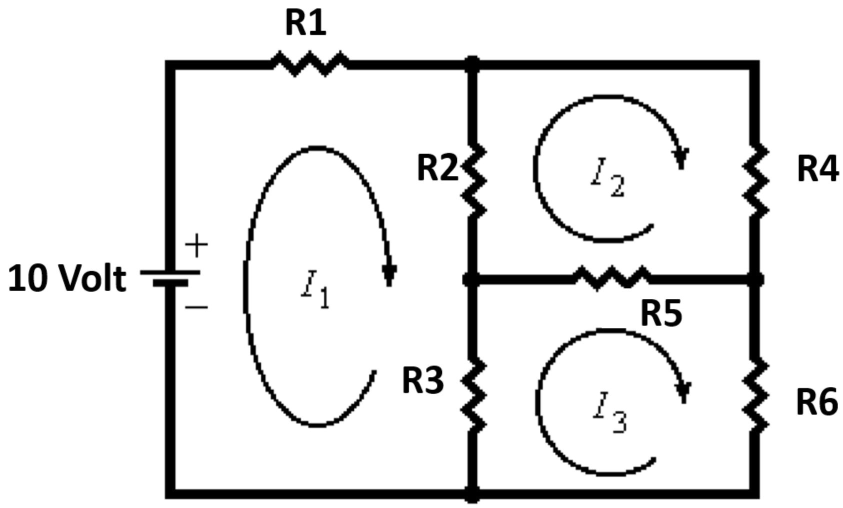
Due: **February 25th**, **11:59 pm**, 2021

1. Consider the resistor circuit in the figure below in which the current at various locations should be determined.



R1	R2	R3	R4	R5	R6
1e5	5	4	100	25	20

(a) Write a set of three equations with three unknown loop currents *(I_1, I_2, I_3)* based on Ohm's law, in the form Ax=b in which A is the coefficient matrix, x is the current, and b is the constant vector.

(b) Determine the values of the three currents by solving the linear system equations (Ax=b) using LU decomposition. Keep 5 significant figures <u>after the decimal point</u> for all of your calculations. (Provide the snippets of your work to calculate L, U and then details of the steps you have taken to calculate the unknowns)

(c) Repeat part (b), but now keeping 6 significant figures <u>after the decimal point,</u> recalculate the currents. (You do not need to provide details of your calculation; Final answer is enough)

(d) Calculate the differences (%) between the current values when the number of significant figures is increased from 5 to 6. (abs (I with 5 significant figures-I with 6 significant figures)/ I with 6 significant figures)))

(e) Could you have anticipated the sensitivity of the system to the rounding off? Explain.

2. Consider the following system of equations:

$$\begin{cases} -x_1 + x_2 + 7x_3 = -6 \\ 4x_1 - x_2 - x_3 = 3 \\ -2x_1 + 6x_2 + x_3 = 9 \end{cases}$$

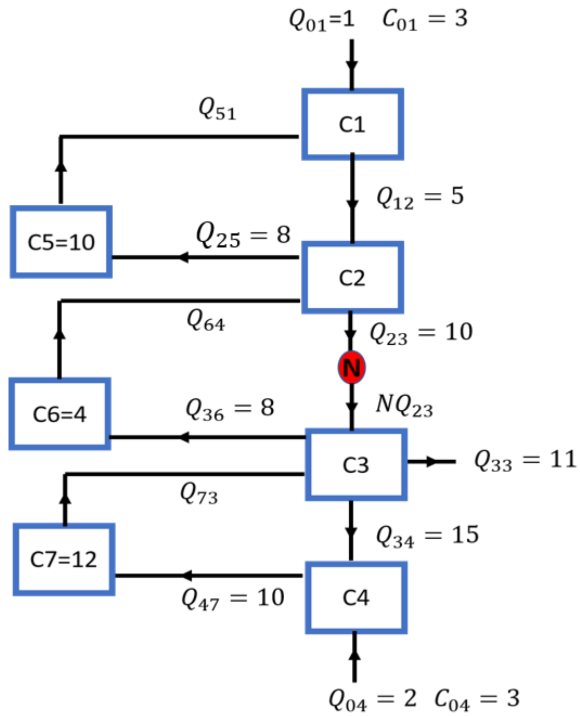
(a) In this form, if you use the Gauss-Seidel method to solve this system of equations, is the solution guaranteed to converge? If not, make the necessary modifications to guarantee convergence. Briefly explain your rationale.

(b) Use the Gauss-Seidel method to solve for the values of x_1, x_2 and x_3 with a stopping criterion of $\varepsilon_s = 0.1\%$. Use $x_1 = x_2 = x_3 = 0$ as your initial guess. Also, use table provided in the answer sheet to store and organize your calculations and carry 6 significant figures after the decimal point (2 figures are enough for error percentages). Write the details of your calculations for iteration 1.

(c) Repeat part (b), but this time apply an over-relaxation factor of λ=1.2. Compare number of iterations in part (b) and (c) and justify your answer.

(d) Write the pseudo-code of your solution in part (b).

3. In the following system of coupled reactors, we can assume that we have conservation of mass. This means that whatever mass (calculated by multiplication of the flow rate Q and concentration c) is coming to the reactor through the inflow pipes will exit at the outflow pipes and the mass is balanced. For instance, at reactor 1, we have the inflow of $Q_{01} \times c_{01} + Q_{51} \times c_5$ which should be equal to outflow of $Q_{12} \times c_1$. ($Q_{01} \times c_{01} + Q_{51} \times c_5 = Q_{12} \times c_1$). The outflow of the reactor 2 to reactor 3 can be controlled by the knob N (changing the N would change the inflow to reactor 3).



a) Write three mass balance equations for reactor 5, 6 and 7. Then, drive Q_{51}, Q_{64} and Q_{73} as a function of c2, c3 and c4 respectively.

b) If N=1, write four mass balance equations for reactor 1, 2, 3 and 4. For Q_{51}, Q_{64} and Q_{73} in your equations use the value you obtained in part (a). Rewrite the four equations in a matrix form. AX=b in which X=$\begin{Bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{Bmatrix}$ and A is a tridiagonal matrix.

(c) Solve the equation obtained in part (b) for the concentration values of c_1 to c_4 using Thomas Algorithm method. (Provide the snippets of your work: e.g. x_i=….etc.)

(d) If N can have a range of 1 to 70. What value of N should be used, so c2 becomes 0.02? (Choose N that give c2 with the nearest value to 0.002)

Hint (Note that N only can be an integer N=1,2,3…,70. You do not need to provide the details of your calculation for part d. Only final answer for N and c2 is needed).