Credit Derivatives: CDS and Swaptions

Goals:

- Discuss some instruments that arise naturally in fixed income markets.
- Not well-known, but very useful instruments to some people; in fact, they comprise markets much bigger than correspodning equity markets.
- Some examples: CDS, CDX, CDX swaptions, CDX tranches

Relevant literature:

- Dominic O'Kane, "Modelling Credit Derivatives": Chapters 3,5,9,10.

Disclaimer

- We will discuss just enough of the instruments to illustrate the ideas.
- There are a lot of details that we will skip over.
- Please don't trade these at home.

Credit Default Swap

CDS are essentially insurance against an issuer defaulting. They have two legs:

Premium leg a cashflow of regular (usually quarterly) “insurance” premium payments, which continue until maturity or a credit event, such as default.

Protection leg also called contingent leg. This leg pays at most once, when there is a credit event.

The contract is over once the contingent payment is made or maturity is reached.

Credit Default Swap: premium leg

At inception of the CDS contract present value of the premium leg is

$$\sum_{n=1}^{N} S\Delta(t_{n-1}, t_n) DF(t_n) P(t_n) \tag{1}$$

where S is the contractual spread (annualized premium), $\Delta(t_{n-1}, t_n)$ is the fraction of the year in the payment period (e.g. $1/4$ if quarterly), $DF(t_n)$ is the discount factor from time t_n to the present, and $P(t_n)$ is the probability that the issuer survives until t_n.

This is correct if the default happens on a payment day. Since it could happen at any time during the period, there will be a partial payment to cover the insurance between the last full payment and the credit event. We approximate this partial payment as if it happened in the middle of the period. (Remember the midpoint rule?)

Credit Default Swap: premium leg

So to (1) we must add the present value of that partial payment:

$$\frac{1}{2}\sum_{n=1}^{N} S\Delta(t_{n-1}, t_n)DF(t_n)(P(t_{n-1}) - P(t_n)) \tag{2}$$

Note that $P(t_{n-1}) - P(t_n)$ is the probability that default will happen after t_{n-1} but before t_n.

We will ignore daycount convention, and other niceties.

Credit Default Swap: premium leg

Therefore, the premium leg is the sum of (1) and (2):

$$\frac{S}{2}\sum_{n=1}^{N}\Delta(t_{n-1},t_n)DF(t_n)(P(t_{n-1})+P(t_n)) = S\times RPV01, \quad (3)$$

where $RPV01$ is defined by (3) and is called *risky annuity* or *risky present value of 1 bp*.

Credit Default Swap: contingent leg

Present value of the protection leg is

$$(1 - R) \sum_{n=1}^{N} DF(t_n)(P(t_{n-1}) - P(t_n)) \tag{4}$$

where R is the expected recovery, which is set *a priori*. Actual recovery is decided after the default via auction.

At inception of the contract, the contractual spread S is chosen so that the premium and the contingent legs are equal, i.e., (3) equals (4):

$$S \times RPV01 = (1 - R) \sum_{n=1}^{N} DF(t_n)(P(t_{n-1}) - P(t_n)) \tag{5}$$

Credit Default Swap: Pricing after inception

Once the contract has been entered into and time passes, we need to be able to value it, either to able to get out of it or to keep it on the books, i.e., to mark it to market.

Let us denote by $S(0)$ the contractual spread defined by S in (5). Let t be the time at which we are valuing the CDS, and let $S(t)$ be the contractual spread of the contracts being issued at time t which have the same payment dates as the remaining payment dates of our original contract. (They do not necessarily exist, but let's assume they do, for now.) Then

$$S(t) \times RPV01(t) = (1-R) \sum_{t_n > t} DF(t_n)(P(t_{n-1}) - P(t_n)) \qquad (6)$$

where

$$RPV01(t) = \frac{1}{2} \sum_{t_n > t} \Delta(t_{n-1}, t_n) DF(t_n)(P(t_{n-1}) + P(t_n)) \qquad (7)$$

Credit Default Swap: Mark-to-Market and pricing after inception

Note that $P(t_n)$ in (6) and (7) will have changed because the dates t_n are closer and also because the market conditions which determine $P(t_n)$s will change, too. This is completely analogous to the reason why $DF(t_n)$s will change.

Equation (6) tell us that the payments for the contract should be $S(t)$ per year as of now. But we are paying $S(0)$ per year. Therefore, current value of the contract, i.e., the mark-to-market value, or the amount that we would want to get out of the contract is

$$(S(t) - S(0)) \times RPV01(t). \tag{8}$$

Credit Default Swap: survival curve

- In order to compute S_t, we need to know $DF(t_n)$, which we get from LIBOR or OIS curves.

  But where do we get the survival probabilities $P(t_n)$? These probabilities are implied by other CDS in the market.

  Each issuer usually has a number of CDS with different maturities. They can be bootstrapped into a curve in a way similar to bootstrapping a yield curve.

- The analogue to the instantaneous forward rate in interest rates is the instantaneous hazard rate, sometimes called default intensity. The hazard rate $h(s)$ can then be defined by

$$P(t) = \exp\left(-\int_0^t h(s)\,ds\right) \tag{9}$$

Credit Default Swap: survival curve

- A natural assumption is that hazard rate is piecewise-constant, i.e., stays constant at all payment dates between available CDS maturities.

- A hazard rate can then be defined as:

$$h(t_{m-1}) = \frac{\log P(t_{m-1}) - \log P(t_m)}{t_m - t_{m-1}} \tag{10}$$

  between maturities t_{m-1} and t_m. The hazard rate is the analogue of the instantaneous forward rate in interest rates. It is sometimes called default intensity.

Once the survival curve is bootstrapped, we have our $P(t_n)$s and are able to price (i.e., compute the S in (4) on slide 7).

CDS - Bond basis

- If you buy a bond and a CDS, in theory it is a riskless instrument. So, you can compare the cost of setting this package up to the cost of holding a real riskless instrument and, if there is a difference, you've got arbitrage.
- In reality, things are a little more complicated. Maturities, coupon payments, and financing terms, for example, don't line up.
- The difference between bond yield and CDS spread is known as a basis, is often non-zero and is not necessarily arbitragable.

CDS Indices

- CDS indices are packages of (usually 100 or 125) most liquid CDS. New indices are created twice a year and have maturities of 3,5,7, and 10 years, with 5 and 10 most common.
- The indices are not reconstituted and slowly shrink as their constituent CDS default.
- They are more complicated than single-name CDS, because the contingent leg is subject to many, not one default.
- Two main kinds are

  CDX For North America

  iTraxx For Europe, Asia, Australia
- There are several types: IG (investment grade), HY (high yield), XO(crossover: moved from IG to HY)

CDX pricing: protection leg

First let's figure out the value of the protection leg of a CDS index. (I'll refer to CDS indices as CDX for short: this is not strictly-speaking correct, because there are other indices, but it is convenient.)

Since the right-hand-side of (6) equals the left-hand-side, we can write the value contributed to the protection leg of the index by the mth constituent as

$$S_m(t) \times RPV01_m(t) \tag{11}$$

where $RPV01_m(t)$ is made out of the survival probabilities of the mth CDS in the index. Therefore, the protection leg of the index is:

$$\frac{1}{M} \sum_{m=1}^{M} S_m(t) \times RPV01_m(t) \tag{12}$$

CDX pricing: premium leg

Let C denote the contractual spread of the index. Once a particular CDS in the index defaults it is taken out of the index (after a protection payment is made), its premium payments cease, and the new contractual payment of the index becomes

$$\left(1 - \frac{1}{M}\right) C \tag{13}$$

In other words, we can think of each of the M constituent CDS contributing C/M to the premium for as long as that that CDS stays alive. Yet another way to say this is that the value of the index premium leg is

$$\frac{C}{M} \sum_{m=1}^{M} RPV01_m(t) \tag{14}$$

CDX pricing: value

Therefore, long protection position in a CDX is protection leg minus the premium leg:

$$\frac{1}{M}\sum_{m=1}^{M}\left(S_m(t)-C\right)RPV01_m(t) \tag{15}$$

Normally, the value is not computed this way. Instead, people construct a single survival curve for the whole index. It is not as straightforward as it sounds, since different CDS have different recoveries and, as a result, it would be wrong to just add the individual spreads.

CDX basis

There is also a notion of CDX basis, which is the difference between the index spread and the composite spread of the underlying CDS. It is also usually non-zero, partly due to difference in liquidity and subtle differences in the contracts.

CDS options

CDS options, also called default swaptions give the holder the right to enter into a CDS at a future date t_E at a spread fixed today, K.

There are two kinds of these swaptions

Knockout swaption Cancels if a credit event occurs prior to the option expiry t_E

Non-knockout swaption Does not cancel prior to option expiry.

Clearly, the non-knockout option is worth more than the knockout one.

The difference is called front end protection and it is

$$(1 - R)DF(t_E)(1 - P(t_E)) \tag{16}$$

CDS Options

There are two kinds of swaptions

Payer swaption Gives the right to enter the CDS contract long protection, i.e., pay the premium.

(acts like a call on CDS spread)

Receiver swaption Gives the right to enter the CDS contract short protection, i.e., receive the premium.

(acts like a put on CDS spread)

CDS Options

Let's try to price a knockout payer.

The value at time t_E of the long protection CDS that will start at time t_E with contractual spread K is:

$$(S(t_E) - K)^+ \times RPV01(t_E) \tag{17}$$

where $S(t_E)$ is the market spread of the CDS at t_E . Therefore, the value of the knockout payer at expiry is

$$V_{pay}(t_E) = 1_{\{\tau > t_E\}} \times (S(t_E) - K)^+ \times RPV01(t_E) \tag{18}$$

and the value at present is

$$V_{pay}(0) = \mathbb{E}^{\mathbb{Q}}\left[\frac{1}{B(t_E)} 1_{\{\tau > t_E\}} \times (S(t_E) - K)^+ \times RPV01(t_E)\right] \tag{19}$$

where $B(t)$ is the value of the money-market account.

CDS Swaptions

We use an important trick of changing the numeraire from $B(t)$ to $RPV01(t)$. Of course, we must also change to the corresponding measure (basically absorbing $RPV01(t)$ into the new measure $\mathbb{P}$).

We then get

$$\begin{aligned} V_{pay}(0) &= \mathbb{E}^{\mathbb{Q}}\left[\frac{1}{B(t_E)}1_{\{\tau>t_E\}} \times (S(t_E)-K)^+ \times RPV01(t_E)\right] \\ &= RPV01(0,t_E)\mathbb{E}^{\mathbb{P}}\left[(S(t_E)-K)^+\right] \end{aligned} \tag{20}$$

where $RPV01(0, t_E)$ is $RPV01(t_E)$ viewed from time 0. O'Kane has a very good discussion on numeraires in Ch.9.

CDS Swaptions

Once we have $V_{pay}(0)$ expressed as an expectation in (20), we can make reasonable assumptions about distribution of $S(t_E)$ such as normality or log-normality, which would allow us to price the swaption.

CDX Swaptions

We can view CDS index itself as an underlying: and, yes, there are derivatives on this, too.

Today, we look at CDX options, aka swaptions.

They are the right to enter into a CDS index swap with the spread equal to K, the strike. The exercise price of the option is

$$G(K) = (K - C)RPV01_I(t_E, K) \tag{21}$$

where C is the contractual spread of the underlying index, whereas $S_I(t_E)$ is the market spread of the index at option's maturity t_E. $RPV01_I(t_E, K)$ is the index's risky annuity at time t_E. Note that it depends on the strike K, because the default probabilities depend on the spread.

CDX Swaptions

There are two kinds of swaptions

Payer swaption Gives the right to enter the CDX contract long protection, i.e., pay the premium.

(acts like a call on CDX spread)

Receiver swaption Gives the right to enter the CDX contract short protection, i.e., receive the premium.

(acts like a put on CDX spread)

CDX Swaptions

At exercise time, the holder of the payer swaption will (at his option)

- Receive the CDX index (with defaulted issuers removed) at the market spread of $S_I(t_E)$. Note that the contractual spread of the index is C. The value of this index is

$$(S_I(t_E) - C) \times RPV01_I(t_E, S_I(t_E)) \tag{22}$$

- Pay the exercise price

$$G(K) = (K - C)RPV01_I(t_E, K) \tag{23}$$

- Receive payment from the individual CDS that had defaulted between option initiation and expiry. The value of each of these is $(1 - R)$ times its weight in the index.

For pricing these CDX swaptions we can use methods similar to CDS swaptions, but calculations are more delicate due to multiple defaults.