The Economics of FinTech, Lecture 9

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Outline

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1. Introduction

2. Robo-Advising: A Dynamic Mean Variance Approach

Topics to be covered

- Black-Litterman Model
- Dynamic Mean Variance

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Motivation

- Based on: M. Dai, H. Jin, S. Kou, and Y. Xu (2019). A dynamic Mean Variance Analysis for Log-Returns. To appear, *Management Science*.

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- A robo-advisor aims at giving general financial advices to large number of clients.
- Some difficulties:
  - The traditional investment theory is based on utility functions. But it is difficult to measuring risk preference of clients automatically.
  - Consistent advice to financial advisors:
    1. A client should rebalance portfolios dynamically, preferably in a time consistent way.
    2. A rich client should invest more (dollar amount) in stocks than a poor client.
    3. No short sale of stocks in the long run, due to positive equity premium.

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- Due to Criertion 1, we will focus on dynamic asset allocation.

Motivation: Two Theories of Dynamic Asset Allocation

- Utility maximization, Merton (1969, 1971):
  - Exponential utility $-\exp(-\tilde{\gamma}W)$, resulting in a constant dolloar amount in stock.
  - Power utility $\frac{W^{1-\tilde{\gamma}}}{1-\tilde{\gamma}}$, resulting in a constant proportion of wealth in stock.
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  - It is difficult to estimate $\tilde{\gamma}$ for investors.
- Dynamic mean variance analysis is an extension to the one-period mean variance analysis by Markowitz (1952).
  - One-period mean variance: $\min Var(R_1)$, s.t. $E(R_1) = a$..
  - Dynamic mean variance: $\min Var(W_T)$, s.t. $E(W_T) = a$, where $W(T)$ is the total wealth.
  - It is easier to get the model input from the client, as only the expected portfolio return is required from the client.

Motivation: Time-Inconsistency in Dynamic Mean Variance

- The dynamic mean variance analysis has an issue of time inconsistency: A strategy that is optimal today may not be optimal tomorrow.
- More precisely, at time 0, a robo advisor gives a optimal strategy for the investor to do at time 1; say a function $f(X_1)$, where X_1 is the wealth at time 1. However, when time 1 arrives, the optimal strategy will be $g(X_1)$, $g \neq f$.
- This will create serious confusions among the investor.
- Time inconsistency is a broad issue, e.g. non-exponential discounting.
- Here the time inconsistency arises because the objective function is not a expectation, but a nonlinear function of expectation.
- More precisely, the objective function:

$$J(\pi_t; t) = \mathrm{E}_t(W_T) - \frac{\gamma}{2}\mathrm{Var}_t(W_T),$$

but $\mathrm{Var}_t(W_T) = \mathrm{E}_t(W_T^2), -(\mathrm{E}_t(W_T))^2$.

Two Solutions: Time-Inconsistency in Dynamic Mean Variance

- Pre-committed strategy: $\min Var(W_T)$, s.t. $E(W_T) = a$, at time 0.
- An equilibrium strategy $\hat{\pi}(t)$ (Bjork and Murgoci, 2010): for any perturbation v in $[t, t+h)$

$$
{}_{h,v}(\tau) = \begin{cases} v & \text{for } t \le \tau < t+h \\ \hat{\pi}(\tau) & \text{for } t+h \le \tau \le T, \end{cases}
$$

$$
\liminf_{h \to 0^+} \frac{J(\hat{\pi}; t) - J(\pi_{h,v}; t)}{h} \ge 0 \quad \text{for any } h, v.
$$

The associated equilibrium value function $V(t) =: J(\hat{\pi}; t)$.

Previous Literature

- Basak and Chabakauri (2010, RFS):
  - Objective function: $J(\pi_t; t) = \mathrm{E}_t(W_T) - \frac{\gamma}{2}\mathrm{Var}_t(W_T)$, where γ is a constant.
  - Complete market: The optimal strategy is to invest, after discounting, a constant dollar amount in stock, which is against Criteria 2. Incomplete markets: Analytical solutions are available in some cases of incomplete markets, and the resulting strategies are also against Criteria 2.

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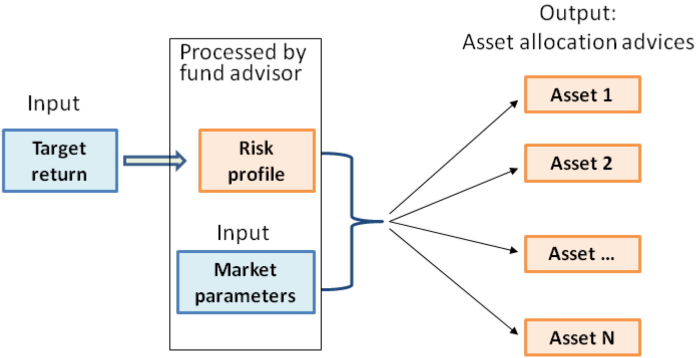
- Bjork, Murgoci and Zhou (2014, MF):
  - Objective function: $E_t(W_T) - \frac{\gamma}{2W_t} Var_t(W_T)$.
  - Complete market: The optimal strategy depends on the wealth level:

$$\frac{y_t}{W_t} = c(t)\frac{\mu - r}{\gamma\sigma^2},$$

    where $c(t)$ solves an integral equation. Incomplete markets: N.A.
  - Unfortunately, $c(t) \to -\gamma < 0$ as $T - t \to \infty$, which is against Criterion 3.

Our Solution



Our Formulation

We propose a dynamic mean-variance model on log return.

- Our formulation: min $Var_{t,T}(R_{t,T})$, s.t. $\frac{1}{T-t}E_t(R_{t,T}) = a$, or equivalently, $E_t(R_{t,T}) - \frac{\gamma}{2}Var_t(R_{t,T})$, where $R_{t,T} = \log(\frac{W_T}{W_t})$, and γ is a constant dependent on a.
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- The risk averse parameter is obtained automatically as $\tilde{\gamma} = 1 + \gamma$.
- Analytical solutions are available in many cases of incomplete markets.

A comparison (complete market)

Complete Market	Comparison (with $\mu > r$)	
	rich invests more	no short-sale in long run
Basak and Chabakauri (2010)	No	Yes
Bjork, Murgoci and Zhou (2014)	Yes	No
This paper	Yes	Yes

Incomplete Markets	Comparison (with $\mu > r$)	
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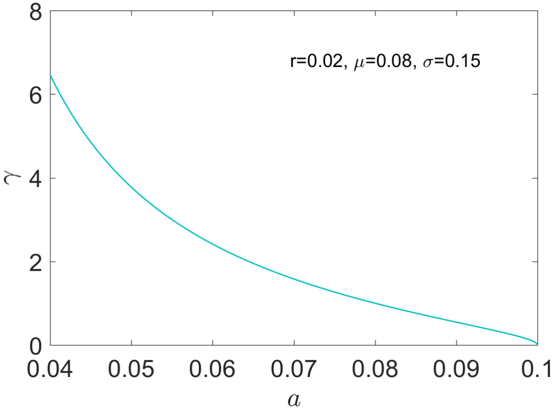
Implied γ



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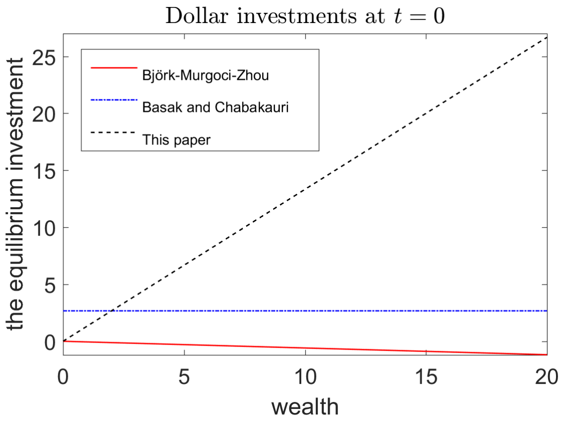

Figure: $a = 0.08$ (equivalently $\gamma = 1$), $\sigma = 0.15$, $\mu = 0.08$, $r = 0.02$, $T = 30$

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