

Homework3 report

Haoyu Guan¹

¹Questrom School of Business, Boston University

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1 Problem Background

Option Pricing via FFT Techniques The Heston Model is defined by the following system of stochastic differential equations:

$$\begin{aligned}dS_t &= rS_t dt + \sqrt{\nu_t} S_t dW_t^1 \\d\nu_t &= \kappa(\theta - \nu_t)dt + \sigma\sqrt{\nu_t}dW_t^2 \\Cov(dW_t^1, dW_t^2) &= \rho dt\end{aligned}$$

The characteristic function for the Heston Model is known to be:

$$\begin{aligned}\omega(u) &= \frac{\exp(iu \ln S_0 + iu(r - q)t + \frac{\kappa\theta t(\kappa - i\rho\sigma u)}{\sigma^2})}{(\cosh \frac{\lambda t}{2} + \frac{\kappa - i\rho\sigma u}{\lambda} \sinh \frac{\lambda t}{2})^{\frac{2\kappa\theta}{\sigma^2}}} \\ \Phi(u) &= \omega(u) \exp\left(\frac{-(u^2 + iu)\nu_0}{\lambda \coth \frac{\lambda t}{2} + \kappa - i\rho\sigma u}\right) \\ \lambda &= \sqrt{\sigma^2(u^2 + iu) + (\kappa - i\rho\sigma u)^2}\end{aligned}$$

Assume the risk-free rate is 2%, the initial asset price is 250 and that the asset pays no dividends

2 Problem Detail

(a) **Exploring FFT Technique Parameters** Consider a European Call Option with strike 250 expiring in six months. Additionally, assume you know that the parameters of the Heston Model are:

$$\sigma = 0.2$$

$$\mu_0 = 0.08$$

$$\kappa = 0.7$$

$$\rho = -0.4$$

$$\theta = 0.1$$

- i. Calculate the price of the European Call option with many values for the damping factor, . What values of seem to lead to the most stable price?

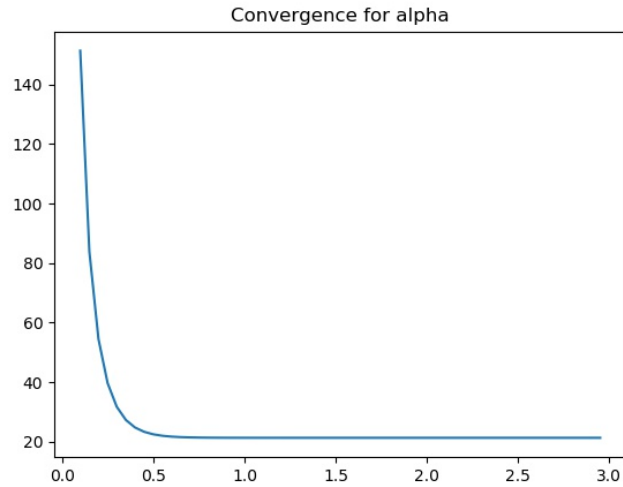


Figure 1: Convergence for alpha

The price of the call option is 21.274373625509146. We can see the picture. When $b = S_0 + S_0 * 7 * \sigma$. If $\alpha > 1.15$, It will be the most stable. So I will choose $\alpha = 1.5$ as my choice

ii. Using the results above, choose a reasonable value of α and calculate the price of the same European Call with various values of N and Δk (or equivalently N and B). Comment on what values seem to lead to the most accurate prices, and the efficiency of each parameterization.

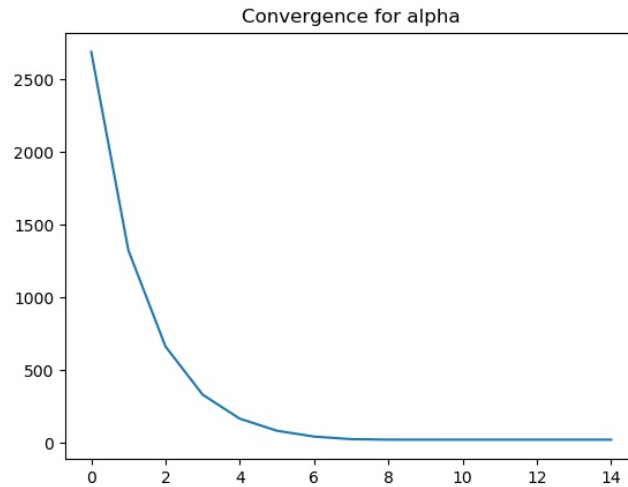


Figure 2: Convergence for alpha

When $b = S_0 + S_0 * 7 * \sigma$. I choose $\alpha = 1.5$. I consider N have to bigger than 2^7 to be accurate

iii. Calculate the price of a European Call with strike 260 using various values of N and Δk (or N and B). Do the same sets of values for N , B and Δk produce the best results? Comment

on any differences that arise

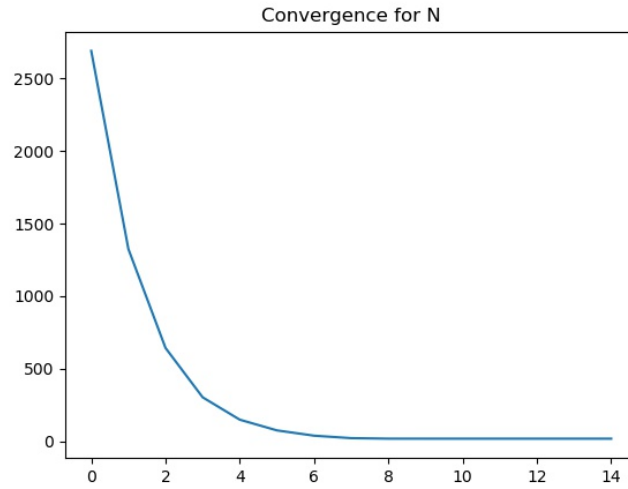


Figure 3: Convergence for N

I think this values of N,B and Δk produce the best results

(b) Exploring Heston Parameters Assume the risk-free rate is 2.5%, the initial asset price is 150 and that the asset pays no dividends.

$$\sigma = 0.4$$

$$\mu_0 = 0.09$$

$$\kappa = 0.5$$

$$\rho = 0.25$$

$$\theta = 0.12$$

i. Using these parameters, calculate Heston Model prices for three-month options at a range of strikes and extract the implied volatilities for each strike. Plot the implied volatility $\sigma(K)$ as a function of strike.

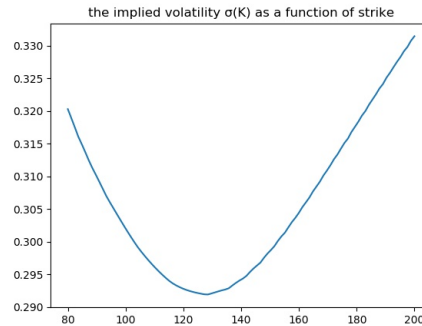


Figure 4: $\sigma(K)$

ii. Use the FFT pricing technique to obtain prices of 150 strike calls at many expiries. Extract the implied volatility for each and plot the term structure of volatility by plotting time to expiry on the x-axis and implied volatility on the y-axis.

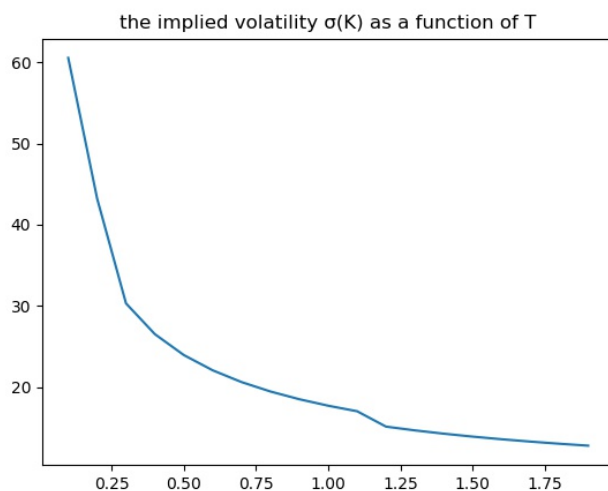


Figure 5: $\sigma(T)$

iii. Holding all other parameters constant, vary each of the model parameters and plot the updated volatility skews and term structures. Comment on the impact that each parameter has on the skew and term structure.

(1) Change the σ

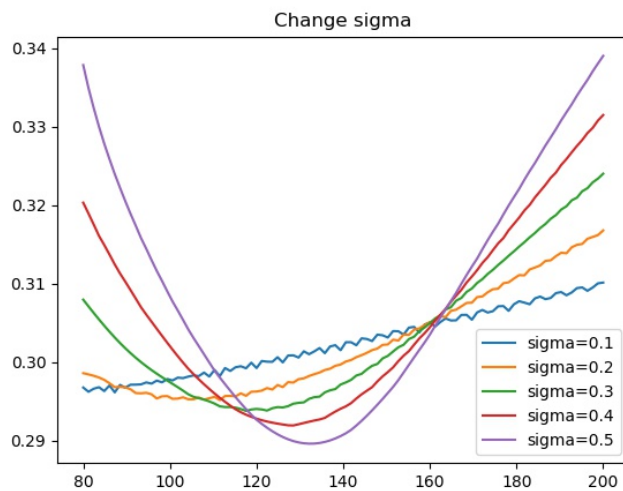


Figure 6: Change σ

(2) Change the ν_0

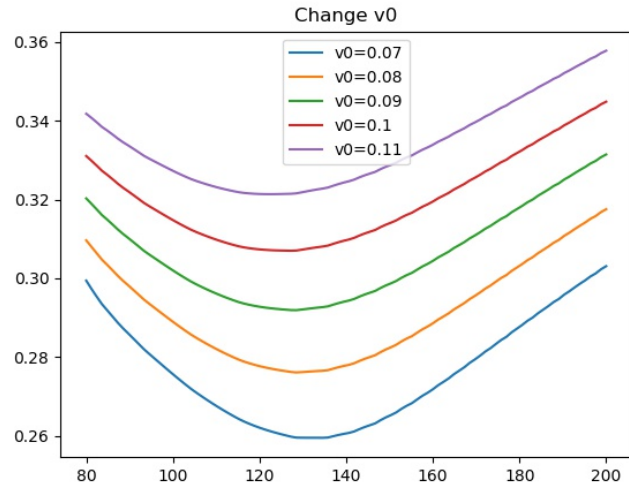


Figure 7: Change ν_0

(3)Change the κ

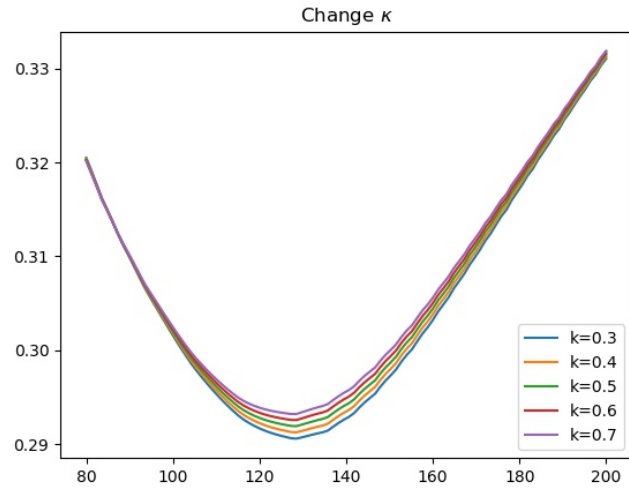


Figure 8: Change κ

(4)Change the ρ

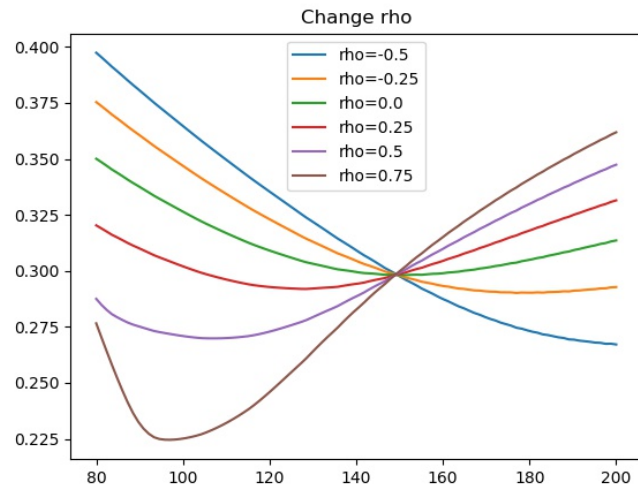


Figure 9: Change ρ

(5) Change the θ

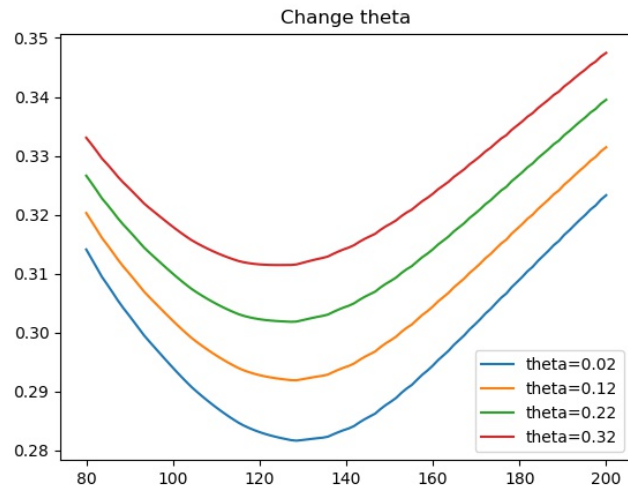


Figure 10: Change θ