Задача 20. Решете смесената задача за уравнението на струната

$$\begin{cases} u_{tt} = u_{xx},\ 0 < x < 1,\ t > 0 \\ u(x,0) = sin(3\pi x),\ u_t(x,0) = 0,\ 0 \le x \le 1 \\ u(0,t) = 0,\ u(1,t) = 0,\ t \ge 0. \end{cases}$$

Решение:

Струна със закрепени краища:

$$\begin{cases} u_{tt} - a^2 u_{xx} = 0,\ 0 < x < L,\ t > 0 \\ u\Big|_{t=0} = \varphi(x),\ u_t\Big|_{t=0} = \psi(x),\ 0 \le x \le L \\ u\Big|_{x=0} = 0,\ u\Big|_{x=L} = 0,\ t \ge 0 \end{cases} \quad \begin{cases} \varphi(x) \in C^2[0,L],\ \psi(x) \in C^1[0,L] \\ \varphi(x) = \varphi''(0) = \psi(0) = 0 \\ \varphi(L) = \varphi''(L) = \psi(L) = 0 \end{cases}$$

$u_{tt} - 1u_{xx} = 0 \Rightarrow a^2 = 1$ и $a > 0 \Rightarrow a = 1, \quad L = 1$
$\varphi(x) = sin(3\pi x),\ \ \varphi'(x) = 2\pi cos(3\pi x),\ \ \varphi''(x) = -9\pi^2 sin(3\pi x)$
$\psi(x) = \psi'(x) = 0; \quad \varphi(0) = sin0 = 0; \quad \varphi(1) = sin(3\pi) = sin(\pi) = 0;$
$\varphi''(0) = -9\pi^2 sin0 = 0; \quad \varphi''(5) = -9\pi^2 sin(16\pi) = -9\pi^2 sin(3\pi) = 0;$
$\psi(0) = 0; \quad \psi(1) = 0.$