PHYS 3142 Spring 2021
Computational Methods in Physics
Assignment 7
Due: 11:59 p.m. 28th March 2021

Before you submit your assignment, do remember:

1. the due day
2. submit a report which contains your figures and results along with your code
3. make sure your code can run
4. do not forget to write comments in your codes.
5. label your figures and describle your results

The basic scoring rubric is :

1. **If you submit the assignment after the deadline or do not submit the report, you can only get up to 80% of grade**
2. **If there is any kind of plagiarism, all of the student involving will get zero mark! (except that the one can really prove the code is written by himself or herself and others copied it without telling him or her)**

Consider a long chain of resistors wired up like this:

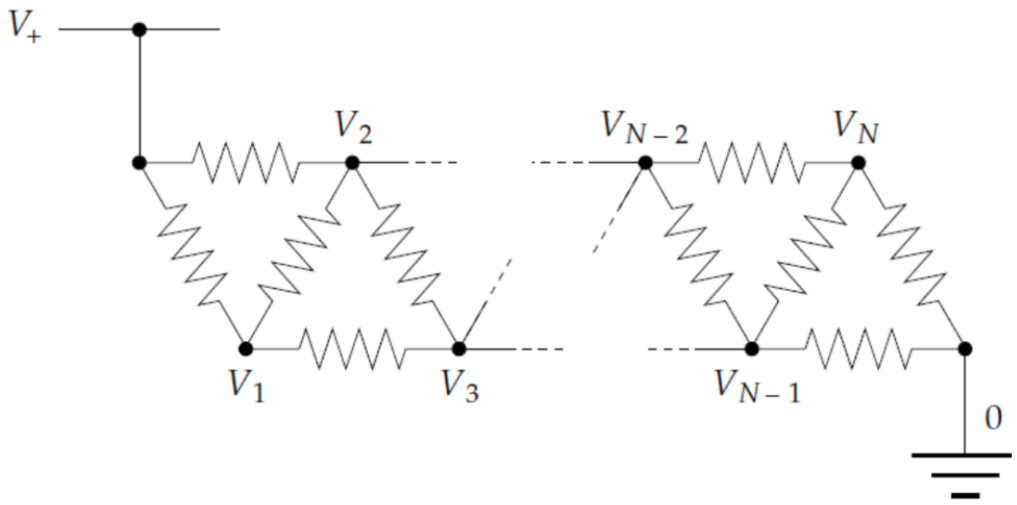

Figure 1:

All the resistors have the same resistance R. The power rail at the top is at voltage $V_+ = 5V$. The problem is to find the voltages $V_1 \ldots V_N$ at the internal points in the circuit.

(a) Using Ohm's law and the Kirchhoff current law, which says that for any junction in an electrical circuit, the sum of currents flowing into that junction is equal to the sum of currents flowing out of that junction. Please show that when N=3, the voltages V_1, V_2, V_3 satisfy the equations

$$\begin{aligned} 3V_1 - V_2 - V_3 &= V_+ \\ 4V_2 - V_1 - V_3 &= V_+ \\ 3V_3 - V_1 - V_2 &= 0 \end{aligned} \tag{1}$$

Express these equations in vector form $Av = w$ and find the values of matrix A and the vector w. Fractorize the matrix A analytically using the LU decomposition. Then solve the simulaneous equations by either using the Gaussian elimination or LU decomposition. Also write a simple program and use numpy.linalg to check your result. [you may upload your hand writing by .jpg or .png files]

(b) If there are N internal junctions, show that the voltages $V_1 \ldots V_N$

satisfy the equations:

$$\begin{gathered} 3V_1 - V_2 - V_3 = V_+ \\ -V_1 + 4V_2 - V_3 - V_4 = V_+ \\ -V_1 - V_2 + 4V_3 - V_4 - V_5 = 0 \\ \cdots \\ -V_{i-2} - V_{i-1} + 4V_i - V_{i+1} - V_{i+2} = 0 \\ \cdots \\ -V_{N-3} - V_{N-2} + 4V_{N-1} - V_N = 0 \\ -V_{N-2} - V_{N-1} + 3V_N = 0 \end{gathered} \tag{2}$$

Express these equations in vector form $Av = w$ and define a function to return the matrix A and the vector w for given N automatically. Show A and w when N=5 and 6.

(c) Write a program to use **solve** in **numpy.linalg** to solve the values of the V_i when there are $\mathrm{N} = 5, 6$ internal junctions with unknown voltages.

(d) Now solve V_i for the case where there are $N = 10000$ internal junctions. Use the **solve** function in **numpy.linalg** and also the **banded** function. Compare these two method by the calculation time.[find the banded.py file in the Lec. 13 folder]