The Economics of FinTech

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Chapter 5

Additional Notes on Stable Coins

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5.1 A Model-Free Bound for Black Swan Events

To describe the cash flow of Class A and B coins, we introduce the net asset dollar values of Class A and B coins, V_A and V_B. Thanks to the exchange between ETH and Class A/B coins, the following parity relation holds at any time:

$$V_A^t + V_B^t = \frac{2P_t}{\beta_t P_0}, \tag{5.1}$$

where P_t is the prevailing price of underlying ETH in USD. The net asset value of Class A coins at time t is defined as

$$V_A^t = 1 + R \cdot v_t, \tag{5.2}$$

where R is the *daily* coupon rate, and v_t is the number of days from the inception, last reset, or last regular coupon payout date, whichever is most recent. The above design ensures that the initial net asset values of Class A and B coins are both equal to one dollar.

We can split Class A into two sub-classes: Class A′ and B′ . Both classes invest in Class A coins. At any time, two Class A coins can be split into one Class A′ coin and one Class B′ coin. Conversely, one Class A′ coin and one B′ coin can be merged into two Class A coins. The split structure for Class A′ and B′ resembles that for Class A and B: Class B′ borrows money from

Class A$'$ at the rate R' to invest in Class A. Here R' is set to be close to the risk-free rate r, whereas the rate R for Class A is generally much higher.

Class A$'$ and B$'$ resets *when and only when* Class A resets or gets regular payout. Class A$'$ gets coupon at the rate R' on regular payouts, upward and downward reset (provided that the net asset value of Class B is positive then), and Class B$'$ gets coupon at the rate $2R - R'$ on upward reset. On downward resets, each share of both Class A$'$ and B$'$ is reduced to $(V_B^t)^+$ share, and Class A$'$ gets the value of the liquidated shares (i.e., $1 - (V_B^t)^+$ shares). In the extreme case where $V_B^t \leq 0$, then both Class A$'$ and B$'$ are fully liquidated, and A$'$ receives its full net asset value $1 + R'v_t$, or the remaining total asset for A$'$ and B$'$, $2(1 + Rv_t - |V_B^t|)$, whichever is smaller.

As we will see later, Class A$'$ behaves like a money market account, except in the extreme case in which the underlying asset suddenly suffers huge losses in a very short period (e.g. a sudden 60% or more drop within a day or within a hour). Here is an intuition on this. The stability of Class A$'$ coin price comes from its stable future cash-flow, as Class A, B and B$'$ coins act like buffers for Class A$'$. Indeed, the volatility of ETH spreads to Class A$'$ coin only via the timing of coupons (on regular payment or upward resets) or the partial payback on principal (on downward reset dates). Since the accrued amount of coupon grows linearly with respect to time, and the coupon is expected to be paid at least once per 100 days, the timing of the coupon payment does not cause a large volatility on Class A$'$ nominal price; the volatility of the total return (taking into account the coupon) is even smaller. By setting the coupon rate to be close to the risk-free rate, the value of future coupons is close to 1. Therefore, on a downward reset date, the partial payback of principal (at book value) also does not cause large impact to the cash flow.

Despite the protection provided by the downward reset mechanism, when there is a sudden drop in ETH value Class A and A$'$ still can take a loss, i.e., losing part of the accrued coupon or even the principal. Here we give a model-free bound on how large the jump in ETH should be to trigger such a loss.

We take Class A$'$ as an example. According to the contract design, Class A$'$ takes a loss if and only if the net asset value of Class B becomes negative, i.e. $V_B^t \leq 0$, and the payment to Class A$'$ (i.e. the net asset value of 2 shares of Class A) is lower than the net asset value of Class A$'$, i.e. $2(1+Rv_t-|V_B^t|) < 1+R'v_t$, on a downward reset time t.

These two conditions are equivalent to $V_B^t \leq 0$ and $V_B^t < \frac{R'v_t-1}{2} - Rv_t$. In other words, assuming $(R' - 2R)T - 1 \leq 0$, Class A$'$ takes a loss if and only if V_B *jumps* from $\mathcal{H}_d$ or above to a negative level below $\frac{R'v_t-1}{2} - Rv_t$ (if V_B drops downward without a jump, a downward reset will be triggered instead and Class A$'$ will not take a loss).

Exercise 1. Assume $(R' - 2R)T - 1 \leq 0$ and if the jump over $\mathcal{H}_d$ occurs Class A shares get no additional coupon payment.

(1) Compute the value of ETH when $V_B = \mathcal{H}_d$ and when $V_B = \frac{R'v_t-1}{2} - Rv_t$.

(2) Show that to incur a loss in Class A$'$ coins, the ETH price must have a

single-day (negative) return lower than

$$\frac{1}{2}\frac{R'v_t+1}{Rv_t+1+\mathcal{H}_d}-1.$$

In our design example, we set $R = 0.02\%$ (7.3% p.a.), $R' = 0.0082\%$ (3% p.a.), $\mathcal{H}_u = 2$, $\mathcal{H}_d = 0.25$, $r = 0.0082\%$ (3% p.a.), $T = 100$. With these contract parameters, we have

$$\max_{0\le v\le T}\frac{1}{2}\frac{R'v+1}{Rv+1+\mathcal{H}_d}-1=-60\%,$$

meaning that unless there is a sudden downward jump of more than -60% between the monitoring time points, Class A′ will not take a loss. In comparison, the historical maximal single-day loss of ETH is recorded at -60% on its second trading day (8 Aug 2015), at which time the market was yet to be familiar with it. The maximum single-day loss thereafter is only -26.67% recorded on 18 Jun 2016 when the DAO hacking occurred, which is not large enough to trigger a loss in Class A′ coin. In a practical implementation, the ETH price can be monitored at a higher frequency, e.g. hourly. Therefore, Class A′ would take a loss only when ETH price jumps downward at this magnitude or more within an hour, which is even less likely.

In comparison, the DAI token starts to take loss when its total collateral value suddenly drops from 150% of the DAI value to below 100%, i.e. with a -33% downward jump. Therefore, Class A′ coin may withstand a larger sudden downward jump in ETH than the DAI token does. By further splitting Class A′ to get A″, the resulting new stable coin A″ can withstand even larger downward jumps before taking any loss.

5.2 A Contract Design with the Subsidy from Class A Coins to Class B Coins

One might worry about the demand of Class B coins, as they are leveraged products. Of course, if the demand of Class B coins is low, then the price of Class A coins may be overpriced. If one wants to provide extra incentive for investors to hold Class B coins, one can modify the contract so that Class B coin holders receive coupon payment from Class A coins, as described below.

The modified contract for Class A, B, A′ and B′ differs from the one introduced previously only in the payoff on downward resets. Specifically, on a downward reset time t, if $V_B^t \ge 0$, Class A holders receive a payment with dollar value $V_A^t - (V_B^t)^+ - (V_A^t - 1)\frac{\tilde{R}}{R}$,[1] Class B holders receive a payment with dollar value $(V_A^t - 1)\frac{\tilde{R}}{R}$, and each share of Class A and B is reduced to $(V_B^t)^+$ share

[1]This equals $V_A^t - (V_B^t)^+ - \tilde{R}v_t$ (recall that v_t is the time from the inception, last reset or regular coupon payout. We require $0 \le \tilde{R} \le \frac{1-\mathcal{H}_d}{2T} + R - \frac{R'}{2}$ to ensure that Class A, B, A′ and B′ all receive nonnegative payments in normal stances where $V_B \ge 0$. Under the contract parameter given before, this means $0 \le \tilde{R} \le 0.387\%$ (142.67% p.a.).

of Class A and B, respectively. Class A′ holders still receive accrued coupon payment $R'v_t$ plus the value $1-(V_B^t)^+$ of the liquidated shares. Class B′ holders receive coupon payment $(2R-R')v_t$ plus the value $1-(V_B^t)^+$, *minus* $2(V_A^t-1)\frac{\tilde{R}}{R}$; and then each share of both Class A′ and B′ is reduced to $(V_B)^+$ share.

In extreme events the net asset value of Class B can jump to a negative value, that is $V_B^t \leq 0$. In this case, all four shares are fully liquidated. Class A receives a payment with dollar value $(V_A^t - |V_B^t| - (V_A^t-1)\frac{\tilde{R}}{R})^+$, and Class B receives $\min\{(V_A^t-1)\frac{\tilde{R}}{R}, V_A^t - |V_B^t|\}$. That is, Class B receives the full subsidy if Class A can still cover it; otherwise, Class B receives the remaining net asset value of Class A, if any. Class A′ receives its full asset value $1+R'v_t$, or the remaining total asset for A′ and B′ , $2(V_A^t - |V_B^t| - (V_A^t-1)\frac{\tilde{R}}{R})^+$, whichever is smaller. Finally, Class B′ receives the remaining total asset value for A′ and B′ minus the payment to A′ .

In this case we can also get a model-free bound on how large the jump in ETH should be to trigger such a loss. Class A′ will take a loss when and only when $V_B^t \leq 0$ and $2(V_A^t - |V_B^t| - (V_A^t-1)\frac{\tilde{R}}{R})^+ < 1+R'v_t$, i.e. the payment to Class A′ is lower than the net asset value of Class A′ , on a downward reset date.

Exercise 2. Use a similar argument as in Exercise 1 to show that, assuming $(R'-2R+2\tilde{R})T-1 \leq 0$ and when the jump over $\mathcal{H}_d$ occurs Class A shares get no additional coupon payment, A′ will loss money only when the ETH price has a sudden (negative) return lower than

$$\frac{1}{2}\frac{R'v_t+1+2\tilde{R}v_t}{Rv_t+1+\mathcal{H}_d}-1.$$

Using the contract parameter as before as well as $\tilde{R}=0.0274\%$ (10% p.a.), we have

$$\max_{0\leq v\leq T}\frac{1}{2}\frac{R'v_t+1+2\tilde{R}v_t}{Rv_t+1+\mathcal{H}_d}-1=-52.4\%,$$

meaning that unless there is a sudden downward jump of more than −52.4%, Class A′ will not take a loss.

5.3 Differences between Our Design and the Design of the Dual-Purpose Fund

There are four main differences between our stable coin with dual-purpose funds in China. First, in China a dual-purpose fund and its underlying fund share the same fund managers, hence the fund managers re-scale the value of the underlying fund upon upward and downward resets and regular payouts, in order to easily ensure the no arbitrage parity relation between the dual-purpose fund and the underlying fund. Since we cannot change the underlying ETH price, we instead change the exchange ratio of the shares between the underlying ETH and Class A and B coins in our case, to maintain no-arbitrage across upward and downward resets and regular payouts.

Second, for the dual-purpose funds, the upward reset is triggered by the underlying up-crossing $\mathcal{H}_u$ while the downward reset is triggered by the net asset value of B share down-crossing $\mathcal{H}_d$. In contrast, for our stable coin, the triggering conditions of both upward and downward resets are all based on the net asset value of Class B coins. This is because unlike the re-scaled underlying fund value in China, the underlying ETH price is not so appropriate as the net asset value of Class B to measure the leverage ratio of Class B.

Third, the underlying funds of Chinese dual-purpose funds incur management fees, whereas the underlying ETH does not. Finally, the periodic payout of dual-purpose fund is annually at a fixed date (e.g. first trading day of each year), while the periodic payout of our Class A coins happens when a pre-specified time has passed from the last reset or payout event, which reduces the frequency of payouts, making the coins more stable.

5.4 Differences between Our Design and the Design of the DAI token

An issue with real asset backed stable coins is: how to verify the deposit account in real time? In other words, instead of asking an accountant to check the value in deposit account every few months, can an investor see the deposit value any time in a timely fashion? There are two ways to solve this issue. One is an issuance backed by over-collateralized cryptocurrencies with automatic exogenous liquidation, such as in the DAI token. In this case, one can see the market value of the deposit account, which is in ETH, instantaneously as one wishes. The second one is what we propose here to create stable coins by using tranches with carefully designed resets.

The main difference between the DAI design and our design is that the DAI design needs a relatively large collateral size, while our design does not require this. More precisely, to generate \$100 worth of stable coins the DAI design needs to deposit \$150 worth of ETH. For our design, the firm mainly serves to do the conversion, with all the A, B, A′ and B′ tokens traded outside the firm and in exchanges. In particular, the main function of the firm is to give a person A and B (A′ and B′ tokens) tokens if the person brings the ETH (Class A token) to convert. This way the capital requirement of our design may be much less, compared to that of the DAI token.

There are also two technical differences. First, they are different financial instruments. By design, the DAI token is an instrument pegged to US dollar. It maintains its level by the promise of one dollar during the settlement, and an auto-liquidation mechanism when the collateral-to-debt ratio drops below certain threshold (150% at present). In contrast, Class A and A′ coins are more like a bond and a money market account, respectively. The stability of their values comes from the stability of their future cash flows. The volatility of the underlying ETH spreads to Class A and A′ coin via the timing of coupons or the partial payback on principal (on downward resets); previous numerical examples demonstrate that their volatility is indeed small, especially for Class A$'$ coin which is almost pegged to US dollar. Secondly, compared with the DAI token, our design has an additional feature of tranching with carefully chosen reset barriers to reduce the risk and to enhance stability. Since Class A$'$ coins are backed by both Class B and B$'$ coins, they are likely more stable and more robust to Black Swan events than Class A coins, as will be demonstrated by the previous numerical examples. Based on the same idea, one can further split Class A$'$ to get an even stabler type of coins, so on and so forth, if one wishes.

It should be noted that the tranche structure in our design is not standard; for example, when the value of Class B tranche falls below the lower threshold, triggering the downward reset, the remaining tranches are restructured and partially payback. This adds an additional layer of protection.