Homework6 report

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1 Numerical PDEs:

Suppose that the underlying security SPY evolves according to standard geometric brownian motion. Then its derivatives obey the Black-Scholes equation:

$$\frac{\partial c}{\partial t} + \frac{1}{2}\sigma^2 s^2 \frac{\partial^2 c}{\partial s^2} + rs\frac{\partial c}{\partial s} - rc = 0$$

Use SPY's closing price on March $4, 2020$.

We are going to find the price of a call spread with the right of early exercise. The two strikes of the call spread are $K_1 = 315$ and $K_2 = 320$ and the expiry is September $30, 2020$

1. Explain why this instrument is not the same as being long an American call with strike 315 and short an American call with strike 320 , both with expiry September $30, 2020$.

It is a totally different thing. Because short an American option that means you do not have the right of early exercise but your opponent does. So that is different from a call spread with the right of early exercise.

2. For riskless rate r, use the 3-month US Treasury bill at the close of March 4, 2020. Say where you got the rate and why you consider it a reliable source.

$r = 0.72\%$ which is from the government website.

3. Let's assume that we are not able to find σ by calibrating to the European call spread price and must find it by other means. Find a way to pick the σ, explain why you chose this method, and then find the σ.

So we apply linear interpolation to acquire the implied volatility of 315 and 320. And then take the average of this 2 volatility. so the σ is 0.2183167.

4. Set up an explicit Euler discretization of (1). You will need to make decisions about the choice of $s_{\max}, h_s, h_t$, etc. Please explain how you arrived at each of these choices.

As we all know the S_0 is near the 320. So I would choose $S_{max} = 600$ which would be big enough to calculate the price. And $T = \frac{7}{12}$. So I built a $M \times N$ scheme where $M = 300, N = 3000$. By that way, we can get $h_s = \frac{S_{max}}{M} = 1$ and $h_t = \frac{T}{N} = \frac{\frac{7}{12}}{3000}$

5. Let A be the update matrix that you created in the previous step. Find out its eigenvalues and check their absolute values.

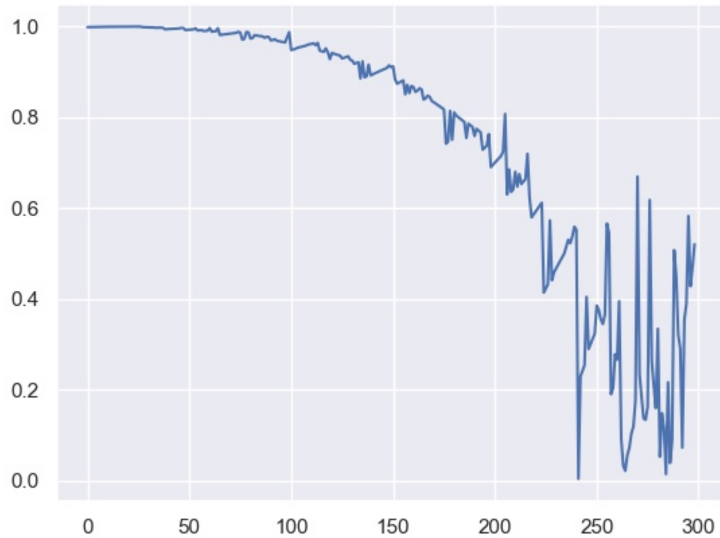

Figure 1: eiegnvalue of A

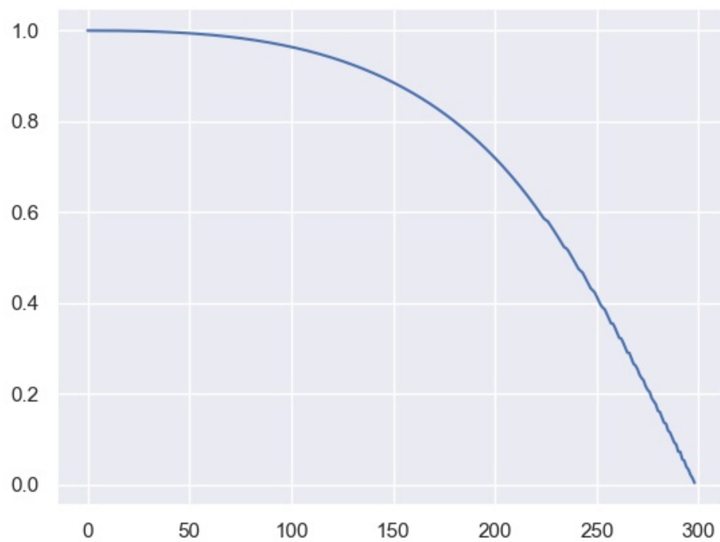

Figure 2: ordered eigenvalue of A

6. Apply your discretization scheme to find today's price of the call spread without the right of early exercise. The scheme will produce a whole vector of prices at time 0. Explain

how you chose the one for today's price.

I get $C_0 = 2.1996583210559235$ by using

$$\begin{bmatrix} a_1 & u_1 & & & \\ l_2 & a_2 & u_2 & & \\ & \ddots & \ddots & \ddots & \\ & & l_{M-2} & a_{M-2} & u_{M-2} \\ & & & l_{M-1} & a_{M-1} \end{bmatrix} \begin{bmatrix} C(s_1, t_j) \\ C(s_2, t_j) \\ \vdots \\ C(s_{M-2}, t_j) \\ C(s_{M-1}, t_j) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ b_j \end{bmatrix}$$

where

$$b_j = u_{M-1}\left(s_M - Ke^{-r(t_N - t_j)}\right)$$

7. Modify your code in the previous step to calculate the price of the call spread with the right of early exercise. What is the price?

I get $C_0 = 4.424891870026654$ and the eigenvalue

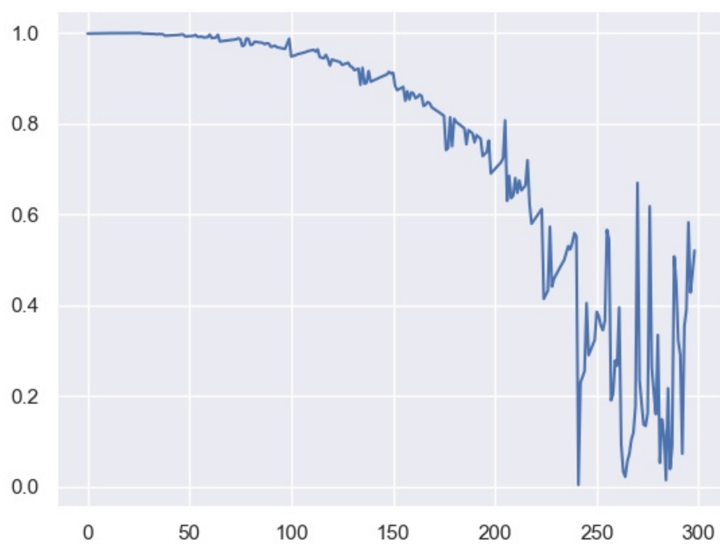

Figure 3: eigenvalue of A

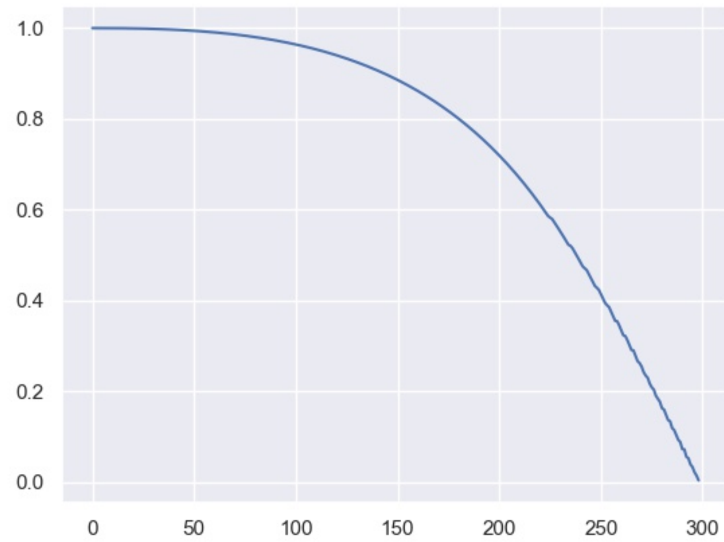

Figure 4: ordered eigenvalue of A

8. Calculate the early exercise premium as the difference between the American and European call spreads. Is it reasonable?

I get $premium = 2.2252335489707304$ which is reasonable because price of early premium is larger than price without it.