Economics of FinTech, Ch. 5
HWK Solution

Exercise 1.

(1) Since Class A coin gets no additional coupon payment during the jump, we have the following NAV table

	Before the jump	After the jump
Class A	$1 + Rv_t$	$1 + Rv_t$
Class B	$\mathcal{H}_d$	$\frac{R'v_t - 1}{2} - Rv_t$
ETH Price $\left(\frac{1}{2}\left(V_A^t + V_B^t\right)\beta_t P_0\right)$	$\frac{1}{2}\left(1 + Rv_t + \mathcal{H}_d\right)\beta_t P_0$	$\frac{1}{2}\left(1 + Rv_t + \frac{R'v_t-1}{2} - Rv_t\right)\beta_t P_0$

(2) From the above table, the required return of ETH is at most

$$
\begin{aligned}
& \frac{\frac{1}{2}\left(1 + Rv_t + \frac{R'v_t-1}{2} - Rv_t\right)\beta_t P_0}{\frac{1}{2}\left(1 + Rv_t + \mathcal{H}_d\right)\beta_t P_0} - 1 \\
= \; & \frac{1 + Rv_t + \frac{R'v_t-1}{2} - Rv_t}{1 + Rv_t + \mathcal{H}_d} - 1 \\
= \; & \frac{1}{2}\frac{R'v_t + 1}{1 + Rv_t + \mathcal{H}_d} - 1
\end{aligned}
$$

Exercise 2.

Note that before the downward reset $V_A^t = 1 + Rv_t$. The conditions $V_B^t \leq 0$ and

$$
2\left(V_A^t - \left|V_B^t\right| - \left(V_A^t - 1\right)\frac{\tilde{R}}{R}\right)^+ < 1 + R'v_t
$$

hold if and only if $V_B^t \leq 0$ and

$$
2\left(V_A^t + V_B^t - \tilde{R}v_t\right)^+ < 1 + R'v_t,
$$

i.e.

$$
2\left(1 + V_B^t + (R - \tilde{R})v_t\right)^+ < 1 + R'v_t.
$$

There are two cases:

(a) $V_B^t \leq 0$ and $1 + V_B^t + (R - \tilde{R})v_t > 0$ and

$$
2\left(1 + V_B^t + (R - \tilde{R})v_t\right) < 1 + R'v_t,
$$

i.e.

$$
V_B^t < \frac{1 + R'v_t}{2} - (R - \tilde{R})v_t - 1 = \frac{\left\{R' + 2(\tilde{R} - R)\right\}v_t - 1}{2}.
$$

Note that $R' + 2(\tilde{R} - R) \geq 0$ by the assumption in the footnote 1. By our assumption in the problem, this implies $V_B^t \leq 0$ is automatically satisfied and that

$$V_B^t < \frac{\left\{R' + 2(\tilde{R} - R)\right\} v_t - 1}{2} = (\tilde{R} - R)v_t - 1 + \frac{R'v_t + 1}{2}.$$

(b) $V_B^t \leq 0$ and $1 + V_B^t + (R - \tilde{R})v_t \leq 0$ and

$$0 < 1 + R'v_t.$$

In this case

$$V_B^t \leq (\tilde{R} - R)v_t - 1.$$

In both case, we have at least

$$V_B^t < (\tilde{R} - R)v_t - 1 + \frac{R'v_t + 1}{2}.$$

(1) Since Class A coin gets no additional coupon payment during the jump, we have the following NAV table

	Before the jump	After the jump
Class A	$1 + Rv_t$	$1 + Rv_t$
Class B	$\mathcal{H}_d$	$(\tilde{R} - R)v_t - 1 + \frac{R'v_t+1}{2}$
ETH Price $\left(\frac{1}{2}\left(V_A^t + V_B^t\right)\beta_t P_0\right)$	$\frac{1}{2}\left(1 + Rv_t + \mathcal{H}_d\right)\beta_t P_0$	$\frac{1}{2}\left(\tilde{R}v_t + \frac{R'v_t+1}{2}\right)\beta_t P_0$

(2) From the above table, the required return of ETH is at most

$$\begin{aligned}
& \frac{\frac{1}{2}\left(\tilde{R}v_t + \frac{R'v_t+1}{2}\right)\beta_t P_0}{\frac{1}{2}\left(1 + Rv_t + \mathcal{H}_d\right)\beta_t P_0} - 1 \\
= \ & \frac{\tilde{R}v_t + \frac{R'v_t+1}{2}}{1 + Rv_t + \mathcal{H}_d} - 1 \\
= \ & \frac{1}{2}\frac{2\tilde{R}v_t + R'v_t + 1}{1 + Rv_t + \mathcal{H}_d} - 1.
\end{aligned}$$