Peer-to-Peer Equity Financing: Contract Design

Abstract

In peer-to-peer equity financing entrepreneurs can raise capital privately (e.g. hedge fund financing) or publicly (e.g. via initial coin offerings) without traditional financial intermediaries. To overcome the information asymmetry between entrepreneurs and funders, we propose an incentive contract that can attract some entrepreneurs who are attractive from the funders' perspective and deter all entrepreneurs who are unappealing to the funders. In contrast to standard screening contracts, our contract neither depends explicitly on the utilities of the principal and agent nor has a menu of choices. Two case studies are given, one for hedge funds and one for initial coin offerings.

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Keywords: P2P financing, contract design, performance fees, first-loss, liquidation boundary

1 Introduction

The traditional approach for entrepreneurs to raise money from capital markets is to issue equity and/or debt via financial intermediaries, typically investment banks which do due diligence and take on the risk of fund raising by conducting the underwriting. This standard way can mitigate the risk associated with asymmetric information between the entrepreneurs and the investors, at the cost of fees paid to the investment banks. Thanks to the rapid development of information technology, peer-to-peer (P2P) financing, also called crowdfunding, arises as an alternative way of raising capital, which does not involve financial intermediaries[1]. There are three basic forms of P2P financing: P2P lending, which is mostly about the debt issuing, P2P equity financing[2], and reward-based crowdfunding[3]. The first two types are called investment-based crowdfunding.

This paper focuses on P2P equity financing, which has two types, private and public. Private P2P equity financing, such as the ones used by hedge funds and venture capital, has been around for many years. However, what is new and recent is that even less experienced fund managers/entrepreneurs (instead of usually seasoned managers) may still raise capital via the P2P framework (see, e.g., the case study of TopWater Captial in this paper). Public P2P equity financing, particularly as a form of initial coin offering (ICO), is almost entirely new and has been growing rapidly, thanks to the recent blockchain technology which can execute smart contracts free of human intervention. ICOs witnessed an explosive growth in 2017 and declined in popularity in the second half of 2018 due to regulatory issues, fraud cases, and poor financial performance. Currently there are thousands of coins and tokens publicly traded in exchanges globally, as the results of ICOs.

Without the financial intermediaries, a main challenge faced by the P2P equity financing is the adverse selection caused by the information asymmetry between entrepreneurs and funders. Note

[1] According to the 2015CF Crowdfunding Industry Report published by Massolution (available at `http://reports.crowdsourcing.org/index.php?route=product/product&path=0&product_id=54`), the global crowdfunding volume increased by 84% in 2012, 125% in 2013, and 167% in 2014, and the volume 2014 totals $16.2 billion. On the other hand, Adhami, Giudici, and Martinazzi (2018) estimate, based on data from `Tokendata.io`, that until 2017 ICOs raised about $5.3 billion globally.

[2] A P2P lending can have small equity components (similar to those of convertible bonds) and a P2P equity issuing can have small debt/cash components, such as deposits from the entrepreneurs. Here we mainly classify them according to their main components and objectives.

[3] In reward-based crowdfunding, an entrepreneur commits to produce a good as long as the crowdfunding campaign is successful, and each funder can receive the good. For theoretical studies on reward-based crowdfunding, one can see for instance Strausz (2017), Ellman and Hurkens (2015), and Chemla and Tinn (2017).

it may be difficult to avoid information asymmetry using the historical data, even if such data is available. For example, as pointed out by Lo (2001), it is difficult to tell whether a fund manager is skillful, i.e., able to beat the market, from the manager's past record, as the fund manager can create skewed payoffs with very high probability of good returns but disastrous returns with very small probability (e.g. by short-selling out-of-money put options), resulting in possibly negative expected returns[4]. Thus, without due diligence and the underwriting by investment banks, adverse selection easily arises and puts the investors at significant disadvantage, which in turn can jeopardize the functioning of the P2P financing market.

The research question of this paper is whether payoff contracts can be designed for entrepreneurs, so that "good" P2P projects will be attracted while "bad" projects will be automatically screened out.

1.1 Our Contribution

To our best knowledge, this is the first paper that studies contract designs for P2P equity financing that can mitigate the risk of information asymmetry. The contribution of the paper is fourfold.

(1) We propose a profit-sharing contract, namely the first-loss scheme with a liquidation boundary, for P2P financing that can attract some entrepreneurs (each with a project) who are attractive from the funders' perspective and deter all entrepreneurs who are unappealing to the funders; see Theorems 3 and 4. This profit-sharing contract differs from the standard incentive scheme used in the US hedge funds in two respects: the first-loss deposit (meaning that any losses from the project will go to the entrepreneurs first before go to the investors, until the first-loss capital is exhausted) and the liquidation boundary.

(2) We show that both the first-loss deposit and the liquidation boundary are necessary in our contract design; see Theorems 1 and 2. The intuition is as follows. (a) Without the first-loss deposit, the entrepreneurs do not cover the funders' losses, so whether or not there is a liquidation boundary, the performance fee always attracts some entrepreneurs who are unappealing to the funders. (b) Without the liquidation boundary, if the first-loss deposit is fully invested, the project

[4]More precisely, Lo (2001) examined the risk exposures of hedge fund investments and showed that a hypothetical strategy of short-selling out-of-money put options on the S&P 500 index generates positive returns most of the time, but also leads to potential extreme losses. Thus, due to the high probability of obtaining reasonable returns, it is difficult for the outside investors to find out the true risk-return profile of the above strategy based on the historical performance data only.

can lose all its value, so effectively the entrepreneur does not cover funders' losses; as a result, the performance fee attracts some entrepreneurs who are unappealing to the funders. If the first-loss deposit is partially held in the risk-free asset, the performance fee taken by an entrepreneur having a project with a high return must be larger than a threshold so as to counter the opportunity cost of the deposit; however, in the absence of a liquidation boundary such a high performance fee in turn attracts some entrepreneurs who have projects with low returns.

(3) We give two case studies in Section 5, one for private P2P financing using TopWater Capital (an asset management company), and one for public P2P financing using Crudecoin (an ICO) as an example. The two case studies bring additional insights. The case of TopWater Capital shows the need of considering possible overshoots over the liquidation boundary; we find that our proposed scheme still works in the presence of random losses over the liquidation boundary. The case of Crudecoin is interesting because the whole contract can be written as part of a smart contract in the blockchain without human intervention, and the case motivates us to consider the situation when the liquidation boundary is not observed directly; we show that the first-loss scheme with the liquidation boundary defined on certain observable variable, that is reasonably correlated the unobservable project value, can still mitigate the risk of adverse selection.

(4) We also present various extensions of our model in the appendix, taking into account other possible practical contract features, such as management fees, different hurdle and reference rates, and limited liability.

It should be emphasized that even with the proposed contract scheme investors must do their own due diligence before investing. For example, it might well be that an entrepreneur truly believe (subjectively) about a project, and therefore is attractive to the scheme; however, objectively the project can yield bad returns, leading to bad outcomes for investors. Thus, our proposed scheme is only a step forward to mitigate the risk of information asymmetry, and is not meant to solve the difficult problem completely.

1.2 Literature Review

Our paper is broadly related to the literature on P2P financing or crowdfunding. Hakenes and Schlegel (2014) compare the debt-based crowdfunding with the traditional debt financing. Chang (2016) study the optimal crowdfunding mechanism for an entrepreneur who has a project with common value. Chen, Huang, and Liu (2016) compare the pure-crowd design and the hybrid-crowd design in crowdfunding. Åstebro, Fernández Sierra, Lovo, and Vulkan (2017), Asami (2018), and Cong and Xiao (2018) study the herding behavior and information cascading in investment-based crowdfunding. All the above works, except Hakenes and Schlegel (2014), assume that there is no information asymmetry. Their paper considers two types of entrepreneurs, good ones and bad ones, who seek funding on a crowdfunding platform or by standard debt financing channels; they assume that a bad entrepreneur always mimic good entrepreneurs by setting the same interest rate of debt, that the crowdfunding scheme does not exclude bad entrepreneurs, and that the proportion of good entrepreneurs is fixed. In other words, their model of P2P debt financing does not include adverse selection. We complement their paper by proposing a contract that can address the issue of adverse selection in P2P financing by screening out bad entrepreneurs.

As an example of public P2P equity financing, ICO's have been growing rapidly in recent years[5]. In an ICO, a firm attempts to raise capital by issuing a digital token via a blockchain smart contract. There are two types of ICO tokens: security tokens and utility tokens. The former are similar to financial securities, giving the holders entitlement to dividend or interest payments, and the latter can be used by the holders to exchange for the firm's products or services.

Our paper is related to the literature on ICO financing. The only theoretical paper that we are aware of to study ICOs with asymmetric information is Chod and Lyandres (2018), which compares ICO financing and traditional venture capital financing of entrepreneurial ventures and studies the "least-cost separating" equilibrium, in which every entrepreneur truthfully reports his/her type. Li and Mann (2018) and Bakos and Halaburda (2018) show that issuing platform-specific digital tokens can overcome the coordination failure in platform adoption. Cong, Li, and Wang (2018) use a continuous-time model to study how the token valuation depends on the platform productivity. Catalini and Gans (2018) show that ICO is useful to reveal product quality. There are also papers focusing on utility ICO tokens: Malinova and Park (2018) show that such token issuing can induce over- or under-production, compared to the traditional equity financing. Lee and Parlour (2018) find that utility token financing differs from bank funding, as consumers consider the financial impact of the project and consumption of products. Garratt and van Oordt (2018) study whether

[5]See, e.g., the empirical studies by Adhami et al. (2018), Benedetti and Kostovetsky (2018), Bianchi and Dickerson (2018), De Jong, Roosenboom, and Van der Kolk (2018), Dittmar and Wu (2018), Howell, Niessner, and Yermack (2018), Hu, Parlour, and Rajan (2018), Lee, Li, and Shin (2018).

a utility ICO can result in a better alignment of the interests of the entrepreneur and the investors compared with conventional financing; interestingly they find that there are projects that will not take place at all unless being financed through an ICO. [6]

Our paper complements these papers in two ways. First, we study the contract design to mitigate information asymmetry from the investors' perspective. More precisely, we consider a contract that deters entrepreneurs who are unappealing from the investors' perspective, while attracts good entrepreneurs. Second, we do not assume specific utility functions for investors and entrepreneurs, and only requires certain classes of utility functions. On the other hand, we do not discuss issues related to capital structures, coordination, production and we do not study the valuation of tokens.

Designing an incentive scheme is closely related to the classical principal-agent problems. There are three main types of principal-agent problems in the literature: problems with symmetric information, problems with hidden actions, and problems with hidden types. Our problem is related to principal-agent problems with hidden types. However, there are two key differences. First, the standard approach to hidden types is to offer a menu of different contracts to all agents, so that these agents reveal their types by choosing suitable contracts from the menu; in contrast, in our problem the same contract is offered to all entrepreneurs with various types. Secondly, we do not assume particular functional forms of utility functions, and interested in the class of contracts separating different types of entrepreneurs, while traditional approaches assume particular forms of utility functions and aim at finding optimal contracts.

The contract design in this paper is related to the screening contracts in Sannikov (2007) or the shutdown contracts in Cvitanić, Wan, and Yang (2013), where they study optimal contracts to screen out all bad agents in the market. Our work differs from these two papers in four respects. First, both papers assume specific utility functions for the agent and principal, while in our study, we do not make such assumption and thus our result is robust to the choice of preference models. Second, both papers consider two types of agents, while we consider multiple types with different project returns. Third, both papers assume that the ability of the two types of agent is known, but the expected project return rates are unknown in our work. Fouth, due to the general framework

[6] There is a stream of literature studying the theoretical values of cryptocurrencies and tokens; see, e.g., Bolt and Van Oordt (2019), Garratt and Wallace (2018), Athey, Parashkevov, Sarukkai, and Xia (2016), and Sockin and Xiong (2018).

here, we do not discuss optimal contracts, and focus instead on the existence of a set of contracts that can screen out bad projects and attract good projects.

The setting of attractive and unappealing entrepreneurs is also broadly related to the setting of sound and unsound banks in the study of banking regulation by Morrison and White (2005), and to the setting of the executive hiring problem faced by a firm (see, e.g., Thanassoulis (2013)). Another related work is Foster and Young (2010), where the authors consider two types of hedge fund managers, perfectly skilled managers and mimic managers, and show that the traditional incentive schemes without a liquidation boundary in the US hedge fund market cannot separate these two types of managers. Here, with three types of managers (perfectly skilled, skilled, and unskilled managers), we show that by combining the first-loss scheme and a liquidation boundary, one can indeed separate different types of managers.

Hedge fund incentive schemes that involve a first-loss deposit were first studied theoretically by He and Kou (2018). They compare different profit sharing schemes in terms of utility gains based on cumulative prospect theory, and find that in most cases both fund managers' and investors' utilities can be improved and fund risk can be reduced simultaneously by replacing the traditional scheme with a first-loss scheme. They did not address the issue of information asymmetry and contract design. Here our focus is on designing contracts using the first-loss scheme to mitigate information asymmetry by separating good and bad entrepreneurs.

Chassang (2013) proposed a limit-liability hurdled contract, in which the agent needs to pay a participation fee at the start of the contract and gets rewarded at the end of the contract if the agent's performance passes a hurdle. The author showed that this contract can deter uninformed agents (e.g., unskilled entrepreneurs) while attract informed agents (skilled entrepreneurs). Note that the participation fee is similar to first-loss capital: the manager shares risk with investors by paying the participation fee. A key difference is that here we have an opportunity cost and a liquidation boundary. Indeed, we complement the results there in two ways: First, with a positive opportunity cost (which may be reasonable if the manager can have an alternative choice of investing the participation fee elsewhere), we show in the appendix that without a liquidation boundary, the limited-liability hurdled contract cannot separate attractive entrepreneurs from unappealing ones for any funder. Secondly, we show that with a liquidation boundary, the separation can be achieved.

First-loss deposits are related to 'fulcrum' fees that are used by some financial advisors and in some mutual funds; see e.g., Cuoco and Kaniel (2011) and Basak, Pavlova, and Shapiro (2007). Similar to a first-loss deposit, with a fulcrum fee fund managers not only take a performance fee when the fund return exceeds certain benchmark but also takes a hit when the fund underperforms the benchmark. However, with a fulcrum fee the manager's exposure to the gain and to the loss of the fund relative to the benchmark is usually symmetric, while with a first-loss deposit, the exposure is asymmetric. Moreover, with a fulcrum fee, the gain or loss of the fund affects the management fee only. With a first-loss deposit, however, the loss can hit the manager's own capital.

In a broad sense, Buraschi, Kosowski, and Sritrakul (2014) considered a hedge fund investment problem where the fund manager's payoff has an embedded component similar to first-loss deposits, and this component is used to model the manager's concern about the short positions in the funding and redemption options. The funding option refers to the involuntary deleveraging of fund assets in bad states of the world due to credit limits and the redemption option refers to the willingness of investors to redeem their shares in bad states of the world; see e.g., Dai and Sundaresan (2009).

When a hedge fund suffers a loss, some of its investors may withdrawal their assets. In the existing literature, outside liquidation boundaries are used to model a complete withdrawal of investors' assets when the fund suffers a big loss; see e.g., Goetzmann, Ingersoll, and Ross (2003) and Hodder and Jackwerth (2007). Different from these works, the liquidation boundary considered in the present paper is explicitly written in the contract.

In long-term financial contracting, the lenders, who do not have access to the project nor observe the return of the project, offer the entrepreneur a take-it-or-leave-it contract that specifies sharing of the cash flows and a liquidation policy contingent on the information available to the lenders; see, e.g., Quadrini (2004), Clementi and Hopenhayn (2006), and DeMarzo and Fishman (2007). These works differ from the present paper in that they consider one entrepreneur of fixed type and focus on principal-agent problems with hidden actions. The present paper, however, considers projects with hidden types, i.e. different levels of project risk and entrepreneurs with different risk aversion degrees.

The remainder of the paper is organized as follows: In Section 2, we propose the project investment model and the first-loss scheme, introduce liquidation boundaries, and define the notion of good and bad projects. We then present the main result in Sections 3 and 4, followed by two case studies in Section 5. A variety of extensions of our model are discussed in the appendices. All proofs are placed in the online supplement.

2 The Model

2.1 Project

For simplicity, we shall first consider an entrepreneur who has a project (or a fund manager who has an investment strategy) that is managed for period from 0 to 1. The case of multiple periods is studied later in Appendix F. At time t, the net return of the project is denoted to be $\tilde{m}_t$, and the total value of the project is X_t. Assume the risk-free rate is a constant r (continuously compounded). The entrepreneur can raise outside capital from investors and use her own money in the project. We denote by τ the liquidation time of the project; here, $\tau > 1$ means that the project is not liquidated. Thus, the management of the project ends at $\tau \wedge 1$. Here and hereafter, $x \wedge y := \min(x, y)$.The entrepreneur can take a performance fee at time 1, as long as the project is not liquidated before that time. The liquidation time τ is determined endogenously by the project and by the managerial compensation contract. We assume that $\tilde{m}_\tau \leq 0$ when $\tau \leq 1$; i.e., the project will not be liquidated when it yields a gain.

2.2 Traditional Scheme

In a typical hedge fund, the manager does not cover any investment loss from the investors' capital, but are responsible for the gain and loss of the manager's own equity stake of the fund. Following He and Kou (2018), we call this compensation scheme the *traditional scheme*, and the scheme can be applied to P2P financing as well.

In the traditional scheme, suppose that the entrepreneur's initial investment of the fund is $Z_0 := wX_0$ for some $w \in [0, 1)$, so that the funders invest the remaining $(1 - w)X_0$. If the project is not liquidated, then at time 1 the entrepreneur gets $wX_0(\tilde{m}_1 + 1)$ (the payoff of her own stake) and a performance fee based on the return on the funders' stake in the project. A standard contract of performance fees is for the entrepreneur to take a proportion of the gain generated by the funders' capital in excess of certain hurdle rate. Here, we assume the hurdle rate to be the risk-free rate, and the more general case is treated in Section B. As a result, the performance fee is $\alpha(1 - w)X_0(\tilde{m}_1 + 1 - e^r)^+$, where $\alpha \in (0, 1)$ is the *incentive rate* for the entrepreneur and $x^+ := \max(x, 0)$. The entrepreneur receives a payoff $Z_{\tau\wedge 1}$ at time $\tau \wedge 1 \in (0, 1]$. If the project is liquidated at $\tau \le 1$, the value of the entrepreneur's stake at that time is $wX_0(\tilde{m}_\tau + 1)$, where $\tilde{m}_\tau < 0$, and there is no performance fee.

Suppose the entrepreneur evaluates this payoff by accumulating it at the risk-free rate from $\tau \wedge 1$ until the terminal time 1. Denote $\tilde{Z}_1 := e^{r(1-\tau\wedge 1)} Z_{\tau\wedge 1}$, the entrepreneur's payoff accumulated at time 1. We have

$$\tilde{Z}_1 = Z_0 \Big\{ \tilde{R} + \alpha \frac{1-w}{w} \big(\tilde{R} - e^r\big)^+ \Big\} = Z_0 \Big\{ \Big[\big(1 + \alpha(1-w)/w\big)\tilde{R} - \alpha \frac{1-w}{w} e^r \Big] + \tilde{D} \Big\}, \tag{1}$$

where

$$\tilde{R} : = e^{r(1-\tau\wedge 1)}(\tilde{m}_{\tau\wedge 1} + 1), \tag{2}$$

$$\tilde{D} : = \big(\alpha(1-w)/w\big)(e^r - \tilde{R})^+. \tag{3}$$

Note that $\tilde{R}$ represents the gross return rate in the period $[0,1]$ of the project. On the other hand, taking a performance fee from the funders effectively enables the entrepreneur (i) to borrow money from the funders at the risk-free rate and invest the money in the project, leading to the gross return in the bracket in (1), and (ii) not to pay back the debt when the project under-performs the risk-free asset, leading to a value represented by $\tilde{D}$. We call $\tilde{D}$ the *option-to-default* as it resembles the option of the shareholders of a firm to default when the firm's asset value is lower than the debt amount. Therefore, under the traditional scheme, the entrepreneur (i) increases the leverage of the project, and (ii) acquires the option-to-default.

We can also find the funders' payoff under the traditional scheme. Consider a funder who pledges $Y_0 := (1-w)X_0$ to the project at time 0 and receives a payoff $Y_{\tau\wedge 1}$ at time $\tau \wedge 1 \in (0,1]$. Suppose that he evaluates this payoff by accumulating it at the risk-free rate until the terminal time 1. Denote $\tilde{Y}_1 := e^{r(1-\tau\wedge 1)} Y_{\tau\wedge 1}$, the funders' payoff at time 1, which can be written as follows:

$$\tilde{Y}_1 = \tilde{R} - \tilde{Z}_1 = Y_0 \Big\{ \tilde{R} - \alpha \left(\tilde{R} - e^r \right)^+ \Big\} = Y_0 \Big\{ \Big[\alpha e^r + (1-\alpha)\tilde{R} \Big] - \big(w/(1-w)\big)\tilde{D} \Big\}. \tag{4}$$

2.3 First-Loss Scheme

Next, consider a new scheme, referred to as the *first-loss* scheme for P2P financing. In this scheme, the entrepreneur's investment in a project, wX_0, is used to set up a *first-loss deposit*. Part of the first-loss deposit, e.g., $\gamma w X_0$ for some $\gamma \in [0,1]$, is cash to be held in the risk-free asset, while the remaining amount $(1-\gamma)wX_0$ is invested in the project. The entrepreneur's first-loss deposit is used to cover the losses of the funders, until the first-loss deposit is used up.

If $\gamma = 1$, all the first-loss deposit is cash, and the first-loss schemes becomes the safety scheme in Foster and Young (2010). If $\gamma = 0$, all the first-loss deposit consists of the equities of the project only, and this special type of first-loss scheme is discussed in He and Kou (2018). This type of contracts with $\gamma = 0$ is popular in China private equities, and is also emerging in the US hedge fund industry.[7]

Note that both the funders' stake and the entrepreneur's equity deposit earn net return $\tilde{m}_t$, but the entrepreneur's cash deposit earns the risk-free return. Consequently, at time t, if the project has not been liquidated yet, the value of the entrepreneur's deposit is $\gamma w X_0 e^{rt} + (1-\gamma)wX_0(\tilde{m}_t+1)$ and the value of the funders' stake is $(1-w)X_0(\tilde{m}_t+1)$. Suppose at the terminal time 1, the project has not been liquidated. If the return of the project passes the hurdle rate, i.e., $\tilde{m}_1 + 1 > e^r$, the entrepreneur takes a performance fee equal to $\alpha(1-w)X_0(\tilde{m}_1+1-e^r)$. We assume that the funders' stake is at a loss if and only if its net return rate is less than 0, i.e., $\tilde{m}_1 < 0$; the general case is treated in Section B. As a result, the entrepreneur's payoff at time 1 consists of the performance fee and the remaining value of the first-loss deposit after covering the funders' loss. On the other hand, if the project is liquidated at $\tau \leq 1$, in which case $\tilde{m}_\tau \leq 0$, the payoff of the entrepreneur at that time is the remaining value of the first-loss deposit after covering the funders' loss.

Under the first-loss scheme, the entrepreneur's total payoff accumulated to time 1 is

$$\tilde{Z}_1 = Z_0 \left\{ \left[\big(1-\gamma+\alpha(1-w)/w\big)\tilde{R} + \big(\gamma - \alpha(1-w)/w\big)e^r \right] + \tilde{D} - \tilde{F} \right\}, \tag{5}$$

where

$$\tilde{F} := \min\left\{ \gamma e^r + (1-\gamma)\tilde{R}, \big((1-w)/w\big)\left(e^{r(1-\tau\wedge 1)} - \tilde{R}\right)^+ \right\}. \tag{6}$$

Note that $\tilde{F}$, so-called *first-loss coverage*, represents the amount of the funders' loss that the entrepreneur needs to cover (per unit of the entrepreneur's initial capital in the project). Therefore, by raising capital with the first-loss scheme, the entrepreneur effectively (i) locks up part of her money in the risk-free asset due to the cash deposit, (ii) increases the leverage of the project, (iii) acquires the option-to-default, and (iv) pays the first-loss coverage. Similarly, the funder's payoff

[7] See a report from CBS Marketwatch on May 23, 2011 about the first-loss capital in the United States.

$\tilde{Y}_1$ can be represented as follows:

$$\tilde{Y}_1 = Y_0 \left\{ \left[\alpha e^r + (1-\alpha)\tilde{R} \right] - \big(w/(1-w)\big)\tilde{D} + \big(w/(1-w)\big)\tilde{F} \right\}. \tag{7}$$

In both the traditional and the first-loss schemes, the manager receives a payoff at a single time. However, it is still necessary for us to model the management and asset value of the fund continuously, to have the possibility of liquidating the fund as soon as its value hits some lower bound. Such a liquidation boundary will play an important role in the following contract design to address the issue of adverse selection.

2.4 Definitions of Attractiveness to Entrepreneurs and Funders

The objective of the paper is to find a scheme such that all entrepreneurs whom are disliked by funders are screened out and some entrepreneurs whom are liked by funders are attracted. To this end, we have to define when an entrepreneur is willing to accept an incentive contract and when an entrepreneur is indeed attractive to a funder. We do this within the decision science framework without explicitly assuming particular functional forms of utility functions. In particular, the utility functions of funders and the entrepreneur are only assumed to be in certain classes and can be quite general, e.g. having a discontinuous kink.

We evaluate the investment in terms of the gross growth rate, to make our results independent of the specification of the initial investment of the entrepreneurs and funders. We assume that both entrepreneurs and funders have some preferences for random gross growth rates in the period $[0, 1]$ of their investment in the fund, and we denote the utility functions for an entrepreneur and for a funder as u_E and u_F, respectively, without assuming particular forms of u_E and u_F. More precisely, for a given entrepreneur with utility function $\bar{u}_E$ for the value of her investment in the fund at time 1 and with initial investment Z_0, the corresponding utility function for the gross growth rate $u_E(x) = \bar{u}_E(Z_0 x)$, and a similar transformation can be made for funders' utility functions.

In general we assume that $u_i \in \mathcal{U}$, $i \in \{E, F\}$, where $\mathcal{U}$ denotes the set of functions: $\mathbb{R} \to \mathbb{R} \cup \{-\infty\}$ that are strictly increasing, continuous, and concave in its domain and are twice-continuously differentiable on $(0, e^r) \cup (e^r, +\infty)$. Note that we allow the possible kink of u_i at e^r, to model the loss aversion as in prospect theory (Kahneman and Tversky, 1979), where the risk-free gross return rate e^r is a natural reference point.

In some specific cases, we shall consider a smaller class of utility functions. More precisely, for each $\delta \geq 0$ and $\lambda \geq 1$, denote by $\mathcal{U}_{\delta,\lambda}$ the set of utility functions u with the local relative risk aversion degree (local RRAD) $-xu''(x)/u'(x) \leq \delta, \forall x \in (0, e^r) \cup (e^r, +\infty)$, and the loss aversion degree (LAD) $\lim_{x\uparrow e^r} u'(x)/\lim_{x\downarrow e^r} u'(x) \leq \lambda$. In particular, $\mathcal{U}_{0,1}$ is the set of linear utility functions.

The entrepreneur is *attracted* by a P2P financing scheme if she strictly prefers raising capital through this scheme to investing on her own by a mixture of her project and the risk-free asset; i.e., if

$$\mathbb{E}\left[u_E\big(\tilde{Z}_1/Z_0\big)\right] > \mathbb{E}\left[u_E\big(a\tilde{R} + (1-a)e^r\big)\right], \quad \text{for all } a \in [0,1]. \tag{8}$$

Otherwise, the entrepreneur is *deterred*. Conversely, from the funder's viewpoint, the entrepreneur is *attractive* to this funder if the funder strictly prefers pledging money to the project to investing in the risk-free asset, i.e., if

$$\mathbb{E}\left[u_F\big(\tilde{Y}_1/Y_0\big)\right] > u_F(e^r). \tag{9}$$

Otherwise, the entrepreneur is *unappealing* to this funder.

2.5 Thresholds and Effective Seperation

Define

$$\bar{\alpha}_E(u_E, \tilde{R}, \tau) := \inf\left\{\alpha \in [0,1] : \mathbb{E}\left[u_E\big(\tilde{Z}_1/Z_0\big)\right] > \mathbb{E}\left[u_E\big(a\tilde{R} + (1-a)e^r\big)\right] \text{ for all } a \in [0,1]\right\}$$

with the convention that $\inf \emptyset = 1$. It is clear from (8) that the entrepreneur is attracted to raise capital for her project if and only if the inventive rate is large enough, i.e. $\alpha > \bar{\alpha}_E(u_E, \tilde{R}, \tau)$. Similarly, from the funder's viewpoint, we can define

$$\bar{\alpha}_F(u_F, \tilde{R}, \tau) := \sup\left\{\alpha \in [0,1] : \mathbb{E}\left[u_F\big(\tilde{Y}_1/Y_0\big)\right] > u_F(e^r)\right\}$$

with the convention that $\sup \emptyset = 0$. By (9), the entrepreneur is attractive to the funder if and only if the inventive rate is small enough, i.e. $\alpha < \bar{\alpha}_F(u_F, \tilde{R}, \tau)$.

Due to asymmetric information, the funders do not know the type of the entrepreneurs, which is represented by the entrepreneurs' utility function u_E and their project profiles $(\tilde{R}, \tau)$. Thus, we attempt to find a range of incentive rate α for an incentive scheme satisfying two requirements: (i) All entrepreneurs under consideration who are unappealing to any funder are deterred; more precisely, if $\bar{\alpha}_F(u_F, \tilde{R}, \tau) \leq \alpha$ for any $u_F \in \mathcal{U}_F$, then $\bar{\alpha}_E(u_E, \tilde{R}, \tau) \geq \alpha$ for all $u_E \in \mathcal{U}_E$ and

$(\tilde{R},\tau)\in\mathcal{R}$. (ii) Some entrepreneurs under consideration are attracted; more precisely, there exist $u_E\in\mathcal{U}_E$ and $(\tilde{R},\tau)\in\mathcal{R}$ such that $\bar{\alpha}_E(u_E,\tilde{R},\tau)<\alpha$. Here the possible utility sets $\mathcal{U}_E$ and $\mathcal{U}_F$, and the possible set of the project return profile $\mathcal{R}$ can take different forms in different schemes. Note that if both requirements (i) and (ii) are satisfied, then any entrepreneur who are attracted to the scheme is automatically attractive to all funders in $\mathcal{U}_F$, thanks to the requirement (i).

For a particular incentive scheme, if we can find a range of such incentive rate α, then we say that the incentive scheme with α in this range is *separation-effective* for any funder with arbitrary utility $u_F\in\mathcal{U}_F$ regarding entrepreneurs with arbitrary utility $u_E\in\mathcal{U}_E$ and project return profile $(\tilde{R},\tau)\in\mathcal{R}$.

3 Incentive Schemes without a Liquidation Boundary: A Negative Result

In this section, we consider the case that the entrepreneur's project will never be liquidated; as a result, the value of the project can touch zero at any time, and the entrepreneur and funders receive their payoffs at the terminal time 1. To ease exposition, consider first a simple case that the set of the return profiles of entrepreneurs' projects is

$$\mathcal{R}_0:=\left\{(\tilde{R},+\infty):\mathbb{P}(\tilde{R}=m_1+1)=1-\mathbb{P}(\tilde{R}=0)>0\text{ for some constant }m_1\geq e^r-1\right\}.$$

Here m_1 is assumed to be a constant for simplicity; the case of random gains (i.e. m_1 is random) is studied in Appendix E. Instead of just being 0 when there is a loss, it is also possible that conditional on having a loss the gross return of the project may be random; see the extension of random losses in Appendix D. Note that in this simple case $\tau=+\infty$ because there is no liquidation.

Based on the return performance of the project, all entrepreneurs are classified into three categories: *perfectly skilled entrepreneurs*, *skilled entrepreneurs*, and *unskilled entrepreneurs*, whose return profiles of the projects are denoted as $\mathcal{R}_P$, $\mathcal{R}_{0,S}$, and $\mathcal{R}_{0,U}$, respectively, where

$$\begin{aligned}
\mathcal{R}_P &= \big\{(\tilde{R}, +\infty) : \tilde{R} = m_1 + 1 \text{ for some constant } m_1 > e^r - 1\big\}, \\
\mathcal{R}_{0,S} &= \big\{(\tilde{R}, +\infty) : \mathbb{P}(\tilde{R} = m_1 + 1) = 1 - \mathbb{P}(\tilde{R} = 0) \in (0,1) \text{ for some constant } m_1 > e^r - 1 \\
&\qquad\qquad \text{and } \mathbb{E}[\tilde{R}] > e^r\big\}, \\
\mathcal{R}_{0,U} &= \big\{(\tilde{R}, +\infty) : \mathbb{P}(\tilde{R} = m_1 + 1) = 1 - \mathbb{P}(\tilde{R} = 0) > 0 \text{ for some constant } m_1 \geq e^r - 1 \\
&\qquad\qquad \text{and } \mathbb{E}[\tilde{R}] \leq e^r\big\}.
\end{aligned}$$

A perfectly skilled entrepreneur (related to $\mathcal{R}_P$) is one who has a project generating a deterministic return rate higher than the risk-free rate. A skilled entrepreneur (related to $\mathcal{R}_{0,S}$) is one who has a project generating expected return rate higher than the risk-free rate, and there is still a positive probability that the project return is lower than the risk-free asset. An unskilled entrepreneur (related to $\mathcal{R}_{0,U}$) has a project generating expected return rate lower than or equal to the risk-free rate.

We show a negative result in the following theorem: Without a liquidation boundary, for any funder, neither the traditional scheme nor the first-loss scheme is able to deter all entrepreneurs who are unappealing to the funder and to attract some entrepreneurs at the same time, where a mix of cash and equity deposits is considered under the first-loss scheme. This theorem generalizes a result in Foster and Young (2010), which covers two (instead of three here) types of entrepreneurs (perfectly skilled and unskilled) with $\gamma = 1$ (instead of general $\gamma \in [0,1]$ here).[8] $u_\delta(x)$ is defined by

$$u_\delta(x) := \big(x^{1-\delta} - 1\big)/(1-\delta), \quad \delta \neq 1, \quad u_\delta(x) := \log(x), \quad \delta = 1.$$

Theorem 1. *Suppose that there is no liquidation boundary.*

(i) The traditional scheme always attracts some entrepreneur who is unappealing to any funder; more precisely, fixing any $\alpha \in (0,1)$ and any $u_E \in \mathcal{U}_{\delta,\lambda}$ with $0 < \lambda/(1-\delta) < 1 + \alpha(1-w)/w$, there exists $(\tilde{R}, \tau) \in \mathcal{R}_{0,U}$ such that $\bar{\alpha}_E(u_E, \tilde{R}, \tau) < \alpha$ and $\bar{\alpha}_F(u_F, \tilde{R}, \tau) \leq \alpha$ for any $u_F \in \mathcal{U}$.

(ii) Either the incentive rate so low that the first loss scheme deters all entrepreneurs, or the incentive rate is so high that the first loss scheme attracts some entrepreneur who is unappealing to any funder. More precisely, fixing any $\alpha \in \big(0, \min\{\gamma w/(1-w), e^{-r}\}\big]$, then $\bar{\alpha}_E(u_E, \tilde{R}, \tau) \geq \alpha$ for

[8] Note that in that paper perfectly skilled and unskilled entrepreneurs are named differently (skilled and mimic managers, respectively).

all $u_E \in \mathcal{U}$ and $(\tilde{R}, \tau) \in \mathcal{R}_0$; fixing any $\alpha \in \big(\min\{\gamma w/(1-w), e^{-r}\}, 1\big)$ and any $u_E \in \mathcal{U}_{\delta,\lambda}$ with δ and λ such that [9]

$$-\lambda u_\delta\left(\gamma - \min(\gamma, e^{-r}(1-w)/w)\right) < 1 - \gamma + \alpha(1-w)/w, \tag{10}$$

there exists $(\tilde{R}, \tau) \in \mathcal{R}_{0,U}$ such that $\bar{\alpha}_E(u_E, \tilde{R}, \tau) < \alpha$ and $\bar{\alpha}_F(u_F, \tilde{R}, \tau) \le \alpha$ for any $u_F \in \mathcal{U}$.

Theorem 1-(i) shows that without the liquidation boundary, given a utility function with sufficiently low local RRAD and small LAD, the traditional scheme always attracts some entrepreneur with that utility function and certain project such that she is unappealing to any funder. The intuition is as follows: Because of the performance fee, an unskilled entrepreneur is attracted under the traditional scheme if she is not very risk averse and her project yields a return with a positive probability to outperform the risk-free asset, and this entrepreneur is unappealing to any funder.

Theorem 1-(ii) shows that without the liquidation boundary, in the first-loss scheme when the incentive rate is low, i.e., when $\alpha \le \min\{\gamma w/(1-w), e^{-r}\}$, all entrepreneurs are deterred. The intuition is as follows: In the first-loss scheme, an entrepreneur may have to lock up part of her first-loss deposit in the risk-free asset, which effectively lowers the leverage of her project; on the other hand, the entrepreneur can effectively increase the leverage as a result of taking performance fees. The net change of the leverage then depends on the magnitude of the cash component of the first-loss deposit and the incentive rate. According to (5), the net change is positive if and only if $\alpha > \gamma w/(1-w)$. In particular, when $\alpha \le \min\{\gamma w/(1-w), e^{-r}\}$, this net change is nonpositive, so the entrepreneur is deterred.

On the other hand, in the first-loss scheme when the incentive rate is high, i.e., when $\alpha > \min\{\gamma w/(1-w), e^{-r}\}$, according to (3) and (6), the net value of the option-to-default and the first-loss coverage is positive, so an unskilled entrepreneur with certain project is attracted if she is not very risk averse. However, Theorem 1-(ii) shows that without the liquidation boundary this entrepreneur is unappealing to any funder. Intuitively, this is because (a) the entrepreneur's project does not bring a positive expected excess return to the funder and (b) the option-to-default paid by the funder is larger than the first-loss coverage provided by the entrepreneur.

[9] Note that the left-hand side of (10) is increasing in λ and δ, and is strictly less than the right-hand side when $\lambda = 1$ and $\delta = 0$ because $\alpha > \min\{\gamma w/(1-w), e^{-r}\}$, so there exists a range of values of (λ, δ) such that (10) holds.

4 Incentive Schemes with Liquidation Boundaries: Positive and Negative Results

In this section we shall study whether a compensation scheme with a liquidation boundary is separation-effective.

4.1 Liquidation Boundaries

Denote by $X_{I,t}$ the value of the funders' stake and $X_{M,t}$ the value of the entrepreneur's stake at time t, respectively, so the total value of the project at time t is $X_t = X_{I,t} + X_{M,t}$. Moreover, at the initial time 0, $X_{I,0} = (1-w)X_0$ and $X_{M,0} = wX_0$.

A *liquidation boundary* is a constant $b \in (0,1)$ such that the project is liquidated once the value of the funders' stake hits b proportion of its initial value. In other words, the liquidation time τ is

$$\tau = \inf\{t \in (0,1] | X_{I,t} \le bX_{I,0}\} = \inf\{t \in (0,1] | \tilde{m}_t \le b - 1\}.$$

Here, we set $\inf \emptyset := +\infty$, so the project is liquidated if and only if $\tau \le 1$. Liquidation boundaries have been widely used in many private equities. The asset values in these equities are usually monitored by a third party on a daily basis. If the asset values hit the boundaries, the equities are liquidated. In practice, the boundaries are usually set to be a constant proportion, e.g., 85%, of the initial value of the investors' stake.

We denote the set of the return profiles of entrepreneurs's projects with liquidation by

$$\begin{aligned}\mathcal{R}_b := \big\{ (\tilde{R}, \tau) : &\ \tau \text{ is a random time, } \tilde{R} = m_1 + 1 \text{ on } \tau > 1 \text{ for some constant } m_1 \ge e^r - 1, \\ &\ \tilde{R} = be^{r(1-\tau)} \text{ on } \tau \le 1 \big\}\,.\end{aligned}$$

Note that the gross return of the project is b at the liquidation time.[10] Based on the return performance of the project, all entrepreneurs are again classified into three categories: perfectly skilled entrepreneurs, skilled entrepreneurs, and unskilled entrepreneurs, whose return profiles of projects with liquidation are denoted as $\mathcal{R}_P$, $\mathcal{R}_{b,S}$, and $\mathcal{R}_{b,U}$, respectively.

The definition of $\mathcal{R}_P$ is the same as the one in the case of no liquidation, because the projects of these entrepreneurs deliver deterministic return rates that are higher than the risk-free return

[10]In practice, it is possible that the gross return of the project may overshot the liquidation boundary, or the project may lead to a loss without touching the liquidation boundary, so that the gross return of the project, conditional on having a loss, may no longer be b; see detailed analysis on overshoot and random losses in Appendix D.

rate, while

$$\mathcal{R}_{b,S} = \big\{(\tilde{R}, \tau) : \tau \text{ is a random time, } \tilde{R} = m_1 + 1 \text{ on } \tau > 1 \text{ for some constant } m_1 > e^r - 1,$$
$$\tilde{R} = be^{r(1-\tau)} \text{ on } \tau \le 1,\ \mathbb{P}(\tau \le 1) > 0, \text{ and } \mathbb{E}[\tilde{R}] > e^r\big\},$$
$$\mathcal{R}_{b,U} = \big\{(\tilde{R}, \tau) : \tau \text{ is a random time, } \tilde{R} = m_1 + 1 \text{ on } \tau > 1 \text{ for some constant } m_1 \ge e^r - 1,$$
$$\tilde{R} = be^{r(1-\tau)} \text{ on } \tau \le 1, \text{ and } \mathbb{E}[\tilde{R}] \le e^r\big\}.$$

A skilled entrepreneur (related to $\mathcal{R}_{b,S}$) either delivers net return $m_1 > e^r - 1$ at time 1 or leads to liquidation at time τ; in the latter case, the net return at the liquidation time is $b - 1$; moreover, the expected gross return discounted by the risk-free rate is strictly larger than 1. An unskilled entrepreneur (related to $\mathcal{R}_{b,U}$) either delivers return $m_1 \ge e^r - 1$ at time 1 or leads to liquidation at time τ; in the latter case, the net return at the liquidation time is $b - 1$; moreover, the expected gross return discounted by the risk-free rate is less than or equal to 1.

4.2 Traditional Scheme with Liquidation: A Negative Result

The following theorem shows that even with a liquidation boundary, for any funder, the traditional scheme is unable to deter all entrepreneurs who are unappealing to the funder and attract some entrepreneurs at the same time.

Theorem 2. *Suppose that there is a liquidation boundary $b \in (0, 1)$. Then the traditional scheme always attracts some entrepreneur who is unappealing to any funder; more precisely, fixing any $\alpha \in (0, 1)$ and $u_E \in \mathcal{U}_{\delta,\lambda}$ with*

$$\lambda \frac{-u_\delta(e^{-r}b)}{1 - e^{-r}b} < 1 + \alpha(1 - w)/w, \tag{11}$$

there exists $(\tilde{R}, \tau) \in \mathcal{R}_{b,U}$ such that $\bar{\alpha}_E(u_E, \tilde{R}, \tau) < \alpha$ and $\bar{\alpha}_F(u_F, \tilde{R}, \tau) \le \alpha$ for any $u_F \in \mathcal{U}$.

The intuition behind the above theorem is the same as the one for Theorem 1-(i). Note that (11) holds for $\lambda = 1$ and $\delta = 0$ and also holds for λ and δ in neighbours of 1 and 0, respectively.

4.3 First-Loss Scheme with Liquidation for Risk-Neutral Funders: A Positive Result

The following theorem shows that with a liquidation boundary, for risk-neutral funders, the first-loss scheme can attract some skilled entrepreneurs while deterring all entrepreneurs who are unappealing to the funders.

Theorem 3 (Risk-Neutral Funders). *Suppose that there is a liquidation boundary $b \in (0,1)$ and consider the first-loss scheme. Denote*

$$L := \frac{\gamma w}{1-w}, \quad U := \min\left\{\frac{\gamma w}{1-w} + \frac{wb}{(1-w)(e^r - b)}, \frac{1-b}{e^r - b}\right\}. \tag{12}$$

Suppose $L < U$. Then, for any given $\alpha \in (L, U)$, the following are true:

(i) All perfectly skilled entrepreneurs and some skilled entrepreneurs are attracted, all unskilled entrepreneurs are deterred; more precisely, fixing any $u_E \in \mathcal{U}$, $\bar{\alpha}_E(u_E, \tilde{R}, \tau) < \alpha$ for all $(\tilde{R}, \tau) \in \mathcal{R}_P$ and for some $(\tilde{R}, \tau) \in \mathcal{R}_{b,S}$, and $\bar{\alpha}_E(u_E, \tilde{R}, \tau) \geq \alpha$ for all $(\tilde{R}, \tau) \in \mathcal{R}_{b,U}$.

(ii) Any entrepreneur who is attracted by the first-loss scheme is attractive to risk-neutral funders; more precisely, $\bar{\alpha}_F(u_F, \tilde{R}, \tau) > \alpha$ for $u_F \in \mathcal{U}_{0,1}$ and any $u_E \in \mathcal{U}$ and $(\tilde{R}, \tau) \in \mathcal{R}_b$ with $\bar{\alpha}_E(u_E, \tilde{R}, \tau) < \alpha$.

Note that $L < U$ as long as $\frac{1-b}{e^r - b} > \frac{\gamma w}{1-w}$, which holds for reasonable parameter values, e.g., for $\gamma = 0$. In the following, we explain the intuition of Theorem 3. For simplicity, we assume $r = 0$ in the following discussion; in this case $U = \min\{\gamma w/(1-w) + wb/((1-w)(1-b)), 1\}$. With a liquidation boundary b, the net return rate of a project is $b - 1$ at the liquidation time. Consequently, the option-to-default is $\alpha((1-w)/w)(1-b)$ and the first-loss coverage is $\min\{\gamma + (1-\gamma)b, ((1-w)/w)(1-b)\}$ when the fund is liquidated. When $\alpha < U$, the net value of these two components is negative from the entrepreneur's point of view. Consequently, for an entrepreneur, who is assumed to be risk averse, to be attracted to raise capital for a project, it is necessary that (i) the project earns a positive expected excess return and (ii) she can increase the leverage of her project by P2P financing. The necessary condition (i) implies that this project must be attractive to a risk-neutral funder because the funder has a positive net value of the first-loss coverage and the option-to-default and shares the positive expected excess return generated by the project. The necessary condition (i) also implies that all unskilled entrepreneurs will be deterred. The necessary condition (ii) implies $\alpha > L$, as we can see from (5), and it is also sufficient for all perfectly skilled entrepreneurs to be attracted because their projects always outperform the risk-free asset and never yield a loss. Moreover, given $\alpha \in (L, U)$ and utility function u_E, a skilled entrepreneur will be attracted as long as her project yields a sufficiently high expected return and the probability of her project underperforming the risk-free asset is sufficiently small.

To summarize, the incentive rate in the first-loss scheme need to be (i) larger than a threshold, so-called the *attracting threshold*, to attract some entrepreneurs and (ii) smaller than another threshold, so-called the *deterring threshold*, to deter all entrepreneurs who are unappealing to the funder. The attracting threshold L is one that sets equal the increase in the leverage of the project due to performance fees and the decrease in the leverage due to the cash component in the first-loss deposit. The deterring threshold U is one that sets equal the option-to-default and the first-loss coverage.

With a higher liquidation boundary b, the value of the entrepreneur's first-loss deposit becomes higher and the loss of the funders' stake becomes smaller when $r = 0$. This explains why the deterring threshold U, which makes the option-to-default and the first-loss coverage equal, is increasing in b.

With a larger γ, the entrepreneur's first-loss deposit consists of more cash and thus becomes more valuable at liquidation. Consequently, the deterring threshold U, which makes the option-to-default and the first-loss coverage equal, becomes higher. On the other hand, a larger cash component also locks more of a entrepreneur's capital in the risk-free asset, so the attracting threshold L, which sets the net change of the leverage of the project by P2P financing to be zero, becomes larger. Similar reasoning explains why L and U are increasing in w.

4.4 First-Loss Scheme with Liquidation for Risk-Averse Funders: A Positive Result

When a funder is risk-averse, there is a misalignment between the risk attitude of this funder and a risk-neutral entrepreneur. A risk-neutral entrepreneur can take a highly risky strategy but is still attracted to raise capital for a project as long as the expected excess return of the project is sufficiently high. Thus, it is expected that if (i) the entrepreneur's project entails a large amount of risk or (ii) the funder is extremely risk averse, the project is unappealing to the funder. In other words, we cannot find a compensation scheme that is separation-effective for *any* risk-averse funder. Consequently, we have to impose a bound on the level of riskiness of projects and a bound on the degree of risk aversion of the funder. The risk aversion degree is quantified by the local RRAD and the LAD in the small, and the riskiness of a project with return $(\tilde{R}, \tau)$ is quantified by the *liquidation odds*, i.e., by $\mathbb{P}(\tau \le 1)/\mathbb{P}(\tau > 1)$.

Theorem 4 (Risk-Averse Funders)**.** *Suppose that there is a liquidation boundary* $b \in (0, 1)$ *and* *consider the first-loss scheme. Fix $\delta \geq 0$, $\lambda \geq 1$, and $\mathcal{O} > 0$, and define*

$$\mathbb{U}(\delta, \lambda, \mathcal{O}) := \min \left\{ L + \frac{(U-L)(1-b)(1-L)\mathcal{O}}{u_\delta^{-1}\big(-\lambda u_\delta\big(1-(1-U)(1-e^{-r}b)\big)\mathcal{O}\big) - 1 + (U-L)(1-b)\mathcal{O}}, U \right\}. \quad (13)$$

Suppose $L < \mathbb{U}(\delta, \lambda, \mathcal{O})$. Then, for any given $\alpha \in (L, \mathbb{U}(\delta, \lambda, \mathcal{O}))$, the following are true:

(i) All perfectly skilled entrepreneurs are attracted, some skilled entrepreneurs are attracted, and all unskilled entrepreneurs are deterred; [11] *more precisely, fixing any $u_E \in \mathcal{U}$, $\bar{\alpha}_E(u_E, \tilde{R}, \tau) < \alpha$ for all $(\tilde{R}, \tau) \in \mathcal{R}_P$ and for some $(\tilde{R}, \tau) \in \mathcal{R}_{b,S}$ with $\mathbb{P}(\tau \leq 1)/\mathbb{P}(\tau > 1) \leq \mathcal{O}$, and $\bar{\alpha}_E(u_E, \tilde{R}, \tau) \geq \alpha$ for any $(\tilde{R}, \tau) \in \mathcal{R}_{b,U}$.*

(ii) Any entrepreneur who takes limited risk and is attracted by the first-loss scheme is attractive to all funders with bounded risk aversion; more precisely, $\bar{\alpha}_F(u_F, \tilde{R}, \tau) > \alpha$ for all $u_F \in \mathcal{U}_{\delta,\lambda}$ and any $u_E \in \mathcal{U}$ and $(\tilde{R}, \tau) \in \mathcal{R}_b$ with $\mathbb{P}(\tau \leq 1)/\mathbb{P}(\tau > 1) \leq \mathcal{O}$ and $\bar{\alpha}_E(u_E, \tilde{R}, \tau) < \alpha$.

Note that $\mathbb{U}(\delta, \lambda, \mathcal{O}) > L$ as long as $U > L$. Theorem 4 shows that the first-loss scheme is able to attract some skilled entrepreneurs who take limited risk in that the odds of liquidation of their projects are bounded, and at the same time to deter all the entrepreneurs who take limited risk and are unappealing to some funder with bounded risk aversion degree.

5 Case Studies

5.1 A Case of P2P Private Financing: TopWater Capital

TopWater Capital, an asset management firm in the U.S., attracts many young fund managers. The fund managers are required to fund 10% of their account from their own pockets, and the managers' stake is placed in a first-loss position to absorb potential fund losses. The manager is compensated for potentially covering some of the investors' loss: The incentive rate for the manager is 40%, much higher than 20%, a common incentive rate in the traditional scheme. The whole project is liquidated once the manager's stake depletes to 10% of her initial stake in the account.[12]

[11] The theorem indicates that projects with high odds of liquidation are screened out automatically by the first-loss scheme with a liquidation boundary. Sometimes, projects that lead to high odds of liquidation might be screened out without designing a scheme, if enough historical data is available. For example, consider the context of private equity financing in the hedge fund industry. Suppose that the odds of fund liquidation of a fund manager's strategy are higher than 2. Then, the probability that the fund is liquidated in one period, e.g., one year, is more than 2/3. Suppose that investors can observe the record of the manager's performance for three years. Then, the chance of observing a bad performance at least once is higher than $1-(1/3)^3 = 96.3\%$. Thus, investors are more than 96.3% certain that fund managers whose strategies have the odds of fund liquidation more than 2 can be screened out based on a three-year performance record only.

[12] If the manager's stake is less than 10% of the total equity due to losses in previous periods, then any profit that the manager received in the current period is first used to top up the manager's stake back to

For example, if an account amounts to \$50 million at the beginning, then \$5 million in this account must be funded by the manager. The manager's stake is an equity deposit. Suppose that after the first period the account loses \$1 million. Then, the manager covers the loss completely, so the managers' stake decreases to \$4 million, while the investors' stake remains at \$45 million. The whole account is liquidated once the manager's stake depletes to 0.5 million.

Note that in the single-period setting, the scheme used by TopWater Capital is a first-loss scheme with a 100% equity deposit, except that the liquidation boundary is contracted on the manager's stake rather than on the investor's stake. This, however, is the special case of our previous setting. Indeed, in the TopWater Capital scheme, because the manager's first-loss deposit consists of equities only, the manager's stake depletes to 10% of her initial stake if and only if her strategy incurs a loss of $(1-10\%)\times 10\% = 9\%$. Thus, the TopWater Capital scheme is the special case of the general first-loss scheme with $w = 10\%$, $\gamma = 0$, $b = 1 - 9\% = 0.91$, and $\alpha = 40\%$. Also note that in the following discussion, the fund manager plays the role of the entrepreneur and the fund investors are the project funders.

In the following, we study whether the compensation scheme used by TopWater Capital is separation-effective, i.e., whether it can deter all managers who are unappealing to the investors while attract at least some skilled managers. We assume that the utility function of an entrepreneur is quite general, $u_E \in \mathcal{U}$, i.e., concave in its domain.

We consider three cases of the utility function of the investors: the case of risk-neutral investors, the case of investors with preferences represented by the classical expected utility theory (i.e. the local loss aversion is no more than 1), and the case of investors with preferences represented by both the classical expected utility and the prospect theory (i.e. the local loss aversion can be larger than 1). In the first case, the range of incentive rates to make the scheme separation-effective is (L, U), where L and U are given as in (12). In the second and third cases, such a range is $(L, \mathbb{U}(\delta, \lambda, \mathcal{O}))$, where $\mathbb{U}$ is given by (13). Using the existing evidence of the effects of wage changes, Chetty (2006) estimated the local RRAD in the EU preferences to be bounded by 2. Thus, we set $\delta = 2$ and $\lambda = 1$ in case two. On the other hand, we set $\delta = 0$ and $\lambda = 2.75$ in case three, to have two piecewise linear utility functions as is typical in the literature of loss aversion.

10%, and then the remaining profit is shared by the manager and the investors. See more discussions on TopWater Capital at http://www.hedgefundlawblog.com/hedge-fund-risk-sharing-capital-programs.html and https://www.rothschild.com/NewsPopup.aspx?Id=2147486623.

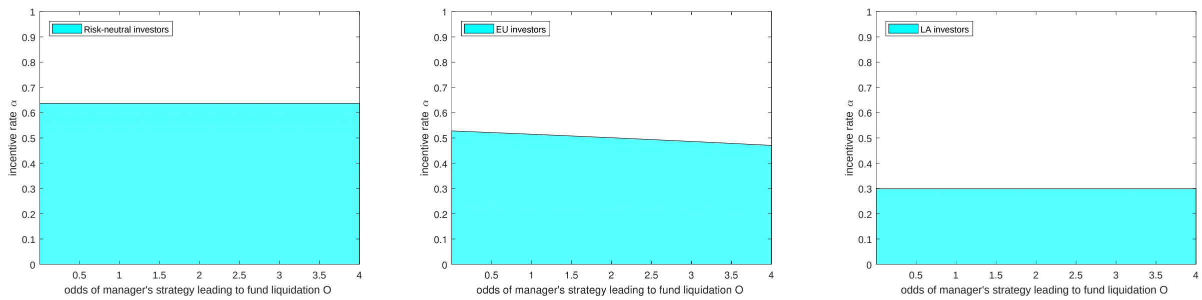

Figure 1. Range of separation-effective incentive rates in the first-loss scheme with respect to $\mathcal{O}$, the bound on the odds of managers' strategies leading to fund liquidation. Set $w = 10\%$, $\gamma = 0$, $b = 0.91$, and $r = 5\%$. The utility function of an entrepreneur $u_E \in \mathcal{U}$. The shaded areas in the left, middle, and right panes represent the ranges corresponding to the cases of risk-neutral investors, EU investors ($\delta = 2$ and $\lambda = 1$), and LA investors ($\delta = 0$ and $\lambda = 2.75$), respectively. Note that for LA investors, even if the incentive rate range is wide, the 40% incentive rate used by TopWater does not lie in the range.

We first plot the incentive rate range with respect to $\mathcal{O}$, the bound on the odds of managers' strategies leading to fund liquidation. We set $r = 5\%$, a reasonable number. The ranges corresponding to the cases of risk-neutral investors, EU investors, and PT investors, are depicted by the shaded areas in the left, middle, and right panes of Figure 1, respectively. We can see that for risk-neutral and EU investors, the first-loss scheme with 40% incentive rate, which is the current practice of TopWater Capital, is separation-effective. It is interesting to note that for PT investors, even if the incentive rate range is wide, the 40% incentive rate used by TopWater does not lie in the range!

We then study the effect of the interest rate r by plotting the range of separation-effective incentive rates with respect to the interest rate r. The ranges corresponding the cases of risk-neutral investors, EU investors, and PT investors, are depicted by shaded areas in the left, middle, and right panes of Figure 2, respectively. We can see that the ranges shrink as r becomes larger; more precisely, the deterring threshold become smaller. This can be explained as follows: with a higher risk-free return rate, the option-to-default becomes larger but the first-loss coverage, which is equal to the investors' loss in the case we plot in Figure 2, remains the same. Thus, one needs to lower the incentive rate to deter those managers who are unattractive to investors.

We also vary the contract parameters γ and w and depict their effect on the range of separation-effective incentive rates in the top panes and bottom panes of Figure 3, respectively. As discussed in Section 4.3, a larger value of γ shifts the range upwards and a larger value of w expands the

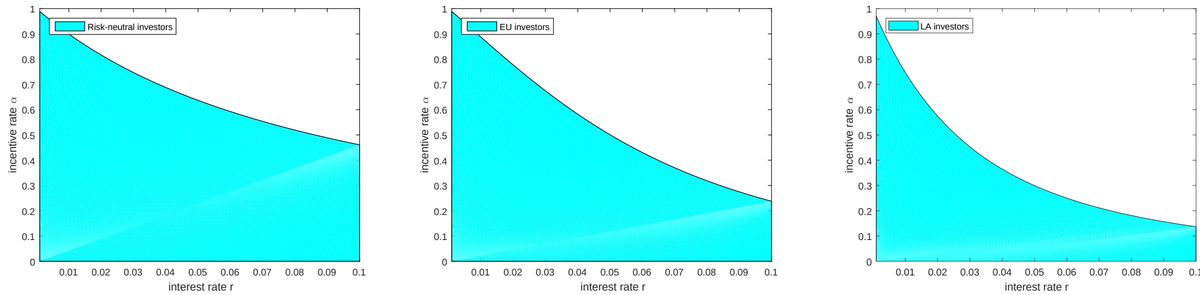

Figure 2. Range of separation-effective incentive rates in the first-loss scheme with respect to r, the risk-free rate. Set $w = 10\%$, $\gamma = 0$, $b = 0.91$, and $\mathcal{O} = 2$. The utility function of an entrepreneur $u_E \in \mathcal{U}$. The shaded areas in the left, middle, and right panes represent the ranges corresponding to the cases of risk-neutral investors, EU investors ($\delta = 2$ and $\lambda = 1$), and LA investors ($\delta = 0$ and $\lambda = 2.75$), respectively.

range.

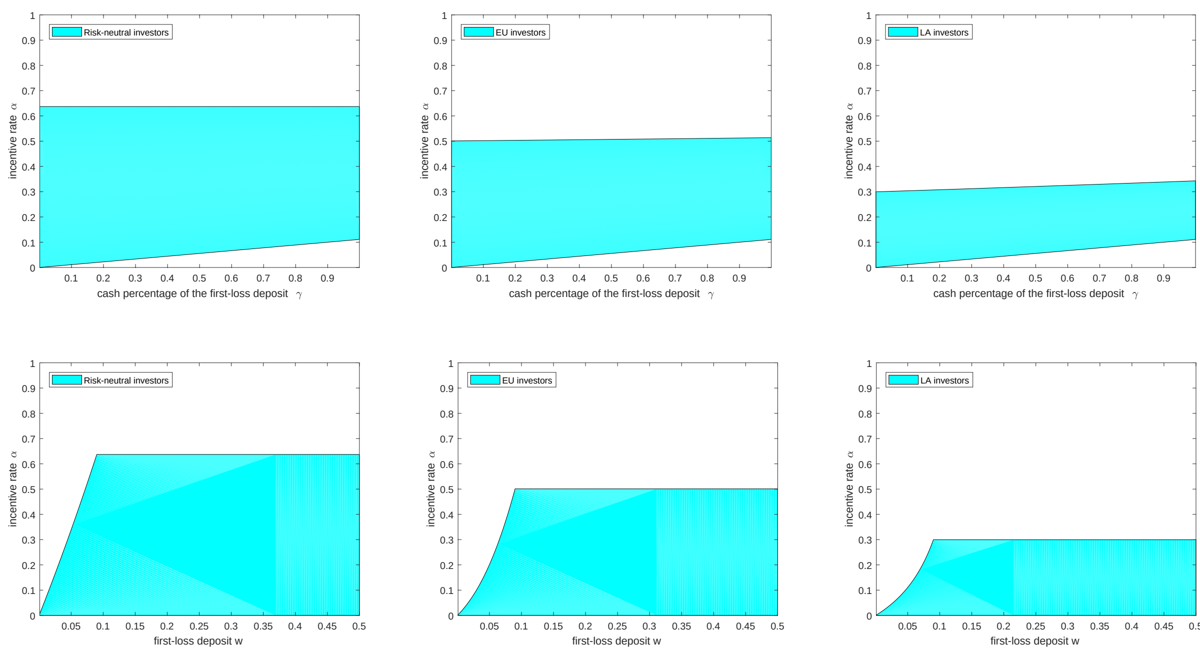

Figure 3. Range of separation-effective incentive rates in the first-loss scheme with respect to the cash percentage of the first-loss deposit γ (top panes) and with respect to the first-loss deposit w (bottom panes). Set $b = 0.91$, $r = 5\%$, and $\mathcal{O} = 2$. Set $w = 10\%$ when we variate γ. Set $\gamma = 0$ when we variate w. The utility function of an entrepreneur $u_E \in \mathcal{U}$. The ranges corresponding to the cases of risk-neutral investors, EU investors ($\delta = 2$ and $\lambda = 1$), and LA investors ($\delta = 0$ and $\lambda = 2.75$), are depicted as shaded areas in the left, middle, and right panes, respectively. Note that for LA investors, even if the incentive rate range is wide, the 40% incentive rate used by TopWater does not lie in the range.

Next, we plot the range of separation-effective incentive rates with respect to the liquidation boundary b in Figure 4. Note that for PT investors, even if the incentive rate range is wide, the

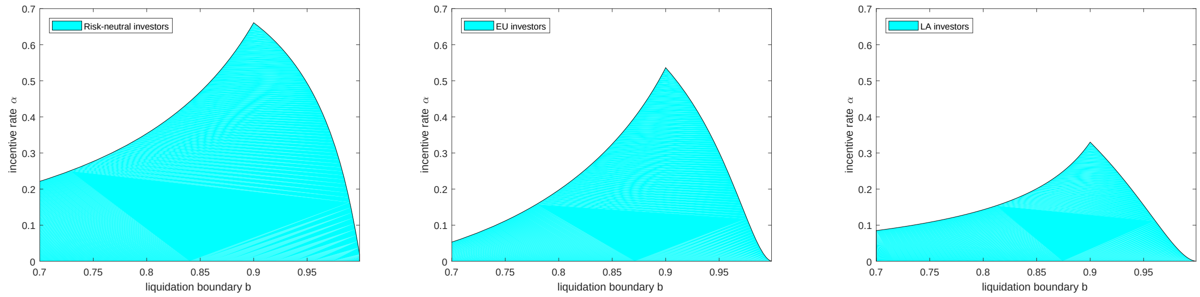

Figure 4. Range of separation-effective incentive rates in the first-loss scheme with respect to the liquidation boundary b. Set $w = 10\%$, $\gamma = 0$, $r = 5\%$, and $\mathcal{O} = 2$. The utility function of an entrepreneur $u_E \in \mathcal{U}$. The ranges corresponding to the cases of risk-neutral investors, EU investors ($\delta = 2$ and $\lambda = 1$), and LA investors ($\delta = 0$ and $\lambda = 2.75$), are depicted as shaded areas in the left, middle, and right panes, respectively.

40% incentive rate used by TopWater does not lie in the range.

We can see that the upper bound of the range, i.e., the deterring threshold, is first increasing and then decreasing with respect to b. This can be explained as follows: When b is small, the manager's first-loss deposit is insufficient to cover investors' losses, so the first-loss coverage is equal to the first-loss deposit and thus is increasing with respect to b. Because the option-to-default is decreasing with respect to b, the deterring threshold, which sets equal the first-loss coverage and option to default, is increasing with respect to b. When b is large, the manager's first-loss capital is sufficient to cover the investors' loss, so the first-loss coverage is equal to the investors' loss. Note that the calculation of the investors' loss is benchmarked to the initial value of the investors' stake while the option-to-default is the shortage of the investors' stake relative to the risk-free return of the investors' initial stake. Consequently, the ratio of the option-to-default and the first-loss capital is decreasing with respect to b and thus the deterring threshold is decreasing with respect to b. In particular, when b approaches to 1, the option-to-default converges to a positive constant and the first-loss coverage goes to zero, so the former always dominate the latter and thus some unskilled managers will be attracted.

Finally, the most interesting aspect of the TopWater capital is that the liquidation boundary is $b = 0.91$, which means that if the fund can be liquidated exactly at $b = 0.91$ then the investors have no losses because the first-loss deposit is 10%. How can this be? The main reason for this is that when the fund is liquidated the final value obtained is not 0.91 but something lower than

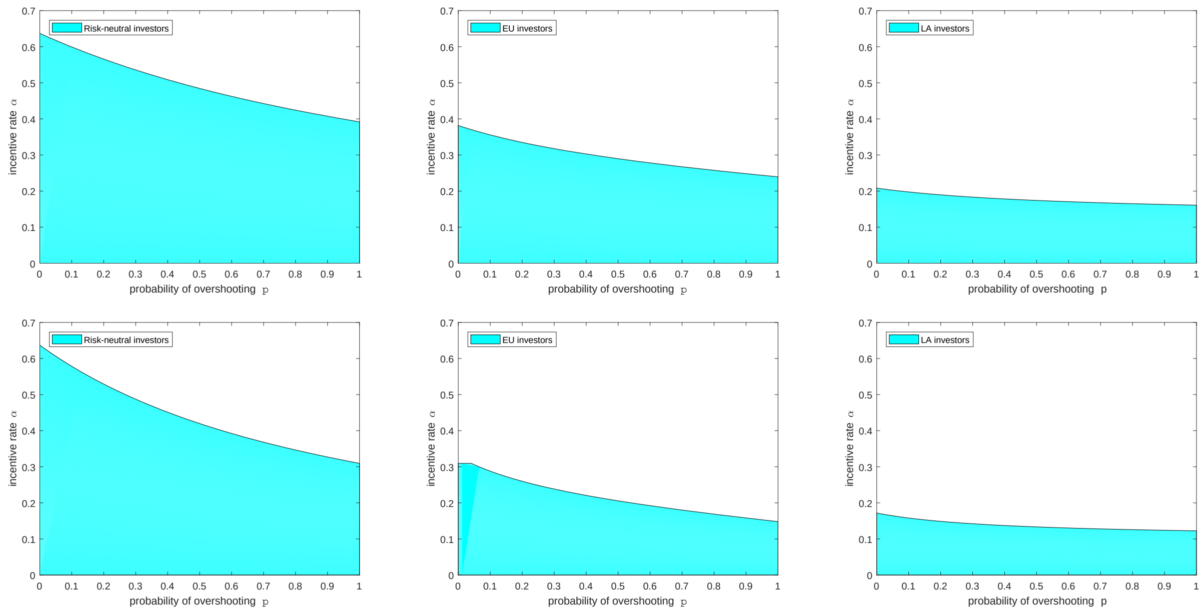

Figure 5. Range of separation-effective incentive rates in the first-loss scheme with respect to the probability of overshooting. Set $w = 10\%$, $\gamma = 0$, $b = 0.91$, $r = 5\%$, and $\mathcal{O} = 2$. Set $k = 0.1$ in the first row and $k = 0.15$ in the second row, i.e., the overshoot percentage below the liquidation boundary. The utility function of an entrepreneur $u_E \in \mathcal{U}$. The ranges corresponding to the cases of risk-neutral investors, EU investors (with local RRAD bounded by $\delta = 2$), and LA investors (with local RRAD bounded by $\delta = 0$ and LAD in the small bounded by $\lambda = 2.75$), are depicted as shaded areas in the left, middle, and right panes, respectively.

that, due to the possibility of overshoot over the boundary and liquidation costs.

In Appendix D, we consider the extension that the liquidation value is a random variable smaller than b. In particuarl, Corollary 12, 13 and 14 in Appendix D provide a range of separation-effective incentive rates when the loss distribution is random. Suppose that conditional on fund liquidation, the overshoot size $\tilde{\xi}$ is a binary random variable taking values in $\{0, kb\}$, where $k \in (0, 1)$. We again consider the parameter values in the TopWater Capital scheme, namely $w = 10\%$, $\gamma = 0$, $b = 0.91$, and $r = 5\%$. Set $\mathcal{O} = 2$. Set $k = 0.1$ and $k = 0.15$, i.e., the overshoot percentage below the liquidation boundary. We plot the range of separation-effective incentive rates with respect to the probability of overshooting p in Figure 5, according to (D.1), (D.2) for risk-neutral, EU and LA investors, respectively. We can observe that the range is shrinking with respect to the probability of overshooting. This is because the larger the overshooting probability is, the smaller the average return rate at liquidation is, and thus a smaller incentive rate is needed to deter unskilled entrepreneurs.

5.2 A Case of P2P Public Financing with an Unobservable Liquidation Boundary: Crudecoin

The Initial Coin Offerings (ICOs) use blockchain technology to do fund raising and to manage profit sharing of the project. Our proposed contract scheme is especially suitable for this application. More precisely, the blockchain technology used in ICOs can be powerful enough to track the project value, to enforce the implementation of the liquidation boundary, and to contract the profit-sharing in the first-loss scheme, via smart contracts without human intervention. Our main results, namely Theorem 3 and 4, provide a quantitative guideline to design the first-loss scheme in ICOs to avoid adverse selection. More precisely, for each choice of the retention of tokens w, the proportion γ of the entrepreneur's capital held in the risk-free asset, and the liquidation boundary b, (12) and (13) provide a range of incentive rates for the entrepreneur that are effective to avoid adverse selection.

In some situations, one cannot observe the project value directly or it is difficult to mark the project to the market value. Consequently, the liquidation time τ cannot be contracted on the project value directly. It is, however, still possible to find a market variable that is observable and correlated to the project value, and then we can contract the liquidation based on the observed market variable.

For example, Crudecoin is a cryptocurrency issued by Wellsite, a social network platform with an integrated marketplace built specifically for the oil and gas industry.[13] On the platform, oil providers and operators can find each other with ease and transact with each other with Crudecoin. Unlike the case of TopWater Capital, the asset/market value of Wellsite cannot be directly observed, so we cannot contract liquidation on the Wellsite's asset value directly. The asset value, however, is correlated to certain oil index, such as crude oil price, in the market, because the firm's business relies on the oil industry. Then, we can contract liquidation on the oil index. In the following, we study whether such a strategy can screen out unappealing oil-related projects.

We now present a method to calculate the range of incentive rates, such that a given entrepreneur is willing to raise capital for her project and a funder finds the project to be attractive. Consider an entrepreneur with a project generating gross return $1 + m_t$, and assume $1 + m_t$ follows a geometric

[13] See `https://bitcoinexchangeguide.com/crudecoin-ico-wcc-token/` for more details.

Brownian motion:

$$d(1+m_t)/(1+m_t) = \mu dt + \sigma dW_t.$$

Suppose that the project is related to some observable market variable, whose value P_t follows

$$dP_t/P_t = \mu_P dt + \sigma_P dW_t^P,$$

where $dW_t^P dW_t = \rho dt$ for some $\rho \in (0,1)$. We contract the liquidation time to be $\tau := \inf\{t \in [0,1] \mid P_t/P_0 \leq b\}$ for some $b \in (0,1)$. Denote by Δ the set in which μ and σ taking values. Fixing μ_P and σ_P, we define

$$L \equiv L_\Delta := \inf\{\alpha_E(\mathbb{I},\mu,\sigma) : (\mu,\sigma) \in \Delta \text{ such that } \alpha_E(\mathbb{I},\mu,\sigma) \leq \alpha_F(\mathbb{I},\mu,\sigma)\},$$

where $\mathbb{I}$ denotes the identity function and thus represents risk-neutral attitude, $\alpha_E(\mathbb{I},\mu,\sigma)$ is the lower bound of incentive rate such that a risk-neutral entrepreneur is willing to raise capital for her project (μ,σ), and $\alpha_F(\mathbb{I},\mu,\sigma)$ is the upper bound of incentive rate such that all the risk-neutral funders find this project attractive. Consequently, L is the smallest incentive rate α such that a risk-neutral entrepreneur is willing to raise the capital for her project while the risk-neutral funders also find the project attractive. Define

$$U \equiv U_\Delta := \sup\{\alpha \in (0,1) : \alpha \geq \bar{\alpha}_E(\mathbb{I},\mu,\sigma) \text{ implies } \alpha \leq \bar{\alpha}_F(\mathbb{I},\mu,\sigma) \text{ for all } (\mu,\sigma) \in \Delta\}.$$

Hence, U is the largest incentive rate α such that once a risk-neutral entrepreneur is willing to raise capital for her project (μ,σ) then all the risk-neutral funders find it attractive. Suppose $L < U$. Then, for any $\alpha \in (L,U)$, there exists a risk-neutral entrepreneur who is attracted and any risk-neutral entrepreneurs who are unappealing to risk-neutral funders are deterred. For risk-averse funders with local RRAD bounded by δ and LAD in the small bounded by λ, define

$$\mathbb{L} \equiv \mathbb{L}_\Delta := \inf\{\alpha_E(\mathbb{I},\mu,\sigma) : (\mu,\sigma) \in \Delta \text{ such that } \alpha_E(\mathbb{I},\mu,\sigma) \leq \alpha_F(u_F,\mu,\sigma) \text{ for all } u_F \in \mathcal{U}_{\delta,\lambda}\},$$

where $\mathcal{U}_{\delta,\lambda}$ denotes the set of utility functions with local RRAD bounded by δ and LAD in the small bounded by λ. Define

$$\mathbb{U} \equiv \mathbb{U}_\Delta := \sup\{\alpha \in (0,1) : \alpha \geq \alpha_E(\mathbb{I},\mu,\sigma) \text{ implies } \alpha \leq \alpha_F(u_F,\mu,\sigma) \text{ for all } u_F \in \mathcal{U}_{\delta,\lambda} \text{ and } (\mu,\sigma) \in \Delta\}.$$

Suppose $\mathbb{L} < \mathbb{U}$. Then, for any $\alpha \in (\mathbb{L}, \mathbb{U})$, there exists a risk-neutral entrepreneur who is attracted and any risk-neutral entrepreneurs who are unappealing to risk-averse funders whose local RRAD bounded by δ and LAD in the small bounded by λ are deterred.

Below is a numerical example for risk-neutral, EU and LA funders. Set $w = 10\%$, $\gamma = 0$, $b = 0.9$, $\mu_P = r = 5\%$, and $\sigma_P = 15\%$. For the range of μ and σ, we set $\Delta = \{(\mu, \sigma) | \mu = k\%, k \in \{100r + 1, ...100\}, \sigma = j\%, j \in \{0, 1, ...50\}, \text{ and } \mu - \sigma^2/2 \geq 0\}$. Set the correlation $\rho = 0.9$, 0.7, and 0.5, respectively. To determine L and U, fixing $(\mu, \sigma) \in \Delta$ we first derive the lower bound $\alpha_E(\mathbb{I}, \mu, \sigma)$ of the incentive rate such that the expected return of raising capital for the entrepreneur is no smaller than $\max(e^r, e^\mu)$, the expected return of investing on her own. Then we derive the upper bound $\alpha_F(\mathbb{I}, \mu, \sigma)$ of the incentive rate such that the expected return from investing in the project for the funder is no smaller than e^r. Consequently, L is the smallest $\alpha_E(\mathbb{I}, \mu, \sigma)$ for $(\mu, \sigma) \in \Delta$ such that an entrepreneur is attracted and her project is attractive to all funders, i.e., $\alpha_E(\mathbb{I}, \mu, \sigma) \leq \alpha_F(\mathbb{I}, \mu, \sigma)$. On one hand, U should be no larger than the smallest $\alpha_F(\mathbb{I}, \mu, \sigma)$ for $(\mu, \sigma) \in \Delta$ such that $\alpha_E(\mathbb{I}, \mu, \sigma) \leq \alpha_F(\mathbb{I}, \mu, \sigma)$, so any entrepreneur who enters has an attractive project for the funders. On the other hand, U should be no larger than the largest $\alpha_E(\mathbb{I}, \mu, \sigma)$ for $(\mu, \sigma) \in \Delta$ such that $\alpha_E(\mathbb{I}, \mu, \sigma) > \alpha_F(\mathbb{I}, \mu, \sigma)$, so the entrepreneurs whose projects are certainly unattractive to the funders are excluded. For risk-averse funders, $\mathbb{L}$ and $\mathbb{U}$ are derived in a same way. It shows that our proposed first-loss scheme with a liquidation boundary is separation-effective for risk-neutral funders, even when the fund asset value is not directly observable. For EU and LA funders, the separation-effective range still exists but shrinks significantly.

	Risk-neutral funders	EU funders	LA funders
$\rho = 0.9$	(3.39%, 40.4%)	(3.39%, 14.3%)	(3.39%, 11.5%)
$\rho = 0.7$	(3.29%, 40.4%)	(3.29%, 14.4%)	(3.29%, 11.7%)
$\rho = 0.5$	(3.18%, 39.2%)	(3.18%, 13.7%)	(3.18%, 9.66%)

Table 1. We set $w = 10\%$, $\gamma = 0$, $b = 0.9$, $\mu_P = r = 5\%$, and $\sigma_P = 15\%$. We set $\Delta = \{(\mu, \sigma) | \mu = k\%, k \in \{100r + 1, ...100\}, \sigma = j\%, j \in \{0, 1, ...50\}, \text{ and } \mu - \sigma^2/2 \geq 0\}$. The separation-effective ranges corresponding to the cases of risk-neutral funders, EU funders (with local RRAD bounded by $\delta = 2$), and LA funders (with local RRAD bounded by $\delta = 0$ and LAD in the small bounded by $\lambda = 2.75$), are shown in the left, middle, and right columns, respectively. Finally, we set the correlation $\rho = 0.9$, 0.7, and 0.5, respectively.

6 Conclusion

In P2P financing, a main challenge is the information asymmetry between the entrepreneur of a project and the poential funders. We proposed an incentive contract that can automatically screen out entrepreneurs who are unappealing to the funders and attract some entrepreneurs who are attractive to the funders. In contrast to standard screening contracts in principal-agent problems, our contract neither depends explicitly on the utilities of the principal and agent nor has a menu of choices.

There are two crucial components in this contract: First, there is a liquidation boundary, meaning that the project funders have the right to be informed and to withdraw their investment as soon as the return on their stake hits the boundary. Second, the entrepreneurs make a first-loss deposit (which can be a combination of cash and equity) to cover the funders' loss. Our two case studies for private and public P2P equity financing show that such a contract is effective to separate entrepreneurs with different types of skills.

Within our general framework of model-free risk preference, we cannot easily discuss optimal contracts. It is an interesting open research problem to study the optimal contract design by assuming particular risk preferences.

Appendix

In the appendix, we shall study some extensions of the model.

A Management Fees and Costs

As a common market practice in the hedge fund industry, in addition to performance fees, an entrepreneur can also take a management fee, e.g., 2% of the project funders' capital, in the traditional scheme. The management fee can be invested, so it can be regarded as the entrepreneur's equity stake in the project. Managing a project also incurs a management cost, which can be interpreted as physical costs occurred in the course of managing the project or as a participation constraint for the entrepreneur. Note that the management fee is paid from the funders' capital to the entrepreneur, while the management cost is borne by the entrepreneur and not credited to the funders' account. We assume that the management cost occurs at the terminal time 1.

Corollary 1. *Suppose that the management fee is z_1 proportion of the funders' initial capital and* *that the amount of the management cost is equal to $z_0 e^r$ times the funders' initial capital. Denote by $b \in (0,1)$ the liquidation boundary or set $b = 0$ when there is no liquidation boundary.*

(i) If $z_0 \le z_1/(1-z_1)$ and assuming that $z_1 b - z_0 e^r < \frac{w}{1-w}(e^r - b)$, then the traditional scheme always attracts some entrepreneur who is unappealing to any funder; more precisely, fixing any $\alpha \in (0,1)$ and any $u_E \in \mathcal{U}_{\delta,\lambda}$ with

$$\left(1+\frac{1-w}{w}\left(z_1-z_0\right)\right)^{\delta}\left(u_\delta\left(1+\frac{1-w}{w}\left(z_1-z_0\right)\right)-u_\delta\left(e^{-r}b+\frac{1-w}{w}(z_1 b e^{-r}-z_0)\right)\lambda\right)$$
$$< (1-e^{-r}b)\left(1+(\alpha+z_1)\frac{1-w}{w}\right), \tag{A.1}$$

there exists $(\tilde{R},\tau) \in \mathcal{R}_b$ such that $\bar{\alpha}_E(u_E,\tilde{R},\tau) < \alpha$ and $\bar{\alpha}_F(u_F,\tilde{R},\tau) \le \alpha$ for any $u_F \in \mathcal{U}$.

(ii) If $z_0 > z_1/(1-z_1)$, then given any

$$\alpha \in \left(0, \frac{z_0(1-z_1)-z_1}{1-be^{-r}+z_0}\right), \tag{A.2}$$

we have the following:

(a) Any entrepreneur who is attracted by the traditional scheme is attractive to risk-neutral funders; more precisely, $\bar{\alpha}_F(u_F,\tilde{R},\tau) > \alpha$ for $u_F \in \mathcal{U}_{1,0}$ and any $u_E \in \mathcal{U}$ and $(\tilde{R},\tau) \in \mathcal{R}_b$ with $\bar{\alpha}_E(u_E,\tilde{R},\tau) < \alpha$; and

(b) some skilled entrepreneur is attracted; more precisely, fixing any $u_E \in \mathcal{U}$, $\bar{\alpha}_E(u_E,\tilde{R},\tau) < \alpha$ for some $(\tilde{R},\tau) \in \mathcal{R}_{b,S}$.

(iii) If $z_0 > z_1/(1-z_1)$ and $\alpha > \frac{z_0(1-z_1)-z_1}{1-be^{-r}+z_0}$, then the traditional scheme always attracts some risk-neutral entrepreneur who is unappealing to all funders; more precisely, there exists $(\tilde{R},\tau) \in \mathcal{R}$ such that $\bar{\alpha}_E(u_E,\tilde{R},\tau) < \alpha$ for $u_E \in \mathcal{U}_{0,1}$ and $\bar{\alpha}_F(u_F,\tilde{R},\tau) \le \alpha$ for all $u_F \in \mathcal{U}$.

Corollary 1 shows that the traditional scheme can be separation-effective as long as the management cost dominates the management fee (i.e., $z_0 > z_1/(1-z_1)$) and the incentive rate is within a range as specified in (A.2), and this condition is nearly necessary. Intuitively, when the management cost dominates the management fee, the entrepreneur's project must be able to yield a high return if she is attracted, so the entrepreneur can be attractive to the funders. The range (A.2), however, tends to be narrow because z_0 is typically a small number (e.g., 4%), so practically the traditional scheme is still not very effective to separate entrepreneurs who are attractive to funders from those who are unappealing to funders. For example, suppose $w = 10\%$, $z_0 = 4\%$, $z_1 = 2\%$, $r = 5\%$, and $b = 0.91$, then the interval (A.2) becomes $(0, 11.01\%)$, which is much narrower than the range of separation-effective incentive rates in the first-loss scheme; see Corollary 2 and Figure 6.

Corollary 2. *Consider the first-loss scheme, where the management fee is z_1 proportion of the funders' initial capital and the amount of the management cost is equal to $z_0 e^r$ times the funders' initial capital. Suppose there is no liquidation boundary, then the following are true:*

(i) When $z_0 < z_1$, the first-loss scheme attracts some risk-neutral entrepreneur who is unappealing to all funders; more precisely, fixing any $\alpha \in (0,1)$ and $u_E \in \mathcal{U}_{0,1}$, there exists $(\tilde{R}, \tau) \in \mathcal{R}_{0,U}$ such that $\bar{\alpha}_E(u_E, \tilde{R}, \tau) < \alpha$ and $\bar{\alpha}_F(u_F, \tilde{R}, \tau) \le \alpha$ for all $u_F \in \mathcal{U}$.

(ii) When $z_0 \ge z_1$ and fixing any $\alpha \le \min\{L - z_1, e^{-r}\}$, all entrepreneurs are deterred; more precisely, $\bar{\alpha}_E(u_E, \tilde{R}, \tau) \ge \alpha$ for all $u_E \in \mathcal{U}$ and $(\tilde{R}, \tau) \in \mathcal{R}_0$.

(iii) When $z_0 \ge z_1$ and fixing any $\alpha > L + \left(\frac{(1-L)z_0}{1-L+z_0} - z_1\right)^+$, the first-loss scheme attracts some risk-neutral entrepreneur who is unappealing to all funders; more precisely, there exists $(\tilde{R}, \tau) \in \mathcal{R}_0$ such that $\bar{\alpha}_E(u_E, \tilde{R}, \tau) < \alpha$ for $u_E \in \mathcal{U}_{0,1}$ and $\bar{\alpha}_F(u_F, \tilde{R}, \tau) \le \alpha$ for all $u_F \in \mathcal{U}$.

Corollary 2 shows that without a liquidation boundary, the first-loss scheme is not separation-effective unless $z_0 \ge z_1$ and

$$\min\{L - z_1, e^{-r}\} < \alpha \le L + \left(\frac{(1-L)z_0}{1-L+z_0} - z_1\right)^+. \tag{A.3}$$

The latter condition holds for a narrow range of incentive rates because z_0 is small. For instance, if $\gamma = 0$ and thus $L = 0$, condition (A.3) becomes $\alpha \le \left(z_0/(1+z_0) - z_1\right)^+$, where the right-hand side of the inequality is a small number. Thus, without a liquidation boundary, the first-loss scheme is not effective to separate attractive entrepreneurs from unappealing ones.

Corollary 3. *Consider the first-loss scheme, where the management fee is z_1 proportion of the funders' initial capital and the amount of the management cost is equal to $z_0 e^r$ times the funders' initial capital. Suppose there is a liquidation boundary $b \in (0,1)$ and $z_0 > z_1 \max\{(1-L)/(1-z_1), 1\}$,* *then for any*

$$\alpha \in \big(\max(L - z_1, 0), \min\{1 - z_1 - (z_1/z_0)(1 - L), U\}\big), \tag{A.4}$$

the following are true:

(i) Any entrepreneur who is attracted by the first-loss scheme is attractive to risk-neutral funders; more precisely, $\bar{\alpha}_F(u_F, \tilde{R}, \tau) > \alpha$ *for* $u_F \in \mathcal{U}_{0,1}$ *and any* $u_E \in \mathcal{U}$ *and* $(\tilde{R}, \tau) \in \mathcal{R}_b$ *with* $\bar{\alpha}_E(u_E, \tilde{R}, \tau) < \alpha$.

(ii) Some skilled entrepreneurs are attracted and all unskilled entrepreneurs are deterred; more precisely, fixing any $u_E \in \mathcal{U}$, $\bar{\alpha}_E(u_E, \tilde{R}, \tau) < \alpha$ *for some* $(\tilde{R}, \tau) \in \mathcal{R}_{b,S}$, *and* $\bar{\alpha}_E(u_E, \tilde{R}, \tau) \geq \alpha$ *for all* $(\tilde{R}, \tau) \in \mathcal{R}_{b,U}$.

Corollary 3 is parallel to Theorem 3, showing that the first-loss scheme with the liquidation boundary is separation-effective for risk-neutral funders. For risk-averse funders, the following corollary shows that the first-loss scheme with the liquidation boundary can also be separation-effective.

Corollary 4. *Consider the first-loss scheme with liquidation boundary* $b \in (0, 1)$. *Suppose that the management fee is* z_1 *proportion of the funders' initial capital and the amount of the management cost is equal to* $z_0 e^r$ *times the funders' initial capital. Assume* $z_0 > z_1 \max\{(1 - L)/(1 - z_1), 1\}$ *and* $z_1 < 1 - U$. *For any* $\delta \geq 0$, $\lambda > 0$, *and* $\mathcal{O} > 0$, *define*

$$\begin{aligned}
\bar{V}(\delta, \lambda, \mathcal{O}) :&= L - z_1 + \big[\big((U - L)(1 - b) + z_0 - z_1 b\big)(1 - L)\mathcal{O}\big] \times \\
&\quad \Big[u_\delta^{-1}\Big(- \lambda u_\delta\Big(1 - \big((1 - U)(1 - e^{-r} b) + z_1 e^{-r} b\big)\Big)\mathcal{O}\Big) - 1 \\
&\qquad + \big((U - L)(1 - b) + z_0 - z_1 b\big)\mathcal{O}\Big]^{-1}, \qquad \text{(A.5)} \\
\bar{\mathbb{U}}(\delta, \lambda, \mathcal{O}) :&= \min\{\bar{V}(\delta, \lambda, \mathcal{O}), 1 - z_1 - (z_1/z_0)(1 - L), U\}. \qquad \text{(A.6)}
\end{aligned}$$

Suppose $L - z_1 < \bar{\mathbb{U}}(\delta, \lambda, \mathcal{O})$. *Then, for any* $\alpha \in \big(\max(L - z_1, 0), \bar{\mathbb{U}}(\delta, \lambda, \mathcal{O})\big)$, *the following are true:*

(i) Any entrepreneur who takes limited risk and is attracted by the first-loss scheme is attractive to all funders with bounded risk aversion; more precisely, $\bar{\alpha}_F(u_F, \tilde{R}, \tau) > \alpha$ *for all* $u_F \in \mathcal{U}_{\delta,\lambda}$, $u_E \in \mathcal{U}$, *and* $(\tilde{R}, \tau) \in \mathcal{R}_b$ *with* $\mathbb{P}(\tau \leq 1)/\mathbb{P}(\tau > 1) \leq \mathcal{O}$ *and* $\bar{\alpha}_E(u_E, \tilde{R}, \tau) < \alpha$.

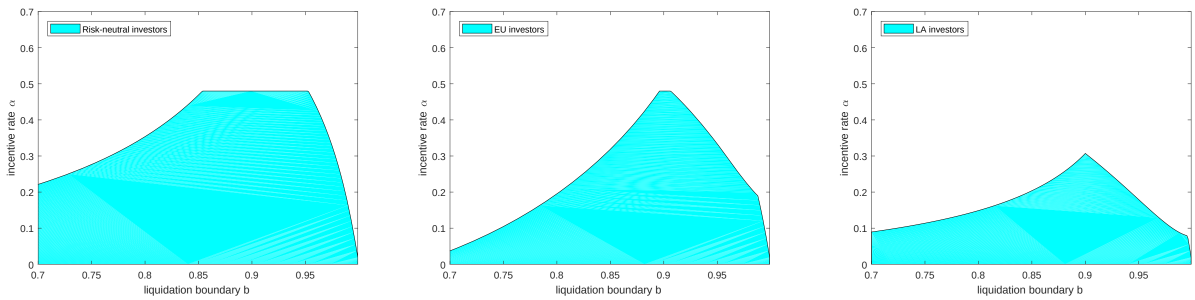

Figure 6. Range of separation-effective incentive rates in the first-loss scheme with respect to the liquidation boundary b. Set $w = 10\%$, $\gamma = 0$, $r = 5\%$, $\mathcal{O} = 2$, $z_0 = 4\%$, and $z_1 = 2\%$. The utility function of an entrepreneur $u_E \in \mathcal{U}$. The ranges corresponding to the cases of risk-neutral funders, EU funders (with local RRAD bounded by $\delta = 2$), and LA funders (with local RRAD bounded by $\delta = 0$ and LAD in the small bounded by $\lambda = 2.75$), are depicted as shaded areas in the left, middle, and right panes, respectively.

(ii) *Some skilled entrepreneurs are attracted and all unskilled entrepreneurs are deterred; more precisely, fixing any $u_E \in \mathcal{U}$, $\bar{\alpha}_E(u_E, \tilde{R}, \tau) < \alpha$ for some $(\tilde{R}, \tau) \in \mathcal{R}_{b,S}$ with $\mathbb{P}(\tau \le 1)/\mathbb{P}(\tau > 1) \le \mathcal{O}$, and $\bar{\alpha}_E(u_E, \tilde{R}, \tau) \ge \alpha$ for any $(\tilde{R}, \tau) \in \mathcal{R}_{b,U}$.*

Corollary 4 is parallel to Theorem 4. Note that $\bar{\mathbb{U}}(\delta, \lambda, \mathcal{O}) = \min\{\bar{V}(\delta, \lambda, \mathcal{O}), U\}$ when $z_1 = 0$, which is the same as $\mathbb{U}(\delta, \lambda, \mathcal{O})$ as defined in (13) when $z_0 = 0$ as well, and the former is increasing in z_0. Consequently, assuming a zero management fee, the presence of a management cost makes it easier to separate entrepreneurs who are attractive to funders from those who are unappealing.

Assume the same parameter values as in the Topwater Capital scheme, i.e., $w = 10\%$ and $\gamma = 0$. Set $r = 5\%$ and $\mathcal{O} = 2$, and assume $z_1 = 2\%$ and $z_0 = 4\%$. Figure 6 plots the range of separation-effective incentive rate with respect to b for risk-neutral funders (left pane), for EU funders whose local RRAD is bounded by $\delta = 2$ (middle range), and for PT funders whose local RRAD is bounded by $\delta = 1$ and whose LAD in the small is bounded by $\lambda = 3.25$ (right pane). We can see that the range is wide when b is around 90%, e.g., $b = 91\%$ as in the Topwater Capital scheme.

B Different Hurdle and Reference Rates

In practice the hurdle rate that triggers performance fee payment may be higher than the risk-free rate. The hurdle rate can be contracted directly or indirectly as a result of other provisions. For instance, under the high-water mark provision, the manager of a fund cannot collect performance fees until the fund has recovered past losses. As a result, if the fund has suffered a loss in previous periods, the hurdle rate can be higher than the risk-free rate.

On the other hand, the reference rate that triggers the first-loss payment can be different from 0. If the reference rate is contracted to be less than 0, the entrepreneur covers the funders' loss in excess of a threshold. This feature is similar to deductibles in insurance contracts. If the reference rate is contracted to be larger than 0, such as the risk-free rate, and the liquidation boundary is set to be one such that the entrepreneur's first-loss deposit never dries up (such as in Topwater Capital), the reference rate actually becomes a return guaranteed by the entrepreneur to the funders.

In the following, we assume a continuously compounded hurdle rate h so that the entrepreneur takes a performance fee at time 1 if and only if the gross return of the funders' stake is higher than e^h. We then assume a continuously compounded reference rate θ and the entrepreneur needs to cover any loss of the funders' capital relative to this reference rate. In other words, at time $\tau \wedge 1$, where τ is the liquidation time of the fund, the entrepreneur needs to cover the loss amount of $(1-w)X_0\big(e^{\theta(\tau\wedge 1)} - (\tilde{m}_{\tau\wedge 1}+1)\big)^+$ until her first-loss deposit dries up, where $\tilde{m}$ is the net return process generated by the project. We assume that $h \geq r \geq \theta$.

Corollary 5. *Assume a continuously compounded hurdle rate h and a continuously compounded reference rate θ such that $h \geq r \geq \theta$.*

(i) The traditional scheme, whether there is a liquidation boundary or not (i.e., whether $b > 0$ or $b = 0$), always attracts some risk-neutral entrepreneur who is unappealing to all funders; more precisely, there exists $(\tilde{R}, \tau) \in \mathcal{R}_{b,U}$ such that $\bar{\alpha}_E(u_E, \tilde{R}, \tau) < \alpha$ for $u_E \in \mathcal{U}_{0,1}$ and $\bar{\alpha}_F(u_F, \tilde{R}, \tau) \leq \alpha$ for all $u_F \in \mathcal{U}$.

(ii) In the first-loss scheme without a liquidation boundary, when the incentive rate is low, all the entrepreneurs are deterred; more precisely, for $\alpha \in (0, \gamma w/(1-w)]$, $\bar{\alpha}_E(u_E, \tilde{R}, \tau) \geq \alpha$ for all $u_E \in \mathcal{U}$ and $(\tilde{R}, \tau) \in \mathcal{R}_0$. When the incentive rate is high, the first-loss scheme always attracts some risk-neutral entrepreneur who is unappealing to all funders; more precisely, for $\alpha \in (\gamma w/(1-w), 1)$, there exists $(\tilde{R}, \tau) \in \mathcal{R}_{0,U}$ such that $\bar{\alpha}_E(u_E, \tilde{R}, \tau) < \alpha$ for $u_E \in \mathcal{U}_{0,1}$ and $\bar{\alpha}_F(u_F, \tilde{R}, \tau) \leq \alpha$ for all $u_F \in \mathcal{U}$.

Corollary 5 shows that even when the hurdle rate and the reference rate are different from the risk-free rate, the traditional scheme, whether with or without a liquidation boundary, and the first-loss scheme without the liquidation boundary are not separation-effective for any funder. This can be explained as follows: In the traditional scheme, even with a high hurdle rate, some unskilled entrepreneurs are still willing to raise capital so as to take the performance fee as long as their projects are able to outperform the hurdle rate with a positive probability, and these projects are unappealing to funders. In the first-loss scheme, taking a performance fee in excess of a high hurdle rate effectively allows the entrepreneur to borrow money from the funders at a rate that is higher than the risk-free rate. The rate difference incurs a cost for the entrepreneur relative to the case in which the hurdle rate is the same as the risk-free rate, and the cost is in effect only when the return of the entrepreneur's project is above the risk-free rate. If the net value of the option-to-default and first-loss coverage is negative, the incentive rate must be lower than $\gamma w/(1-w)$. This implies that the lock-up of part of the entrepreneur's capital in the risk-free asset cannot be offset by the increase in leverage due to the performance fee, and the cost incurred by the high hurdle rate makes the entrepreneur even more unwilling to raise capital for her project, so the entrepreneur is deterred. When the incentive rate is high, the net value of the option-to-default and the first-loss coverage is positive. On the other hand, the cost incurred by the high hurdle rate is negligible as long as the probability that the return of the project beats the risk-free return is small. Thus, an entrepreneur who has a project that is very risky in a sense that the probability of its return outperforming the risk-free return is very small is attracted, but this project is unappealing to any funder.

Corollary 6. *Consider the first-loss scheme and assume a continuously compounded hurdle rate h and a continuously compounded reference rate θ such that $h \geq r \geq \theta$. Suppose that there is a liquidation boundary $b \in (0,1)$. Recall L as defined in (12) and define*

$$\hat{U} := \min\left\{\frac{\gamma w}{1-w} + \frac{wb}{(1-w)(e^r - b)}, \frac{e^\theta - b}{e^r - b}\right\}. \tag{B.1}$$

Suppose $L < \hat{U}$. Then, for any $\alpha \in (L, \hat{U})$, the following are true:

(i) Any entrepreneur who is attracted by the first-loss scheme is attractive to risk-neutral funders; more precisely, $\bar{\alpha}_F(u_F, \tilde{R}, \tau) > \alpha$ for $u_F \in \mathcal{U}_{0,1}$ and any $u_E \in \mathcal{U}$ and $(\tilde{R}, \tau) \in \mathcal{R}_b$ with $\bar{\alpha}_E(u_E, \tilde{R}, \tau) < \alpha$.

(ii) Some skilled entrepreneurs are attracted and all unskilled entrepreneurs are deterred; more

precisely, fixing any $u_E \in \mathcal{U}$, $\bar{\alpha}_E(u_E, \tilde{R}, \tau) < \alpha$ for some $(\tilde{R}, \tau) \in \mathcal{R}_{b,S}$, and $\bar{\alpha}_E(u_E, \tilde{R}, \tau) \geq \alpha$ for all $(\tilde{R}, \tau) \in \mathcal{R}_{b,U}$.

Corollary 7. *Consider the first-loss scheme and assume a continuously compounded hurdle rate h and a continuously compounded reference rate θ such that $h \geq r \geq \theta$. Suppose that there is a liquidation boundary $b \in (0, 1)$. For each $\delta \geq 0$, $\lambda > 0$, and $\mathcal{O} > 0$, define $\hat{\mathbb{U}}(\delta, \lambda, \mathcal{O})$ by replacing U in (13) with $\hat{U}$ as defined in (B.1). Suppose $L < \hat{\mathbb{U}}(\delta, \lambda, \mathcal{O})$. Then, for any given $\alpha \in (L, \hat{\mathbb{U}}(\delta, \lambda, \mathcal{O}))$, the following are true:*

(i) Any entrepreneur who takes limited risk and is attracted by the first-loss scheme is attractive to all funders with bounded risk aversion; more precisely, $\bar{\alpha}_F(u_F, \tilde{R}, \tau) > \alpha$ for all $u_F \in \mathcal{U}_{\delta,\lambda}$, $u_E \in \mathcal{U}$, and $(\tilde{R}, \tau) \in \mathcal{R}_b$ with $\mathbb{P}(\tau \leq 1)/\mathbb{P}(\tau > 1) \leq \mathcal{O}$ and $\bar{\alpha}_E(u_E, \tilde{R}, \tau) < \alpha$.

(ii) Some skilled entrepreneurs are attracted and all unskilled entrepreneurs are deterred; more precisely, fixing any $u_E \in \mathcal{U}$, $\bar{\alpha}_E(u_E, \tilde{R}, \tau) < \alpha$ for some $(\tilde{R}, \tau) \in \mathcal{R}_{b,S}$ with $\mathbb{P}(\tau \leq 1)/\mathbb{P}(\tau > 1) \leq \mathcal{O}$, and $\bar{\alpha}_E(u_E, \tilde{R}, \tau) \geq \alpha$ for all $(\tilde{R}, \tau) \in \mathcal{R}_{b,U}$.

Corollaries 6 and 7 are parallel to Theorems 3 and 4, showing that the first-loss scheme with a liquidation boundary is separation-effective. Setting $w = 10\%$, $\gamma = 0$, $b = 0.91$, $r = 5\%$, and $\mathcal{O} = 2$ we plot in Figure 7 the range of separation-effective incentive rates with respect to the reference rate θ for risk-neutral funders (left pane), EU funders with local RRAD less than or equal to $\delta = 2$ (middle pane), and PT funders with local RRAD less than or equal to $\delta = 0$ and LAD in the small less than or equal to $\lambda = 3.25$ (right pane). We can observe that the upper bound of the range, namely the deterring threshold of the incentive rate, is increasing with respect to θ. This is because with a larger value of θ, the first-loss coverage becomes larger and thus the deterring threshold, which sets equal the first-loss coverage and option-to-default, becomes higher.

Finally, we consider a special case in which $\theta = r$ and $b \geq (1 - w - \gamma w)e^r/(1 - \gamma w)$. In this case, when the entrepreneur's project underperforms the risk-free asset, she needs to cover any shortage of the return of the funders' capital relative to the risk-free return because $\theta = r$. The entrepreneur's first-loss deposit is sufficient to cover the shortage because $b \geq (1 - w - \gamma w)e^r/(1 - \gamma w)$. Consequently, the funder's payoff is always above the risk-free return, so he finds any entrepreneur to be attractive.

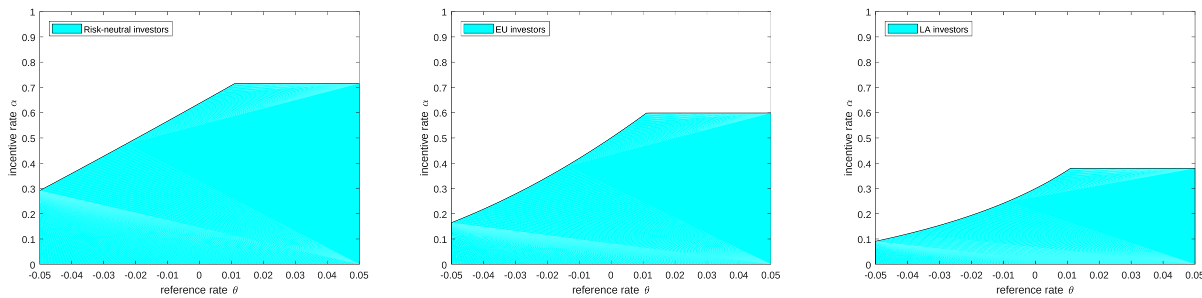

Figure 7. Range of separation-effective incentive rates in the first-loss scheme with respect to the reference rate θ. Set $w = 10\%$, $\gamma = 0$, $b = 0.91$, $r = 5\%$, and $\mathcal{O} = 2$. The utility function of an entrepreneur $u_E \in \mathcal{U}$. The ranges corresponding to the cases of risk-neutral funders, EU funders (with local RRAD bounded by $\delta = 2$), and LA funders (with local RRAD bounded by $\delta = 0$ and LAD in the small bounded by $\lambda = 2.75$), are given in the left, middle, and right panes, respectively.

Corollary 8. *Consider the first-loss scheme, assume a continuously compounded hurdle rate $h \geq r$, and suppose that the reference rate is the same as the risk-free rate. Suppose that there is a liquidation boundary $b \geq (1 - w - \gamma w)e^r/(1 - \gamma w)$. Then, the funder's payoff is larger than or equal to $Y_0 e^r$.*

C A Limited-Liability Hurdled Contract

Chassang (2013, Section 5) proposed a limit-liability hurdled contract in which the agent needs to pay a participation fee at the start of the contract and gets rewarded at the end of the contract only if her performance passes a hurdle. Assuming a zero opportunity cost for the participation fee, the author shows that unskilled agents will be self-screened out under this contract. We study whether such a contract can separate entrepreneurs who are attractive to funders from those who are unappealing to funders in our P2P financing model.

Consider the following limited-liability hurdled scheme for entrepreneurs: to raise capital for a project, at time 0, the entrepreneur needs to pay $cX_{I,0} > 0$ where $X_{I,0}$ is the initial value of the funders' stake. At the terminal time, the entrepreneur takes a performance fee as in the traditional and first-loss schemes if the project is at a gain. Consequently, the entrepreneur's payoff is

$$\tilde{Z}_1 = \alpha X_{I,0}(\tilde{R} - e^r)^+, \tag{C.1}$$

where $\tilde{R}$ is the gross return of the entrepreneur's project at time 1; see (2). The entrepreneur is attracted by the limited-liability hurdled contract if and only if $\mathbb{E}\big[u_E\big(\tilde{Z}_1/(cX_{I,0})\big)\big] > \mathbb{E}\big[u_E\big(a\tilde{R} +$

$(1-a)e^r)\big]$ for any $a \in [0,1]$; in other words, the entrepreneur is attracted by the contract if and only if she would be worse off if she invested the participation fee on her own. Thus, the participation fee incurs an opportunity cost, but this cost is not taken into account in Chassang (2013); see also the discussion in Footnote 21 of Chassang (2013).

On the other hand, Chassang (2013) does not study the issue of whether the entrepreneur is attractive to the funders, so the author did not model specifically the physical meaning of the participation cost. Here, we assume that the entrepreneur pays the participation cost to the funder and the funder can invest it in the risk-free asset. Consequently, the funder's payoff is

$$\tilde{Y}_1 = X_{I,0}\left[\tilde{R} - \alpha(\tilde{R} - e^r)^+ + ce^r\right], \tag{C.2}$$

and the funder finds the entrepreneur to be attractive if and only if $\mathbb{E}\big[u_F(\tilde{Y}_1/X_{I,0})\big] > \mathbb{E}\big[u_F(e^r)\big]$. We define $\bar{\alpha}_E$ and $\bar{\alpha}_F$ in the same way as in Section 2, so an entrepreneur with utility u_E and project return $(\tilde{R}, \tau)$ is attracted by the limited-liability hurdled contract if and only if $\bar{\alpha}_E(u_E, \tilde{R}, \tau) < \alpha$, and this entrepreneur is attractive to a funder with utility function u_F if and only if $\bar{\alpha}_F(u_F, \tilde{R}, \tau) > \alpha$.

The limited-liability hurdled contract is similar to the first-loss scheme with cash-deposit only: in both schemes the entrepreneur's capital is not invested in the entrepreneur's project, incurring an opportunity cost for skilled entrepreneurs. These two schemes, however, differ: in the limited-liability hurdled contract, the entrepreneur's initial capital is transferred to the funder as a participation fee regardless of the return of the project; in the first-loss scheme, however, the entrepreneur's capital is used to cover the funder's loss and thus effectively transferred to the funder only when the entrepreneur's project leads to a loss. Thus, comparing the limited-liability hurdled contract and the first-loss scheme with cash deposit only, the entrepreneur prefers the latter and the funder prefers the former.

Corollary 9. *In the limited-liability hurdled contract without a liquidation boundary, the following are true:*

(i) Suppose $\alpha \le c$, then all the entrepreneurs are deterred; more precisely, $\bar{\alpha}_E(u_E, \tilde{R}, \tau) \ge \alpha$ for all $u_E \in \mathcal{U}$ and $(\tilde{R}, \tau) \in \mathcal{R}_0$.

(ii) Suppose $\alpha \in (c, 1)$, then the limited-liability hurdled contract attracts some risk-neutral en-

trepreneur who is unappealing to all funders; more precisely, there exists $(\tilde{R}, \tau) \in \mathcal{R}_{0,U}$ *such that* $\bar{\alpha}_E(u_E, \tilde{R}, \tau) < \alpha$ *for* $u_E \in \mathcal{U}_{0,1}$ *and* $\bar{\alpha}_F(u_F, \tilde{R}, \tau) \le \alpha$ *for all* $u_F \in \mathcal{U}$*.*

Corollary 10. *In the limited-liability hurdled contract with liquidation boundary* $b \in (0,1)$*, the following are true:*

(i) *Suppose* $\alpha \le c$*, then all the entrepreneurs are deterred; more precisely,* $\bar{\alpha}_E(u_E, \tilde{R}, \tau) \ge \alpha$ *for all* $u_E \in \mathcal{U}$ *and* $(\tilde{R}, \tau) \in \mathcal{R}_0$*.*

(ii) *Suppose* $\alpha \in (c, 1)$ *and* $c \ge 1 - e^{-r}b$*. Then, the funder's payoff is larger than or equal to* $X_{I,0}e^r$*. Moreover, any entrepreneur who is attracted by the limited-liability hurdled contract is attractive to all funders; more precisely,* $\bar{\alpha}_F(u_F, \tilde{R}, \tau) > \alpha$ *for all* $u_F \in \mathcal{U}$*,* $u_E \in \mathcal{U}$*, and* $(\tilde{R}, \tau) \in \mathcal{R}_b$ *with* $\bar{\alpha}_E(u_E, \tilde{R}, \tau) < \alpha$*. Furthermore, some skilled entrepreneurs are attracted, that is, for any* $u_E \in \mathcal{U}$*, there exists* $(\tilde{R}, \tau) \in \mathcal{R}_{b,S}$ *such that* $\bar{\alpha}_E(u_E, \tilde{R}, \tau) < \alpha$*.*

(iii) *Suppose* $\alpha \in (c, 1)$ *and* $c < 1 - e^{-r}b$*. When*

$$\alpha \in \left(c, \frac{c}{1 - e^{-r}b}\right), \tag{C.3}$$

any entrepreneur who is attracted by the limited-liability hurdled contract is attractive to risk-neutral funders; more precisely, we have $\bar{\alpha}_F(u_F, \tilde{R}, \tau) > \alpha$ *for any* $u_F \in \mathcal{U}_{0,1}$*,* $u_E \in \mathcal{U}$*, and* $(\tilde{R}, \tau) \in \mathcal{R}_b$ *with* $\bar{\alpha}_E(u_E, \tilde{R}, \tau) < \alpha$*. Moreover, some skilled entrepreneurs are attracted, that is, for any* $u_E \in \mathcal{U}$*, there exists* $(\tilde{R}, \tau) \in \mathcal{R}_{b,S}$ *such that* $\bar{\alpha}_E(u_E, \tilde{R}, \tau) < \alpha$*.*

(iv) *Suppose* $\alpha \in (c, 1)$ *and* $c < 1 - e^{-r}b$*. When* $\alpha > c/(1 - e^{-r}b)$*, the limited-liability hurdled contract attracts some risk-neutral entrepreneur who is unappealing to all funders; more precisely, there exists* $(\tilde{R}, \tau) \in \mathcal{R}_{0,U}$ *such that* $\bar{\alpha}_E(u_E, \tilde{R}, \tau) < \alpha$ *for* $u_E \in \mathcal{U}_{0,1}$ *and* $\bar{\alpha}_F(u_F, \tilde{R}, \tau) \le \alpha$ *for all* $u_F \in \mathcal{U}$*.*

Corollary 9 shows that without a liquidation boundary, the limited-liability hurdled contract is not separation-effective. On the other hand, Corollary 10 shows that with a liquidation boundary, if the incentive rate α is less than or equal to the participation rate c, all entrepreneurs are deterred because the performance fee is less valuable than the participation fee. If $\alpha > c$ and the participation fee is sufficiently high (i.e., $c \ge 1 - e^{-r}b$), the limited-liability hurdled contract is separation-effective for any funder regardless of his risk attitude. Indeed, in this case, when the fund is at a loss, the participation fee paid by the entrepreneur to the funder is more than the loss incurred by the entrepreneur's project, so the funder is always willing to invest his money in the project. If the participation fee is low (i.e., $c < 1 - e^{-r}b$), the participation fee cannot fully cover the funders' loss, so whether an entrepreneur is attractive to a funder depends on the funder's risk attitude and the riskiness of the project. For risk-neutral funders, the limited-liability hurdled contract is separation-effective if and only if the incentive rate falls into the range (C.3). For risk-averse funders, the following corollary, which is parallel to Theorem 4, shows that the limited-liability hurdled contract can also be separation-effective .

Corollary 11. *Consider the limited-liability hurdled contract with a liquidation boundary $b \in (0, 1)$. Suppose $c < 1 - be^{-r}$. For each $\delta \geq 0$, $\lambda > 0$, and $\mathcal{O} > 0$, define*

$$\tilde{\mathbb{U}}(\delta, \lambda, \mathcal{O}) := \min \left\{ \frac{c(1 + \mathcal{O})}{u_\delta^{-1}\Big(- \lambda u_\delta\big(1 - (1 - c - e^{-r}b)\big)\mathcal{O}\Big) - 1 + c\mathcal{O}}, \frac{c}{1 - e^{-r}b} \right\}. \tag{C.4}$$

Suppose $c < \tilde{\mathbb{U}}(\delta, \lambda, \mathcal{O})$. Then, for any given $\alpha \in (c, \tilde{\mathbb{U}}(\delta, \lambda, \mathcal{O}))$, the following are true:

(i) Any entrepreneur who takes limited risk and is attracted by the limited-liability hurdled contract is attractive to all funders with bounded risk aversion; more precisely, $\bar{\alpha}_F(u_F, \tilde{R}, \tau) > \alpha$ for all $u_F \in \mathcal{U}_{\delta,\lambda}$, $u_E \in \mathcal{U}$, and $(\tilde{R}, \tau) \in \mathcal{R}_b$ with $\mathbb{P}(\tau \leq 1)/\mathbb{P}(\tau > 1) \leq \mathcal{O}$ and $\bar{\alpha}_E(u_E, \tilde{R}, \tau) < \alpha$.

(ii) Some skilled entrepreneurs are attracted; more precisely, fixing any $u_E \in \mathcal{U}$, $\bar{\alpha}_E(u_E, \tilde{R}, \tau) < \alpha$ for some $(\tilde{R}, \tau) \in \mathcal{R}_{b,S}$ with $\mathbb{P}(\tau \leq 1)/\mathbb{P}(\tau > 1) \leq \mathcal{O}$.

D Overshoot over the Liquidation Boundary

In the previous sections, we assume that when a liquidation boundary $b \in (0, 1)$ is contracted, the gross return of the project is b as long as the project is at a loss. We make this assumption for two reasons. First, if the project value evolves continuously in time and is monitored continuously as well, then the project is liquidated once the gross return hits b; consequently, the project's gross return cannot be lower than b. Second, if an entrepreneur attempts to maximize the probability of achieving a target, she does not concern the magnitude of her loss; consequently, the loss amount is maximized, i.e., the gross return is b, when the project incurs a loss. This is indeed the case when the entrepreneur trades in a complete market; see e.g. Browne (1999) and He and Zhou

(2011, Section 3.1). In Foster and Young (2010), the authors also assume that in the absence of liquidation boundaries, the gross return of a mimic manager is 0 as long as a loss is incurred.

In practice, the asset value of a project can be monitored discretely in time only, e.g., daily. In this case, even a liquidation boundary $b \in (0,1)$ is contracted, it is possible that the gross return of the project may overshot the liquidation boundary. On the other hand, the project may lead to a loss without touching the liquidation boundary, which may be due to the incompleteness of the market or a result of the entrepreneur's risk attitude. In both cases, the gross return of the project, conditional on having a loss, may no longer be b; instead, it can be a random variable.

Recall that $\tilde{R} := (\tilde{m}_{\tau\wedge 1} + 1)e^{r(1-\tau\wedge 1)}$ is the gross return of the project. Now, we assume that conditional on $\tilde{R} < e^r$, $\tilde{R}$ is a random variable, and show that the first-loss scheme with a liquidation boundary can still be separation-effective.

Corollary 12. *Consider the first-loss scheme with a liquidation boundary $b \in (0,1)$. Consider an entrepreneur whose project has the return profile $(\tilde{R}, \tau)$. Assume that $\tilde{R} > e^r$ if and only if $\tau > 1$ and that conditional on $\tau \le 1$, the overshoot size $b - (\tilde{m}_\tau + 1)$ is identically distributed as $\tilde{\xi}$, which is a random variable taking values in $[0,b]$. Denote by $\bar{\mathcal{R}}_{b,S}$ the set of the above $(\tilde{R}, \tau)$ with $\mathbb{E}[e^{-r}\tilde{R}] > 1$ and $\mathbb{P}(\tilde{R} < e^r) > 0$, by $\bar{\mathcal{R}}_{b,U}$ the set of the above $(\tilde{R}, \tau)$ with $\mathbb{E}[e^{-r}\tilde{R}] \le 1$, and $\bar{\mathcal{R}}_b := \mathcal{R}_P \cup \bar{\mathcal{R}}_{b,S} \cup \bar{\mathcal{R}}_{b,U}$. Denote*

$$U_{\tilde{\xi}} := \mathbb{E}\left[\min\left\{\frac{\gamma w}{1-w}\cdot\frac{e^r - b + \tilde{\xi}}{e^r - b + \mathbb{E}[\tilde{\xi}]} + \frac{w}{1-w}\cdot\frac{b - \tilde{\xi}}{e^r - b + \mathbb{E}[\tilde{\xi}]}, \frac{1 - b + \tilde{\xi}}{e^r - b + \mathbb{E}[\tilde{\xi}]}\right\}\right]. \tag{D.1}$$

Suppose $L < U_{\tilde{\xi}}$. Then, for each fixed $\alpha \in (L, U_{\tilde{\xi}})$, the following are true:

(i) Any entrepreneur who is attracted by the first-loss scheme is attractive to risk-neutral funders; more precisely, $\bar{\alpha}_F(u_F, \tilde{R}, \tau) > \alpha$ for any $u_F \in \mathcal{U}_{0,1}$, $u_E \in \mathcal{U}$, and $(\tilde{R}, \tau) \in \bar{\mathcal{R}}_b$ with $\bar{\alpha}_E(u_E, \tilde{R}, \tau) < \alpha$.

(ii) Some skilled entrepreneurs are attracted; more precisely, fixing any $u_E \in \mathcal{U}$, $\bar{\alpha}_E(u_E, \tilde{R}, \tau) < \alpha$ for some $(\tilde{R}, \tau) \in \bar{\mathcal{R}}_{b,S}$.

Corollary 13. *Consider the first-loss scheme with a liquidation boundary $b \in (0,1)$. Consider an entrepreneur whose project has the return profile $(\tilde{R}, \tau)$. Assume that $\tilde{R} > e^r$ if and only if $\tau > 1$ and that conditional on $\tau \le 1$, the overshoot size $b - (\tilde{m}_\tau + 1)$ is identically distributed* *as $\tilde{\xi}$, which is a binary random variable taking values in $\{0, kb\}$ for some $k \in (0,1)$. Denote by $p := 1 - \mathbb{P}(\tilde{\xi} = 0) = \mathbb{P}(\tilde{\xi} = kb)$. Recall $\bar{\mathcal{R}}_{b,S}$, $\bar{\mathcal{R}}_{b,U}$, and $\bar{\mathcal{R}}_b$ in Corollary 12. Denote*

$$
\begin{aligned}
&U_k := \min\left\{\tfrac{\gamma w}{1-w} + \tfrac{w(1-k)b}{(1-w)(e^r-(1-k)b)}, \tfrac{1-(1-k)b}{e^r-(1-k)b}\right\}, \quad V_{\tilde{\xi}}(\delta,\lambda,\mathcal{O}) := \\
&L + \frac{(1-L)(U\wedge U_k - L)\Big((1-b)(1-p)+(1-(1-k)b)p\Big)\mathcal{O}}{u_\delta^{-1}\Big(-\lambda\Big(u_\delta\Big(1-(1-U)(1-e^{-r}b)\Big)(1-p)+u_\delta\Big(1-(1-U_k)(1-e^{-r}(1-k)b)\Big)p\Big)\mathcal{O}\Big)-1+(U\wedge U_k-L)\Big((1-b)(1-p)+(1-(1-k)b)p\Big)\mathcal{O}}, \\
&\mathbb{U}_{\tilde{\xi}}(\delta,\lambda,\mathcal{O}) := \min\Big\{V_{\tilde{\xi}}(\delta,\lambda,\mathcal{O}), U_{\tilde{\xi}}, U, U_k\Big\}.
\end{aligned}
\tag{D.2}
$$

Suppose $L < \mathbb{U}_{\tilde{\xi}}(\delta,\lambda,\mathcal{O})$. Then, for each fixed $\alpha \in (L, \mathbb{U}_{\tilde{\xi}}(\delta,\lambda,\mathcal{O}))$, the following are true:

(i) Any entrepreneur who takes limited risk and is attracted by the first-loss scheme is attractive to all funders with bounded risk aversion; more precisely, $\bar{\alpha}_F(u_F, \tilde{R}, \tau) > \alpha$ for all $u_F \in \mathcal{U}_{\delta,\lambda}$ and any $u_E \in \mathcal{U}$ and $(\tilde{R}, \tau) \in \bar{\mathcal{R}}_b$ with $\mathbb{P}(\tau \le 1)/\mathbb{P}(\tau > 1) \le \mathcal{O}$ and $\bar{\alpha}_E(u_E, \tilde{R}, \tau) < \alpha$.

(ii) Some skilled entrepreneurs are attracted; more precisely, fixing any $u_E \in \mathcal{U}$, $\bar{\alpha}_E(u_E, \tilde{R}, \tau) < \alpha$ for some $(\tilde{R}, \tau) \in \bar{\mathcal{R}}_{b,S}$ with $\mathbb{P}(\tau \le 1)/\mathbb{P}(\tau > 1) \le \mathcal{O}$.

Corollary 14. *Consider the first-loss scheme with a liquidation boundary $b \in (0,1)$. Consider an entrepreneur whose project has the return profile $(\tilde{R}, \tau)$. In general, suppose that conditional on $\tilde{R} < e^r$, the gross return $\tilde{R}$ is identically distributed as $\tilde{b}$, where $\tilde{b}$ is a random variable taking values in $[0, e^r]$. Denote by $\hat{\mathcal{R}}_{b,S}$ the set of the above $(\tilde{R}, \tau)$ with $\mathbb{E}[e^{-r}\tilde{R}] > 1$ and $\mathbb{P}(\tilde{R} < e^r) > 0$, by $\hat{\mathcal{R}}_{b,U}$ the set of the above $(\tilde{R}, \tau)$ with $\mathbb{E}[e^{-r}\tilde{R}] \le 1$, and $\hat{\mathcal{R}}_b := \mathcal{R}_P \cup \hat{\mathcal{R}}_{b,S} \cup \hat{\mathcal{R}}_{b,U}$. Denote*

$$
U'_{\tilde{b}} := \mathbb{E}\left[\min\left\{\frac{\gamma w}{1-w} \cdot \frac{e^r - \tilde{b}}{\mathbb{E}[e^r - \tilde{b}]} + \frac{w}{1-w} \cdot \frac{\tilde{b}}{\mathbb{E}[e^r - \tilde{b}]}, \frac{\left(1-\tilde{b}\right)^+}{\mathbb{E}[e^r - \tilde{b}]}\right\}\right]. \tag{D.3}
$$

Suppose $L < U'_{\tilde{b}}$. Then, for each fixed $\alpha \in (L, U'_{\tilde{b}})$, the following are true:

(i) Any entrepreneur who is attracted by the first-loss scheme is attractive to risk-neutral funders; more precisely, $\bar{\alpha}_F(u_F, \tilde{R}, \tau) > \alpha$ for any $u_F \in \mathcal{U}_{0,1}$, $u_E \in \mathcal{U}$, and $(\tilde{R}, \tau) \in \hat{\mathcal{R}}_b$ with $\bar{\alpha}_E(u_E, \tilde{R}, \tau) < \alpha$.

(ii) Some skilled entrepreneurs are attracted; more precisely, fixing any $u_E \in \mathcal{U}$, $\bar{\alpha}_E(u_E, \tilde{R}, \tau) < \alpha$ for some $(\tilde{R}, \tau) \in \hat{\mathcal{R}}_{b,S}$.

Moreover, when $\tilde{b} \ge \big(1 - w - \gamma w e^r\big)/(1 - \gamma w)$, $U'_{\tilde{b}} = \mathbb{E}[(1-\tilde{b})^+]/\mathbb{E}[e^r - \tilde{b}]$.

E Random Gains

It is possible that the gross return $\tilde{R}$ of a project, conditional on the project outperforming the risk-free asset, is random. The presence of this randomness, however, does not affect the preference of a risk-neutral funder for a particular entrepreneur. Thus, all the previous results, except for Theorem 4, Corollaries 4, 7, and 11 that concern risk-averse funders, still hold if we enlarge the set of gross returns by allowing for random gains. In general, however, this randomness makes an entrepreneur to be less attractive to risk-averse funders, so it becomes more difficult for the first-loss scheme to achieve the separation.

F Multiple Periods

In this section, we show that the first-loss scheme with a liquidation boundary is separation-effective in a multi-period setting as well. The following model setting is completely parallel to the single-period case.

Consider T periods of investment in a project. Assume the continuously compounded risk-free rate to be constant r. The project value is denoted as $X_t, t \in [0, T]$. In addition, capital can be injected into or withdrawn from the project at the beginning of each period, and X_{n-1} stands for the project value at the beginning of period n *after* capital injection and withdrawal. Note that X_{n-1} is not necessarily known before time $n-1$; i.e., we allow for random capital injection and withdrawal. We assume that at the beginning of each period, the first-loss deposit of the entrepreneur is reset to be a *constant* proportion w of the project value, where γ proportion of the deposit is cash and the remaining $1-\gamma$ proportion is an equity. The reset is achieved by capital injection or withdrawal.

Consider an entrepreneur with a project and denote the liquidation boundary of her project as $b \in [0, 1)$, where $b = 0$ actually refers to the case of no liquidation boundary. Suppose that the project is still alive at time $n-1$. Denote by $\tilde{m}_{n,t}$ the net return that the project generates in the period $n-1$ to t, $t \in (n-1, n]$. Assume that conditional on $\tilde{m}_{n,t} > b-1$, $t \in (n-1, n]$, $\tilde{m}_{n,n} = m_n \geq e^r - 1$, where m_n is a quantity known at time $n-1$. When there is a liquidation boundary, i.e., $b \in (0, 1)$, denote by τ the liquidation time of the project, namely

$$\tau := \inf\{t \in (0, T] \mid \tilde{m}_{\lceil t \rceil, t} \leq b - 1\},$$

where $\inf \emptyset := +\infty$ and $\lceil x \rceil$ denotes the smallest integer that dominates x. Then, conditional on $\{\tau \le T\}$, $\tilde{m}_{\lceil \tau \rceil, \tau} = b - 1$. Moreover, conditional on $\{\tau > n\}$, $\tilde{m}_{i,i} \ge e^r - 1$ for $i = 1, \ldots, n$. When there is no liquidation boundary, i.e., $b = 0$, set $\tau := \inf\{n = 1, ..., T \mid \tilde{m}_{n,t} \le -1, t \in (n-1, n]\}$, and this is reasonable because there is no monitoring at intermediate time and the project will be reviewed and terminated only at the end of each period.

We assume that there are only three types of entrepreneurs in the market. The project of a *perfectly skilled entrepreneur* yields $\tau = +\infty$ almost surely and $\tilde{m}_{n,n} > e^r - 1$ for $n = 1, \ldots, T$. The project of a *skilled entrepreneur* generates a return such that for each $n = 1, \ldots, T$, $\mathbb{P}_{n-1}(\tau \le n \mid \tau > n-1) > 0$ and $\mathbb{E}_{n-1}\left[e^{-r(\tau \wedge n - (n-1))}(\tilde{m}_{\tau \wedge n} + 1) \mid \tau > n-1\right] > 1$, where $\mathbb{P}_{n-1}$ and $\mathbb{E}_{n-1}$ denote the probability and expectation, respectively, conditional on the information at time $n-1$. For the project return delivered by an *unskilled entrepreneur*, for each $n = 1, \ldots, T$, $\mathbb{E}_{n-1}\left[e^{-r(\tau \wedge n - (n-1))}(\tilde{m}_{\tau \wedge n} + 1) \mid \tau > n-1\right] \le 1$.

If the project is liquidated at $\tau \in (n-1, n]$ for some $n = 1, \ldots, T$, we assume that both the entrepreneur and the funders accumulate their payoffs to the end of the corresponding period at the risk-free rate. Denote $\tilde{R}_n := e^{r(n - \tau \wedge n)}(\tilde{m}_{\tau \wedge n} + 1)$ as the gross return of the entrepreneur's project in the n-th period. Then, in the traditional scheme, the entrepreneur's payoff $\tilde{Z}_n$ and the funders' payoff $\tilde{Y}_n$ at time n are, respectively,

$$\tilde{Z}_n = \left[w X_{n-1} \tilde{R}_n + \alpha(1-w) X_{n-1} \left(\tilde{R}_n - e^r\right)^+ \right] \mathbf{1}_{\{\tau > n-1\}}, \tag{F.1}$$

$$\tilde{Y}_n = \left[(1-w) X_{n-1} \tilde{R}_n - \alpha(1-w) X_{n-1} \left(\tilde{R}_n - e^r\right)^+ \right] 1_{\{\tau > n-1\}}. \tag{F.2}$$

Following the discussion in Section 2.2, in the first-loss scheme, the entrepreneur's payoff $\tilde{Z}_n$ and the funders' payoff $\tilde{Y}_n$ at time n are, respectively,

$$\tilde{Z}_n = w X_{n-1} \left\{ \left[\big(1 - \gamma + \alpha(1-w)/w\big) \tilde{R}_n + \big(\gamma - \alpha(1-w)/w\big) e^r \right] + \tilde{D}_n - \tilde{F}_n \right\} \mathbf{1}_{\{\tau > n-1\}}, \tag{F.3}$$

$$\tilde{Y}_n = (1-w) X_{n-1} \left\{ \left[\alpha e^r + (1-\alpha) \tilde{R}_n \right] - \big(w/(1-w)\big) \tilde{D}_n + \big(w/(1-w)\big) \tilde{F}_n \right\} \mathbf{1}_{\{\tau > n-1\}}, \tag{F.4}$$

where

$$\tilde{D}_n := \big(\alpha(1-w)/w\big)(e^r - \tilde{R}_n)^+,$$
$$\tilde{F}_n := \min\left\{ \gamma e^r + (1-\gamma) \tilde{R}_n, \big((1-w)/w\big) \left(e^{r(n - \tau \wedge n)} - \tilde{R}_n\right)^+ \right\}.$$

Suppose that the entrepreneur's preferences for cash flow streams at $n = 1, \ldots, T$ are represented by the relation $\succcurlyeq_M$ (with $\succ_M$ standing for the strict preference relation). Similarly, suppose that the funder's preferences for cash flow streams at $n = 1, \ldots, T$ are represented by the relation $\succcurlyeq_I$ (with $\succ_I$ standing for the strict preference relation). Suppose that the entrepreneur is attracted by a P2P financing scheme if the cash flow stream generated by raising capital for the project under this scheme, namely $(\tilde{Z}_1, \ldots, \tilde{Z}_T)$, is strictly preferred to the cash flow stream generated by investing on her own, namely $wX_{n-1}[a_{n-1}\tilde{R}_n + (1-a_{n-1})e^r]\mathbf{1}_{\{\tau>n-1\}}$, $n = 1, \ldots, T$ for any investing strategy $a_{n-1} \in [0,1], n = 1, \ldots, T$. Otherwise, the entrepreneur is deterred. The entrepreneur is attractive to the funder if the funder strictly prefers the cash flow stream generated by pledging money to the entrepreneur's project, namely $(\tilde{Y}_1, \ldots, \tilde{Y}_T)$, to the cash flow stream of investing on his own, namely $(1-w)X_{n-1}e^r\mathbf{1}_{\{\tau>n-1\}}$, $n = 1, \ldots, T$. Otherwise, the entrepreneur is unappealing to the funder.

The entrepreneur is weakly risk-averse if $(\mathbb{E}_0[X_1], \mathbb{E}_1[X_2], \ldots, \mathbb{E}_{T-1}[X_T]) \succcurlyeq_M (X_1, X_2, \ldots, X_T)$ for any cash flow stream $(X_1, \ldots, X_T)$, and is risk-neutral if $\succcurlyeq_M$ in the above is replaced by $\sim_M$. Weak risk-aversion and risk-neutrality for the funder is defined similarly. We also assume that both the entrepreneur and the funder's preferences are consistent with the first-order stochastic dominance; i.e., if $X_n \geq X_n'$, $n = 1, \ldots, T$ and $\mathbb{P}(X_{n_0} > X_{n_0}') > 0$ for some n_0, then $(X_1, \ldots, X_T)$ is strictly preferred to $(X_1', \ldots, X_T')$. Note that different from the setting in the previous section, here we do not assume expected utility preferences or any model for the entrepreneur and funder's intertemporal preferences.

Corollary 15. *Whether there is a liquidation boundary or not, the traditional scheme attracts some unskilled entrepreneurs who are unappealing to all weakly risk-averse funders.*

Corollary 16. *Suppose there is no liquidation boundary, i.e., $b = 0$. For any weakly risk-averse funder, the first-loss scheme cannot deter all entrepreneurs who are unappealing to this funder and attract some entrepreneurs who are attractive to this funder at the same time.*

Corollary 17. *Suppose there is a liquidation boundary, i.e., $b \in (0,1)$. Recall L and U as defined in (12) and assume $L < U$. Then, for any given $\alpha \in (L, U)$, any entrepreneur who is attracted by the first-loss scheme must be attractive to all risk-neutral funders, and the first-loss scheme attracts all perfectly skilled entrepreneurs and some skilled entrepreneurs.*

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Online Supplement

Peer-to-Peer Equity Financing: Preference-Free and Menuless Screening Contracts

This online supplement contains all the proofs in the paper. The following results will be useful in the subsequent proof. Straightforward calculation shows that for any $u \in \mathcal{U}_{\delta,\lambda}$, we have

$$u(x) - u(y) \geq u'(y)y^{\delta}\big[u_\delta(x) - u_\delta(y)\big], \quad \forall x, y \in (0, e^r) \text{ and } x, y \in (e^r, +\infty), \tag{O–1}$$

$$u'(e^r-)/u'(e^r+) \leq \lambda, \tag{O–2}$$

where $u'(x+)$ and $u'(x-)$ denotes the right- and left-derivatives of u at x, respectively, and $u_\delta(x)$ given by

$$u_\delta(x) := \big(x^{1-\delta} - 1\big)/(1-\delta), \quad \delta \neq 1, \quad u_\delta(x) := \log(x), \quad \delta = 1.$$

We can see from (O–1) that for $\delta < 1$, any $u \in \mathcal{U}_{\delta,\lambda}$ satisfies $u(0) = \lim_{x \downarrow 0} u(x) > -\infty$.

Proof of Theorem 1. We first consider the traditional scheme. Consider $u_E \in \mathcal{U}_{\delta,\lambda}$ with $0 < \lambda/(1-\delta) < 1 + \alpha(1-w)/w$ and $(\tilde{R}, \tau) \in \mathcal{R}_{0,U}$ with $\mathbb{P}(\tilde{R} = e^r/p) = p$ and $\mathbb{P}(\tilde{R} = 0) = 1 - p$ for certain $p \in (0, 1]$. Denote by

$$f(p) := \mathbb{E}[u_E(\tilde{Z}_1/Z_0)] = u_E\left(\frac{e^r}{p} + \alpha\frac{1-w}{w}\left(\frac{e^r}{p} - e^r\right)\right)p + u_E(0)(1-p).$$

Straightforward calculation yields that

$$\begin{aligned} f'(1) &= u_E(e^r) - u_E(0) - e^r\big(1 + \alpha(1-w)/w\big)u_E'(e^r+) \\ &\leq u_E'(e^r-)e^r/(1-\delta) - e^r\big(1 + \alpha(1-w)/w\big)u_E'(e^r+) \\ &\leq u_E'(e^r+)e^r\big[\lambda/(1-\delta) - \big(1 + \alpha(1-w)/w\big)\big] < 0, \end{aligned}$$

where the first inequality is the case due to (O–1), the second inequality is the case due to (O–2), and the last inequality is the case because $0 < \lambda/(1-\delta) < 1 + \alpha(1-w)/w$. As a result, there exists $p \in (0, 1)$ such that

$$f(p) > f(1) = u_E(e^r) = \max_{a \in [0,1]} \mathbb{E}\left[u_E\big(a\tilde{R} + (1-a)e^r\big)\right], \tag{O–3}$$

where the last equality is the case because $\mathbb{E}[\tilde{R}] = e^r$ and u_E is concave. From (O–3), we immediately have $\bar{\alpha}_E(u_E, \tilde{R}, \tau) < \alpha$. On the other hand, for any $u_F \in \mathcal{U}$,

$$\mathbb{E}\left[u_F\big(\tilde{Y}_1/Y_0\big)\right] \le \mathbb{E}\left[u_F\big(\alpha e^r + (1-\alpha)\tilde{R}\big)\right] \le u_F\big(\mathbb{E}\left[\alpha e^r + (1-\alpha)\tilde{R}\right]\big) = u_F(e^r), \tag{O–4}$$

where the first inequality is the case due to (4), the second is the case due to the concavity of u_F, and the equality is the case because $\mathbb{E}[\tilde{R}] = e^r$. As a result, from (O–4), we have $\bar{\alpha}_F(u_F, \tilde{R}, \tau) \le \alpha$.

Next, we consider the first-loss scheme. From (3) and (6), for any $(\tilde{R}, \tau) \in \mathcal{R}_0$, we have

$$\tilde{D} - \tilde{F} = \frac{1-w}{w} e^r \left[\alpha - \min\left\{\frac{\gamma w}{1-w}, e^{-r}\right\}\right] \mathbf{1}_{\{\tilde{R} < e^r\}}. \tag{O–5}$$

If $\alpha \le \min\{\gamma w/(1-w), e^{-r}\}$, we have $\tilde{D} - \tilde{F} \le 0$, so by (5), the entrepreneur's payoff

$$\tilde{Z}_1 \le Z_0\left[\big(1-\gamma+\alpha(1-w)/w\big)\tilde{R} + \big(\gamma - \alpha(1-w)/w\big)e^r\right].$$

Note that $\alpha \le \min\{\gamma w/(1-w), e^{-r}\}$ implies $\gamma - \alpha(1-w)/w \in [0,1]$. As a result, for any $u_E \in \mathcal{U}$,

$$\begin{aligned}
\mathbb{E}[u_E(\tilde{Z}_1/Z_0)] &\le \mathbb{E}\left\{u_E\left[\big(1-\gamma+\alpha(1-w)/w\big)\tilde{R} + \big(\gamma-\alpha(1-w)/w\big)e^r\right]\right\} \\
&\le \max_{a\in[0,1]} \mathbb{E}\left[u_E\big(a\tilde{R} + (1-a)e^r\big)\right],
\end{aligned}$$

showing that $\bar{\alpha}_E(u_E, \tilde{R}, \tau) \ge \alpha$.

Next, suppose $\alpha > \min\{\gamma w/(1-w), e^{-r}\}$. Consider $u_E \in \mathcal{U}_{\delta,\lambda}$ with δ and λ satisfying (10) and $(\tilde{R}, \tau) \in \mathcal{R}_{0,U}$ with $\mathbb{P}(\tilde{R} = e^r/p) = p$ and $\mathbb{P}(\tilde{R} = 0) = 1 - p$ for certain $p \in (0,1]$. Denote by

$$\begin{aligned}
f(p) := \mathbb{E}[u_E(\tilde{Z}_1/Z_0)] = u_E&\left(\left(1-\gamma+\alpha\frac{1-w}{w}\right)\frac{e^r}{p} + \left(\gamma - \alpha\frac{1-w}{w}\right)e^r\right)p \\
&+ u_E\left(\left(\gamma - \min\left(\gamma, \frac{1-w}{w}e^{-r}\right)\right)e^r\right)(1-p).
\end{aligned}$$

Straightforward calculation yields that

$$\begin{aligned}
f'(1) &= u_E(e^r) - u_E\left(\big(\gamma - \min\big(\gamma, e^{-r}(1-w)/w\big)\big)e^r\right) - e^r\big(1-\gamma+\alpha(1-w)/w\big)u_E'(e^r+) \\
&\le -u_\delta\left(\gamma - \min(\gamma, e^{-r}(1-w)/w)\right)u_E'(e^r-)e^r - e^r\big(1-\gamma+\alpha(1-w)/w\big)u_E'(e^r+) \\
&\le u_E'(e^r+)e^r\left[-\lambda u_\delta\left(\gamma - \min(\gamma, e^{-r}(1-w)/w)\right) - \big(1-\gamma+\alpha(1-w)/w\big)\right] < 0,
\end{aligned}$$

where the first inequality is the case due to (O–1), the second inequality is the case due to (O–2), and the last inequality is the case due to (10). As a result, there exists $p \in (0,1)$ such that (O–3) holds and thus $\bar{\alpha}_E(u_E, \tilde{R}, \tau) < \alpha$. On the other hand, from (O–5) and $\alpha > \min\{\gamma w/(1-w), e^{-r}\}$, $\tilde{D} - \tilde{F} > 0$. Then, for any $u_F \in \mathcal{U}$, due to (7), we have (O–4) and thus $\alpha \ge \bar{\alpha}_F(u_F, \tilde{R}, \tau)$. $\square$

Proof of Theorem 2 Consider $u_E \in \mathcal{U}_{\delta,\lambda}$ with δ and λ satisfying (11) and $(\tilde{R}, \tau) \in \mathcal{R}_{b,U}$ with $\mathbb{P}(\tilde{R} = b) = \mathbb{P}(\tau \le 1) = \mathbb{P}(\tau = 1) = 1 - p$ and $\mathbb{P}(\tilde{R} = e^r/p - b(1-p)/p) = \mathbb{P}(\tau > 1) = p$ and for certain $p \in (0, 1]$. One can verify that $\mathbb{E}[\tilde{R}] = e^r$. Denote by

$$f(p) := \mathbb{E}[u_E(\tilde{Z}_1/Z_0)] = u_E\left(\frac{e^r}{p} - \frac{b(1-p)}{p} + \alpha\frac{1-w}{w}\left(\frac{e^r}{p} - \frac{b(1-p)}{p} - e^r\right)\right)p + u_E(b)(1-p).$$

Straightforward calculation yields that

$$\begin{aligned} f'(1) &= u_E(e^r) - u_E(b) - (e^r - b)\big(1 + \alpha(1-w)/w\big)u_E'(e^r+) \\ &\le -e^r u_\delta(e^{-r}b)u_E'(e^r-) - (e^r - b)\big(1 + \alpha(1-w)/w\big)u_E'(e^r+) \\ &\le u_E'(e^r+)(e^r - b)\left[\lambda\frac{-u_\delta(e^{-r}b)}{1 - e^{-r}b} - \big(1 + \alpha(1-w)/w\big)\right] < 0, \end{aligned}$$

where the first inequality is the case due to (O–1), the second inequality is the case due to (O–2), and the last inequality is the case due to (11). The remaining proof is the same as the proof of Theorem 1. $\square$

Proof of Theorem 3 Fix $u_E \in \mathcal{U}$ and $(\tilde{R}, \tau) \in \mathcal{R}_b$ and recall the option-to-default in (3) and the first-loss coverage in (6), and note that $\tilde{R} < e^r$ if and only if $\tilde{R} < e^{r(1-\tau\wedge 1)}$ and if and only if $\tau \le 1$, in which case $\tilde{R} = e^{r(1-\tau\wedge 1)}b$. Consequently,

$$\begin{aligned} \tilde{D} - \tilde{F} &= \mathbf{1}_{\{\tau \le 1\}} \cdot \Big[\alpha\big((1-w)/w\big)\left(e^r - \tilde{R}\right) \\ &\qquad - \min\left\{\gamma e^r + (1-\gamma)\tilde{R}, \big((1-w)/w\big)\left(e^{r(1-\tau\wedge 1)} - \tilde{R}\right)\right\}\Big] \\ &= \mathbf{1}_{\{\tau \le 1\}}\big((1-w)/w\big)\left(e^r - \tilde{R}\right) \cdot \Bigg[\alpha \\ &\qquad - \min\left\{\big(w/(1-w)\big)\left(\gamma + \frac{\tilde{R}}{e^r - \tilde{R}}\right), \frac{\tilde{R}(1/b - 1)}{e^r - \tilde{R}}\right\}\Bigg]. \end{aligned}$$

Note that on $\{\tau \le 1\}$, we have $\tilde{R} = e^{r(1-\tau\wedge 1)}b$ and thus

$$\frac{b}{e^r - b} \le \frac{\tilde{R}}{e^r - \tilde{R}} \le \frac{b}{1-b}, \quad \frac{1-b}{e^r - b} \le \frac{\tilde{R}(1/b - 1)}{e^r - \tilde{R}} \le 1.$$

Consequently, we have

$$\frac{1-w}{w}\left(e^r-\tilde{R}\right)(\alpha-U')\mathbf{1}_{\{\tau\le 1\}} \le \tilde{D}-\tilde{F} \le \frac{1-w}{w}\left(e^r-\tilde{R}\right)(\alpha-U)\mathbf{1}_{\{\tau\le 1\}}, \tag{O–6}$$

where

$$U' := \min\left\{\frac{\gamma w}{1-w}+\frac{wb}{(1-w)(1-b)},1\right\}.$$

Now, suppose $\alpha < U$. Then, we conclude $\tilde{D}-\tilde{F}\le 0$ from (O–6). Consequently, according to (5), the entrepreneur's payoff satisfies

$$\tilde{Z}_1 \le Z_0\left[\left(1-\gamma+\alpha(1-w)/w\right)\tilde{R}+\left(\gamma-\alpha(1-w)/w\right)e^r\right].$$

Suppose that the entrepreneur is attracted, i.e., $\bar{\alpha}_E(u_E,\tilde{R},\tau)<\alpha$. Then, we have

$$\begin{aligned} u_E(e^r) &< \mathbb{E}\left[u_E(\tilde{Z}_1/Z_0)\right] \le \mathbb{E}\left[u_E\left(\left(1-\gamma+\alpha(1-w)/w\right)\tilde{R}+\left(\gamma-\alpha(1-w)/w\right)e^r\right)\right] \\ &\le u_E\left(\mathbb{E}\left[\left(1-\gamma+\alpha(1-w)/w\right)\tilde{R}+\left(\gamma-\alpha(1-w)/w\right)e^r\right]\right), \end{aligned}$$

where the last inequality is the case because u_E is concave. Because $1-\gamma+\alpha(1-w)/w>0$, we conclude that $\mathbb{E}[\tilde{R}]>e^r$. On the other hand, recalling (7) and that $\tilde{D}-\tilde{F}\le 0$, we conclude that the funder's payoff satisfies

$$\tilde{Y}_1 \ge Y_0\left[\alpha e^r+(1-\alpha)\tilde{R}\right].$$

Then, for $u_F\in\mathcal{U}_{0,1}$, we have

$$\mathbb{E}\left[u_F\left(\tilde{Y}_1/Y_0\right)\right] \ge \mathbb{E}\left[u_F\left(\alpha e^r+(1-\alpha)\tilde{R}\right)\right] = u_F\left(\alpha e^r+(1-\alpha)\mathbb{E}[\tilde{R}]\right) > u_F(e^r),$$

where the last inequality is the case because $\mathbb{E}[\tilde{R}]>e^r$ and $\alpha>0$. As a result, $\bar{\alpha}_F(u_F,\tilde{R},\tau)>\alpha$, i.e., the funder finds the entrepreneur to be attractive.

The above proof shows that if an entrepreneur is attracted, we have $\mathbb{E}[\tilde{R}]>e^r$, which then implies that all unskilled entrepreneur are deterred, i.e., for any $u_E\in\mathcal{U}$ and $(\tilde{R},\tau)\in\mathcal{R}_{b,U}$, we have $\bar{\alpha}_E(u_E,\tilde{R},\tau)\ge\alpha$ because $\mathbb{E}[\tilde{R}]\le e^r$ for $(\tilde{R},\tau)\in\mathcal{R}_{b,U}$.

Next, suppose we also have $\alpha>L$. It is straightforward to see from (5) that for all perfectly skilled entrepreneurs, i.e., for all $u_E\in\mathcal{U}$ and all $(\tilde{R},\tau)\in\mathcal{R}_P$, $\bar{\alpha}_E(u_E,\tilde{R},\tau)<\alpha$ because $\gamma-\alpha(1-w)/w=(L-\alpha)(1-w)/w<0$.

Finally, given $\alpha \in (L, U)$ and $u_E \in \mathcal{U}$, consider $(\tilde{R}, \tau) \in \mathcal{R}_{b,S}$ with $\mathbb{P}(\tau = 1) = \mathbb{P}(\tau \le 1) = 1 - p \in (0,1)$ and $\tilde{R} = x$ for some constant $x > e^r$ when $\tau > 1$. Then, by (5), we have

$$\mathbb{E}[u_E(\tilde{Z}_1/Z_0)] - \mathbb{E}[u_E(\tilde{R})] = \left[u_E\left(\left(1-\gamma+\alpha(1-w)/w\right)x + \left(\gamma - \alpha(1-w)/w\right)e^r\right) - u_E(x)\right]p$$
$$- \left[u_E(b) - u_E\big(\max(0, \gamma e^r + (1-\gamma)b - (1-b)(1-w)/w)\big)\right](1-p). \tag{O–7}$$

We also compute the derivative of $\mathbb{E}[u_E(a\tilde{R} + (1-a)e^r)]$ with respect to a and denote by $h(a) := \mathbb{E}[u_E'(a\tilde{R} + (1-a)e^r)(\tilde{R} - e^r)]$, $a \in [0,1]$. Due to the concavity of u_E,

$$h(a) \ge h(1) = \mathbb{E}[u_E'(\tilde{R})(\tilde{R} - e^r)] = u_E'(x)(x - e^r)p + u_E'(b)(b - e^r)(1-p). \tag{O–8}$$

Because $\alpha > L$, for any fixed $x > e^r$, $\left(1-\gamma+\alpha(1-w)/w\right)x + \left(\gamma - \alpha(1-w)/w\right)e^r > x$, so there exists $p \in (0,1)$ such that the right-hand side of both (O–7) and (O–8) is positive and thus

$$\mathbb{E}[u_E(\tilde{Z}_1/Z_0)] > \mathbb{E}[u_E(\tilde{R})] = \max_{a \in [0,1]} \mathbb{E}[u_E(a\tilde{R} + (1-a)e^r)].$$

As a result, $\bar{\alpha}_E(u_E, \tilde{R}, \tau) < \alpha$. $\square$

Proof of Theorem 4 Because $\mathbb{U}(\delta, \lambda, \mathcal{O}) \le U$, assertion (i) of the theorem follows from the proof of Theorem 3 immediately. In the following, we prove assertion (ii).

Consider any entrepreneur with $u_E \in \mathcal{U}$ and $(\tilde{R}, \tau) \in \mathcal{R}_b$ such that $\bar{\alpha}_E(u_E, \tilde{R}, \tau) < \alpha$; i.e., this entrepreneur is attracted. Following (5) and (O–6) and noting that $\tau \le 1$ if and only if $\tilde{R} < e^r$, we conclude that the entrepreneur's payoff satisfies

$$\begin{aligned}\tilde{Z}_1 &= Z_0\Big\{\tilde{R} + \left((1-w)/w\right)(\alpha - L)(\tilde{R} - e^r) + \tilde{D} - \tilde{F}\Big\} \\ &\le Z_0\Big\{\tilde{R} + \left((1-w)/w\right)(\alpha - L)(\tilde{R} - e^r) + \mathbf{1}_{\{\tilde{R} < e^r\}}\left((1-w)/w\right)\left(e^r - \tilde{R}\right)(\alpha - U)\Big\} \\ &=: \tilde{Z}_1'.\end{aligned}$$

Because $\alpha > L$ and $\alpha < \mathbb{U}(\delta, \lambda, \mathcal{O}) \le U$, $\tilde{Z}_1'/Z_0 - \tilde{R}$ is an increasing function of $\tilde{R}$ and thus is comonotone with respect to $\tilde{R}$. On the other hand, because the entrepreneur is attracted, she must prefer $\tilde{Z}_1/Z_0$ and thus $\tilde{Z}_1'/Z_0$ to $\tilde{R}$. Because u_E is concave and thus the entrepreneur is monotone risk-averse, we must have $\mathbb{E}[\tilde{Z}_1'/Z_0 - \tilde{R}] > 0$; see for instance Quiggin (1991), Cohen (1995), and Chateauneuf, Cohen, and Meilijson (2005) for more details about the definition of monotone risk aversion and its properties. As a result,

$$\mathbb{E}\left[\left((1-w)/w\right)(\alpha - L)(\tilde{R} - e^r) + \mathbf{1}_{\{\tilde{R} < e^r\}}\left((1-w)/w\right)\left(e^r - \tilde{R}\right)(\alpha - U)\right] > 0.$$

Recalling that $\tilde{R} < e^r$ if and only if $\tau \le 1$, in which case $\tilde{R} = e^{r(1-\tau\wedge 1)}b \in [b, e^r b]$, and $\tilde{R} > e^r$ if and only if $\tau > 1$, in which case $\tilde{R} = m_1 + 1$ for some constant $m_1 > e^r - 1$, the above condition implies

$$(\alpha - L)(m_1 + 1 - e^r)\mathbb{P}(\tau > 1) > (U - L)\mathbb{E}\left[(e^r - \tilde{R})\mathbf{1}_{\{\tilde{R} < e^r\}}\right] \ge (U - L)e^r(1 - b)\mathbb{P}(\tau \le 1).$$

In consequence, we have

$$m_1 + 1 - e^r > \frac{e^r(U - L)(1 - b)}{\alpha - L} \times \frac{\mathbb{P}(\tau \le 1)}{\mathbb{P}(\tau > 1)} \tag{O–9}$$

Next, consider a funder with utility function $u_F \in \mathcal{U}_{\delta,\lambda}$. Recall that the funder's payoff as in (7). From (O–6) and recalling that $\tilde{R} < e^r$ if and only if $\tau \le 1$, in which case $\tilde{R} = e^{r(1-\tau\wedge 1)}b \in [b, e^r b]$, we conclude

$$\begin{aligned}
\tilde{Y}_1 &= Y_0\Big\{e^r + (1 - \alpha)(\tilde{R} - e^r) - \big(w/(1 - w)\big)(\tilde{D} - \tilde{F})\Big\} \\
&\ge Y_0\Big\{e^r + (1 - \alpha)(\tilde{R} - e^r) - \mathbf{1}_{\{\tilde{R} < e^r\}}\left(e^r - \tilde{R}\right)(\alpha - U)\Big\} \\
&\ge Y_0\Big\{e^r + (1 - \alpha)(m_1 + 1 - e^r)\mathbf{1}_{\{\tau > 1\}} - (1 - U)(e^r - b)\mathbf{1}_{\{\tau \le 1\}}\Big\} \\
&=: \tilde{Y}_1'.
\end{aligned}$$

As a result, if $\mathbb{E}[u_F(\tilde{Y}_1'/Y_0)] > u_F(e^r)$, then $\bar{\alpha}_F(u_F, \tilde{R}, \tau) > \alpha$, i.e., the entrepreneur is attractive to the funder. Straightforward calculation yields

$$\begin{aligned}
\mathbb{E}[u_F(\tilde{Y}_1'/Y_0)] &= u_F\big(e^r + (1 - \alpha)(m_1 + 1 - e^r)\big)\mathbb{P}(\tau > 1) \\
&\quad + u_F\big(e^r - (1 - U)(e^r - b)\big)\mathbb{P}(\tau \le 1) \\
&\ge u_F\left(e^r + (1 - \alpha)\frac{e^r(U - L)(1 - b)}{\alpha - L} \cdot \frac{\mathbb{P}(\tau \le 1)}{\mathbb{P}(\tau > 1)}\right)\mathbb{P}(\tau > 1) \\
&\quad + u_F\big(e^r - (1 - U)(e^r - b)\big)\mathbb{P}(\tau \le 1),
\end{aligned}$$

where the inequality follows from (O–9). Denote by $O_\tau := \mathbb{P}(\tau \le 1)/\mathbb{P}(\tau > 1)$ the odds of the entrepreneur's project leading the fund to liquidate. Then, $\bar{\alpha}_F(u_F, \tilde{R}, \tau) > \alpha$ if the right-hand side of the above inequality is strictly larger than $u_F(e^r)$, which is the case if and only if

$$\begin{aligned}
&u_F\left(e^r\left(1 + (1 - \alpha)\frac{(U - L)(1 - b)}{\alpha - L}O_\tau\right)\right) - u_F(e^r) \\
&+\Big(u_F\big(e^r(1 - (1 - U)(1 - e^{-r}b))\big) - u_F(e^r)\Big)O_\tau > 0.
\end{aligned}$$

Then, because $u_F \in \mathcal{U}_{\delta,\lambda}$, (O–1) and (O–2) yield that

$$
\begin{aligned}
& u_F\left(e^r\left(1+(1-\alpha)\frac{(U-L)(1-b)}{\alpha-L}O_\tau\right)\right)-u_F(e^r) \\
& \quad + \Big(u_F\big(e^r(1-(1-U)(1-e^{-r}b))\big)-u_F(e^r)\Big)O_\tau \\
& \geq u_F'(e^r+)(e^r)^\delta\Big\{u_\delta\left(e^r\left(1+(1-\alpha)\frac{(U-L)(1-b)}{\alpha-L}O_\tau\right)\right)-u_\delta(e^r) \\
& \quad + \lambda\Big[u_\delta\big(e^r(1-(1-U)(1-e^{-r}b))\big)-u_\delta(e^r)\Big]O_\tau\Big\}.
\end{aligned}
$$

Thus, given $\alpha \in (L, U)$, $\bar{\alpha}_F(u_F, \tilde{R}, \tau) > \alpha$ if the right-hand side of the above inequality is strictly larger than 0, which is the case if and only if $f(\alpha; \delta, \lambda, O_\tau) > 0$, where

$$
f(\alpha;\delta,\lambda,O_\tau) := u_\delta\left(1+(1-\alpha)\frac{(U-L)(1-b)}{\alpha-L}O_\tau\right)+\lambda u_\delta\big(1-(1-U)(1-e^{-r}b)\big)O_\tau.
$$

Noting that f is continuous and strictly decreasing in α, $f(1;\delta,\lambda,O_\tau)<0$, and $\lim_{\alpha\downarrow L} f(\alpha;\delta,\lambda,O_\tau) = +\infty$, we conclude that $f(\alpha;\delta,\lambda,\mathcal{O}_\tau)>0$ if and only if $\alpha < V(\delta,\lambda,O_\tau)$, where

$$
V(\delta,\lambda,O_\tau) := L+\frac{(U-L)(1-b)(1-L)O_\tau}{u_\delta^{-1}\big(-\lambda u_\delta\big(1-(1-U)(1-e^{-r}b)\big)O_\tau\big)-1+(U-L)(1-b)O_\tau}.
$$

Denote $a := -\lambda u_\delta\big(1-(1-U)(1-e^{-r}b)\big) > 0$ and note that

$$
V(\delta,\lambda,O_\tau) = L + a^{-1}(U-L)(1-b)(1-L)\Big[g(aO_\tau)+a^{-1}(U-L)(1-b)\Big]^{-1},
$$

where $g(z) := \left[u_\delta^{-1}(z)-1\right]/z$. Straightforward calculation yields

$$
g'(z) = z^{-1}/u_\delta'(u_\delta^{-1}(z)) - \left[u_\delta^{-1}(z)-1\right]z^{-2} = z^{-2}h(u_\delta^{-1}(z)),
$$

where $h(y) := u_\delta(y)/u_\delta'(y) - y + 1 = y^\delta u_\delta(y) - y + 1$. One can verify that h is strictly convex in $y \in (0,+\infty)$ and $h(1) = h'(1) = 0$, so $h(y) > 0$ for $y \in (1,+\infty)$. Consequently, $g'(z) > 0$ and thus g is strictly increasing for $z \in (0,+\infty)$. Because $a > 0$, we conclude that $V(\delta,\lambda,O_\tau)$ is strictly decreasing in $O_\tau \in [0,+\infty)$. Consequently, $\alpha < \mathbb{U}(\delta,\lambda,\mathcal{O})$ implies $\alpha < \min\{V(\delta,\lambda,O_\tau), U\}$ for any $O_\tau \leq \mathcal{O}$, so the proof completes. □

Proof of Corollary 1 In the traditional scheme, the entrepreneur's payoff is

$$
\tilde{Z}_1 = Z_0\left\{\tilde{R}+\frac{1-w}{w}\alpha(\tilde{R}-e^r)^+ + \frac{1-w}{w}(z_1\tilde{R}-z_0e^r)\right\} \tag{O–10}
$$

and the funder's payoff is

$$\tilde{Y}_1 = Y_0 \left\{ \tilde{R} - \alpha(\tilde{R} - e^r)^+ - z_1 \tilde{R} \right\}. \tag{O–11}$$

The case in which $z_0 = z_1 = 0$ has been discussed in Theorems 1 and 2, so in the following, we first consider the case in which $z_0 \le z_1$ and $z_1 > 0$. In this case, consider $u_E \in \mathcal{U}_{\delta,\lambda}$ with δ and λ satisfying (A.1) and $(\tilde{R}, \tau) \in \mathcal{R}_{b,U}$ with $\mathbb{P}(\tilde{R} = e^{r(1-\tau\wedge 1)}b) = \mathbb{P}(\tau \le 1) = \mathbb{P}(\tau = 1) = 1 - p$ and $\mathbb{P}(\tilde{R} = e^r/p - b(1-p)/p) = \mathbb{P}(\tau > 1) = p$ and for certain $p \in (0, 1]$. One can verify that $\mathbb{E}[\tilde{R}] = e^r$. Denote by

$$\begin{aligned}
f(p) := \mathbb{E}[u_E(\tilde{Z}_1/Z_0)] = u_E\bigg(& \frac{e^r}{p} - \frac{b(1-p)}{p} + \alpha\frac{1-w}{w}\left(\frac{e^r}{p} - \frac{b(1-p)}{p} - e^r\right) \\
& + \frac{1-w}{w}\left(z_1\left(\frac{e^r}{p} - \frac{b(1-p)}{p}\right) - z_0 e^r\right)\bigg)p + u_E\left(b + \frac{1-w}{w}(z_1 b - z_0 e^r)\right)(1-p).
\end{aligned}$$

Straightforward calculation yields that

$$\begin{aligned}
f'(1) = & \left(u_E\left(e^r + \frac{1-w}{w}(z_1 e^r - z_0 e^r)\right) - u_E(e^r)\right) + \left(u_E(e^r) - u_E\left(b + \frac{1-w}{w}(z_1 b - z_0 e^r)\right)\right) \\
& - (e^r - b)\big(1 + (\alpha + z_1)(1-w)/w\big)u_E'\left(\left(e^r + \frac{1-w}{w}(z_1 e^r - z_0 e^r)\right)+\right) \\
\le & \; u_E'\left(\left(e^r + \frac{1-w}{w}(z_1 e^r - z_0 e^r)\right)+\right)\left(e^r + \frac{1-w}{w}(z_1 e^r - z_0 e^r)\right)^\delta \\
& \times \left[u_\delta\left(e^r + \frac{1-w}{w}(z_1 e^r - z_0 e^r)\right) - u_\delta(e^r)\right] - e^r u_\delta\left(e^{-r}b + \frac{1-w}{w}(z_1 b e^{-r} - z_0)\right)u_E'(e^r-) \\
& - (e^r - b)\big(1 + (\alpha + z_1)(1-w)/w\big)u_E'\left(\left(e^r + \frac{1-w}{w}(z_1 e^r - z_0 e^r)\right)+\right) \\
\le & \; e^r u_E'\left(\left(e^r + \frac{1-w}{w}(z_1 e^r - z_0 e^r)\right)+\right)\bigg[-(1 - e^{-r}b)\big(1 + (\alpha + z_1)(1-w)/w\big) \\
& + \left(1 + \frac{1-w}{w}(z_1 - z_0)\right)^\delta \left(u_\delta\left(1 + \frac{1-w}{w}(z_1 - z_0)\right) - u_\delta\left(e^{-r}b + \frac{1-w}{w}(z_1 b e^{-r} - z_0)\right)\lambda\right)\bigg] \\
< & \; 0,
\end{aligned}$$

where the first inequality is the case due to (O–1), the second inequality is the case due to (O–2) and because $u_E'(x)/u_E'(y) \ge (x/y)^{-\delta}$ for any $x > y > e^r$, and the last inequality is the case due to (A.1). Thus, for some $p \in (0, 1)$, we have

$$f(p) > f(1) = u_E\left(\left(1 + \frac{1-w}{w}(z_1 - z_0)\right)e^r\right) \ge u_E(e^r) = \max_{a\in[0,1]} \mathbb{E}\left[u_E\big(a\tilde{R} + (1-a)e^r\big)\right],$$

where the last equality is the case because $\mathbb{E}[\tilde{R}] = e^r$ and u_E is concave. Thus, we have $\bar{\alpha}_E(u_E, \tilde{R}, \tau) < \alpha$. On the other hand, for any $u_F \in \mathcal{U}$,

$$\mathbb{E}\left[u_F\big(\tilde{Y}_1/Y_0\big)\right] \le \mathbb{E}\left[u_F\big(\alpha e^r + (1-\alpha)\tilde{R}\big)\right] \le u_F\big(\mathbb{E}\left[\alpha e^r + (1-\alpha)\tilde{R}\right]\big) = u_F(e^r), \tag{O–12}$$

where the first inequality is the case due to (O–11), the second is the case due to the concavity of u_F, and the equality is the case because $\mathbb{E}[\tilde{R}] = e^r$. As a result, $\bar{\alpha}_F(u_F, \tilde{R}, \tau) \le \alpha$.

Next, we consider the case in which $z_1 < z_0 \le z_1/(1-z_1)$ and $z_1 > 0$. In this case, consider $(\tilde{R}, \tau) \in \mathcal{R}_P$ with $\tilde{R} = (z_0/z_1)e^r > e^r$ and any $u_E \in \mathcal{U}$. Straight forward calculation yields

$$\tilde{Z}_1/Z_0 = (z_0/z_1)e^r + \frac{1-w}{w}\alpha((z_0/z_1)e^r - e^r)^+ + \frac{1-w}{w}(z_1(z_0/z_1)e^r - z_0e^r) > (z_0/z_1)e^r,$$

showing that $\bar{\alpha}_E(u_E, \tilde{R}, \tau) < \alpha$. On the other hand,

$$\tilde{Y}_1/Y_0 = (z_0/z_1)e^r - \alpha((z_0/z_1)e^r - e^r)^+ - z_1(z_0/z_1)e^r < e^r,$$

where the inequality is the case because $z_0 \le z_1/(1-z_1)$. Thus, $\bar{\alpha}_F(u_F, \tilde{R}, \tau) \le \alpha$ for any $u_F \in \mathcal{U}$.

Next, we consider the case in which $z_0 > z_1/(1-z_1)$ and (A.2) holds. Consider an entrepreneur with $u_E \in \mathcal{U}$ and $(R, \tau) \in \mathcal{R}_b$, and suppose she is attracted, i.e., $\bar{\alpha}_E(u_E, \tilde{R}, \tau) < \alpha$. Recalling (O–10), $\tilde{Z}_1/Z_0 - \tilde{R}$ is an increasing function of $\tilde{R}$ and thus is comonotone with respect to $\tilde{R}$. Since the entrepreneur is monotone risk-averse (as shown in the proof of Theorem 4), we have $\mathbb{E}[\tilde{Z}_1/Z_0 - \tilde{R}] > 0$ and immediately conclude that

$$\mathbb{E}[\tilde{R} - e^r] > \Big((z_0 - z_1)e^r - \alpha\mathbb{E}\left[(e^r - \tilde{R})^+\right]\Big)/(\alpha + z_1). \tag{O–13}$$

Consequently,

$$\begin{aligned}
\mathbb{E}[\tilde{Y}_1/Y_0] &= (1 - z_1 - \alpha)\mathbb{E}[\tilde{R} - e^r] - \alpha\mathbb{E}\left[(e^r - \tilde{R})^+\right] + (1 - z_1)e^r \\
&> (1 - z_1 - \alpha)(\alpha + z_1)^{-1}\left((z_0 - z_1)e^r - \alpha\mathbb{E}\left[(e^r - \tilde{R})^+\right]\right) \\
&\quad - \alpha\mathbb{E}\left[(e^r - \tilde{R})^+\right] + (1 - z_1)e^r \\
&= (\alpha + z_1)^{-1}\left[\big(z_0 - z_1 - z_0(\alpha + z_1)\big)e^r - \alpha\mathbb{E}\left[(e^r - \tilde{R})^+\right]\right] + e^r \\
&\ge (\alpha + z_1)^{-1}\left[\big(z_0 - z_1) - z_0(\alpha + z_1)\big)e^r - \alpha(e^r - b)\right] + e^r \\
&> e^r,
\end{aligned}$$

where the first inequality is due to (O–13), the second inequality is the case because $\tilde{R} \geq b$, and the third inequality is the case due to (A.2). Thus, this entrepreneur is attractive to all risk-neutral funders, i.e., $\bar{\alpha}_F(u_F, \tilde{R}, \tau) > \alpha$ for $u_F \in \mathcal{U}_{0,1}$. Following the same proof as in the last part of the proof of Theorem 3, we can show that for any fixed $u_E \in \mathcal{U}$, there exists $(\tilde{R}, \tau) \in \mathcal{R}_{b,S}$ such that $\bar{\alpha}_E(u_E, \tilde{R}, \tau) < \alpha$.

Finally, we consider the case in which $z_0 > z_1/(1 - z_1)$ and $\alpha > \frac{z_0(1-z_1)-z_1}{1-be^{-r}+z_0}$. Given a sufficiently small $\epsilon > 0$, we can find a risk-neutral entrepreneur (i.e., $u_E \in \mathcal{U}_{0,1}$) with $(\tilde{R}, \tau) \in \mathcal{R}_b$ such that

$$\mathbb{E}\left[(e^r - \tilde{R})^+\right] > e^r - b - \epsilon, \tag{O–14}$$

$$\mathbb{E}\left[\tilde{R} - e^r\right] > \left((z_0 - z_1)e^r - \alpha\mathbb{E}\left[(e^r - \tilde{R})^+\right]\right)^+ / (\alpha + z_1), \tag{O–15}$$

$$\mathbb{E}\left[\tilde{R} - e^r\right] < \left((z_0 - z_1)e^r - \alpha\mathbb{E}\left[(e^r - \tilde{R})^+\right]\right)^+ / (\alpha + z_1) + \epsilon. \tag{O–16}$$

Then, (O–15) implies that $\mathbb{E}[\tilde{R}] > e^r$. Together with (O–10), (O–15) also implies that $\mathbb{E}[\tilde{Z}_1/Z_0] > \mathbb{E}[\tilde{R}]$. Due to $u_E \in \mathcal{U}_{0,1}$, $\max_{a\in[0,1]} \mathbb{E}[u_E(a\tilde{R} + (1 - a)e^r)] < \mathbb{E}[u_E(\tilde{Z}_1/Z_0)]$, showing that the entrepreneur is attracted, i.e., $\bar{\alpha}_E(u_E, \tilde{R}, \tau) < \alpha$. On the other hand, (O–16) implies that

$$\begin{aligned}
\mathbb{E}\left[\tilde{Y}_1/Y_0\right] &= (1 - z_1 - \alpha)\mathbb{E}[\tilde{R} - e^r] - \alpha\mathbb{E}\left[(e^r - \tilde{R})^+\right] + (1 - z_1)e^r \\
&< (1 - z_1 - \alpha)\left[\left((z_0 - z_1)e^r - \alpha\mathbb{E}\left[(e^r - \tilde{R})^+\right]\right)^+ / (\alpha + z_1) + \epsilon\right] \\
&\quad - \alpha\mathbb{E}\left[(e^r - \tilde{R})^+\right] + (1 - z_1)e^r.
\end{aligned} \tag{O–17}$$

As a result, if $(z_0 - z_1)e^r - \alpha\mathbb{E}\left[(e^r - \tilde{R})^+\right] \leq 0$, we conclude from (O–17) that

$$\begin{aligned}
\mathbb{E}\left[\tilde{Y}_1/Y_0\right] &< (1 - z_1 - \alpha)\epsilon - \alpha\mathbb{E}\left[(e^r - \tilde{R})^+\right] + (1 - z_1)e^r \\
&\leq (1 - z_1 - \alpha)\epsilon - \alpha\left(e^r - b - \epsilon\right) + (1 - z_1)e^r < e^r
\end{aligned}$$

for sufficiently small ϵ, where the second inequality is the case due to (O–14); consequently, the entrepreneur is unappealing to all funders. If $(z_0 - z_1)e^r - \alpha\mathbb{E}\left[(e^r - \tilde{R})^+\right] > 0$, we conclude from

(O–17) that

$$\begin{aligned}\mathbb{E}\left[\tilde{Y}_1/Y_0\right] &< (\alpha+z_1)^{-1}\left[\big(z_0-z_1-z_0(\alpha+z_1)\big)e^r-\alpha\mathbb{E}\left[(e^r-\tilde{R})^+\right]\right]\\ &\qquad+e^r+(1-z_1-\alpha)\epsilon\\ &< (\alpha+z_1)^{-1}\left[\big(z_0-z_1-z_0(\alpha+z_1)\big)e^r-\alpha\left(e^r-b-\epsilon\right)\right]\\ &\qquad+e^r+(1-z_1-\alpha)\epsilon\\ &= (\alpha+z_1)^{-1}(e^r-b+z_0e^r)\left[\frac{z_0(1-z_1)-z_1}{1-be^{-r}+z_0}-\alpha\right]\\ &\qquad+e^r+(1-z_1-\alpha)\epsilon+(\alpha+z_1)^{-1}\alpha\epsilon\\ &\le e^r,\end{aligned}$$

for sufficiently small $\epsilon>0$, where the second inequality is the case due to (O–14) and the last inequality is because $\alpha>\frac{z_0(1-z_1)-z_1}{1-be^{-r}+z_0}$, and thus this entrepreneur is unappealing to all funders, i.e., $\bar{\alpha}_F(u_F,\tilde{R},\tau)\le\alpha$ for all $u_F\in\mathcal{U}$. $\square$

Proof of Corollary 2 With the management fee and cost, the entrepreneur's gross growth is

$$\begin{aligned}\tilde{Z}_1/Z_0 = \Big[&\big(1-\gamma+\alpha(1-w)/w\big)\tilde{R}+\big(\gamma-\alpha(1-w)/w\big)e^r\Big]\\ &+\tilde{D}-\tilde{F}+\big((1-w)/w\big)\big(z_1\tilde{R}-z_0e^r\big)\end{aligned}\tag{O–18}$$

and the funder's gross growth is

$$\tilde{Y}_1/Y_0=\left[\alpha e^r+(1-\alpha)\tilde{R}\right]-\big(w/(1-w)\big)\tilde{D}+\big(w/(1-w)\big)\tilde{F}-z_1\tilde{R}.\tag{O–19}$$

We first consider the case of no liquidation boundaries. Then, we have

$$\begin{aligned}\tilde{D}-\tilde{F}&=\frac{1-w}{w}e^r\left(\alpha-\min(L,e^{-r})\right)\mathbf{1}_{\{\tilde{R}<e^r\}},\\ \mathbb{E}[\tilde{Z}_1/Z_0]&=\mathbb{E}[\tilde{R}]+\frac{1-w}{w}\Big[(\alpha-L+z_1)\mathbb{E}[\tilde{R}-e^r]+(z_1-z_0)e^r\\ &\qquad+\big(\alpha-\min(L,e^{-r})\big)\mathbb{P}(\tilde{R}<e^r)e^r\Big],\end{aligned}\tag{O–20}$$

$$\mathbb{E}[\tilde{Y}_1/Y_0]=e^r+(1-\alpha-z_1)\mathbb{E}[\tilde{R}-e^r]-\big(\alpha-\min(L,e^{-r})\big)\mathbb{P}(\tilde{R}<e^r)e^r-z_1e^r,\tag{O–21}$$

due to (O–5), (O–18), and (O–19), respectively. As the case of $z_0=z_1=0$ has been studied in Theorem 1, we assume in the following that one of z_0 and z_1 is positive. Suppose $z_1>z_0$. For any $u_E\in\mathcal{U}_{0,1}$, consider $(\tilde{R},\tau)\in\mathcal{R}_{0,U}$ such that $\mathbb{P}(\tilde{R}=e^r/p)=p$ and $\mathbb{P}(\tilde{R}=0)=1-p$ for certain

$p \in (0,1)$. It is obvious that $\mathbb{E}[\tilde{R}] = e^r$. Because $z_1 > z_0$, it is straightforward to show from (O–20) that when p is sufficiently close to 1, $\bar{\alpha}_E(u_E, \tilde{R}, \tau) < \alpha$, showing that the entrepreneur is attracted. On the other hand, from (O–21), we have

$$\mathbb{E}[\tilde{Y}_1/Y_0] = e^r - \left(\alpha - \min(L, e^{-r})\right) \mathbb{P}(\tilde{R} < e^r)e^r - z_1 e^r < e^r,$$

showing that $\bar{\alpha}_F(u_F, \tilde{R}, \tau) \le \alpha$ for all $u_F \in \mathcal{U}$.

Next, suppose $z_1 \le z_0$ and $\alpha \le \min(L - z_1, e^{-r})$, and consider any entrepreneur with $u_E \in \mathcal{U}$ and $(\tilde{R}, \tau) \in \mathcal{R}_0$. Because $\alpha \le \min(L - z_1, e^{-r}) \le \min(L, e^{-r})$ and thus $\tilde{D} - \tilde{F} \le 0$, we conclude from (O–18) that

$$\begin{aligned}
\tilde{Z}_1/Z_0 &\le \left[\left(1 - \frac{1-w}{w}(L - \alpha - z_1)\right)\tilde{R} + \frac{1-w}{w}(L - \alpha - z_1)e^r\right] + \frac{1-w}{w}(z_1 - z_0)e^r \\
&\le \left[\left(1 - \frac{1-w}{w}(L - \alpha - z_1)\right)\tilde{R} + \frac{1-w}{w}(L - \alpha - z_1)e^r\right].
\end{aligned}$$

Because $\alpha \le \min(L - z_1, e^{-r}) \le L - z_1$, we have $\frac{(1-w)}{w}(L - \alpha - z_1) = \gamma - \frac{(1-w)}{w}(\alpha + z_1) \in [0,1]$ and thus the right-hand side of the above inequality is the return of a portfolio of the entrepreneur's project and risk-free asset. Thus, the entrepreneur is deterred, i.e., $\bar{\alpha}_E(u_E, \tilde{R}, \tau) \ge \alpha$.

Next, suppose $z_1 \le z_0$ and $\alpha > L + \left(\frac{(1-L)z_0}{1-L+z_0} - z_1\right)^+$; the latter implies that $\alpha > L$. Given a sufficiently small $\epsilon > 0$, consider $u_E \in \mathcal{U}_{0,1}$ and $(\tilde{R}, \tau) \in \mathcal{R}_0$ such that $\mathbb{P}(\tilde{R} < e^r) \in (1 - \epsilon, 1)$ and

$$\mathbb{E}[\tilde{R} - e^r] - \frac{1}{\alpha - L + z_1}\left((z_0 - z_1)e^r - \left(\alpha - \min(L, e^{-r})\right)\mathbb{P}(\tilde{R} < e^r)e^r\right)^+ \in (0, \epsilon). \qquad \text{(O–22)}$$

Then, (O–22) implies that $\mathbb{E}[\tilde{R}] > e^r$ and, together with (O–20), also implies that $\mathbb{E}[\tilde{Z}_1/Z_0] > \mathbb{E}[\tilde{R}]$, showing that the entrepreneur is attracted, i.e., $\bar{\alpha}_E(u_E, \tilde{R}, \tau) < \alpha$. On the other hand, if $\alpha \ge \max\{1 - z_1, L + \left(\frac{(1-L)z_0}{1-L+z_0} - z_1\right)^+\}$, (O–21) and (O–22) lead to

$$\mathbb{E}[\tilde{Y}_1/Y_0] - e^r \le -\left(\alpha - \min(L, e^{-r})\right) \mathbb{P}(\tilde{R} < e^r)e^r - z_1 e^r < 0,$$

showing that this entrepreneur is unappealing to all funders, i.e., $\bar{\alpha}_F(u_F, \tilde{R}, \tau) \le \alpha$ for all $u_F \in \mathcal{U}$;

if $L+\big(\frac{(1-L)z_0}{1-L+z_0}-z_1\big)^+<\alpha<1-z_1$,

$$\begin{aligned}
&\mathbb{E}[\tilde{Y}_1/Y_0]-e^r\\
&<(1-\alpha-z_1)\epsilon+\frac{1-\alpha-z_1}{\alpha-L+z_1}\Big((z_0-z_1)e^r-\big(\alpha-\min(L,e^{-r})\big)\mathbb{P}(\tilde{R}<e^r)e^r\Big)^+\\
&\quad-\big(\alpha-\min(L,e^{-r})\big)\,\mathbb{P}(\tilde{R}<e^r)e^r-z_1e^r\\
&=(1-\alpha-z_1)\epsilon+\frac{e^r}{\alpha-L+z_1}\Big[\max\Big\{-z_0\alpha+(1-z_1)(z_0-z_1)+(L-z_1)z_1\\
&\quad-(1-L)\big(\alpha-\min(L,e^{-r})\big)\mathbb{P}(\tilde{R}<e^r),-\Big(\big(\alpha-\min(L,e^{-r})\big)\mathbb{P}(\tilde{R}<e^r)+z_1\Big)\,(\alpha-L+z_1)\Big\}\Big]\\
&\leq(1-\alpha-z_1)\epsilon+\frac{e^r}{\alpha-L+z_1}\Big[\max\Big\{-z_0\alpha+(1-z_1)(z_0-z_1)+(L-z_1)z_1\\
&\quad-(1-L)\big(\alpha-L\big)(1-\epsilon),0\Big\}\Big],
\end{aligned}$$

where the second inequality is the case because $\mathbb{P}(\tilde{R}<e^r)\in(1-\epsilon,1)$ and $\alpha>L$. The right-hand side of the above inequality is strictly less than zero for sufficiently small ϵ because $\alpha>L+\big(\frac{(1-L)z_0}{1-L+z_0}-z_1\big)^+$. Consequently, this entrepreneur is unappealing to all funders, i.e., $\bar{\alpha}_F(u_F,\tilde{R},\tau)\leq\alpha$ for all $u_F\in\mathcal{U}$. $\square$

Proof of Corollary 3 We consider the case of a liquidation boundary $b\in(0,1)$. Suppose $z_0>z_1\max\big\{(1-L)/(1-z_1),1\big\}$, assume (A.4) holds, and consider any entrepreneur who has a return profile $(\tilde{R},\tau)\in\mathcal{R}_b$. Then, because $\alpha<U$, we conclude from (O–6) that $\tilde{D}-\tilde{F}\leq 0$. Consequently, due to (O–18), the entrepreneur's payoff satisfies

$$\tilde{Z}_1/Z_0\leq\tilde{R}+\big((1-w)/w\big)(\alpha-L)(\tilde{R}-e^r)+\big((1-w)/w\big)\big(z_1\tilde{R}-z_0e^r\big)=:\tilde{Z}_1'/Z_0.$$

Suppose the entrepreneur is attracted. Then, because $\alpha>L-z_1$ and the entrepreneur is monotone risk-averse (as shown in the proof of Theorem 4), we must have $\mathbb{E}[\tilde{Z}_1'/Z_0-\tilde{R}]>0$, which implies

$$(\alpha-L+z_1)\mathbb{E}[\tilde{R}-e^r]>(z_0-z_1)e^r.\tag{O–23}$$

On the other hand, due to (O–19) and $\tilde{D}-\tilde{F}\leq 0$, the funder's payoff satisfies

$$\tilde{Y}_1/Y_0\geq e^r+(1-\alpha)(\tilde{R}-e^r)-z_1\tilde{R}=:\tilde{Y}_1'/Y_0.$$

Consequently, we have

$$\begin{aligned}\mathbb{E}[\tilde{Y}_1'/Y_0] &= e^r + (1-\alpha-z_1)\mathbb{E}[\tilde{R}-e^r] - z_1 e^r \\ &> e^r + (1-\alpha-z_1)\cdot\frac{z_0-z_1}{\alpha-L+z_1}e^r - z_1 e^r \\ &= e^r + \frac{1}{\alpha-L+z_1}\big[-z_0\alpha + z_0(1-z_1) - z_1(1-L)\big]e^r \\ &> e^r,\end{aligned}$$

where the first inequality is due to (O–23) and $1-\alpha-z_1>0$, and the last inequality is the case because $\alpha < 1-z_1-(z_1/z_0)(1-L)$. Thus, $\bar{\alpha}_F(u_F,\tilde{R},\tau)>\alpha$ for $u_F\in\mathcal{U}_{0,1}$.

Note that (O–23) implies that for any entrepreneur with $u_E\in\mathcal{U}$ to be attracted by the first-loss scheme, the expected excess return of her project must be positive; consequently, unskilled entrepreneurs are deterred, i.e., $\bar{\alpha}_E(u_E,\tilde{R},\tau)\geq\alpha$ for all $(\tilde{R},\tau)\in\mathcal{R}_{b,U}$.

Finally, following the same argument as in the proof of Theorem 3, for each $u_E\in\mathcal{U}$, there exists $(\tilde{R},\tau)\in\mathcal{R}_{b,S}$ with $\mathbb{P}(\tau=1)=\mathbb{P}(\tau\leq 1)=1-p\in(0,1)$ and $\tilde{R}=x$ for some constant $x>(\alpha-L+z_0)e^r/(\alpha-L+z_1)$ when $\tau>1$, showing that $\bar{\alpha}_E(u_E,\tilde{R},\tau)<\alpha$. □

Proof of Corollary 4 Assertion (ii) follows from the proof of Corollary 3, so we prove assertion (i) in the following.

By (O–18) and (O–6), the entrepreneur's payoff satisfies

$$\begin{aligned}\tilde{Z}_1/Z_0 \leq \tilde{R} &+ \big((1-w)/w\big)(\alpha-L)(\tilde{R}-e^r) + \big((1-w)/w\big)(z_1\tilde{R}-z_0e^r) \\ &+\big((1-w)/w\big)(e^r-\tilde{R})(\alpha-U)\mathbf{1}_{\{\tilde{R}<e^r\}} =: \tilde{Z}_1'/Z_0.\end{aligned}$$

Because $\alpha>L-z_1$ and $\alpha<\bar{\mathbb{U}}(\delta,\lambda,\mathcal{O})\leq U$, $\tilde{Z}_1'/Z_0-\tilde{R}$ is an increasing function of $\tilde{R}$ and thus is comonotone with $\tilde{R}$. Suppose $\bar{\alpha}_E(u_E,\tilde{R},\tau)<\alpha$, i.e., the entrepreneur is attracted. Then, because the entrepreneur is monotone risk-averse (as shown in the proof of Theorem 4), we must have $\mathbb{E}[\tilde{Z}_1'/Z_0-\tilde{R}]>0$, which implies that

$$\mathbb{E}\left[(\alpha-L)(\tilde{R}-e^r)+(z_1\tilde{R}-z_0e^r)+(e^r-\tilde{R})(\alpha-U)\mathbf{1}_{\{\tilde{R}<e^r\}}\right]>0.$$

Together with the observation that $\tilde{R}=e^{r(1-\tau\wedge 1)}b\in[b,e^rb]$ when $\tau\leq 1$,

$$\big((\alpha-L+z_1)(m_1+1)-(\alpha-L+z_0)e^r\big)\mathbb{P}(\tau>1)>e^r\left[U-L+z_0-\big((U-L)+z_1\big)b\right]\mathbb{P}(\tau\leq 1).$$

Because $\alpha > L - z_1$,

$$m_1 + 1 > \frac{e^r\left[U - L + z_0 - \big((U-L)+z_1\big)b\right]}{\alpha - L + z_1} \times \frac{\mathbb{P}(\tau \le 1)}{\mathbb{P}(\tau > 1)} + \frac{(\alpha - L + z_0)e^r}{\alpha - L + z_1}. \tag{O–24}$$

Next, recall the funder's payoff in (O–19). From (O–6) and $1 - U - z_1 \ge 0$, and recalling that $\tilde{R} < e^r$ if and only if $\tau \le 1$, in which case $\tilde{R} = e^{r(1-\tau\wedge 1)}b \in [b, e^r b]$, we conclude

$$\begin{aligned}\tilde{Y}_1/Y_0 &\ge e^r + (1-\alpha)(\tilde{R} - e^r) - \mathbf{1}_{\{\tilde{R}<e^r\}}\left(e^r - \tilde{R}\right)(\alpha - U) - z_1\tilde{R}\\ &\ge e^r + \big((1-\alpha-z_1)(m_1+1) - (1-\alpha)e^r\big)\mathbf{1}_{\{\tau>1\}}\\ &\qquad - \left[(1-U)e^r - (1-U-z_1)b\right]\mathbf{1}_{\{\tau\le 1\}}\\ &=: \tilde{Y}_1'/Y_0.\end{aligned}$$

As a result, if $\mathbb{E}[u_F(\tilde{Y}_1'/Y_0)] > \mathbb{E}[u_F(e^r))]$, then the entrepreneur is attractive to a funder with utility function u_F, i.e., $\bar{\alpha}_F(u_F, \tilde{R}, \tau) > \alpha$.

Denote $O_\tau := \mathbb{P}(\tau \le 1)/\mathbb{P}(\tau > 1)$. Then, (O–24) implies

$$\begin{aligned}&(1-\alpha-z_1)(m_1+1) - (1-\alpha)e^r\\ >&\frac{1-\alpha-z_1}{\alpha-L+z_1}\Big\{e^r\left[U-L+z_0-\big(U-L+z_1\big)b\right]O_\tau + (\alpha-L+z_0)e^r\Big\} - (1-\alpha)e^r\\ =&e^r\left\{\frac{1-\alpha-z_1}{\alpha-L+z_1}\left[(U-L)(1-b)+z_0-z_1b\right]O_\tau + \frac{z_0\big(-\alpha+1-z_1-(z_1/z_0)(1-L)\big)}{\alpha-L+z_1}\right\}\\ >&e^r\frac{(1-\alpha-z_1)\big((U-L)(1-b)+z_0-z_1b\big)}{\alpha-L+z_1}O_\tau,\end{aligned}$$

where the last inequality is the case because $\alpha < 1 - z_1 - (z_1/z_0)(1-L)$. Consequently,

$$\begin{aligned}\mathbb{E}[u_F(\tilde{Y}_1'/Y_0)] \ge u&\left(e^r + e^r\frac{(1-\alpha-z_1)\big((U-L)(1-b)+z_0-z_1b\big)}{\alpha-L+z_1}O_\tau\right)\mathbb{P}(\tau>1)\\ &+ u\left(e^r - e^r\big((1-U)(1-e^{-r}b)+z_1e^{-r}b\big)\right)\mathbb{P}(\tau\le 1).\end{aligned}$$

Thus, the entrepreneur is attractive to the funder if the right-hand side of the above inequality is strictly larger than $u_F(e^r)$, and this is the case if and only if

$$\begin{aligned}u_F&\left(e^r\left(1+\frac{(1-\alpha-z_1)\big((U-L)(1-b)+z_0-z_1b\big)}{\alpha-L+z_1}O_\tau\right)\right) - u_F(e^r)\\ &+\left[u_F\left(e^r\Big(1-\big((1-U)(1-e^{-r}b)+z_1e^{-r}b\big)\Big)\right) - u_F(e^r)\right]O_\tau > 0.\end{aligned}$$

Noting that $z_0 > z_1 > z_1 b$ and using the same argument in the proof of Theorem 4, for $u_F \in \mathcal{U}_{\delta,\lambda}$, we can show that the above inequality is implied by $\bar{f}(\alpha; \delta, \lambda, O_\tau) > 0$, where

$$\bar{f}(\alpha; \delta, \lambda, O_\tau) := u_\delta \left(1 + \frac{(1-\alpha-z_1)\big((U-L)(1-b)+z_0-z_1 b\big)}{\alpha - L + z_1} O_\tau\right) \\ +\lambda u_\delta \Big(1 - \big((1-U)(1-e^{-r}b) + z_1 e^{-r} b\big)\Big) O_\tau.$$

Noting that $\bar{f}$ is continuous and strictly decreasing in α, and because $\bar{f}(1-z_1; \delta, \lambda, O_\tau) < 0$ and $\lim_{\alpha \downarrow L - z_1} \bar{f}(\alpha; \delta, \lambda, O_\tau) = +\infty$, we conclude that $\bar{f} > 0$ if and only if $\alpha < \bar{V}(\delta, \lambda, O_\tau)$, where $\bar{V}$ is given by (A.5). As in the proof of Theorem 4, we can show that $\bar{V}(\delta, \lambda, O_\tau)$ is strictly decreasing in $O_\tau \in [0, +\infty)$. Consequently, $\alpha < \bar{\mathbb{U}}(\delta, \lambda, \mathcal{O})$ implies $\alpha < \bar{\mathbb{U}}(\delta, \lambda, O_\tau)$ for any $O_\tau \le \mathcal{O}$, so the proof of (i) completes. $\square$

Proof of Corollary 5 We first consider the traditional scheme. In the traditional scheme, the entrepreneur's gross growth is

$$\tilde{Z}_1/Z_0 = \tilde{R} + \big((1-w)/w\big)\alpha(\tilde{R} - e^h)^+ \tag{O–25}$$

and the funder's gross growth is

$$\tilde{Y}_1/Y_0 = \tilde{R} - \alpha(\tilde{R} - e^h)^+. \tag{O–26}$$

Consider an entrepreneur with $u_E \in \mathcal{U}_{0,1}$ and $(\tilde{R}, \tau) \in \mathcal{R}_{b,U}$ with $\mathbb{E}[\tilde{R}] = e^r$ and $\mathbb{P}(\tilde{R} > e^h) > 0$. Then, $\mathbb{E}[\tilde{Z}_1/Z_0] > \mathbb{E}[\tilde{R}] = e^r$, showing that the entrepreneur is attracted, i.e., $\bar{\alpha}_E(u_E, \tilde{R}, \tau) < \alpha$. On the other hand, $\mathbb{E}[\tilde{Y}_1/Y_0] < e^r$, showing that this entrepreneur is unappealing to any weakly risk-averse funders, i.e., $\bar{\alpha}_F(u_F, \tilde{R}, \tau) \le \alpha$ for all $u_F \in \mathcal{U}$.

Next, we consider the first-loss scheme. Recalling that (i) $\tilde{R} \ge e^r$ if and only if $\tau > 1$, (ii) $h \ge r$, and (iii) $\theta \le r$, we can write the entrepreneur's gross growth as

$$\tilde{Z}_1/Z_0 = \Big[\big(1 - \gamma + \alpha(1-w)/w\big)\tilde{R} + \big(\gamma - \alpha(1-w)/w\big)e^r\Big] - \tilde{C} + \tilde{D} - \tilde{F}', \tag{O–27}$$

where the option-to-default $\tilde{D}$ is the same as in (3) and

$$\tilde{C} := \big((1-w)/w\big)\alpha\left[\left(\tilde{R} - e^r\right)^+ - \left(\tilde{R} - e^h\right)^+\right], \tag{O–28}$$

$$\tilde{F}' := \min\left\{\gamma e^r + (1-\gamma)\tilde{R}, \big((1-w)/w\big)\left(e^{\theta(\tau \wedge 1)} e^{r(1-\tau\wedge 1)} - \tilde{R}\right)^+\right\}. \tag{O–29}$$

Here, $\tilde{F}'$ is the first-loss coverage, which is different from the one in (6) because the reference rate θ can be nonzero. Because the entrepreneur's hurdle rate is higher than the risk-free rate, taking a performance fee in excess of the hurdle rate effectively allows the entrepreneur to borrow money from the funders at a rate that is higher than the risk-free rate. The rate difference incurs a cost for the entrepreneur relative to the case in which the hurdle rate is the same as the risk-free rate, and $\tilde{C} \geq 0$ stands for this cost. On the other hand, the funder's gross growth is

$$\tilde{Y}_1/Y_0 = \left[\alpha e^r + (1-\alpha)\tilde{R}\right] + \big(w/(1-w)\big)\left[\tilde{C} - \tilde{D} + \tilde{F}'\right]. \tag{O–30}$$

Now, we assume that there is no liquidation boundary in the first-loss scheme. Then, $\tau > 1$. Moreover, $\tilde{R} = 0$ as long as $\tilde{R} < e^r$. Consequently,

$$\begin{aligned}\tilde{D} - \tilde{F}' &= \left[\big((1-w)/w\big)\alpha e^r - \min\left\{\gamma e^r, \big((1-w)/w\big)e^{\theta}\right\}\right]\mathbf{1}_{\{\tilde{R}<e^r\}}\\ &= \big((1-w)/w\big)e^r\left[\alpha - \min\left\{\frac{\gamma w}{1-w}, e^{\theta-r}\right\}\right]\mathbf{1}_{\{\tilde{R}<e^r\}}.\end{aligned}$$

When $\alpha \leq \min\left\{\gamma w/(1-w), e^{\theta-r}\right\}$, we have $\tilde{D} - \tilde{F}' \leq 0$. Consequently,

$$\tilde{Z}_1/Z_0 \leq \left[\big(1-\gamma+\alpha(1-w)/w\big)\tilde{R} + \big(\gamma - \alpha(1-w)/w\big)e^r\right].$$

The right-hand side of the above inequality cannot be strictly preferred to $a\tilde{R} + (1-a)e^r$ for any $a \in [0,1]$ because $\gamma - \alpha(1-w)/w \in [0,1]$. Thus, the entrepreneur is deterred, i.e., $\bar{\alpha}_E(u_E, \tilde{R}, \tau) \geq \alpha$ for all $u_E \in \mathcal{U}$. When $\alpha > \min\left\{\gamma w/(1-w), e^{\theta-r}\right\}$, consider $(\tilde{R}, \tau) \in \mathcal{R}_{0,U}$ such that that $\tilde{R}$ takes values of either 0 or $m_1 + 1$ for some constant $m_1 > e^h - 1$ and $\mathbb{E}[\tilde{R}] = e^r$. Then, we have

$$\begin{aligned}\mathbb{E}\left[-\tilde{C} + \tilde{D} - \tilde{F}'\right] &= \mathbb{E}\left[\big((1-w)/w\big)e^r\left[\alpha - \min\left\{\frac{\gamma w}{1-w}, e^{\theta-r}\right\}\right]\mathbf{1}_{\{\tilde{R}<e^r\}}\right]\\ &\quad - \mathbb{E}\left[\big((1-w)/w\big)(e^h - e^r)\mathbf{1}_{\{\tilde{R}\geq e^r\}}\right]\\ &= \big((1-w)/w\big)\left[\alpha - \min\left\{\frac{\gamma w}{1-w}, e^{\theta-r}\right\}\right]\mathbb{P}(\tilde{R} < e^r)\\ &\quad - \big((1-w)/w\big)(e^h - e^r)\mathbb{P}(\tilde{R} \geq e^r).\end{aligned}$$

As m_1 goes to infinity, we have $\mathbb{P}(\tilde{R} < e^r)$ goes to one and $\mathbb{P}(\tilde{R} \geq e^r)$ goes to zero because $\mathbb{E}[\tilde{R}] = e^r$. Consequently, when m_1 is sufficiently large, because $\alpha > \min\left\{\gamma w/(1-w), e^{\theta-r}\right\}$, $\mathbb{E}\left[-\tilde{C} + \tilde{D} - \tilde{F}'\right] > 0$ and thus $\mathbb{E}[\tilde{Z}_1/Z_0] > e^r$; as a result, this entrepreneur is attracted as long as he is risk-neutral, i.e., $\bar{\alpha}_E(u_E, \tilde{R}, \tau) < \alpha$ for $u_E \in \mathcal{U}_{0,1}$. On the other hand, because

$\mathbb{E}\left[-\tilde{C}+\tilde{D}-\tilde{F}'\right] > 0$, we have $\mathbb{E}[\tilde{Y}_1/Y_0] < e^r$, showing that this entrepreneur is unappealing to any weakly risk-averse funder, i.e., $\bar{\alpha}_F(u_F,\tilde{R},\tau) \le \alpha$ for all $u_F \in \mathcal{U}$. $\square$

Proof of Corollary 6 Recall the entrepreneur's and funder's payoffs (O–27) and (O–30), respectively, in the first-loss scheme. Recall that $\tilde{R} < e^r$ if and only if $\tau \le 1$ and if and only if $\tilde{R} = be^{r(1-\tau\wedge 1)}$. Then, straightforward calculation yields

$$\begin{aligned}\tilde{D}-\tilde{F}' &= \mathbf{1}_{\{\tau\le 1\}}\cdot\Big[\alpha\big((1-w)/w\big)\left(e^r-\tilde{R}\right)\\ &\qquad -\min\left\{\gamma e^r+(1-\gamma)\tilde{R},\big((1-w)/w\big)\left(e^{\theta(\tau\wedge 1)}e^{r(1-\tau\wedge 1)}-\tilde{R}\right)^+\right\}\Big]\\ &= \mathbf{1}_{\{\tau\le 1\}}\big((1-w)/w\big)\left(e^r-\tilde{R}\right)\cdot\Bigg[\alpha\\ &\qquad -\min\left\{\big(w/(1-w)\big)\left(\gamma+\frac{\tilde{R}}{e^r-\tilde{R}}\right),\frac{\tilde{R}(e^{\theta(\tau\wedge 1)}/b-1)^+}{e^r-\tilde{R}}\right\}\Bigg].\end{aligned}$$

Note that on $\{\tau\le 1\}$, we have $\tilde{R} = be^{r(1-\tau\wedge 1)} \in [b, be^r]$ and thus

$$\frac{b}{e^r-b} \le \frac{\tilde{R}}{e^r-\tilde{R}} \le \frac{b}{1-b},\quad \frac{e^\theta-b}{e^r-b} \le \frac{\tilde{R}(e^{\theta(\tau\wedge 1)}/b-1)}{e^r-\tilde{R}} = \frac{e^{(\theta-r)(\tau\wedge 1)}-be^{-r(\tau\wedge 1)}}{1-be^{-r(\tau\wedge 1)}} \le 1.$$

Consequently, we have

$$\frac{1-w}{w}\left(e^r-\tilde{R}\right)(\alpha-U')\mathbf{1}_{\{\tau\le 1\}} \le \tilde{D}-\tilde{F}' \le \frac{1-w}{w}\left(e^r-\tilde{R}\right)(\alpha-\hat{U})\mathbf{1}_{\{\tau\le 1\}}, \tag{O–31}$$

where $\hat{U}$ is given by (B.1) and

$$U' := \min\left\{\frac{\gamma w}{1-w}+\frac{wb}{(1-w)(1-b)},1\right\}.$$

Now suppose $\alpha \in (L,\hat{U})$. Because $\alpha < \hat{U}$, we conclude from (O–31) that $\tilde{D}-\tilde{F}' \le 0$. Consequently, we conclude from (O–27) that the entrepreneur's payoff satisfies

$$\tilde{Z}_1/Z_0 \le \big(1-\gamma+\alpha(1-w)/w\big)\tilde{R}+\big(\gamma-\alpha(1-w)/w\big)e^r.$$

Suppose that the entrepreneur is attracted, i.e., $\bar{\alpha}_E(u_E,\tilde{R},\tau) < \alpha$, then the expectation of the right-hand side of the above equation must be strictly larger than e^r, which, together with $\alpha > L$, implies that $\mathbb{E}[\tilde{R}] > e^r$. Then, recalling the funder's payoff (O–30) and $\tilde{D}-\tilde{F}' \le 0$, we have

$$\tilde{Y}_1/Y_0 \ge \alpha e^r + (1-\alpha)\tilde{R},$$

so $\mathbb{E}[\tilde{Y}_1/Y_0] > e^r$, showing that the entrepreneur is attractive to all risk-neutral funders, i.e., $\bar{\alpha}_F(u_F, \tilde{R}, \tau) > \alpha$ for $u_F \in \mathcal{U}_{0,1}$.

Finally, Assertion (ii) follows from the same argument as the one used in the proof of Theorem 3. $\square$

Proof of Corollary 7 Because $\hat{\mathbb{U}}(\delta, \lambda, \mathcal{O}) \leq \hat{U}$, assertion (ii) follows from Corollary 6. On the other hand, by (O–27), (O–30), and (O–31), and noting $\tilde{C} \geq 0$, we conclude

$$\begin{aligned}
\tilde{Z}_1/Z_0 \leq & \Big[\big(1-\gamma+\alpha(1-w)/w\big)\tilde{R} + \big(\gamma - \alpha(1-w)/w\big)e^r\Big] \\
& + \frac{1-w}{w}\left(e^r - \tilde{R}\right)(\alpha - \hat{U})\mathbf{1}_{\{\tau \leq 1\}}, \\
\tilde{Y}_1/Y_0 \geq & \Big[\alpha e^r + (1-\alpha)\tilde{R}\Big] - \left(e^r - \tilde{R}\right)(\alpha - \hat{U})\mathbf{1}_{\{\tau \leq 1\}}.
\end{aligned}$$

Then, the same proof as the one of Theorem 4 yields assertion (i). $\square$

Proof of Corollary 8 Because $\theta = r$, recalling (O–29), we have

$$\tilde{F}' = \min\left\{\gamma e^r + (1-\gamma)\tilde{R}, \big((1-w)/w\big)\left(e^r - \tilde{R}\right)^+\right\} = \big((1-w)/w\big)\left(e^r - \tilde{R}\right)^+,$$

where the second equality is the case because $\tilde{R} = e^{r(1-\tau\wedge 1)}b \geq b$ on $\tilde{R} < e^r$ and $b \geq (1-w-\gamma w)e^r/(1-\gamma w)$. Consequently,

$$\tilde{F}' - \tilde{D} = \big((1-w)/w\big)\left(e^r - \tilde{R}\right)^+ - \alpha\big((1-w)/w\big)\left(e^r - \tilde{R}\right)^+ = (1-\alpha)\big((1-w)/w\big)\left(e^r - \tilde{R}\right)^+.$$

Recalling the funder's payoffs in (O–30), and noting $\tilde{C} \geq 0$, we then conclude that

$$\begin{aligned}
\tilde{Y}_1 &\geq Y_0\left\{\Big[\alpha e^r + (1-\alpha)\tilde{R}\Big] + \big(w/(1-w)\big)\Big[-\tilde{D} + \tilde{F}'\Big]\right\} \\
&= Y_0\left\{\Big[\alpha e^r + (1-\alpha)\tilde{R}\Big] + (1-\alpha)\left(e^r - \tilde{R}\right)^+\right\} \\
&= Y_0\left\{e^r + (1-\alpha)(\tilde{R} - e^r)^+\right\} \\
&\geq Y_0 e^r,
\end{aligned}$$

where the last inequality becomes strict when $\mathbb{P}(\tilde{R} > e^r) > 0$. The proof then completes. $\square$

Proof of Corollary 9 Consider the case in which there is no liquidation boundary. When $\alpha \leq c$, it is straightforward to see from (C.1) that $\tilde{Z}_1 \leq cX_{I,0}\tilde{R}$, so all entrepreneurs are deterred, i.e.,

$\bar{\alpha}_E(u_E, \tilde{R}, \tau) \geq \alpha$ for all $u_E \in \mathcal{U}$ and $(\tilde{R}, \tau) \in \mathcal{R}_0$. When $\alpha > c$, consider an entrepreneur with $u_E \in \mathcal{U}_{0,1}$ and $(\tilde{R}, \tau) \in \mathcal{R}_{0,U}$ such that $\mathbb{E}[\tilde{R}] = e^r$ and $\mathbb{P}(\tilde{R} = 0) > c/\alpha$. Then, we have

$$\mathbb{E}[(e^r - \tilde{R})^+] \geq e^r \mathbb{P}(\tilde{R} = 0) > (c/\alpha)e^r.$$

As a result,

$$\mathbb{E}[\tilde{Z}_1] = X_{I,0}\mathbb{E}\left[\alpha(\tilde{R} - e^r) + \alpha(e^r - \tilde{R})^+\right] = X_{I,0}\alpha\mathbb{E}\left[(e^r - \tilde{R})^+\right] > cX_{I,0}e^r,$$

showing that the entrepreneur is attracted, i.e., $\bar{\alpha}_E(u_E, \tilde{R}, \tau) < \alpha$. On the other hand,

$$\mathbb{E}[\tilde{Y}_1] = X_{I,0}\mathbb{E}\left[\tilde{R} - \alpha(\tilde{R} - e^r) - \alpha(e^r - \tilde{R})^+ + ce^r\right] = X_{I,0}\left[e^r - \alpha\mathbb{E}[(e^r - \tilde{R})^+] + ce^r\right] < e^r X_{I,0},$$

showing that the entrepreneur is unappealing to all funders, i.e., $\bar{\alpha}_F(u_F, \tilde{R}, \tau) \leq \alpha$ for all $u_F \in \mathcal{U}$.

□

Proof of Corollary 10 Consider the case in which there is a liquidation boundary $b \in (0, 1)$. As in the case of no liquidation boundary, we can show that all entrepreneurs are deterred when $\alpha \leq c$, so we focus in the following on the case in which $\alpha > c$.

If $c \geq 1 - e^{-r}b$, we conclude from (C.2) that

$$\begin{aligned}
\tilde{Y}_1 &= X_{I,0}\left[\tilde{R} - \alpha\left(\tilde{R} - e^r\right)^+ + ce^r\right] \geq X_{I,0}\left[\tilde{R} - \alpha\left(\tilde{R} - e^r\right)^+ + (e^r - \tilde{R})^+\right] \\
&= X_{I,0}\left[e^r + (1 - \alpha)\left(\tilde{R} - e^r\right)^+\right] \geq X_{I,0}e^r,
\end{aligned}$$

and the second inequality becomes strict as long as $\mathbb{P}(\tilde{R} > e^r) > 0$. From (C.1), we conclude that if the gross return of the project $\tilde{R} \leq e^r$ with probability one then the entrepreneur's payoff $\tilde{Z}_1 = 0$ and thus she is deterred. Thus, the limited-liability hurdled contract deters all entrepreneurs who are unappealing to the funder and attracts some skilled entrepreneurs.

Next, suppose $c < 1 - e^{-r}b$ and $\alpha \in \left(c, \frac{c}{1 - e^{-r}b}\right)$. Recalling that

$$\tilde{Z}_1 = X_{I,0}\left[\alpha(\tilde{R} - e^r)^+\right] = X_{I,0}\left[\alpha(\tilde{R} - e^r) + \alpha(e^r - \tilde{R})^+\right].$$

Suppose an entrepreneur with $u_E \in \mathcal{U}$ and $(\tilde{R}, \tau) \in \mathcal{R}_b$ is attracted, i.e., $\bar{\alpha}_E(u_E, \tilde{R}, \tau) < \alpha$. Then, we must have $\mathbb{E}[\tilde{Z}_1] > cX_{I,0}e^r$, which leads to

$$\mathbb{E}[\tilde{R}] > \frac{\alpha + c}{\alpha}e^r - \mathbb{E}[(e^r - \tilde{R})^+]. \tag{O–32}$$

Consequently,

$$
\begin{aligned}
\mathbb{E}[\tilde{Y}_1] &= X_{I,0}\mathbb{E}\left[\tilde{R} - \alpha(\tilde{R} - e^r) - \alpha(e^r - \tilde{R})^+ + ce^r\right] \\
&> X_{I,0}\left\{(1-\alpha)\left(\frac{\alpha + c}{\alpha}e^r - \mathbb{E}[(e^r - \tilde{R})^+]\right) - \alpha\mathbb{E}[(e^r - \tilde{R})^+] + (\alpha + c)e^r\right\} \\
&= X_{I,0}\left\{e^r + (c/\alpha)e^r - \mathbb{E}[(e^r - \tilde{R})^+]\right\} \\
&\geq X_{I,0}\left\{e^r + (c/\alpha)e^r - (e^r - b)\right\} \\
&> X_{I,0}e^r,
\end{aligned}
$$

where the second inequality is the case because $\tilde{R} \geq b$ and the third inequality is the case because $\alpha < ce^r/(e^r - b)$. Thus, this entrepreneur is attractive to all risk-neutral funders, i.e., $\bar{\alpha}_F(u_F, \tilde{R}, \tau) > \alpha$ for $u_F \in \mathcal{U}_{0,1}$. Moreover, fixing any $u_E \in \mathcal{U}$, the same argument as the one in the proof of Theorem 3 yields that there exists $(\tilde{R}, \tau) \in \mathcal{R}_{b,S}$ such that $\bar{\alpha}_E(u_E, \tilde{R}, \tau) < \alpha$.

Finally, suppose $c < 1 - e^{-r}b$ and $\alpha > \frac{c}{1-e^{-r}b}$. Consider an entrepreneur with $u_E \in \mathcal{U}_{0,1}$ and $(\tilde{R}, \tau) \in \mathcal{R}_{b,U}$ such that $\mathbb{E}[\tilde{R}] = e^r$ and $\mathbb{P}(\tilde{R} = b) = \mathbb{P}(\tau \leq 1) = \mathbb{P}(\tau = 1) > c/\big(\alpha(1 - e^{-r}b)\big)$. Then, we have

$$
\mathbb{E}[(e^r - \tilde{R})^+] = (e^r - b)\mathbb{P}(\tilde{R} = b) > (c/\alpha)e^r.
$$

As a result,

$$
\mathbb{E}[\tilde{Z}_1] = X_{I,0}\mathbb{E}\left[\alpha(\tilde{R} - e^r) + \alpha(e^r - \tilde{R})^+\right] = X_{I,0}\alpha\mathbb{E}\left[(e^r - \tilde{R})^+\right] > cX_{I,0}e^r,
$$

showing that the entrepreneur is attracted, i.e., $\bar{\alpha}_E(u_E, \tilde{R}, \tau) < \alpha$. On the other hand,

$$
\mathbb{E}[\tilde{Y}_1] = X_{I,0}\mathbb{E}\left[\tilde{R} - \alpha(\tilde{R} - e^r) - \alpha(e^r - \tilde{R})^+ + ce^r\right] = X_{I,0}\left[e^r - \alpha\mathbb{E}[(e^r - \tilde{R})^+] + ce^r\right] < e^r X_{I,0},
$$

showing that the entrepreneur is unappealing to all funders, i.e., $\bar{\alpha}_F(u_F, \tilde{R}, \tau) \leq \alpha$ for all $u_F \in \mathcal{U}$. $\square$

Proof of Corollary 11 Because $\tilde{\mathbb{U}}(\delta, \lambda, \mathcal{O}) \leq c/(1 - e^{-r}b)$, assertion (ii) follows from Corollary 10, so in the following we prove assertion (i).

Consider an entrepreneur with utility function u_E and her project yielding gross return $\tilde{R}$ that takes value $m_1 + 1 \geq e^r$ when $\tau > 1$ and $be^{r(1-\tau)}$ when $\tau \leq 1$, and suppose that she is attracted, i.e.,

$\bar{\alpha}_E(u_E, \tilde{R}, \tau) < \alpha$. Then, we have $\alpha > c$ because otherwise all entrepreneurs are deterred. From the necessary condition (O–32) for an entrepreneur to be attracted, we conclude that

$$m_1 + 1 - e^r > \frac{(c/\alpha)e^r}{\mathbb{P}(\tau > 1)} = (c/\alpha)e^r\left(1 + O_\tau\right), \tag{O–33}$$

where $O_\tau := \mathbb{P}(\tau \le 1)/\mathbb{P}(\tau > 1)$. Consequently, for a funder with utility function $u_F \in \mathcal{U}_{\delta,\lambda}$,

$$\begin{aligned}\mathbb{E}\left[u_F\left(\tilde{Y}_1/X_{I,0}\right)\right] &\ge u_F\Big((1-\alpha)(m_1 + 1 - e^r) + (1+c)e^r\Big)\mathbb{P}(\tau > 1)\\ &\quad + u_F\Big(b + ce^r\Big)\mathbb{P}(\tau \le 1)\\ &\ge u_F\Big(e^r + (c/\alpha)e^r + \big(c(1-\alpha)/\alpha\big)e^r O_\tau\Big)\mathbb{P}(\tau > 1)\\ &\quad + u_F\Big(e^r - ((1-c)e^r - b)\Big)\mathbb{P}(\tau \le 1),\end{aligned}$$

where the first inequality is because $\tilde{R} = be^{r(1-\tau)} \ge b$ when $\tau \le 1$ and the second inequality is due to (O–33). Then, $\bar{\alpha}_F(u_F, \tilde{R}, \tau) > \alpha$ if the right-hand side of the above inequality is strictly larger than $u_F(e^r)$, and this is the case if and only if

$$\begin{aligned}&u_F\Big(e^r\big(1 + (c/\alpha) + \big(c(1-\alpha)/\alpha\big)O_\tau\big)\Big) - u_F(e^r)\\ &\quad + \Big(u_F\Big(e^r\big(1 - (1 - c - e^{-r}b)\big)\Big) - u_F(e^r)\Big)O_\tau > 0.\end{aligned}$$

Noting $1 - c - e^{-r}b > 0$, the same argument as the one in the proof of Theorem 4 yields that the entrepreneur is attractive to the funder if $\tilde{f}(\alpha; \delta, \lambda, O_\tau) > 0$, where

$$\tilde{f}(\alpha; \delta, \lambda, O_\tau) := u_\delta\Big(1 + (c/\alpha) + \big(c(1-\alpha)/\alpha\big)O_\tau\Big) + \lambda u_\delta\Big(1 - \big(1 - c - e^{-r}b\big)\Big)O_\tau.$$

Note that $\tilde{f}$ is strictly decreasing in α, so $\tilde{f} > 0$ if and only if

$$\alpha < \frac{c(1 + O_\tau)}{u_\delta^{-1}\Big(-\lambda u_\delta\big(1 - (1 - c - e^{-r}b)\big)O_\tau\Big) - 1 + cO_\tau}. \tag{O–34}$$

Noting $-\lambda u_\delta\big(1 - (1 - c - e^{-r}b)\big) > 0$, one can verify that the right-hand side of the above inequality is strictly decreasing in $O_\tau > 0$, so the proof completes. $\square$

Proof of Corollary 12 Recalling the option-to-default (3) and the first-loss coverage (6) in the first-loss scheme, we have

$$
\begin{aligned}
\mathbb{E}[\tilde{D}-\tilde{F}] &= \mathbb{E}\Bigg[\mathbf{1}_{\{\tau\le 1\}}\Big(\big(\alpha(1-w)/w\big)\left(e^r-(\tilde{m}_\tau+1)e^{r(1-\tau)}\right) \\
&\qquad - \min\left\{\gamma e^r+(1-\gamma)(\tilde{m}_\tau+1)e^{r(1-\tau)}, \big((1-w)/w\big)\left(e^{r(1-\tau)}-(\tilde{m}_\tau+1)e^{r(1-\tau)}\right)\right\}\Big)\Bigg] \\
&\le \mathbb{E}\Bigg[\mathbf{1}_{\{\tau\le 1\}}\Big(\big(\alpha(1-w)/w\big)\left(e^r-(\tilde{m}_\tau+1)\right) \\
&\qquad - \min\left\{\gamma e^r+(1-\gamma)(\tilde{m}_\tau+1), \big((1-w)/w\big)\left(1-(\tilde{m}_\tau+1)\right)\right\}\Big)\Bigg] \\
&= \mathbb{P}(\tau\le 1)\big((1-w)/w\big)(e^r-b+\mathbb{E}[\tilde{\xi}])\left(\alpha-U_{\tilde{\xi}}\right),
\end{aligned}
$$

where $U_{\tilde{\xi}}$ is defined in (D.1), and the last equality is the case because conditional on $\tau\le 1$, $b-(\tilde{m}_\tau+1)$ is identically distributed as $\tilde{\xi}\in[0,b]$. Consequently, we have $\mathbb{E}[\tilde{D}-\tilde{F}]\le 0$ because $\alpha<U_{\tilde{\xi}}$. Now, suppose an entrepreneur with utility function u_E is s attracted, i.e., $\bar{\alpha}_E(u_E,\tilde{R},\tau)<\alpha$. Then, we must have $\mathbb{E}[\tilde{Z}_1/Z_0]>e^r$ because the entrepreneur is risk-averse, and this condition, together with the entrepreneur's payoff (5) and $\mathbb{E}[\tilde{D}-\tilde{F}]\le 0$, implies that $\mathbb{E}[\tilde{R}-e^r]>0$. Consequently, we conclude from the funder's payoff (7) that $\mathbb{E}[\tilde{Y}_1/Y_0]>e^r$ and thus the entrepreneur is attractive to any risk-neutral funders, i.e., $\bar{\alpha}_F(u_F,\tilde{R},\tau)>\alpha$ for $u_F\in\mathcal{U}_{0,1}$.

Fixing $u_E\in\mathcal{U}$, following the same argument in the proof of Theorem 3, one can show that there exists $(\tilde{R},\tau)\in\bar{\mathcal{R}}_{b,S}$ such that $\bar{\alpha}_E(u_E,\tilde{R},\tau)<\alpha$. □

Proof of Corollary 13 Because $\mathbb{U}_{\tilde{\xi}}(\delta,\lambda,\mathcal{O})\le U_{\tilde{\xi}}$, assertion (ii) follows from Corollary 12, so in the following we prove assertion (i).

Note that conditional on $\tau\le 1$, $\mathbb{P}(\tilde{m}_\tau+1=b)=1-\mathbb{P}(\tilde{m}_\tau+1=(1-k)b)$. Recalling option-to-default (3) and the first-loss coverage (6), we have

$$
\begin{aligned}
\tilde{D}-\tilde{F} &= \mathbf{1}_{\{\tau\le 1,\tilde{m}_\tau+1=b\}}\Big(\big(\alpha(1-w)/w\big)\left(e^r-\tilde{R}\right) \\
&\qquad - \min\left\{\gamma e^r+(1-\gamma)be^{r(1-\tau)}, \big((1-w)/w\big)\left(e^{r(1-\tau)}-be^{r(1-\tau)}\right)\right\}\Big) \\
&\qquad + \mathbf{1}_{\{\tau\le 1,\tilde{m}_\tau+1=(1-k)b\}}\Big(\big(\alpha(1-w)/w\big)\left(e^r-\tilde{R}\right) \\
&\qquad - \min\left\{\gamma e^r+(1-\gamma)(1-k)be^{r(1-\tau)}, \big((1-w)/w\big)\left(e^{r(1-\tau)}-(1-k)be^{r(1-\tau)}\right)\right\}\Big) \\
&\le \big((1-w)/w\big)\left(e^r-\tilde{R}\right)(\alpha-U)\mathbf{1}_{\{\tau\le 1,\tilde{m}_\tau+1=b\}} \\
&\qquad + \big((1-w)/w\big)\left(e^r-\tilde{R}\right)(\alpha-U_k)\mathbf{1}_{\{\tau\le 1,\tilde{m}_\tau+1=(1-k)b\}},
\end{aligned}
$$

where U_k is defined in (D.2). Recalling the entrepreneur's payoff (5), we have

$$
\begin{aligned}
\tilde{Z}_1 &= Z_0\Big\{\tilde{R} + \big((1-w)/w\big)(\alpha - L)(\tilde{R} - e^r) + \tilde{D} - \tilde{F}\Big\} \\
&\le Z_0\Big\{\tilde{R} + \big((1-w)/w\big)(\alpha - L)(\tilde{R} - e^r)\mathbf{1}_{\{\tau>1\}} + \big((1-w)/w\big)(L-U)(e^r - \tilde{R})\mathbf{1}_{\{\tau\le 1, \tilde{m}_\tau+1=b\}} \\
&\qquad + \big((1-w)/w\big)(L-U_k)(e^r - \tilde{R})\mathbf{1}_{\{\tau\le 1, \tilde{m}_\tau+1=(1-k)b\}}\Big\} \\
&\le Z_0\Big\{\tilde{R} + \big((1-w)/w\big)(\alpha - L)(\tilde{R} - e^r)\mathbf{1}_{\{\tau>1\}} + \big((1-w)/w\big)(L-U\wedge U_k)(e^r - \tilde{R})\mathbf{1}_{\{\tau\le 1\}}\Big\} \\
&=: \tilde{Z}_1'.
\end{aligned}
$$

Note that $\tilde{Z}_1'/Z_0 - \tilde{R}$ is comonotone with respect to $\tilde{R}$, because $\alpha > L$ and $\alpha < \mathbb{U}_{\tilde{\xi}}(\delta, \lambda, \mathcal{O}) \le U \wedge U_k$. On the other hand, because the entrepreneur is attracted, we must have $\mathbb{E}[\tilde{Z}_1'/Z_0 - \tilde{R}] > 0$. As a result,

$$
\mathbb{E}\left[\big((1-w)/w\big)(\alpha - L)(\tilde{R} - e^r)\mathbf{1}_{\{\tau>1\}} + \big((1-w)/w\big)(L-U\wedge U_k)(e^r - \tilde{R})\mathbf{1}_{\{\tau\le 1\}}\right] > 0.
$$

Then the above condition implies

$$
\begin{aligned}
(\alpha - L)(m_1 + 1 - e^r)\mathbb{P}(\tau > 1) &> (U\wedge U_k - L)\mathbb{E}\left[(e^r - \tilde{R})\mathbf{1}_{\{\tau\le 1\}}\right] \\
&\ge (U\wedge U_k - L)e^r(1-b)\mathbb{P}(\tau \le 1, \tilde{m}_\tau + 1 = b) \\
&\quad + (U\wedge U_k - L)e^r(1-(1-k)b)\mathbb{P}(\tau \le 1, \tilde{m}_\tau + 1 = (1-k)b).
\end{aligned}
$$

In consequence, we have

$$
\begin{aligned}
m_1 + 1 - e^r &> \frac{e^r(U\wedge U_k - L)(1-b)}{\alpha - L} \times \frac{\mathbb{P}(\tau \le 1, \tilde{m}_\tau + 1 = b)}{\mathbb{P}(\tau > 1)} \\
&\quad + \frac{e^r(U\wedge U_k - L)(1-(1-k)b)}{\alpha - L} \times \frac{\mathbb{P}(\tau \le 1, \tilde{m}_\tau + 1 = (1-k)b)}{\mathbb{P}(\tau > 1)}.
\end{aligned} \tag{O–35}
$$

On the other hand, recalling the funder's payoff (7), we have

$$
\begin{aligned}
\tilde{Y}_1 &= Y_0\Big\{e^r + (1-\alpha)(\tilde{R} - e^r) - \big(w/(1-w)\big)(\tilde{D} - \tilde{F})\Big\} \\
&\ge Y_0\Big\{e^r + (1-\alpha)(\tilde{R} - e^r) - \Big(e^r - \tilde{R}\Big)(\alpha - U)\mathbf{1}_{\{\tau\le 1, \tilde{m}_\tau+1=b\}} - \Big(e^r - \tilde{R}\Big)(\alpha - U_k)\mathbf{1}_{\{\tau\le 1, \tilde{m}_\tau+1=(1-k)b\}}\Big\} \\
&\ge Y_0\Big\{e^r + (1-\alpha)(m_1 + 1 - e^r)\mathbf{1}_{\{\tau>1\}} - (1-U)(e^r - b)\mathbf{1}_{\{\tau\le 1, \tilde{m}_\tau+1=b\}} \\
&\qquad - (1-U_k)(e^r - (1-k)b)\mathbf{1}_{\{\tau\le 1, \tilde{m}_\tau+1=(1-k)b\}}\Big\} \\
&=: \tilde{Y}_1'.
\end{aligned}
$$

Then $\bar{\alpha}_F(u_F, \tilde{R}, \tau) > \alpha$ if $\mathbb{E}[u_F(\tilde{Y}_1'/Y_0)] > u_F(e^r)$. Straightforward calculation yields

$$\begin{aligned}
\mathbb{E}[u_F(\tilde{Y}_1'/Y_0)] &= u_F\big(e^r + (1-\alpha)(m_1 + 1 - e^r)\big)\mathbb{P}(\tau > 1) \\
&\quad + u_F\big(e^r - (1-U)(e^r - b)\big)\mathbb{P}(\tau \le 1, \tilde{m}_\tau + 1 = b) \\
&\quad + u_F\big(e^r - (1-U_k)(e^r - (1-k)b)\big)\mathbb{P}(\tau \le 1, \tilde{m}_\tau + 1 = (1-k)b) \\
&\ge u_F\bigg(e^r + (1-\alpha)\frac{e^r(U \wedge U_k - L)(1-b)}{\alpha - L} \cdot \frac{\mathbb{P}(\tau \le 1, \tilde{m}_\tau + 1 = b)}{\mathbb{P}(\tau > 1)} \\
&\quad + (1-\alpha)\frac{e^r(U \wedge U_k - L)(1-(1-k)b)}{\alpha - L} \cdot \frac{\mathbb{P}(\tau \le 1, \tilde{m}_\tau + 1 = (1-k)b)}{\mathbb{P}(\tau > 1)}\bigg)\mathbb{P}(\tau > 1) \\
&\quad + u_F\big(e^r - (1-U)(e^r - b)\big)\mathbb{P}(\tau \le 1, \tilde{m}_\tau + 1 = b) \\
&\quad + u_F\big(e^r - (1-U_k)(e^r - (1-k)b)\big)\mathbb{P}(\tau \le 1, \tilde{m}_\tau + 1 = (1-k)b),
\end{aligned}$$

where the inequality follows from (O–35). Recall that conditional on $\tau \le 1$, $b - (\tilde{m}_\tau + 1)$ is identically distributed as $\tilde{\xi}$. Hence, $\mathbb{P}(\tau \le 1, \tilde{m}_\tau + 1 = b) = \mathbb{P}(\tau \le 1)\mathbb{P}(\tilde{\xi} = 0)$ and $\mathbb{P}(\tau \le 1, \tilde{m}_\tau + 1 = (1-k)b) = \mathbb{P}(\tau \le 1)\mathbb{P}(\tilde{\xi} = kb)$. Denote by $O_\tau := \mathbb{P}(\tau \le 1)/\mathbb{P}(\tau > 1)$ the odds of the entrepreneur's project leading to the fund to liquidate. Recall that $p = 1 - \mathbb{P}(\tilde{\xi} = 0) = \mathbb{P}(\tilde{\xi} = kb)$. Then, $\mathbb{E}[u_F(\tilde{Y}_1'/Y_0)] > u_F(e^r)$ if the right-hand side of the above inequality is strictly larger than $u_F(e^r)$, which is the case if and only if

$$\begin{aligned}
& u_F\bigg(e^r\bigg(1 + (1-\alpha)\frac{(U \wedge U_k - L)(1-b)}{\alpha - L}O_\tau(1-p) + (1-\alpha)\frac{(U \wedge U_k - L)(1-(1-k)b)}{\alpha - L}O_\tau p\bigg)\bigg) - u_F(e^r) \\
&\quad + \Big(u_F\big(e^r(1 - (1-U)(1 - e^{-r}b))\big) - u_F(e^r)\Big)O_\tau(1-p) \\
&\quad + \Big(u_F\big(e^r(1 - (1-U_k))(1 - e^{-r}(1-k)b)\big) - u_F(e^r)\Big)O_\tau p > 0.
\end{aligned}$$

Then (O–1) and (O–2) yield that

$$\begin{aligned}
& u_F\left(e^r\left(1 + (1-\alpha)\frac{(U \wedge U_k - L)(1-b)}{\alpha - L}O_\tau(1-p) + (1-\alpha)\frac{(U \wedge U_k - L)(1-(1-k)b)}{\alpha - L}O_\tau p\right)\right) - u_F(e^r) \\
&\quad + \Big(u_F\big(e^r(1 - (1-U)(1 - e^{-r}b))\big) - u_F(e^r)\Big)O_\tau(1-p) \\
&\quad + \Big(u_F\big(e^r(1 - (1-U_k)(1 - e^{-r}(1-k)b))\big) - u_F(e^r)\Big)O_\tau p \\
& \ge u_F'(e^r+)(e^r)^\delta \\
&\quad \bigg\{u_\delta\left(e^r\left(1 + (1-\alpha)\frac{(U \wedge U_k - L)(1-b)}{\alpha - L}O_\tau(1-p) + (1-\alpha)\frac{(U \wedge U_k - L)(1-(1-k)b)}{\alpha - L}O_\tau p\right)\right) - u_\delta(e^r) \\
&\quad + \lambda\Big[u_\delta\big(e^r(1 - (1-U)(1 - e^{-r}b))\big) - u_\delta(e^r)\Big]O_\tau(1-p) \\
&\quad + \lambda\Big[u_\delta\big(e^r(1 - (1-U_k)(1 - e^{-r}(1-k)b))\big) - u_\delta(e^r)\Big]O_\tau p\bigg\}.
\end{aligned}$$

Note that the right-hand side of the above inequality is strictly larger than 0 if and only if $f_{\tilde{\xi}}(\alpha; \delta, \lambda, O_\tau) > 0$, where

$$f_{\tilde{\xi}}(\alpha; \delta, \lambda, O_\tau) := u_\delta \left(1 + (1-\alpha)\frac{(U \wedge U_k - L)\big((1-b)(1-p) + (1-(1-k)b)p\big)}{\alpha - L} O_\tau\right) + \lambda\big(u_\delta\big(1-(1-U)(1-e^{-r}b)\big)(1-p) + u_\delta\big(1-(1-U_k)(1-e^{-r}(1-k)b)\big)p\big)O_\tau.$$

Note that $f_{\tilde{\xi}}(\alpha; \delta, \lambda, \mathcal{O}_\tau) > 0$ if and only if $\alpha < V_{\tilde{\xi}}(\delta, \lambda, O_\tau)$, where $V_{\tilde{\xi}}(\delta, \lambda, O_\tau)$ is defined in (D.2). Straightforward calculation yields that $V_{\tilde{\xi}}(\delta, \lambda, O_\tau)$ is strictly decreasing in $O_\tau \in [0, +\infty)$. Consequently, $\alpha < \mathbb{U}_{\tilde{\xi}}(\delta, \lambda, \mathcal{O})$ implies $\alpha < V_{\tilde{\xi}}(\delta, \lambda, O_\tau)$ for any $O_\tau \le \mathcal{O}$, so the proof completes. □

Proof of Corollary 14 Consider any entrepreneur whose project has the return profile $(\tilde{R}, \tau)$ and conditional on $\tilde{R} < e^r$, $\tilde{R}$ is identically distributed as $\tilde{b}$. Then, we have

$$\begin{aligned}
\mathbb{E}[\tilde{D} - \tilde{F}] &= \mathbb{E}\bigg[\mathbf{1}_{\{\tilde{R}<e^r\}}\Big(\big(\alpha(1-w)/w\big)\left(e^r - \tilde{R}\right) \\
&\qquad - \min\left\{\gamma e^r + (1-\gamma)\tilde{R}, \big((1-w)/w\big)\left(e^{r(1-\tau\wedge 1)} - \tilde{R}\right)^+\right\}\Big)\bigg] \\
&\le \mathbb{E}\bigg[\mathbf{1}_{\{\tilde{R}<e^r\}}\Big(\big(\alpha(1-w)/w\big)\left(e^r - \tilde{R}\right) \\
&\qquad - \min\left\{\gamma e^r + (1-\gamma)\tilde{R}, \big((1-w)/w\big)\left(1 - \tilde{R}\right)^+\right\}\Big)\bigg] \\
&= \mathbb{P}(\tilde{R} < e^r)\big((1-w)/w\big)(e^r - \mathbb{E}[\tilde{b}])\left(\alpha - U'_{\tilde{b}}\right).
\end{aligned}$$

Consequently, we have $\mathbb{E}[\tilde{D} - \tilde{F}] \le 0$ because $\alpha < U'_{\tilde{b}}$, so the same argument as in the proof of Corollary 12 shows that if the entrepreneur is attracted (i.e., $\bar{\alpha}_E(u_E, \tilde{R}, \tau) < \alpha$), she must be attractive to risk-neutral funders (i.e., $\bar{\alpha}_F(u_F, \tilde{R}, \tau) > \alpha$ for $u_F \in \mathcal{U}_{0,1}$), and that fixing $u_E \in \mathcal{U}$, there exists $(\tilde{R}, \tau) \in \hat{\mathcal{R}}_{b,S}$ such that $\bar{\alpha}_E(u_E, \tilde{R}, \tau) < \alpha$. Moreover, when $\tilde{b} \ge \big(1 - w - \gamma w e^r\big)/(1-\gamma w)$, one can check that

$$1 - \tilde{b} \le \frac{\gamma w}{1-w}(e^r - \tilde{b}) + \frac{w}{1-w}\tilde{b},$$

showing that $U'_{\tilde{b}} = \mathbb{E}[(1-\tilde{b})^+]/\mathbb{E}[e^r - \tilde{b}]$, so the proof completes. □

Proof of Corollary 15 Consider a risk-neutral, unskilled entrepreneur whose project yields return that satisfies $\mathbb{E}_{n-1}[e^{-r}\tilde{R}_n \mid \tau > n-1] = 1$ and $\mathbb{P}(\tilde{R}_n > e^r \mid \tau > n-1) > 0$ for all n. Under the traditional scheme, the entrepreneur's payoff stream is given by (F.1). Because $\alpha > 0$,

$$\begin{aligned}\mathbb{E}_{n-1}\left[\tilde{Z}_n \mid \tau > n-1\right] &> \mathbb{E}_{n-1}\big[wX_{n-1}\tilde{R}_n|\tau > n-1\big]\\ &= \mathbb{E}_{n-1}\big[wX_{n-1}\big(a_{n-1}\tilde{R}_n + (1-a_{n-1})e^r\big)|\tau > n-1\big]\end{aligned}$$

for any $a_{n-1} \in [0,1]$. Thus, the entrepreneur is attracted by the scheme because she is risk-neutral. On the other hand, the funder's payoff stream is given by (F.2). As a result,

$$\mathbb{E}_{n-1}[\tilde{Y}_n \mid \tau > n-1] < \mathbb{E}_{n-1}\big[(1-w)X_{n-1}\tilde{R}_n|\tau > n-1\big] = (1-w)X_{n-1}e^r,$$

showing that the entrepreneur is unappealing to any weakly risk-averse funder. $\square$

Proof of Corollary 16 Under the first-loss scheme, the entrepreneur's payoff stream is given by (F.3). Suppose $\tau > n-1$. Because there is no liquidation boundary (i.e., $b = 0$), $\tilde{R}_n < e^r$ if and only if $\tilde{R}_n = 0$. As a result,

$$\tilde{D}_n - \tilde{F}_n = \frac{1-w}{w}e^r\left[\alpha - \min\left\{\frac{\gamma w}{1-w}, e^{-r}\right\}\right]\mathbf{1}_{\{\tilde{R}_n < e^r\}}.$$

If $\alpha \le \min\{\gamma w/(1-w), e^{-r}\}$, we have $\tilde{D}_n - \tilde{F}_n \le 0$, so the entrepreneur's payoff at time n satisfies

$$\tilde{Z}_n \le wX_{n-1}\left[\big(1-\gamma+\alpha(1-w)/w\big)\tilde{R}_n + \big(\gamma - \alpha(1-w)/w\big)e^r\right]\mathbf{1}_{\{\tau > n-1\}}.$$

Note that $\alpha \le \min\{\gamma w/(1-w), e^{-r}\}$ implies $\gamma - \alpha(1-w)/w \in [0,1]$, so the right-hand side of the above inequality stands for the payoff of investing $\gamma - \alpha(1-w)/w$ proportion of the entrepreneur's wealth at the beginning of the n-th period in the risk-free asset and the remaining in the her project. Consequently, the entrepreneur is deterred by the first-loss scheme.

If $\alpha > \min\{\gamma w/(1-w), e^{-r}\}$, then $\tilde{D}_n - \tilde{F}_n \ge 0$ and the inequality becomes strict as long as $\tilde{R}_n < e^r$. Consequently, for a risk-neutral, unskilled entrepreneur whose project yields $\mathbb{E}_{n-1}[e^{-r}\tilde{R}_n \mid \tau > n-1] = 1$ and $\mathbb{P}_{n-1}(\tilde{R}_n < e^r \mid \tau > n-1) > 0$ for all n, we have

$$\mathbb{E}_{n-1}\left[\tilde{Z}_n \mid \tau > n-1\right] = wX_{n-1}\left\{e^r + \mathbb{E}_{n-1}\left[\tilde{D}_n - \tilde{F}_n \mid \tau > n-1\right]\right\} > wX_{n-1}e^r,$$

showing that this entrepreneur is attracted by the first-loss scheme because she is risk-neutral. This entrepreneur, however, is unappealing to any weakly risk-averse funder. Indeed, because $\tilde{D}_n - \tilde{F}_n \ge 0$, we conclude from (F.4) that

$$\tilde{Y}_n \le (1-w)X_{n-1}\left[\alpha e^r + (1-\alpha)\tilde{R}_n\right]\mathbf{1}_{\{\tau > n-1\}}.$$

The expectation of the right-hand side, conditional on $\tau > n-1$ and other information at time $n-1$, is equal to $(1-w)X_{n-1}e^r$. Consequently, any weakly risk-averse funder finds this entrepreneur to be unappealing. $\square$

Proof of Corollary 17 Under the first-loss scheme with the liquidation boundary, consider any entrepreneur who is attracted. Then,

$$
\begin{aligned}
(\tilde{Z}_1, \tilde{Z}_2, ..., \tilde{Z}_n, ..., \tilde{Z}_T) \succ_M \Big(wX_0 \left[a_0\tilde{R}_1 + (1-a_0)e^r\right], wX_1 \left[a_1\tilde{R}_2 + (1-a_1)e^r\right] \mathbf{1}_{\{1<\tau\}}, \\
..., wX_{n-1} \left[a_{n-1}\tilde{R}_n + (1-a_{n-1})e^r\right] \mathbf{1}_{\{n-1<\tau\}}, \\
..., wX_{T-1} \left[a_{T-1}\tilde{R}_T + (1-a_{T-1})e^r\right] \mathbf{1}_{\{T-1<\tau\}}\Big)
\end{aligned}
$$

for any $a_{n-1} \in [0,1]$. Now, suppose $\alpha < U$. Then, we conclude $\tilde{D}_n - \tilde{F}_n \le ((1-w)/w)(e^r - \tilde{R}_n)(\alpha - U)\mathbf{1}_{\{\tilde{R}_n < e^r\}} \le 0$. Thus, the entrepreneur's payoff

$$
\tilde{Z}_n \le wX_{n-1} \left[\big(1-\gamma+\alpha(1-w)/w\big)\tilde{R}_n + \big(\gamma - \alpha(1-w)/w\big)e^r\right] \mathbf{1}_{\{\tau>n-1\}}.
$$

Then, noting $1-\gamma+\alpha(1-w)/w > 0$, because the entrepreneur is weakly risk-averse and attracted, she cannot be unskilled, i.e., we must have $\mathbb{E}_{n-1}[\tilde{R}_n|\tau > n-1] > e^r$ for all n.

On the other hand, noting $\tilde{D}_n - \tilde{F}_n \le 0$, we have

$$
\tilde{Y}_n \ge (1-w)X_{n-1} \left[\alpha e^r + (1-\alpha)\tilde{R}_n\right] \mathbf{1}_{\{\tau>n-1\}}.
$$

Because $\mathbb{E}_{n-1}[\tilde{R}_n \mid \tau > n-1] > e^r$ for all n, the risk-neutral funders find the entrepreneur to be attractive because

$$
\begin{aligned}
(\tilde{Y}_1, \tilde{Y}_2\mathbf{1}_{\{1<\tau\}}, ..., \tilde{Y}_n\mathbf{1}_{\{n-1<\tau\}}, ..., \tilde{Y}_T\mathbf{1}_{\{T-1<\tau\}}) \succ_I ((1-w)X_0e^r, (1-w)X_1e^r\mathbf{1}_{\{1<\tau\}}, \\
..., (1-w)X_{n-1}e^r\mathbf{1}_{\{n-1<\tau\}}, \\
..., (1-w)X_{T-1}e^r\mathbf{1}_{\{T-1<\tau\}}).
\end{aligned}
$$

Finally, it is straightforward to see that because $\alpha > L$, all perfectly skilled entrepreneurs are attracted, and some skilled entrepreneurs whose projects yields sufficiently high expected returns are also attracted. $\square$