

Задача 19.

Да се намери редицата $\{a_n\}_0^\infty$, която е зададена рекурентно с уравнението $a_{n+1} = -3a_n + (2n+1)5^n$ и $a_0 = 0$.

Решение:

Пресмятаме:

$$a_1 = -3a_0 + (2 \cdot 0 + 1)5^0 = 1; \quad a_2 = -3a_1 + (2 \cdot 1 + 1)5^1 = -3 + 3 \cdot 5 = 12$$

Хомогенна част: $p(-3)^n$

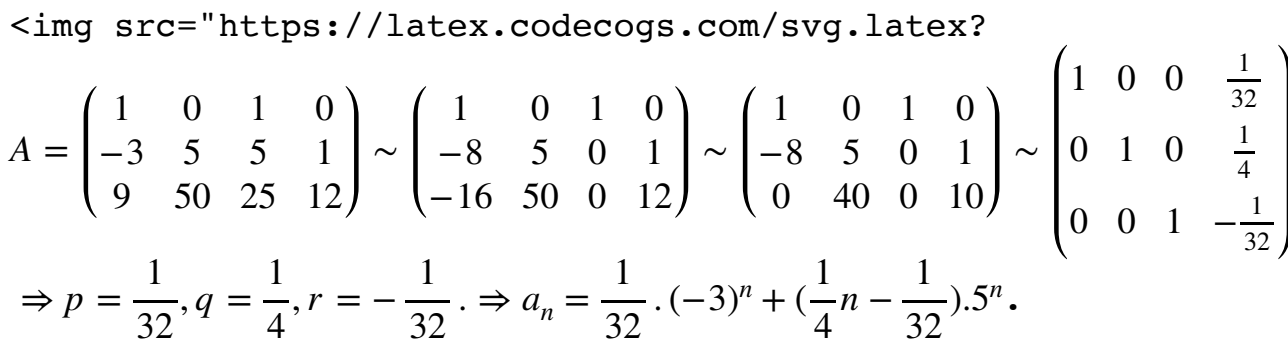
Нехомогенна част: $(qn+r)5^n$

Следователно общия вид е: $a_n = \underbrace{p(-3)^n + (qn+r)5^n}$

$$0 = a_0 = p + r$$

$$1 = a_1 = -3p + 5q + 5r$$

$$12 = a_2 = 9p + (2q+r) \cdot 25 = 9p + 50q + 25r$$

 $\text{src}=\text{"https://latex.codecogs.com/svg.latex?"}$

$$A = \begin{pmatrix} 1 & 0 & 1 & 0 \\ -3 & 5 & 5 & 1 \\ 9 & 50 & 25 & 12 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 1 & 0 \\ -8 & 5 & 0 & 1 \\ -16 & 50 & 0 & 12 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 1 & 0 \\ -8 & 5 & 0 & 1 \\ 0 & 40 & 0 & 10 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 0 & \frac{1}{32} \\ 0 & 1 & 0 & \frac{1}{4} \\ 0 & 0 & 1 & -\frac{1}{32} \end{pmatrix}$$

$$\Rightarrow p = \frac{1}{32}, q = \frac{1}{4}, r = -\frac{1}{32} \Rightarrow a_n = \frac{1}{32} \cdot (-3)^n + \left(\frac{1}{4}n - \frac{1}{32}\right) \cdot 5^n.$$

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