PHYS 3142 Spring 2021
Computational Methods in Physics
Assignment 8
Due: 11:59 p.m. 25th Apr. 2021

Before you submit your assignment, do remember:

1. the due day
2. submit a report which contains your figures and results along with your code
3. make sure your code can run
4. do not forget to write comments in your codes.
5. label your figures and describe your results

The basic scoring rubric is :

1. **If you submit the assignment after the deadline or do not submit the report, you can only get up to 80% of grade**
2. **If there is any kind of plagiarism, all of the student involving will get zero mark! (except that the one can really prove the code is written by himself or herself and others copied it without telling him or her)**

1. Planetary Orbits

The equations of motion for the position x,y of a planet in its orbital plane are the same as those for any orbiting body :

$$\frac{d^2x}{dt^2} = -GM\frac{x}{r^3}$$
$$\frac{d^2y}{dt^2} = -GM\frac{y}{r^3}$$

where $G = 6.6738 \times 10^{-11} m^3 kg^{-1} s^{-2}$ is Newton's gravitational constant, $M = 1.9891 \times 10^{30} kg$ is the mass of the sun, and $r = \sqrt{x^2 + y^2}$.

Let us first solve these equations for the orbit of the Earth. The Earth's orbit is not perfectly circular, but rather slightly elliptical. When it is at its closest approach to the Sun, its perihelion, it is moving precisely tangentially (i.e., perpendicular to the line between itself and the Sun) and it has distance $1.4710 \times 10^{11} m$ from the Sun and linear velocity $3.0287 \times 10^4 m/s$.

1. Write a program to calculate the orbit of the Earth using the Bulirsch-Stoer method to a positional accuracy of **1 km per year**. Divide the orbit into intervals of length **H = 1 week** and then calculate the solution for each interval using the combined modified midpoint method. Make a plot of the orbit, showing at least one complete revolution about the Sun.

2. Modify your program to calculate the orbit of the dwarf planet Pluto. The distance between the Sun and Pluto at perihelion is $4.4368 \times 10^{12} m$ and the linear velocity is $6.1218 \times 10^3 m/s$. Choose a suitable value for H to make your calculation run in reasonable time, while once again giving a solution accurate to **1 km per year**. You should find that the orbit of Pluto is significantly elliptical-much more than the orbit of the Earth. Pluto is a Kuiper belt object, similar to a comet, and (unlike true planets) it's typical for such objects to have quite elliptical orbits.

The following questions are optional, just for exercise and will not be marked.

2. Quantum oscillators

Consider the one-dimensional,time-independent Schrodinger equation in a harmonic (i.e. quadratic) potential $V(x) = V_0 x^2/a^2$, where V_0 and a are constants.

1. Write down the dimensionless version of the Schrodinger equation for this problem and convert it from a second-order equation to two first-order ones. Write a program using the 4th order Runge Kutta method to find the energies of the ground state and the first two excited states for these equations when m is the electron mass, $V_0 = 50eV$, and $a = 10^{-11}m$. Note that in theory the wavefunction goes all the way out to $x = \pm\infty$, but you can get good answers by using a large but finite interval. Try using $x = -10a$ to $+10a$, with the wavefunction $\Psi = 0$ at both boundaries. (In effect, you are putting the harmonic oscillator in a box with impenetrable walls.) The wavefunction is real everywhere (only in one-dimensional system), so you don't need to use complex variables, and you can use evenly spaced points for the solution. Hint: Please refer to the lecture note about the secant method for solving eigen energies.

   The quantum harmonic oscillator is known to have energy states that are equally spaced. Check that this is true, to the precision of your calculation, for your answers. (Hint: The ground state has energy in the range 100 to 200 eV.)

2. Now modify your program to calculate the same three energies for the an harmonic oscillator with $V(x) = V_0 x^4/a^4$, with the same parameter values.