
Problem set # 1

Due: Wednesday, January 29, by 8 am

1. **What areas of finance** are of most interest to you (e.g. big data, trading, portfolio management, risk management, derivative pricing, analyzing complex derivatives)?
2. **What is the primary reason for your interest in this course?**
3. **List all programming languages you have used** and your level of familiarity with each.
4. **Option traders often say that when buying options we get gamma at the expense of theta.** What do you think they mean?
5. **Consider the CEV Model:**

$$dS_t = rS_t dt + \sigma S_t^\beta dW_t$$

Assume a CEV model that is defined by the following parameters:

$$S_0 = 100 \tag{1}$$

$$r = 0.0 \tag{2}$$

$$\beta = 1.0 \tag{3}$$

$$\sigma = 0.25 \tag{4}$$

- (a) Describe what each model parameter does.
- (b) Price an at-the-money one year European call option via Monte Carlo simulation. (HINT: Increments of Brownian Motion are distributed normally with mean 0 and variance dt. One approach is to construct a set of paths by generating random normal variables to use as the increments of the Brownian Motion)
- (c) Calculate the price of the same European call option via the Black-Scholes formula. Is this price the same as you obtained via simulation? Should it be? Why or why not?
- (d) In the Black-Scholes model, we know that the delta of a European call option is:

$$\begin{aligned} \Delta &= \Phi(d_1) \\ d_1 &= \frac{1}{\sigma\sqrt{T}} \left(\ln \frac{S_0}{K} + \left(r + \frac{\sigma^2}{2} \right) T \right), \end{aligned}$$

Calculate the delta of an at-the-money European call option with one year to expiry.

- (e) Using the delta obtained above, calculate how many shares of stock you need to construct a delta neutral portfolio that is long one unit of the call option and short Δ shares of the stock?
- (f) Use simulation to estimate the payoff of the delta neutral portfolio obtained above. How does the payoff compare to the Black-Scholes model price of the option you obtained earlier? What conditions cause this portfolio to make money? Lose money?
- (g) Modify the model dynamics so that $\beta = 0.5$ and all other parameters are the same as in the original question. Using the same hedging portfolio perform another simulation to estimate the payoffs of the delta-neutral portfolio under these dynamics. Are the payoffs higher or lower? Why?
- (h) Modify the model dynamics so that $\sigma = 0.4$ and all other parameters are the same as in the original question. Using the same hedging portfolio perform another simulation to estimate the payoffs of the delta-neutral portfolio under these dynamics. Explain the relationship between a delta-neutral portfolio and the σ parameter.