Homework4 report

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1 Covariance Matrix Decomposition

Download historical data from your favorite source for 5 years and at least 100 companies or ETFs. In this problem we will look at the covariance matrix for these assets and its properties

1. Clean the data so that your input pricing matrix is as full as possible. Fill in any gaps using a reasonable method of your choice. Explain why you chose that particular method.
I choose 106 stocks from 2015 to 2020 as my data. And we cleaned the data already.

2. Generate a sequence of daily returns for each asset for each date

	Date	AAPL_Close	MSFT_Close	AMZN_Close	FB_Close	XOM_Close	JNJ_Close	JPM_Close	GOOGL_Close	GOOG_Close	GE_Close	WFC_Close
0	2015-02-12	126.459999	43.090000	377.170013	76.230003	92.370003	98.440002	59.570000	546.010010	541.443481	23.932692	54.860001
1	2015-02-13	127.080002	43.869999	381.829987	75.739998	93.370003	99.620003	59.669998	551.159973	547.506836	24.182692	55.330002
2	2015-02-17	127.830002	43.580002	375.429993	75.599998	93.050003	100.440002	60.099998	545.010010	541.353699	24.201923	55.369999
3	2015-02-18	128.720001	43.529999	373.369995	76.709999	91.010002	99.959999	59.369999	542.650024	538.222290	24.278847	54.520000
4	2015-02-19	128.449997	43.500000	379.000000	79.419998	89.440002	100.699997	59.230000	546.450012	541.383606	24.048077	54.560001
5	2015-02-20	129.500000	43.860001	383.660004	79.900002	89.919998	100.260002	59.799999	541.799988	537.474365	24.240385	54.830002
6	2015-02-23	133.000000	44.150002	380.140015	78.839996	89.010002	100.180000	59.349998	535.000000	530.453613	24.201923	55.099998

Figure 1: daily return

3. Calculate the covariance matrix of daily returns and perform an eigenvalue decomposition on the matrix. How many positive eigenvalues are there? How many were negative? If any are negative, what happened?

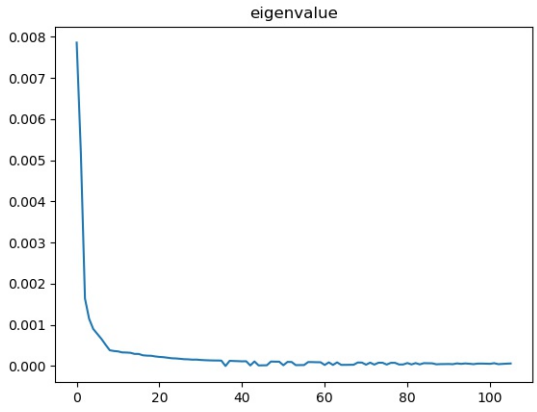

Figure 2: eigenvalue

All of the eigenvalue is positive.

4. How many eigenvalues are required to account for 50% of the variance? How about 90%? Does this make sense to you?

By the calculating, 4 eigenvalues account for 50%. And 57 eigenvalues account for 90%

5. Using the number of eigenvalues in the 90% threshold above, create a return stream that represents the residual return after the principal components that correspond to these eigenvalues have been removed. Plot this return stream and comment on its properties.

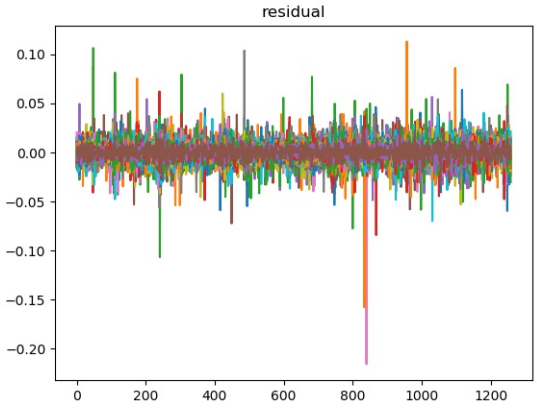

Figure 3: residual

2 Portfolio Construction

In Lecture 7, we defined a Lagrangian for portfolio with constraints in matrix form by

$$L(\omega, \lambda) =< R, \omega > -a < \omega, C\omega > - < \lambda, G\omega - c >$$

1. Form the matrix G by imposing the budget constraint, which is $< 1, \omega >= 1$, and another constraint that allocates 10% of the portfolio to the first 17 securities (to simulate sector allocation). Using C from Problem 1, use your favorite method and the software package of your choice to invert $GC^{-1}G^T$ in a nice, stable way. (Hint: consider my favorite method).

This is G

```
array([[1., 1., 1., 1., 1., 1., 1., 1., 1., 1., 1., 1., 1., 1., 1.,
        1., 1., 1., 1., 1., 1., 1., 1., 1., 1., 1., 1., 1., 1., 1.,
        1., 1., 1., 1., 1., 1., 1., 1., 1., 1., 1., 1., 1., 1., 1.,
        1., 1., 1., 1., 1., 1., 1., 1., 1., 1., 1., 1., 1., 1., 1.,
        1., 1., 1., 1., 1., 1., 1., 1., 1., 1., 1., 1., 1., 1., 1.,
        1., 1., 1., 1., 1., 1., 1., 1., 1., 1., 1., 1., 1., 1., 1.,
        1., 1., 1., 1., 1., 1., 1., 1., 1., 1.],
       [1., 1., 1., 1., 1., 1., 1., 1., 1., 1., 1., 1., 1., 1., 1.,
        1., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0.,
        0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0.,
        0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0.,
        0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0.,
        0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0.,
        0., 0., 0., 0., 0., 0., 0., 0., 0., 0.]])
```

Figure 4: G

2. What does the resulting portfolio look like? Would it be acceptable to most mutual funds? If not, what would you to do to fix that?

This is the weight of 106 stocks of my portfolio.

```
array([-0.96064993,  0.28868458,  1.1874172 , -0.77447179, -2.51970897,
       -0.15598044,  6.43855495,  3.28968316, -3.32086406, -0.83190957,
       -4.46864308,  2.29512692,  0.10533886,  0.45737787,  0.35207608,
       -1.6613883 ,  0.37935653,  0.73053877,  0.49111406,  0.02732587,
        0.17446919,  0.25231184,  0.40641674, -0.62456878,  0.37926214,
       -0.13524507,  2.53766292, -0.607784  , -0.54471574, -1.37284424,
       -0.83818765, -0.57017203,  0.43435699, -2.15934471,  1.27958974,
       -0.74551242, -0.55197561,  0.48090621,  0.03864235,  0.75771569,
        2.0386442 , -1.01010942, -0.3624534 , -0.18620963,  0.55494972,
       -0.79664632,  0.64846429, -1.77035724, -1.81563124, -2.34455411,
       -0.7857506 , -0.53451964, -0.29277194,  0.13520503,  2.13198742,
        1.21903187, -0.81706722, -0.63575208,  0.47460501, -0.49269103,
        0.55090195, -0.92984887,  0.3765037 ,  0.94047655,  1.28963197,
       -0.24314484, -0.46907995,  1.26800512,  0.22083543,  0.58332595,
        1.61585514, -0.4008009 , -1.99109571, -0.48193736,  6.2756795 ,
        1.20872649, -0.48958785,  2.9069463 ,  1.41108758, -0.24833733,
        0.81850296, -1.5729114 , -1.26664468, -0.27392664, -0.87666848,
       -3.13476521, -0.34925625,  1.63737456, -1.82688683,  0.50245527,
        0.84621959, -0.95017381,  2.89441191, -1.27299547,  0.37512392,
       -0.05266634, -2.49174851,  0.94946601, -0.26064011,  0.36249316,
       -0.17712174, -1.57914761,  0.23813215, -1.46012923,  1.63996513,
       -0.41094113])
```

Figure 5: weight

I think this it not acceptable enough. Because we long about half of the stocks and short the other half. This portfolio may not be accepted by mutual funds because they do not usually short stock.