PHYS 3142 Spring 2021
Computational Methods in Physics
Assignment 2
Due: 11:59 p.m. 21th Feb 2021

Before you submit your assignment, do remember:

1. the due day
2. submit a report which contains your figures and results along with your code
3. make sure your code can run
4. do not upload a compressed file(e.g. rar, zip, . . .)

1. Implement Numerical Integral Algorithms
Calculate the following integral numerically:

$$I = \int_0^1 (\frac{\sin(x) + 3 \times \cos(x) - 1}{\sqrt{x}})\,\mathrm{dx}$$

using 5 different methods. Including:

1. **Rectangle′s rule**
2. **Adaptive Trapezoidal**
3. **Adaptive Simpson**
4. **Romberg**
5. **Gaussian Quadrature**

And the requirement is following:

- For **Adaptive Trapezoidal** and **Adaptive Simpson**, you can estimate the error in the iteration. For other methods, you can compare with the accurate value you get from other method to calculate the error.

- Achieve the accuracy of 10^{-5}, i.e. the error is at the order of 10^{-5}. See how many slices you used for each method.

- Make a plot like the one in Lec 4(e.g. page 10. the integral value v.s. number of slices used) and see how the numerical integral converge.

OPTIONAL

2. Weierstrass Function

Calculate the following integral numerically:

$$I = \int_0^{0.5} W(x)\,\mathrm{dx}$$

Where $W(x)$ is the Weierstrass function:

$$W(x) = \sum_{n=0}^{\infty} a^n \cos\left(b^n \pi x\right)$$

Here you just set a = 0.5 and b = 15, and cut off the summation at n = 100.(I tested it, which is sufficient for the integral to converge, or you can test by yourself).

Try **Adaptive Trapezoidal**, **Adaptive Simpson** and **Gaussian Quadrature** method, and achieve the accuracy of $\epsilon = 10^{-6}$. Compare the result to find which method is the fast one in different accuracy like the first problem.

Note: You may find some methods are hard to achieve the accuracy. For this situation, you can compare these three methods in **the same point number** like 10000 to show your result and analyze it.

HINTS

- When encounters singularities of function, you can change the integral limit a little bit to avoid divergence of integral(e.g. minus a very small number).
- You can find a file named "gaussxw.py" on Canvas which you will use when you do Gaussian quadrature.
- Use integrate functions from “Scipy” to check your results.