Designing Stable Coins

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Based on the paper with Yizhou Cao, Min Dai, Lewei Li, and Chen Yang

Outline

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1. Background of Cryptocurrencies and Our Contribution
2. Design Details
   - Vanilla A and B Coins
   - A' and B' Coins
3. Black Swan Events
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Background

- The difficulties of inventing electronic cash:
  - Duplication problem.
  - Double spending problem.

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- The difficulties of inventing electronic cash:
    - Duplication problem.
    - Double spending problem.
- Impossibility theorems for reaching consensus over distributed networks.
- A solution is to use blockchains (Nakamoto, 2008), e.g. Bitcoin.
    - A blockchain is a decentralized (peer to peer) and distributed network that is used to record, after miners' verification, all transactions which can be viewed by every users.
    - The records cannot be easily altered retroactively.
    - A peer-to-peer network. Anonymous payment.
    - A blockchain confirms with very high probability that each unit of value was transferred only once.
    - See `https://anders.com/blockchain/hash.html` for a demonstration of blockchains in action.

Background

- Another breakthrough came in late 2013 when Vitalik Buterin extends the idea of Bitcoin to create the Ethereum platform on which people can write smart contracts.
  - This is a very important technology advance, as many types of clerk works, such as public notary, import and export paper works, certain legal and accounting documentations, can be programmed as smart contracts which can be tracked and executed automatically on the Ethereum platform.
  - The cryptocurrency generated and circulated on the Ethereum platform is Ether (with the trading symbol ETH).
  - ETH is needed to pay for basic programming operations, such as $+$, *, max, min, etc.

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  - ETH is needed to pay for basic programming operations, such as $+$, *, max, min, etc.
- Currently, there are thousands crytocurrencies traded in exchanges; see, e.g, the list on coinmarketcap.com.
- The extremely large volatility means that a cryptocurrency like ETH cannot be used as a reliable means to store value.

ETH Coin

ETH/USD price data from 1 Oct 2017 to 31 Aug 2018.

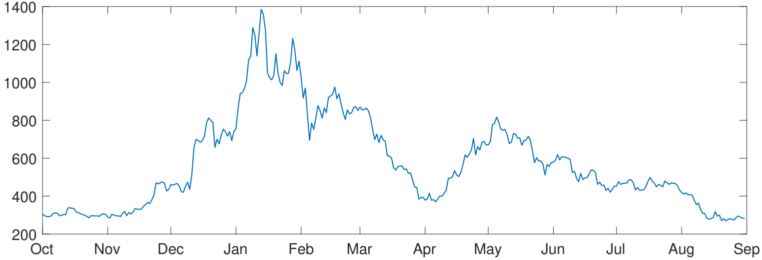

Figure: ETH/USD Price from 1 Oct 2017 to 31 Aug 2018. Annualized volatility during this period 107%, as compared to 13% of S&P500.

The Need for Stable Coins

- A stable coin is a crypto coin that keeps stable market value against a specific index or asset, most noticeably U.S. dollar.
- Stable coins are desirable for at least three reasons:
    - They can be used within blockchain systems to settle payments (e.g. by lawyer or accountants) for the services they provide within the system, without being bothered by the exchange fees from a cryptocurrency to U.S. dollar.
    - They can be used to form crypto money market accounts.
    - They can be used by miners or other people who provide essential services to maintain blockchain systems to store values, as it may be difficult and expensive for them to convert mined coins into traditional currencies.

The Need for Stable Coins

- How to create stable coins?

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- "The Holy Grail of Cryptocurrency" (Forbes, March 12, 2018, The Sydney Morning Herald, Feb 22, 2018, Yahoo Finance, Oct 14, 2017)

The Need for Stable Coins

- How to create stable coins?
- "The Holy Grail of Cryptocurrency" (Forbes, March 12, 2018, The Sydney Morning Herald, Feb 22, 2018, Yahoo Finance, Oct 14, 2017)
- There are at least 4 existing ways to issue stable coins in Blockchain networks.

Existing Stable Coins (1)

- The first type is an issuance backed by accounts in real assets such as U.S. dollars, gold, oil, etc.
- More precisely, these stable coins represent claims on the underlying assets.
  - The most famous one is Tether, which 100% backed by USD. The conversion rate is 1 tether USDT equals 1 USD. Disadvantage: The total assets is about \$62.46 million, but the total liabilities is \$63.014 million, with a negative equity more than half million.
  - Tokens are claimed to link to gold, although it is difficult to verify the claims, e.g. Digix, GoldMint, Royal Mint Gold, OzCoinGold, ONEGRAM.
  - Petro, issued by Venezuela and backed by one barrel of oil.

Existing Stable Coins (2)

- The second type is the seigniorage shares system, which has automatic adjustment of the quantity of coin supply.
- When the coin price is too high, new coins are issued; when the coin price is too low, bonds are issued to remove tokens from circulation.
- A typical example of this type is Basecoin.

Existing Stable Coins (3)

- The third type is an issuance backed by over-collateralized cryptocurrencies with automatic exogenous liquidation.
  - For example, one can generate \$100 worth of stable coins by depositing \$150 worth of Ether.
  - The collateral will be sold automatically by a smart contract, if the Ether price reaches \$110.
  - Examples of this type include the DAI token issued by MakerDAO.
- One can also combine the idea of over-collateralization and seigniorage by issuing more coins if the coin price is too high, and allow people to borrow the coin, which gives borrower to buy back the coin if the coin price is too low, thus pushing the price higher.
- A drawback of this type is the relative large collateral size.

Existing Stable Coins (4)

- The last type is government-backed stable coins.
- Besides Venezuela, other countries are considering issuing cryptocurrencies, including Russia and China.
- CAD-coin: Canadian government also did "Project Japser", in which a Blockchain network is built for domestic interbank payments settlement.
- Fedcoin: There is a virtual currency working group under the Federal Reserve System in U.S. The Fedcoin is used internally.
- "The goal is to create a stable (less price volatility) and dependable cryptocurrency that delivers the practical advantages of Bitcoin even if this means involving the central government and abandoning the Libertarian principles that many believe underlay Bitcoin's creation." Rod Garrett (2016).

Government Backed Stable Coins

There are several advantages of issuing stable coins by governments

- They are cheaper to produce than the cash in bills or coins, and stable coins are never worn out.
- They can be tracked and taxed automatically by the Blockchain technology.
- They can facilitate statistical works, such as GDP calculation and collecting consumer data.
- They cannot be forged (at least in theory).
- They can simplify legal money transfers inside and outside Blockchains.
- Bech and Garratt (2017), the main benefit of a retail central bank backed cryptocurrency is that it would have the potential to provide the anonymity of cash.

Our Contribution: 1

- We use the option pricing theory to design several dual-class structures that offer entitlements to fixed income stable coins (Class A coins) pegged to a traditional currency as well as leveraged investment opportunities (Class B coins).
- The design is inspired by the dual-purpose funds popular in the U.S. and China.

Our Contribution: 1

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- The design is inspired by the dual-purpose funds popular in the U.S. and China.
  - More precisely, due to downward resets, a vanilla A coin behaves like a bond with the collateral amount being reset automatically.
  - To reduce volatility, the vanilla A coin can be further split into additional coins, A' and B'.
  - Unlike traditional currencies, these new class A coins record all transactions on a blockchain without centralized counterparties.

Our Contribution: 2

- We show that the proposed stable coins have very low volatility.
- Volatility comparison (from Oct 1, 2017 to Aug 31, 2018)
  - ETH coin, 107.03%
  - S&P 500, 12.96%
  - Gold, 9.55%
  - U.S. Dollar Index, 5.86%
  - Class A coin, 2.69% (GBM model), 4.93% to 15.67% (DEJD model)
  - Class A' coin, 1.14% (both models)
- The volatility of A' is so low that it is essentially pegged to U.S. dollar.

Our Contribution: 3 and 4

- A policy implication of this paper is that a public-private partnership may be formed to issue stable coins backed by a government.
    - More precisely, by designing a set of stable coins using the option pricing theory via private market forces, the government only needs to back up the stable coins in extreme cases of Black swan events, just like what the U.S. government does for the FDIC insurance for private money market accounts in U.S.
- Technically, in contrast to the standard Black-Scholes partial differential equation (PDE), the pricing equation here is a periodic PDE with a time-dependent upper barrier, which requires a new theoretical derivation of the stochastic representation of the PDE (Theorem 1) and a new numerical procedure (Theorem 2).

Literature Review (1)

- Contract and Model Comparison of Our Stable Coins and Dual-Purpose Fund in U.S. and China
  - Dual-Purpose Fund in U.S.: Dividend, single payment, no reset barriers.
  - Dual-Purpose Fund in China: Fixed income, payments affect the underlying asset but not the exchange ratio, reset barriers.
  - Our Vanilla A and B coins: Fixed income, payments affect the exchange ratio but the underlying asset but not the underlying asset, reset barriers.
- Model comparison
  - Dual-Purpose Fund in U.S., Ingersoll (1976) and Jarrow and O'Hara (1989): Black-Scholes PDE
  - Dual-Purpose Fund in China, Dai et al. (2018): Periodic PDE, a constant upper barrier.
  - Our Vanilla A and B coins: Periodic PDE, a time-dependent upper barrier.
- Extensive empirical studies of cryptocurrencies are conducted in Chen et al. (2016), Trimborn and Hardle (2016), and Chen et al. (2018)

Literature Review (2)

- There are many papers and media articles discussed advantages of cryptocurrencies. Harvey (2016), Catalini and Gans (2016), Nakamoto (2008), Grunspan and Perez-Marco (2018), Khapko and Zoican (2018), Rogoff (2015).
- There are also some criticisms of cryptocurrencies. Grinberg (2011), and Garratt and Wallace (2018).
- It is generally agreed that the blockchain technologies are here to stay.
- However, blockchain technologies automatically generate cryptocurrencies for the purpose of charging the services provided by the system (such as fees incurred by all programming codes which are run on the Ethereum network), crediting essential services to the system (such as the verification services provided by miners), and of exchanging credits for services.
- Therefore, cryptocurrencies will not disappear as long as blockchain technologies exist;
- Thus, designing suitable stable coins is essential.

Outline

Vanilla A Coin, Initial Split

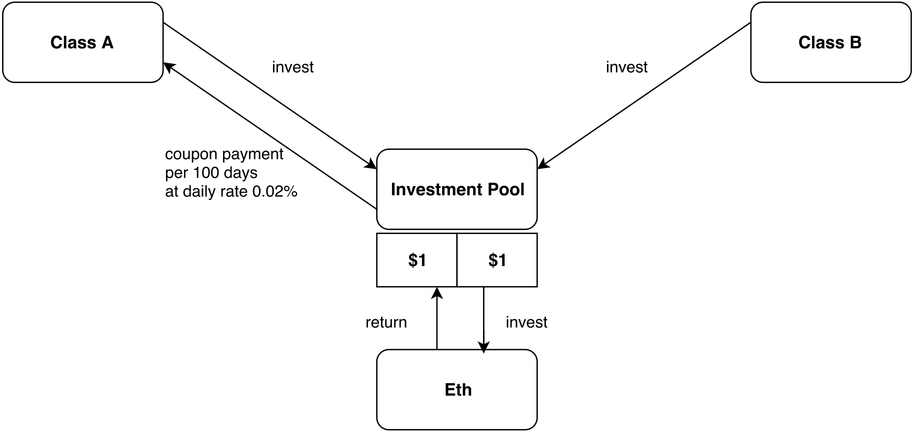


Figure: Initial Split. At the beginning, one share of Class A and B each invests $1 in ETH. The ETH price is $500, so two shares of ETH correspond to 500 shares of Class A coins and 500 shares of Class B coins. $\beta = 1$.

Vanilla A Coin, Regular Payout

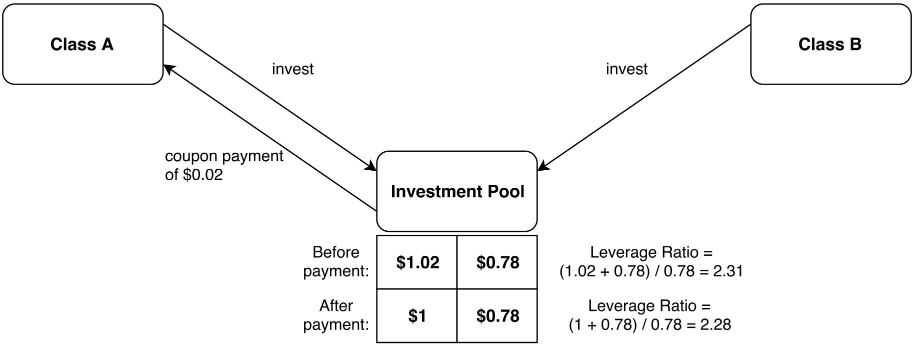


Figure: Regular payout. After 100 days, the ETH price drops to $450, so that total investment of one Class A coin and one Class B coin becomes $1.8, within which $1.02 belongs to Class A. A regular payout takes place, and Class A receives $0.02 coupon payment. New exchange ratio: 2 shares of ETH now correspond to 505.62 ($= 500 \times \frac{2\times 450}{2\times 450 - 500 * 0.02}$) shares of Class A and 505.62 shares of Class B, yielding $\beta = 1.01$.

Vanilla A Coin, Upward Reset

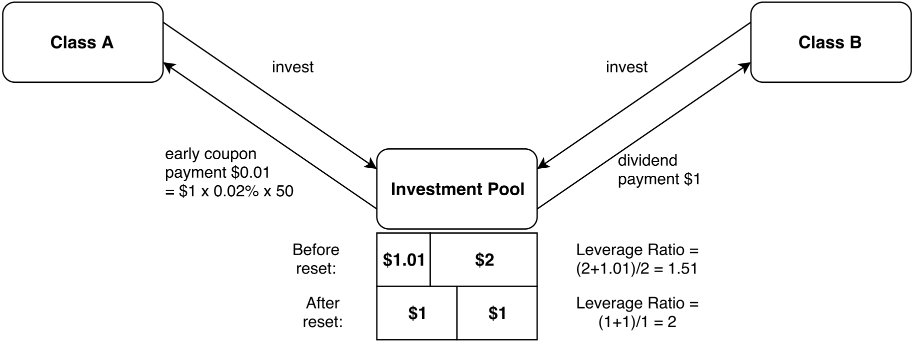


Figure: Upward Reset. After 50 days, the ETH price grows to $760.95, and Class B NAV grows to $2, triggering an upward reset. Class A NAV equals $1.01, where $0.01 is half-year accrued coupon. On this date, Class A receives $0.01 coupon payment, and Class B receives $1 dividend payment. New exchange ratio: 2 shares of ETH now correspond to 760.95 shares of Class A and 760.95 shares of Class B, yielding $\beta = 1$.

Vanilla A Coin, Downward Reset

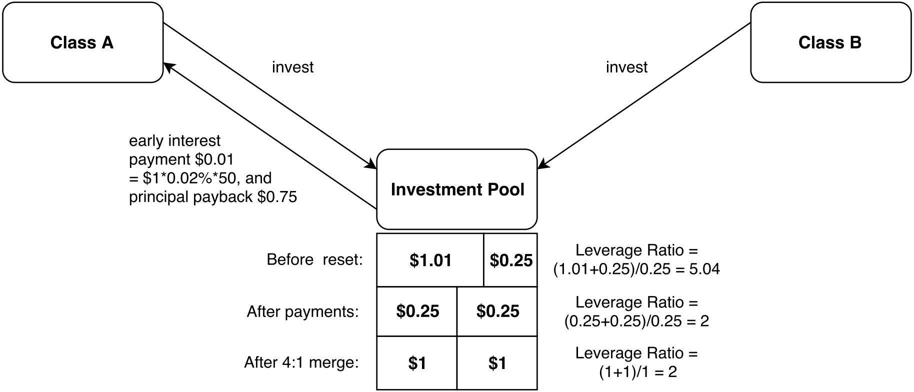


Figure: Downward Reset. After another 50 days, the ETH price drops to \$479.40, and Class B NAV drops to \$0.25, triggering an downward reset. Again, Class A NAV equals \$1.01, where \$0.01 is half-year accrued coupon. On this date, Class A receives \$0.01 coupon payment, as well as \$0.75 principal payback. Then, Class A and B each undergo a 4:1 merger, so that both have NAV equal to \$1. New exchange ratio: 2 shares of ETH now correspond to 479.40 shares of Class A and 479.40 shares of Class B, yielding $\beta = 1$.

Remarks

- No arbitrage implies that

$$W_A^t + W_B^t = 2P_t/(\beta_t P_0),$$

  where P_t is the price of the underlying cryptocurrency, W_A and W_B are the price of the Class A and B coins, and β_t is the conversion ratio, initially equal to 1, and will be changed on regular reset.
- Coin A behaves like a corporate bond.
  - Although Class A has a fixed coupon rate and its coupon payment is periodic and protected by the resets, its value is still volatile on non-coupon dates.
  - The main risk of Class A is not credit risk, but the risk of a downward reset. On a downward reset, a portion of Class A coin will be liquidated, so the investor will lose the value of future coupons that would be generated from this portion.
  - Therefore, a downward reset will reduce the value of Class A.
- We propose another class of stable coins, the class of A' and B' coins.

Overview of A' and B'

- This second extension splits Class A into two sub-classes: Class A' and B'.
- Class A' and B' invest in Class A coin.
- At any time, two Class A coins can be split into one Class A' and one Class B' coin. Conversely, one Class A' and B' coin can be merged into 2 Class A coins.
- Class B' borrow money from Class A' at the rate R' to invest in Class A.
- R' is set to close to the risk-free rate r, where the rate R for Class A is generally much higher.
- Class A' and B' resets *when and only when* Class A resets and Class A gets regular payout.
- A' behaves like cash, except in extreme case, when the underlying asset suddenly jumps (not smooth transit) to close to zero.

A' Coin, Regular Payout

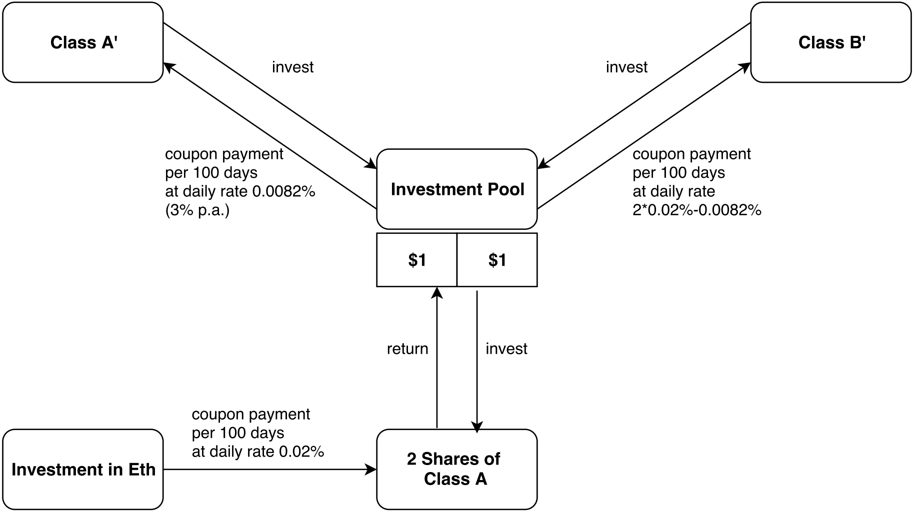


Figure: What happens to A' on a regular payout date of A. On regular payout dates for A (per 100 days), 2 shares of A receives coupon payment $0.04, i.e. at daily rate 0.02%.

A' Coin, Upward Reset

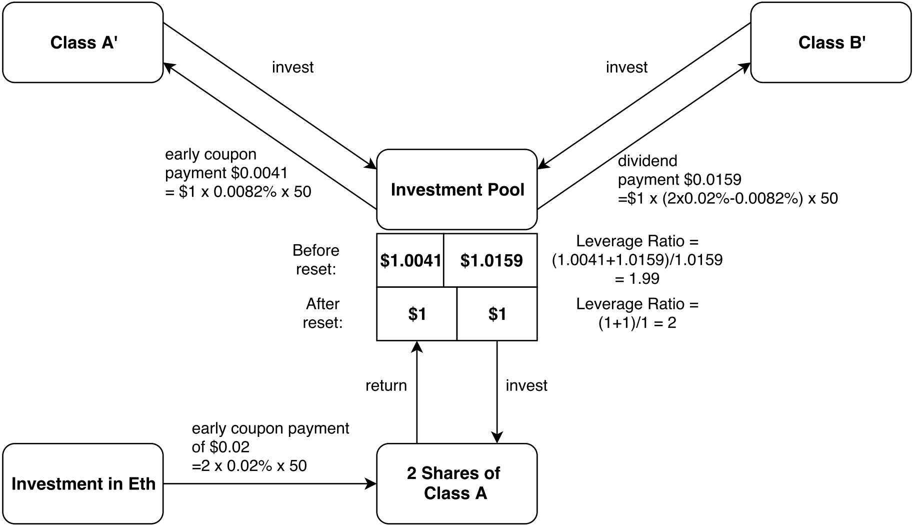


Figure: Upward Reset of Class A′. After 50 days, Class B's NAV grows to $2, triggering an upward reset.

A' Coin, Downward Reset

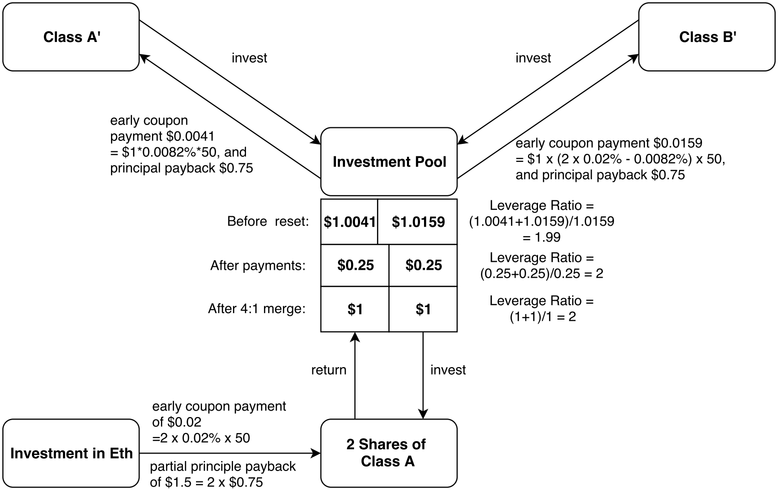


Figure: After another 50 days, Class B NAV drops to $0.25, triggering an downward reset.

A Coin with Subsidy to B, Downward Reset

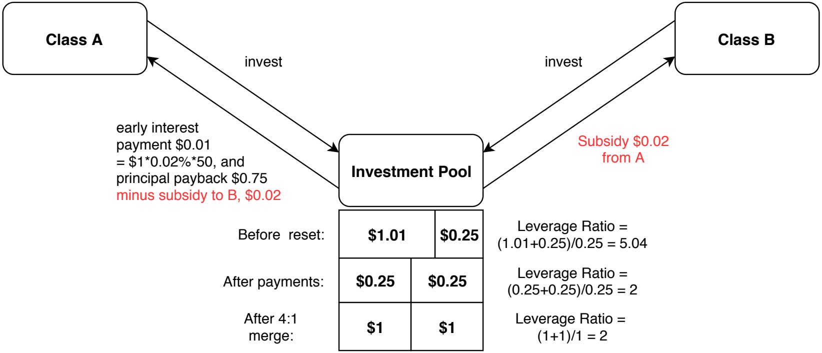


Figure: Downward Reset. After another 50 days, the ETH price drops to $479.40, and Class B NAV drops to $0.25, triggering an downward reset. Again, Class A NAV equals $1.01, where $0.01 is half-year accrued coupon. On this date, Class A receives $0.01 coupon payment, as well as $0.75 principal payback, but pays $0.02 to B shares as subsidy. Then, Class A and B each undergo a 4:1 merger, so that both have NAV equal to $1. New exchange ratio: 2 shares of ETH now correspond to 479.40 shares of Class A and 479.40 shares of Class B, yielding $\beta = 1$.

Outline

Black Swan Events (1)

- During a black swan event when the ETH price jumps down suddenly, Class A$'$ may still take a loss.
- From its stochastic representation, Class A$'$ will take a loss if and only if V_B jumps from above $\mathcal{H}_d$ to below $\frac{R'\eta}{2} - R\eta$. Therefore, A$'$ will only take a loss if ETH has a single-day return of $\frac{1}{2}\frac{R'\eta+1}{R\eta+1+\mathcal{H}_d} - 1$, depending only on contract parameters (not model parameters). Here η is the time from the last dividend payout date to the jump date.
- In our design example, we set

$$\begin{aligned} R &= 0.02\% \text{ (7.3\% p.a.)} & R' &= 0.0082\% \text{ (3\% p.a.)} \\ \mathcal{H}_d &= 0.25 & T &= 100. \end{aligned}$$

- Under the current contract parameters, the above quantity has an upper bound

$$\max_{0\le\eta\le T} \frac{1}{2}\frac{R'\eta+1}{R\eta+1+\mathcal{H}_d} - 1 = -60\%.$$

Black Swan Events (2)

- This means that unless there is a sudden jump of more than 60%, A$'$ will not take a loss.
- In comparison, the historical maximal single-day loss of ETH is recorded at -60% on its second trading day (8 Aug 2015), at which time the market was still not familiar with it. The maximum single-day loss thereafter is only -26.67% recorded on 18 Jun 2016, which is not large enough to trigger a loss in Class A$'$ coin.
- In a real life implement, the ETH price can be monitored at a higher frequency, e.g. hourly. Therefore, Class A$'$ will take a loss only when ETH price sees a downward jump at this magnitude within an hour, which is even less likely.
- For the contract with subsidy from A to B (with $\tilde{R} = 10\%$ p.a.),

$$\max_{0 \leq v \leq T} \frac{1}{2} \frac{R' v_t + 1 + 2\tilde{R} v_t}{R v_t + 1 + \mathcal{H}_d} - 1 = -52.4\%,$$

Outline

Monetary Equilibrium Model with ETH Only

- The total ETH supply is

$$M^S = P^E E, \tag{1}$$

where E is the total stock of ETH, and P^E is the price of ETH denominated by USD.

- The demand of ETH comes from two sources: (1) using ETH as a medium of exchange for purchasing goods or buying service over the Ethereum network; (2) for speculation.
- The demand of ETH as medium of exchange is (c.f. Barro (1979)):

$$M^D_{Exchange} = \frac{PG}{V^E}, \tag{2}$$

where P is the price level of goods and services, G is the size of cryptocurrency economy, and V^E is the velocity of ETH circulation.

- The demand of ETH for speculation depends on its current price level P^E, and its investment attractiveness a, i.e.

$$M^D_{Speculation} = f(P^E, a). \tag{3}$$

Monetary Equilibrium Model with ETH Only

Here, we impose the following conditions on f:

- $f \geq 0$ and is twice differentiable in P^E and differentiable in a;
- For a given P^E, f is increasing in a, since the demand of ETH is higher if it has a greater attractiveness (e.g. from news);
- Also, for a given a, the marginal demand $\frac{\partial f}{\partial P^E}$ is negative, increasing in P^E, and increases to 0 when P^E goes to infinity. This is typical for the demand function: the demand of ETH is lower if its price is higher, since it is too risky for investment. On the other hand, when the price is very high, the marginal decrease in demand with respect to price becomes lower and eventually vanishes. When P^E goes to infinity, the marginal demand vanishes.
- Finally, $f(\cdot, 0) = 0$. That is, if there is no speculative attractiveness of ETH, its speculative demand is zero.
- For illustrate, we take $f(P^E, a) = k \cdot (1 + P^E)^{-\eta_1} \cdot a^{\eta_2}$ with $\eta_1 > 0, \eta_2 \in (0, 1)$.

Monetary Equilibrium with ETH Only

In equilibrium, the supply of ETH M^S equals its demand M^D. Therefore, all of the equations above give

$$f(P^E, a) + \frac{PG}{V^E} = P^E E. \qquad (4)$$

Theorem

Assuming $E, P, G, V^E > 0$, $a \geq 0$, there exists a unique positive equilibrium price of ETH $P^{E,*}$ satisfying (4).

Monetary Equilibrium Model with ETH and A+B

- Assume that one can convert ETH into Class A and Class B with equal quantity.
- In the total real crypto-economy PG (for ETH and A), $\gamma_1\gamma_2 PG$ goes to A and the remaining goes to ETH.
- $\gamma_1\gamma_2 < 1$, since ETH is always needed for payment of services on the Ethereum network.

Monetary Equilibrium Model with ETH and A+B

- Denote q as the proportion of ETH converted to stable coins, so that the total ETH supply is reduced to

$$M^S = P^E E(1-q). \tag{5}$$

- Denote the speculator's demand of ETH and B as $f(P^E, a)$, and they want to invest γ_2 fraction in B and remaining in ETH. Therefore, the demand of B

$$D^B = \gamma_2 f(P^E, a), \tag{6}$$

  Since one unit of ETH can be converted into B of USD value $0.5P^E$, the demanded stock of ETH that needs to converted is $\frac{D^B}{0.5P^E}$.
- Let

$$q = g\left(\frac{D^B}{0.5P^E}\right), \tag{7}$$

  where (i) g is concave and increasing, so that the proportion of ETH converted, q, increases with the demands stock of ETH to be converted (ii) $g(0) = 0$ and $g'(0) = 1/E$, meaning that no stable coins will be created if there is no demand for it, and the demand of B will be completely met initially when there is no existing B and (iii) $g(+\infty) = 1$, indicating that the fraction of ETH converted to stable coins is limited by 1. To illustrate, we assume that $g(x) = \frac{x}{1+x}$.

Monetary Equilibrium with ETH and A+B

- The total supply of A is

$$0.5P^E E \cdot g\left(\frac{D^B}{0.5P^E}\right),$$

  within which $\gamma_1\gamma_2\frac{PG}{V^S}$ is for the real demand on A as medium of exchange.
- γ_1 is typically very small (less than 5%).

Monetary Equilibrium with ETH and A+B

The following table describes the proportion of these types of demand.

Table: Demand Proportion of the Whole Market

	Real Economy Demand	Speculative Demand
ETH	$(1-\gamma_1\gamma_2)\frac{PG}{V^E}$	$(1-\gamma_2)f(P^E,a)+\gamma_1\gamma_2\frac{PG}{V^E}-0.5P^E Eg\left(\frac{\gamma_2 f(P^E,a)}{0.5P^E}\right)$
A	$\gamma_1\gamma_2\frac{PG}{V^E}$	$0.5P^E Eg\left(\frac{\gamma_2 f(P^E,a)}{0.5P^E}\right)-\gamma_1\gamma_2\frac{PG}{V^E}$
B	0	$\gamma_2 f(P^E,a)$
Total	$\frac{PG}{V^E}$	$f(P^E,a)$

After the introduction of A+B, the total real economy demand and speculative demand are reallocated, but the total demand within each type remains unchanged. The total demands of A and B are equal. Within the real economy demand, $\gamma_1\gamma_2\frac{PG}{V^E}$ belongs to A, where $\gamma_1 > 0$ is small. Within the total speculative demand, γ_2 fraction goes to B. Speculative demand of A refers to the demand for asset allocation purpose.

Monetary Equilibrium with ETH and A+B

In equilibrium, the demand and supply of ETH are equal, meaning

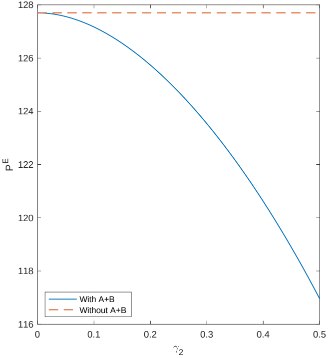
$$(1-\gamma_2)f(P^E, a) + \frac{PG}{V^E} = P^E E\left(1 - 0.5g\left(\frac{\gamma_2 f(P^E, a)}{0.5P^E}\right)\right). \qquad (8)$$

Theorem

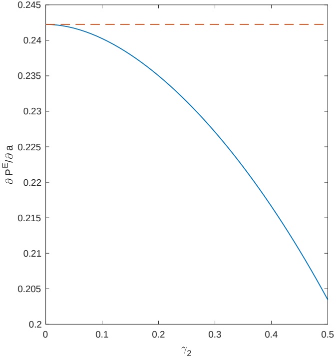
Assuming $E, P, G, V^E > 0$, $a \geq 0$, $0 \leq \gamma_1 \leq 1$, $0 \leq \gamma_2 \leq 0.5$, there exists a unique positive equilibrium price of ETH, $P^{E,*}$ satisfying (8).

Numerical Illustration

Equilibrium ETH Price

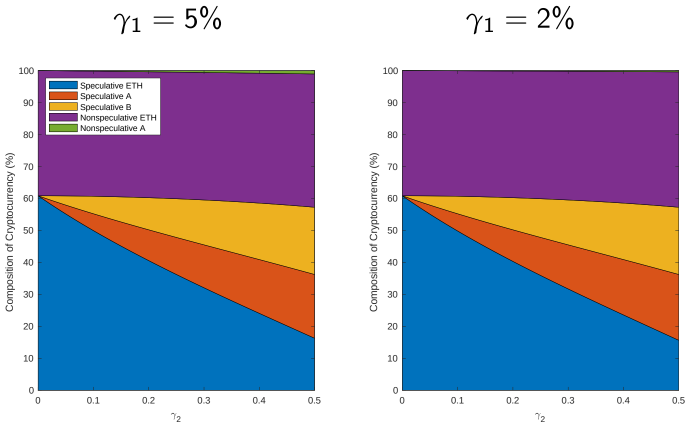


Sensitivity of ETH Price to Attractiveness



Parameters: $V^E = 0.2, P = G = 1, a = k = 100, \eta_1 = 1, \eta_2 = 0.5, E = 0.1, \gamma_1 = 0.05, \gamma_2 = 0.5$.
γ_2 is the fraction of speculators' demand in B. $\gamma_1\gamma_2$ is the fraction of crypto-economy using A.

Numerical Illustration



Parameters: $V^E = 0.2, P = G = 1, a = k = 100, \eta_1 = 1, \eta_2 = 0.5, E = 0.1, \gamma_2 = 0.5.$, $\gamma_1 = 0.05$ or 0.02. γ_2 is the fraction of speculators' demand in B. $\gamma_1\gamma_2$ is the fraction of crypto-economy using A.

Comparison with DAI token

- Our stable coin does not require over-collateralization, as opposed to DAI token that requires a relatively large collateral size. Upon creation, the holder gets Class A and B coins (resp. A' and B' coins), and the B coins (resp. B' coins) can then be sold on the exchange.
- The split design of our stable coins opens up the possibility of tranching. One can create even more stable coins by further splitting Class A' coins, just as the split of Class A coins does.
- Class A' resembles a money market account, and has a stable future cash flow, which has a present value close to 1. In contrast, DAI token is essentially an instrument pegged to USD and does not have a cash flow. The stability of DAI token comes from its automatic liquidation when the collateralization ratio drops below a certain threshold, and the global settlement guaranteeing 1$ for each DAI token.

Outline

The Vanilla A and B Coins, Overview

- Let the downward reset boundary be $H_d(t) = \frac{1}{2}(1 + Rt) + \frac{1}{2}\mathcal{H}_d$, and upward reset boundary $H_u(t) = \frac{1}{2}(1 + Rt) + \frac{1}{2}\mathcal{H}_u$.
- We have both stochastic representation and PDE for vanilla A and B coins.

The Vanilla A and B Coins, Stochastic Representation

- $W_A(t,S)$ denotes the market value of Class A coin with time from last interest payment $0 \le t \le T$, with $S_t = P_t/(\beta_t P_0)$, $H_d(t) \le S_t \le H_u(t)$.
- $W_A(t,S)$ is given recursively by

$$E_t\Big[e^{-r(T-t)}[RT + W_A(0, S_T - RT/2)] \cdot \mathbf{1}_{\{T<\tau\wedge\eta\}}$$
$$+ e^{-r(\tau-t)}[R\tau + W_A(0,1)] \cdot \mathbf{1}_{\{\tau\le T\wedge\eta\}}$$
$$+ e^{-r(\eta-t)}[R\eta + 1 - |V_B^\eta| + (V_B^\eta)^+ W_A(0,1)] \cdot \mathbf{1}_{\{\eta\le T\wedge\tau\}}\Big],$$

  where E_t is the expectation computed under the risk-neutral measure and under the initial condition $S_{t-} = S$,
- The random times ζ, τ and η represent the first regular payout, upward and downward reset date from t, respectively.
- Once we calculate W_A, the value of Class B coin can be calculated as $W_B = 2S - W_A$.

The Vanilla A and B Coins, PDE

We assume that P_t follows a geometric Brownian motion under the risk neutral measure:

$$dP_t = rP_t dt + \sigma P_t dW_t,$$

where W_t is a one-dimensional standard Brownian motion.
$W_A(t, S)$ is the unique solution of the following periodic PDE with nonlocal boundary and nonlocal terminal conditions,

$$\begin{aligned}
-\frac{\partial W_A}{\partial t} &= \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 W_A}{\partial S^2} + rS\frac{\partial W_A}{\partial S} - rW_A, \\
& \quad t \in [0, T),\ S \in (H_d(t), H_u(t)) \\
W_A(T, S) &= RT + W_A(0, S - \frac{1}{2}RT), \\
W_A(t, H_u(t)) &= Rt + W_A(0, 1) \\
W_A(t, H_d(t)) &= Rt + 1 - \mathcal{H}_d + \mathcal{H}_d W_A(0, 1).
\end{aligned}$$

The A' and B' Coins, Stochastic Representation

Under the risk-neutral pricing framework, the value $W_{A'}(t,S)$ of Class A' is given as

$$
\begin{aligned}
E_t \Bigg[& e^{-r(T-t)}(R'T + W_{A'}\ (0, S_T - RT/2)) \cdot \mathbf{1}_{\{T<\tau\wedge\eta\}} \\
& + e^{-r(\tau-t)}(R'\tau + W_{A'}(0,1)) \cdot \mathbf{1}_{\{\tau\le T\wedge\eta\}} \\
& + e^{-r(\eta-t)}\left(\min\{R'\eta + 1 - (V_B^{\eta})^+, 2(R\eta + 1 + V_B^{\eta})^+\} + (V_B^{\eta})^+ W_{A'}(0,1)\right) \mathbf{1}_{\{\eta\le T\wedge\tau\}} \Bigg],
\end{aligned}
$$

where τ and η are the first upward and downward reset of Class A (or equivalently, Class A' and B') after t, respectively. On downward reset, if $V_B^{\eta-} > 0$, A receives coupon $R'\eta$, $1 - V_B^{\eta-}$ shares of A is liquidated, and A receives the liquidation value; if $\frac{R'\eta - 1}{2} - R\eta \le V_B^{\eta-} \le 0$, A is fully liquidated, and still receives full NAV; otherwise, A is fully liquidated and takes a loss by receiving $2(1 + R\eta + V_B^{\eta-})^+$ which is smaller than the NAV $1 + R\eta$.

We assume that P_t follows a geometric Brownian motion.
$W_{A'}$ satisfies the following PDE, which is the same as that for W_A, except changing R to R' for the coupon payment:

$$-\frac{\partial W_{A'}}{\partial t} = \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 W_{A'}}{\partial S^2} + rS\frac{\partial W_{A'}}{\partial S} - rW_{A'},$$
$$t \in [0, T), \; S \in (H_d(t), H_u(t))$$
$$W_{A'}(T, S) = R'T + W_{A'}(0, S - \frac{1}{2}RT),$$
$$W_{A'}(t, H_u(t)) = R't + W_{A'}(0, 1)$$
$$W_{A'}(t, H_d(t)) = R't + 1 - \mathcal{H}_d + \mathcal{H}_d W_{A'}(0, 1).$$

A and B Coins with Subsidy, Stochastic Representation

- $W_A(t, S)$ denotes the market value of Class A coin with time from last interest payment $0 \leq t \leq T$, with $S_t = P_t/(\beta_t P_0)$, $H_d(t) \leq S_t \leq H_u(t)$.
- $W_A(t, S)$ is given recursively by

$$
\begin{aligned}
E_t \Bigg[& e^{-r(T-t)}[RT + \tilde{W}_A(0, S_T - RT/2)] \cdot \mathbf{1}_{\{T<\tau\wedge\eta\}} \\
& + e^{-r(\tau-t)}[R\tau + \tilde{W}_A(0,1)] \cdot \mathbf{1}_{\{\tau\leq T\wedge\eta\}} \\
& + e^{-r(\eta-t)}[(R\eta - \tilde{R}\eta + 1 - |V_B^{\eta}|)^+ + (V_B^{\eta})^+ \tilde{W}_A(0,1)] \cdot \mathbf{1}_{\{\eta\leq T\wedge\tau\}} \Bigg],
\end{aligned}
$$

- Once we calculate W_A, the value of Class B coin can be calculated as $W_B = 2S - W_A$.

A and B Coins with Subsidy, PDE

We assume that P_t follows a geometric Brownian motion under the risk neutral measure:

$$dP_t = rP_t dt + \sigma P_t dW_t,$$

where W_t is a one-dimensional standard Brownian motion.
$W_A(t, S)$ is the unique solution of the following periodic PDE with nonlocal boundary and nonlocal terminal conditions,

$$-\frac{\partial W_A}{\partial t} = \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 W_A}{\partial S^2} + rS\frac{\partial W_A}{\partial S} - rW_A,$$
$$t \in [0, T), \; S \in (H_d(t), H_u(t))$$
$$W_A(T, S) = RT + W_A(0, S - \frac{1}{2}RT),$$
$$W_A(t, H_u(t)) = Rt + W_A(0, 1)$$
$$W_A(t, H_d(t)) = (R - \tilde{R})t + 1 - \mathcal{H}_d + \mathcal{H}_d W_A(0, 1).$$

A' and B' Coins with Subsidy, Stochastic Representation

Under the risk-neutral pricing framework, the value $W_{A'}(t,S)$ of Class A' is given as

$$E_t\Bigg[e^{-r(T-t)}(R'T + W_{A'}\ (0, S_T - RT/2)) \cdot \mathbf{1}_{\{T<\tau\wedge\eta\}}$$
$$+ e^{-r(\tau-t)}(R'\tau + W_{A'}(0,1)) \cdot \mathbf{1}_{\{\tau\le T\wedge\eta\}}$$
$$+ e^{-r(\eta-t)}\left(\min\{R'\eta + 1 - (V_B^\eta)^+, 2(R\eta - \tilde{R}\eta + 1 + V_B^\eta)^+\} + (V_B^\eta)^+ W_{A'}(0,1)\right)\mathbf{1}_{\{\eta\le T\wedge\tau\}}\Bigg],$$

where τ and η are the first upward and downward reset of Class A (or equivalently, Class A' and B') after t, respectively. On downward reset, if $V_B^{\eta-} > 0$, A receives coupon $R'\eta$, $1 - V_B^{\eta-}$ shares of A is liquidated, and A receives the liquidation value; if $\frac{R'\eta-1}{2} - R\eta + \tilde{R}\eta \le V_B^{\eta-} \le 0$, A is fully liquidated, and still receives full NAV; otherwise, A is fully liquidated and takes a loss by receiving $2(1 + R\eta - \tilde{R}\eta + V_B^{\eta-})^+$ which is smaller than the NAV $1 + R\eta$.

Outline

The Setting

- For illustration, we hereby uses Ether (ETH) as the underlying coin and apply below parameter values:

$$
\begin{aligned}
R &= 0.02\% \text{ (7.3\% p.a.)} & R' &= 0.0082\% \text{ (3\% p.a.)} \\
\mathcal{H}_u &= 2 & \mathcal{H}_d &= 0.25 \\
r &= 0.0082\% \text{ (3\% p.a.)} & \tilde{R} &= 0.027\% \text{ (10\% p.a.)} \\
\sigma &= 0.0882 \text{ (169\% p.a.)} & T &= 100
\end{aligned}
$$

- Parameter estimation period: 7 Aug 2015 – 30 Sept 2017. Pricing simulation period: 1 Oct 2017 – 31 Aug 2018.
- The following assumptions are used:
  1. Price is monitored on daily basis
  2. Resets are performed according to end-of-day prices
  3. Reinvestment of ETH payout from resets is not considered

Class A, Simulated Market Values

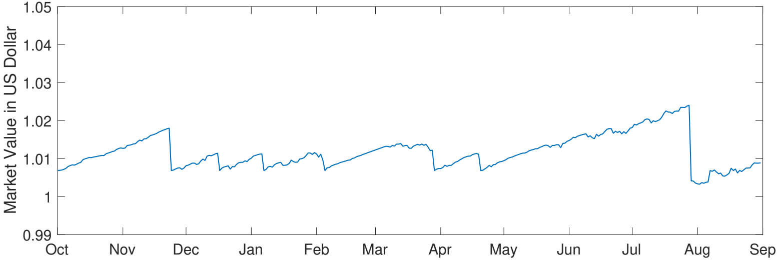


Figure: Simulated Class A Market Value. Estimated parameter $\sigma = 169\%$ p.a. Volatility of Class A market value: 2.69%.

Upward reset takes place on 24 Nov 2017, 17 Dec 2017, 7 Jan 2018, and 20 Apr 2018. Downward reset takes place on 5 Feb 2018, 29 Mar 2018 and 07 Aug 2018. Regular payout takes place on 28 Jul 2018.

Class B, Simulated Market Values

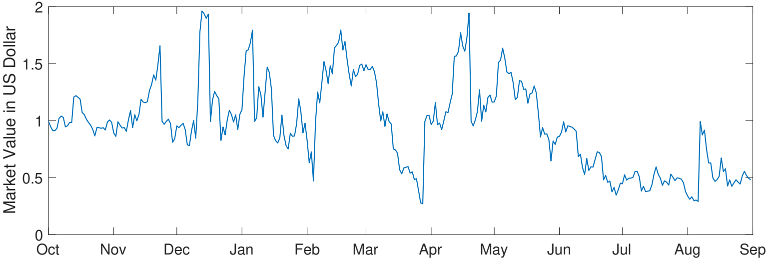


Figure: Simulated Class B Market Value. Estimated parameter $\sigma = 169\%$ p.a. Volatility of Class B market value: 327.22%.

Upward reset takes place on 24 Nov 2017, 17 Dec 2017, 7 Jan 2018, and 20 Apr 2018 with dividend payments \$1.0846, \$1.0467, \$1.1106 and \$1.2108. Downward reset takes place on 5 Feb 2018, 29 Mar 2018 and 07 Aug 2018. Regular payout takes place on 28 Jul 2018.

Class A', Simulated Market Values

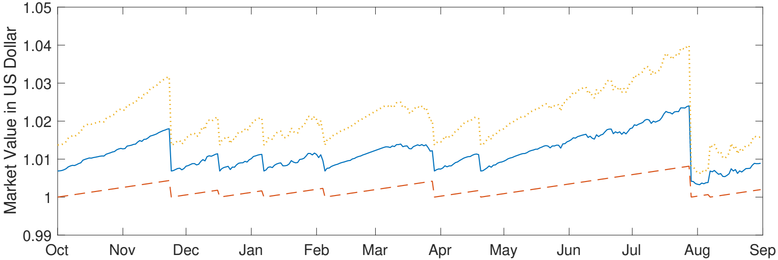


Figure: Market Value of Class A$'$ (red) and B$'$ (yellow), compared with Class A (blue).

Annualized market value volatility of Class A$'$ and B$'$ are 1.14% and 4.30%, respectively.

Upward reset takes place on 24 Nov 2017, 17 Dec 2017, 7 Jan 2018, and 20 Apr 2018. Downward reset takes place on 5 Feb 2018, 29 Mar 2018 and 07 Aug 2018. Regular payout takes place on 28 Jul 2018.

Class A', Simulated Market Values

- The 8 downward jumps in the red curve correspond to the coupon payment of Class A' on 3 downward, 4 upward reset dates and 1 regular payout date of Class A.
- If we consider the total return from holding Class A' (by assuming that all coupons are reinvested into Class A'), it has a volatility of 0.0025%, which is much smaller than that of Class A (0.76%).

Jump Risk

- To reflect the sudden changes in the ETH price, We extend the underlying model from geometric Brownian motion to include jumps.
- We model the jumps using Jump Diffusion (c.f. Liu, Longstaff and Pan, JF 2003), interpreted as the price jumps triggered by event risk. Specifically, the risk-neutral dynamics is,

$$dP_t/P_{t-} = (r - \lambda(EU - 1))dt + \sigma dW_t + d\sum_{\tau_j \le t}(U_j - 1).$$

- The jump sizes $(U_j)_{j\geq 1}$ are i.i.d. and modeled by a Double Exponential distribution, which features both a high peak and heavy tails, reflecting underreaction and overreaction to outside news, respectively. Specifically,

$$f_U(x) = p\eta_1 e^{-\eta_1 x}\mathbf{1}_{\{x\geq 0\}} + (1-p)\eta_2 e^{\eta_2 x}\mathbf{1}_{\{x<0\}}.$$

Parameter Estimation

- We follow the two-step procedure in Dang and Forsyth (EJOR 2016):
  1. jointly identify jumps and estimate volatility, where jumps are defined as the returns with absolute value greater than k times the volatility;
  2. fit the jump returns and the remaining non-jump returns to the double exponential distribution and normal distribution, respectively.
- Parameter estimation

k	p	η_1	η_2	λ	σ
2.5	0.6555	6.0134	4.9574	0.1516	0.0435
3.0	0.5333	3.8657	3.3078	0.0382	0.0639
3.5	0.4118	3.1776	2.9385	0.0217	0.0691

Stability of Class A and A'

- Under different k, the following table reports the out-of-the-sample annualized volatility of Class A and A$'$.

	Nominal Value		Total Return	
k	A	A$'$	A	A$'$
2.5	4.93%	1.14%	6.22%	0.0025%
3.0	10.76%	1.14%	12.73%	0.0026%
3.5	15.67%	1.14%	13.91%	0.0696%
No Jump	2.69%	1.14%	0.75%	0.0025%

Volatility of the nominal value of Class A and A$'$, as well as the total return of holding Class A and A$'$. Time period: 1 Oct 2017 – 31 Aug 2018.

- As a comparison, during the same period
  - volatility of S&P U.S. AAA Investment Grade Corporate Bond Index: 4.05%.
  - volatility of S&P U.S. Treasury Bill 0–3 Month Index: 0.0752%.

Stability of Class A and A' with Subsidy

- Under different k, the following table reports the out-of-the-sample annualized volatility of Class A and A$'$ with subsidy.

	Nominal Value		Total Return	
k	A	A$'$	A	A$'$
2.5	5.18%	1.14%	6.55%	0.0025%
3.0	10.73%	1.14%	13.12%	0.0026%
3.5	15.07%	1.13%	14.04%	0.1161%
No Jump	2.65%	1.14%	1.34%	0.0025%

Volatility of the nominal value of Class A and A$'$ with subsidy, as well as the total return of holding Class A and A$'$. Time period: 1 Oct 2017 – 31 Aug 2018.

- As a comparison, during the same period
  - volatility of S&P U.S. AAA Investment Grade Corporate Bond Index: 4.05%.
  - volatility of S&P U.S. Treasury Bill 0–3 Month Index: 0.0752%.

Outline

Conclusion

- Within the ecology system of Blockchains, stable coins are needed to settle transactions, to pay miners, and to do asset allocation.
- We design a class of stable coins, A', based on vanilla A, which are inspired by the dual purpose funds in China and U.S.
- These stable coins can be used as a basis for sovereign stable coins, if a government can provide the insurance in extreme events.

Thank you.