PHYS 3142 Spring 2021
Computational Methods in Physics
Assignment 9
Due: 11:59 p.m. 18th April 2021

Before you submit your assignment, do remember:

1. the due day
2. submit a report which contains your figures and results along with your code
3. make sure your code can run
4. do not forget to write comments in your codes.
5. label your figures and describle your results

The basic scoring rubric is :

1. **If you submit the assignment after the deadline or do not submit the report, you can only get up to 80% of grade**
2. **If there is any kind of plagiarism, all of the student involving will get zero mark! (except that the one can really prove the code is written by himself or herself and others copied it without telling him or her)**

1. Relaxation

Consider the equation $x = 1 - e^{-cx} + 0.2x$, where c is a known parameter and x is unknown. This equation arises in a variety of situations, including the physics of contact processes, mathematical models of epidemics, and the theory of random graphs.

(a) Write a program to solve this equation for x using the relaxation method for the case $c = 1$. Calculate your solution to an accuracy of at least 10^{-8} (Please start with a positive initial x)

(b) Modify your program to calculate the solution for values of c from 0 to 2 in steps of 0.01 and make a plot of x as a function of c. You should see a clear transition from a regime in which $x = 0$ to a regime of nonzero x. (Please start with a positive initial x)

This is an example of a phase transition.

2. The Lagrange point

There is a magical point between the Earth and the Moon,called the L_1 Lagrange point, at which a satellite will orbit the Earth in perfect synchrony with the Moon, staying always in between the two. This works beacause the inward pull of the Earth and the outward pull of the Moon combine to create exactly the needed centripetal force that keeps the satellite in its orbit. Here's the setup:

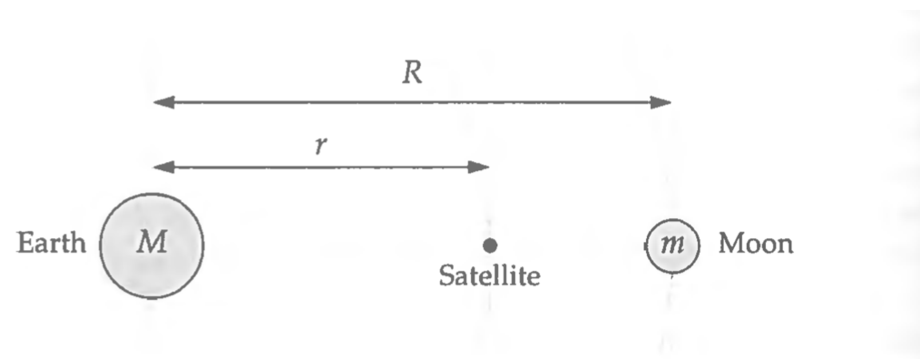

Figure 1:

(a) Assuming circular orbits, and assuming that the Earth is much more maasive than either the Moon or the satellite, please prove that the distance r from the center of the Earth to the L_1 point satisfies:

$$\frac{GM}{r^2} - \frac{Gm}{(R-r)^2} = w^2 r$$

Where R is the distance from the Earth to the Moon, M and m are the Earth and Moon masses, G is Newton's gravitational constant, and w is the angular velocity of both the Moon and the satellite.

(b) The equation above is a degree-five polynomial equation in r (also called a quintic equation). Such equations cannot be solved exactly in closed form, but it's straightforward to solve them numerically. Write a program that uses either Newton's method or the secant method to solve for the distance r from the Earth to the L_1 point. Compute a solution accurate to at least four significant figures. The values of the various parameters are :

$$\begin{gathered} G = 6.674 * 10^{-11} m^3 kg^{-1} s^{-2} \\ M = 5.974 * 10^{24} kg \\ m = 7.348 * 10^{22} kg \\ R = 3.844 * 10^{8} m \\ w = 2.662 * 10^{-6} s^{-1} \end{gathered}$$

You will also need to choose a suitable starting value for r, or two starting values if you use the secant method.