Min-Cuts Algorithms

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Motivation

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This here is the introduction of Cut Problems.

Definition

Karger's Algorithm

- Contraction method is used.
- Randomized selection of Edges.
- Running multiple times of the algorithm will provide more accurate result.

Karger's Algorithm

- Basically one run of Karger's Algo takes $O(n^2)$ time.
- It achieves error probability of $\frac{1}{\mathrm{poly}(n)}$ with $O(n^4 \log n)$ time.

Derivation will be given in the later part.

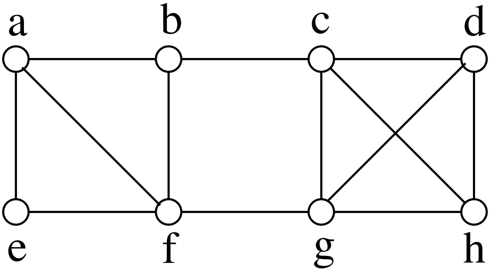
Karger's Algorithm

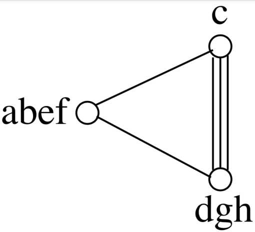
Algorithm

Karger's Algorithm:

repeat
  choose an edge (v, w) uniformly at random from G;
  let $G \leftarrow \frac{G}{(v,w)}$
until G has 2 vertices;

Algorithm





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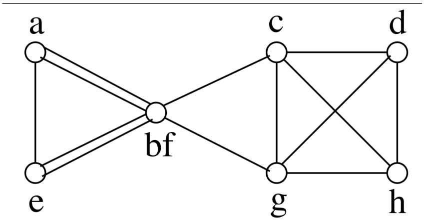

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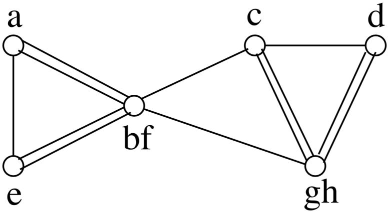

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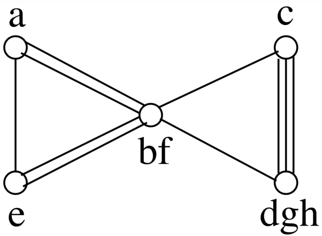

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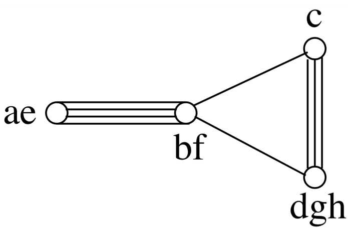
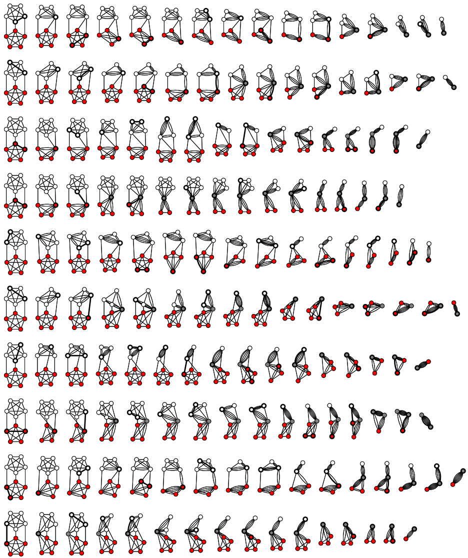


Results



Fact 1 – Sum of Degrees

$$\sum_{u \in V} \mathrm{degree}(u) = 2|E|$$

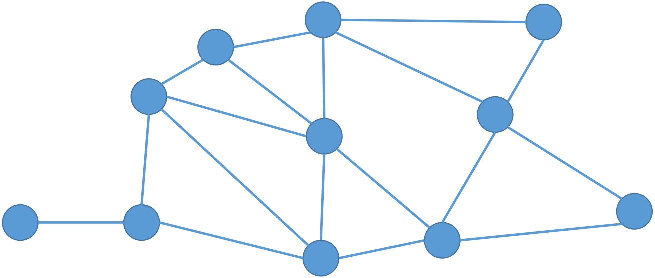
Every edge contributes exactly once to the degree of exactly two nodes.

Fact 2 – Average Degree

$$\begin{aligned} E(\mathrm{degree}(X)) &= \sum_{u \in V} \Pr(X = u) \cdot \mathrm{degree}(u) \\ &= \frac{1}{n} \sum_{u} \mathrm{degree}(u) = \frac{2|E|}{n} \end{aligned} \tag{1}$$

Fact 3 – Min-cut Size

The size of a min-cut is at most $\frac{2|E|}{n}$.



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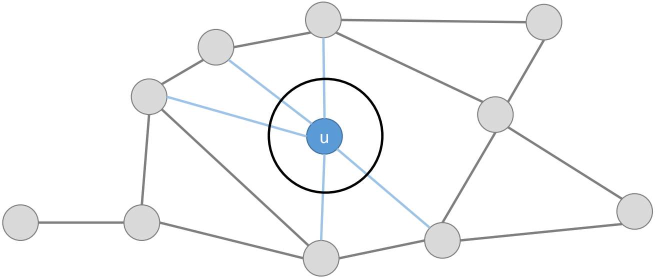

Fact 3 – Min-cut Size

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Proof

- For every node u, we have a cut of size $\mathsf{degree}(u)$.
- Not all nodes can have degree above average, i.e.

$$\exists u \in V : \ \mathsf{degree}(u) \leqslant \frac{2|E|}{n}$$

Fact 4 – Pr(edge across min-cut)

- Fix a certain min-cut in a graph.
- At most $2|E|/n$ of all edges are part of this min-cut.
- Choose a random edge out of all $|E|$ edges.
- $\Pr(\text{edge crosses the cut}) = \frac{2|E|/n}{|E|} = \frac{2}{n}$

References