

# Yield: Statistical Modeling and Enhancement Techniques

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## Preliminaries – Defects

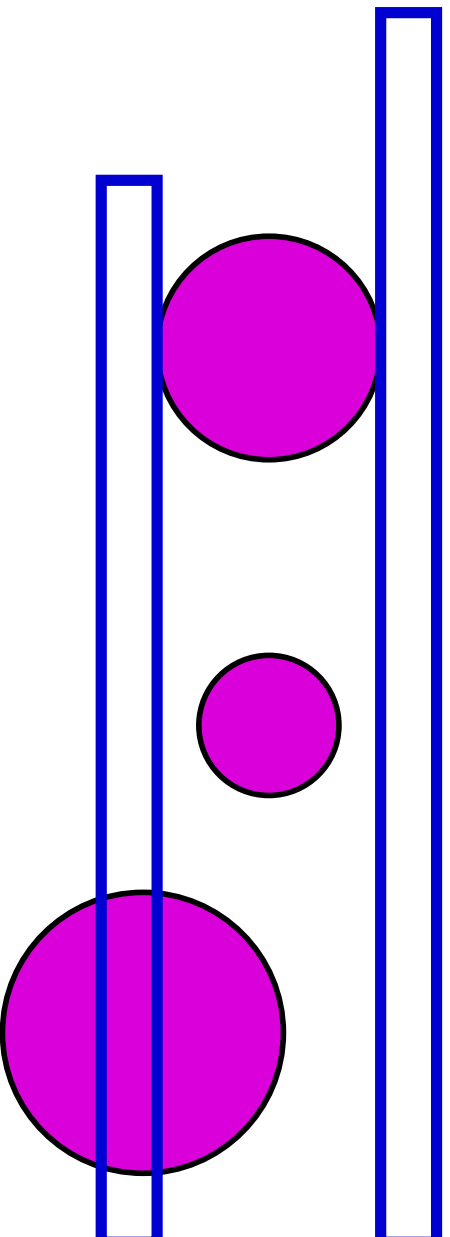
- **Yield** – fraction of operational ICs manufactured
- **Yield losses** – caused mainly by systematic defects and random spot defects
- **Systematic defects** – parametric defects, mask misalignments dealt with during the fabrication process
- **Random spot defects** – result of local contamination (dust particles) – cannot be eliminated
- Their effect can be minimized during the design process
- **We will focus on spot defects – their statistical modeling and ways to minimize their harm**

## Defect Types

- Spot defects classification:
  - Missing material – may result in open circuits
  - Extra patterns – may result in short circuits
  - intra-layer defects – missing or extra material between adjacent wires
  - Inter-layer defects – missing or extra material between two separate layers
- Not all spot defects result in structural faults
- Only defects that turn into faults cause yield losses
- What is the percentage of defects that become faults?

## Defect vs. Fault

- Assumption – A defect is circle shaped with diameter  $x$  ( $x$  is a random variable)
- Other geometrical shapes can be considered
- To become a **fault**, the defect needs to connect two disjoint conductors or disconnect a continuous pattern



## Critical Area

- $po f$  –  
**Probability Of Failure**: percentage of defects that cause faults
  - depends on the type  $i$  and diameter  $x$  of the defect
- A defect will cause a fault if its center is located “properly”
- $A_i^c(x)$  – the **critical area** for defects of **type  $i$**  and **diameter  $x$**  : the size of the area in which the center of a defect of **type  $i$**  and **diameter  $x$**  must fall in order to cause a circuit fault
- Assuming that the centers of the defects are uniformly distributed over the chip area, then

$$po f_i(x) = \frac{A_i^c(x)}{A_{chip}}$$

$A_{chip}$  – area of the whole chip

## Defect Size Distribution

- To compute the yield, critical area must be averaged over all defect sizes  $x$  and defect types  $i$
  - To average over  $x$ ,  $f_d(x)$  – the probability density function of the defect size  $x$ , is necessary
  - Experimental data shows that  $f_d(x)$  decreases as  $1/x^p$  between  $x_0$  – the resolution limit of the lithography process, and  $x_M$  – the maximum size of a defect
  - $p$  and  $x_M$  are determined empirically and may depend on the defect type  $i$  (Typically,  $2 \leq p \leq 3.5$ )
  - Therefore,  $f_d(x) = \begin{cases} k/x^p & \text{if } x_0 \leq x \leq x_M \\ 0 & \text{otherwise} \end{cases}$
- where  $k = (p-1)x_0^{p-1}x_M^{p-1} / (x_M^{p-1} - x_0^{p-1})$

## Averaging Critical Areas

### ■ Averaging over defect diameters $x$ :

- $prof_i = \int_{x_0}^{x_M} prof_i(x) f_d(x) dx$
- $A_i^c = \int_{x_0}^{x_M} A_i^c(x) f_d(x) dx$

### ■ Averaging over defect types $i$ :

- $d_i$  – type  $i$  defect density – average number of defects of type  $i$  per unit area
- $\lambda$  – average number of circuit faults (of all types) on the chip

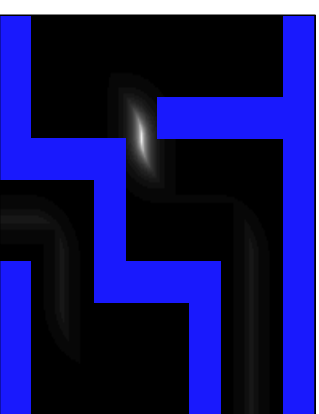
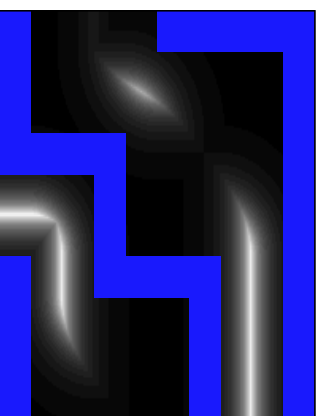
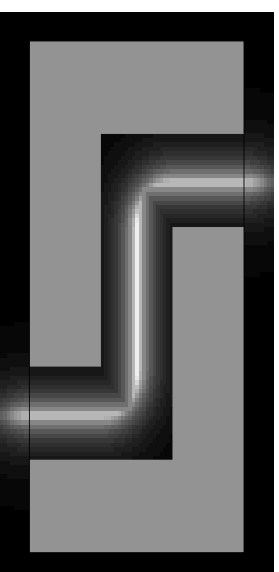
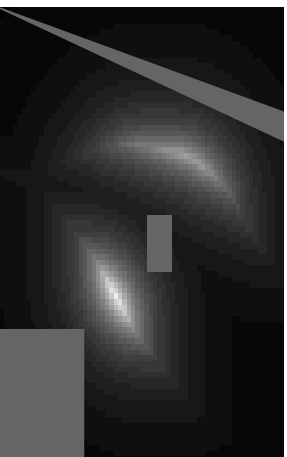
- $$\lambda = \sum_i A_{chip} \cdot d_i \cdot prof_i = \sum_i A_i^{(c)} \cdot d_i$$

## Calculating Critical Areas

- Calculating either  $po$  or critical area of chip – essential for yield prediction
- Several methods of calculation:
  - Geometry-based methods for calculating  $A_i^c(x)$
  - Monte-Carlo methods for calculating  $po f_i(x)$
- Geometrical methods – polygon expansion technique, etc.
- Monte Carlo approach – simulated circles representing defects of different sizes are placed at random locations in the layout. The fraction of defects of type  $i$  and diameter  $x$  which cause a fault is an estimate for  $po f_i(x)$
- Commercial and academic tools for calculating critical areas exist



## Critical Area Maps



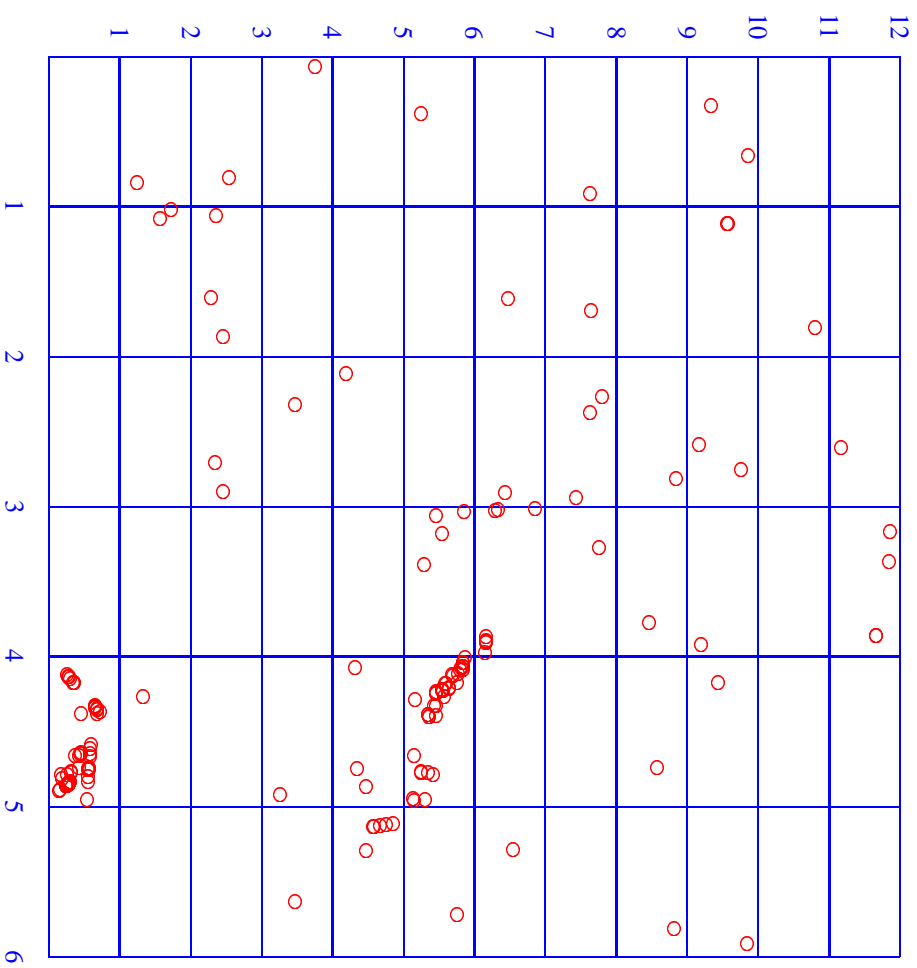
**Source:** I. A. Wagner and I. Koren, “An Interactive VLSI CAD Tool for Yield Estimation,” *IEEE Trans. on Semiconductor Manufacturing*, Vol. 8, pp. 130-138, May 1995.

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## Yield Modeling

- $\lambda$  – The average number of faults – main parameter in yield modeling
- In simplistic models –  $\lambda$  of the whole chip is used
- In more advanced models –  $\lambda_i$  for chip component  $i$  or for chip layer  $i$
- **Yield model** – a probabilistic model describing the distribution of faults over the chip, using the parameter  $\lambda$  and possibly additional parameters
- **Which probabilistic model should be used?**

# Wafer Defect Map



## Defect Maps

- The previous picture showed a **defect map**
- **Faults** are a percentage of the defects
- Their distribution on the wafer will look similar, only with fewer points
- **How do we describe the distribution of defects/faults in a probabilistic manner?**

## Basic Yield Models

- In the past, the **spatial Poisson distribution** was used to describe the defect and fault distribution
- It assumes a uniform distribution of defects and faults over the wafer resulting in

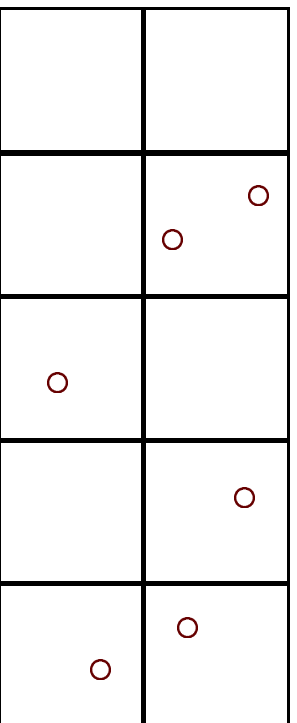
$$Prob(\text{Number of Faults on Chip} = k) = e^{-\lambda} \frac{\lambda^k}{k!}$$

$$\text{Chip Yield} = Prob(\text{Number of Faults on Chip} = 0) = e^{-\lambda}$$

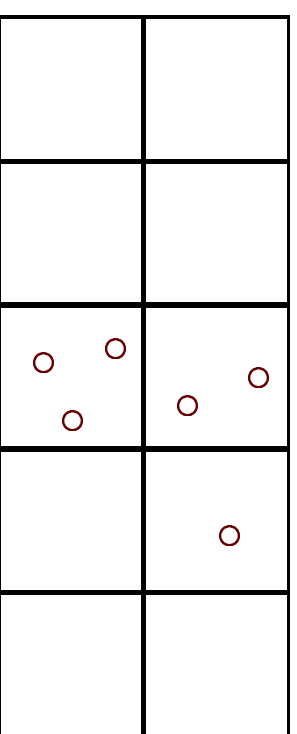
- This is a very simplistic model and does not fit empirical manufacturing data
- In practice, both defects and faults are more clustered than predicted by the pure Poisson distribution

## Yield Models – Clustering

Effect of fault clustering on chip yield



(a) Non-clustered faults,  
 $Y_{chip} = 0.5$



(b) Clustered faults,  
 $Y_{chip} = 0.7$

Clustering increases the yield, and must be incorporated into the fault distribution model

## Introducing Clustering

- Clustering is introduced by regarding  $\lambda$ 
  - the average number of faults – as a random variable with a given probability density function
- Different density functions for  $\lambda$  result in different fault distributions
- The most common model –  $\lambda$  has a Gamma density function

## Negative Binomial Yield Model

- Integrating

$$Chip\ Yield = e^{-\lambda}$$

over  $\lambda$  with regard to the Gamma distribution results in

$$Chip\ Yield = \left( \frac{1 + \lambda}{\alpha} \right)^{-\alpha}$$

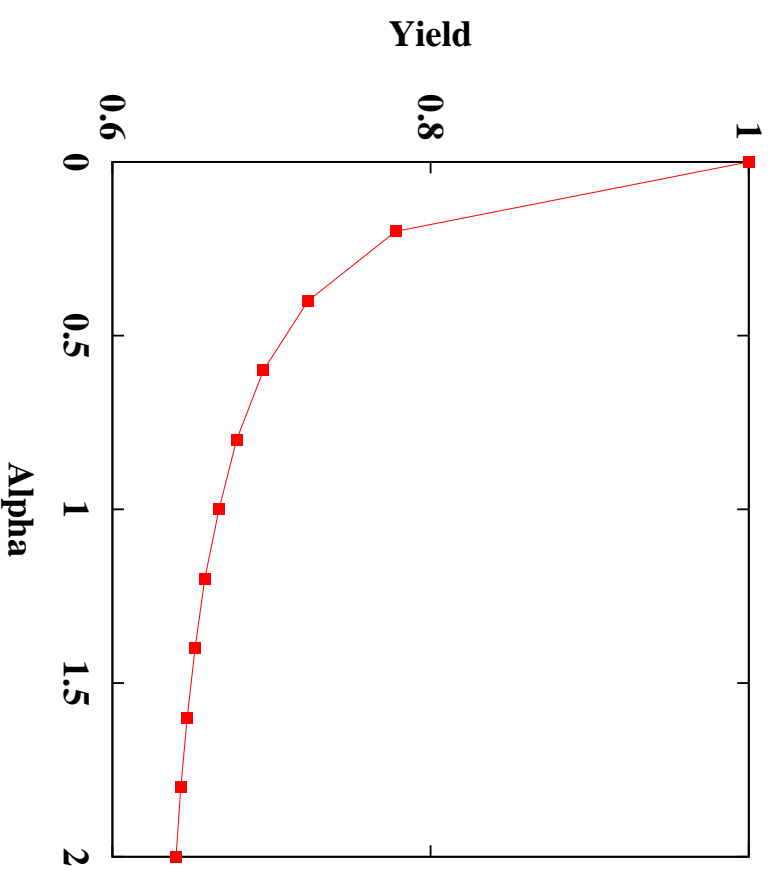
- This is called the Negative Binomial model



## Negative Binomial Model – cont.

- It is characterized by the two parameters:
  - $\lambda$  – the average number of faults per chip
  - $\alpha$  – the clustering parameter
- Typically,  $0.5 \leq \alpha \leq 5$
- The smaller the value of  $\alpha$ , the more pronounced the clustering
- As  $\alpha \rightarrow \infty$ , the Negative Binomial distribution approaches the Poisson distribution
- The use of the Negative Binomial distribution for yield projection is now the industry standard

## Effect of Clustering on Yield



The effect of  $\alpha$  on the chip yield ( $\lambda = 0.5$ )

## Negative Binomial Distribution – Variations

Variations on the basic model:

- Including the effect of systematic defects in addition to spot defects

$$Chip\ Yield = Y_0 \left( \frac{1 + \lambda}{\alpha} \right)^{-\alpha}$$

$Y_0$  is called the gross yield factor and is the probability that there are no systematic defects on the chip

- Dealing with varying fault densities such as for different chip components or for different chip layers

$$Chip\ Yield = \Pi \left( \frac{1 + \lambda_i}{\alpha} \right)^{-\alpha}$$

## Yield Enhancement

- All the models mentioned so far assume no redundancy in the chip – to be operational, the whole chip must be fault-free
- The yield of a chip can be enhanced through
  - Architecture choice (i.e., redundancy)
  - Decreasing the critical area at the design stage (during placement, routing and compaction)
  - Decreasing the defect density
- We will concentrate on the first two options

## Effect of Redundancy on Yield

- **Redundancy** – effective for identical, replicated circuits
- **Most common example** – large memory chips
- **Basic model:**  $N$  modules needed,  $R$  spares added, all identical
- For proper operation - at least  $N$  out of the  $N + R$  must be fault-free
- $\lambda$  – average number of faults per module

$$Chip\ Yield = \sum_{i=N}^{N+R} \sum_{k=0}^{N+R-i} (-1)^k \binom{N+R}{i} \binom{N+R-i}{k} \left( \frac{1+(i+k)\lambda}{\alpha} \right)^{-\alpha}$$

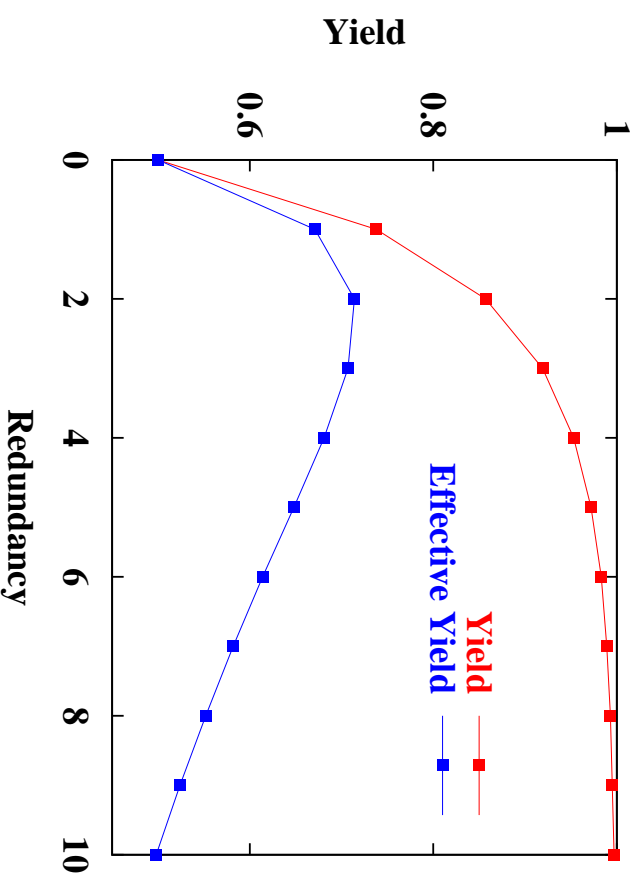
## Effective Yield

- Adding spare modules increases the yield
- At the same time it also increases the chip area – less chips out of the wafer
- Even with a higher yield, we may end up with fewer operational chips per wafer
- Yield may not be the right measure for circuits with redundancy
- **Effective Yield** takes into account the increase in chip area

$$Y_{chip}^{eff} = Y_{chip} \frac{\textit{Area of Chip Without Redundancy}}{\textit{Area of Chip With Redundancy}}$$

## Yield vs. Effective Yield

$N = 10$ ,  $\lambda = 0.1$ ,  $\alpha = 1$ ,  $R = 0, 1, 2, \dots, 10$



The maximum value of  $Y_{chip}^{eff}$  determines the optimal amount of redundancy to be incorporated into the chip

## Yield of Defect-Tolerant Memories

- Large memories must have some redundancy for yield enhancement
- Conventional method of redundancy – spare rows and columns
- Memory chip includes some components (mainly control circuitry) which are uncorrectable
- $Chip\ Yield = Y_0 \cdot Y_c \cdot Y_{uc}$ 
  - $Y_0$  – gross yield factor
  - $Y_c$  – yield of correctable part (with redundancy)
  - $Y_{uc}$  – yield of uncorrectable part



## New Defect-Tolerant Memories

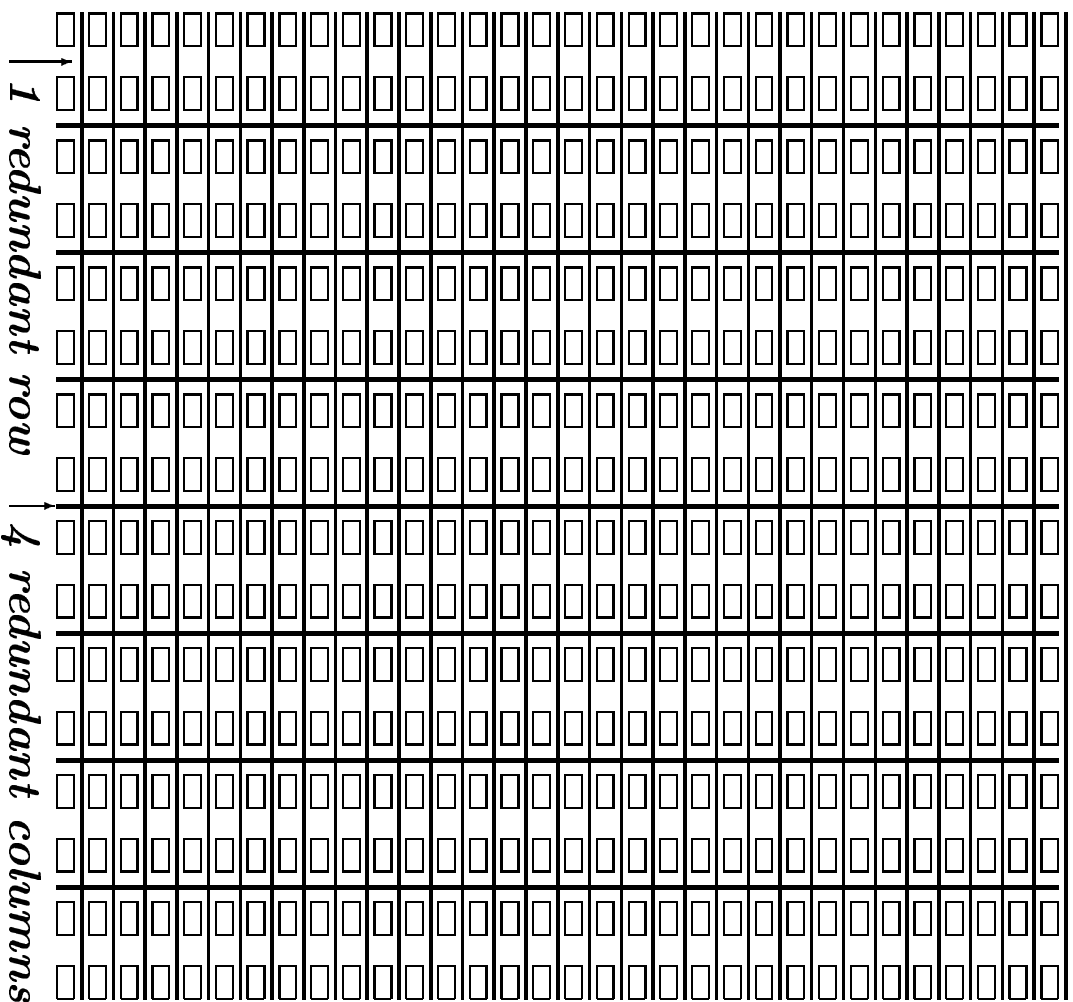
- Memory ICs have become very large
- Conventional redundancy of rows and columns is not sufficient
- Partitioning into sub-arrays is a must
  - Decrease the current
  - Shorten bit & word lines to reduce access time
- Disadvantages of conventional techniques
  - Inefficient use of redundant lines
  - Unable to deal with chip-kill defects
- New defect tolerance techniques are necessary

## Memory with Redundant Blocks

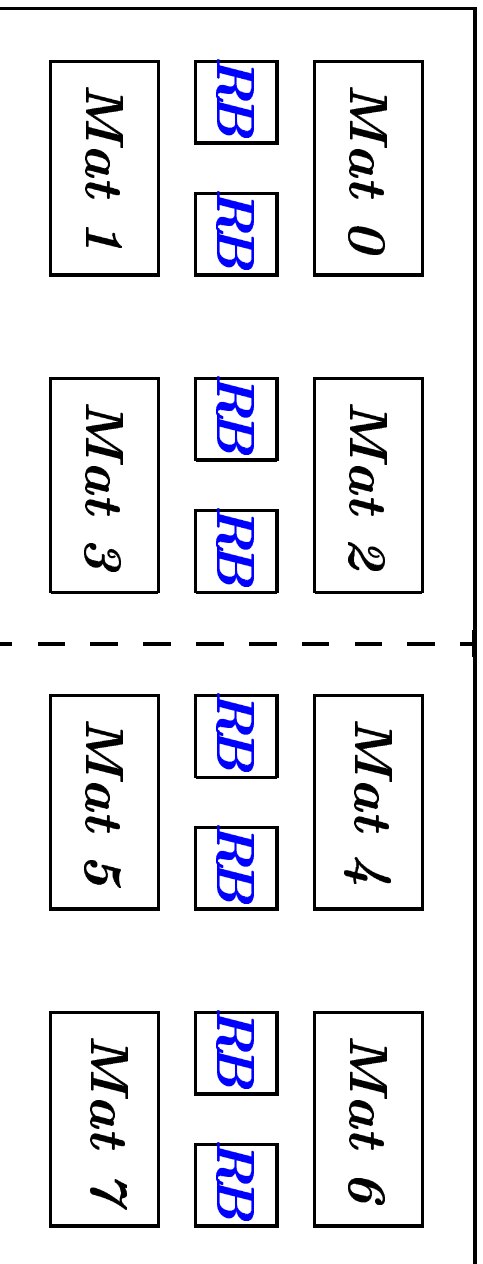
J-H. Yoo *et al.*, “A 32-Bank 1Gb Self-Strobing Synchronous DRAM with 1GB/s Bandwidth,” *IEEE J. of Solid-State Circuits*, vol. 31, pp. 1635-1643, Nov. 1996.

- 1 Gb DRAM is partitioned into eight 128 Mb mats
  - 512 basic arrays of size 256Kbit ( $32 \times 16$  matrix)
  - 32 spare rows and 32 spare columns
  - Four spare rows are allocated to a 16Mbit portion of the mat
  - Eight spare columns are allocated to a 32 Mbit portion of the mat
- Eight redundant blocks of size 1Mbit each
  - Four basic 256Kbit arrays
  - Eight spare rows + four spare columns

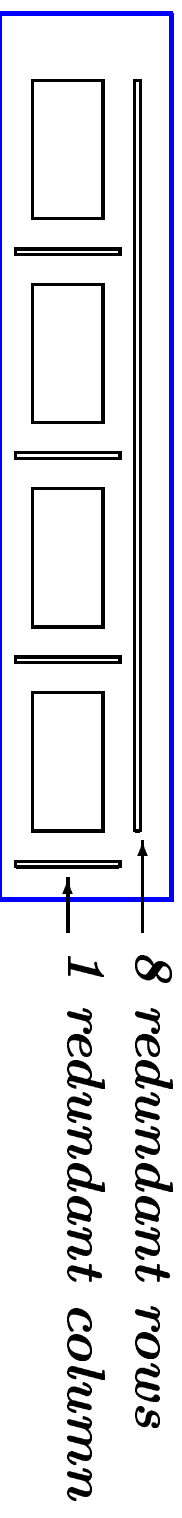
# 128Mb mat (32×16 256Kbit arrays)



## Block Diagram

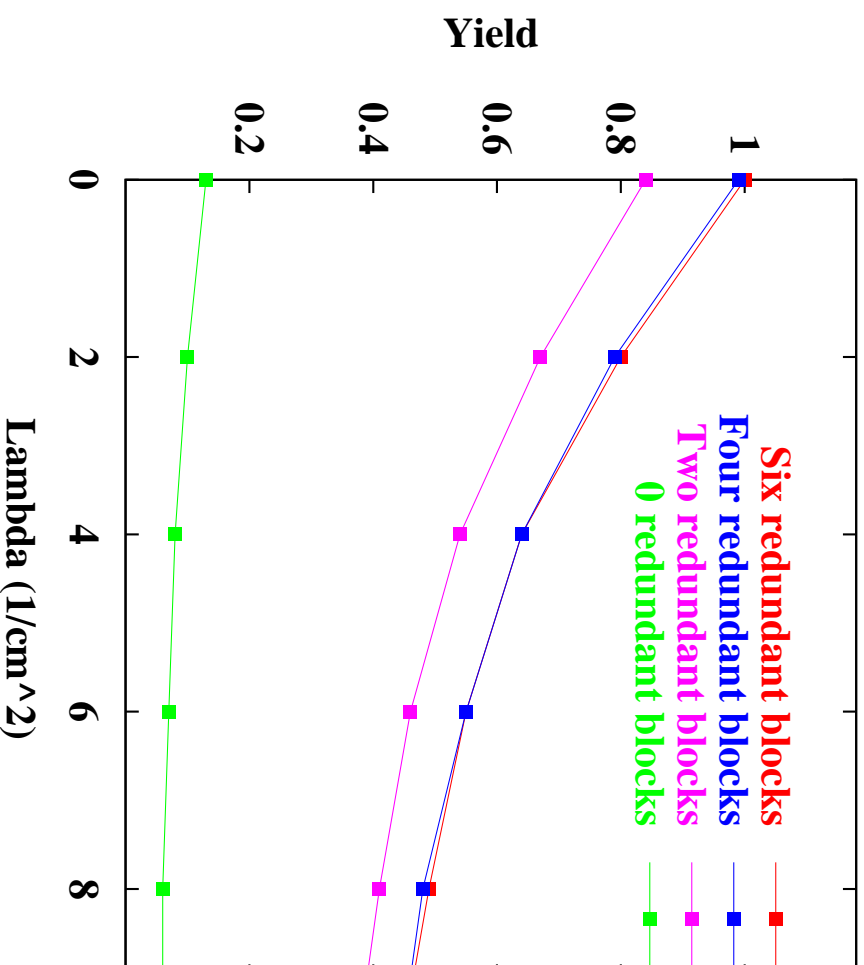


Eight mats (128Mbit each) + eight redundant blocks (*RB*) (1Mbit each).



A redundant block including four 256Kbit arrays, eight redundant rows and four redundant columns.

## Yield Comparison



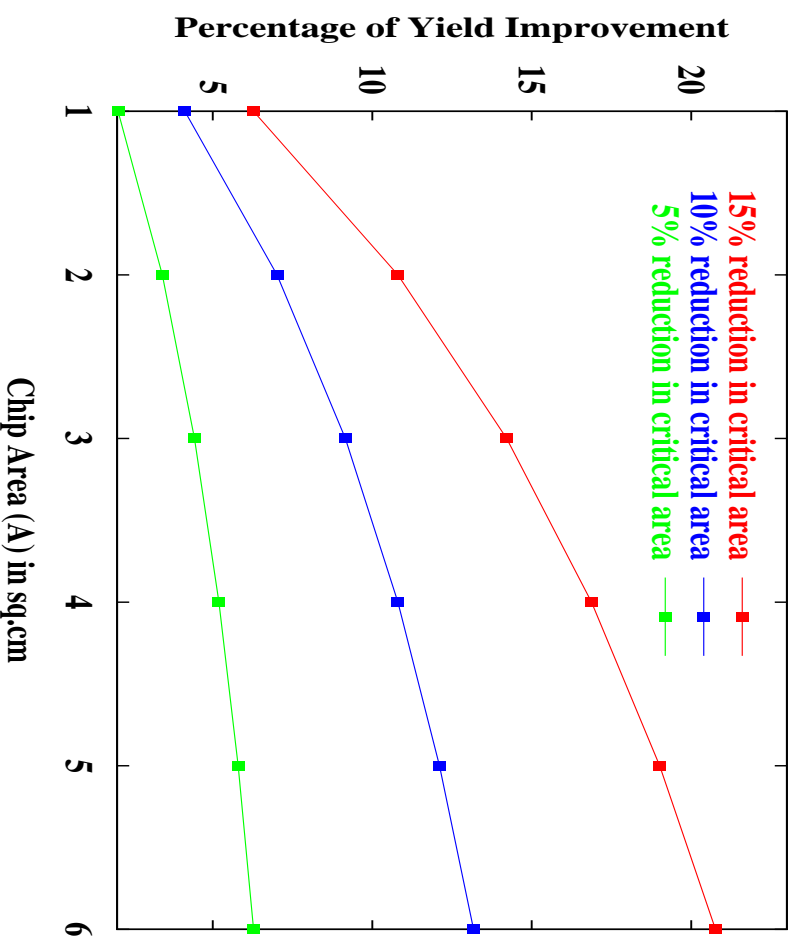
**Source:** I. Koren and Z. Koren, “Analysis of a Hybrid Defect-Tolerance Scheme for High-Density Memory ICs,” *Proc. of the 1997 IEEE Intern. Symp. on Defect and Fault Tolerance in VLSI Systems*, pp. 166-174, Oct. 1997.

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## Yield Enhancement At The Design Stage

- Adding redundancy increases the chip size
- Modifications at the design stage can decrease the critical area without increasing the chip area
- Decreasing the critical area increases the yield

## Effect of Decrease in Critical Area



$$\text{Percentage of Yield Improvement} = \frac{\text{New Yield} - \text{Old Yield}}{\text{Old Yield}} \times 100$$

**Source:** V.K.R. Chiluvuri and I. Koren, “Layout Synthesis Techniques for Yield Enhancement,” *IEEE Trans. on Semicond. Manufacturing*, pp. 178-187, May 1995

## Critical Area Reduction During Compaction

Two approaches to yield enhancement at the compaction stage:

- Local modification in the layout as a post-compaction step
- Modification of the compaction algorithm for critical area reduction

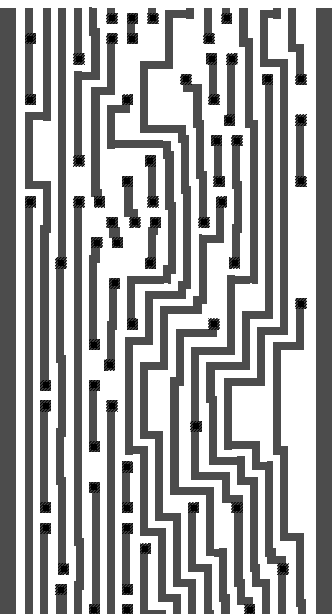
Reductions in critical area of about 8%



# Minimization of Short-Circuit Type Faults

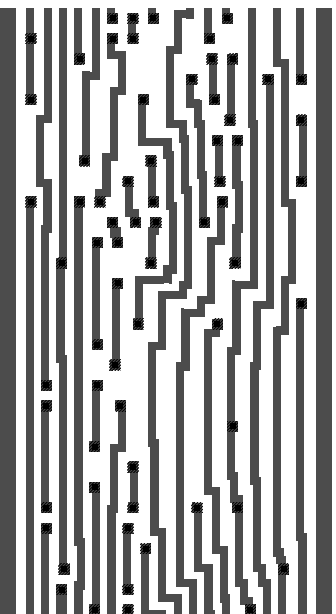
(a)

Original  
layout



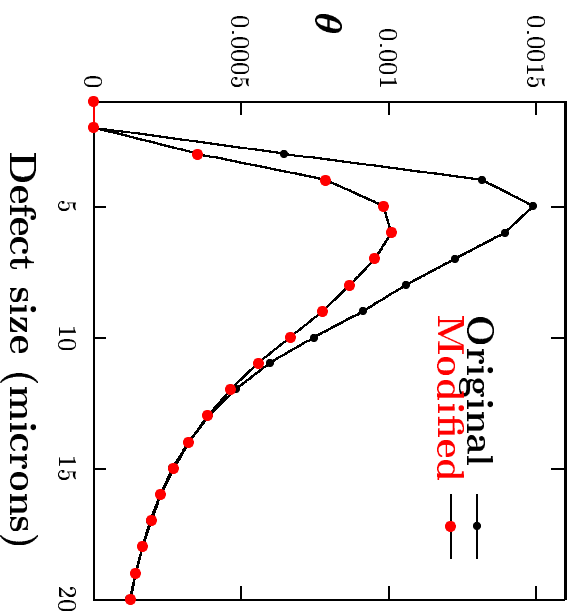
(b)

Modified  
layout

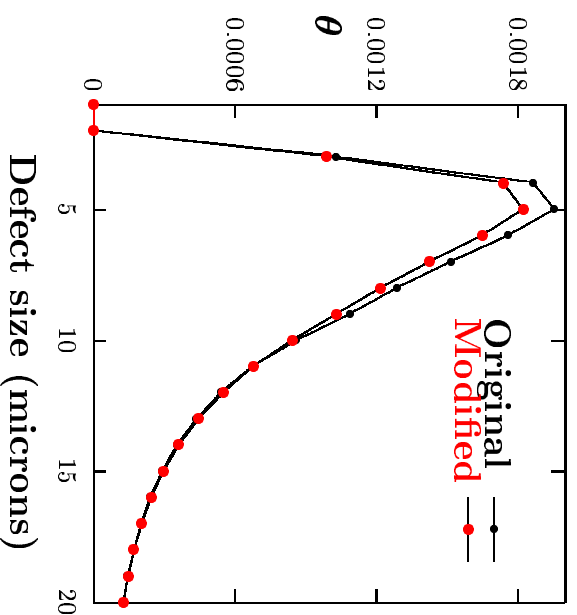


## Effect of Uniform Spacing

Probability of failure ( $\theta$ ) vs. Defect size



(a) Short-circuit faults

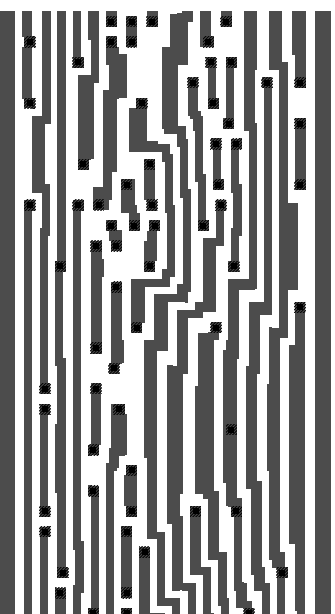


(b) Open-circuit faults

## Minimization of Open-Circuit Type Faults

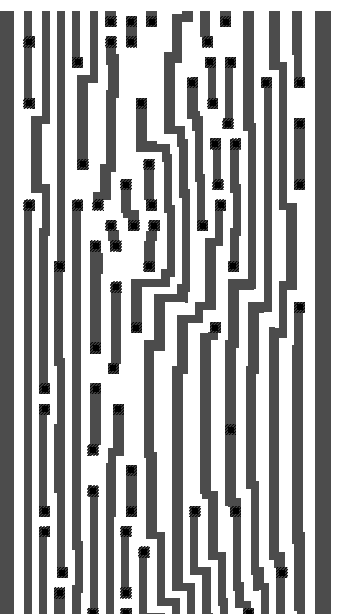
(a)

Modified for  
same defect  
density



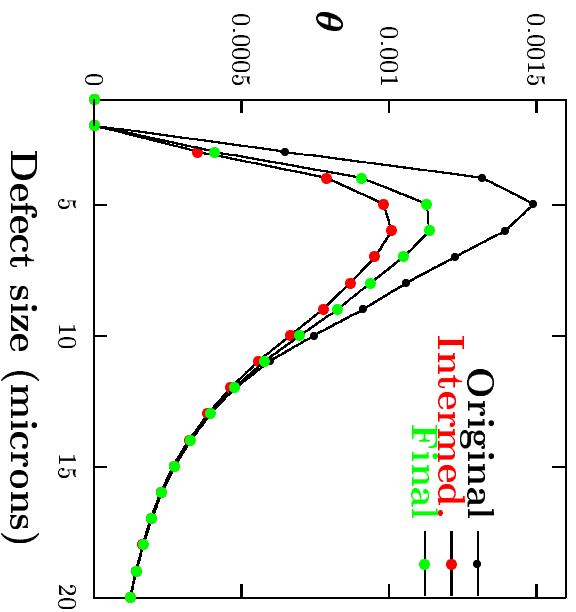
(b)

Modified for  
higher  
short-circuit  
defect density

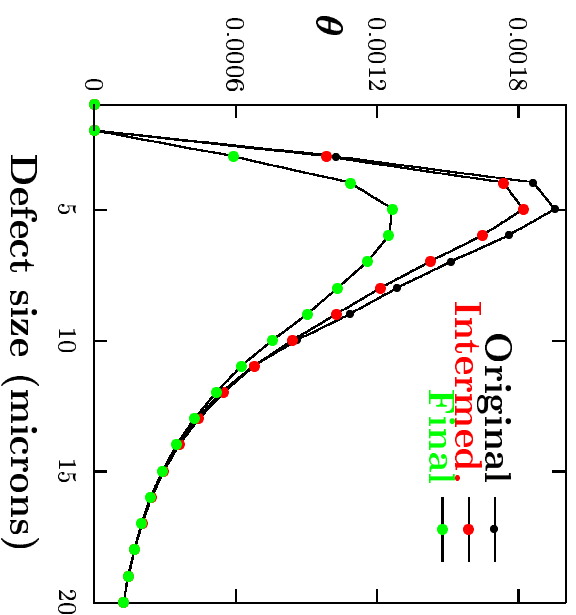


## Effect of Wider Wires

Probability of failure ( $\theta$ ) vs. Defect size



(a) Short-circuit faults



(b) Open-circuit faults

## Critical Area Reduction During Routing

The effectiveness of enhancements during compaction depends on the previously obtained layout

Two approaches to yield enhancement at the routing stage:

- **Local modification** in the layout as a post-routing step
- **Modification of the routing algorithm** for yield enhancement

**Reductions in critical area of about 6%**

## Conclusions

- As density and size of integrated circuits increase, so do the importance and difficulty of achieving high yield
- Accurate yield projection at every stage of the design is of utmost importance
- Yield enhancement efforts must be part of the physical design stage