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MIE334: Numerical Methods   **Assignment 3**

Due: **March 25th**, **11:59 pm**, 2021

(1)

$a1 =$	-0.7805	$a0=$	31.0589	;	$s_{y/x} =$	4.4763	;	$r^2 =$	0.8127

x_i	y_i	x^2	$x \times y$	y^2	$(y_i - \bar{y})^2$	$(y_i - a_0 - a_1 x_i)^2$
6	29	36	174	841	211.570248	6.88590081
7	21	49	147	441	42.8429752	21.1177012
11	29	121	319	841	211.570248	42.5965076
15	14	225	210	196	0.20661157	28.637482
17	21	289	357	441	42.8429752	10.3015322
21	15	441	315	225	0.29752066	0.10995856
23	7	529	161	49	55.5702479	37.3003348
29	7	841	203	49	55.5702479	2.02891536
29	13	841	377	169	2.11570248	20.9361154
37	0	1369	0	0	208.933884	4.75414416
39	3	1521	117	9	131.206612	5.66725636
Σ 234	159	6262	2380	3261	962.727273	180.335848

Calculations and plot:

$$\bar{y} = \frac{159}{11} = 14.4545$$

$$\bar{x} = \frac{234}{11} = 21.2727$$

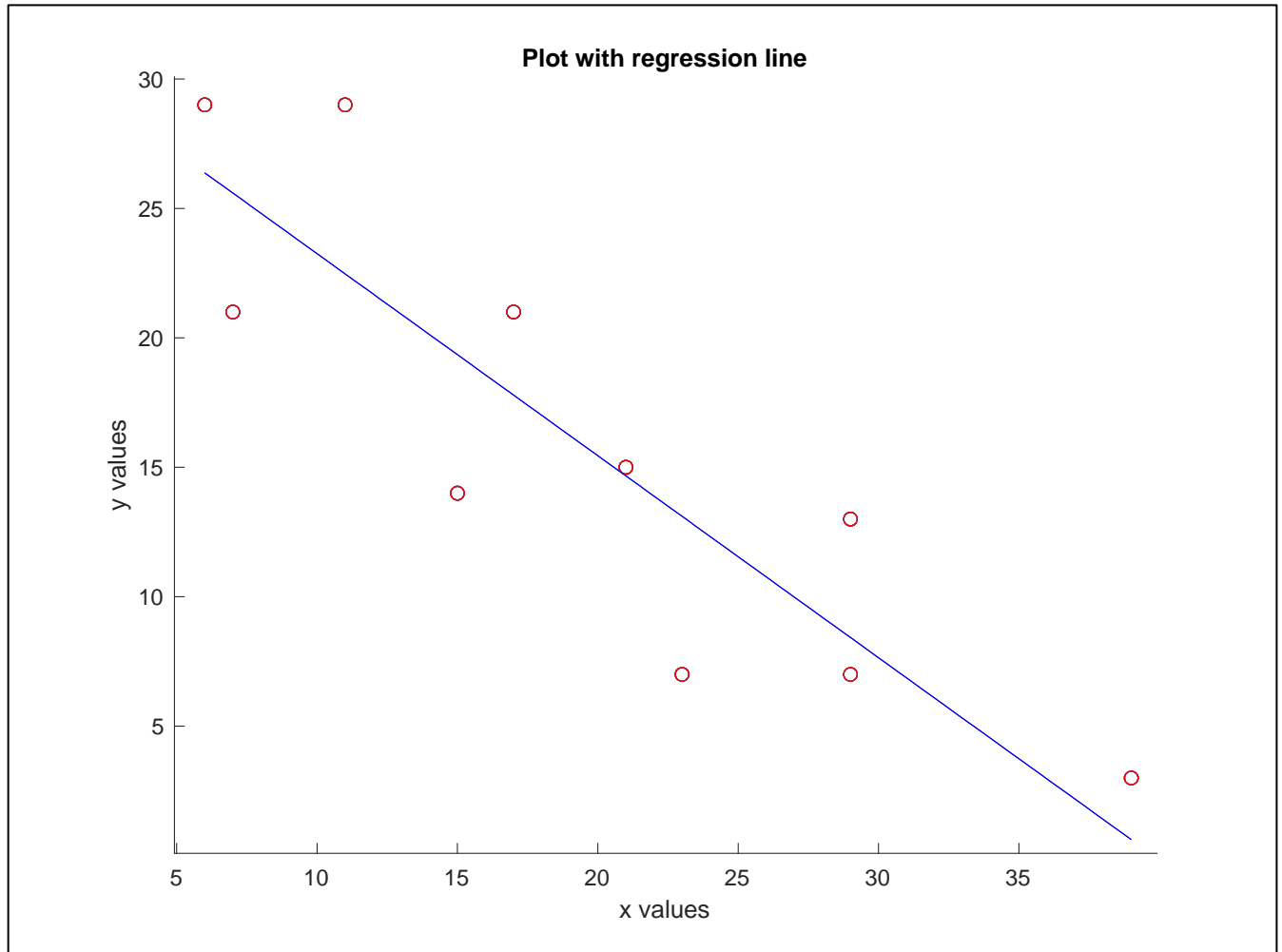$$a_1 = \frac{(11)(2380) - (234)(159)}{(11)(6262) - (234)^2} = -0.7805$$

$$a_0 = 14.4545 - (-0.7805)(21.2727) = 31.0589$$

$$S_y = \sqrt{\frac{962.727}{11-1}} = 9.8119$$

$$S_{y/x} = \sqrt{\frac{180.336}{11-2}} = 4.4763$$

$$R^2 = \frac{962.727 - 180.336}{962.727} = 0.8127$$

Plot with regression line

Adding the point (x,y)=(10,10) would be erroneous, as it is far below the regression line and thus would not be a good fit within our current measured data.

(2)

i	x_i	$f(x_i)$	1st	2nd	3rd
0	300	846.3	0.9840	-0.00112	1.86667E-06
1	350	895.5	0.8720	-0.00084	
2	400	939.1	0.7880		
3	450	978.5			

Answer: 922.2424 J/kgK

Calculations:

$$b_0 = 846.3$$

$$b_1 = \frac{895.5 - 846.3}{350 - 300} = 0.9840$$

$$b_2 = \frac{0.8720 - 0.9840}{400 - 300} = -0.00112$$

$$b_3 = \frac{-0.00084 - (-0.00112)}{450 - 300} = 1.8667E - 6$$

$$f_3(x) = 846.3 + (0.984)(380 - 300) + (-0.00112)(380 - 300)(380 - 350) + (1.8667E - 6)(380 - 300)(380 - 350)(380 - 400)$$
$$f_3(x) = 922.2424$$

(3)

$a2 =$	-0.4149	$a1 =$	4.6331	$a0 =$	-8.4519	;	$s_{y/x} =$	0.2484	;	$r^2 =$	0.9757

x_i	y_i	$(y_i - \bar{y})^2$	$(y_i - a_0 - a_1 x_i - a_2 x_i^2)^2$	x^2	x^3	x^4	$x \times y$	$x^2 \times y$
3	1.6	2.0736	0.0128	9	27	81	4.8	14.4
4	3.6	0.3136	0.0249	16	64	256	14.4	57.6
5	4.4	1.8496	0.0035	25	125	625	22	110
7	3.4	0.1296	0.0624	49	343	2401	23.8	166.6
8	2.2	0.7056	0.0198	64	512	4096	17.6	140.8
27	15.2	5.072	0.1234	163	1071	7459	82.6	489.4

Calculations:

$$\bar{y} = \frac{15.2}{5} = 3.04$$

$$\bar{x} = \frac{27}{5} = 5.4$$

$$\begin{bmatrix} 5 & 27 & 163 \\ 27 & 163 & 1071 \\ 163 & 1071 & 7459 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 15.2 \\ 82.6 \\ 489.4 \end{bmatrix}$$

$$\begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} -8.4519 \\ 4.6331 \\ -0.4149 \end{bmatrix}$$

$$S_y = \sqrt{\frac{5.072}{5-1}} = 1.1261$$

$$S_{y/x} = \sqrt{\frac{0.1234}{5-3}} = 0.2484$$

$$R^2 = \frac{5.072 - 0.1234}{5.072} = 0.9757$$