Exercise 1:

if $u_i(c) = \frac{c^{1-r_i}}{1-r_i}$

$$\Rightarrow \max_{x,y} p_i u_i(w_i + x - \bar{p}x - \bar{q}y) + q_i u_i(w_i + y - \bar{p}x - \bar{q}y)$$

$$\Rightarrow \max_z u(z) = \max_z p_i u_i(w_i + (1-\bar{p})z) + (1-p_i) u_i(w_i - \bar{p}z)$$

if $z = x - y$

$$\Rightarrow \frac{\partial u}{\partial z} \Rightarrow p_i(1-\bar{p}) u_i'(w_i + (1-\bar{p})z) = (1-p_i)\bar{p} u_i'(w_i - \bar{p}z)$$

put $u_i(c) = \frac{c^{1-r_i}}{1-r_i}$

after solve the equation $$z = w_i \frac{e^{\frac{1}{r_i}(\log\frac{p_i}{1-p_i} - \log\frac{\bar{p}}{1-\bar{p}})} - 1}{\bar{p} e^{\frac{1}{r_i}(\log\frac{p_i}{1-p_i} - \log\frac{\bar{p}}{1-\bar{p}})} + 1 - \bar{p}}$$

$$\Rightarrow x_i = w_i \frac{e^{\frac{1}{r_i}(\log\frac{p_i}{1-p_i} - \log\frac{\bar{p}}{1-\bar{p}})}}{\bar{p} e^{\frac{1}{r_i}(\log\frac{p_i}{1-p_i} - \log\frac{\bar{p}}{1-\bar{p}})} + 1 - \bar{p}}$$

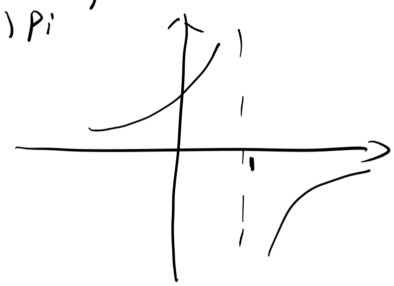
$$y_i = w_i \frac{1}{\bar{p} e^{\frac{1}{r_i}(\log\frac{p_i}{1-p_i} - \log\frac{\bar{p}}{1-\bar{p}})} + 1 - \bar{p}}$$

Exercise 2:

$$f_2(x) = \sum_{i=1}^{N} \frac{1 - \left(\frac{x(1-p_i)}{(1-x)p_i}\right)^{\frac{1}{r_i}}}{\left[\left(\frac{x(1-p_i)}{(1-x)p_i}\right)^{\frac{1}{r_i}} - 1\right](1-x)+1} w_i \qquad r_i > 0$$

$$f_2(x) = \sum_{i=1}^{N} \frac{1}{x-1+\frac{1}{1-\left(\frac{x(1-p_i)}{(1-x)p_i}\right)^{\frac{1}{r_i}}}} w_i$$

let $g(x) = \frac{x}{1-x}$ $\Rightarrow$



We know $0 < x < 1$

so $g(x)$ is strict increasing

$$\Rightarrow \quad f_2(x) = \sum_{i=1}^{N} \frac{1}{x-1+\frac{1}{1-\left(g(x)\frac{1-p_i}{p_i}\right)^{\frac{1}{r_i}}}}$$

$h_i(x) = \frac{1}{1-\left(g(x)\frac{1-p_i}{p_i}\right)^{\frac{1}{r_i}}}$ $\Rightarrow h(\cdot)$ is strict increasing

$$\Rightarrow \quad f_2(x) = \sum_{i=1}^{n} \frac{1}{x-1+h_i(x)}$$

$\Rightarrow$ $f_2(x)$ is strick decreasing