Examples for the WZ–method from "Generatingfunctionology" by Herbert Wilf

Theorem. $\sum_{k=0}^{n}(-1)^k\frac{\binom{n}{k}}{\binom{k+a}{k}}=\frac{a}{n+a},\ (n\geq 0)$

Proof: Take $R(n,k)=\frac{k}{n+a}$.

Theorem. $\sum_{k=0}^{n}(-1)^{n-k}\binom{2n}{k}^2=\binom{2n}{n},\ (n\geq 0)$

Proof: Take $R(n,k)=-\frac{10n^2-6kn+17n+k^2-5k+7}{2(2n-k+2)^2}$.

Theorem. (Dixon's identity)

$$\sum_{k=0}^{n}(-1)^k\binom{n+b}{n+k}\binom{n+c}{c+k}\binom{b+c}{b+k}=\frac{(n+b+c)!}{n!b!c!}$$

Proof: Take $R(n,k)=\frac{(c+1-k)(b+1-k)}{2(n+k)(n+b+c+1)}$