

# THIS IS FULL TITLE

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ABSTRACT. This is an example L<sup>A</sup>T<sub>E</sub>X article. This can be used as a template for new articles. Abstracts must be able to stand alone and so cannot contain citations to the paper's references, equations, etc. An abstract must consist of a single paragraph and be concise. Because of online formatting, abstracts must appear as plain as possible. Any equations should be inline.

## 1. INTRODUCTION

The introduction introduces the context and summarizes the manuscript. It is importantly to clearly state the contributions of this piece of work.

In this paper we present a new method for solving the model equation

$$\begin{cases} \partial_t u - \varepsilon^2 \Delta u + u^3 - u = 0, & \text{in } \Omega \times \mathcal{T}, \\ u(x, y, t) = g(t), & \text{on } \partial\Omega, \\ u(x, y, 0) = \varphi(x, y), & \text{on } \Omega. \end{cases} \quad (1.1)$$

where  $\varepsilon$  is a small parameter.

This is an example of quoting an equation (1.1).

The merits of our method are as follows:

- item one
- item two
- (1) item one
- (2) item two

The outline is not required, but we show an example here.

The paper is organized as follows. Our main results are in 3, our new algorithm is in 4, experimental results are in 5, discussion is in 6 and the conclusions follow in 7.

## 2. PRELIMINARIES

### 2.1. This is subsection.

#### 2.1.1. This is subsubsection.

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*E-mail addresses:* list of emails.

*Date:* April 25, 2023.

2000 *Mathematics Subject Classification.* 65L60, 65L05, 65L70.

*Key words and phrases.* Keyword 1, keyword 2, error analysis.

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The first author was supported in part by NSF Grant #000000.

Support information for the second author.

Support information for the third author.

## 3. MAIN RESULTS

We interleave text filler with some example theorems and theorem-like items.

**Definition 3.1.** This is a definition environment.

**Lemma 3.1.** *This is a lemma environment.*

**Theorem 3.1.** *This is a theorem environment.*

*Proof.* This is a proof environment. □

**Lemma 3.2.** *This is a lemma environment*

- (i) *item A*
- (ii) *item B*

$$\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n = e. \quad (3.1)$$

This is a citation example [1]. This statement requires citation [3–5].

Here we state our main result as 3.2.

**Theorem 3.2** (*LDL<sup>T</sup> Factorization* [2]). *If  $A \in \mathbb{R}^{n \times n}$  is symmetric and the principal submatrix  $A(1 : k, 1 : k)$  is nonsingular for  $k = 1 : n - 1$ , then there exists a unit lower triangular matrix  $L$  and a diagonal matrix*

$$D = \text{diag}(d_1, \dots, d_n),$$

*such that  $A = LDL^T$ . The factorization is unique.*

**Theorem 3.3** (*Mean Value Theorem*). *Suppose  $f$  is a function that is continuous on the closed interval  $[a, b]$ . and differentiable on the open interval  $(a, b)$ . Then there exists a number  $c$  such that  $a < c < b$  and*

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

*In other words,*

$$f(b) - f(a) = f'(c)(b - a).$$

*Remark 3.1.* Observe that 3.2, 3.3 correctly mix references to multiple labels.

**Corollary 3.1.** *Let  $f(x)$  be continuous and differentiable everywhere. If  $f(x)$  has at least two roots, then  $f'(x)$  must have at least one root.*

*Proof.* Let  $a$  and  $b$  be two distinct roots of  $f$ . By 3.3, there exists a number  $c$  such that

$$f'(c) = \frac{f(b) - f(a)}{b - a} = \frac{0 - 0}{b - a} = 0.$$

□

Note that it may require two L<sup>A</sup>T<sub>E</sub>X compilations for the proof marks to show.

Display matrices can be rendered using environments from `amsmath`:

$$S = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \quad \text{and} \quad C = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \quad (3.2)$$

Equation 3.2 shows some example matrices.

We calculate the Fréchet derivative of  $F$  as follows:

$$F'(U, V)(H, K) = \langle R(U, V), H\Sigma V^T + U\Sigma K^T - P(H\Sigma V^T + U\Sigma K^T) \rangle \quad (3.3a)$$

$$\begin{aligned}
&= \langle R(U, V), H\Sigma V^T + U\Sigma K^T \rangle \\
&= \langle R(U, V)V\Sigma^T, H \rangle + \langle \Sigma^T U^T R(U, V), K^T \rangle.
\end{aligned} \tag{3.3b}$$

3.3a is the first line, and 3.3b is the last line.

#### 4. ALGORITHM

Our analysis leads to the algorithm in 1.

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**Algorithm 1** Build tree

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Define  $P := T := \{\{1\}, \dots, \{d\}\}$ 
while  $\#P > 1$  do
  Choose  $C' \in \mathcal{C}_p(P)$  with  $C' := \operatorname{argmin}_{C \in \mathcal{C}_p(P)} \varrho(C)$ 
  Find an optimal partition tree  $T_{C'}$ 
  Update  $P := (P \setminus C') \cup \{\bigcup_{t \in C'} t\}$ 
  Update  $T := T \cup \{\bigcup_{t \in \tau} t : \tau \in T_{C'} \setminus \mathcal{L}(T_{C'})\}$ 
end while
return  $T$ 

```

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Adjust the width of the algorithm environment

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**Algorithm 2** Euclid's algorithm

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```

1: procedure EUCLID( $a, b$ ) ▷ The g.c.d. of a and b
2:    $r \leftarrow a \bmod b$ 
3:   while  $r \neq 0$  do ▷ We have the answer if r is 0
4:      $a \leftarrow b$ 
5:      $b \leftarrow r$ 
6:      $r \leftarrow a \bmod b$ 
7:   end while
8:   return  $b$  ▷ The gcd is b
9: end procedure

```

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#### 5. EXPERIMENTAL RESULTS

Some experimental results here.

**Example 1.** This is example environment.

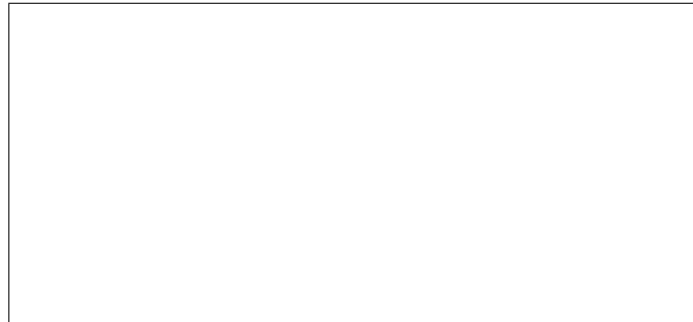


FIGURE 5.1. This is an example of a figure caption with text.

Figure 5.2 shows some example results.



FIGURE 5.3. Left: Caption 1, Right: Caption 2.

Use `minipage` package to set images side-by-side, each with its own title, as shown in Figure 5.4 and Figure 5.5.

## 6. DISCUSSION OF $Z = X \cup Y$

Some discussions here. Some discussions here. Some discussions here. Some discussions here.  
Some discussions here. Some discussions here. Some discussions here. Some discussions here.

TABLE 5.1. Table description

| N  | A        | B        | C        | D        | E        |
|----|----------|----------|----------|----------|----------|
| 2  | 9.20E-05 | 9.90E-05 | 1.00E-06 | 8.00E-06 | 1.50E-05 |
| 4  | 9.80E-05 | 8.00E-05 | 7.00E-06 | 1.40E-05 | 1.60E-05 |
| 6  | 4.00E-06 | 8.10E-05 | 8.80E-05 | 2.00E-05 | 2.20E-05 |
| 8  | 8.50E-05 | 8.70E-05 | 1.90E-05 | 2.10E-05 | 3.00E-06 |
| 10 | 8.60E-05 | 9.30E-05 | 2.50E-05 | 2.00E-06 | 9.00E-06 |
| 12 | 1.70E-05 | 2.40E-05 | 7.60E-05 | 8.30E-05 | 9.00E-05 |
| 14 | 2.30E-05 | 5.00E-06 | 8.20E-05 | 8.90E-05 | 9.10E-05 |

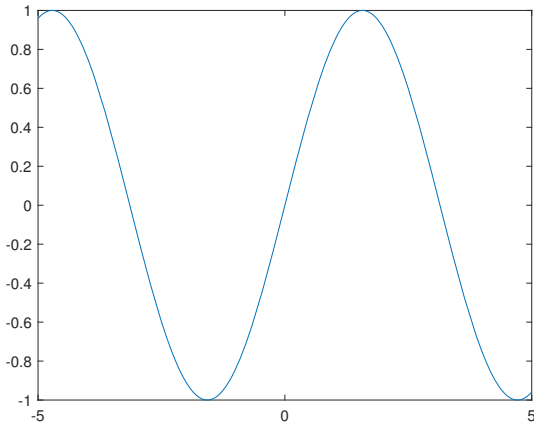


FIGURE 5.4. Caption A

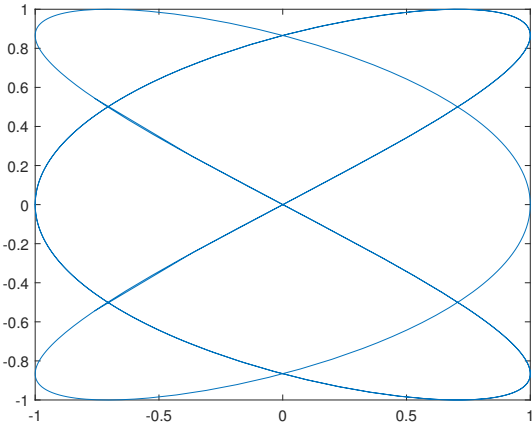


FIGURE 5.5. Caption B

TABLE 5.2. Numerical error

| degree | step-size $h$ | $L^2$ -errors | order | $H^1$ -errors | order | $L^\infty$ -errors | order |
|--------|---------------|---------------|-------|---------------|-------|--------------------|-------|
| 1      | 1/128         | 9.18E-06      | 2.02  | 7.70E-03      | 1.01  | 6.46E-07           | 2.02  |
|        | 1/256         | 2.29E-06      | 2.01  | 1.92E-03      | 1.00  | 1.61E-07           | 2.01  |
|        | 1/512         | 5.70E-07      | 2.00  | 9.56E-04      | 1.00  | 4.01E-08           | 2.00  |
| 2      | 1/128         | 1.39E-08      | 3.01  | 1.15E-05      | 2.01  | 3.48E-12           | 4.02  |
|        | 1/256         | 1.73E-09      | 3.01  | 2.88E-06      | 2.01  | 3.27E-13           | 3.94  |
|        | 1/512         | 2.17E-10      | 3.00  | 7.24E-06      | 2.00  | 6.66E-13           | 1.55  |
| 3      | 1/32          | 2.28E-09      | 4.05  | 6.92E-07      | 3.04  | 1.45E-15           | 8.21  |
|        | 1/64          | 1.42E-10      | 4.03  | 8.65E-08      | 3.02  | 2.06E-14           | 3.85  |
|        | 1/128         | 8.91E-12      | 4.01  | 1.08E-08      | 3.01  | 3.86E-14           | 0.91  |

## 7. CONCLUSIONS

Some conclusions here. Some conclusions here. Some conclusions here. Some conclusions here.  
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Some conclusions here. Some conclusions here. Some conclusions here. Some conclusions here.

## APPENDIX A. AN EXAMPLE APPENDIX

The contents of the appendix are here.

**Lemma A.1.** *Test Lemma.*

This is a equation in appendix.

$$a^2 + b^2 = c^2. \tag{A.1}$$

## ACKNOWLEDGMENTS

We would like to acknowledge the assistance of volunteers in putting together this example manuscript and supplement.

## REFERENCES

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