Review of GEMM Implementation

GEMM general structure

infrastructure

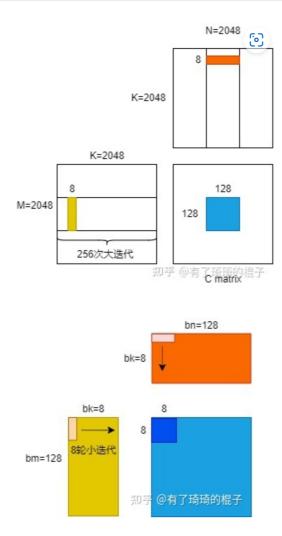
```
1. A -> mxk & B -> kxn & C -> mxn  
2. C = \alpha * A \times B^T + \beta * C
```

solution

```
1. m = M * bm & n = N * bn & k = K * bk
```

- 2. a grid = M * N blocks, each block calculate a bm * bn area
- 3. for a block, each time load bm * bk data from A and bk * bn data from B to shared memory
- 4. there are K rounds for a block to calculate the target block
- 5. bm = X * rm & bn = Y * rn
- 6. there are X * Y threads, and each thread calculate a block of rm * rn
- implement

```
// Device function to compute a thread block's accumulated matrix product
__device__ void block_matrix_product(int K_dim) {
   // Fragments used to store data fetched from SMEM
    value_t frag_a[ThreadItemsY];
   value_t frag_b[ThreadItemsX];
   // Accumulator storage
    accum_t accumulator[ThreadItemsX][ThreadItemsY];
    // GEMM Mainloop - iterates over the entire K dimension - not unrolled
    for (int kblock = 0; kblock < K_dim; kblock += BlockItemsK) {</pre>
        // Load A and B tiles from global memory and store to SMEM
        // (not shown for brevity - see the CUTLASS source for more detail)
        . . .
        __syncthreads();
        // warp tile structure - iterates over the Thread Block tile
        #pragma unroll
        for (int warp_k = 0; warp_k < BlockItemsK; warp_k += WarpItemsK) {</pre>
            // Fetch frag_a and frag_b from SMEM corresponding to k-index
            // (not shown for brevity - see CUTLASS source for more detail)
```



• now the above optimization can reach 80% cublas

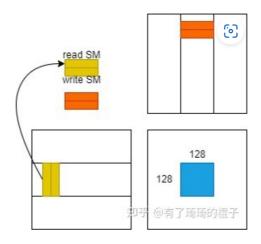
Advance Optimization

- transpose matrix A (smemA) and use LDS.128 (vector read instruction)
- try LDGSTS?
- prefetch & read/write into different buffer (double buffer)
 we use read SM & write SM to represent the two shared memory
 - 1. load the data to write SM and write reg

```
for k in 256 big_loop:
    prefetch next loop data to write_SM

// compute in read_SM

for iter in 8 small_loop:
    prefetch next loop data to write_REG
    compute in read_REG
```



• implement

```
#define TILE_K 16
   __shared__ float4 smemA[2][TILE_K * 128 / 4];
    float4 c[8][2] = \{\{make\_float4(0.f, 0.f, 0.f, 0.f)\}\};
   float4 ldg_a_reg[2];
   float4 ldg_b_reg[2];
   float4 a_reg[2][2];
   float4 b_reg[2][2];
   // transfer first tile from global mem to shared mem
   load_gmem_tile_to_reg(A, 0, ldg_a_reg);
   load_gmem_tile_to_reg(B, 0, ldg_b_reg);
   store_reg_to_smem_tile_transpose(ldg_a_reg, 0, smemA[0]);
   store_reg_to_smem_tile(ldg_b_reg, 0, smemB[0]);
   __syncthreads();
   // load first tile from shared mem to register
   load_smem_tile_to_reg(smemA[0], 0, a_reg[0]);
   load_smem_tile_to_reg(smemB[0], 0, b_reg[0]);
   int write_stage_idx = 1; //ping pong switch
   do {
       i += TILE_K;
       // load next tile from global mem
       load_gmem_tile_to_reg(A, i, ldg_a_reg);
       load_gmem_tile_to_reg(B, i, ldg_b_reg);
       int load_stage_idx = write_stage_idx ^ 1;
   #pragma unroll
```

```
for(int j = 0; j < TILE_K - 1; ++j) {
            // load next tile from shared mem to register
            load_smem_tile_to_reg(smemA[load_stage_idx], j + 1, a_reg[(j +
1) % 2]);
            load_smem_tile_to_reg(smemB[load_stage_idx], j + 1, b_reg[(j +
1) % 2]);
            // compute matrix multiply accumulate 8x8
            mma8x8(a_reg[j % 2], b_reg[j % 2], c);
        }
        if(i < K) {
            // store next tile to shared mem
            store_reg_to_smem_tile_transpose(ldg_a_reg, 0,
smemA[write_stage_idx]);
            store_reg_to_smem_tile(ldg_b_reg, 0, smemB[write_stage_idx]);
            // use double buffer, only need one sync
            __syncthreads();
            // switch
            write_stage_idx ^= 1;
        }
        // load first tile from shared mem to register of next iter
        load_smem_tile_to_reg(smemA[load_stage_idx ^ 1], 0, a_reg[0]);
        load_smem_tile_to_reg(smemB[load_stage_idx ^ 1], 0, b_reg[0]);
        // compute last tile mma 8x8
        mma8x8(a_reg[1], b_reg[1], c);
    } while (i < K);
    store_c(c, C);
```

• reach 97.5% cublas

surpass cutlass

- SASS (Shader Assembly)
 - o register bank conflict
 - o register reuse

Cutlass GEMM Implementation

code structure

interface

```
cutlass::gemm::device::Gemm<A_type,A_save,B_type,B_save,C_type,C_save>
```

• select gemm shape

cutlass/include/cutlass/gemm/device/gemm.h

```
template <
```

```
/// Element type for A matrix operand
    typename ElementA_,
    /// Layout type for A matrix operand
    typename LayoutA_,
    /// Element type for B matrix operand
    typename ElementB_,
    /// Layout type for B matrix operand
    typename LayoutB_,
    /// Element type for C and D matrix operands
    typename ElementC_,
    /// Layout type for C and D matrix operands
    typename LayoutC_,
    /// Element type for internal accumulation
    typename ElementAccumulator_ = ElementC_,
    /// Operator class tag
    typename OperatorClass_ = arch::OpClassSimt,
    /// Tag indicating architecture to tune for
    typename ArchTag_ = arch::Sm70,
    /// Threadblock-level tile size (concept: GemmShape)
    typename ThreadblockShape_ = typename DefaultGemmConfiguration<</pre>
        OperatorClass_, ArchTag_, ElementA_, ElementB_, ElementC_,
        ElementAccumulator_>::ThreadblockShape,
    /// Warp-level tile size (concept: GemmShape)
    typename WarpShape_ = typename DefaultGemmConfiguration<
        OperatorClass_, ArchTag_, ElementA_, ElementB_, ElementC_,
        ElementAccumulator_>::WarpShape,
    /// Instruction-level tile size (concept: GemmShape)
    typename InstructionShape_ = typename DefaultGemmConfiguration<
        OperatorClass_, ArchTag_, ElementA_, ElementB_, ElementC_,
        ElementAccumulator_>::InstructionShape,
    /// Epilogue output operator
    typename EpilogueOutputOp_ = typename DefaultGemmConfiguration<
        OperatorClass_, ArchTag_, ElementA_, ElementB_, ElementC_,
        ElementAccumulator_>::EpilogueOutputOp,
    /// Threadblock-level swizzling operator
    typename ThreadblockSwizzle_ =
        typename threadblock::GemmIdentityThreadblockSwizzle<>,
    /// Number of stages used in the pipelined mainloop
    int Stages =
        DefaultGemmConfiguration<OperatorClass_, ArchTag_, ElementA_,
ElementB_,
                                 ElementC_, ElementAccumulator_>::kStages,
    /// Access granularity of A matrix in units of elements
    int AlignmentA =
        DefaultGemmConfiguration<OperatorClass_, ArchTag_, ElementA_,</pre>
ElementB_,
                                 ElementC_,
ElementAccumulator_>::kAlignmentA,
    /// Access granularity of B matrix in units of elements
    int AlignmentB =
        DefaultGemmConfiguration<OperatorClass_, ArchTag_, ElementA_,</pre>
ElementB_,
                                 ElementC_,
ElementAccumulator_>::kAlignmentB,
    /// If true, kernel supports split-K with serial reduction
```

```
bool SplitKSerial = false,
/// Operation performed by GEMM

typename Operator_ = typename DefaultGemmConfiguration<
        OperatorClass_, ArchTag_, ElementA_, ElementB_, ElementC_,
        ElementAccumulator_>::Operator,
/// Gather operand A by using an index array
bool GatherA = false,
/// Gather operand B by using an index array
bool GatherB = false,
/// Scatter result D by using an index array
bool ScatterD = false>
class Gemm;
```

cutlass/include/cutlass/gemm/device/default_gemm_configuration.h

```
template <
  typename OperatorClass,
  typename ArchTag,
  typename ElementA,
  typename ElementB,
  typename ElementC,
  typename ElementAccumulator
>
struct DefaultGemmConfiguration;
```

Kstages: 2 KAlignmentA: 1 KAlignmentB: 1 KAlignmentC: 1

kernel call

cutlass/include/cutlass/gemm/device/gemm.h

```
if (result != cudaSuccess) {
    return Status::kErrorInternal;
}

// kernel call
cutlass::Kernel<GemmKernel><<<grid, block, smem_size, stream>>>
(params_);
// kernel call

result = cudaGetLastError();

return result == cudaSuccess ? Status::kSuccess :
Status::kErrorInternal;
}
```

cutlass/include/cutlass/device_kernel.h

```
template <typename Operator>
    __global__
void Kernel(typename Operator::Params params) {
    // Dynamic shared memory base pointer
    extern __shared__ int SharedStorageBase[];

    // Declare pointer to dynamic shared memory.
    typename Operator::SharedStorage *shared_storage =
        reinterpret_cast<typename Operator::SharedStorage *>
    (SharedStorageBase);

    Operator op;
    op(params, *shared_storage);
}
```

• DefaultGemm structure

cutlass/include/cutlass/gemm/kernel/default_gemm.h

```
template <
 /// Element type for A matrix operand
 typename ElementA,
 /// Layout type for A matrix operand
 typename LayoutA,
 /// Access granularity of A matrix in units of elements
 int kAlignmentA,
 /// Element type for B matrix operand
 typename ElementB,
 /// Layout type for B matrix operand
 typename LayoutB,
 /// Access granularity of B matrix in units of elements
 int kAlignmentB,
  /// Element type for C and D matrix operands
  typename ElementC,
  /// Element type for internal accumulation
```

```
typename ElementAccumulator,
  /// Threadblock-level tile size (concept: GemmShape)
  typename ThreadblockShape,
  /// Warp-level tile size (concept: GemmShape)
 typename WarpShape,
  /// Warp-level tile size (concept: GemmShape)
 typename InstructionShape,
  /// Epilogue output operator
 typename EpilogueOutputOp,
  /// Threadblock-level swizzling operator
 typename ThreadblockSwizzle,
  /// If true, kernel is configured to support serial reduction in the
epilogue
 bool SplitKSerial,
 /// Operation performed by GEMM
 typename Operator,
 /// Use zfill or predicate for out-of-bound cp.async
 SharedMemoryClearOption SharedMemoryClear,
 /// Gather operand A by using an index array
 bool GatherA,
  /// Gather operand B by using an index array
 bool GatherB,
 /// Scatter result D by using an index array
 bool ScatterD
struct DefaultGemm<
  ElementA, LayoutA, kAlignmentA,
 ElementB, LayoutB, kAlignmentB,
  ElementC, layout::RowMajor,
 ElementAccumulator,
 arch::OpclassTensorOp,
 arch::Sm75,
 ThreadblockShape,
 WarpShape,
 InstructionShape,
  EpilogueOutputOp,
 ThreadblockSwizzle,
 2,
 SplitKSerial,
 Operator,
 SharedMemoryClear,
 GatherA,
 GatherB,
 ScatterD
> {
 /// Define the threadblock-scoped matrix multiply-accumulate
 using Mma = typename cutlass::gemm::threadblock::DefaultMma<</pre>
    ElementA,
    LayoutA,
    kAlignmentA,
    ElementB,
    LayoutB,
    kAlignmentB,
    ElementAccumulator,
```

```
layout::RowMajor,
    arch::OpClassTensorOp,
    arch::Sm75,
   ThreadblockShape,
   WarpShape,
    InstructionShape,
   2,
   Operator,
    false,
    SharedMemoryClear,
   GatherA,
   GatherB
 >::ThreadblockMma;
 static const int kPartitionsK = ThreadblockShape::kK / WarpShape::kK;
 /// Define the epilogue
  using Epilogue = typename
cutlass::epilogue::threadblock::DefaultEpilogueTensorOp<
   ThreadblockShape,
    typename Mma::Operator,
    kPartitionsK,
    EpilogueOutputOp,
    EpilogueOutputOp::kCount,
    ScatterD
 >::Epilogue;
 /// Define the kernel-level GEMM operator.
  using GemmKernel = kernel::Gemm<Mma, Epilogue, ThreadblockSwizzle,</pre>
SplitKSerial>;
};
```

cutlass/include/cutlass/gemm/kernel/gemm.h

```
CUTLASS_DEVICE
void operator()(Params const &params, SharedStorage &shared_storage) {
    // Compute threadblock location
   ThreadblockSwizzle threadblock_swizzle;
    cutlass::gemm::GemmCoord threadblock_tile_offset =
        threadblock_swizzle.get_tile_offset(params.swizzle_log_tile);
    // Early exit if CTA is out of range
   if (params.grid_tiled_shape.m() <= threadblock_tile_offset.m() ||</pre>
      params.grid_tiled_shape.n() <= threadblock_tile_offset.n()) {</pre>
     return;
    }
   // Compute initial location in logical coordinates
    cutlass::MatrixCoord tb_offset_A{
     threadblock_tile_offset.m() * Mma::Shape::kM,
      threadblock_tile_offset.k() * params.gemm_k_size,
    };
```

```
cutlass::MatrixCoord tb_offset_B{
      threadblock_tile_offset.k() * params.gemm_k_size,
      threadblock_tile_offset.n() * Mma::Shape::kN
   };
   // Problem size is a function of threadblock index in the K dimension
    int problem_size_k = min(
      params.problem_size.k(),
      (threadblock_tile_offset.k() + 1) * params.gemm_k_size);
    // Compute threadblock-scoped matrix multiply-add
    int gemm_k_iterations = (problem_size_k - tb_offset_A.column() +
Mma::Shape::kK - 1) / Mma::Shape::kK;
    // Compute position within threadblock
    int thread_idx = threadIdx.x;
    // Construct iterators to A and B operands
    typename Mma::IteratorA iterator_A(
      params.params_A,
      params.ref_A.data(),
      {params.problem_size.m(), problem_size_k},
      thread_idx,
      tb_offset_A,
      params.gather_A_indices);
    typename Mma::IteratorB iterator_B(
      params.params_B,
      params.ref_B.data(),
      {problem_size_k, params.problem_size.n()},
      thread_idx,
      tb_offset_B,
      params.gather_B_indices);
   // Broadcast the warp_id computed by lane 0 to ensure dependent code
    // is compiled as warp-uniform.
   int warp_idx = __shfl_sync(0xffffffff, threadIdx.x / 32, 0);
    int lane_idx = threadIdx.x % 32;
   //
    // Main loop
    //
    // Construct thread-scoped matrix multiply
   Mma mma(shared_storage.main_loop, thread_idx, warp_idx, lane_idx);
    typename Mma::FragmentC accumulators;
    accumulators.clear();
   if (!kSplitKSerial || gemm_k_iterations > 0) {
      // Compute threadblock-scoped matrix multiply-add
     mma(gemm_k_iterations, accumulators, iterator_A, iterator_B,
accumulators);
```

```
//
    // Epilogue
    //
   OutputOp output_op(params.output_op);
   //
    // Masked tile iterators constructed from members
    threadblock_tile_offset =
        threadblock_swizzle.get_tile_offset(params.swizzle_log_tile);
    //assume identity swizzle
   MatrixCoord threadblock_offset(
      threadblock_tile_offset.m() * Mma::Shape::kM,
     threadblock_tile_offset.n() * Mma::Shape::kN
    );
    int block_idx = threadblock_tile_offset.m() +
threadblock_tile_offset.n() * params.grid_tiled_shape.m();
    // Construct the semaphore.
    Semaphore semaphore(params.semaphore + block_idx, thread_idx);
   // If performing a reduction via split-K, fetch the initial
synchronization
    if (kSplitKSerial && params.grid_tiled_shape.k() > 1) {
     // Fetch the synchronization lock initially but do not block.
      semaphore.fetch();
      // Indicate which position in a serial reduction the output operator
is currently updating
      output_op.set_k_partition(threadblock_tile_offset.k(),
params.grid_tiled_shape.k());
    }
    // Tile iterator loading from source tensor.
    typename Epilogue::OutputTileIterator iterator_C(
      params.params_C,
      params.ref_C.data(),
      params.problem_size.mn(),
      thread_idx,
      threadblock_offset,
      params.scatter_D_indices
   );
    // Tile iterator writing to destination tensor.
    typename Epilogue::OutputTileIterator iterator_D(
      params.params_D,
      params.ref_D.data(),
      params.problem_size.mn(),
```

```
thread_idx,
      threadblock_offset,
      params.scatter_D_indices
    );
    Epilogue epilogue(
      shared_storage.epilogue,
      thread_idx,
      warp_idx,
      lane_idx);
    // Wait on the semaphore - this latency may have been covered by
iterator construction
   if (kSplitKSerial && params.grid_tiled_shape.k() > 1) {
      // For subsequent threadblocks, the source matrix is held in the 'D'
tensor.
      if (threadblock_tile_offset.k()) {
        iterator_C = iterator_D;
      }
      semaphore.wait(threadblock_tile_offset.k());
    }
    // Execute the epilogue operator to update the destination tensor.
    epilogue(output_op, iterator_D, accumulators, iterator_C);
    //
    // Release the semaphore
    if (kSplitKSerial && params.grid_tiled_shape.k() > 1) {
      int lock = 0;
      if (params.grid_tiled_shape.k() == threadblock_tile_offset.k() + 1) {
        // The final threadblock resets the semaphore for subsequent grids.
       lock = 0;
      }
      else {
        // Otherwise, the semaphore is incremented
       lock = threadblock_tile_offset.k() + 1;
      }
      semaphore.release(lock);
    }
}
```

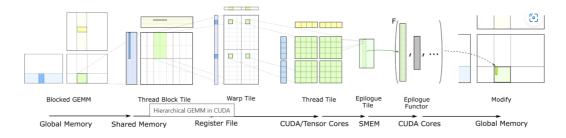
cutlass/include/cutlass/gemm/threadblock/mma_pipelined.h

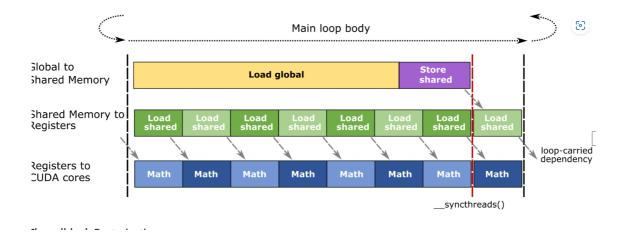
```
///< destination
    FragmentC &accum,
accumulator tile
   IteratorA iterator_A,
                                                      ///< iterator over A
operand in global memory
   IteratorB iterator_B,
                                                      ///< iterator over B
operand in global memory
    FragmentC const &src_accum,
                                                      ///< source
accumulator tile
    TransformA transform_A = TransformA(),
                                                      ///< transformation
applied to A fragment
   TransformB transform_B = TransformB()) {
                                                    ///< transformation
applied to B fragment
   //
   // Prologue
   //
   // Perform accumulation in the 'd' output operand
    accum = src_accum;
    FragmentA tb_frag_A;
    FragmentB tb_frag_B;
   tb_frag_A.clear();
   tb_frag_B.clear();
   // The last kblock is loaded in the prolog
    iterator_A.load(tb_frag_A);
    iterator_B.load(tb_frag_B);
   ++iterator_A;
   ++iterator_B;
    this->smem_iterator_A_.store(transform_A(tb_frag_A));
    this->smem_iterator_B_.store(transform_B(tb_frag_B));
   ++this->smem_iterator_A_;
   ++this->smem_iterator_B_;
    __syncthreads();
   // Pair of fragments used to overlap shared memory loads and math
instructions
   WarpFragmentA warp_frag_A[2];
   WarpFragmentB warp_frag_B[2];
    this->warp_tile_iterator_A_.set_kgroup_index(0);
    this->warp_tile_iterator_B_.set_kgroup_index(0);
    this->warp_tile_iterator_A_.load(warp_frag_A[0]);
    this->warp_tile_iterator_B_.load(warp_frag_B[0]);
   ++this->warp_tile_iterator_A_;
    ++this->warp_tile_iterator_B_;
```

```
Operator warp_mma;
    int smem_write_stage_idx = 1;
    // Avoid reading out of bounds
    iterator_A.clear_mask(gemm_k_iterations <= 1);</pre>
    iterator_B.clear_mask(gemm_k_iterations <= 1);</pre>
   // Issue loads during the first warp-level matrix multiply-add *AFTER*
issuing
   // shared memory loads (which have the tighest latency requirement).
   //
   // Mainloop
   //
   // Note: The main loop does not support Base::kWarpGemmIterations == 2.
   CUTLASS_GEMM_LOOP
    for (; gemm_k_iterations > 0; --gemm_k_iterations) {
     // Loop over GEMM K dimension
     //
      CUTLASS_PRAGMA_UNROLL
      for (int warp_mma_k = 0; warp_mma_k < Base::kwarpGemmIterations;</pre>
++warp_mma_k) {
       // Load warp-level tiles from shared memory, wrapping to k offset if
this is the last group
       // as the case may be.
       if (warp_mma_k == Base::kWarpGemmIterations - 1) {
          // Write fragments to shared memory
          this->smem_iterator_A_.store(transform_A(tb_frag_A));
          this->smem_iterator_B_.store(transform_B(tb_frag_B));
          __syncthreads();
          ++this->smem_iterator_A_;
          ++this->smem_iterator_B_;
          // Add negative offsets to return iterators to the 'start' of the
circular buffer in shared memory
          if (smem_write_stage_idx == 1) {
            this->smem_iterator_A_.add_tile_offset({0, -Base::kStages});
            this->smem_iterator_B_.add_tile_offset({-Base::kStages, 0});
          }
          else {
            this->warp_tile_iterator_A_.add_tile_offset(
                {0, -Base::kStages * Policy::kPartitionsK *
Base::kWarpGemmIterations});
            this->warp_tile_iterator_B_.add_tile_offset(
```

```
{-Base::kStages * Policy::kPartitionsK *
Base::kWarpGemmIterations,
                 0});
          }
          smem_write_stage_idx ^= 1;
        }
        this->warp_tile_iterator_A_.set_kgroup_index((warp_mma_k + 1) %
Base::kWarpGemmIterations);
        this->warp_tile_iterator_B_.set_kgroup_index((warp_mma_k + 1) %
Base::kwarpGemmIterations);
        this->warp_tile_iterator_A_.load(warp_frag_A[(warp_mma_k + 1) % 2]);
        this->warp_tile_iterator_B_.load(warp_frag_B[(warp_mma_k + 1) % 2]);
        ++this->warp_tile_iterator_A_;
        ++this->warp_tile_iterator_B_;
        if (warp_mma_k == 0) {
          iterator_A.load(tb_frag_A);
          iterator_B.load(tb_frag_B);
          ++iterator_A;
          ++iterator_B;
          // Avoid reading out of bounds if this was the last loop iteration
          iterator_A.clear_mask(gemm_k_iterations <= 2);</pre>
          iterator_B.clear_mask(gemm_k_iterations <= 2);</pre>
        }
        warp_mma(accum, warp_frag_A[warp_mma_k % 2],
                 warp_frag_B[warp_mma_k % 2], accum);
      }
    }
 }
```

• analysis on code





Reference

• <u>CUTLASS: Fast Linear Algebra in CUDA C++ | NVIDIA Technical Blog</u>

<u>CUTLASS: Fast Linear Algebra in CUDA C++ - 知平 (zhihu.com)</u> (translated version)

- 深入浅出GPU优化系列: GEMM优化 (一) 知乎 (zhihu.com)
- 深入浅出GPU优化系列: GEMM优化 (二) 知乎 (zhihu.com)
- 深入浅出GPU优化系列: GEMM优化(三) 知乎 (zhihu.com)
- SGEMM · NervanaSystems/maxas Wiki (github.com)
- CUDA 矩阵乘法终极优化指南 知乎 (zhihu.com)
- <u>cuda sgemm/gemm.cu at master · niuhope/cuda sgemm (github.com)</u>
- Why use CUTLASS instead of CUBLAS for GEMM? What are the advantages of CUTLASS?
 Issue #109 · NVIDIA/cutlass (github.com)
- <u>Liu-xiandong/How to optimize in GPU: This is a series of GPU optimization topics. Here we</u> <u>will introduce how to optimize the CUDA kernel in detail. I will introduce several basic kernel optimizations, including: elementwise, reduce, sgemv, sgemm, etc. The performance of these kernels is basically at or near the theoretical limit. (github.com)</u>
- (27 封私信 / 80 条消息) 自己写的CUDA矩阵乘法能优化到多快? 知乎 (zhihu.com)
- CUDA矩阵乘法的优化·wu-kan
- cutlass/efficient_gemm.md at master · NVIDIA/cutlass (github.com)