augmented Dickey–Fuller test. * Denotes the rejection of the null hypothesis at the significance level of 5%.

significance level. In addition, the ADF test shows the stationarity of daily VIX changes, daily VXD changes and daily trading volume fluctuations of two kinds of stock markets.

IV. Empirical results

Cross-correlation test

For the two time series, $\{x_t,\ t = 1, \ldots N\}$ and $\{y_t,\ t = 1, \ldots N\}$, the statistical test

$$Q_{cc}(m) = N^2 \sum\nolimits_{t=1}^{m} \frac{{c_t}^2}{N-t} \tag{17}$$

where their cross-correlation function is:

$$c_t = \frac{\sum_{k=t+1}^{N} x_k y_{k-t}}{\sqrt{\sum_{k=1}^{N} x_k^2 \sum_{k=1}^{N} y_k^2}} \tag{18}$$

The cross-correlation statistic $Q_{cc}(m)$ is approximately $\chi^2(m)$ distributed with m degrees of freedom. We select any degree of freedom ranging from 1 to 1000 using $\chi^2(m)$ distribution. If there is no cross-correlation between the two time series, the cross-correlations statistic $Q_{cc}(m)$ is well aligned with the $\chi^2(m)$ distribution. If the value of $Q_{cc}(m)$ deviates from the value of $\chi^2(m)$ distribution at the 5% level of significance, nonlinear cross-correlations exist between the two time series.

Figure 2 shows the cross-correlation statistic for the daily trading volume fluctuations of two different stock markets and the daily changes in each investor fear gauge compared to the critical values for the $\chi^2(m)$ distribution at the 5% level of significance with the degree of freedom m ranging from 1 to 1000. Each line is deviated from the critical value of $\chi^2_{0.95}(m)$. Therefore, we find that a nonlinear cross-correlated relationship exists between the two time series. Nonlinear cross-correlations exist in these four pairs of series, and we need to use the MF-DCCA method to further investigate the cross-correlations between daily trading volume fluctuations of two different stock markets and investor fear gauges movements.

DCCA coefficient

The DCCA coefficient was proposed by Zebende (2011), who offered a method to investigate how coefficients vary with different time scales. Then, Reboredo, Rivera-Castro, and Zebende (2014), Wang et al. (2017) adopt the detrended fluctuation analysis to quantify the level of dynamic relationship between two different markets. The coefficient ρ_{DCCA} is expressed as follows:

$$\rho_{DCCA} = \frac{F^2_{DCCA}(s)}{F_{DFA1}(s) F_{DFA2}(s)} \tag{19}$$

where $F^2_{DCCA}(s)$, $F_{DFA1}(s)$, $F_{DFA2}(s)$ are calculated using Equations (4 and 5) while q= 2, polynomial order m= 1 in this article. The value of ρ_{DCCA} ranges from −1 to 1. If $\rho_{DCCA} = 0$, there is no cross-correlation between the two time series. If $\rho_{DCCA} = 1$, the two series are perfect persistently cross-correlated. If $\rho_{DCCA} = -1$, there is perfect anti-cross-correlation between the two time series. Different values of ρ_{DCCA} based on different values of window size s (s= 8, 16, 32, 64, 128, 256, 512) are shown in Table 2.

As seen in Table 2, with each different s, the values of DCCA coefficient ρ_{DCCA} are all within the range from −1 to 0. This shows that the dynamic relationship between daily trading volume fluctuations and daily changes in two different kinds of investor fear gauges are all anti-persistent, respectively. These results are consistent with the results from previous cross-correlation test. The anti-cross-correlated relationship between daily trading volume fluctuations of stock markets and daily changes investor fear gauges are significant.

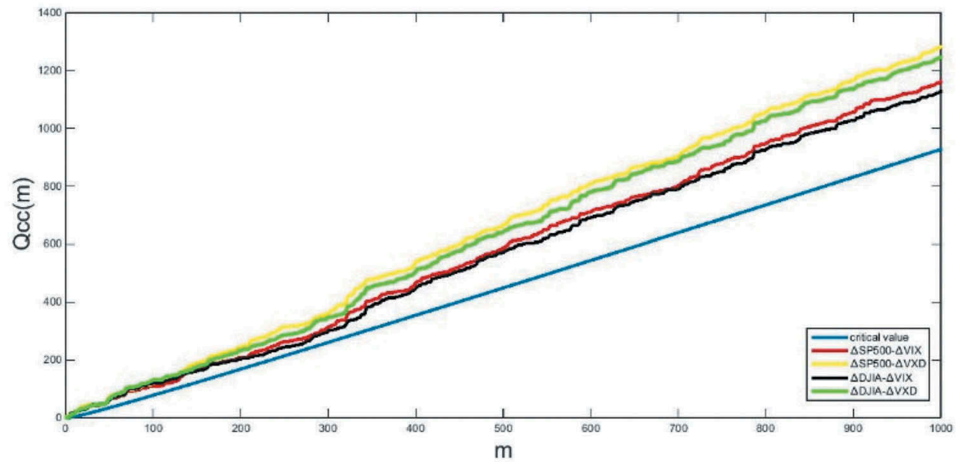


Figure 2. (Colors online) the cross-correlation statistic $Q_{cc}(m)$ for daily trading volume fluctuations of stock markets (ΔS&P500, ΔDJIA) and daily investor fear gauges movements. The blue line denotes the critical value of the cross-correlation statistic. The red line denotes the cross-correlation statistic for daily trading volume fluctuations of S&P500 and daily changes in investor fear gauge VIX, the yellow line denotes the cross-correlation statistic for ΔSP500-ΔVXD, the black line denotes the cross-correlation statistic for ΔDJIA-ΔVIX, and the green line denotes the cross-correlation statistic for ΔDJIA-ΔVXD, respectively.

Table 2. The value of ρ_{DCCA} for different window sizes s while m= 1.

Size $s(m = 1)$	8	16	32	64	128	256	512
ΔSP500-ΔVIX	−0.1201	−0.2071	−0.2777	−0.3154	−0.2915	−0.2853	−0.2854
ΔSP500-ΔVXD	−0.1088	−0.2031	−0.2660	−0.2629	−0.2242	−0.1521	−0.1667
ΔDJIA-ΔVIX	−0.1248	−0.2293	−0.2961	−0.3396	−0.3376	−0.3583	−0.3569
ΔDJIA-ΔVXD	−0.0893	−0.2031	−0.2769	−0.2848	−0.2806	−0.2546	−0.2665

MF-DFA and MF-DCCA analyses

In order to test the effectiveness of a single financial market, we use MF-DFA method to obtain the multifractality degree ΔH and the width of the multifractal spectra $\Delta\alpha$ of the single financial market. While to further observe the dynamic relationship between trading volume fluctuations of stock markets and investor fear gauges, we adopt MF-DCCA to model the scaling behaviour of fluctuations and multifractality of cross-correlations between different time series in a quantitative way.

From equations (1–9), we set $-10 \leq q \leq 10$, $8 \leq s \leq N/4$, polynomial order m= 1, and calculate the slope of the fluctuation function $F_q(s)$ by ordinary least squares to obtain the Hurst exponents, each single financial market effectiveness and cross-correlations between daily trading volume fluctuations of stock markets and daily changes in two different kinds of investor fear gauges are shown in Tables 3 and 4.

The larger the values ΔH of a market, the stronger is the multifractality, as shown in Table 3, the strongest multifractal market is investor fear gauge VXD, and the lowest multifractal market is trading volume fluctuations of DJIA. That means the investor fear gauge VXD is the most irregular and disordered in these four financial time series, and based on the historical trend, predicting the future is not effective.

In Table 4, while q= 2, the scaling exponent $H_{xy}(2)$ for ΔSP500-ΔVIX is 0.1988, which shows that the cross-correlations of S&P500 daily trading volume fluctuations and VIX gauge movements are strongly anti-persistent. The Hurst exponent $H_{xy}(2)$ for ΔSP500-ΔVXD is 0.2039, and this also shows that the cross-correlations of DJIA trading volume fluctuations and VXD gauge movements are strongly anti-persistent. The scaling exponent for ΔDJIA-ΔVIX、ΔDJIA-ΔVXD all satisfies $H_{xy}(2) < 0.5$, which shows a strong anti-persistent cross-correlation between daily trading volume fluctuations of DJIA and investor fear gauges movements. The

Table 3. The Hurst exponents $H(q)$ for ΔSP500, ΔDJIA, ΔVIX and ΔVXD.

q	ΔSP500	ΔDJIA	ΔVIX	ΔVXD
−10	0.2625	0.2218	0.4804	0.4924
−9	0.2553	0.2157	0.4724	0.4836
−8	0.2468	0.2089	0.4631	0.4733
−7	0.2369	0.2010	0.4522	0.4613
−6	0.2252	0.1922	0.4394	0.4474
−5	0.2117	0.1821	0.4246	0.4315
−4	0.1964	0.1710	0.4079	0.4139
−3	0.1799	0.1590	0.3900	0.3959
−2	0.1635	0.1468	0.3729	0.3795
−1	0.1486	0.1353	0.3590	0.3670
0	0.1353	0.1247	0.3483	0.3569
1	0.1233	0.1146	0.3321	0.3389
2	0.1118	0.1047	0.2987	0.3023
3	0.1003	0.0946	0.2542	0.2549
4	0.0877	0.0842	0.2132	0.2119
5	0.0743	0.0732	0.1806	0.1780
6	0.0607	0.0621	0.1555	0.1519
7	0.0480	0.0513	0.1359	0.1315
8	0.0367	0.0413	0.1203	0.1154
9	0.0268	0.0324	0.1077	0.1022
10	0.0182	0.0244	0.0972	0.0914
MF-degree	**0.2443**	**0.1974**	**0.3832**	**0.4010**

Note: 'MF-degree' is the degree of multifractality ΔH.

Table 4. Cross-correlation exponents $H_{xy}(q)$ for ΔSP500-ΔVIX,ΔSP500-ΔVXD,ΔDJIA-ΔVIX and ΔDJIA-ΔVXD.

q	ΔSP500-ΔVIX	ΔSP500-ΔVXD	ΔDJIA-ΔVIX	ΔDJIA-ΔVXD
−10	0.3078	0.3213	0.2967	0.3055
−9	0.3022	0.3149	0.2917	0.3000
−8	0.2962	0.3077	0.2864	0.2939
−7	0.2896	0.2998	0.2808	0.2875
−6	0.2828	0.2913	0.2750	0.2806
−5	0.2761	0.2826	0.2692	0.2735
−4	0.2698	0.2742	0.2635	0.2664
−3	0.2643	0.2671	0.2581	0.2594
−2	0.2590	0.2611	0.2527	0.2526
−1	0.2522	0.2544	0.2463	0.2454
0	0.2409	0.2440	0.2373	0.2364
1	0.2232	0.2273	0.2237	0.2236
2	**0.1988**	**0.2039**	**0.2041**	**0.2050**
3	0.1697	0.1760	0.1795	0.1809
4	0.1395	0.1471	0.1531	0.1542
5	0.1118	0.1205	0.1284	0.1290
6	0.0884	0.0978	0.1072	0.1073
7	0.0693	0.0790	0.0895	0.0895
8	0.0538	0.0636	0.0750	0.0749
9	0.0410	0.0510	0.0630	0.0629
10	0.0305	0.0405	0.0530	0.0529
MF-degree	**0.2773**	**0.2808**	**0.2437**	**0.2526**

Note: 'MF-degree' is the degree of multifractality ΔH.

above results show that the cross-correlations of small and large fluctuations are anti-persistent. That is, a rise trading volume fluctuations of stock markets will be accompanied by a fall in changes of investor fear gauges movements. From Tables 3 and 4, we can see $H_{xy}(q)$ changes versus different values of q, which means the value of $H_{xy}(q)$ is dependent on q, and the cross-correlation between daily trading volume fluctuations of stock markets and two different investor fear gauges movements are multifractal. A strong anti-persistent cross-correlation between each daily trading volume fluctuations of stock market and each investor fear gauge movement is established. That means a rise of daily trading volume fluctuations of stock market will be accompanied by a fall in changes of investor fear gauges movements. This is consistent with our perception of the market, when the market goes down, the trading volume tends to be smaller, the panic index will be bigger, and vice versa.

If the exponents $H(q)$ depend on q, the cross-correlations between the two series are multifractal. To investigate the dependence of the scaling exponents on different values of q, the degree of multifractality is quantified by the method of the value $\Delta H(q)$, which obtains the largest values of $H(q)$ minus the smallest values of $H(q)$ based on Equation (14). As we can see from Table 4, the strongest multifractality is ΔSP500-ΔVXD, and the weakest multifractality is ΔDJIA-ΔVIX in four pairs of financial time series.

The Renyi exponent $\tau(q)$ can be calculated using Equation (10). Figure 3 shows $\tau_{xy}(q)$, $\tau_{xx}(q)$, $\tau_{yy}(q)$ versus different values of q. As seen from Figure 3, the value of $\tau_{xy}(q)$ is non-linearly dependent on q, which also confirms that stock markets trading volume fluctuations and each fear gauge movements has a multifractal dynamic relationship. These results are further evidence of what we have previously obtained.

Figure 4 shows that the multifractal spectra of the two time series is not a point, and is 'bell type', indicating that the cross-correlation between stock markets trading volume fluctuations and investor fear gauges movements has multifractal characteristics. The strength of multifractality can be estimated by the width of the multifractal spectrum and can be calculated using the value of $\Delta\alpha$ in Table 5.

In looking at Figure 4 and Table 5, we find that stock markets trading volume fluctuations and the investor fear gauges changes all have multifractal features. The strongest multifractal strength is ΔSP500-ΔVXD, and the weakest multifractal strength is ΔDJIA-ΔVIX in four pairs of financial time series as previously obtained, meaning that the volatility of ΔSP500-ΔVXD is the most dramatic, and the fluctuation of ΔDJIA-ΔVIX is the least dramatic. This dynamic relationship between stock markets trading volume fluctuations and the investor fear gauges changes can help investors to find capital risk and opportunity. It is possible to avoid risk by using the strong linkages between stock markets trading volume fluctuations and the investor fear gauges changes, and quantitative analysis suggests that linkage of ΔSP500-ΔVXD should be applied first, the linkage of ΔDJIA-ΔVIX should be applied last. While the lower the trading volume, and the more panic the stock market is, the real investment opportunities may have arrived.

Cross-correlations across time

To explore whether the cross-correlations between stock market trading volume fluctuations and investor fear gauges have changed across time, particularly around financial crisis in 2008, we divide the whole sample into three important sub-periods: Period 1 denotes pre-crisis, Period 2 denotes during-crisis, and Period 3 denotes post-crisis.

Our data are separated into three subsets as follows: (1) pre-crisis from 2 January 2004 to 31 July 2007; (2) during the financial crisis from 1 August 2007 to 31 December 2009; (3) post-crisis from 2 January 2010 to 31 July 2018.

Using MF-DCCA method, the cross-correlation exponents between the stock market trading volume fluctuations and investor fear gauges during different periods are showed in Table 6. The cross-correlation exponents decrease with the q from -10 to 10, suggesting that the cross-correlations between the stock market trading volume fluctuations and investor fear gauges are multifractal. When $q = 2$, the cross-correlation exponent for period 1 is the smallest among the three periods across the four pairs of financial time series, while the largest cross-correlation exponent is the period 2. Such cross-correlations can be expressed as follows:

$$\left|H(2)_{period\ 1} - 0.5\right| > \left|H(2)_{period\ 3} - 0.5\right| > \left|H(2)_{period\ 2} - 0.5\right|$$

Therefore, the linkage between stock market trading volume fluctuations and investor fear gauges appear to get stronger across time except for the financial crisis time. This is partly due to more and more company are listed (IPO) across

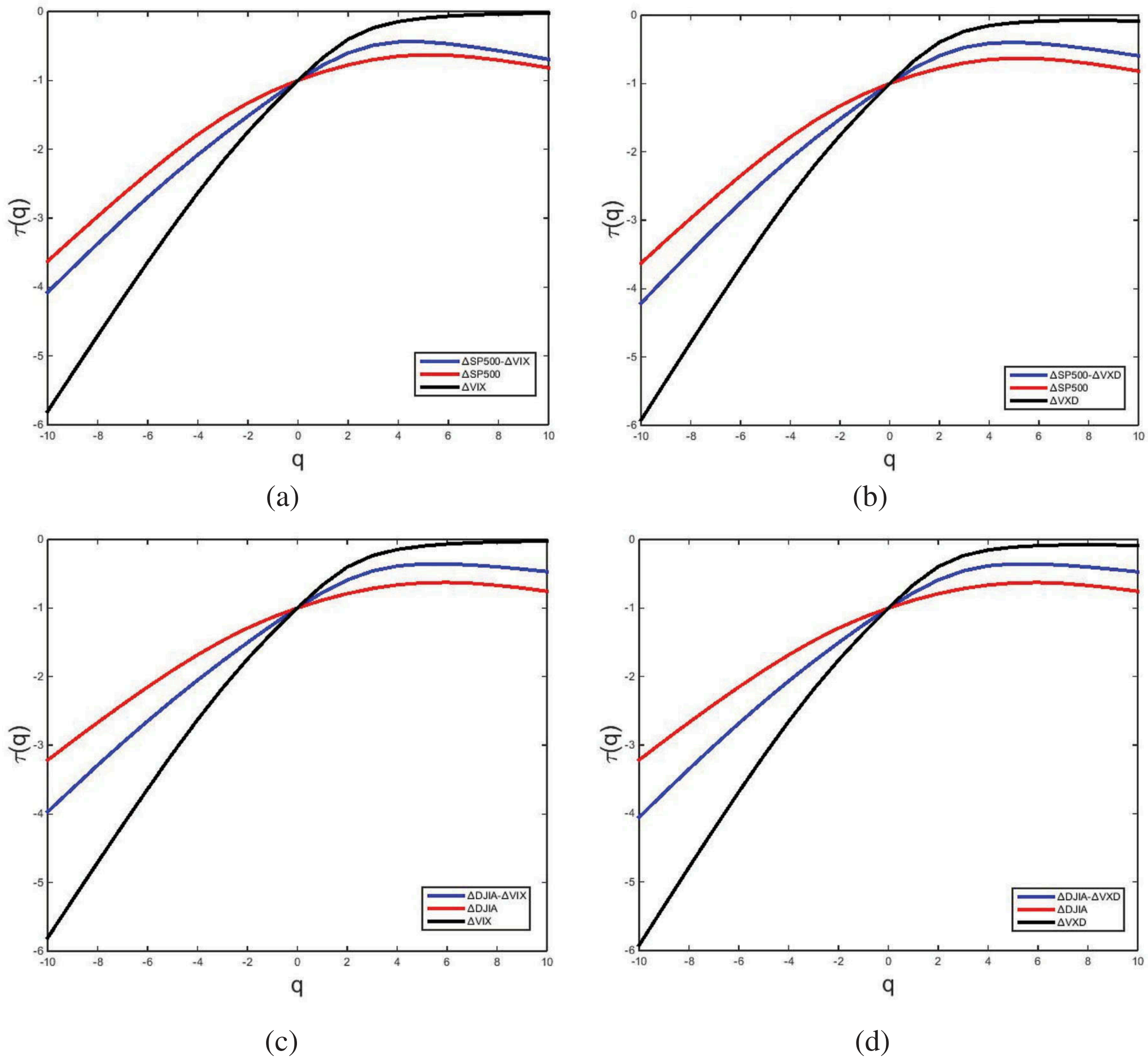


Figure 3. (Colors online) The Renyi exponent $\tau_{xy}(q)$ versus q for daily trading volume fluctuations of stock markets and daily changes in two different fear gauges. The blue curves denote the $\tau_{xy}(q)$ by MF-DCCA, the red curves denote $\tau_{xx}(q)$ of each daily trading volume fluctuations of stock market using MF-DFA, and the black curves denote $\tau_{yy}(q)$ of each daily investor fear gauge changes using MF-DFA.

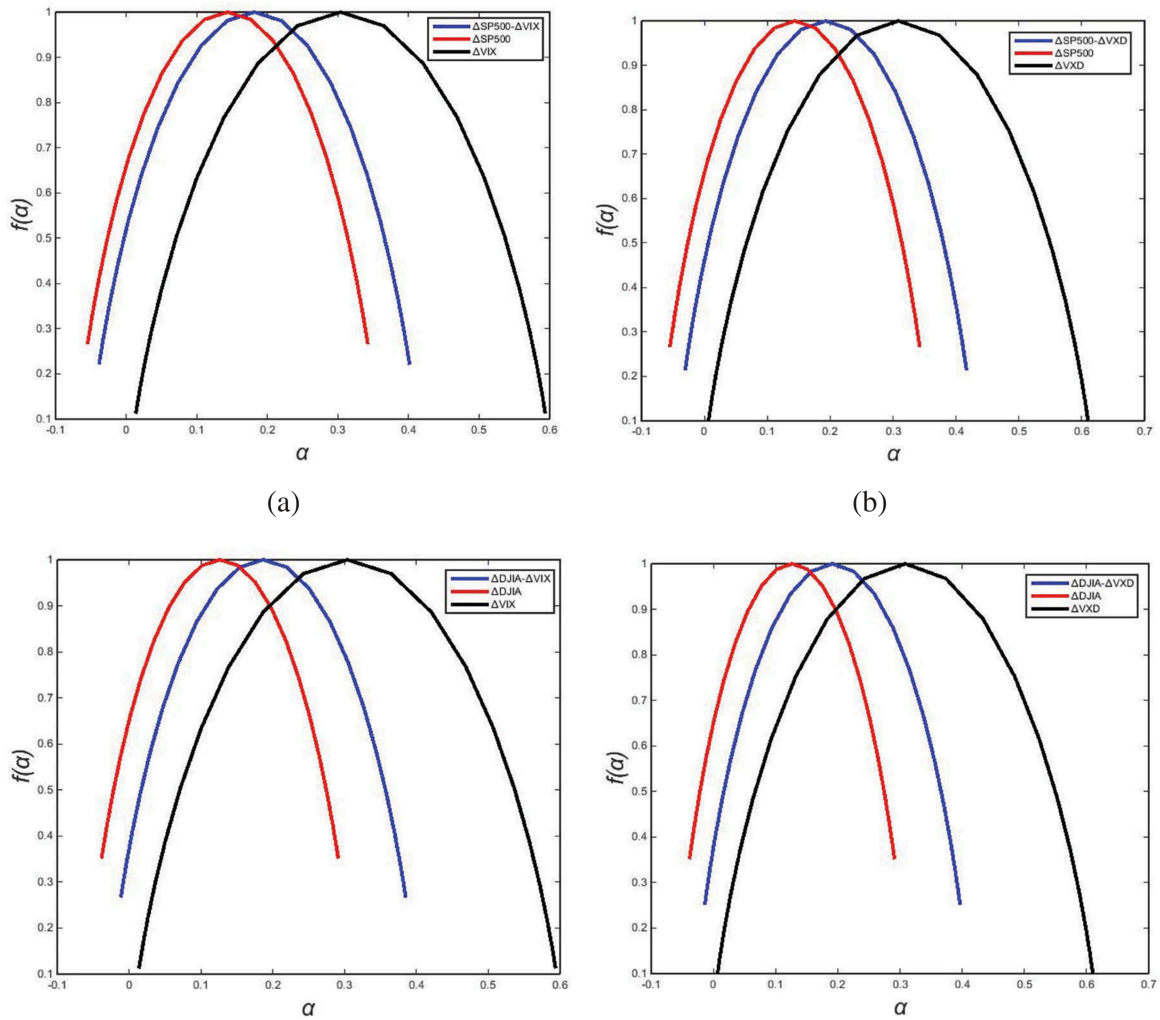
the time, resulting in uncertainties are more influential, and market sentiment is becoming increasingly unstable.

The multifractal spectra in the cross-correlated markets for different periods are showed in Figure 5. According to Table 6 and Figure 5, there are some common properties for the widths of multifractal spectra in four pairs of financial time series. The span of spectra for period 2 is the largest among all three periods. That means the cross-correlations show the strongest multifractality during the global financial crisis. Global financial crisis has seriously affected market sentiment, and the cross-correlations between the stock market trading volume fluctuations and investor fear gauges are the most susceptible to each other.

The widths of spectra $\Delta\alpha$ across time for four pairs of financial time series satisfy the following relation:

$$\Delta\alpha_{period\ 2} > \Delta\alpha_{period\ 3} > \Delta\alpha_{period\ 1}$$

It shows that the strongest multifractal characteristics time is period 2 (during the crisis), this is the further evidence of what we have previously obtained that the cross-correlated is the most susceptible to each other.



(a) (b)

(c) (d)

Figure 4. (Colors online) Nonlinear relationship of $f_{xy}(\alpha)$ versus α for daily stock market trading volume fluctuations and daily changes in two kinds of investor fear gauges movements ((a) ΔSP500-ΔVIX, (b) ΔSP500-ΔVXD, (c) ΔDJIA-ΔVIX, (d)ΔDJIA-ΔVXD). The blue curves denote $f_{xy}(\alpha)$ by MF-DCCA, the red curves denote $f(\alpha)$ of each daily stock market fluctuations using MF-DFA, and the black curves denote $f(\alpha)$ of daily investor fear gauges movements using MF-DFA.

Table 5. Estimated parameters of multifractal spectrum.

Time series	α_{min}	α_{max}	$\Delta\alpha$
ΔSP500-ΔVIX	−0.0552	0.4183	0.4735
ΔSP500-ΔVXD	−0.0468	0.4330	0.4798
ΔDJIA-ΔVIX	−0.0314	0.4051	0.4365
ΔDJIA-ΔVXD	−0.0328	0.4156	0.4484
ΔSP500	−0.0745	0.3622	0.4367
ΔDJIA	−0.0635	0.3163	0.3798
ΔVIX	0.0048	0.6027	0.5979
ΔVXD	−0.0014	0.6182	0.6196

Rolling-window analysis

The rolling-windows method is often used to further investigate the dynamic relationship between two financial time series. The windows length can be adjusted to suitable segments that fit the research needs. Inoue, Jin, and Rossi (2017) used macroeconomic time series to provide evidence that the choice of estimated window size is

Table 6. $H_{xy}(q)$ for ΔSP500-ΔVIX, ΔSP500-ΔVXD,ΔDJIA-ΔVIX,ΔDJIA-ΔVXD during different periods.

	ΔSP500-ΔVIX			ΔSP500-ΔVXD			ΔDJIA-ΔVIX			ΔDJIA-ΔVXD		
q	P 1	P 2	P 3	P 1	P 2	P 3	P 1	P 2	P 3	P 1	P 2	P 3
−10	0.3314	0.5576	0.3712	0.3332	0.5595	0.3650	0.3202	0.5622	0.3416	0.3201	0.5411	0.3498
−9	0.3243	0.5498	0.3635	0.3254	0.5515	0.3581	0.3125	0.5531	0.3349	0.3121	0.5326	0.3426
−8	0.3162	0.5408	0.3547	0.3166	0.5423	0.3502	0.3038	0.5424	0.3272	0.3030	0.5228	0.3344
−7	0.3071	0.5301	0.3445	0.3065	0.5313	0.3410	0.2939	0.5298	0.3183	0.2926	0.5114	0.3249
−6	0.2966	0.5172	0.3328	0.2951	0.5182	0.3303	0.2825	0.5147	0.3082	0.2805	0.4980	0.3139
−5	0.2847	0.5015	0.3193	0.2821	0.5021	0.3178	0.2696	0.4966	0.2963	0.2667	0.4823	0.3012
−4	0.2710	0.4820	0.3041	0.2674	0.4820	0.3032	0.2551	0.4748	0.2826	0.2512	0.4637	0.2864
−3	0.2554	0.4575	0.2872	0.2511	0.4563	0.2864	0.2390	0.4494	0.2667	0.2341	0.4417	0.2693
−2	0.2375	0.4287	0.2686	0.2330	0.4257	0.2673	0.2213	0.4208	0.2484	0.2158	0.4157	0.2497
−1	0.2176	0.3967	0.2476	0.2132	0.3936	0.2454	0.2022	0.3883	0.2276	0.1965	0.3849	0.2276
0	0.1957	0.3599	0.2235	0.1915	0.3583	0.2204	0.1817	0.3502	0.2037	0.1759	0.3483	0.2030
1	0.1722	0.3174	0.1959	0.1684	0.3174	0.1917	0.1603	0.3065	0.1763	0.1542	0.3058	0.1752
2	**0.1476**	**0.2720**	**0.1644**	**0.1449**	**0.2732**	**0.1600**	**0.1386**	**0.2602**	**0.1454**	**0.1320**	**0.2599**	**0.1443**
3	0.1229	0.2291	0.1303	0.1220	0.2306	0.1270	0.1178	0.2165	0.1126	0.1104	0.2158	0.1113
4	0.0994	0.1923	0.0974	0.1008	0.1938	0.0959	0.0986	0.1791	0.0809	0.0901	0.1775	0.0787
5	0.0781	0.1625	0.0691	0.0820	0.1638	0.0690	0.0814	0.1486	0.0525	0.0717	0.1464	0.0499
6	0.0598	0.1389	0.0460	0.0656	0.1400	0.0470	0.0664	0.1242	0.0286	0.0556	0.1216	0.0260
7	0.0443	0.1200	0.0276	0.0515	0.1211	0.0292	0.0532	0.1046	0.0089	0.0415	0.1019	0.0068
8	0.0313	0.1049	0.0127	0.0394	0.1059	0.0150	0.0418	0.0886	−0.0072	0.0294	0.0860	−0.0087
9	0.0202	0.0925	0.0006	0.0290	0.0935	0.0034	0.0319	0.0755	−0.0204	0.0190	0.0730	−0.0212
10	0.0108	0.0823	−0.0094	0.0202	0.0833	−0.0061	0.0233	0.0646	−0.0314	0.0100	0.0622	−0.0316
ΔH	**0.3206**	**0.4753**	**0.3806**	**0.3130**	**0.4762**	**0.3711**	**0.2969**	**0.4976**	**0.3730**	**0.3101**	**0.4789**	**0.3814**

Note: ΔH denotes the degree of multifractality during different periods.

sensitive and proposed that an optimal size should be used for forecasting. Ruan, Jiang, and Ma (2016) set the length of each window at 250 trading days (approximately one year) to research the cross-correlations between price and volume in the Chinese gold markets. We fix the length of each window at 250 business days (approximately one year), set the rolling step as one day, and calculate the scaling exponents for the four pairs of series in each window when q= 2. The results are shown in Figure 6.

Figure 6 shows that almost all scaling exponents are less than 0.5, indicating the strong anti-persistent cross-correlations between daily stock markets trading volume fluctuations and two kinds of investor fear gauges movements. Around the time of the 2008 financial crisis, scaling exponents are almost the largest value through the whole period, investors' emotions were significantly affected strongly, and the trading volume fluctuations of stock market is affected by many complex factors, as the window moves forward, the dynamics of Hurst exponents exhibit great fluctuations.

Our analysis uncovers the important of using MF-DCCA in studying the dynamic relationship between trading volume fluctuations and two kinds of investor fear gauges movements. This dynamic relationship can be very important for investors and Securities and Exchange Commission (SEC). First, since the investor fear gauges represent the market's expectations of 30-day volatility, investors can use the cross-correlations in order to avoid market risk. Second, since the strength of the anti-persistence between trading volume and investor fear gauges is closely related to the market sentiment, SEC should incorporate the anti-persistence strength into its policy decisions and pay attention to the oscillation of the anti-persistence strength, this can help to stabilize the market sentiment. Third, as for stock market policy adjusting is concerned, SEC may need to pay close attention to the time-varying characteristics of the policy impact on the changing stock markets and timely implement dynamic monitoring in different financial markets. This would help the policymakers to provide suggestions for adjusting the policy just in case global financial crisis come again.

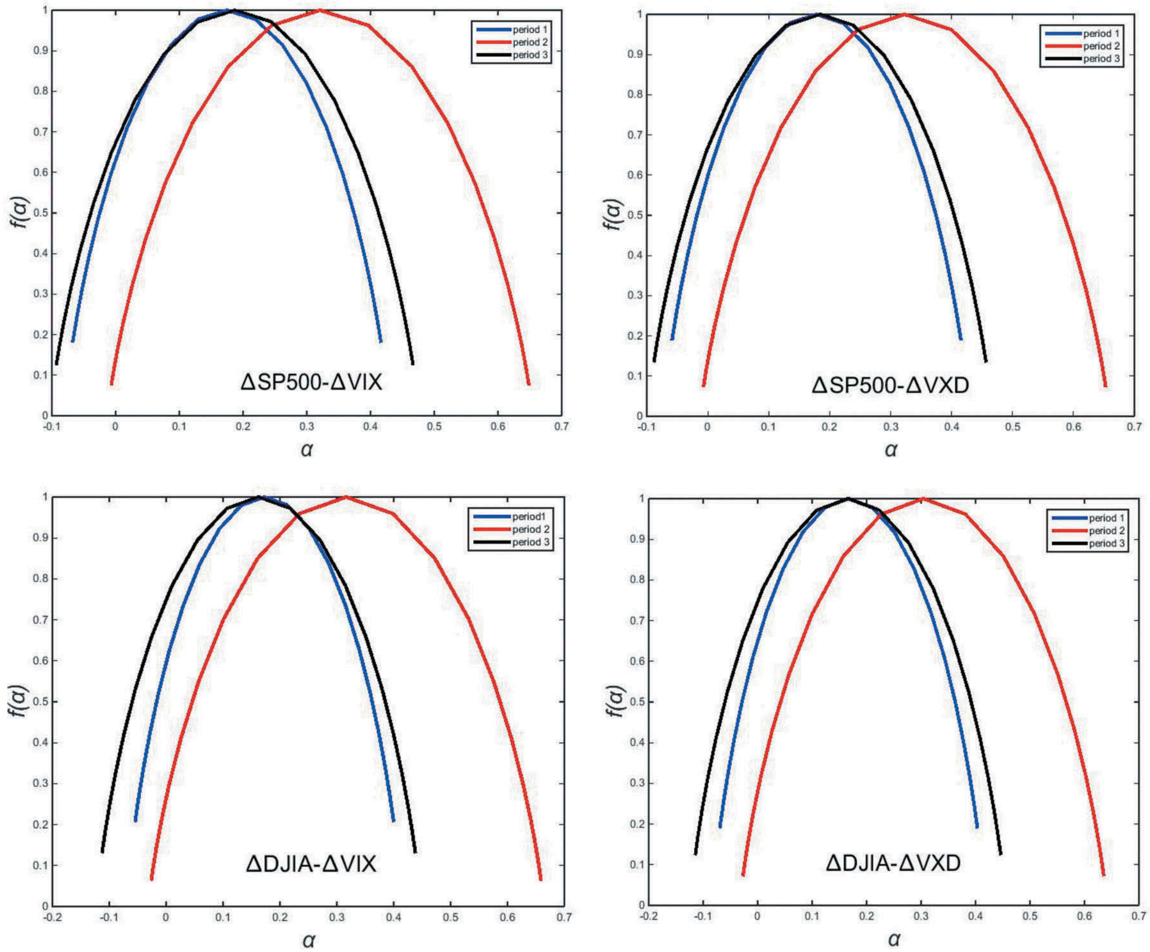


Figure 5. (Colors online) The multifractal spectra for ΔSP500-ΔVIX, ΔSP500-ΔVXD, ΔDJIA-ΔVIX, ΔDJIA-ΔVXD during different periods.

V. Conclusions

In this article, the dynamic relationship between stock markets trading volume fluctuations and two kinds of investor fear gauges movements are investigated using MF-DFA and MF-DCCA analyses. The conclusions are as follows:

First, both the qualitative and the quantitative analyses confirmed that ΔSP500, ΔDJIA, ΔVIX, ΔVXD financial markets movements are strongly multifractality, respectively, and the dynamic relationship between daily trading volume fluctuations of stock markets and daily investor fear gauges movements for each pair of series are also strongly multifractal.

Second, the empirical results show that the dynamic relationship between daily stock markets trading volume fluctuations and daily movements in two kinds of investors fear gauges for each pair of series are strongly anti-persistent.

Third, after dividing the whole sample into three important sub-periods: pre-crisis, during-crisis and post-crisis. The Hurst exponents and the multifractal spectra show that during-crisis period is the strongest multifractal characteristics period, secondly is the post-crisis period and lastly is the pre-crisis period. It shows that the cross-correlated is the most susceptible to each other duringcrisis period.

Fourth, using the analysis of the multifractal spectrum, we find that daily stock market trading volume fluctuations and each daily investor fear gauge change all have multifractal characteristics. The multifractal strength of dynamic relationships between ΔSP500-ΔVXD is the strongest in four pairs of two different financial time series, followed by ΔSP500-ΔVIX, then by ΔDJIA-ΔVXD, and last by ΔDJIA-ΔVIX.

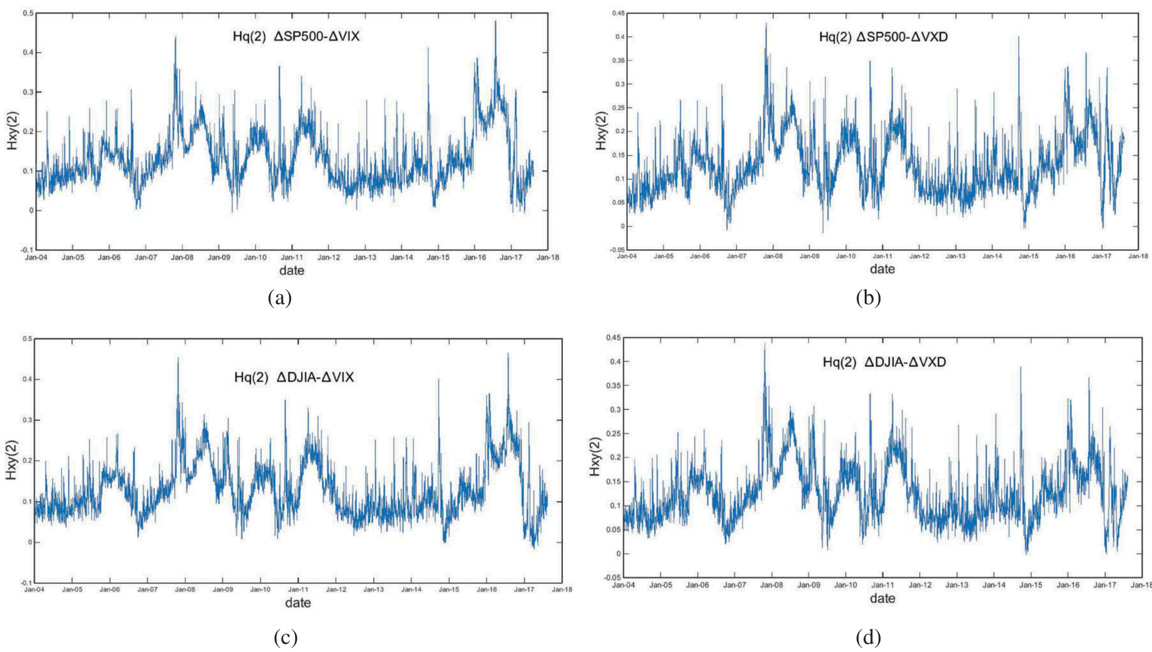


Figure 6. Dynamics of scaling exponents for q= 2 with window moving. (a) denotes $H_{xy}(2)$ of ΔSP500-ΔVIX, (b) denotes $H_{xy}(2)$ of ΔSP500-ΔVXD, and (c), (d) express $H_{xy}(2)$ of ΔDJIA-ΔVIX, ΔDJIA-ΔVXD respectively.

Last, the rolling-window analysis suggests that the daily stock market trading volume fluctuations and two kinds of daily investor fear gauges movements are all anti-persistent cross-correlated, stock market trading volume and investor fear gauges are susceptible to each other.

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