

— Handbook of —

PI and PID Controller Tuning Rules

2nd Edition



Aidan O'Dwyer

Imperial College Press

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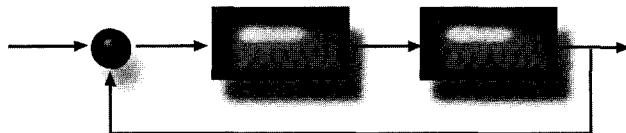
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Dedication

Once again, this book is dedicated with love to Angela, Catherine and Fiona and to my parents, Sean and Lillian.

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Preface

Proportional Integral (PI) and Proportional Integral Derivative (PID) controllers have been at the heart of control engineering practice for seven decades. However, in spite of this, the PID controller has not received much attention from the academic research community until the past fifteen years, when work by K.J. Åström, T. Hägglund and F.G. Shinskey, among others, has sparked a revival of interest in the use of this “workhorse” of controller implementation.

There is strong evidence that PI and PID controllers remain poorly understood and, in particular, poorly tuned in many applications. It is clear that the *many* controller tuning rules proposed in the literature are not having an impact on industrial practice. One reason is that the tuning rules are not very accessible, being scattered throughout the control literature; in addition, the notation used is not unified. The purpose of this book is to bring together and summarise, using a unified notation, tuning rules for PI and PID controllers. The author restricts the work to tuning rules that may be applied to the control of processes with time delays (dead times); in practice, this is not a significant restriction, as most process models have a time delay term.

It is the author’s belief that this book will be useful to control and instrument engineering practitioners and will be a useful reference for students and educators in universities and technical colleges.

I would like to thank the School of Control Systems and Electrical Engineering, Dublin Institute of Technology, for providing the facilities needed to complete the book. Finally, I am deeply grateful to my mother, Lillian and my father, Sean, for their inspiration and support over many

years and to my family Angela, Catherine and Fiona, for their love and understanding.

Aidan O'Dwyer

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Chapter 1

Introduction

1.1 Preliminary Remarks

The ability of proportional integral (PI) and proportional integral derivative (PID) controllers to compensate most practical industrial processes has led to their wide acceptance in industrial applications. Koivo and Tanttu (1991), for example, suggest that there are perhaps 5-10% of control loops that cannot be controlled by single input, single output (SISO) PI or PID controllers; in particular, these controllers perform well for processes with benign dynamics and modest performance requirements (Hwang, 1993; Åström and Hägglund, 1995). It has been stated that 98% of control loops in the pulp and paper industries are controlled by SISO PI controllers (Bialkowski, 1996) and that, in process control applications, more than 95% of the controllers are of PID type (Åström and Hägglund, 1995). The PI or PID controller implementation has been recommended for the control of processes of low to medium order, with small time delays, when parameter setting must be done using tuning rules and when controller synthesis is performed either once or more often (Isermann, 1989).

However, despite decades of development work, surveys indicating the state of the art of control industrial practice report sobering results. For example, Ender (1993) states that, in his testing of thousands of control loops in hundreds of plants, it has been found that more than 30% of installed controllers are operating in manual mode and 65% of loops operating in automatic mode produce less variance in manual than in automatic (i.e. the automatic controllers are poorly tuned). The situation

does not appear to have improved in recent years as Van Overschee and De Moor (2000) report that 80% of PID controllers are badly tuned; 30% of PID controllers operate in manual with another 30% of the controlled loops increasing the short term variability of the process to be controlled (typically due to too strong integral action). The authors state that 25% of all PID controller loops use default factory settings, implying that they have not been tuned at all.

These and other surveys (well summarised by Yu, 1999, pages 1-2) show that the determination of PI and PID controller tuning parameters is a vexing problem in many applications. The most direct way to set up controller parameters is the use of tuning rules; obviously, the wealth of information on this topic available in the literature has been poorly communicated to the industrial community. One reason is that this information is scattered in a variety of media, including journal papers, conference papers, websites and books over a period of seventy years. The author has recorded 408 separate sources of tuning rules since the first such rule was published by Callender *et al.* (1935/6). In a striking statistic, 293 sources of tuning rules have been recorded since 1992, reflecting the upsurge of interest in the use of the PID controller recently.

The purpose of this book is to bring together, in summary form, the tuning rules for PI and PID controllers that have been developed to compensate SISO processes with time delay. Tuning rules for the variations that have been proposed in the ‘ideal’ PI and PID controller structure are included. Considerable variations in the ideal PID controller structure, in particular, are encountered; these variations are explored in more detail in Chapter 2.

1.2 Structure of the Book

Tuning rules are set out in the book in tabular form. This form allows the rules to be represented compactly. The tables have four or five columns, according to whether the controller considered is of PI or PID form, respectively. The first column in all cases details the author of the rule and other pertinent information. The final column in all cases is labelled “Comment”; this facilitates the inclusion of information about the tuning

rule that may be useful in its application. The remaining columns detail the formulae for the controller parameters.

Chapter 2 explores the range of PI and PID controller structures proposed in the literature. It is often forgotten that different manufacturers implement different versions of the PID controller algorithm (in particular); therefore, controller tuning rules that work well in one PID architecture may work poorly on another. This chapter also details the process models used to define the controller tuning rules.

Chapters 3 and 4 of the book detail, in tabular form, tuning rules for setting up, respectively, PI controllers and PID controllers (and their variations), for a wide variety of process models. P controller and I controller tuning rules are also defined (as subsets of PI controller tuning rules), and PD controller tuning rules are defined (as a subset of PID controller tuning rules), for processes whose model includes an integrator. One hundred and eighty-three such tables are provided altogether. To allow the reader to access data readily, the author has arranged that each table start on its own page of the book; each table is preceded by the controller used, together with a block diagram showing the unity feedback closed loop arrangement of the controller and process model.

In Chapter 5 of the book, analytical calculations of the gain and phase margins of a large sample of PI and PID controller tuning rules are determined, when the process is modelled in first order lag plus time delay (FOLPD) form, at a range of ratios of time delay to time constant of the process model. Results are given in graphical form.

An important feature of the book is the unified notation that is used for the tuning rules; a glossary of the symbols used is provided in Appendix 1. Appendix 2 outlines the range of methods that are used to determine process model parameters; this information is presented in summary form, as this topic could provide data for a book in itself. However, sufficient information, together with references, is provided for the interested reader.

Finally, a comprehensive reference list is provided. In particular, the author would like to recommend the contributions by McMillan (1994), Åstrom and Hagglund (1995), Shinskey (1994), (1996), Tan *et al.* (1999a), Yu (1999), Lelic and Gajic (2000) and Ang *et al.* (2005) to the

interested reader, which treat comprehensively the wider perspective of PID controller design and application.

Chapter 2

Controller Architecture

2.1 Introduction

The ideal continuous time domain PID controller for a SISO process is expressed in the Laplace domain as follows:

$$U(s) = G_c(s)E(s) \quad (2.1)$$

with

$$G_c(s) = K_c \left(1 + \frac{1}{T_i s} + T_d s \right) \quad (2.2)$$

and with K_c = proportional gain, T_i = integral time constant and T_d = derivative time constant. If $T_i = \infty$ and $T_d = 0$ (i.e. P control), then it is clear that the closed loop measured value, y , will always be less than the desired value, r (for processes without an integrator term, as a positive error is necessary to keep the measured value constant, and less than the desired value). The introduction of integral action facilitates the achievement of equality between the measured value and the desired value, as a constant error produces an increasing controller output. The introduction of derivative action means that changes in the desired value may be anticipated, and thus an appropriate correction may be added prior to the actual change. Thus, in simplified terms, the PID controller allows contributions from present controller inputs, past controller inputs and future controller inputs.

Many variations of the PI and, in particular, the PID controller structure have been proposed. As Tan *et al.* (1999a) suggest, one

important reason for the non-standard structures is due to the transition of the controllers from pneumatic implementation through electronic implementation to the present microprocessor implementation. The variations in the controller structures are detailed below.

2.2 PI Controller Structures

Tuning rules have been detailed for seven PI controller structures:

1. Ideal controller $G_c(s) = K_c \left(1 + \frac{1}{T_i s} \right)$ (2.3)

2. Ideal controller in series with a first order lag:

$$G_c(s) = K_c \left(1 + \frac{1}{T_i s} \right) \frac{1}{1 + T_f s} \quad (2.4)$$

3. Ideal controller in series with a second order filter (also labelled the ‘generalised PID’ controller (Lee and Shi, 2002)):

$$G_c(s) = K_c \left(1 + \frac{1}{T_i s} \right) \frac{1 + b_{f1}s + b_{f2}s^2}{1 + a_{f1}s + a_{f2}s^2} \quad (2.5)$$

4. Controller with set-point weighting:

$$U(s) = K_c \left(1 + \frac{1}{T_i s} \right) E(s) - \alpha K_c R(s) \quad (2.6)$$

5. Controller with proportional term acting on the output 1:

$$U(s) = \frac{K_c}{T_i s} E(s) - K_c Y(s) \quad (2.7)$$

6. Controller with proportional term acting on the output 2:

$$U(s) = K_c \left(1 + \frac{1}{T_i s} \right) E(s) - K_1 Y(s) \quad (2.8)$$

7. Controller with a double integral term:

$$G_c(s) = K_c \left(1 + \frac{1}{T_{i1}s} + \frac{1}{T_{i2}s^2} \right) \quad (2.9)$$

2.3 PID Controller Structures

Forty-six such structures have been considered. The labelling of these structures has not been consistent in the literature; for example, the first PID controller structure considered (labelled the ideal controller below) has also been labelled the ‘non-interacting’ controller (McMillan, 1994), the ‘ISA’ algorithm (Gerry and Hansen, 1987) or the ‘parallel non-interacting’ controller (Visioli, 2001). The controller structures have been divided into four types, as described below.

2.3.1 Ideal PID controller structure and its variations

1. Ideal controller: $G_c(s) = K_c \left(1 + \frac{1}{T_i s} + T_d s \right)$ (2.10)

A variation of the controller is labelled the ‘parallel’ controller structure (McMillan, 1994). This variation has also been labelled the ‘ideal parallel’, ‘noninteracting’, ‘independent’ or ‘gain independent’ algorithm:

$$G_c(s) = K_c + \frac{1}{T_i s} + T_d s \quad (2.11)$$

The controller structure is used in the following products:

- (a) Allen Bradley PLC5 product (McMillan, 1994)
- (b) Bailey FC19 PID algorithm (EZYtune, 2003)
- (c) Fanuc Series 90-30 and 90-70 independent form PID algorithm (EZYtune, 2003)
- (d) Intellution FIX products (McMillan, 1994)
- (e) Honeywell TDC3000 Process Manager Type A, non-interactive mode product (ISMC, 1999)
- (f) Leeds and Northrup Electromax 5 product (Åström and Hägglund, 1988)

(g) Yokogawa Field Control Station (FCS) PID algorithm (EZYtune, 2003).

2. Ideal controller in series with a first order lag:

$$G_c(s) = K_c \left(1 + \frac{1}{T_i s} + T_d s \right) \frac{1}{T_f s + 1} \quad (2.12)$$

3. Ideal controller in series with a first order filter:

$$G_c(s) = K_c \left(1 + \frac{1}{T_i s} + T_d s \right) \frac{b_{f1}s + 1}{a_{f1}s + 1} \quad (2.13)$$

4. Ideal controller in series with a second order filter:

$$G_c(s) = K_c \left(1 + \frac{1}{T_i s} + T_d s \right) \frac{1 + b_{f1}s}{1 + a_{f1}s + a_{f2}s^2} \quad (2.14)$$

5. Ideal controller with weighted proportional term:

$$G_c(s) = K_c \left(b + \frac{1}{T_i s} + T_d s \right) \quad (2.15)$$

6. Ideal controller with first order filter and setpoint weighting 1:

$$U(s) = K_c \left(1 + \frac{1}{T_i s} + T_d s \right) \frac{1}{T_f s + 1} \left[R(s) \frac{1 + 0.4T_r s}{1 + sT_r} - Y(s) \right] \quad (2.16)$$

7. Ideal controller with first order filter and setpoint weighting 2:

$$U(s) = K_c \left(1 + \frac{1}{T_i s} + T_d s \right) \frac{1}{T_f s + 1} \left[R(s) \frac{1 + 0.4T_r s}{1 + sT_r} - Y(s) \right] - K_0 Y(s) \quad (2.17)$$

8. Ideal controller with first order filter and dynamics on the controlled variable:

$$U(s) = K_c \left(1 + \frac{1}{T_i s} + T_d s \right) \frac{1}{T_f s + 1} \frac{Ts + (1 + K)}{Ts + 1} E(s) - \frac{K}{Ts + 1} Y(s) \quad (2.18)$$

9. Controller with filtered derivative:

$$G_c(s) = K_c \left(1 + \frac{1}{T_i s} + \frac{T_d s}{1 + s \frac{T_d}{N}} \right) \quad (2.19)$$

This structure is used in the following products:

- (a) Bailey Net 90 PID error input product with $N = 10$ (McMillan, 1994) and FC156 Independent Form PID algorithm (EZYtune, 2003)
- (b) Concept PIDP1 and PID1 PID algorithms (EZYtune, 2003)
- (c) Fischer and Porter DCU 3200 CON PID algorithm with $N = 8$ (EZYtune, 2003)
- (d) Foxboro EXACT I/A series PIDA product (in which it is an option labelled ideal PID) (Foxboro, 1994).
- (e) Hartmann and Braun Freelance 2000 PID algorithm (EZYtune, 2003)
- (f) Modicon 984 product with $2 \leq N \leq 30$ (McMillan, 1994; EZYtune, 2003)
- (g) Siemens Teleperm/PSC7 ContC/PCS7 CTRL PID products with $N = 10$ (ISMC, 1999) and the S7 FB41 CONT_C PID product (EZYtune, 2003).

10. Controller with filtered derivative in series with a second order filter:

$$G_c(s) = K_c \left(1 + \frac{1}{T_i s} + \frac{T_d s}{1 + \frac{T_d}{N} s} \right) \left(\frac{1 + b_{f1}s + b_{f2}s^2}{1 + a_{f1}s + a_{f2}s^2} \right) \quad (2.20)$$

11. Controller with filtered derivative with setpoint weighting 1:

$$G_c(s) = K_c \left(1 + \frac{1}{T_i s} + \frac{T_d s}{1 + \frac{T_d}{N} s} \right) \left(\frac{1 + b_{f1}s}{1 + a_{f1}s} R(s) - Y(s) \right) \quad (2.21)$$

12. Controller with filtered derivative with setpoint weighting 2:

$$G_c(s) = K_c \left(1 + \frac{1}{T_i s} + \frac{T_d s}{1 + \frac{T_d}{N} s} \right) \left(\frac{1 + b_{f1}s + b_{f2}s^2}{1 + a_{f1}s + a_{f2}s^2} R(s) - Y(s) \right) \quad (2.22)$$

13. Controller with filtered derivative with setpoint weighting 3:

$$G_c(s) = K_c \left(1 + \frac{1}{T_i s} + \frac{T_d s}{1 + \frac{T_d}{N} s} \right) \left(\frac{1 + b_{f1}s + b_{f2}s^2 + b_{f3}s^3}{1 + a_{f1}s + a_{f2}s^2 + a_{f3}s^3} R(s) - Y(s) \right) \quad (2.23)$$

14. Controller with filtered derivative with setpoint weighting 4:

$$G_c(s) = K_c \left(1 + \frac{1}{T_i s} + \frac{T_d s}{1 + \frac{T_d}{N} s} \right) \left(\frac{1 + b_{f1}s + b_{f2}s^2 + b_{f3}s^3 + b_{f4}s^4}{1 + a_{f1}s + a_{f2}s^2 + a_{f3}s^3 + a_{f4}s^4} R(s) - Y(s) \right) \quad (2.24)$$

15. Controller with filtered derivative and dynamics on the controlled variable:

$$G_c(s) = K_c \left(1 + \frac{1}{T_i s} + \frac{T_d s}{1 + s \frac{T_d}{N}} \right) E(s) - K_0 Y(s) \quad (2.25)$$

16. Ideal controller with setpoint weighting 1:

$$U(s) = K_c (F_p R(s) - Y(s)) + \frac{K_c}{T_i s} (F_i R(s) - Y(s)) + K_c T_d s (F_d R(s) - Y(s)) \quad (2.26)$$

17. Ideal controller with setpoint weighting 2:

$$U(s) = K_c \left(1 + \frac{1}{T_i s} + T_d s \right) \left[\frac{1}{T_i T_d s^2 + T_i s + 1} R(s) - Y(s) \right] \quad (2.27)$$

18. Blending controller:

$$G_c(s) = K_c \left(1 + \frac{1}{T_i s} + T_d s \right) \frac{1}{s} \quad (2.28)$$

This structure is used in the Yokogawa EFCS/EFCD Field Control Station product (Chen *et al.*, 2001).

2.3.2 Classical PID controller structure and its variations

19. Classical controller 1: This controller is also labelled the ‘cascade’ controller (Witt and Waggoner, 1990), the ‘interacting’ or ‘series’ controller (Poulin and Pomerleau, 1996), the ‘interactive’ controller (Tsang and Rad, 1995), the ‘rate-before-reset’ controller (Smith and Corripio, 1997), the ‘analog’ controller (St. Clair, 2000) or the ‘commercial’ controller (Luyben, 2001).

$$G_c(s) = K_c \left(1 + \frac{1}{T_i s} \right) \frac{1 + s T_d}{1 + s \frac{T_d}{N}} \quad (2.29)$$

The structure is used in the following products:

- (a) Honeywell TDC Basic/Extended/Multifunction Types A and B products with $N = 8$ (McMillan, 1994)
- (b) Toshiba TOSDIC 200 product with $3.33 \leq N \leq 10$ (McMillan, 1994)
- (c) Foxboro EXACT Model 761 product with $N = 10$ (McMillan, 1994)
- (d) Honeywell UDC6000 product with $N = 8$ (Åström and Hägglund, 1995)
- (e) Honeywell TDC3000 Process Manager product – Type A, interactive mode with $N = 10$ (ISMC, 1999)
- (f) Honeywell TDC3000 Universal, Multifunction and Advanced Multifunction products with $N = 8$ (ISMC, 1999)
- (g) Foxboro EXACT I/A Series PIDA product (in which it is an option labelled series PID) (Foxboro, 1994).

20. Classical controller 2: $G_c(s) = K_c \left(1 + \frac{1}{T_i s}\right) \left(\frac{1 + NT_d s}{1 + T_d s}\right)$ (2.30)

21. Series controller (Classical controller 3): This controller is also labelled the ‘interacting’ controller, the ‘analog algorithm’ (McMillan, 1994) or the ‘dependent’ controller (EZYtune, 2003).

$$G_c(s) = K_c \left(1 + \frac{1}{T_i s}\right) (1 + s T_d) \quad (2.31)$$

The structure is used in the following products:

- (a) Turnbull TCS6000 series product (McMillan, 1994)
- (b) Alfa-Laval Automation ECA400 product (Åström and Hägglund, 1995)
- (c) Foxboro EXACT 760/761 product (Åström and Hägglund, 1995).

22. Classical controller 4: This controller is also labelled the ‘interacting’ controller (Fertik, 1975).

$$G_c(s) = K_c \left(1 + \frac{1}{T_i s}\right) \left(1 + \frac{T_d s}{1 + s \frac{T_d}{N}}\right) \quad (2.32)$$

The structure is used in the following products:

- (a) Bailey FC156 Classical Form PID product (EZYtune, 2003)
- (b) Fischer and Porter DCI 4000 PID algorithm (EZYtune, 2003)

2.3.3 Non-interacting PID controller structure and its variations

23. Non-interacting controller 1: This controller is also labelled the ‘reset-feedback’ controller (Huang *et al.*, 1996).

$$U(s) = K_c \left(1 + \frac{1}{T_i s}\right) \left(E(s) - \frac{T_d s}{1 + \frac{T_d s}{N}} Y(s)\right) \quad (2.33)$$

This structure is used in the following products:

- (a) Bailey Fisher and Porter 53SL6000 and 53MC5000 products with $N = 0$ (ISMC, 1999).
- (b) Moore Model 352 Single-Loop Controller product (Wade, 1994).

24. Non-interacting controller based on the parallel controller structure:

24a. Non-interacting controller 2a:

$$U(s) = K_c \left(1 + \frac{1}{T_i s} \right) E(s) - \frac{T_d s}{1 + \frac{s T_d}{N}} Y(s) \quad (2.34)$$

24b. Non-interacting controller 2b:

$$U(s) = \left(K_c + \frac{1}{T_i s} \right) E(s) - \frac{T_d s}{1 + \frac{s T_d}{N}} Y(s) \quad (2.35)$$

25. Non-interacting controller based on the two degree of freedom structure 1: This controller is also labelled the ‘m-PID’ controller (Huang *et al.*, 2000) the ‘ISA-PID’ controller (Leva and Colombo, 2001) and the ‘P-I-PD (only P is DOF) incomplete 2DOF algorithm’ by Mizutani and Hiroi (1991).

$$U(s) = K_c \left(1 + \frac{1}{T_i s} + \frac{T_d s}{1 + \frac{T_d}{N} s} \right) E(s) - K_c \left(\alpha + \frac{\beta T_d s}{1 + \frac{T_d}{N} s} \right) R(s) \quad (2.36)$$

This structure is used in the following products:

- (a) Bailey Net 90 PID PV and SP product (McMillan, 1994)
- (b) Yokogawa SLPC products with $\alpha = -1$, $\beta = -1$, $N=10$ (McMillan, 1994)
- (c) Omron E5CK digital controller with $\beta = 1$ and $N=3$ (ISMC, 1999).

26. Non-interacting controller based on the two degree of freedom structure 2:

$$U(s) = K_c \left(1 + \frac{1}{T_i s} + \frac{T_d s}{1 + \frac{T_d}{N} s} \right) E(s) - K_c \left(\alpha + \frac{\beta T_d s}{1 + \frac{T_d}{N} s} - \frac{\chi}{1 + T_i s} \right) R(s) \quad (2.37)$$

27. Non-interacting controller based on the two degree of freedom structure 3:

$$\begin{aligned} U(s) = & K_c [(b-1) + (c-1)T_d s] R(s) \\ & + K_c \left(1 + \frac{1}{T_i s} + T_d s \right) \left[R(s) - \frac{1}{(1+sT_f)^2} Y(s) \right] \end{aligned} \quad (2.38)$$

28. PD-I-PD (only PD is DOF) incomplete 2DOF algorithm (Mizutani and Hiroi, 1991):

$$\begin{aligned} U(s) = & K_c \left[\alpha + \frac{1}{T_i s} \left(\frac{\chi T_d s}{1 + \frac{T_d}{N} s} - \frac{1}{1 + T_i s} \right) \right] R(s) \\ & - K_c \left(1 + \frac{1}{T_i s} + \frac{T_d s}{1 + \frac{T_d}{N} s} \right) Y(s) \end{aligned} \quad (2.39)$$

However, no tuning rules are available for this controller structure.

29. PI-PID (only PI is DOF) incomplete 2DOF algorithm (Mizutani and Hiroi, 1991):

$$U(s) = K_c \left[\alpha + \frac{1}{T_i s} - \frac{\beta}{T_i s (1 + T_i s)} \right] R(s) - K_c \left(1 + \frac{1}{T_i s} + \frac{T_d s}{1 + \frac{T_d}{N} s} \right) Y(s) \quad (2.40)$$

However, no tuning rules are available for this controller structure.

30. Super (or complete) 2DOF PID algorithm (Mizutani and Hiroi, 1991):

$$U(s) = K_c \left[\alpha + \frac{1}{T_i s} \left(1 - \frac{\beta}{1+T_i s} + \frac{\chi T_d s}{1+\frac{T_d}{N} s} \right) \right] R(s) - K_c \left(1 + \frac{1}{T_i s} + \frac{T_d s}{1+\frac{T_d}{N} s} \right) Y(s) \quad (2.41)$$

Mizutani and Hiroi (1991) state that $0 \leq \alpha \leq 1$, $\beta \geq 1$ and $\chi \geq 1$, and suggest starting values of $\alpha = 0.4$, $\beta = 1.35$ and $\chi = 1.25$. However, no tuning rules are available for this controller structure.

31. Non-interacting controller 3:

$$U(s) = K_c \left(1 + \frac{1}{T_i s} \right) E(s) - \frac{K_c T_d s}{1 + \frac{s T_d}{N}} Y(s) \quad (2.42)$$

This structure is used in the following products:

- (a) Allen Bradley SLC5/02, SLC5/03, SLC5/04, PLC5 and Logix5550 products (EZYtune, 2003).
- (b) Modcomp product with $N = 10$ (McMillan, 1994).

32. Non-interacting controller 4: This controller is also labelled the ‘PI+D’ controller (Chen, 1996).

$$U(s) = K_c \left(1 + \frac{1}{T_i s} \right) E(s) - K_c T_d s Y(s) \quad (2.43)$$

This structure is used in the following products:

- (a) ABB 53SL6000 Product (ABB, 2001).
- (b) Genesis product (McMillan, 1994)
- (c) Honeywell TDC3000 Process Manager Type B, non-interactive mode product (ISMC, 1999)
- (d) Square D PIDR PID product (EZYtune, 2003)

33. Non-interacting controller 5:

$$U(s) = K_c \left(b + \frac{1}{T_i s} \right) E(s) - (c + T_d s) Y(s) \quad (2.44)$$

34. Non-interacting controller 6 (I-PD controller):

$$U(s) = \frac{K_c}{T_i s} E(s) - K_c (1 + T_d s) Y(s) \quad (2.45)$$

This structure is used in the Toshiba AdTune TOSDIC 211D8 product (Shigemasa *et al.*, 1987) and the Honeywell TDC3000 Process Manager Type C non-interactive mode product (ISMC, 1999).

35. Non-interacting controller 7:

$$U(s) = \frac{K_c}{T_i s} E(s) - K_c \left(1 + \frac{T_d s}{1 + \frac{T_d}{N} s} \right) Y(s) \quad (2.46)$$

36. Non-interacting controller 8:

$$U(s) = K_c \left(1 + \frac{1}{T_i s} + T_d s \right) E(s) - K_i (1 + T_{di} s) Y(s) \quad (2.47)$$

37. Non-interacting controller 9:

$$U(s) = K_c \left(1 + \frac{1}{T_i s} + T_d s \right) E(s) - \frac{T_d}{N} s Y(s) \quad (2.48)$$

38. Non-interacting controller 10:

$$U(s) = K_c \left(1 + \frac{1}{T_i s} \right) E(s) - K_f \left(\frac{T_d}{T_m} s + 1 \right) Y(s) \quad (2.49)$$

39. Non-interacting controller 11:

$$U(s) = K_c \left(1 + \frac{1}{T_i s} \right) (1 + T_d s) E(s) - K_i (1 + T_{di} s) Y(s) \quad (2.50)$$

40. Non-interacting controller 12:

$$U(s) = K_c \left(1 + \frac{1}{T_i s} + T_d s \right) \frac{1}{1 + T_d s} E(s) - K_i Y(s) \quad (2.51)$$

41. Non-interacting controller 13:

$$U(s) = K_c \left(1 + \frac{1}{T_i s} \right) \left(E(s) - \frac{T_d s}{1 + \frac{T_d}{N} s + \frac{T_d}{2N} s^2} Y(s) \right) \quad (2.52)$$

This structure is used in the Foxboro IA PID product, with $10 \leq N \leq 50$ (EZTune, 2003); however, no tuning rules are available for this controller structure.

2.3.4 Other PID controller structures

42. Industrial controller (Kaya and Scheib, 1988):

$$U(s) = K_c \left(1 + \frac{1}{T_i s} \right) \left(R(s) - \frac{1 + T_d s}{1 + \frac{T_d}{N} s} Y(s) \right) \quad (2.53)$$

This structure is used in the following products:

- (a) Fisher-Rosemount Provox product with $N = 8$ (ISM, 1999; McMillan, 1994)
- (b) Foxboro Model 761 product with $N=10$ (McMillan, 1994)
- (c) Fischer-Porter Micro DCI product with $N = 0$ (McMillan, 1994)
- (d) Moore Products Type 352 controller with $1 \leq N \leq 30$ (McMillan, 1994)
- (e) SATT Instruments EAC400 product with $N=8.33$ (McMillan, 1994)
- (f) Taylor Mod 30 ESPO product with $N=16.7$ (McMillan, 1994)

- (g) Honeywell TDC3000 Process Manager Type B, interactive mode product with $N = 10$ (ISMC, 1999)

$$43. \text{ Alternative controller 1: } G_c(s) = K_c \left(\frac{1 + T_i s}{1 + \frac{T_d s}{N}} \right) \left(\frac{1 + T_d s}{1 + \frac{T_d s}{N}} \right) \quad (2.54)$$

44. Alternative controller 2:

$$G_c(s) = K_c \left(\frac{1 + T_i s}{1 + \frac{T_d s}{N}} \right) \left(\frac{1 + 0.5\tau_m s + 0.0833\tau_m^2 s^2}{1 + \frac{T_d s}{N}} \right) \quad (2.55)$$

45. Alternative controller 3:

$$U(s) = \frac{K_c}{T_i s} R(s) - K_c \left(1 + \frac{1}{T_i s} \right) \left(\frac{1 + sT_d}{1 + \frac{sT_d}{N}} \right) Y(s) \quad (2.56)$$

This structure is used in the following products:

- (a) Honeywell TDC3000 Process Manager Type C, interactive mode product with $N = 10$ (ISMC, 1999)
 - (b) Honeywell TDC3000 Universal, Multifunction and Advanced Multifunction products with $N = 8$ (ISMC, 1999)
- However, no tuning rules are available for this controller structure.

46. Alternative controller 4:

$$U(s) = K_c \left(\frac{1 + sT_d}{1 + \frac{sT_d}{N}} \right) E(s) + \frac{R(s)}{K_m} \quad (2.57)$$

2.3.5 Comments on the PID controller structures

In some cases, one controller structure may be transformed into another; clearly, the ideal and parallel controller structures (equations (2.10) and (2.11)) are very closely related. It is shown by McMillan (1994), among others, that the parameters of the ideal PID controller may be worked out from the parameters of the series PID controller, and vice versa. The ideal PID controller is given in Equation (2.58) and the series PID controller is given in Equation (2.59).

$$G_{cp}(s) = K_{cp} \left(1 + \frac{1}{T_{ip}s} + T_{dp}s \right) \quad (2.58)$$

$$G_{cs}(s) = K_{cs} \left(1 + \frac{1}{T_{is}s} \right) (1 + sT_{ds}) \quad (2.59)$$

Then, it may be shown that

$$K_{cp} = \left(1 + \frac{T_{ds}}{T_{is}} \right) K_{cs}, \quad T_{ip} = (T_{is} + T_{ds}), \quad T_{dp} = \left(\frac{T_{is}}{T_{is} + T_{ds}} \right) T_{ds}.$$

Similarly, it may be shown that, provided $T_{ip} > 4T_{dp}$,

$$K_{cs} = 0.5K_{cp} \left(1 + \sqrt{1 - 4 \frac{T_{dp}}{T_{ip}}} \right), \quad T_{is} = 0.5T_{ip} \left(1 + \sqrt{1 - 4 \frac{T_{dp}}{T_{ip}}} \right),$$

$$T_{ds} = 0.5T_{dp} \left(1 - \sqrt{1 - 4 \frac{T_{dp}}{T_{ip}}} \right).$$

Åström and Hägglund (1996) point out that the ideal controller admits complex zeroes and is thus a more flexible controller structure than the series controller, which has real zeroes; however, in the frequency domain, the series controller has the interesting interpretation that the zeroes of the closed loop transfer function are the inverse values of T_{is} and T_{ds} . O'Dwyer (2001b) developed a comprehensive set of tuning rules for the series PID controller, based on the ideal PID controller; as these are not strictly original tuning rules, only representative examples are included in the relevant tables.

In a similar manner, the parameters of an ideal controller in series with a first order filter may be determined from the parameters of the classical controller 1 (Witt and Waggoner, 1990) and the parameters of

the non-interacting controller 10 structure may be determined from the parameters of the ideal PID controller (Kaya *et al.*, 2003).

2.4 Process Modelling

Processes with time delay may be modelled in a variety of ways. The modelling strategy used will influence the value of the model parameters, which will in turn affect the controller values determined from the tuning rules. The modelling strategy used in association with each tuning rule, as described in the original papers, is indicated in the tables (see Chapters 3 and 4). These modelling strategies are outlined in Appendix 2. The following models are defined:

1. First order lag plus time delay (FOLPD) model ($G_m(s) = \frac{K_m e^{-sT_m}}{1 + sT_m}$)
2. FOLPD model with a positive zero ($G_m(s) = \frac{K_m (1 - sT_{m3}) e^{-sT_m}}{1 + sT_{m1}}$)
3. FOLPD model with a negative zero ($G_m(s) = \frac{K_m (1 + sT_{m3}) e^{-sT_m}}{1 + sT_{m1}}$)
4. Non-model specific
5. Integral plus time delay (IPD) model ($G_m(s) = \frac{K_m e^{-sT_m}}{s}$)
6. First order lag plus integral plus time delay (FOLIPD) model

$$(G_m(s) = \frac{K_m e^{-sT_m}}{s(1 + sT_m)})$$
7. Second order system plus time delay (SOSPD) model

$$(G_m(s) = \frac{K_m e^{-sT_m}}{T_{m1}^2 s^2 + 2\xi_m T_{m1} s + 1}) \text{ or } G_m(s) = \frac{K_m e^{-sT_m}}{(1 + T_{m1} s)(1 + T_{m2} s)}$$
8. Integral squared plus time delay (I²PD) model ($G_m(s) = \frac{K_m e^{-sT_m}}{s^2}$)
9. Second order system (repeated pole) plus integral plus time delay (SOSIPD) model ($G_m(s) = \frac{K_m e^{-sT_m}}{s(1 + sT_m)^2}$)

10. SOSPD model with a positive zero

$$(G_m(s) = \frac{K_m(1-sT_{m3})e^{-s\tau_m}}{(1+sT_{m1})(1+sT_{m2})} \text{ or } G_m(s) = \frac{K_m(1-sT_{m3})e^{-s\tau_m}}{T_{m1}^2 s^2 + 2\xi_m T_{m1}s + 1})$$

11. SOSPD model with a negative zero

$$(G_m(s) = \frac{K_m(1+sT_{m3})e^{-s\tau_m}}{(1+sT_{m1})(1+sT_{m2})} \text{ or } G_m(s) = \frac{K_m(1+sT_{m3})e^{-s\tau_m}}{T_{m1}^2 s^2 + 2\xi_m T_{m1}s + 1})$$

12. Third order system plus time delay model

$$(G_m(s) = \frac{K_m(1+b_1s+b_2s^2+b_3s^3)e^{-s\tau_m}}{(1+a_1s+a_2s^2+a_3s^3)} \text{ or }$$

$$G_m(s) = \frac{K_m e^{-s\tau_m}}{(1+sT_{m1})(1+sT_{m2})(1+sT_{m3})}$$

13. Fifth order system plus delay model

$$(G_m(s) = \frac{K_m(1+b_1s+b_2s^2+b_3s^3+b_4s^4+b_5s^5)e^{-s\tau_m}}{(1+a_1s+a_2s^2+a_3s^3+a_4s^4+a_5s^5)})$$

14. Unstable FOLPD model ($G_m(s) = \frac{K_m e^{-s\tau_m}}{T_m s - 1}$)

15. Unstable FOLPD model with a positive zero

$$(G_m(s) = \frac{K_m(1-sT_{m3})e^{-s\tau_m}}{T_m s - 1})$$

16. Unstable SOSPD model (one unstable pole)

$$(G_m(s) = \frac{K_m e^{-s\tau_m}}{(T_{m1}s - 1)(1+sT_{m2})})$$

17. Unstable SOSPD model with a positive zero

$$(G_m(s) = \frac{K_m(1-sT_{m3})e^{-s\tau_m}}{T_{m1}^2 s^2 - 2\xi_m T_{m1}s + 1})$$

18. Unstable SOSPD model (two unstable poles)

$$(G_m(s) = \frac{K_m e^{-s\tau_m}}{(T_{m1}s - 1)(T_{m2}s - 1)})$$

19. Delay model ($G_m(s) = K_m e^{-s\tau_m}$)

20. General model with a repeated pole ($G_m(s) = \frac{K_m e^{-s\tau_m}}{(1+sT_m)^n}$)

21. General model with integrator

$$(G_m(s) = \frac{K_m}{s} \frac{\prod_i (T_{m_1i}s + 1) \prod_i (T_{m_2i}^2 s^2 + 2\xi_{m_ni} T_{m_2i}s + 1)}{\prod_i (T_{m_3i}s + 1) \prod_i (T_{m_4i}^2 s^2 + 2\xi_{m_di} T_{m_4i}s + 1)} e^{-s\tau_m})$$

22. General stable non-oscillating model with a time delay

Tables 1 and 2 show the number of tuning rules defined for each PI or PID controller structure and each process model. The following data is key to the process model type in Tables 1 and 2:

Model 1: Stable FOLPD model

Model 2: Non-model specific

Model 3: IPD model

Model 4: FOLIPD model

Model 5: SOSPD model

Model 6: Other models

Table 1: PI controller structure and tuning rules – a summary

<i>Process model Controller Equation</i>	<i>1</i>	<i>2</i>	<i>3</i>	<i>4</i>	<i>5</i>	<i>6</i>	<i>Total</i>
(2.3)	172	42	46	10	32	62	364
(2.4)	3	0	2	0	0	2	7
(2.5)	2	0	0	0	0	0	2
(2.6)	7	2	4	3	3	13	32
(2.7)	1	0	9	14	0	7	31
(2.8)	2	0	1	1	0	2	6
(2.9)	0	0	1	0	0	0	1
<i>Total</i>	187	44	63	28	35	86	443

Table 2: PID controller structure and tuning rules – a summary

<i>Process model Controller Equation</i>	<i>1</i>	<i>2</i>	<i>3</i>	<i>4</i>	<i>5</i>	<i>6</i>	<i>Total</i>
<i>Ideal structure + variations</i>							
(2.10,2.11)	100	51	22	12	55	36	276
(2.12)	7	2	1	5	13	9	37
(2.13)	0	0	0	0	1	0	1
(2.14)	2	2	0	0	1	0	5
(2.15)	2	2	0	1	0	0	5
(2.16)	1	0	0	0	0	0	1
(2.17)	0	0	1	0	0	0	1
(2.18)	1	0	0	0	0	0	1
(2.19)	16	11	2	2	3	5	39
(2.20)	0	0	0	0	1	0	1
(2.21)	0	0	0	1	0	0	1
(2.22)	0	0	0	0	0	1	1
(2.23)	0	0	0	2	0	0	2
(2.24)	0	0	0	0	0	2	2
(2.25)	0	0	1	0	0	0	1
(2.26)	0	1	0	1	1	9	12
(2.27)	0	0	0	0	2	0	2
(2.28)	1	0	0	0	0	0	1
<i>Subtotal</i>	130	69	27	24	77	62	389
<i>Classical structure + variations</i>							
(2.29)	54	5	8	5	20	9	101
(2.30)	1	0	2	1	2	0	6
(2.31)	6	5	0	1	7	6	25
(2.32)	2	1	1	1	0	2	7
<i>Subtotal</i>	63	11	11	8	29	17	139
<i>Non-interacting structure + variations</i>							
(2.33)	1	0	0	0	5	2	8
(2.34)	5	0	0	0	0	0	5
(2.35)	6	0	0	0	0	0	6
(2.36)	12	1	5	6	7	13	44
(2.37)	4	0	0	0	0	0	4
(2.38)	2	0	1	0	0	0	3
(2.42)	0	0	0	0	0	4	4
(2.43)	10	3	4	2	3	0	22
(2.44)	2	0	0	0	1	0	3
(2.45)	10	0	8	11	1	0	30
(2.46)	1	0	0	0	0	1	2
(2.47)	1	0	1	2	0	4	8
(2.48)	0	1	0	0	0	0	1

<i>Process model Controller Equation</i>	<i>1</i>	<i>2</i>	<i>3</i>	<i>4</i>	<i>5</i>	<i>6</i>	<i>Total</i>
<i>Non-interacting structure + variations (continued)</i>							
(2.49)	0	0	1	0	0	2	3
(2.50)	0	0	0	1	0	0	1
(2.51)	1	0	1	1	0	1	4
<i>Subtotal</i>	55	5	21	23	17	27	148
<i>Other controller structures</i>							
(2.53)	8	0	0	1	0	1	10
(2.54)	0	0	0	2	0	0	2
(2.55)	0	0	0	1	0	0	1
(2.57)	0	0	0	0	2	0	2
<i>Subtotal</i>	8	0	0	4	2	1	15
<i>Total</i>	256	85	59	59	125	107	691

Table 1 shows that 82% of tuning rules have been defined for the ideal PI controller structure, with 42% of tuning rules based on a FOLPD process model.

The range of PID controller variations has lead to a less homogenous situation than for the PI controller; Table 2 shows that 40% of tuning rules have been defined for the ideal PID controller structure, with 37% of tuning rules based on a FOLPD process model. Many of the controller structures are used in a variety of manufacturers' products (as outlined in Section 2.3); clearly a range of tuning rules are available for use with these products.

2.5 Organisation of the Tuning Rules

The tuning rules are organised in tabular form in Chapters 3 and 4. Within each table, the tuning rules are classified further; the main subdivisions made are as follows:

- (i) Tuning rules based on a measured step response (also called process reaction curve methods).
- (ii) Tuning rules based on minimising an appropriate performance criterion, either for optimum regulator or optimum servo action.
- (iii) Tuning rules that give a specified closed loop response (direct synthesis tuning rules). Such rules may be defined by specifying the

desired poles of the closed loop response, for instance, though more generally, the desired closed loop transfer function may be specified. The definition may be expanded to cover techniques that allow the achievement of a specified gain margin and/or phase margin.

- (iv) Robust tuning rules, with an explicit robust stability and robust performance criterion built in to the design process.
- (v) Tuning rules based on recording appropriate parameters at the ultimate frequency (also called ultimate cycling methods).
- (vi) Other tuning rules, such as tuning rules that depend on the proportional gain required to achieve a quarter decay ratio or to achieve magnitude and frequency information at a particular phase lag.

Some tuning rules could be considered to belong to more than one subdivision, so the subdivisions cannot be considered to be mutually exclusive; nevertheless, they provide a convenient way to classify the rules. In the tables, all symbols used are defined in Appendix 1.

Chapter 3

Tuning Rules for PI Controllers

3.1 FOLPD Model $G_m(s) = \frac{K_m e^{-s\tau_m}}{1 + sT_m}$

3.1.1 Ideal controller $G_c(s) = K_c \left(1 + \frac{1}{T_i s} \right)$

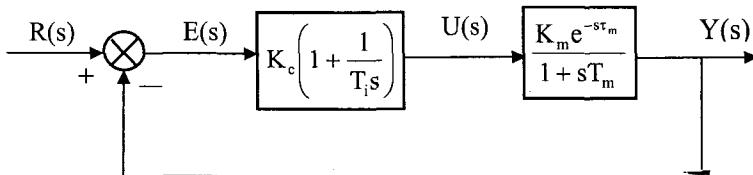


Table 3: PI controller tuning rules - FOLPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{1 + sT_m}$

Rule	K_c	T_i	Comment
Process reaction			
Callender <i>et al.</i> (1935/6). <i>Model: Method 1</i>	¹ $0.568/K_m \tau_m$	$3.64\tau_m$	$\frac{\tau_m}{T_m} = 0.3$
	² $0.690/K_m \tau_m$	$2.45\tau_m$	

¹ Decay ratio = 0.015; Period of decaying oscillation = $10.5\tau_m$

² Decay ratio = 0.043; Period of decaying oscillation = $6.28\tau_m$

Rule	K_c	T_i	Comment
Ziegler and Nichols (1942). Model: Method 2	$\frac{0.9T_m}{K_m \tau_m}$	$3.33\tau_m$	Quarter decay ratio. $\frac{\tau_m}{T_m} \leq 1$
Hazebroek and Van der Waerden (1950). Model: Method 2	$\frac{x_1 T_m}{K_m \tau_m}$	$x_2 \tau_m$	
Coefficient values			
	$\frac{\tau_m}{T_m}$	x_1	x_2
	0.2	0.68	7.14
	0.3	0.70	4.76
	0.4	0.72	3.70
	0.5	0.74	3.03
	0.6	0.76	2.50
	0.7	0.79	2.17
	0.8	0.81	1.92
	0.9	0.84	1.75
	1.0	0.87	1.61
	$x_1 = 0.5 \frac{\tau_m}{T_m} + 0.1$	$x_2 = \frac{\tau_m}{1.6\tau_m - 1.2T_m}$	$\frac{\tau_m}{T_m} > 3.5$
Oppelt (1951). Model: Method 2	${}^3 K_c^{(1)}$	$1.66\tau_m$	$\tau_m >> T_m$
		$3.32\tau_m$	$\tau_m \ll T_m$
Moros (1999). Model: Method 1	$0.8T_m / K_m \tau_m$	$3\tau_m$	attributed to Oppelt
	$0.91T_m / K_m \tau_m$	$3.3\tau_m$	attributed to Rosenberg
Chien <i>et al.</i> (1952) – regulator. Model: Method 2; $0.1 < \frac{\tau_m}{T_m} < 1.0$	$\frac{0.6T_m}{K_m \tau_m}$	$4\tau_m$	0% overshoot
	$\frac{0.7T_m}{K_m \tau_m}$	$2.33\tau_m$	20% overshoot
Chien <i>et al.</i> (1952) – servo. Model: Method 2; $0.1 < \frac{\tau_m}{T_m} < 1.0$	$\frac{0.35T_m}{K_m \tau_m}$	$1.17T_m$	0% overshoot
	$\frac{0.6T_m}{K_m \tau_m}$	T_m	20% overshoot

$${}^3 K_c^{(1)} \leq \frac{1}{K_m} \left[1.5708 \frac{T_m}{\tau_m} + 1 \right]; \text{ recommended } K_c^{(1)} = \frac{1}{K_m} \left[0.77 \frac{T_m}{\tau_m} - 1 \right].$$

Rule	K_c	T_i	Comment					
Reswick (1956). Model: Method 2; K_c, T_i deduced from graphs. $\frac{\tau_m}{T_m} = 6.25$	$0.20/K_m$	$0.20\tau_m$	0% overshoot – regulator					
	$0.30/K_m$	$0.22\tau_m$	20% overshoot – regulator					
	$0.15/K_m$	$0.16\tau_m$	0% overshoot – servo					
	$0.20/K_m$	$0.14\tau_m$	20% overshoot – servo					
Cohen and Coon (1953). Model: Method 2	$\frac{1}{K_m} \left(0.9 \frac{T_m}{\tau_m} + 0.083 \right)$	${}^4 T_i^{(2)}$	Quarter decay ratio; $0 < \frac{\tau_m}{T_m} \leq 1.0$					
Two constraints method – Wolfe (1951). Model: Method 4	$\frac{x_1 T_m}{K_m \tau_m}$	$x_2 \tau_m$	Decay ratio is as small as possible; minimum error integral (regulator mode).					
	Coefficient values							
	$\frac{\tau_m}{T_m}$	x_1	x_2	$\frac{\tau_m}{T_m}$	x_1	x_2		
	0.2	4.4	3.23	1.0	0.78	1.28		
	0.5	1.8	2.27	5.0	0.30	0.53		
Two constraints criterion – Murrill (1967) – page 356. Model: Method 5	$\frac{0.928}{K_m} \left(\frac{T_m}{\tau_m} \right)^{0.946}$	$\frac{T_m}{1.078} \left(\frac{\tau_m}{T_m} \right)^{0.583}$	Quarter decay ratio; minimum error integral (servo mode). $0.1 \leq \frac{\tau_m}{T_m} \leq 1.0$					
Heck (1969). Model: Method 2	$\frac{0.8T_m}{K_m \tau_m}$	$3\tau_m$						
	0	$x_2 K_m \tau_m$	Integral controller					
	Representative coefficient values – from graph – "Störung"							
	$\frac{\tau_m}{T_m}$	x_2	$\frac{\tau_m}{T_m}$	x_2	$\frac{\tau_m}{T_m}$	x_2	$\frac{\tau_m}{T_m}$	x_2
	0.11	2.94	0.20	2.50	0.40	2.17	0.71	1.96
	0.13	2.86	0.25	2.38	0.50	2.13		
	0.14	2.78	0.29	2.35	0.56	2.08		
	0.17	2.63	0.33	2.30	0.63	2.00		

$${}^4 T_i^{(2)} = T_m \left(\frac{3.33 \frac{\tau_m}{T_m} + 0.31 \left(\frac{\tau_m}{T_m} \right)^2}{1 + 2.22 \frac{\tau_m}{T_m}} \right).$$

Rule	K_c		T_i		Comment		
Heck (1969)-continued.	Representative coefficient values – from graph – “Führung”						
	$\frac{\tau_m}{T_m}$	x_2	$\frac{\tau_m}{T_m}$	x_2	$\frac{\tau_m}{T_m}$	x_2	$\frac{\tau_m}{T_m}$
	0.11	3.92	0.20	3.33	0.40	2.63	0.71
	0.13	3.85	0.25	3.08	0.50	2.47	
	0.14	3.64	0.29	2.94	0.56	2.41	
	0.17	3.51	0.33	2.82	0.63	2.35	
Fertik and Sharpe (1979).	$\frac{0.56}{K_m}$		$0.65T_m$		Model: Method 3		
Sakai <i>et al.</i> (1989). Model: Method 2	$\frac{1.2408K_m}{T_m \tau_m}$		$0.5\tau_m$				
Borresen and Grindal (1990). Model: Method 2	$\frac{1.0T_m}{K_m \tau_m}$		$3\tau_m$		Also described by ABB (2001)		
Klein <i>et al.</i> (1992). Model: Method 1	$\frac{0.28T_m}{K_m(\tau_m + 0.1T_m)}$		$0.53T_m$				
McMillan (1994) – page 25. Model: Method 5	$\frac{K_m}{3}$		τ_m		Time delay dominant processes		
St. Clair (1997) – page 22. Model: Method 5	$\frac{0.333T_m}{K_m \tau_m}$		T_m		$\frac{\tau_m}{T_m} \geq 0.33$		
Hay (1998) – pages 197-198. Model: Method 1; <i>Coefficients of K_c, T_i deduced from graphs</i>	x_1/K_m		$x_2 \tau_m$				
	Coefficient values – servo tuning						
	τ_m/T_m	x_1	x_2	τ_m/T_m	x_1	x_2	
	0.1	6.0	9.5	0.1	4.0	10.0	
	0.125	4.5	7.5	0.125	3.2	8.0	
	0.167	3.0	5.4	0.167	2.2	6.0	
	0.25	2.0	3.4	0.25	1.3	4.0	
	0.5	0.8	1.7	0.5	0.5	2.0	
	Coefficient values – regulator tuning						
	τ_m/T_m	x_1	x_2	τ_m/T_m	x_1	x_2	
Shinskey (2000), (2001). Model: Method 2	$\frac{0.667T_m}{K_m \tau_m}$		$3.78\tau_m$		$\frac{\tau_m}{T_m} = 6$		

Rule	K_c	T_i	Comment		
Lipták (2001). Model: Method 2	$\frac{0.95T_m}{K_m \tau_m}$	$4\tau_m$			
Minimum performance index: regulator tuning					
Minimum IAE – Murrill (1967) – pages 358-363.	$\frac{0.984}{K_m} \left(\frac{T_m}{\tau_m} \right)^{0.986}$	$\frac{T_m}{0.608} \left(\frac{\tau_m}{T_m} \right)^{0.707}$	Model: Method 5; $0.1 \leq \frac{\tau_m}{T_m} \leq 1.0$		
Minimum IAE – Frank and Lenz (1969). Model: Method 1	$\frac{1}{K_m} \left(x_1 + x_2 \frac{T_m}{\tau_m} \right)$	$\frac{\tau_m}{x_2} \left(x_1 + x_2 \frac{T_m}{\tau_m} \right)$			
Representative coefficient values – deduced from a graph					
	τ_m / T_m	x_1	x_2	τ_m / T_m	x_1
	0.2	0.37	0.77	1.0	0.46
	0.4	0.42	0.85	1.2	0.46
	0.6	0.44	0.91	1.4	0.46
	0.8	0.45	0.98		
Minimum IAE – Pemberton (1972a), Smith and Corripio (1997) – page 345.	$\frac{T_m}{K_m \tau_m}$	T_m	Model: Method 1; $0.1 \leq \frac{\tau_m}{T_m} \leq 0.5$		
Minimum IAE – Yu (1988). Model: Method 1. Load disturbance model = $\frac{K_L e^{-sT_L}}{1+sT_L}$.	⁵ $K_c^{(3)}$	$T_i^{(3)}$	$\frac{\tau_m}{T_m} \leq 0.35$		
	⁶ $K_c^{(4)}$	$T_i^{(4)}$			

$${}^5 K_c^{(3)} = \frac{0.685}{K_m} \left(\frac{T_L}{T_m} \right)^{0.214 - 0.346 \frac{\tau_m}{T_m}} \left(\frac{\tau_m}{T_m} \right)^{-1.256},$$

$$T_i^{(3)} = \frac{T_m}{0.214} \left(\frac{T_L}{T_m} \right)^{1.977 \frac{\tau_m}{T_m} - 0.55} \left(\frac{\tau_m}{T_m} \right)^{1.123}, \quad \frac{T_L}{T_m} \leq 2.641 \frac{\tau_m}{T_m} + 0.16.$$

$${}^6 K_c^{(4)} = \frac{0.874}{K_m} \left(\frac{T_L}{T_m} \right)^{-0.099 + 0.159 \frac{\tau_m}{T_m}} \left(\frac{\tau_m}{T_m} \right)^{-1.041},$$

$$T_i^{(4)} = \frac{T_m}{0.415} \left(\frac{T_L}{T_m} \right)^{-4.515 \frac{\tau_m}{T_m} + 0.067} \left(\frac{\tau_m}{T_m} \right)^{0.876}, \quad 2.641 \frac{\tau_m}{T_m} + 0.16 \leq \frac{T_L}{T_m} \leq 1.$$

Rule	K_c	T_i	Comment
Minimum IAE – Yu (1988) – continued.	${}^7 K_c^{(5)}$	$T_i^{(5)}$	
	${}^8 K_c^{(6)}$	$T_i^{(6)}$	$\tau_m/T_m > 0.35$
Minimum IAE – Shinskey (1988) – page 123. Model: Method 1	$1.00T_m/K_m\tau_m$	$3.0\tau_m$	$\tau_m/T_m = 0.2$
	$1.04T_m/K_m\tau_m$	$2.25\tau_m$	$\tau_m/T_m = 0.5$
	$1.08T_m/K_m\tau_m$	$1.45\tau_m$	$\tau_m/T_m = 1$
	$1.39T_m/K_m\tau_m$	τ_m	$\tau_m/T_m = 2$
Minimum IAE – Shinskey (1996) – page 38. Model: Method 1	$0.95T_m/K_m\tau_m$	$3.4\tau_m$	$\tau_m/T_m = 0.1$
	$0.95T_m/K_m\tau_m$	$2.9\tau_m$	$\tau_m/T_m = 0.2$
Minimum IAE – Marlin (1995) – pages 301-307. Model: Method 1.	$1.4/K_m$	$0.24\tau_m$	$\tau_m/T_m = 0.11$
	$1.8/K_m$	$0.52\tau_m$	$\tau_m/T_m = 0.25$
	$1.4/K_m$	$0.75\tau_m$	$\tau_m/T_m = 0.43$
	$1.0/K_m$	$0.68\tau_m$	$\tau_m/T_m = 0.67$
K_c, T_i deduced from graphs; robust to $\pm 25\%$ process model parameter changes.	$0.8/K_m$	$0.71\tau_m$	$\tau_m/T_m = 1.0$
	$0.55/K_m$	$0.60\tau_m$	$\tau_m/T_m = 1.50$
	$0.45/K_m$	$0.54\tau_m$	$\tau_m/T_m = 2.33$
	$0.35/K_m$	$0.49\tau_m$	$\tau_m/T_m = 4.0$
Minimum IAE – Edgar et al. (1997) – pages 8-14, 8-15. Model: Method 1	$0.4/K_m$	$0.5\tau_m$	Time delay dominant
	$0.94T_m/K_m\tau_m$	$4\tau_m$	Time constant dominant
Minimum IAE – Edgar et al. (1997) – page 8-15.	$0.67T_m/K_m\tau_m$	$3.5\tau_m$	Model: Method 2

$${}^7 K_c^{(5)} = \frac{0.871}{K_m} \left(\frac{T_L}{T_m} \right)^{-0.015 + 0.384 \frac{\tau_m}{T_m}} \left(\frac{\tau_m}{T_m} \right)^{-1.055},$$

$$T_i^{(5)} = \frac{T_m}{0.444} \left(\frac{T_L}{T_m} \right)^{-0.217 \frac{\tau_m}{T_m} - 0.213} \left(\frac{\tau_m}{T_m} \right)^{0.867}, \quad 1 < \frac{T_L}{T_m} \leq 3.$$

$${}^8 K_c^{(6)} = \frac{0.513}{K_m} \left(\frac{T_L}{T_m} \right)^{0.218 \frac{\tau_m}{T_m}} \left(\frac{\tau_m}{T_m} \right)^{-1.451}, \quad T_i^{(6)} = \frac{T_m}{0.670} \left(\frac{T_L}{T_m} \right)^{-0.003 \frac{\tau_m}{T_m} - 0.084} \left(\frac{\tau_m}{T_m} \right)^{0.56}.$$

Rule	K_c	T_i	Comment
Minimum IAE – Huang <i>et al.</i> (1996). <i>Model: Method 1</i>	$^9 K_c^{(7)}$	$T_i^{(7)}$	$0.1 \leq \frac{\tau_m}{T_m} \leq 1$
Minimum IAE – Shinskey (1988) – <i>page 148.</i> <i>Model: Method 1</i>	$0.58K_u$	$0.81T_u$	$\tau_m/T_m = 0.2$
	$0.54K_u$	$0.66T_u$	$\tau_m/T_m = 0.5$
	$0.48K_u$	$0.47T_u$	$\tau_m/T_m = 1$
	$0.46K_u$	$0.37T_u$	$\tau_m/T_m = 2$
Minimum IAE – Shinskey (1994) – <i>page 167.</i>	$\frac{K_u}{3.05 - 0.35 \frac{T_u}{\tau_m}}$	$^{10} T_i^{(8)}$	<i>Model: Method 1</i>
Minimum IAE – Shinskey (1996) – <i>page 121.</i>	$0.55K_u$	$0.78T_u$	<i>Model: Method 1;</i> $\tau_m/T_m = 0.2$
Minimum ISE – Hazebroek and Van der Waerden (1950). <i>Model: Method 2</i>	$^{11} K_c^{(9)}$	$1.43T_m$	$\tau_m/T_m < 0.2$

$$^9 K_c^{(7)} = \frac{1}{K_m} \left[6.4884 + 4.6198 \frac{\tau_m}{T_m} + 0.8196 \left(\frac{\tau_m}{T_m} \right)^{-0.9077} - 5.2132 \left(\frac{\tau_m}{T_m} \right)^{-0.063} \right] \\ + \frac{1}{K_m} \left[- 7.2712 \left(\frac{\tau_m}{T_m} \right)^{0.5961} - 0.7241 e^{\frac{\tau_m}{T_m}} \right],$$

$$T_i^{(7)} = T_m \left[0.0064 + 3.9574 \frac{\tau_m}{T_m} - 6.4789 \left(\frac{\tau_m}{T_m} \right)^2 + 9.4348 \left(\frac{\tau_m}{T_m} \right)^3 - 10.7619 \left(\frac{\tau_m}{T_m} \right)^4 \right] \\ + T_m \left[7.5146 \left(\frac{\tau_m}{T_m} \right)^5 - 2.2236 \left(\frac{\tau_m}{T_m} \right)^6 \right].$$

$$^{10} T_i^{(8)} = T_u \left(0.87 - 0.855 \frac{T_u}{\tau_m} + 0.172 \left[\frac{T_u}{\tau_m} \right]^2 \right).$$

$$^{11} K_c^{(9)} = \frac{T_m}{K_m \tau_m} \left(0.74 + 0.3 \frac{\tau_m}{T_m} \right).$$

Rule	K_c	T_i	Comment						
Hazeboek and Van der Waerden (1950) continued.	$\frac{x_1 T_m}{K_m \tau_m}$	$x_2 \tau_m$							
Coefficient values									
	$\frac{\tau_m}{T_m}$	x_1	x_2	$\frac{\tau_m}{T_m}$	x_1	x_2	$\frac{\tau_m}{T_m}$	x_1	x_2
	0.2	0.80	7.14	0.7	0.96	2.44	2.0	1.46	1.18
	0.3	0.83	5.00	1.0	1.07	1.85	3.0	1.89	0.95
	0.5	0.89	3.23	1.5	1.26	1.41	5.0	2.75	0.81
Minimum ISE – Haalman (1965). Model: Method 1	$\frac{0.67 T_m}{K_m \tau_m}$	T_m			$M_{\max} = 1.9; A_m = 2.36; \phi_m = 50^0$				
Minimum ISE – Murrill (1967) – pages 358-363. Model: Method 5	$\frac{1.305}{K_m} \left(\frac{T_m}{\tau_m} \right)^{0.959}$	$\frac{T_m}{0.492} \left(\frac{\tau_m}{T_m} \right)^{0.739}$		$0.1 \leq \frac{\tau_m}{T_m} \leq 1.0$					
Minimum ISE – Frank and Lenz (1969). Model: Method 1	$\frac{1}{K_m} \left(x_1 + x_2 \frac{T_m}{\tau_m} \right)$	$\frac{\tau_m}{x_2} \left(x_1 + x_2 \frac{T_m}{\tau_m} \right)$							
Representative coefficient values – deduced from a graph									
	τ_m/T_m	x_1	x_2	τ_m/T_m	x_1	x_2			
	0.2	0.53	0.82	1.0	0.63	1.12			
	0.4	0.58	0.88	1.2	0.63	1.17			
	0.6	0.61	0.97	1.4	0.62	1.20			
	0.8	0.62	1.05						
Minimum ISE – Zhuang and Atherton (1993). Model: Method 1	$\frac{1.279}{K_m} \left(\frac{T_m}{\tau_m} \right)^{0.945}$	$\frac{T_m}{0.535} \left(\frac{\tau_m}{T_m} \right)^{0.586}$		$0.1 \leq \frac{\tau_m}{T_m} \leq 1.0$					
	$\frac{1.346}{K_m} \left(\frac{T_m}{\tau_m} \right)^{0.675}$	$\frac{T_m}{0.552} \left(\frac{\tau_m}{T_m} \right)^{0.438}$		$1.1 \leq \frac{\tau_m}{T_m} \leq 2.0$					
Minimum ISE – Yu (1988). Model: Method 1.	$^{12} K_c^{(10)}$	$T_i^{(10)}$		$\frac{\tau_m}{T_m} \leq 0.35$					

12 Load disturbance model = $\frac{K_L e^{-sT_L}}{1+sT_L} \cdot K_c^{(10)} = \frac{0.921}{K_m} \left(\frac{T_L}{T_m} \right)^{0.181-0.205 \frac{\tau_m}{T_m}} \left(\frac{\tau_m}{T_m} \right)^{-1.214},$

$$T_i^{(10)} = \frac{T_m}{0.430} \left(\frac{T_L}{T_m} \right)^{0.954 \frac{\tau_m}{T_m} - 0.49} \left(\frac{\tau_m}{T_m} \right)^{0.639}, \quad \frac{T_L}{T_m} \leq 2.310 \frac{\tau_m}{T_m} + 0.077.$$

Rule	K_c	T_i	Comment			
Minimum ISE – Yu (1988) – continued.	$^{13} K_c^{(11)}$	$T_i^{(11)}$	$\frac{\tau_m}{T_m} \leq 0.35$			
	$^{14} K_c^{(12)}$	$T_i^{(12)}$				
	$^{15} K_c^{(13)}$	$T_i^{(13)}$	$\frac{\tau_m}{T_m} > 0.35$			
Minimum ITAE - Murrill (1967) – pages 358-363. Model: Method 5	$\frac{0.859}{K_m} \left(\frac{T_m}{\tau_m} \right)^{0.977}$	$\frac{T_m}{0.674} \left(\frac{\tau_m}{T_m} \right)^{0.680}$	$0.1 \leq \frac{\tau_m}{T_m} \leq 1.0$			
Minimum ITAE – Frank and Lenz (1969). Model: Method 1	$\frac{1}{K_m} \left(x_1 + x_2 \frac{T_m}{\tau_m} \right)$	$\frac{\tau_m}{x_2} \left(x_1 + x_2 \frac{T_m}{\tau_m} \right)$	Representative coefficient values – deduced from a graph			
	τ_m/T_m	x_1	x_2	τ_m/T_m	x_1	x_2
	0.2	0.32	0.77	1.0	0.38	1.00
	0.4	0.35	0.80	1.2	0.39	1.05
	0.6	0.37	0.86	1.4	0.39	1.10
	0.8	0.38	0.93			

$$^{13} K_c^{(11)} = \frac{1.157}{K_m} \left(\frac{T_L}{T_m} \right)^{-0.045+0.344 \frac{\tau_m}{T_m}} \left(\frac{\tau_m}{T_m} \right)^{-1.014},$$

$$T_i^{(11)} = \frac{T_m}{0.359} \left(\frac{T_L}{T_m} \right)^{-2.532 \frac{\tau_m}{T_m}-0.292} \left(\frac{\tau_m}{T_m} \right)^{0.899}, \quad 2.310 \frac{\tau_m}{T_m} + 0.077 \leq \frac{T_L}{T_m} \leq 1.$$

$$^{14} K_c^{(12)} = \frac{1.07}{K_m} \left(\frac{T_L}{T_m} \right)^{-0.065+0.234 \frac{\tau_m}{T_m}} \left(\frac{\tau_m}{T_m} \right)^{-1.047},$$

$$T_i^{(12)} = \frac{T_m}{0.347} \left(\frac{T_L}{T_m} \right)^{-1.112 \frac{\tau_m}{T_m}-0.094} \left(\frac{\tau_m}{T_m} \right)^{0.898}, \quad 1 < \frac{T_L}{T_m} \leq 3.$$

$$^{15} K_c^{(13)} = \frac{1.289}{K_m} \left(\frac{T_L}{T_m} \right)^{0.04+0.067 \frac{\tau_m}{T_m}} \left(\frac{\tau_m}{T_m} \right)^{-0.889}, \quad T_i^{(13)} = \frac{T_m}{0.596} \left(\frac{T_L}{T_m} \right)^{0.372 \frac{\tau_m}{T_m}-0.44} \left(\frac{\tau_m}{T_m} \right)^{0.46}.$$

Rule	K_c	T_i	Comment
Minimum ITAE – Yu (1988). <i>Model: Method 1.</i> Load disturbance model = $\frac{K_L e^{-sT_L}}{1+sT_L}$.	$^{16} K_c^{(14)}$	$T_i^{(14)}$	$\frac{\tau_m}{T_m} \leq 0.35$
	$^{17} K_c^{(15)}$	$T_i^{(15)}$	
	$^{18} K_c^{(16)}$	$T_i^{(16)}$	
	$^{19} K_c^{(17)}$	$T_i^{(17)}$	$\frac{\tau_m}{T_m} > 0.35$
Minimum ITSE – Frank and Lenz (1969). <i>Model: Method 1</i>	$\frac{1}{K_m} \left(x_1 + x_2 \frac{T_m}{\tau_m} \right)$	$\frac{\tau_m}{x_2} \left(x_1 + x_2 \frac{T_m}{\tau_m} \right)$	Representative coefficient values – deduced from a graph
	τ_m/T_m	x_1	
	0.2	0.46	
	0.4	0.50	
	0.6	0.53	
	0.8	0.55	
	x_2	τ_m/T_m	
		1.0	0.55
		1.2	0.54
		1.4	0.54
			1.20

$$\begin{aligned}
 ^{16} K_c^{(14)} &= \frac{0.598}{K_m} \left(\frac{T_L}{T_m} \right)^{0.272-0.254 \frac{\tau_m}{T_m}} \left(\frac{\tau_m}{T_m} \right)^{-1.341}, \\
 T_i^{(14)} &= \frac{T_m}{0.805} \left(\frac{T_L}{T_m} \right)^{0.304 \frac{\tau_m}{T_m}-0.112} \left(\frac{\tau_m}{T_m} \right)^{0.196}, \quad \frac{T_L}{T_m} \leq 2.385 \frac{\tau_m}{T_m} + 0.112. \\
 ^{17} K_c^{(15)} &= \frac{0.735}{K_m} \left(\frac{T_L}{T_m} \right)^{-0.011-1.945 \frac{\tau_m}{T_m}} \left(\frac{\tau_m}{T_m} \right)^{-1.055}, \\
 T_i^{(15)} &= \frac{T_m}{0.425} \left(\frac{T_L}{T_m} \right)^{-5.809 \frac{\tau_m}{T_m}+0.241} \left(\frac{\tau_m}{T_m} \right)^{0.901}, \quad 2.385 \frac{\tau_m}{T_m} + 0.112 \leq \frac{T_L}{T_m} \leq 1. \\
 ^{18} K_c^{(16)} &= \frac{0.787}{K_m} \left(\frac{T_L}{T_m} \right)^{0.084+0.154 \frac{\tau_m}{T_m}} \left(\frac{\tau_m}{T_m} \right)^{-1.042}, \\
 T_i^{(16)} &= \frac{T_m}{0.431} \left(\frac{T_L}{T_m} \right)^{-0.148 \frac{\tau_m}{T_m}-0.365} \left(\frac{\tau_m}{T_m} \right)^{0.901}, \quad 1 < \frac{T_L}{T_m} \leq 3. \\
 ^{19} K_c^{(17)} &= \frac{0.878}{K_m} \left(\frac{T_L}{T_m} \right)^{0.172-0.057 \frac{\tau_m}{T_m}} \left(\frac{\tau_m}{T_m} \right)^{-0.909}, \\
 T_i^{(17)} &= \frac{T_m}{0.794} \left(\frac{T_L}{T_m} \right)^{0.228 \frac{\tau_m}{T_m}-0.257} \left(\frac{\tau_m}{T_m} \right)^{0.489}.
 \end{aligned}$$

Rule	K_c	T_i	Comment
Minimum ISTSE – Zhuang and Atherton (1993). <i>Model: Method 1</i>	$\frac{1.015}{K_m} \left(\frac{T_m}{\tau_m} \right)^{0.957}$	$\frac{T_m}{0.667} \left(\frac{\tau_m}{T_m} \right)^{0.552}$	$0.1 \leq \frac{\tau_m}{T_m} \leq 1.0$
	$\frac{1.065}{K_m} \left(\frac{T_m}{\tau_m} \right)^{0.673}$	$\frac{T_m}{0.687} \left(\frac{\tau_m}{T_m} \right)^{0.427}$	$1.1 \leq \frac{\tau_m}{T_m} \leq 2.0$
Minimum ISTSE – Zhuang and Atherton (1993).	$^{20} K_c^{(18)}$	$T_i^{(18)}$	<i>Model: Method 1;</i> $0.1 \leq \frac{\tau_m}{T_m} \leq 2.0$
Minimum ISTSE – Zhuang and Atherton (1993).	$^{21} K_c^{(19)}$	$T_i^{(19)}$	<i>Model: Method 31;</i> $0.1 \leq \frac{\tau_m}{T_m} \leq 2.0$
Minimum ISTES – Zhuang and Atherton (1993). <i>Model: Method 1</i>	$\frac{1.021}{K_m} \left(\frac{T_m}{\tau_m} \right)^{0.953}$	$\frac{T_m}{0.629} \left(\frac{\tau_m}{T_m} \right)^{0.546}$	$0.1 \leq \frac{\tau_m}{T_m} \leq 1.0$
	$\frac{1.076}{K_m} \left(\frac{T_m}{\tau_m} \right)^{0.648}$	$\frac{T_m}{0.650} \left(\frac{\tau_m}{T_m} \right)^{0.442}$	$1.1 \leq \frac{\tau_m}{T_m} \leq 2.0$
Nearly minimum IAE, ISE, ITAE – Hwang (1995).	$^{22} K_c^{(20)}$	$K_c^{(20)} / \mu_2 K_u \omega_u$	<i>Model: Method 35;</i> $0.1 \leq \frac{\tau_m}{T_m} \leq 2.0$

$$^{20} K_c^{(18)} = \left(\frac{1.892 K_m K_u + 0.244}{3.249 K_m K_u + 2.097} \right) K_u, \quad T_i^{(18)} = \left(\frac{0.706 K_m K_u - 0.227}{0.7229 K_m K_u + 1.2736} \right) T_u.$$

$$^{21} K_c^{(19)} = \left(\frac{4.126 K_m \hat{K}_u - 2.610}{5.848 K_m \hat{K}_u - 1.06} \right) \hat{K}_u, \quad T_i^{(19)} = \left(\frac{5.352 K_m \hat{K}_u - 2.926}{5.539 K_m \hat{K}_u + 5.036} \right) \hat{T}_u.$$

$$^{22} K_c^{(20)} = (1 - \mu_1) K_u, \quad \mu_1 = \frac{1.14(1 - 0.482 \omega_u \tau_m + 0.068 \omega_u^2 \tau_m^2)}{K_u K_m / 1 + K_u K_m},$$

$$\mu_2 = \frac{0.0694(-1 + 2.1 \omega_u \tau_m - 0.367 \omega_u^2 \tau_m^2)}{K_u K_m / 1 + K_u K_m}, \quad \text{Decay ratio} = 0.15.$$

Rule	K_c	T_i	Comment	
Nearly minimum IAE and ITAE – Hwang and Fang (1995). Model: Method 25	$^{23} K_c^{(21)}$	$T_i^{(21)}$	$0.1 \leq \frac{\tau_m}{T_m} \leq 2.0 ;$ decay ratio = 0.12	
Gerry (2003) – minimum error: step load change. Model: Method 1	$\frac{0.3}{K_m}$	$0.42\tau_m$	$\frac{\tau_m}{T_m} \geq 5$	
Minimum IAE or ISE – ECOSSE Team (1996a). Model: Method 1.	x_1/K_m	$x_2(\tau_m + T_m)$	Coefficient values	
<i>Coefficients of K_c, T_i deduced from graphs</i>	τ_m/T_m	x_1	x_2	
	0.11	1.1	0.23	
	0.25	1.8	0.23	
	0.43	1.1	0.72	
	0.67	1.0	0.72	
	1.0	0.8	0.70	
	1.50	0.59	0.67	
	2.33	0.42	0.60	
	4.0	0.32	0.53	
Minimum IE – Devanathan (1991). Model: Method 1.	$^{24} K_c^{(22)}$	$x_2 T_u$		
<i>Coefficients of K_c, T_i deduced from graphs</i>		Coefficient values		
		τ_m/T_m	x_2	
		0.2	0.35	
		0.4	0.29	
		0.6	0.26	
		0.8	0.23	
		1.0	0.21	

$$^{23} K_c^{(21)} = \left(0.515 - 0.0521 \frac{\tau_m}{T_m} + 0.0254 \left[\frac{\tau_m}{T_m} \right]^2 \right) K_u ,$$

$$T_i^{(21)} = \frac{(K_c^{(21)} / K_u \omega_u)}{\left(0.0877 + 0.0918 \frac{\tau_m}{T_m} - 0.0141 \left[\frac{\tau_m}{T_m} \right]^2 \right)} .$$

$$^{24} K_c^{(22)} = \frac{\xi \cos \left[\tan^{-1} \left(1.57 D_R - \frac{0.16}{D_R} \right) \right]}{K_m \cos \left[\tan^{-1} \left(\frac{6.28 T_m}{T_u} \right) \right]} .$$

Rule	K_c	T_i	Comment			
Minimum performance index: servo tuning						
Minimum IAE – Gallier and Otto (1968). <i>Model: Method 47</i>	x_1/K_m	$x_2(\tau_m + T_m)$				
Representative coefficient values – deduced from graphs						
	τ_m/T_m	x_1	x_2	τ_m/T_m	x_1	x_2
	0.053	12	0.95	0.67	1.15	0.75
	0.11	6.4	0.91	1.50	0.65	0.66
	0.25	2.9	0.84	4.0	0.45	0.55
Minimum IAE – Rovira <i>et al.</i> (1969). <i>Model: Method 5</i>	$\frac{0.758}{K_m} \left(\frac{T_m}{\tau_m} \right)^{0.861}$	$\frac{T_m}{1.020 - 0.323 \frac{\tau_m}{T_m}}$	$0.1 \leq \frac{\tau_m}{T_m} \leq 1.0$			
Minimum IAE - Bain and Martin (1983). <i>Model: Method 1</i>	$\frac{1}{K_m} \left[0.3 + 0.3 \frac{T_m}{\tau_m} \right]$	$T_m + 0.4\tau_m$	$\frac{\tau_m}{T_m} > 0.2$			
Minimum IAE – Marlin (1995) – pages 301-307. <i>Model: Method 1</i>	$\frac{1.4}{K_m}$	$0.72\tau_m$	$\frac{\tau_m}{T_m} = 0.11$			
K_c , T_i deduced from graphs; robust to $\pm 25\%$ process model parameter changes						
Minimum IAE – Huang <i>et al.</i> (1996). <i>Model: Method 1</i>	$^{25} K_c^{(23)}$	$T_i^{(23)}$	$0.1 \leq \frac{\tau_m}{T_m} \leq 1$			
Minimum IAE – Smith and Corripio (1997) – page 345. <i>Model: Method 1</i>	$\frac{0.6T_m}{K_m \tau_m}$	T_m	$0.1 \leq \frac{\tau_m}{T_m} \leq 1.5$			
Minimum IAE – Huang and Jeng (2002). <i>Model: Method 1</i>	$\frac{0.59T_m}{K_m \tau_m}$	T_m	Minimum IAE = $2.1038\tau_m$; $\tau_m/T_m < 0.2$			

$$\begin{aligned}
 ^{25} K_c^{(23)} &= \frac{1}{K_m} \left[-13.0454 - 9.0916 \frac{\tau_m}{T_m} + 0.3053 \left(\frac{\tau_m}{T_m} \right)^{-1.0169} + 1.1075 \left(\frac{\tau_m}{T_m} \right)^{3.5959} \right] \\
 &\quad + \frac{1}{K_m} \left[-2.2927 \left(\frac{\tau_m}{T_m} \right)^{3.6843} + 4.8259 e^{\frac{\tau_m}{T_m}} \right], \\
 T_i^{(23)} &= T_m \left[0.9771 - 0.2492 \frac{\tau_m}{T_m} + 3.4651 \left(\frac{\tau_m}{T_m} \right)^2 - 7.4538 \left(\frac{\tau_m}{T_m} \right)^3 + 8.2567 \left(\frac{\tau_m}{T_m} \right)^4 \right] \\
 &\quad + T_m \left[-4.7536 \left(\frac{\tau_m}{T_m} \right)^5 + 1.1496 \left(\frac{\tau_m}{T_m} \right)^6 \right].
 \end{aligned}$$

Rule	K_c	T_i	Comment
Minimum IAE – Tavakoli and Fleming (2003). Model: Method 1	$^{26} K_c^{(24)}$	$T_i^{(24)}$	$0.1 \leq \frac{\tau_m}{T_m} \leq 10$
	Minimum $A_m = 6\text{dB}$; minimum $\phi_m = 60^\circ$		
Small IAE – Hwang (1995). Model: Method 35	$^{27} K_c^{(25)}$	$K_c^{(25)} / \mu_2 K_u \omega_u$	Decay ratio = 0.1, $0.1 \leq \frac{\tau_m}{T_m} \leq 2.0$
	$\frac{0.980}{K_m} \left(\frac{T_m}{\tau_m} \right)^{0.892}$	$\frac{T_m}{0.690 - 0.155 \frac{\tau_m}{T_m}}$	$0.1 \leq \frac{\tau_m}{T_m} \leq 1.0$
Minimum ISE – Zhuang and Atherton (1993). Model: Method 1		$\frac{T_m}{0.648 - 0.114 \frac{\tau_m}{T_m}}$	$1.1 \leq \frac{\tau_m}{T_m} \leq 2.0$
Minimum ISE – Khan and Lehman (1996). Model: Method 1	$^{28} K_c^{(26)}$	$T_i^{(26)}$	$0.01 \leq \frac{\tau_m}{T_m} \leq 0.2$

$$\begin{aligned}
 ^{26} K_c^{(24)} &= \frac{1}{K_m} \left[0.4849 \frac{T_m}{\tau_m} + 0.3047 \right], \quad T_i^{(24)} = T_m \left[0.4262 \frac{\tau_m}{T_m} + 0.9581 \right]. \\
 ^{27} K_c^{(25)} &= (1 - \mu_1) K_u, \quad \mu_1 = \frac{1.07(1 - 0.466\omega_u\tau_m + 0.0667\omega_u^2\tau_m^2)}{K_u K_m / 1 + K_u K_m}, \\
 &\quad \mu_2 = \frac{0.0328(-1 + 2.21\omega_u\tau_m - 0.338\omega_u^2\tau_m^2)}{K_u K_m / 1 + K_u K_m}, \quad r = 0.5; \\
 \mu_1 &= \frac{1.11(1 - 0.467\omega_u\tau_m + 0.0657\omega_u^2\tau_m^2)}{K_u K_m / 1 + K_u K_m}, \\
 &\quad \mu_2 = \frac{0.0477(-1 + 2.07\omega_u\tau_m - 0.333\omega_u^2\tau_m^2)}{K_u K_m / 1 + K_u K_m}, \quad r = 0.75; \\
 \mu_1 &= \frac{1.14(1 - 0.466\omega_u\tau_m + 0.0647\omega_u^2\tau_m^2)}{K_u K_m / 1 + K_u K_m}, \\
 &\quad \mu_2 = \frac{0.0609(-1 + 1.97\omega_u\tau_m - 0.323\omega_u^2\tau_m^2)}{K_u K_m / 1 + K_u K_m}, \quad r = 1.0;
 \end{aligned}$$

r = parameter related to the position of the dominant real pole.

$$^{28} K_c^{(26)} = \left(\frac{0.7388}{\tau_m} + \frac{0.3185}{T_m} \right) \frac{T_m}{K_m}, \quad T_i^{(26)} = \tau_m \left[\frac{0.7388 T_m + 0.3185 \tau_m}{-0.0003082 T_m + 0.5291 \tau_m} \right].$$

Rule	K_c	T_i	Comment
Khan and Lehman (1996) – continued.	$^{29} K_c^{(27)}$	$T_i^{(27)}$	$0.2 \leq \frac{\tau_m}{T_m} \leq 20$
Minimum ITAE – Rovira <i>et al.</i> (1969). Model: Method 5	$\frac{0.586}{K_m} \left(\frac{T_m}{\tau_m} \right)^{0.916}$	$\frac{T_m}{1.030 - 0.165 \frac{\tau_m}{T_m}}$	$0.1 \leq \frac{\tau_m}{T_m} \leq 1.0$
Approximately minimum ITAE – Smith (2002).	$\frac{0.586 T_m}{K_m \tau_m}$	T_m	Model: Method 1
Minimum ISTSE – Zhuang and Atherton (1993). Model: Method 1	$\frac{0.712}{K_m} \left(\frac{T_m}{\tau_m} \right)^{0.921}$	$\frac{T_m}{0.968 - 0.247 \frac{\tau_m}{T_m}}$	$0.1 \leq \frac{\tau_m}{T_m} \leq 1.0$
	$\frac{0.786}{K_m} \left(\frac{T_m}{\tau_m} \right)^{0.559}$	$\frac{T_m}{0.883 - 0.158 \frac{\tau_m}{T_m}}$	$1.1 \leq \frac{\tau_m}{T_m} \leq 2.0$
Minimum ISTSE – Zhuang and Atherton (1993).	$0.361 K_u$	$^{30} T_i^{(28)}$	Model: Method 1; $0.1 \leq \frac{\tau_m}{T_m} \leq 2.0$
	$^{31} K_c^{(29)}$	$T_i^{(29)}$	Model: Method 31; $0.1 \leq \frac{\tau_m}{T_m} \leq 2.0$
Minimum ISTES – Zhuang and Atherton (1993). Model: Method 1	$\frac{0.569}{K_m} \left(\frac{T_m}{\tau_m} \right)^{0.951}$	$\frac{T_m}{1.023 - 0.179 \frac{\tau_m}{T_m}}$	$0.1 \leq \frac{\tau_m}{T_m} \leq 1.0$
	$\frac{0.628}{K_m} \left(\frac{T_m}{\tau_m} \right)^{0.583}$	$\frac{T_m}{1.007 - 0.167 \frac{\tau_m}{T_m}}$	$1.1 \leq \frac{\tau_m}{T_m} \leq 2.0$

$$^{29} K_c^{(27)} = \left(\frac{0.808}{\tau_m} + \frac{0.511}{T_m} - \frac{0.255}{\sqrt{T_m \tau_m}} \right) \frac{T_m}{K_m},$$

$$T_i^{(27)} = \tau_m \left[\frac{0.808 T_m + 0.511 \tau_m - 0.255 \sqrt{T_m \tau_m}}{0.095 T_m + 0.846 \tau_m - 0.381 \sqrt{T_m \tau_m}} \right].$$

$$^{30} T_i^{(28)} = 0.083(1.935 K_m K_u + 1) T_u.$$

$$^{31} K_c^{(29)} = \left(\frac{1.506 K_m \hat{K}_u - 0.177}{3.341 K_m \hat{K}_u + 0.606} \right) \hat{K}_u, \quad T_i^{(29)} = 0.055 \left(3.616 K_m \hat{K}_u + 1 \right) \hat{T}_u.$$

Rule	K_c	T_i	Comment		
Nearly minimum IAE and ITAE – Hwang and Fang (1995). Model: Method 25	$^{32} K_c^{(30)}$	$T_i^{(30)}$	$0.1 \leq \frac{\tau_m}{T_m} \leq 2.0$; decay ratio = 0.03		
Minimum performance index: other tuning					
Simultaneous servo/regulator tuning – Hwang and Fang (1995). Model: Method 25	$^{33} K_c^{(31)}$	$T_i^{(31)}$	$0.1 \leq \frac{\tau_m}{T_m} \leq 2.0$; decay ratio = 0.05		
34 Minimum performance index – Wilton (1999). Model: Method 1. <i>Coefficients of K_c estimated from graphs</i>	x_1/K_m	T_m	.		
	Coefficient values				
	τ_m/T_m	x_1	b	τ_m/T_m	x_1
	0.1	750.0	0.0001	0.2	3.3675
	0.1	17.487	0.04	0.2	2.2946
	0.1	4.8990	0.2	0.2	1.7146
	0.1	2.1213	1	0.2	1.1313
				0.2	0.8764
					5
	0.4	1.6837	0.008	0.5	150.01
	0.4	1.5297	0.04	0.5	7.1386
	0.4	1.2247	0.2	0.5	2.6944
	0.4	0.9192	1	0.5	1.1314
	0.4	0.7668	5		

$$^{32} K_c^{(30)} = \left(0.438 - 0.110 \frac{\tau_m}{T_m} + 0.0376 \left[\frac{\tau_m}{T_m} \right]^2 \right) K_u ,$$

$$T_i^{(30)} = \frac{(K_c^{(30)} / K_u \omega_u)}{\left(0.0388 + 0.108 \frac{\tau_m}{T_m} - 0.0154 \left[\frac{\tau_m}{T_m} \right]^2 \right)} .$$

$$^{33} K_c^{(31)} = \left(0.46 - 0.0835 \frac{\tau_m}{T_m} + 0.0305 \left[\frac{\tau_m}{T_m} \right]^2 \right) K_u ,$$

$$T_i^{(31)} = \frac{(K_c^{(31)} / K_u \omega_u)}{\left(0.0644 + 0.0759 \frac{\tau_m}{T_m} - 0.0111 \left[\frac{\tau_m}{T_m} \right]^2 \right)} .$$

$$^{34} \text{Performance index} = \int_0^{\infty} [e^2(t) + b^2 K_m^2 (y(t) - y_{\infty})^2] dt .$$

Rule	K_c	T_i	Comment			
Wilton (1999) - continued	Coefficient values - continued					
	τ_m/T_m	x_1	b	τ_m/T_m	x_1	
	0.6	1.1225	0.008	0.8	0.8980	
	0.6	1.1218	0.04	0.8	0.9178	
	0.6	0.9798	0.2	0.8	0.7348	
	0.6	0.7778	1	0.8	0.7071	
	0.6	0.6572	5	0.8	0.6025	
	1.0	72.004	0.0001	1.0	3.6713	
	1.0	1.6657	0.2	1.0	0.8485	
Minimum ITAE – ABB (2001). Model: Method 7	$\frac{0.8591}{K_m} \left(\frac{T_m}{\tau_m} \right)^{0.977}$	$1.4837 T_m \left(\frac{\tau_m}{T_m} \right)^{0.68}$	$\frac{\tau_m}{T_m} < 0.5$			
Direct synthesis: time domain criteria						
Bryant <i>et al.</i> (1973). Model: Method 1	$\frac{0.37 T_m}{K_m \tau_m}$		T_m	Critically damped dominant pole		
	$\frac{0.40 T_m}{K_m \tau_m}$		T_m	Damping ratio (dominant pole) = 0.6		
	0		$x_2 K_m \tau_m$			
Coefficient values – deduced from graph						
	τ_m/T_m	x_2	τ_m/T_m	x_2		
	0.33	12.5	1.0	6.7	Critically damped dominant pole	
	0.40	11.1	2.0	4.2		
	0.50	11.1	10.0	3.0		
	0.67	9.1				
	0.33	7.1	1.0	3.2	Damping ratio (dominant pole) = 0.6	
	0.40	6.3	2.0	2.1		
	0.50	5.6	10.0	1.6		
	0.67	4.3				
Chiu <i>et al.</i> (1973). Model: Method 1	$\frac{\lambda T_m}{K_m (1 + \lambda \tau_m)}$		T_m	λ variable; suggested values: 0.2, 0.4, 0.6, 1.0.		
	$\lambda = 1.0$, OS<10%, Model: Method 10 (Poulin and Pomerleau, 2004).					
Mollenkamp <i>et al.</i> (1973). Model: Method 1	$\frac{T_m}{K_m (T_{CL} + \tau_m)}$		T_m	Appropriate for “small τ_m ”		

Rule	K_c	T_i	Comment			
Keviczky and Csáki (1973); OS values in first row; τ_m/T_m values in first column; $K_m = 1$.	0	$x_2 \tau_m$	<i>Model: Method 1</i>			
x_2 coefficient values (estimated from graph)						
τ_m/T_m	OS	0%	5%			
0.1	30	22	16			
0.2	18	11	9			
0.5	8	6	5			
1.0	5	4	3.5			
2.0	4	3	2.5			
5.0	3	2.4	2			
10.0	2.5	2.1	1.8			
5% overshoot – servo – Smith <i>et al.</i> (1975) deduced from graph	$\frac{0.52T_m}{K_m \tau_m}$	T_m	$0.04 \leq \frac{\tau_m}{T_m} \leq 1.4$			
<i>Model: Method 1</i>						
Also given by Bain and Martin (1983).						
1% overshoot – servo – Smith <i>et al.</i> (1975) deduced from graph	$\frac{0.44T_m}{K_m \tau_m}$	T_m	<i>Model: Method 1</i> ; $0.04 \leq \frac{\tau_m}{T_m} \leq 1.4$			
Bain and Martin (1983). <i>Model: Method 1</i>	$\frac{0.43T_m}{K_m \tau_m}$	T_m	1% overshoot – servo $-\frac{\tau_m}{T_m} > 0.2$			
	$\frac{1}{K_m}$	T_m	“Very conservative tuning for small delay”			
Suyama (1992). <i>Model: Method 1</i>	$\frac{0.5T_m}{K_m \tau_m}$	T_m	OS = 10%			
5% overshoot – servo – Smith and Corripio (1997) – page 346. <i>Model: Method 1</i>	$\frac{0.5T_m}{K_m \tau_m}$	T_m				
	$\phi_m = 60^\circ; 0.25 \leq \frac{\tau_m}{T_m} \leq 1$ (Voda and Landau (1995))					
Vítěcková (1999), Vítěcková <i>et al.</i> (2000a) (<i>Model: Method 9</i> – Vítěcková <i>et al.</i> (2000b)).	$\frac{x_1 T_m}{K_m \tau_m}$	T_m	Closed loop response overshoot (OS) shown			
Coefficient values						
	x_1	OS (%)	x_1	OS (%)	x_1	OS (%)
	0.368	0	0.696	20	0.906	40
	0.514	5	0.748	25	0.957	45
	0.581	10	0.801	30	1.008	50
	0.641	15	0.853	35		

Rule	K_c	T_i	Comment
Mann <i>et al.</i> (2001). Model: Method 31 or Method 33; Closed loop response overshoot (step input) $< 5\%$	$\frac{0.51T_m}{K_m \tau_m}$	T_m	$0 < \frac{\tau_m}{T_m} < 2$
	$^{35} K_c^{(32)}$	$T_i^{(32)}$	
	$x_1 = 0.4, x_2 = 3.68$		$1 < \frac{\tau_m}{T_m} < 2$
	$x_1 = 0.8, x_2 = 3.28$		$2 < \frac{\tau_m}{T_m} < 4$
	$x_1 = 1.2, x_2 = 2.88$		$4 < \frac{\tau_m}{T_m}$
Vitečková and Viteček (2002) Model: Method 1	0	$^{36} T_i^{(33)}$	
	$^{37} K_c^{(34)}$	$T_i^{(34)}$	
	For $0.05 < \frac{\tau_m}{T_m} < 0.8$, $T_i = T_m$; overshoot is under 2% (Vitečková and Viteček (2003))		
Pinnella <i>et al.</i> (1986). Model: Method 20	0	$^{38} K_I^{(35)}$	Critically damped servo response

$$^{35} K_c^{(32)} = \frac{1}{K_m} \left[0.56 + 1.02 \frac{T_m}{\tau_m} - \sqrt{\frac{T_m}{\tau_m} \left[1.0404 \frac{T_m}{\tau_m} - x_1 \right] + x_2} \right],$$

$$T_i^{(32)} = \left(\frac{\tau_m}{T_m} - 1 \right) \frac{\left[0.56 + 1.02 \frac{T_m}{\tau_m} - \sqrt{\frac{T_m}{\tau_m} \left(1.0404 \frac{T_m}{\tau_m} - x_1 \right) + x_2} \right]}{\left[0.04 - 1.02 \frac{T_m}{\tau_m} + \sqrt{\frac{T_m}{\tau_m} \left(1.0404 \frac{T_m}{\tau_m} - x_1 \right) + x_2} \right]} T_m.$$

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$$T_i^{(33)} = - \frac{K_m}{\left[\sqrt{\frac{1}{\tau_m^2} + \frac{0.25}{T_m^2}} - \frac{1}{\tau_m} - \frac{0.5}{T_m} \right] \left[\sqrt{\left(\frac{T_m}{\tau_m} \right)^2 + 0.25} - \frac{T_m}{\tau_m} + 0.5 \right] e^{\sqrt{1+0.25\left(\frac{\tau_m}{T_m}\right)^2}-\frac{0.5\tau_m}{T_m}-1}}$$

$$^{37} K_c^{(34)} = - \frac{1}{K_m} \left[\tau_m T_m x_1^2 + (2T_m + \tau_m)x_1 + 1 \right] e^{\tau_m x_1},$$

$$x_1 = - \frac{2}{\tau_m} - \frac{0.5}{T_m} + \sqrt{\frac{2}{\tau_m^2} + \frac{0.25}{T_m^2}}, T_i^{(34)} = - \frac{\tau_m T_m x_1^2 + (2T_m + \tau_m)x_1 + 1}{x_1^2 (\tau_m T_m x_1 + T_m + \tau_m)}.$$

$$^{38} K_I^{(35)} = \frac{1}{4K_m \tau_m^2} \left| 2 - \frac{\tau_m}{T_m} + \left(\frac{\tau_m}{T_m} \right)^2 \right| \left| T_m \left[2 - \frac{\tau_m}{T_m} + \left(\frac{\tau_m}{T_m} \right)^2 \right] + 1 \right| e^{0.5 \left[2 - \frac{\tau_m}{T_m} + \left(\frac{\tau_m}{T_m} \right)^2 \right]}.$$

Rule	K_c	T_i	Comment
Schneider (1988), Bekker <i>et al.</i> (1991), Khan and Lehman (1996). <i>Model: Method 1</i>	$0.368 \frac{T_m}{K_m \tau_m}$	T_m	$\xi = 1$
	$0.403 \frac{T_m}{K_m \tau_m}$	T_m	$\xi = 0.6$
	$1.571 \frac{T_m}{K_m \tau_m}$	T_m	$\xi = 0.0$
Regulator – Gorecki <i>et al.</i> (1989). <i>Model: Method 1</i>	$^{39} K_c^{(36)}$	$T_i^{(36)}$	Pole is real and has maximum attainable multiplicity
Hang <i>et al.</i> (1991). <i>Model: Method 1</i>	$^{40} K_c^{(37)}$	$T_i^{(37)}$	$0.16 \leq \frac{\tau_m}{T_m} < 0.96$
	Servo response: 10% overshoot, 3% undershoot		
McAnany (1993). <i>Model: Method 24</i>	$^{41} K_c^{(38)}$	$T_i^{(38)}$	$T_{CL} = 1.67 T_m$
Sklaroff (1992), Åström and Hägglund (1995) – pages 250-251.	$\frac{3}{K_m \left(1 + \frac{3\tau_m}{T_m} \right)}$	T_m	<i>Model: Method 28; Honeywell UDC 6000 controller</i>

$$^{39} K_c^{(36)} = \frac{2}{K_m \tau_m} \frac{T_m}{\tau_m} \left[\sqrt{2 + \left(\frac{\tau_m}{2T_m} \right)^2} - 1 \right] e^{\sqrt{2 + \left(\frac{\tau_m}{2T_m} \right)^2} - 2 - \frac{\tau_m}{2T_m}},$$

$$T_i^{(36)} = \tau_m \frac{1 + \left(\frac{\tau_m}{2T_m} \right)^2}{3 + \left(\frac{\tau_m}{2T_m} \right) + \left(\frac{\tau_m}{T_m} \right)^2 + \left(\frac{\tau_m}{2T_m} \right)^3 - \left[2 + \left(\frac{\tau_m}{2T_m} \right)^2 \right] \sqrt{2 + \left(\frac{\tau_m}{2T_m} \right)^2}}.$$

$$^{40} K_c^{(37)} = \frac{5}{6} \left(\frac{12 + 2 \left[\begin{array}{l} 11 \frac{\tau_m}{T_m} + 13 \\ 37 \frac{\tau_m}{T_m} - 4 \end{array} \right]}{15 + 14 \left[\begin{array}{l} 11 \frac{\tau_m}{T_m} + 13 \\ 37 \frac{\tau_m}{T_m} - 4 \end{array} \right]} \right) K_u, \quad T_i^{(37)} = 0.2 \left(\frac{4}{15} \left[\begin{array}{l} 11 \frac{\tau_m}{T_m} + 13 \\ 37 \frac{\tau_m}{T_m} - 4 \end{array} \right] + 1 \right) T_u.$$

$$^{41} K_c^{(38)} = \frac{(1.44T_m + 0.72T_m\tau_m - 0.43\tau_m - 2.14)}{K_m(0.72\tau_m + 0.36\tau_m^2 + 2)}, \quad T_i^{(38)} = \frac{K_m(5.56 + 2\tau_m + \tau_m^2)}{4T_m + 1.28\tau_m - 2.4}.$$

Rule	K_c	T_i	Comment
Fruehauf <i>et al.</i> (1993). <i>Model: Method 2</i>	$\frac{0.5556T_m}{\tau_m K_m}$	$5\tau_m$	$\frac{\tau_m}{T_m} < 0.33$
	$\frac{0.5T_m}{\tau_m K_m}$	T_m	$\frac{\tau_m}{T_m} \geq 0.33$
Kamimura <i>et al.</i> (1994). <i>Model: Method 1</i>	$^{42} K_c^{(39)}$	$T_i^{(39)}$	Servo response: 10% overshoot
	$^{43} K_c^{(40)}$	$T_i^{(40)}$	Servo response: 0% overshoot
	$^{44} K_c^{(41)}$	$T_i^{(41)}$	“Quick” servo response: “negligible” overshoot
Zhang (1994). <i>Model: Method 1</i>	$\frac{T_m}{K_m \tau_m}$	T_m	“Good” servo response; τ_m is “small”
Davydov <i>et al.</i> (1995). <i>Model: Method 6</i>	$^{45} K_c^{(42)}$	$T_i^{(42)}$	$\xi = 0.9$; $0.2 \leq \tau_m / T_m \leq 1$.
Kuhn (1995). <i>Model: Method 14</i>	$\frac{0.5}{K_m}$	$0.5(T_m + \tau_m)$	“Normale Einstellung”
	$\frac{1}{K_m}$	$0.7(T_m + \tau_m)$	“Schnelle Einstellung”

$$^{42} K_c^{(39)} = \frac{0.5(T_m + \tau_m)}{K_m [0.5\tau_m (2T_m + \tau_m) + 0.10]}, \quad T_i^{(39)} = T_m + \tau_m - \frac{0.5\tau_m (2T_m + \tau_m) + 0.10}{T_m + \tau_m}.$$

$$^{43} K_c^{(40)} = \frac{0.375(T_m + \tau_m)}{K_m [0.5\tau_m (2T_m + \tau_m) + 0.0781]}, \quad T_i^{(40)} = T_m + \tau_m - \frac{0.5\tau_m (2T_m + \tau_m) + 0.0781}{T_m + \tau_m}.$$

$$^{44} K_c^{(41)} = \frac{0.425(T_m + \tau_m)}{K_m [0.5\tau_m (2T_m + \tau_m) + 0.083]}, \quad T_i^{(41)} = T_m + \tau_m - \frac{0.5\tau_m (2T_m + \tau_m) + 0.083}{T_m + \tau_m}.$$

$$^{45} K_c^{(42)} = \frac{1}{K_m \left(1.905 \frac{\tau_m}{T_m} + 0.826 \right)}, \quad T_i^{(42)} = \left(0.153 \frac{\tau_m}{T_m} + 0.362 \right) T_m.$$

Rule	K_c	T_i	Comment
Khan and Lehman (1996). Model: Method 1	$^{46} K_c^{(43)}$	$T_i^{(43)}$	$0.01 \leq \frac{\tau_m}{T_m} \leq 0.2$
	$^{47} K_c^{(44)}$	$T_i^{(44)}$	$0.2 \leq \frac{\tau_m}{T_m} \leq 20$
Abbas (1997). Model: Method 1	$^{48} K_c^{(45)}$	$T_m + 0.5\tau_m$	$0 \leq V \leq 0.2 ;$ $0.1 \leq \frac{\tau_m}{T_m} \leq 5.0$
Valentine and Chidambaram (1997a).	$\frac{1}{K_m} \left[0.43 - 0.97 \frac{\tau_m}{T_m} \right]$	$0.413 + 0.8 \frac{\tau_m}{T_m}$	Model: Method 1. $\xi = 0.707^{49}$
Bi et al. (1999). Model: Method 13	$\frac{0.5064 T_m}{K_m \tau_m}$	T_m	
Bi et al. (2000). Model: Method 13	$\frac{0.5064 T_m}{K_m \tau_m}$	$\frac{T_m K_m}{0.5064}$	
Ettaleb and Roche (2000). Model: Method 29	$\frac{1}{K_m}$	$T_m + \tau_m$	$T_{CL} = T_m + \tau_m$

$$^{46} K_c^{(43)} = \frac{T_m}{K_m} \left(\frac{0.3852}{\tau_m} + \frac{0.723}{T_m} - 0.404 \frac{\tau_m}{T_m^2} \right),$$

$$T_i^{(43)} = \tau_m \frac{-0.404 \tau_m^2 + 0.723 T_m \tau_m - 0.3852 T_m^2}{-0.525 \tau_m^2 + 0.4104 T_m \tau_m - 0.00024 T_m^2}.$$

$$^{47} K_c^{(44)} = \frac{T_m}{K_m} \left(\frac{0.404}{\tau_m} + \frac{0.256}{T_m} - \frac{0.1275}{\sqrt{\tau_m T_m}} \right),$$

$$T_i^{(44)} = \tau_m \frac{0.404 T_m + 0.256 \tau_m - 0.1275 \sqrt{T_m \tau_m}}{0.0808 T_m + 0.719 \tau_m - 0.324 \sqrt{T_m \tau_m}}.$$

$$^{48} K_c^{(45)} = \frac{0.148 + 0.186 \left(\frac{\tau_m}{T_m} \right)^{-1.045}}{K_m (0.497 - 0.464 V^{0.590})}.$$

$$^{49} \pm 1\% T_s = 2.5 T_m, 0.1 \leq \frac{\tau_m}{T_m} \leq 0.4; \pm 1\% T_s = 5 T_m, 0.5 \leq \frac{\tau_m}{T_m} \leq 0.9.$$

Rule	K_c	T_i	Comment
Chen and Seborg (2002).	$^{50} K_c^{(46)}$	$T_i^{(46)}$	<i>Model: Method 1</i>
Skogestad (2003). <i>Model: Method 45</i>	$\frac{0.5T_m}{K_m \tau_m}$	$\min(T_m, 8\tau_m)$	
Gorez (2003). <i>Model: Method 1</i>	$^{51} K_c^{(47)}$	T_m	$0 \leq \frac{\tau_m}{\tau_m + T_m} \leq \tau_m$
	$^{52} K_c^{(48)}$	$(1 - v)\tau_m + T_m$	$\tau_m \leq \frac{\tau_m}{\tau_m + T_m} \leq 1$

$$^{50} K_c^{(46)} = \frac{1}{K_m} \frac{T_m \tau_m + 2T_m T_{CL} - T_{CL}^2}{(T_{CL} + \tau_m)^2}, \quad T_i^{(46)} = \frac{T_m \tau_m + 2T_m T_{CL} - T_{CL}^2}{T_m + \tau_m},$$

$$0 < T_{CL} < T_m + \sqrt{T_m^2 + T_m \tau_m}, \quad \left(\frac{y}{d}\right)_{\text{desired}} = \frac{T_i^{(46)} s e^{-\tau_m s}}{K_c (T_{CL} s + 1)^2}.$$

$$^{51} K_c^{(47)} = \frac{T_m}{\chi K_m \tau_m} \quad \text{with}$$

$$\chi = \frac{1}{M_s(M_s - 1)} + \frac{1.5 M_s^2 - 2}{0.32 M_s^2 (M_s - 1)} \left(\frac{\tau_m}{\tau_m + T_m} \right)^2 \left(3 - \frac{5\tau_m}{(\tau_m + T_m) \sqrt{M_s}} \right) - \frac{0.5}{M_s(M_s - 1)} \left(1 - \frac{2.5\tau_m}{(\tau_m + T_m) \sqrt{M_s}} \right)^{2(M_s^2 - 1)}.$$

$$^{52} K_c^{(48)} = \frac{T_m}{\chi K_m \tau_m} \quad \text{with } v = 1 - 0.5 \left(\frac{\frac{\tau_m}{\tau_m + T_m} - 0.4 \sqrt{M_s}}{1 - 0.4 \sqrt{M_s}} \right)^2,$$

$$\chi = \frac{0.1 M_s}{M_s - 1} \left[7.5 - \left(\frac{\frac{\tau_m}{\tau_m + T_m} - 0.4 \sqrt{M_s}}{1 - 0.4 \sqrt{M_s}} \right)^2 \right].$$

Rule	K_c	T_i	Comment
Sree <i>et al.</i> (2004), Chidambaram <i>et al.</i> (2005). <i>Model: Method 1</i>	$\frac{0.9179}{K_m} \left(\frac{\tau_m}{T_m} \right)^{-0.8915}$	$^{53} T_i^{(49)}$	$0.1 \leq \frac{\tau_m}{T_m} \leq 1$
Haeri (2005). <i>Model: Method 1</i>	$^{54} K_c^{(50)}$	$0.96T_m + 0.46\tau_m$	$0 \leq \frac{\tau_m}{T_m} \leq 4$
Direct synthesis: frequency domain criteria			
Hougen (1979) – page 335. <i>Model: Method 1; Coefficients of K_c estimated from graphs.</i>	x_1/K_m	T_m	Maximise crossover frequency
	Coefficient values		
	τ_m/T_m	x_1	τ_m/T_m
	0.1	7.0	0.6
	0.2	3.5	0.7
	0.3	2.2	0.8
Regulator – Gorecki <i>et al.</i> (1989). <i>Model: Method 1</i>	$^{55} K_c^{(51)}$	$T_i^{(51)}$	Low frequency part of magnitude Bode diagram is flat

$$^{53} T_i^{(49)} = T_m \left[10.59 \left(\frac{\tau_m}{T_m} \right)^2 - 2.3588 \frac{\tau_m}{T_m} + 0.8985 \right], \quad 0.1 \leq \frac{\tau_m}{T_m} \leq 0.4 ;$$

$$T_i^{(49)} = T_m \left[0.7719 \left(\frac{\tau_m}{T_m} \right)^4 - 3.6608 \left(\frac{\tau_m}{T_m} \right)^3 + 6.5791 \left(\frac{\tau_m}{T_m} \right)^2 - 5.1652 \frac{\tau_m}{T_m} + 2.8059 \right],$$

$$0.4 \leq \frac{\tau_m}{T_m} \leq 1.5 .$$

$$^{54} K_c^{(50)} = \frac{1}{K_m} \left[0.407 + \frac{6.282}{1 + 14.956 \left(\frac{\tau_m}{T_m} \right)^{1.195}} \right].$$

$$^{55} K_c^{(51)} = \frac{1}{K_m} \frac{1 + 3(T_m/\tau_m) + 6(T_m/\tau_m)^2 + 6(T_m/\tau_m)^3}{4[1 + 3(T_m/\tau_m) + 3(T_m/\tau_m)^2]},$$

$$T_i^{(51)} = \tau_m \frac{1 + 3(T_m/\tau_m) + 6(T_m/\tau_m)^2 + 6(T_m/\tau_m)^3}{3[1 + 2(T_m/\tau_m) + 2(T_m/\tau_m)^2]} .$$

Rule	K_c	T_i	Comment		
Somani <i>et al.</i> (1992). Model: Method 1; Suggested A_m : $1.75 \leq A_m \leq 3.0$	$x_1/K_m A_m^{56}$	$x_2 \tau_m$			
Coefficient values					
	τ_m/T_m	x_1	x_2	τ_m/T_m	x_1
	0.0375	36.599	3.57	1.869	1.476
	0.1912	7.754	3.33	2.392	1.333
	0.3693	4.361	3.13	3.067	1.226
	0.5764	3.055	2.94	3.968	1.146
	0.8183	2.369	2.78	5.263	1.086
	1.105	1.950	2.63	7.246	1.042
	1.449	1.672	2.50	10.870	1.011
	$^{57} K_c^{(52)}$		$T_i^{(52)}$		$0.1 \leq \frac{\tau_m}{T_m} \leq 1$
Hang <i>et al.</i> (1993a), (1993b) – pages 61- 65.	$\frac{\omega_p T_m}{K_m A_m}$		$^{58} T_i^{(53)}$		Model: Method 31 or Method 33

⁵⁶ For $\frac{\tau_m}{T_m} < 1$, $K_c \approx \frac{0.9806}{K_m A_m} \sqrt{\frac{4}{\left(\sqrt{1.886 + 4 \frac{\tau_m}{T_m}} - 1.373\right)^2} + 1}$ or

$$K_c \approx \frac{0.9806}{K_m A_m} \sqrt{1 + 1.886 \left(\frac{T_m}{\tau_m}\right)^2}; \text{ for } \frac{\tau_m}{T_m} > 1, K_c \approx \frac{0.9806}{K_m A_m} \sqrt{1 + 8.669 \left(\frac{T_m}{\tau_m}\right)^2}.$$

⁵⁷ $K_c^{(52)} = \frac{1}{K_m A_m} \left[0.80 + 1.33 \frac{T_m}{\tau_m} \right]$, $T_i^{(52)} = \frac{5T_m}{\sqrt{1.04 \left[0.80 + 1.33 \frac{T_m}{\tau_m} \right]^2 - 1}}$.

⁵⁸ $T_i^{(53)} = \frac{1}{\frac{4\omega_p^2 \tau_m}{\pi} + \frac{1}{T_m}}$; $\frac{\tau_m}{T_m} \leq 0.3$, Yang and Clarke (1996).

Rule	K_c	T_i	Comment
O'Dwyer (2001a). Model: Method 1; $\frac{\tau_m}{T_m} > 0.3$, Yang and Clarke (1996).	$\frac{x_1 T_m}{K_m \tau_m}$	T_m	$A_m = \pi/2x_1$; $\phi_m = \pi/2 - x_1^{59}$
Representative results			
	x_1	A_m	ϕ_m
	1.048	1.5	30^0
	0.7854	2	45^0
			x_1
			0.524
			0.393
			A_m
			3
			60^0
			4
			67.5^0
O'Dwyer (2001a). Model: Method 1	$\frac{x_1}{\tau_m} \frac{T_m K_u T_u}{\sqrt{T_u^2 + 4\pi^2 T_m^2}}$	T_m	$A_m = \pi/2x_1$; $\phi_m = \pi/2 - x_1$
Chen et al. (1999b). Model: Method 1	$\frac{x_1 T_m}{\tau_m K_m}$	T_m	
Coefficient values			
	x_1	A_m	ϕ_m
	0.50	3.14	61.4^0
	0.61	2.58	55.0^0
	0.67	2.34	51.6^0
	0.70	2.24	50.0^0
			M_s
			1.00
			0.72
			2.18
			48.7^0
			1.30
			1.10
			0.76
			2.07
			46.5^0
			1.40
			1.20
			0.80
			1.96
			44.1^0
			1.50
Gain margin and maximum closed loop magnitude – Chen et al. (1999a).	$\frac{r_l \omega_r T_m}{K_m}$	$\frac{1}{\omega_r - \frac{4}{\pi} \tau_m \omega_r^2 + \frac{1}{T_m}}$	Model: Method 1 ⁶⁰
Symmetrical optimum principle – Voda and Landau (1995). Model: Method 1	$\frac{1}{3.5 G_p(j\omega_{135^0}) }$	$\frac{4.6}{\omega_{135^0}}$	$\frac{\tau_m}{T_m} \leq 0.1$
	$\frac{1}{2.828 G_p(j\omega_{135^0}) }$	$\frac{4}{\omega_{135^0}}$	$0.1 < \frac{\tau_m}{T_m} \leq 0.15$

⁵⁹ Given A_m , ISE is minimised when $\phi_m = 68.8884 - 34.3534A_m + 9.1606\frac{\tau_m}{T_m}$

for servo tuning (Ho et al. (1999)); $2 \leq A_m \leq 5$; $0.1 \leq \tau_m/T_m \leq 1$;

Given A_m , ISE is minimised when $\phi_m = 45.9848A_m^{0.2677}(\tau_m/T_m)^{0.2755}$ for regulator tuning (Ho et al. (1999)); $2 \leq A_m \leq 5$; $0.1 \leq \tau_m/T_m \leq 1$.

⁶⁰ $\omega_r = \frac{\pi}{4\tau_m} \frac{2r_l A_m - 1}{(r_l A_m)^2 - 1}$

Rule	K_c	T_i	Comment
Voda and Landau (1995) – continued.	$^{61} K_c^{(54)}$	$T_i^{(54)}$	$0.15 < \frac{\tau_m}{T_m} \leq 1$
Friman and Waller (1997). Model: Method 1	$\frac{0.1751}{ G_p(j\omega_{135^\circ}) }$	$\frac{1}{\omega_{135^\circ}}$	$\frac{\tau_m}{T_m} > 2 . A_m > 3$
Schaedel (1997), Henry and Schaedel (2005) - Optimal servo response.	$^{62} \frac{0.5T_i^{(55)}}{K_m(T_l - T_i^{(55)})}$	$T_i^{(55)}$	“Normal” design – Butterworth filter. Model: Method 21
	$^{63} \frac{0.375T_i^{(56)}}{K_m(T_l - T_i^{(56)})}$	$T_i^{(56)}$	“Sharp” design – Tschebyscheff filter (0.5dB). Model: Method 21
	$\frac{0.375T_i^{(57)}}{K_m(T_l - T_i^{(57)})}$	$^{63} T_i^{(57)}$	Minimum ITAE (Henry and Schaedel, 2005). Model: Method 22
Henry and Schaedel (2005). Model: Method 22; Optimal regulator response; $\frac{\tau_m^a}{T_m} < 0.2$	$^{64} K_c^{(58)}$	$T_i^{(58)}$	“Normal” design – Butterworth filter

$$^{61} K_c^{(54)} = \frac{1}{4.6|G_p(j\omega_{135^\circ})| - 0.6K_m}, T_i^{(54)} = \frac{1.15|G_p(j\omega_{135^\circ})| + 0.75K_m}{\omega_{135^\circ}[2.3|G_p(j\omega_{135^\circ})| - 0.3K_m]}.$$

$$^{62} T_i^{(55)} = \sqrt{T_1^2 - 2T_2^2},$$

$$T_i = \frac{T_m}{1 + 3.45 \frac{\tau_m^a}{T_m}} + \tau_m, T_2^2 = 1.25T_m\tau_m^a + \frac{T_m}{1 + 3.45 \frac{\tau_m^a}{T_m}}\tau_m + 0.5\tau_m^2, 0 < \frac{\tau_m^a}{T_m} \leq 0.104;$$

$$T_i = \frac{0.7T_m^2}{T_m - 0.7\tau_m^a} + \tau_m, T_2^2 = T_m\tau_m^a + \frac{0.7T_m^2}{T_m - 0.7\tau_m^a} + 0.5\tau_m^2, \frac{\tau_m^a}{T_m} > 0.104.$$

$$^{63} T_i^{(56)} = T_1 - \frac{T_2^2}{T_1}, T_i^{(57)} = -0.64T_1 + 1.64T_1 \sqrt{1 - 1.2 \left(\frac{T_2}{T_1} \right)^2}.$$

$$^{64} K_c^{(58)} = \frac{1}{K_m} \left(0.5 \left[\frac{T_i^{(58)}}{T_2} \right]^2 - 1 \right), T_i^{(58)} = 4 \frac{T_2^2}{T_1} \left(1 - 2 \frac{T_2^2}{T_1^2} \right).$$

Rule	K_c	T_i	Comment
Henry and Schaedel (2005) – continued.	$^{65} K_c^{(59)}$	$T_i^{(59)}$	“Sharp” design – Tschebyscheff filter (0.1 dB)
	$^{66} K_c^{(60)}$	$T_i^{(60)}$	Minimum ITAE
Leva (2001). Model: Method 1 Minimum $\phi_m = 50^\circ$	$^{67} K_c^{(61)}$	T_m	$\omega_c = \frac{20}{\tau_m + 5T_m}$
	$^{68} K_c^{(62)}$		$\omega_c = \frac{20}{\tau_m + 5T_m}$
Potočnik et al. (2001). Model: Method 39	$^{69} K_c^{(63)}$	$T_i^{(63)}$	$\frac{\tau_m}{T_m} \leq 0.1$
	$^{70} K_c^{(64)}$	$T_i^{(64)}$	$0.1 < \frac{\tau_m}{T_m} < 0.15$

$$^{65} K_c^{(59)} = \frac{1}{K_m} \left(0.7 \left[\frac{T_i^{(59)}}{T_2} \right]^2 - 1 \right), \quad T_i^{(59)} = 3.11 \frac{T_2^2}{T_1} \left(1 - 1.43 \frac{T_2^2}{T_1^2} \right).$$

$$^{66} K_c^{(60)} = \frac{1}{K_m} \left(0.69 \left[\frac{T_i^{(59)}}{T_2} \right]^2 - 1 \right), \quad T_i^{(60)} = 3.86 \frac{T_2^2}{T_1} \left(1 - 1.46 \frac{T_2^2}{T_1^2} \right).$$

$$^{67} K_c^{(61)} = \min \left[\frac{0.6981 T_m}{K_m \tau_m}, \frac{20 T_m}{K_m (\tau_m + 5T_m)} \right].$$

$$^{68} K_c^{(62)} = \min \left[\frac{0.6981 T_m}{K_m \tau_m}, \frac{50 T_m}{K_m (\tau_m + 5T_m)} \right].$$

$$^{69} K_c^{(63)} = \frac{0.5}{\left| G_p(j\omega_{135^\circ}) \right| \left[1.14 + 13.24 \frac{\left| G_p(j\omega_{135^\circ}) \right|}{K_m} \right]}, \\ T_i^{(63)} = \frac{4}{\omega_{135^\circ}} \left[1 + 2.48 \frac{\left| G_p(j\omega_{135^\circ}) \right|}{K_m} + 17.40 \frac{\left| G_p(j\omega_{135^\circ}) \right|^2}{K_m^2} \right].$$

$$^{70} K_c^{(64)} = \frac{K_m - 2 \left| G_p(j\omega_{135^\circ}) \right|}{2 K_m \left[K_m - 2 \left| G_p(j\omega_{135^\circ}) \right| \right]}, \\ T_i^{(64)} = \frac{\left[K_m + \sqrt{2} \left| G_p(j\omega_{135^\circ}) \right| \right] \left[2 \left| G_p(j\omega_{135^\circ}) \right| - K_m \right]}{\omega_{135^\circ} \left| G_p(j\omega_{135^\circ}) \right| \left[\left| G_p(j\omega_{135^\circ}) \right| - K_m \right]}.$$

Rule	K_c	T_i	Comment
Potočnik <i>et al.</i> (2001) -- continued.	$^{71} K_c^{(65)}$	$T_i^{(65)}$	$0.1 < \frac{\tau_m}{T_m} < 1$
Cluett and Wang (1997), Wang and Cluett (2000) – page 177. <i>Model: Method 1;</i> $0.01 < \frac{\tau_m}{T_m} \leq 10$	$^{72} K_c^{(66)}$	$T_i^{(66)}$	$T_{CL} = 4\tau_m$. $A_m \in [7.50, 8.28]$, $\phi_m \in [78.5^0, 79.7^0]$
	$^{73} K_c^{(67)}$	$T_i^{(67)}$	$T_{CL} = 2\tau_m$. $A_m \in [4.35, 5.02]$, $\phi_m \in [70.7^0, 74.0^0]$
	$^{74} K_c^{(68)}$	$T_i^{(68)}$	$T_{CL} = 1.33\tau_m$. $A_m \in [3.29, 3.89]$, $\phi_m \in [64.4^0, 70.6^0]$
	$^{75} K_c^{(69)}$	$T_i^{(69)}$	$T_{CL} = \tau_m$. $A_m \in [2.74, 3.30]$, $\phi_m \in [57.8^0, 68.5^0]$
	$^{76} K_c^{(70)}$	$T_i^{(70)}$	$T_{CL} = 0.8\tau_m$. $A_m \in [2.39, 2.93]$, $\phi_m \in [49.6^0, 67.1^0]$

$$^{71} K_c^{(65)} = \frac{2K_m}{\omega_{135^0}} \left[0.75 + 1.15 \frac{|G_p(j\omega_{135^0})|}{K_m} \right], \quad T_i^{(65)} = \frac{1}{\omega_{135^0}} \left[0.75 + 1.15 \frac{|G_p(j\omega_{135^0})|}{K_m} \right].$$

$$^{72} K_c^{(66)} = \frac{0.019952\tau_m + 0.20042T_m}{K_m \tau_m}, \quad T_i^{(66)} = \frac{0.099508\tau_m + 0.99956T_m}{0.99747\tau_m - 8.7425 \cdot 10^{-5} T_m} \tau_m.$$

$$^{73} K_c^{(67)} = \frac{0.055548\tau_m + 0.33639T_m}{K_m \tau_m}, \quad T_i^{(67)} = \frac{0.16440\tau_m + 0.99558T_m}{0.98607\tau_m - 1.5032 \cdot 10^{-4} T_m} \tau_m.$$

$$^{74} K_c^{(68)} = \frac{0.092654\tau_m + 0.43620T_m}{K_m \tau_m}, \quad T_i^{(68)} = \frac{0.20926\tau_m + 0.98518T_m}{0.96515\tau_m + 4.2550 \cdot 10^{-3} T_m} \tau_m.$$

$$^{75} K_c^{(69)} = \frac{0.12786\tau_m + 0.51235T_m}{K_m \tau_m}, \quad T_i^{(69)} = \frac{0.24145\tau_m + 0.96751T_m}{0.93566\tau_m + 2.2988 \cdot 10^{-2} T_m} \tau_m.$$

$$^{76} K_c^{(70)} = \frac{0.16051\tau_m + 0.57109T_m}{K_m \tau_m}, \quad T_i^{(70)} = \frac{0.26502\tau_m + 0.94291T_m}{0.89868\tau_m + 6.9355 \cdot 10^{-2} T_m} \tau_m.$$

Rule	K_c	T_i	Comment
Cluett and Wang (1997), Wang and Cluett (2000) – continued.	$^{77} K_c^{(71)}$	$T_i^{(71)}$	$T_{CL} = 0.67\tau_m$. $A_m \in [2.11, 2.68]$, $\phi_m \in [38.1^\circ, 66.3^\circ]$
Modulus optimum principle – Cox <i>et al.</i> (1997). Model: Method 16	$^{78} K_c^{(72)}$	$T_i^{(72)}$	$\frac{\tau_m}{T_m} \leq 1$
	$\frac{0.5T_m}{K_m \tau_m}$	T_m	$\frac{\tau_m}{T_m} > 1$
Smith (1998). Model: Method 1	$\frac{0.35}{K_m}$	$0.42\tau_m$	6 dB gain margin – dominant delay process
Wang <i>et al.</i> (2000a). Model: Method 31 or 32	$^{79} K_c^{(73)}$	T_m	$1.3 \leq M_s \leq 2$

$$^{77} K_c^{(71)} = \frac{0.19067\tau_m + 0.61593T_m}{K_m \tau_m}, T_i^{(71)} = \frac{0.28242\tau_m + 0.91231T_m}{0.85491\tau_m + 0.15937T_m} \tau_m.$$

$$^{78} K_c^{(72)} = \frac{0.5}{K_m} \left(\frac{T_m^3 + T_m^2\tau_m + 0.5T_m\tau_m^2 + 0.167\tau_m^3}{T_m^2\tau_m + T_m\tau_m^2 + 0.667\tau_m^3} \right),$$

$$T_i^{(72)} = \left(\frac{T_m^3 + T_m^2\tau_m + 0.5T_m\tau_m^2 + 0.167\tau_m^3}{T_m^2 + T_m\tau_m + 0.5\tau_m^2} \right).$$

$$^{79} K_c^{(73)} = \frac{1}{\tau_m} \left[1.451 - \frac{1.508}{M_s} \right] \frac{T_m}{K_m}.$$

Rule	K_c	T_i	Comment
Wang and Shao (2000a). Model: Method 1	$^{80} K_c^{(74)}$	$T_i^{(74)}$	$\lambda \in [1.5, 2.5]$
Chen and Yang (2000). Model: Method 8	$\frac{0.7T_m}{K_m \tau_m}$	T_m	$M_s = 1.26$; $A_m = 2.24$; $\phi_m = 50^\circ$
Hägglund and Åström (2002). Model: Method 5; $M_s = 1.4$	$\frac{0.25T_m}{K_m \tau_m}$	$0.8T_m$	$0.2 \leq \frac{\tau_m}{T_m} < 1$
	$\frac{0.1T_m}{K_m \tau_m} + \frac{0.15}{K_m}$	$0.3\tau_m + 0.5T_m$	$\frac{\tau_m}{T_m} > 1$
	$\frac{0.15}{K_m}$	$0.3\tau_m$	$\frac{\tau_m}{T_m} \rightarrow \infty$
Hägglund and Åström (2002). Model: Method 1	$^{81} K_c^{(75)}$	$T_i^{(75)}$	$M_s = 1.4$; $\frac{\tau_m}{T_m} \geq 0.5$

$$\begin{aligned}
 ^{80} K_c^{(74)} &= \frac{1}{\lambda f_1(\omega_{90^\circ})} \left[f_2(\omega_{90^\circ}) - \frac{1}{\omega_{90^\circ}} \right], \\
 f_1(\omega_{90^\circ}) &= \frac{K_m}{\left(1 + \omega_{90^\circ}^2 T_m^2\right)^{1.5}} \left[(T_m + \{1 + \omega_{90^\circ}^2 T_m^2\} \tau_m) \sin(-\omega_{90^\circ} \tau_m - \tan^{-1} \omega_{90^\circ} T_m) \right] \\
 &\quad - \frac{K_m}{\left(1 + \omega_{90^\circ}^2 T_m^2\right)^{1.5}} \left[\omega_{90^\circ} T_m^2 \cos(-\omega_{90^\circ} \tau_m - \tan^{-1} \omega_{90^\circ} T_m) \right], \\
 f_2(\omega_{90^\circ}) &= -\frac{1}{1 + \omega_{90^\circ}^2 T_m^2} \left[(T_m + \{1 + \omega_{90^\circ}^2 T_m^2\} \tau_m) \cot(-\omega_{90^\circ} \tau_m - \tan^{-1} \omega_{90^\circ} T_m) \right] \\
 &\quad - \frac{\omega_{90^\circ} T_m^2}{1 + \omega_{90^\circ}^2 T_m^2}, \\
 T_i^{(74)} &= \frac{\omega_{90^\circ} [T_m + (1 + \omega_{90^\circ}^2 T_m^2) \tau_m] \cos \theta + (1 + 2\omega_{90^\circ}^2 T_m^2) \sin \theta}{-\omega_{90^\circ}^3 T_m^2 \cos \theta + \omega_{90^\circ}^2 [T_m + (1 + \omega_{90^\circ}^2 T_m^2) \tau_m] \sin \theta}, \\
 \theta &= -\omega_{90^\circ} \tau_m - \tan^{-1} \omega_{90^\circ} T_m. \\
 ^{81} K_c^{(75)} &= \frac{1}{K_m} \left(0.14 + 0.28 \frac{T_m}{\tau_m} \right), \quad T_i^{(75)} = 0.33\tau_m + \frac{6.8\tau_m T_m}{10\tau_m + T_m}.
 \end{aligned}$$

Rule	K_c	T_i	Comment
Leva <i>et al.</i> (2003). <i>Model: Method 1</i>	$^{82} K_c^{(76)}$	T_m	minimum ϕ_m ; maximum ω_c
	$^{83} K_c^{(77)}$	T_m	minimum $\phi_m = 50^0$
Mataušek and Kvaščev (2003). <i>Model: Method 1</i>	$\frac{x_1 T_m}{K_m \tau_m}$	T_m	$x_1 = 0.5\pi - \phi_m$ ⁸⁴
Kristiansson (2003). <i>Model: Method 11</i>	$^{85} K_c^{(78)}$	$\frac{1.25}{T_m}$	$1.6 \leq M_s \leq 1.9$
Leva <i>et al.</i> (1994). <i>Model: Method 43</i>	$^{86} K_c^{(79)}$	$T_i^{(79)}$	

$$^{82} K_c^{(76)} = \min \left[\frac{T_m (0.5\pi - \phi_m)}{K_m \tau_m}, \frac{5T_m}{K_m T_{CL}} \right].$$

$$^{83} K_c^{(77)} = \min \left[\frac{0.698T_m}{K_m \tau_m}, \frac{5aT_m}{K_m (\tau_m + 5T_m)} \right], \quad T_{CL} = \frac{\tau_m + 5T_m}{a}, \quad a \in [4, 10].$$

⁸⁴ ‘Acceptable’ values of x_1 : $0.5 \leq x_1 \leq 3$. ‘Recommended’ values of x_1 : $0.32 \leq x_1 \leq 0.54$ ($59^0 \leq \phi_m \leq 72^0$). Choose $x_1 = 0.32$ when large variations in process parameters are expected; choose $x_1 = 0.54$ to give $6\% \leq OS < 13\%$ and $1.4 < M_s \leq 2$.

$$^{85} K_c^{(78)} = \frac{\omega_{150^0}}{K_m} \left(0.2 + \frac{0.075}{\frac{K_{150^0}}{K_m} + 0.05} \right).$$

$$^{86} K_c^{(79)} = \frac{\omega_{cn} T_i^{(79)}}{K_m} \sqrt{\frac{1 + \omega_{cn}^2 T_m^2}{1 + \omega_{cn}^2 [T_i^{(79)}]^2}}, \quad T_i^{(79)} = \frac{\tan \left[\phi_m - \frac{\pi}{2} + \tau_m \omega_{cn} + \tan^{-1}(\omega_{cn} T_m) \right]}{\omega_{cn}}$$

$$\text{with } \omega_{cn} = \frac{2.82 - \phi_m - \tan^{-1} \left[\frac{T_m}{\tau_m} \left(\frac{\pi}{2} - \phi_m \right) \right]}{\tau_m}.$$

Rule	K_c	T_i	Comment
Tan <i>et al.</i> (1996). <i>Model: Method 30</i>	$^{87} K_c^{(80)}$	$T_i^{(80)}$	
Xu <i>et al.</i> (2004). <i>Model: Method 31</i>	$\frac{T_m}{\lambda K_m \tau_m}$	T_m	$\lambda \in [1.5, 2.5]$
Huang <i>et al.</i> (2005). <i>Model: Method 42</i>	$\frac{0.50T_m + 0.22\tau_m}{K_m \tau_m}$	$0.9T_m + 0.4\tau_m$	
Robust			
Rivera <i>et al.</i> (1986). <i>Model: Method 1</i>	$\frac{T_m}{\lambda K_m}$	T_m	$\lambda \geq 1.7\tau_m, \lambda > 0.1T_m$
	$\lambda \geq 1.7\tau_m, \lambda > 0.2T_m$ (Luyben (2001)); $\lambda = 2\tau_m$ (minimum ISE - de Oliveira <i>et al.</i> , (1995))		
	$\frac{2T_m + \tau_m}{2\lambda K_m}$	$T_m + 0.5\tau_m$	$\lambda \geq 1.7\tau_m, \lambda > 0.1T_m$
Chien (1988). <i>Model: Method 1</i>	$\frac{T_m}{K_m(\tau_m + \lambda)}$	T_m	$\lambda = T_m$ (Chien (1988)) ⁸⁸
Brambilla <i>et al.</i> (1990). <i>Model: Method 1</i>	$\frac{T_m + 0.5\tau_m}{K_m(\lambda + \tau_m)}$	$T_m + 0.5\tau_m$	
	No model uncertainty: $\lambda = \tau_m, 0.1 \leq \tau_m/T_m \leq 1;$ $\lambda = [1 - 0.5 \log_{10}(\tau_m/T_m)]\tau_m, 1 < \tau_m/T_m \leq 10.$		

$$^{87} K_c^{(80)} = \frac{\beta T_i^{(80)} \omega_\phi \sqrt{1 + (\beta T_m \omega_\phi)^2}}{A_m \sqrt{1 + (\beta T_i^{(80)} \omega_\phi)^2}}, T_i^{(80)} = \frac{1}{\beta \omega_\phi \tan[-\tan^{-1} \beta T_m \omega_\phi - \beta \tau_m \omega_\phi - \phi]},$$

$$\omega_\phi < \omega_u. \quad \beta = 0.8, \frac{\tau_m}{T_m} < 0.5; \beta = 0.5, \frac{\tau_m}{T_m} > 0.5.$$

$\lambda = 2\tau_m$ (Bialkowski (1996)). $\lambda > T_m + \tau_m$ (Thomasson (1997)). $\lambda \in [0.45\tau_m, 0.8\tau_m]$ (Huang *et al.* (1998)). $\lambda = 2\tau_m$ (aggressive, less robust tuning) (Gerry, (1999)); $\lambda = 2(T_m + \tau_m)$ (more robust tuning) (Gerry, (1999)); $\lambda = a \cdot \max(T_m, \tau_m)$: $a > 3 \dots$ slow design, $2 \leq a \leq 3 \dots$ normal design, $1 \leq a < 2 \dots$ fast design, $a < 1 \dots$ faster design (Andersson (2000) – page 11). *Model: Method 10*.

$$\lambda \in [0.1T_m, 0.5T_m], \frac{\tau_m}{T_m} \leq 0.25; \lambda = 1.5(\tau_m + T_m), 0.25 < \frac{\tau_m}{T_m} \leq 0.75;$$

$$\lambda = 3(\tau_m + T_m), \frac{\tau_m}{T_m} > 0.75 \text{ (Leva (2001))}. \quad \lambda \in [T_m, \tau_m] \text{ (Smith (2002))}.$$

Rule	K_c	T_i	Comment			
Pulkkinen <i>et al.</i> (1993).	$\frac{0.7T_m}{K_m \tau_m}$	$\lambda(\tau_m + T_m)$	<i>Model: Method 1</i>			
Ogawa (1995). <i>Model: Method 1;</i> <i>Coefficients of K_c, T_i deduced from graphs</i>	x_1/K_m	$x_2 T_m$				
Coefficient values						
$\frac{\tau_m}{T_m}$	x_1	x_2	$\frac{\tau_m}{T_m}$	x_1	x_2	
0.5	0.9	1.3	2.0	0.45	2.0	20% uncertainty in process parameters
1.0	0.6	1.6	10.0	0.4	7.0	
0.5	0.7	1.3	2.0	0.4	2.2	33% uncertainty in process parameters
1.0	0.47	1.7	10.0	0.35	7.5	
0.5	0.47	1.3	2.0	0.32	2.4	50% uncertainty in process parameters
1.0	0.36	1.8	10.0	0.3	8.5	
0.5	0.4	1.3	2.0	0.3	2.4	60% uncertainty in process parameters
1.0	0.33	1.8	10.0	0.29	9.0	
Thomasson (1997). <i>Model: Method 1</i>	$\frac{\tau_m}{2K_m(\tau_m + \lambda)}$	$0.5\tau_m$	$\tau_m \gg T_m$; $\lambda = T_{CL}$			
Chen <i>et al.</i> (1997). <i>Model: Method 26</i>	$\frac{0.5T_m}{K_m \max(\tau_m, 0.5)}$	minimum($T_m, 6\tau_m$) 3	$\tau_m > 0.5$ $\tau_m < 0.5$			
Lee <i>et al.</i> (1998). <i>Model: Method 1</i>	$\frac{T_m + \frac{\tau_m^2}{2(\lambda + \tau_m)}}{K_m(\lambda + \tau_m)}$	$T_m + \frac{\tau_m^2}{2(\lambda + \tau_m)}$	Desired closed loop response = $\frac{e^{-\tau_m s}}{(\lambda s + 1)}$			
Isaksson and Graebe (1999). <i>Model: Method 1</i>	$\frac{T_m + 0.25\tau_m}{K_m \lambda}$	$T_m + 0.25\tau_m$	$T_m > \tau_m$; λ not specified			
Chun <i>et al.</i> (1999). <i>Model: Method 34</i>	$\frac{T_m(\tau_m + 2\lambda) - \lambda^2}{K_m(\tau_m + \lambda)^2}$	$\frac{T_m(\tau_m + 2\lambda) - \lambda^2}{\tau_m + T_m}$	Representative λ : $\lambda = 0.4T_m$			
Zhong and Li (2002). <i>Model: Method 1</i>	$\frac{T_m}{K_m(\alpha + \tau_m)}$	T_m	$\alpha \in [\Delta\tau_m, 1.4\Delta\tau_m]$; $\Delta\tau_m$ = time delay uncertainty.			
Smith (2002). <i>Model: Method 1</i>	$\frac{1}{K_m} \frac{T_m}{\tau_m}$	T_m				

Rule	K_c	T_i	Comment
Ultimate cycle			
McMillan (1984). Model: Method 2 or Method 44	$^{89} K_c^{(81)}$	$T_i^{(81)}$	Tuning rules developed from K_u, T_u
Boe and Chang (1988). Model: Method 1	$0.54K_u$	$1.935T_m \left(\frac{\tau_m}{T_m} \right)^{0.654}$	Quarter decay ratio
Geng and Geary (1993). Model: Method 1	$\frac{0.71T_m}{K_m \tau_m}$	$3.33\tau_m$	$\frac{\tau_m}{T_m} < 0.167$
Perić <i>et al.</i> (1997). Model: Method 40	$\frac{\hat{K}_u \tan^2 \phi_m}{\sqrt{1 + \tan^2 \phi_m}}$	$\frac{0.1592 \hat{T}_u}{\tan \phi_m}$	
Matsuba <i>et al.</i> (1998). Model: Method 1	$\frac{0.99}{K_m} \left(\frac{T_m}{\tau_m} \right)^{0.9}$	$2.63\tau_m \left(\frac{\tau_m}{T_m} \right)^{0.097}$	$0.1 \leq \frac{\tau_m}{T_m} \leq 1.0$
Huang <i>et al.</i> (2005). Model: Method 42	$^{90} K_c^{(82)}$	$T_i^{(82)}$	

$$^{89} K_c^{(81)} = \frac{1.881}{K_m} \frac{T_m}{\tau_m} \left\{ \frac{1}{1 + \left(\frac{T_m}{T_m + \tau_m} \right)^{0.65}} \right\}, \quad T_i^{(81)} = 1.67 \tau_m \left\{ 1 + \left(\frac{T_m}{T_m + \tau_m} \right)^{0.65} \right\}.$$

$$^{90} K_c^{(82)} = \frac{0.55}{K_m} \left[\frac{0.9 \sqrt{\left(\hat{K}_u \right)^2 - 1}}{\pi - \tan^{-1} \left[\sqrt{\left(\hat{K}_u \right)^2 - 1} \right]} + 0.4 \right],$$

$$T_i^{(82)} = 0.159 \hat{T}_u \left[0.9 \sqrt{\left(\hat{K}_u \right)^2 - 1} + 0.4 \left\{ \pi - \tan^{-1} \left[\sqrt{\left(\hat{K}_u \right)^2 - 1} \right] \right\} \right].$$

3.1.2 Ideal controller in series with a first order lag

$$G_c(s) = K_c \left(1 + \frac{1}{T_i s} \right) \frac{1}{T_f s + 1}$$

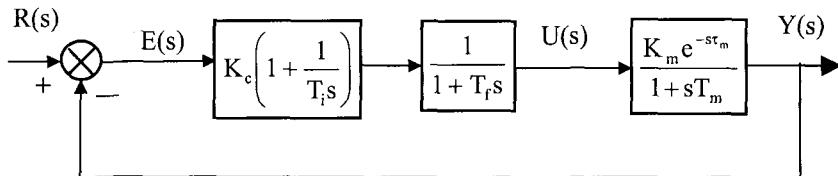


Table 4: PI controller tuning rules – FOLPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{1 + sT_m}$

Rule	K_c	T_i	Comment
Direct synthesis: time domain criteria			
Lim <i>et al.</i> (1985). <i>Model: Method 1</i>	$\frac{T_m}{K_m (2\xi T_{CL2} + \tau_m)}$	T_m	$T_f = \frac{T_{CL2}^2}{2\xi T_{CL2} + \tau_m}$ ¹
Robust			
Rivera and Jun (2000). <i>Model: Method 1</i>	$\frac{T_m}{K_m (2\tau_m + \lambda)}$	T_m	$T_f = \frac{\tau_m \lambda}{2\tau_m + \lambda};$ λ not specified

¹ Desired closed loop transfer function = $\frac{e^{-s\tau_m}}{T_{CL2}^2 s^2 + 2\xi T_{CL2} s + 1}$.

3.1.3 Ideal controller in series with a second order filter

$$G_c(s) = K_c \left(1 + \frac{1}{T_i s} \right) \left(\frac{1 + b_{f1}s + b_{f2}s^2}{1 + a_{f1}s + a_{f2}s^2} \right)$$

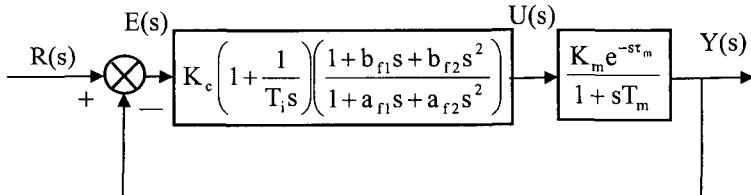


Table 5: PI controller tuning rules – FOLPD model $G_m(s) = \frac{K_m e^{-sT_m}}{1 + sT_m}$

Rule	K_c	T_i	Comment			
Direct synthesis: time domain criteria						
Tsang <i>et al.</i> (1993). <i>Model: Method 15</i>	$\frac{x_1 T_m}{K_m \tau_m}$	T_m	²			
Coefficient values						
	x_1	ξ	x_1	ξ	x_1	ξ
	1.851	0.0	1.028	0.4	0.695	0.8
	1.552	0.1	0.925	0.5	0.622	0.9
	1.329	0.2	0.841	0.6	0.553	1.0
	1.160	0.3	0.768	0.7		
Robust						
Lee and Shi (2002). <i>Model: Method 1</i>	³ $K_c^{(83)}$	$\frac{T_m}{\gamma}$	$1 < \gamma < \frac{2T_m}{\tau_m}$			

² $b_{f1} = 0.5\tau_m$, $b_{f2} = 0.0833\tau_m^2$, $a_{f1} = 0.2\tau_m$, $a_{f2} = 0.01\tau_m^2$

³ $K_c^{(83)} = \frac{\omega_{bw} T_m}{K_m (\omega_{bw} \gamma \tau_m + 1) \left[1 + \frac{2\omega_{bw}}{\omega_g} \tan \left(\frac{\omega_g \tau_m}{2} \right) \right]}$; $b_{f1} = T_m + 0.5\tau_m$; $b_{f2} = 0.5\tau_m T_m$;

$a_{f1} = \frac{\tau_m + 2\omega_{bw} T_m \tau_m + 2T_m}{2(\omega_{bw} \gamma \tau_m + 1)}$; $a_{f2} = \frac{T_m \tau_m}{2(\omega_{bw} \gamma \tau_m + 1)}$; suggested $\omega_g = \frac{0.6}{\tau_m}$.

3.1.4 Controller with set-point weighting

$$U(s) = K_c \left(1 + \frac{1}{T_i s} \right) E(s) - \alpha K_c R(s)$$

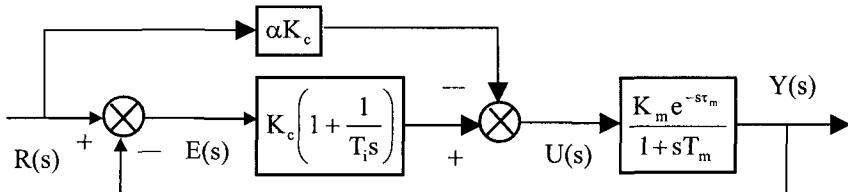


Table 6: PI controller tuning rules – FOLPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{1 + sT_m}$

Rule	K_c	T_i	Comment
Minimum performance index: other tuning			
Servo/regulator optimisation - Taguchi and Araki (2000). <i>Model: Method 1</i>	⁴ $K_c^{(84)}$	$T_i^{(84)}$	$\tau_m/T_m \leq 1.0$. Overshoot (servo step) $\leq 20\%$
Direct synthesis: time domain criteria			
Chidambaram (2000a). <i>Model: Method 1</i>	$\frac{T_m T_i^{(85)}}{T_{CL}^2 + T_i^{(85)} \tau_m}$	$T_i^{(85)}$	Choose T_{CL}, ξ

$$\begin{aligned} {}^4 K_c^{(84)} &= \frac{1}{K_m} \left(0.1098 + \frac{0.7382}{\frac{\tau_m}{T_m} - 0.002434} \right), \\ T_i^{(84)} &= T_m \left(0.06216 + 3.171 \frac{\tau_m}{T_m} - 3.058 \left[\frac{\tau_m}{T_m} \right]^2 + 1.205 \left[\frac{\tau_m}{T_m} \right]^3 \right). \end{aligned}$$

$${}^5 T_i^{(85)} = \frac{T_m \tau_m + 2T_{CL} \xi - T_{CL}^2}{T_m + \tau_m}, \quad \alpha = \frac{T_i^{(85)} - \xi T_{CL}}{T_i^{(85)}} \text{ or } \alpha = \frac{K_c K_m [T_i^{(85)} + \tau_m] - T_i^{(85)}}{2K_c K_m T_i^{(85)}}.$$

Rule	K_c	T_i	Comment
Srinivas and Chidambaram (2001).	$^6 \frac{0.9T_m}{K_m \tau_m}$	$3.33\tau_m$	<i>Model: Method 1</i>
Direct synthesis: frequency domain criteria			
$\ddot{\text{A}}\ddot{\text{s}}\ddot{\text{t}}\ddot{\text{r}}\ddot{\text{o}}\ddot{\text{m}}$ and H��gglund (1995) - dominant pole design – page 204-208. <i>Model: Method 5 or 14</i>	$\frac{0.29e^{-2.7\tau+3.7\tau^2}T_m}{K_m \tau_m}$	$8.9\tau_m e^{-6.6\tau+3.0\tau^2}$ or $0.79T_m e^{-1.4\tau+2.4\tau^2}$	$0.14 \leq \frac{\tau_m}{T_m} \leq 5.5 ;$ $M_s = 1.4$
		$\alpha = 1 - 0.81e^{0.73\tau+1.9\tau^2}$	
	$\frac{0.78e^{-4.1\tau+5.7\tau^2}T_m}{K_m \tau_m}$	$8.9\tau_m e^{-6.6\tau+3.0\tau^2}$ or $0.79T_m e^{-1.4\tau+2.4\tau^2}$	$0.14 \leq \frac{\tau_m}{T_m} \leq 5.5 ;$ $M_s = 2.0$
$\ddot{\text{A}}\ddot{\text{s}}\ddot{\text{t}}\ddot{\text{r}}\ddot{\text{o}}\ddot{\text{m}}$ and H��gglund (1995) - modified Ziegler-Nichols – page 208.		$\alpha = 1 - 0.44e^{0.78\tau-0.45\tau^2}$	
	$\frac{0.4T_m}{K_m \tau_m}$	$0.7T_m$	$0.1 \leq \frac{\tau_m}{T_m} \leq 2$
	$\alpha = 0.5$; <i>Model: Method 5 or 14</i>		
$\ddot{\text{H}}\ddot{\text{��gglund}}$ and $\ddot{\text{A}}\ddot{\text{s}}\ddot{\text{t}}\ddot{\text{o}}\ddot{\text{m}}$ (2002). <i>Model: Method 5;</i> $M_s = 1.4$; $0 < \alpha < 0.5$	$\frac{0.35T_m}{K_m \tau_m} - \frac{0.6}{K_m}$	$7\tau_m$	$\frac{\tau_m}{T_m} < 0.11$
	$\frac{0.35T_m}{K_m \tau_m} - \frac{0.6}{K_m}$	$0.8T_m$	$0.11 < \frac{\tau_m}{T_m} < 0.17$
	$\frac{0.25T_m}{K_m \tau_m}$	$0.8T_m$	$0.17 < \frac{\tau_m}{T_m} < 0.2$
$\ddot{\text{H}}\ddot{\text{��gglund}}$ and $\ddot{\text{A}}\ddot{\text{s}}\ddot{\text{t}}\ddot{\text{o}}\ddot{\text{m}}$ (2002). <i>Model: Method 1</i>	$^7 K_c^{(86)}$	$T_i^{(86)}$	$\frac{\tau_m}{T_m} < 0.5$
	$M_s = 1.4; 0 < \alpha < 0.5$		

⁶ Sample K_c , T_i taken by the authors (corresponding to the process reaction curve tuning rules of Zeigler and Nichols (1942)); $\alpha = \frac{\tau_m}{T_i} - \frac{1}{K_c K_m} = 0.30 - 1.11 \frac{\tau_m}{T_m}$ for these tuning rules.

⁷ $K_c^{(86)} = \frac{1}{K_m} \left(0.14 + 0.28 \frac{T_m}{\tau_m} \right)$, $T_i^{(86)} = 0.33\tau_m + \frac{6.8\tau_m T_m}{10\tau_m + T_m}$.

3.1.5 Controller with proportional term acting on the output 1

$$U(s) = \frac{K_c}{T_i s} E(s) - K_c Y(s)$$

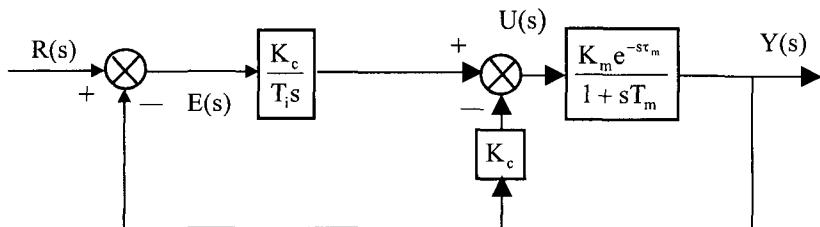


Table 7: PI controller tuning rules – FOLPD model $G_m(s) = \frac{K_m e^{-st_m}}{1 + sT_m}$

Rule	K_c	T_i	Comment
Minimum performance index: other tuning			
Minimum ITAE - Setiawan <i>et al.</i> (2000). Model: Method 1	$\frac{x_1 T_m}{K_m \tau_m}$	$x_2 \tau_m$	
Coefficient values			
	$\frac{\tau_m}{T_m}$	x_1	x_2
		$\frac{\tau_m}{T_m}$	x_1
	0.5	0.554	1.805
	0.75	0.570	1.373
	1.0	0.594	1.117
	1.5	0.645	0.822
	2.0	0.720	0.679
Direct synthesis: time domain criteria			
Chien <i>et al.</i> (1999). Model: Method 1	${}^8 K_c^{(87)}$	$T_i^{(87)}$	Underdamped system response - $\xi = 0.707$. $\tau_m > 0.2T_m$

$${}^8 K_c^{(87)} = \frac{-T_{CL2}^2 + 1.414T_{CL2}T_m + \tau_m T_m}{K_m (T_{CL2}^2 + 1.414T_{CL2}\tau_m + \tau_m^2)}, \quad T_i^{(87)} = \frac{-T_{CL2}^2 + 1.414T_{CL2}T_m + T_m \tau_m}{T_m + \tau_m}.$$

3.1.6 Controller with proportional term acting on the output 2

$$U(s) = K_c \left(1 + \frac{1}{T_i s} \right) E(s) - K_1 Y(s)$$

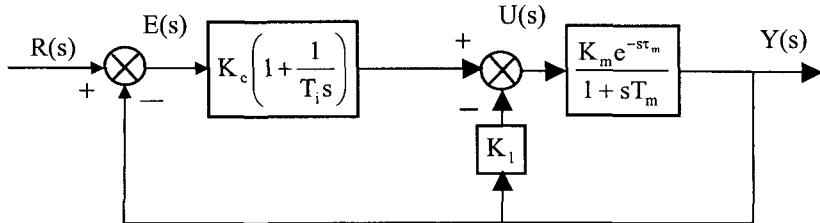


Table 8: PI controller tuning rules -- FOLPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{1 + sT_m}$

Rule	K_c	T_i	Comment
Robust			
Lee and Edgar (2002). Model: Method 1	${}^9 K_c^{(88)}$	$T_i^{(88)}$	$K_1 = 0.25K_u$

$${}^9 K_c^{(88)} = \frac{1 + 0.25K_u K_m}{K_m(\lambda + \tau_m)} \left[\frac{T_m - 0.25K_u K_m \tau_m}{1 + 0.25K_u K_m} + \frac{\tau_m^2}{2(\lambda + \tau_m)} \right],$$

$$T_i^{(88)} = \frac{T_m - 0.25K_u K_m \tau_m}{1 + 0.25K_u K_m} + \frac{\tau_m^2}{2(\lambda + \tau_m)}.$$

3.2 FOLPD Model with a Positive Zero $G_m(s) = \frac{K_m(1-sT_{m3})e^{-sT_m}}{1+sT_{m1}}$

3.2.1 Ideal controller $G_c(s) = K_c \left(1 + \frac{1}{T_i s} \right)$

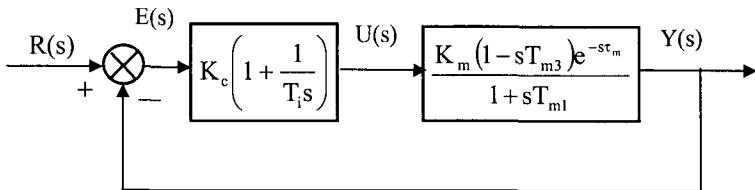


Table 9: PI controller tuning rules - FOLPD model with a positive zero

$$G_m(s) = \frac{K_m(1-sT_{m3})e^{-sT_m}}{1+sT_{m1}}$$

Rule	K_c	T_i	Comment
Direct synthesis: time domain criteria			
Sree and Chidambaram (2003a). Model: Method 1	$\gamma \frac{\gamma T_{m1}}{K_m T_{m3}}$	$T_i^{(89)}$	

¹ γ = value of the inverse jump of the closed loop system; $\gamma < \frac{K_m T_{m3}}{T_{m1}}$.

$$T_i^{(89)} = \frac{\frac{\gamma T_{m1}}{T_{m3}} [T_{m3}(1-x_1) - 0.5\tau_m(1+x_1)]}{\frac{\gamma T_{m1}}{T_{m3}}(1-x_1) - x_1};$$

$$x_1 \in \left[0.95 \frac{\gamma T_{m1}}{\gamma T_{m1} + T_{m3}}, 0.98 \frac{\gamma T_{m1}}{\gamma T_{m1} + T_{m3}} \right], \quad 0.1 \leq \frac{T_{m3}}{T_{m1}} \leq 0.3;$$

$$x_1 \in \left[0.1 \frac{\gamma T_{m1}}{\gamma T_{m1} + T_{m3}}, 0.3 \frac{\gamma T_{m1}}{\gamma T_{m1} + T_{m3}} \right], \quad \frac{T_{m3}}{T_{m1}} > 1.$$

3.2.2 Ideal controller in series with a first order lag

$$G_c(s) = K_c \left(1 + \frac{1}{T_i s} \right) \frac{1}{1 + T_f s}$$

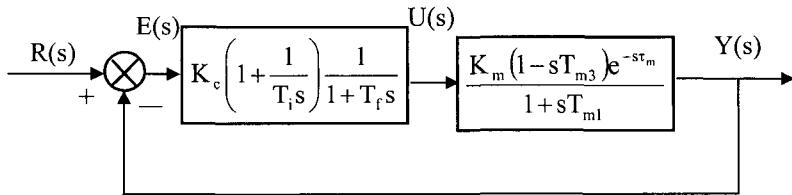


Table 10: PI controller tuning rules - FOLPD model with a positive zero

$$G_m(s) = \frac{K_m (1 - sT_{m3}) e^{-s\tau_m}}{1 + sT_{m1}}$$

Rule	K_c	T_i	Comment
Direct synthesis: time domain criteria			
Sree and Chidambaram (2003a). <i>Model: Method 1</i>	$\frac{T_{m1}}{K_m (2T_{m3} + \lambda + \tau_m)}$	T_{m1}	$T_f = \frac{T_{m3}(\lambda - \tau_m)}{2T_{m3} + \lambda + \tau_m}$

3.3 FOLPD Model with a Negative Zero $G_m(s) = \frac{K_m(1+sT_{m3})e^{-s\tau_m}}{1+sT_{m1}}$

3.3.1 Ideal controller in series with a first order lag

$$G_c(s) = K_c \left(1 + \frac{1}{T_i s} \right) \frac{1}{1 + T_f s}$$

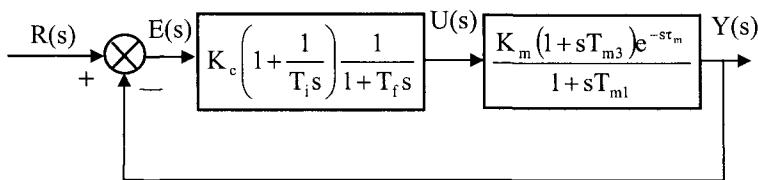


Table 11: PI controller tuning rules – FOLPD model with a negative zero

$$G_m(s) = \frac{K_m(1+sT_{m3})e^{-s\tau_m}}{1+sT_{m1}}$$

Rule	K_c	T_i	Comment
Robust			
Chang <i>et al.</i> (1997). Model: Method 2	$\frac{T_{m1}}{K_m(T_{CL} + \tau_m)}$	T_{m1}	$T_f = T_{m3}$

3.4 Non-Model Specific

3.4.1 Ideal controller $G_c(s) = K_c \left(1 + \frac{1}{T_i s} \right)$

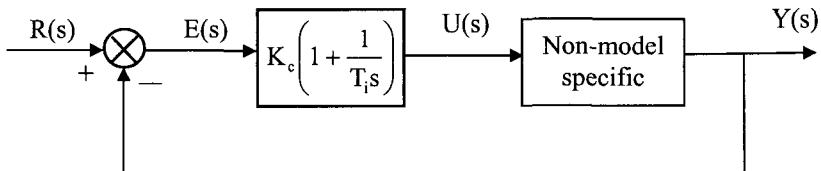


Table 12: PI controller tuning rules – non-model specific

Rule	K_c	T_i	Comment
Ultimate cycle			
Ziegler and Nichols (1942).	$0.45K_u$	$0.83T_u$	Quarter decay ratio
Hwang and Chang (1987).	$0.45K_u$	$\frac{1}{p_1} \left(\frac{5.22}{T_u} - \frac{5.22}{T_1} \right) T_u$	
	p_1, T_1 = decay rate, period measured under proportional control when $K_c = 0.5K_u$		
Parr (1989) – page 191.	$0.5K_u$	$0.43T_u$	
Parr (1989) – page 192.	$0.33K_u$	$2T_u$	
Hang <i>et al.</i> (1993b) – page 59.	$0.25K_u$	$0.25T_u$	dominant time delay process
Pessen (1994).	$0.25K_u$	$0.167T_u$	dominant time delay process
McMillan (1994) – page 90.	$0.3571K_u$	T_u	
Åström and Hägglund (1995) – pages 141-142.	$0.4698K_u$	$0.4373T_u$	$A_m = 2, \phi_m = 20^\circ$
	$0.1988K_u$	$0.0882T_u$	$A_m = 2.44, \phi_m = 61^\circ$
	$0.2015K_u$	$0.1537T_u$	$A_m = 3.45, \phi_m = 46^\circ$

Rule	K_c	T_i	Comment
Calcev and Gorez (1995).	$0.3536K_u$	$0.1592T_u$	$\phi_m = 45^0$, small τ_m $\phi_m = 15^0$, large τ_m
Edgar <i>et al.</i> (1997) – page 8-15.	$0.59K_u$	$0.81T_u$	Minimum IAE - regulator
ABB (2001).	$0.5K_u$	$0.8T_u$	
Robbins (2002).	$0.3K_u$	${}^1 T_i^{(90)}$	Minimum IAE – servo
	$0.3K_u$	${}^1 T_i^{(91)}$	Good response – regulator
Direct synthesis			
Wang <i>et al.</i> (1995b).	${}^2 K_c^{(92)}$	$T_i^{(92)}$	
Vrančić <i>et al.</i> (1996), Vrančić (1996) – page 121.	$\frac{0.5A_3}{(A_1A_2 - K_m A_3)}$	$\frac{A_3}{A_2}$	$A_m \geq 2, \phi_m \geq 60^0$ (Vrančić <i>et al.</i> (1996))
Vrančić (1996) – page 140.	$0.45K_u$	$\frac{A_3}{A_2}$	Modified Ziegler-Nichols method
Friman and Waller (1997).	$\frac{0.4830}{ G_p(j\omega_{150^0}) }$	3.7321 ω_{150^0}	$A_m > 2,$ $\phi_m > 15^0$

$${}^1 T_i^{(90)} = (0.24 + 0.111K_u K_m)T_u ; T_i^{(91)} = (0.27 + 0.054K_u K_m)T_u .$$

² For a stable process, K_m is assumed known; for an integrating process, K_m and τ_m are assumed known.

$$K_c^{(92)} = \frac{1}{\omega_{CL}} \operatorname{Im} \left[\frac{j\omega_{CL} G_{CL}(j\omega_{CL})}{G_p(j\omega_{CL}) \{1 - G_{CL}(j\omega_{CL})\}} \right],$$

$$T_i^{(92)} = \frac{1}{\omega_{CL}} \frac{\operatorname{Im} \left[\frac{j\omega_{CL} G_{CL}(j\omega_{CL})}{G_p(j\omega_{CL}) \{1 - G_{CL}(j\omega_{CL})\}} \right]}{\operatorname{Re} \left[\frac{j\omega_{CL} G_{CL}(j\omega_{CL})}{G_p(j\omega_{CL}) \{1 - G_{CL}(j\omega_{CL})\}} \right]} .$$

Rule	K_c	T_i	Comment
Kristiansson and Lennartson (2000).	$\frac{1.18K_u K_m - 1.72}{K_u K_m^2}$	${}^3 T_i^{(93)}$	$0.1 \leq K_u K_m \leq 0.5$
	$\frac{0.50K_u K_m - 0.36}{K_u K_m^2}$	${}^3 T_i^{(94)}$	$K_u K_m > 0.5$
	$\frac{20K_{135^\circ} K_m - 160}{K_{135^\circ} K_m^2}$	${}^4 T_i^{(95)}$	$K_u K_m < 0.1 ; K_{135^\circ} K_m \leq 0.1$
	$\frac{5.4K_{135^\circ} K_m - 13.6}{K_{135^\circ} K_m^2}$	${}^4 T_i^{(96)}$	$K_u K_m < 0.1 ; K_{135^\circ} K_m > 0.1$
Kristiansson and Lennartson (2002).	$\frac{0.33K_u K_m - 0.15}{K_m (0.18K_u K_m + 1)}$	$\frac{K_u K_m}{\omega_u (0.18K_u K_m + 1)}$	$K_u K_m \leq 10$
	${}^5 K_c^{(97)}$	$T_i^{(97)}$	$K_u K_m > 10$
Kristiansson (2003).	${}^6 K_c^{(98)}$	$T_i^{(98)}$	$1.6 \leq M_s \leq 1.9$
Other tuning rules			
Aikman (1950).	$0.81K_{37\%}$	$1.29T_{37\%}$	37% decay ratio
	$[0.63K_{33\%}, 0.97K_{33\%}]$	$[1.1T_{33\%}, 1.3T_{33\%}]$	33% decay ratio
Rutherford (1950).	$0.90K_{37\%}$	$T_{37\%}$	37% decay ratio
MacLellan (1950).	$1.19K_{37\%}$	$0.92T_{37\%}$	37% decay ratio
	$1.02K_{37\%}$	$0.92T_{37\%}$	37% decay ratio
Young (1955).	$K_{37\%}$	$< 1.1T_u$	37% decay ratio

$${}^3 T_i^{(93)} = \frac{1.18K_u K_m - 1.72}{(0.33K_u K_m - 0.17)\omega_u}, \quad {}^3 T_i^{(94)} = \frac{0.50K_u K_m - 0.36}{(0.33K_u K_m - 0.17)\omega_u}.$$

$${}^4 T_i^{(95)} = \frac{20K_{135^\circ} K_m - 160}{(0.315K_{135^\circ} K_m - 0.175)\omega_u}, \quad {}^4 T_i^{(96)} = \frac{5.4K_{135^\circ} K_m - 13.6}{(1.32K_{135^\circ} K_m - 3.2)\omega_u}.$$

$${}^5 K_c^{(97)} = K_{135^\circ} \frac{0.09K_{135^\circ} K_m + 0.25}{0.09K_{135^\circ} K_m + 1.2}, \quad {}^5 T_i^{(97)} = \frac{K_{135^\circ} K_m}{\omega_{135^\circ} (0.09K_{135^\circ} K_m + 1.2)}.$$

$${}^6 K_c^{(98)} = \frac{\omega_{150^\circ}}{K_m} \left(0.2 + \frac{0.075}{\frac{K_{150^\circ}}{K_m} + 0.05} \right), \quad {}^6 T_i^{(98)} = \omega_{150^\circ} \left[0.06 + 1.6 \frac{K_{150^\circ}}{K_m} - 0.06 \left(\frac{K_{150^\circ}}{K_m} \right)^2 \right].$$

Rule	K_c	T_i	Comment
Chesmond (1982) – page 400.	$\frac{K_{x\%}}{0.5 + 0.361 \ln x}$	$\frac{T_{x\%}}{1.2\sqrt{1 + 0.025 \ln^2 x}}$	$x = 100$ (decay ratio)
	0.999 $K_{25\%}$	0.814 $T_{25\%}$	Example: $x = 25$ i.e. quarter decay ratio
Parr (1989) – page 191.	0.667 $K_{25\%}$	$T_{25\%}$	Quarter decay ratio
McMillan (1994) – pages 42-43.	0.42 $K_{25\%}$	$T_{25\%}$	'Fast' tuning
	0.33 $K_{25\%}$	$T_{25\%}$	'Slow' tuning
Wade (1994) – page 114.	0.7515 $K_{25\%}$	0.7470 $T_{25\%}$	
ECOSSE team (1996b).	0.65 $K_{50\%}$	0.7917 $T_{50\%}$	
Hay (1998) – page 189.	$K_{25\%}$	$T_{25\%}$	
Bateson (2002) – page 616.	$K_{25\%}$	T_u	'Modified' ultimate cycle method
Åström (1982).	$\frac{\sin \phi_m}{ G_p(j\omega_{90^\circ}) }$	$\frac{\tan \phi_m}{\omega_{90^\circ}}$	
Leva (1993).	${}^7 K_c^{(99)}$	$T_i^{(99)}$	$\tan(\phi_m - \phi_o - 0.5\pi) > 0$
Åström and Hägglund (1995) – page 248.	$\frac{0.5}{ G_p(j\omega_{135^\circ}) }$	$\frac{4}{\omega_{135^\circ}}$	Alfa-Laval Automation ECA400 controller
	$\frac{0.25}{ G_p(j\omega_{135^\circ}) }$	$\frac{1.6}{\omega_{135^\circ}}$	Alfa-Laval Automation ECA400 controller – large delay
Hägglund and Åström (1991).	$0.25 \hat{K}_u$	$0.2546 \hat{T}_u$	Model: Method 2
Cox <i>et al.</i> (1994).	$\frac{0.2h T_u \sin \phi_m}{A_p}$	$0.16 T_u \tan \phi_m$	
Jones <i>et al.</i> (1997). Model: Method 2	$0.5787 \frac{h}{A_p}$	$a \hat{T}_u$	$a \in [0.7958, 1.5915]$

$${}^7 K_c^{(99)} = \frac{\tan(\phi_m - \phi_o - 0.5\pi)}{|G_p(j\omega)|\sqrt{1 + \tan^2(\phi_m - \phi_o - 0.5\pi)}}, \quad T_i^{(99)} = \frac{\tan(\phi_m - \phi_o - 0.5\pi)}{\omega}.$$

Rule	K_c	T_i	Comment
NI Labview (2001). <i>Model: Method 2</i>	$\hat{0.4K_u}$	$\hat{0.8T_u}$	Quarter decay ratio
	$\hat{0.18K_u}$	$\hat{0.8T_u}$	'Some' overshoot
	$\hat{0.13K_u}$	$\hat{0.8T_u}$	'Little' overshoot
Tang <i>et al.</i> (2002). <i>Model: Method 4</i>	$\frac{2.5465h \sin \phi_m}{\hat{A_p} \hat{x}_l \hat{\omega}_{90^\circ} \hat{A_m}}$	$\frac{\tan \phi_m}{\hat{\omega}_{90^\circ}}$	$0.81 \leq x_1 \leq 1$; $x_1 = 1$, 'small' τ_m ; $x_1 = 0.91$, 'large' τ_m

3.4.2 Controller with set-point weighting

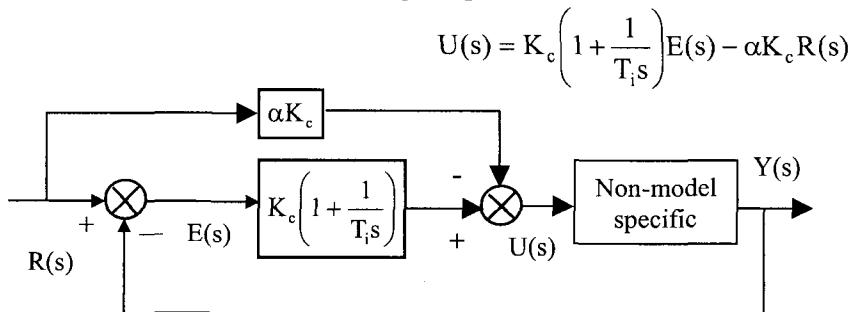


Table 13: PI controller tuning rules – non-model specific

Rule	K_c	T_i	Comment
Direct synthesis			
Åström and Hägglund (1995) – page 215.	$0.053K_u e^{2.9\kappa-2.6\kappa^2}$	$0.90T_u e^{-4.4\kappa+2.7\kappa^2}$	$M_s = 1.4$
	$\alpha = 1 - 1.1e^{-0.0061\kappa+1.8\kappa^2}$; $0 < K_m K_u < \infty$.		
	$0.13K_u e^{1.9\kappa-1.3\kappa^2}$	$0.90T_u e^{-4.4\kappa+2.7\kappa^2}$	$M_s = 2.0$
	$\alpha = 1 - 0.48e^{0.40\kappa-0.17\kappa^2}$; $0 < K_m K_u < \infty$.		
Vrančić (1996) – page 136-137.	${}^8 K_c^{(100)}$	${}^{100} T_i$	$\alpha = [0.2, 0.5]$ - good servo & regulator action (Vrančić (1996), page 139)

$${}^8 K_c^{(100)} = \frac{A_1 A_2 - K_m A_3 - \chi}{(1 - b^2)(K_m^2 A_3 + A_1^3 - 2K_m A_1 A_2)}, \quad K_m A_3 - A_1 A_2 < 0;$$

$$K_c^{(100)} = \frac{A_1 A_2 - K_m A_3 + \chi}{(1 - b^2)(K_m^2 A_3 + A_1^3 - 2K_m A_1 A_2)}, \quad K_m A_3 - A_1 A_2 > 0; \quad b = 1 - \alpha;$$

$$\chi = \sqrt{(K_m A_3 - A_1 A_2)^2 - (1 - b^2)A_3(K_m^2 A_3 + A_1^3 - 2K_m A_1 A_2)};$$

$$T_i^{(100)} = \frac{A_1}{K_m + \frac{1}{2K_c^{(100)}} + \frac{K_c^{(100)} K_m^2}{2} (1 - b^2)}.$$

3.5 IPD Model $G_m(s) = \frac{K_m e^{-st_m}}{s}$

3.5.1 Ideal controller $G_c(s) = K_c \left(1 + \frac{1}{T_i s} \right)$

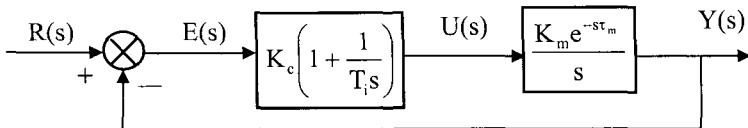


Table 14: PI controller tuning rules – IPD model $G_m(s) = \frac{K_m e^{-st_m}}{s}$

Rule	K_c	T_i	Comment
Process reaction			
Ziegler and Nichols (1942). <i>Model: Method 2</i>	$\frac{0.9}{K_m \tau_m}$	$3.33\tau_m$	Quarter decay ratio
Two constraints method – Wolfe (1951). <i>Model: Method 2</i>	$\frac{0.6}{K_m \tau_m}$	$2.78\tau_m$	Decay ratio = 0.4
	$\frac{0.87}{K_m \tau_m}$	$4.35\tau_m$	Decay ratio is as small as possible
Minimum error integral (regulator mode).			
Coon (1956), (1964). <i>Model: Method 1</i>	$\frac{1.0}{K_m \tau_m}$	0	Quarter decay ratio; K_c, T_i deduced from graphs
Åström and Hägglund (1995) – page 13. <i>Model: Method 1</i>	$\frac{0.63}{K_m \tau_m}$	$3.2\tau_m$	Ultimate cycle Ziegler-Nichols equivalent
Hay (1998) – page 188.	$\frac{0.42}{K_m \tau_m}$	$5.8\tau_m$	<i>Model: Method 3</i>
Hay (1998) – page 199. <i>Model: Method 1</i> K_c and T_i deduced from graphs	7.5	$4.5K_m \tau_m$	$K_m \tau_m = 0.1$
	3.5		$K_m \tau_m = 0.2$
	2.5		$K_m \tau_m = 0.3$
	2.2		$K_m \tau_m = 0.4$

Rule	K_c	T_i	Comment				
Hay (1998) – continued.	2.0	$x_2 \tau_m$	$K_m \tau_m = 0.5$				
	1.7		$K_m \tau_m = 0.6$				
Bunzemeier (1998). <i>Model: Method 4</i>	$\frac{x_1}{K_m \tau_m}$	$x_2 \tau_m$	0% overshoot (servo)				
	$\frac{x_3}{K_m \tau_m}$		10% overshoot (servo)				
$G_p = \frac{K_p}{s(1+sT_p)^n}$							
Coefficient values							
x_1	x_2	x_3	n	x_1	x_2	x_3	n
0.18	1.350	0.23	0	0.20	1.477	0.25	6
0.24	3.050	0.40	1	0.19	1.463	0.24	7
0.22	1.910	0.30	2	0.19	1.453	0.24	8
0.21	1.690	0.28	3	0.18	1.385	0.24	9
0.20	1.584	0.26	4	0.18	1.378	0.24	10
0.20	1.557	0.26	5				
Minimum performance index: regulator tuning							
Minimum IAE – Shinskey (1988) – page 123.	$\frac{0.9524}{K_m \tau_m}$	$4\tau_m$	<i>Model: Method 1</i>				
Minimum IAE – Shinskey (1994) – page 74.	$\frac{0.9259}{K_m \tau_m}$	$4\tau_m$	<i>Model: Method 1</i>				
Minimum IAE – Shinskey (1988) – page 148. <i>Model: Method 1</i>	$0.61K_u$	T_u	Also specified by Shinskey (1996), page 121.				
Minimum ISE – Hazebroek and Van der Waerden (1950).	$\frac{1.5}{K_m \tau_m}$	$5.56\tau_m$	<i>Model: Method 2</i>				
Minimum ISE – Haalman (1965). <i>Model: Method 1</i>	$\frac{0.6667}{K_m \tau_m}$	0	$M_s = 1.9$, $A_m = 2.36$, $\phi_m = 50^\circ$				
Minimum ITAE – Poulin and Pomerleau (1996). <i>Model: Method 1</i>	$\frac{0.5264}{K_m \tau_m}$	$4.5804\tau_m$	Process output step load disturbance				
	$\frac{0.5327}{K_m \tau_m}$	$3.8853\tau_m$	Process input step load disturbance				
Minimum performance index: other tuning							
Skogestad (2001). <i>Model: Method 1</i>	$\frac{0.28}{K_m \tau_m}$	$7\tau_m$	$M_{max} = 1.4$				

Rule	K_c	T_i	Comment
Skogestad (2003). <i>Model: Method 1</i>	$\frac{0.404}{K_m \tau_m}$	$7\tau_m$	$M_{max} = 1.7$
Skogestad (2001). <i>Model: Method 1</i>	$\frac{0.49}{K_m \tau_m}$	$3.77\tau_m$	$M_{max} = 2.0$
Direct synthesis: time domain criteria			
Tyreus and Luyben (1992). <i>Model: Method 1 or 9</i>	$\frac{0.487}{K_m \tau_m}$	$8.75\tau_m$	Maximum closed loop log modulus = 2dB ; $T_{CL} = \tau_m \sqrt{10}$
	$0.31K_u$	$2.2T_u$	
Fruehauf <i>et al.</i> (1993).	$\frac{0.5}{K_m \tau_m}$	$5\tau_m$	<i>Model: Method 2</i>
Rotach (1995). <i>Model: Method 5</i>	$\frac{0.75}{K_m \tau_m}$	$2.41\tau_m$	Damping factor for oscillations to a disturbance input = 0.75.
Wang and Cluett (1997). <i>Model: Method 1</i>	$^1 K_c^{(101)}$	$T_i^{(101)}$	$\xi = 0.707$; $T_{CL} \in [\tau_m, 16\tau_m]$
	$^2 K_c^{(102)}$	$T_i^{(102)}$	$\xi = 1$; $T_{CL} \in [\tau_m, 16\tau_m]$
Cluett and Wang (1997). <i>Model: Method 1</i>	$0.9588/K_m \tau_m$	$3.0425\tau_m$	$T_{CL} = \tau_m$
	$0.6232/K_m \tau_m$	$5.2586\tau_m$	$T_{CL} = 2\tau_m$
	$0.4668/K_m \tau_m$	$7.2291\tau_m$	$T_{CL} = 3\tau_m$
	$0.3752/K_m \tau_m$	$9.1925\tau_m$	$T_{CL} = 4\tau_m$
	$0.3144/K_m \tau_m$	$11.1637\tau_m$	$T_{CL} = 5\tau_m$
	$0.2709/K_m \tau_m$	$13.1416\tau_m$	$T_{CL} = 6\tau_m$
Poulin and Pomerleau (1999). <i>Model: Method 1</i>	$0.34K_u$ or $\frac{2.13}{K_m T_u}$	$1.04T_u$	$M_{max} = 5$ dB

$$^1 K_c^{(101)} = \frac{1}{K_m \tau_m (0.7138 T_{CL} + 0.3904)}, \quad T_i^{(101)} = (1.4020 T_{CL} + 1.2076) \tau_m.$$

$$^2 K_c^{(102)} = \frac{1}{K_m \tau_m (0.5080 T_{CL} + 0.6208)}, \quad T_i^{(102)} = (1.9885 T_{CL} + 1.2235) \tau_m.$$

Rule	K_c	T_i	Comment			
Vítečková (1999), Vítečková <i>et al.</i> (2000a). <i>Model: Method 1</i>	$x_1/K_m \tau_m$	0				
Coefficient values						
	x_1	OS	x_1	OS	x_1	OS
	0.368	0%	0.696	20%	0.906	40%
	0.514	5%	0.748	25%	0.957	45%
	0.581	10%	0.801	30%	1.008	50%
	0.641	15%	0.853	35%		
Chidambaram and Sree (2003). <i>Model: Method 1</i>	$\frac{1.1111}{K_m \tau_m}$	$4.5\tau_m$				
Huba and Žáková (2003). <i>Model: Method 1</i>	$\frac{0.23}{K_m \tau_m}$	$2.914\tau_m$				
	$\frac{0.281}{K_m \tau_m}$	$3.555\tau_m$				
Skogestad (2003), (2004b). <i>Model: Method 1</i>	$\frac{1}{K_m(T_{CL} + \tau_m)}$	$4\xi^2(T_{CL} + \tau_m)$	Suggested $\xi = 0.7$ or 1			
	$\frac{0.5}{K_m \tau_m}$	$8\tau_m$	'good' robustness - $T_{CL} = \tau_m$, $\xi = 1$			
Direct synthesis: frequency domain criteria						
Hougen (1979) – page 333. <i>Model: Method 1</i>	${}^3 K_c^{(103)}$	0	Maximise crossover frequency			
Chidambaram (1994), Srividya and Chidambaram (1997).	$\frac{0.67075}{K_m \tau_m}$	$3.6547\tau_m$	<i>Model: Method 6;</i> $A_m = 2$			
Gain and phase margin – Kookos <i>et</i> <i>al.</i> (1999). <i>Model: Method 1</i>	$\frac{\omega_p}{A_m K_m}$	$\frac{1}{\omega_p(0.5\pi - \omega_p \tau_m)}$				
Representative results						
	$0.942/K_m \tau_m$	$4.510\tau_m$	$A_m = 1.5$; $\phi_m = 22.5^\circ$			
	$0.698/K_m \tau_m$	$4.098\tau_m$	$A_m = 2$; $\phi_m = 30^\circ$			
	$0.491/K_m \tau_m$	$6.942\tau_m$	$A_m = 3$; $\phi_m = 45^\circ$			
	$0.384/K_m \tau_m$	$18.710\tau_m$	$A_m = 4$; $\phi_m = 60^\circ$			

³ Values recorded deduced from a graph:

$K_m \tau_m$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
$K_c^{(103)}$	7.0	3.5	2.2	1.8	1.4	1.1	1.0	0.8	0.75	0.7

Rule	K_c	T_i	Comment				
Chen <i>et al.</i> (1999a). <i>Model: Method 10</i>	$\frac{\phi_m + \frac{\pi}{2}(A_m - 1)}{K_m \tau_m (A_m^2 - 1)}$	${}^4 T_i^{(104)}$	$\phi_m \geq \frac{\pi}{2} \left(1 - \frac{1.0607}{A_m} \right)$				
	${}^5 K_c^{(105)}$	${}^5 T_i^{(105)}$					
Cheng and Yu (2000). <i>Model: Method 1</i>	$0.5236 / K_m \tau_m$	$8\tau_m$	$A_m = 2.83;$ $\phi_m = 46.1^\circ$				
O'Dwyer (2001a). <i>Model: Method 1</i>	$\frac{x_1}{K_m \tau_m}$	$x_2 \tau_m$	[⁶]				
Representative coefficient values							
x_1	x_2	A_m	ϕ_m	x_1	x_2	A_m	ϕ_m
0.558	1.4	1.5	46.2°	0.357	4.3	4.0	60.0°
0.484	1.55	2.0	45.5°	0.305	12.15	5.0	75.0°
0.458	3.35	3.0	59.9°				

$${}^4 T_i^{(104)} = \frac{\pi}{4} \left[\frac{A_m^2 - 1}{\phi_m + 0.5\pi(A_m - 1)} \right] \left[\frac{\tau_m}{0.5\pi - \phi_m - \frac{\phi_m + 0.5\pi(A_m - 1)}{A_m^2 - 1}} \right].$$

$${}^5 K_c^{(105)} = \frac{r_1 \pi (2r_1 A_m - 1)}{4K_m \tau_m (r_1^2 A_m^2 - 1)}, \quad {}^5 T_i^{(105)} = \frac{4\tau_m (r_1^2 A_m^2 - 1)^2}{\pi r_1^2 A_m (r_1 A_m - 2)(2r_1 A_m - 1)}$$

$${}^6 A_m = \frac{\frac{x_2}{4x_1} \left[\frac{\pi}{2} + \sqrt{\frac{\pi^2}{4} - \frac{\pi}{x_2}} \right]^2}{\sqrt{1 + \frac{x_2^2}{4} \left[\frac{\pi}{2} + \sqrt{\frac{\pi^2}{4} - \frac{\pi}{x_2}} \right]^2}}, \quad x_2 > 1.273;$$

$$\phi_m = \tan^{-1} \left[\sqrt{0.5x_1^2 x_2^2 + 0.5x_1 x_2 \sqrt{x_1^2 x_2^2 + 4}} \right] - \sqrt{0.5x_1^2 + 0.5 \frac{x_1}{x_2} \sqrt{x_1^2 x_2^2 + 4}}.$$

Rule	K_c	T_i	Comment
O'Dwyer (2001a). <i>Model: Method 1</i>	aK_u	bT_u	[⁷]
Robust			
Chien (1988). <i>Model: Method 1</i>	$\frac{1}{K_m} \left(\frac{2\lambda + \tau_m}{[\lambda + \tau_m]^2} \right)$	$2\lambda + \tau_m$	⁸
Ogawa (1995). <i>Model: Method 1;</i> <i>Coefficients of K_c and T_i deduced from graphs</i>	$x_1/K_m \tau_m$	$x_2 \tau_m$	
	x_1	x_2	
	0.45	11	20% uncertainty in process parameters
	0.39	12	30% uncertainty in process parameters
	0.34	13	40% uncertainty in process parameters
	0.30	14	50% uncertainty in process parameters
	0.27	15	60% uncertainty in process parameters
Alvarez-Ramirez <i>et al.</i> (1998). <i>Model: Method 1</i>	$\frac{1}{K_m} \left(\frac{1}{T_{CL}} + \frac{1}{T_m} \right)$	$T_m + \tau_m$	$T_{CL} > \tau_m$
Zhong and Li (2002). <i>Model: Method 2</i>	$\frac{2\alpha + \tau_m}{K_m (\alpha + \tau_m)^2}$	$2\alpha + \tau_m$	α not specified

$$^7 A_m = \frac{\frac{2b}{\pi a} \left[\frac{\pi}{2} + \sqrt{\frac{\pi^2}{4} - \frac{\pi}{4b}} \right]^2}{\sqrt{1 + \frac{1}{a^2 \pi^2} \left[\frac{\pi}{2} + \sqrt{\frac{\pi^2}{4} - \frac{\pi}{4b}} \right]^2}},$$

$$\phi_m = \tan^{-1} \left[2\pi^2 a^2 b^2 + 2\pi ab \sqrt{4\pi^2 a^2 b^2 + 4} \right]^{0.5} - \sqrt{0.125\pi^2 a^2 + \frac{\pi a}{16b} \sqrt{4\pi^2 a^2 b^2 + 4}}$$

⁸ $\lambda = [1/K_m, \tau_m]$ (Chien and Fruehauf (1990)); $\lambda = 3\tau_m$ (Bialkowski (1996)); $\lambda > \tau_m + T_m$ (Thomasson (1997)); $\lambda = [1.5\tau_m, 4.5\tau_m]$ (Zhang *et al.* (1999)). From a graph, $\lambda = 1.5\tau_m$...Overshoot = 58%, Settling time = $6\tau_m$

$\lambda = 2.5\tau_m$...Overshoot = 35%, Settling time = $11\tau_m$

$\lambda = 3.5\tau_m$...Overshoot = 26%, Settling time = $16\tau_m$

$\lambda = 4.5\tau_m$...Overshoot = 22%, Settling time = $20\tau_m$.

$\lambda = e_{max}/100K_m$; e_{max} = maximum output error after a step load disturbance

(Andersson (2000) – page 12);

$$\left(\frac{y}{d} \right)_{desired} = \frac{T_i s e^{-\tau_m s}}{K_c (\lambda s + 1)^2} \text{ (Chen and Seborg (2002)); } \lambda = \tau_m \text{ (Smith (2002))}.$$

Rule	K_c	T_i	Comment
Smith (2002). <i>Model: Method 1</i>	$\frac{1}{K_m \tau_m}$	τ_m	
Skogestad (2004a). <i>Model: Method 1</i>	${}^9 K_c^{(106)}$	$\frac{4}{K_c^{(106)} K_m}$	
Other methods			
Penner (1988). <i>Model: Method 1</i>	$\frac{0.58}{K_m \tau_m}$	$10\tau_m$	Maximum closed loop gain = 1.26
	$\frac{0.8}{K_m \tau_m}$	$5.9\tau_m$	Maximum closed loop gain = 2.0

$${}^9 K_c^{(106)} \geq \frac{|\Delta d_0|}{|\Delta y_{\max}|},$$

$|\Delta d_0|$ = maximum magnitude of a sinusoidal disturbance over all frequencies,

$|\Delta y_{\max}|$ = maximum controlled variable change, corresponding to the sinusoidal disturbance.

3.5.2 Ideal controller in series with a first order lag

$$G_c(s) = K_c \left(1 + \frac{1}{T_i s} \right) \frac{1}{1 + T_f s}$$

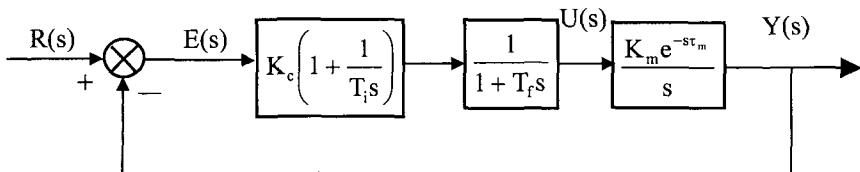


Table 15: PI controller tuning rules – IPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{s}$

Rule	K_c	T_i	Comment
Robust			
H _∞ optimal - Tan <i>et al.</i> (1998b). <i>Model: Method 1</i>	$\frac{0.463\lambda + 0.277}{K_m \tau_m}$	$\frac{\tau_m}{0.238\lambda + 0.123}$	$\lambda = 0.5$; $T_f = \frac{\tau_m}{5.750\lambda + 0.590}$
Rivera and Jun (2000). <i>Model: Method 1</i>	¹ $K_c^{(107)}$	$2(\tau_m + \lambda)$	λ not specified

$$^1 K_c^{(107)} = \frac{2(\tau_m + \lambda)}{K_m(2\tau_m^2 + 4\tau_m\lambda + \lambda^2)}, T_f = \frac{\tau_m\lambda^2}{2\tau_m^2 + 4\tau_m\lambda + \lambda^2}.$$

3.5.3 Controller with set-point weighting

$$U(s) = K_c \left(1 + \frac{1}{T_i s} \right) E(s) - \alpha K_c R(s)$$

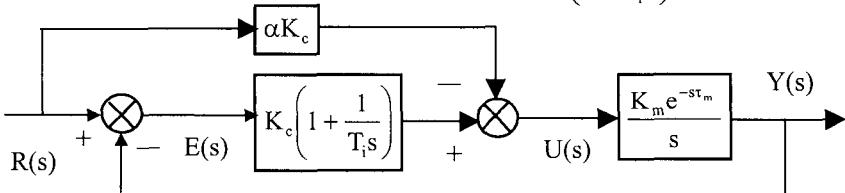


Table 16: PI controller tuning rules – IPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{s}$

Rule	K_c	T_i	Comment					
Minimum performance index: servo/regulator tuning								
Taguchi and Araki (2000). <i>Model: Method 1</i>	$\frac{0.7662}{K_m \tau_m}$	$4.091 \tau_m$	$\alpha = 0.6810$, $\tau_m / T_m \leq 1.0$. Overshoot (servo step) $\leq 20\%$					
Minimum ITAE - Pecharromán (2000). <i>Model: Method 10</i>	$x_1 K_u$	$x_2 T_u$	$K_m = 1$					
	Coefficient values							
	x_1	x_2	α	ϕ_c	x_1	x_2	α	ϕ_c
	0.049	2.826	0.506	-164°	0.218	1.279	0.564	-135°
	0.066	2.402	0.512	-160°	0.250	1.216	0.573	-130°
	0.099	1.962	0.522	-155°	0.286	1.127	0.578	-125°
	0.129	1.716	0.532	-150°	0.330	1.114	0.579	-120°
	0.159	1.506	0.544	-145°	0.351	1.093	0.577	-118°
	0.189	1.392	0.555	-140°				
Direct synthesis								
Chidambaram (2000b). <i>Model: Method 1</i>	$0.64 / K_m \tau_m$	$3.333 \tau_m$	$\alpha = 0.65$					
	$0.487 / K_m \tau_m$	$8.75 \tau_m$	$\alpha = 0.557$					
	In general, $\alpha = 0.6$ (on average) or $\alpha = 0.5 \left[1 - \frac{\tau_m}{T_i} \right]$							
Hägglund and Åström (2002). <i>Model: Method 2</i>	$\frac{0.35}{K_m \tau_m}$	$7 \tau_m$	$M_s = 1.4$; $0.3 < \alpha < 0.5$					

3.5.4 Controller with proportional term acting on the output I

$$U(s) = \frac{K_c}{T_i s} E(s) - K_c Y(s)$$

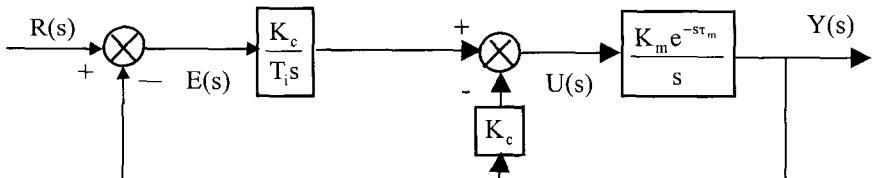


Table 17: PI controller tuning rules – IPD model $G_m(s) = \frac{K_m e^{-st_m}}{s}$

Rule	K_c	T_i	Comment
Minimum performance index: regulator tuning			
Minimum ISE – Arvanitis <i>et al.</i> (2003a).	$\frac{0.6330}{K_m \tau_m}$	$2.3800\tau_m$	<i>Model: Method 1</i>
Arvanitis <i>et al.</i> (2003a). <i>Model: Method 1</i>	$\frac{2}{3} \frac{0.6114}{K_m \tau_m}$	$2.7442\tau_m$	
	$\frac{3}{3} \frac{0.6146}{K_m \tau_m}$	$2.6816\tau_m$	
Minimum performance index: servo tuning			
Minimum ISE – Arvanitis <i>et al.</i> (2003a).	$\frac{0.6270}{K_m \tau_m}$	$2.4688\tau_m$	<i>Model: Method 1</i>
Arvanitis <i>et al.</i> (2003a). <i>Model: Method 1</i>	$\frac{2}{3} \frac{0.6064}{K_m \tau_m}$	$2.8489\tau_m$	
	$\frac{3}{3} \frac{0.6130}{K_m \tau_m}$	$2.7126\tau_m$	

² Minimum $\int_0^{\infty} [e^2(t) + K_m^2 u^2(t)] dt$.

³ Minimum $\int_0^{\infty} \left[e^2(t) + K_m^2 \left(\frac{du}{dt} \right)^2 \right] dt$.

Rule	K_c	T_i	Comment
Direct synthesis			
Chien <i>et al.</i> (1999). <i>Model: Method 2</i>	$^4 K_c^{(108)}$	$1.414T_{CL2} + \tau_m$	Underdamped system response - $\xi = 0.707$.
Arvanitis <i>et al.</i> (2003a). <i>Model: Method 1</i>	$\frac{4}{(8 - \beta)K_m \tau_m}$	$\frac{4\tau_m}{\beta}$	$\beta = \frac{4}{4\xi^2 + 1}$
Robust			
Arvanitis <i>et al.</i> (2003a). <i>Model: Method 1</i>	$^5 K_c^{(109)}$	$\frac{4\pi^2 \tau_m}{\alpha(\pi^2 - \alpha)}$	

$$^4 K_c^{(108)} = \frac{1.414T_{CL2} + \tau_m}{K_m (T_{CL2}^2 + 1.414T_{CL2}\tau_m + \tau_m^2)}.$$

$$^5 K_c^{(109)} = \frac{4\pi^2}{[(8 - \alpha)\pi^2 + \alpha^2]K_m \tau_m} \text{ with } 0 < \alpha < \pi^2;$$

recommended $\alpha = \frac{\pi^2}{2} \left(1 - \sqrt{1 - \frac{16}{\pi^2(4\xi^2 + 1)}} \right)$, $\xi > 0.3941$.

3.5.5 Controller with proportional term acting on the output 2

$$U(s) = K_c \left(1 + \frac{1}{T_i s} \right) E(s) - K_1 Y(s)$$

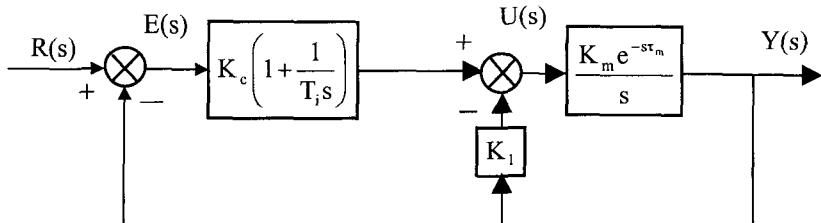


Table 18: PI controller tuning rules – IPD model $G_m(s) = \frac{K_m e^{-st_m}}{s}$

Rule	K_c	T_i	Comment
Robust			
Lee and Edgar (2002). <i>Model: Method 1</i>	${}^6 K_c^{(110)}$	$T_i^{(110)}$	$K_1 = 0.25K_u$

$${}^6 K_c^{(110)} = \frac{0.25K_u}{(\lambda + \tau_m)} \left[\frac{4}{K_u K_m} - \tau_m + \frac{\tau_m^2}{2(\lambda + \tau_m)} \right], \quad T_i^{(110)} = \frac{4}{K_u K_m} - \tau_m + \frac{\tau_m^2}{2(\lambda + \tau_m)}.$$

3.5.6 Controller with a double integral term

$$G_c(s) = K_c \left(1 + \frac{1}{T_{i1}s} + \frac{1}{T_{i2}s^2} \right)$$

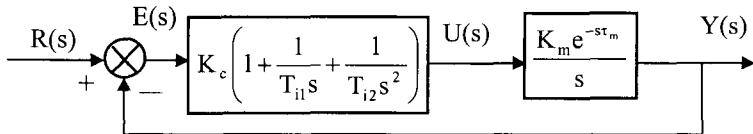


Table 19: PI controller tuning rules – IPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{s}$

Rule	K_c	T_i	Comment
Ultimate cycle			
Belanger and Luyben (1997). Model: Method 2	$0.33K_u$	$T_{i1} = 2.26T_u ;$ $T_{i2} = 20.5T_u^2$	Maximum closed loop log modulus = +2 dB.

3.6 FOLIPD Model $G_m(s) = \frac{K_m e^{-s\tau_m}}{s(1+sT_m)}$

3.6.1 Ideal controller $G_c(s) = K_c \left(1 + \frac{1}{T_i s} \right)$

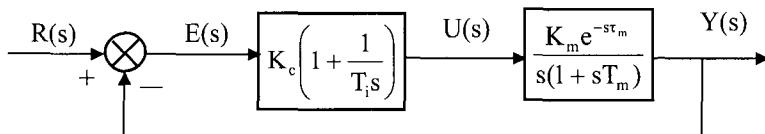


Table 20: PI controller tuning rules – FOLIPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{s(1+sT_m)}$

Rule	K_c	T_i	Comment
Process reaction			
Coon (1956), (1964). Model: Method 1; Coefficients of K_c deduced from graphs		$\frac{x_1}{K_m(\tau_m + T_m)}$	Quarter decay ratio criterion
Coefficient values			
τ_m / T_m	x_1	τ_m / T_m	x_1
0.020	5.0	0.25	2.2
0.053	4.0	0.43	1.7
0.11	3.0	1.0	1.3
Minimum performance index: regulator tuning			
Minimum IAE – Shinskey (1994) – page 75.	$\frac{0.556}{K_m(\tau_m + T_m)}$	$3.7(\tau_m + T_m)$	Model: Method 1
Minimum IAE – Shinskey (1994) – page 158	$\frac{0.952}{K_m(T_m + \tau_m)}$	$4(T_m + \tau_m)$	Model: Method 1

Rule	K_c	T_i	Comment		
Minimum ITAE – Poulin and Pomerleau (1996). <i>Model: Method 1</i>	$^1 K_c^{(111)}$	$x_1(\tau_m + T_m)$	K_c and T_i coefficients deduced from graph		
Coefficient values					
Process output step load disturbance	τ_m/T_m	x_1	x_2	τ_m/T_m	x_1
	0.2	5.0728	0.5231	1.2	4.7565
	0.4	4.9688	0.5237	1.4	4.7293
	0.6	4.8983	0.5241	1.6	4.7107
	0.8	4.8218	0.5245	1.8	4.6837
	1.0	4.7839	0.5249	2.0	4.6669
Process input step load disturbance	0.2	3.9465	0.5320	1.2	4.0337
	0.4	3.9981	0.5315	1.4	4.0278
	0.6	4.0397	0.5311	1.6	4.0278
	0.8	4.0397	0.5311	1.8	4.0218
	1.0	4.0397	0.5311	2.0	4.0099
					0.5314
Direct synthesis					
Hougen (1979) – page 338. <i>Model: Method 1</i>	$^2 \frac{1.26x_1}{K_m \tau_m}$	0	Maximise crossover frequency		
Poulin and Pomerleau (1999). <i>Model: Method 1</i>	$0.34K_u$ or $\frac{2.13}{K_m T_u}$	$1.04T_u$		$M_{\max} = 5$ dB	

$$^1 K_c^{(111)} = \frac{x_2}{K_m(\tau_m + T_m)} \sqrt{\frac{T_m^2}{x_1(\tau_m + T_m)^2}} + 1.$$

² x_1 values recorded deduced from a graph:

τ_m/T_m	1	2	3	4	5	6	7	8	9	10
x_1	0.31	0.19	0.14	0.11	0.09	0.078	0.068	0.06	0.053	0.05

Rule	K_c	T_i	Comment
Huba and Žáková (2003). <i>Model: Method 1</i>	${}^3 K_c^{(112)}$	0	
	Representative tuning rules; tuning rules for other τ_m/T_m values may be obtained from a plot.		
	$0.193/K_m \tau_m$	$3.717\tau_m$	$\tau_m/T_m = 0.5$
	$0.207/K_m \tau_m$	$3.386\tau_m$	$\tau_m/T_m = 1$
	$0.219/K_m \tau_m$	$3.127\tau_m$	$\tau_m/T_m = 2$
Ultimate cycle			
McMillan (1984). <i>Model: Method 1</i>	${}^4 K_c^{(113)}$	$T_i^{(113)}$	Tuning rules developed from K_u, T_u
Perić <i>et al.</i> (1997). <i>Model: Method 8</i>	$\frac{\hat{K}_u \tan^2 \phi_m}{\sqrt{1 + \tan^2 \phi_m}}$	$\frac{0.1592 \hat{T}_u}{\tan \phi_m}$	

$${}^3 K_c^{(112)} = \frac{-2T_m + \sqrt{\tau_m^2 + 4T_m^2}}{\frac{2T_m + \tau_m - \sqrt{\tau_m^2 + 4T_m^2}}{2T_m}} \cdot K_m \tau_m^2 e^{-\frac{2T_m}{\tau_m}}$$

$${}^4 K_c^{(113)} = \frac{1.477}{K_m} \frac{T_m}{\tau_m^2} \left\{ \frac{1}{1 + \left(\frac{T_m}{\tau_m} \right)^{0.65}} \right\}^2, \quad T_i^{(113)} = 3.33 \tau_m \left\{ 1 + \left(\frac{T_m}{\tau_m} \right)^{0.65} \right\}.$$

3.6.2 Controller with set-point weighting

$$U(s) = K_c \left(1 + \frac{1}{T_i s} \right) E(s) - \alpha K_c R(s)$$

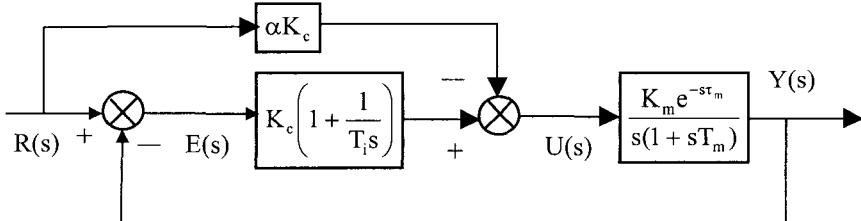


Table 21: PI controller tuning rules – FOLIPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{s(1+sT_m)}$

Rule	K_c	T_i	Comment					
Minimum performance index: servo/regulator tuning								
Taguchi and Araki (2000). <i>Model: Method 1</i>	${}^5 K_c^{(114)}$	$T_i^{(114)}$	$\tau_m/T_m \leq 1.0$. Overshoot (servo step) $\leq 20\%$					
Minimum ITAE - Pecharromán (2000). <i>Model: Method 4</i> $K_m = 1$; $T_m = 1$ $0.05 < \tau_m < 0.8$	$x_1 K_u$	$x_2 T_u$						
	Coefficient values							
	x_1	x_2	α	ϕ_c	x_1	x_2	α	ϕ_c
	0.049	2.826	0.506	-164°	0.218	1.279	0.564	-135°
	0.066	2.402	0.512	-160°	0.250	1.216	0.573	-130°
	0.099	1.962	0.522	-155°	0.286	1.127	0.578	-125°
	0.129	1.716	0.532	-150°	0.330	1.114	0.579	-120°
	0.159	1.506	0.544	-145°	0.351	1.093	0.577	-118°
	0.189	1.392	0.555	-140°				

$${}^5 K_c^{(114)} = \frac{1}{K_m} \left(0.1787 + \frac{0.2839}{\frac{\tau_m}{T_m} + 0.001723} \right),$$

$$T_i^{(114)} = 4.296 + 3.794 \frac{\tau_m}{T_m} + 0.2591 \left(\frac{\tau_m}{T_m} \right)^2, \quad \alpha = 0.6551 + 0.01877 \frac{\tau_m}{T_m}.$$

Rule	K_c	T_i	Comment
Direct synthesis			
Åström and Hägglund (1995) – pages 210- 212. <i>Model: Method 3</i> $0.14 \leq \frac{\tau_m}{T_m} \leq 5.5$	$\frac{0.41e^{-0.23\tau+0.019\tau^2}}{K_m(T_m + \tau_m)}$	$5.7\tau_m e^{1.7\tau - 0.69\tau^2}$ $\alpha = 1 - 0.33e^{2.5\tau - 1.9\tau^2}$	$M_s = 1.4$
	$\frac{0.81e^{-1.1\tau+0.76\tau^2}}{K_m(T_m + \tau_m)}$	$3.4\tau_m e^{0.28\tau - 0.0089\tau^2}$ $\alpha = 1 - 0.78e^{-1.9\tau + 1.2\tau^2}$	$M_s = 2.0$

3.6.3 Controller with proportional term acting on the output I

$$U(s) = \frac{K_c}{T_i s} E(s) - K_c Y(s)$$

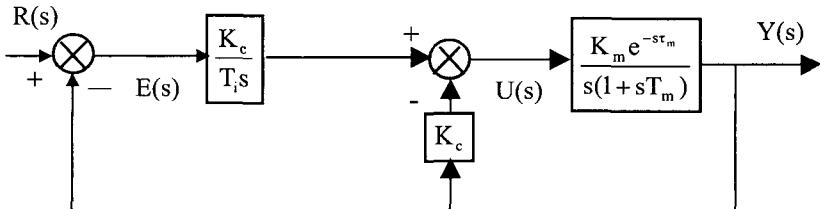


Table 22: PI controller tuning rules – FOLIPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{s(1+sT_m)}$

Rule	K_c	T_i	Comment
Minimum performance index: regulator tuning			
Arvanitis <i>et al.</i> (2003b). Model: Method 1	$\frac{4}{(8-\beta)K_m \tau_m}$	$\frac{4\tau_m}{\beta}$	
Minimum ISE ⁶			

$$\begin{aligned}
 {}^6 \beta &= 2.6302 - 2.1248 \left(\frac{\tau_m}{T_m} \right) + 2.5776 \left(\frac{\tau_m}{T_m} \right)^2 - 1.8511 \left(\frac{\tau_m}{T_m} \right)^3 + 0.82557 \left(\frac{\tau_m}{T_m} \right)^4 \\
 &\quad - 0.23641 \left(\frac{\tau_m}{T_m} \right)^5 + 0.044128 \left(\frac{\tau_m}{T_m} \right)^6 - 0.0053335 \left(\frac{\tau_m}{T_m} \right)^7 + 0.00040201 \left(\frac{\tau_m}{T_m} \right)^8 \\
 &\quad - 1.7164 \cdot 10^{-5} \left(\frac{\tau_m}{T_m} \right)^9 + 3.1685 \cdot 10^{-7} \left(\frac{\tau_m}{T_m} \right)^{10}, \quad 0 < \frac{\tau_m}{T_m} < 4.1; \\
 \beta &= 1.7110073 - 0.003554 \left(\frac{\tau_m}{T_m} - 4.1 \right), \quad \frac{\tau_m}{T_m} \geq 4.1.
 \end{aligned}$$

Rule	K_c	T_i	Comment
Arvanitis <i>et al.</i> (2003b). <i>Model: Method 1</i> (continued)		Minimum performance index ^{7, 8}	
		Minimum performance index ^{9, 10}	

$$^7 \text{Performance index} = \int_0^\infty [e^2(t) + K_m^{-2} u^2(t)] dt .$$

$$^8 \beta = 2.1054 - 0.76462 \left(\frac{\tau_m}{T_m} \right) + 0.79445 \left(\frac{\tau_m}{T_m} \right)^2 - 0.51826 \left(\frac{\tau_m}{T_m} \right)^3 + 0.21771 \left(\frac{\tau_m}{T_m} \right)^4 \\ - 0.06 \left(\frac{\tau_m}{T_m} \right)^5 + 0.010927 \left(\frac{\tau_m}{T_m} \right)^6 - 0.0013006 \left(\frac{\tau_m}{T_m} \right)^7 + 9.717 \cdot 10^{-5} \left(\frac{\tau_m}{T_m} \right)^8 \\ - 4.132 \cdot 10^{-6} \left(\frac{\tau_m}{T_m} \right)^9 + 7.6235 \cdot 10^{-8} \left(\frac{\tau_m}{T_m} \right)^{10}, \quad 0 < \frac{\tau_m}{T_m} < 4.6 ;$$

$$\beta = 1.69667833406 - 0.00148768 \left(\frac{\tau_m}{T_m} - 4.6 \right), \quad \frac{\tau_m}{T_m} \geq 4.6 .$$

$$^9 \text{Performance index} = \int_0^\infty [e^2(t) + K_m^{-2} \left(\frac{du}{dt} \right)^2] dt$$

$$^{10} \beta = 1.6505623 - 0.0034985 \left(\frac{\tau_m}{T_m} - 8 \right), \quad \frac{\tau_m}{T_m} \geq 2 ;$$

$$\beta = 2.1806 - 0.60273 \left(\frac{\tau_m}{T_m} \right) + 0.3875 \left(\frac{\tau_m}{T_m} \right)^2 - 0.14955 \left(\frac{\tau_m}{T_m} \right)^3 + 0.034535 \left(\frac{\tau_m}{T_m} \right)^4 \\ - 0.0042243 \left(\frac{\tau_m}{T_m} \right)^5 + 8.2395 \cdot 10^{-5} \left(\frac{\tau_m}{T_m} \right)^6 + 5.0912 \cdot 10^{-5} \left(\frac{\tau_m}{T_m} \right)^7 - 7.283 \cdot 10^{-6} \left(\frac{\tau_m}{T_m} \right)^8 \\ + 4.2603 \cdot 10^{-7} \left(\frac{\tau_m}{T_m} \right)^9 - 9.5824 \cdot 10^{-9} \left(\frac{\tau_m}{T_m} \right)^{10}, \quad \frac{\tau_m}{T_m} \text{ range not defined.}$$

Rule	K_c	T_i	Comment
Arvanitis <i>et al.</i> (2003b). Model: Method 1	$^{11} K_c^{(115)}$	$T_i^{(115)}$	
Minimum ISE ¹²			
Minimum performance index ^{13, 14}			

$$^{11} K_c^{(115)} = \frac{12.5664(\tau_m + T_m) - 4}{K_m [2(\tau_m + T_m)(12.5664(\tau_m + T_m) - 4) - (1.5\tau_m^2 + 3\tau_m T_m + 2T_m^2)a]} \\ \text{with } a = [0.5708 + \sqrt{\frac{3.4674\tau_m + 3.1416T_m - 1}{\tau_m}}]\chi; T_i^{(115)} = \frac{12.5664(\tau_m + T_m) - 4}{a}.$$

$$^{12} \chi = 0.99315 + 1.9536\left(\frac{\tau_m}{T_m}\right) - 2.5157\left(\frac{\tau_m}{T_m}\right)^2 + 1.9163\left(\frac{\tau_m}{T_m}\right)^3 - 0.8893\left(\frac{\tau_m}{T_m}\right)^4 \\ + 0.26167\left(\frac{\tau_m}{T_m}\right)^5 - 0.049809\left(\frac{\tau_m}{T_m}\right)^6 + 0.0061098\left(\frac{\tau_m}{T_m}\right)^7 - 0.00046593\left(\frac{\tau_m}{T_m}\right)^8 \\ + 2.0083 \cdot 10^{-5}\left(\frac{\tau_m}{T_m}\right)^9 - 3.7368 \cdot 10^{-7}\left(\frac{\tau_m}{T_m}\right)^{10}, \quad 0 < \frac{\tau_m}{T_m} < 6.0; \\ \chi = 1.96705757 + 0.01308189\left(\frac{\tau_m}{T_m} - 6\right), \quad \frac{\tau_m}{T_m} \geq 6.0.$$

$$^{13} \text{Performance index} = \int_0^\infty [e^2(t) + K_m^2 u^2(t)] dt$$

$$^{14} \chi = 0.73114 + 2.3461\left(\frac{\tau_m}{T_m}\right) - 2.8509\left(\frac{\tau_m}{T_m}\right)^2 + 2.0965\left(\frac{\tau_m}{T_m}\right)^3 - 0.95252\left(\frac{\tau_m}{T_m}\right)^4 \\ + 0.27639\left(\frac{\tau_m}{T_m}\right)^5 - 0.052089\left(\frac{\tau_m}{T_m}\right)^6 + 0.0063413\left(\frac{\tau_m}{T_m}\right)^7 - 0.00048064\left(\frac{\tau_m}{T_m}\right)^8 \\ + 2.0611 \cdot 10^{-5}\left(\frac{\tau_m}{T_m}\right)^9 - 3.8178 \cdot 10^{-7}\left(\frac{\tau_m}{T_m}\right)^{10}, \quad 0 < \frac{\tau_m}{T_m} < 4.5; \\ \chi = 1.916423116 + 0.018175439\left(\frac{\tau_m}{T_m} - 4.5\right), \quad \frac{\tau_m}{T_m} \geq 4.5.$$

Rule	K_c	T_i	Comment
Arvanitis <i>et al.</i> (2003b). <i>Model: Method 1</i> (continued)	Minimum performance index ^{15, 16}		
Minimum performance index: servo tuning			
Arvanitis <i>et al.</i> (2003b). <i>Model: Method 1</i>	$\frac{4}{(8-\beta)K_m \tau_m}$	$\frac{4\tau_m}{\beta}$	
Minimum ISE ¹⁷			

$$^{15} \text{Performance index} = \int_0^{\infty} [e^2(t) + K_m^2 \left(\frac{du}{dt} \right)^2] dt$$

$$^{16} \chi = 0.60896 + 3.0593 \left(\frac{\tau_m}{T_m} \right) - 3.9339 \left(\frac{\tau_m}{T_m} \right)^2 + 2.933 \left(\frac{\tau_m}{T_m} \right)^3 - 1.3353 \left(\frac{\tau_m}{T_m} \right)^4 \\ + 0.38696 \left(\frac{\tau_m}{T_m} \right)^5 - 0.072771 \left(\frac{\tau_m}{T_m} \right)^6 + 0.0088395 \left(\frac{\tau_m}{T_m} \right)^7 - 0.00066864 \left(\frac{\tau_m}{T_m} \right)^8 \\ + 2.8623 \cdot 10^{-5} \left(\frac{\tau_m}{T_m} \right)^9 - 5.2941 \cdot 10^{-7} \left(\frac{\tau_m}{T_m} \right)^{10}, \quad 0 < \frac{\tau_m}{T_m} < 4.5;$$

$$\chi = 1.9283751 + 0.016846129 \left(\frac{\tau_m}{T_m} - 4.5 \right), \quad \frac{\tau_m}{T_m} \geq 4.5.$$

$$^{17} \beta = 1.997 - 0.82781 \left(\frac{\tau_m}{T_m} \right) + 1.0002 \left(\frac{\tau_m}{T_m} \right)^2 - 0.72106 \left(\frac{\tau_m}{T_m} \right)^3 + 0.32269 \left(\frac{\tau_m}{T_m} \right)^4 \\ - 0.092452 \left(\frac{\tau_m}{T_m} \right)^5 + 0.017215 \left(\frac{\tau_m}{T_m} \right)^6 - 0.0020706 \left(\frac{\tau_m}{T_m} \right)^7 + 0.00015505 \left(\frac{\tau_m}{T_m} \right)^8 \\ - 6.5691 \cdot 10^{-6} \left(\frac{\tau_m}{T_m} \right)^9 + 1.2025 \cdot 10^{-7} \left(\frac{\tau_m}{T_m} \right)^{10}, \quad 0 < \frac{\tau_m}{T_m} < 5.5;$$

$$\beta = 1.6281745 - 0.001171 \left(\frac{\tau_m}{T_m} - 5.5 \right), \quad \frac{\tau_m}{T_m} \geq 5.5.$$

Rule	K_c	T_i	Comment
Arvanitis <i>et al.</i> (2003b).			Minimum performance index ^{18, 19} ,
Model: Method 1 (continued)			Minimum performance index ^{20, 21} ,

$$^{18} \text{Performance index} = \int_0^{\infty} [e^2(t) + K_m^2 u^2(t)] dt$$

$$\begin{aligned} ^{19} \beta = & 1.5693 + 0.1066 \left(\frac{\tau_m}{T_m} \right) - 0.10101 \left(\frac{\tau_m}{T_m} \right)^2 + 0.05217 \left(\frac{\tau_m}{T_m} \right)^3 - 0.016147 \left(\frac{\tau_m}{T_m} \right)^4 \\ & + 0.0030412 \left(\frac{\tau_m}{T_m} \right)^5 - 0.00032187 \left(\frac{\tau_m}{T_m} \right)^6 + 1.2181 \cdot 10^{-5} \left(\frac{\tau_m}{T_m} \right)^7 + 9.9698 \cdot 10^{-7} \left(\frac{\tau_m}{T_m} \right)^8 \\ & - 1.2274 \cdot 10^{-7} \left(\frac{\tau_m}{T_m} \right)^9 + 3.7365 \cdot 10^{-9} \left(\frac{\tau_m}{T_m} \right)^{10}, \quad 0 < \frac{\tau_m}{T_m} < 4.6. \end{aligned}$$

$$^{20} \text{Performance index} = \int_0^{\infty} \left[e^2(t) + K_m^2 \left(\frac{du}{dt} \right)^2 \right] dt$$

$$^{21} \beta = 0.780651417 + 4.7464131 \left(\frac{\tau_m}{T_m} - 0.1 \right), \quad 0 < \frac{\tau_m}{T_m} < 0.3;$$

$$\beta = 1.729934 - 0.051535 \left(\frac{\tau_m}{T_m} - 0.3 \right), \quad 0.3 \leq \frac{\tau_m}{T_m} < 2.$$

Rule	K_c	T_i	Comment
Arvanitis <i>et al.</i> (2003b). Model: Method 1	$^{22} K_c^{(116)}$	$T_i^{(116)}$	
		Minimum ISE ²³	
		Minimum performance index ^{24, 25} ,	

$$^{22} K_c^{(116)} = \frac{12.5664(\tau_m + T_m) - 4}{K_m [2(\tau_m + T_m) \{12.5664(\tau_m + T_m) - 4\} - (1.5\tau_m^2 + 3\tau_m T_m + 2T_m^2)]},$$

with $a = [0.5708 + \sqrt{\frac{3.4674\tau_m + 3.1416T_m - 1}{\tau_m}}]\chi$, $T_i^{(116)} = \frac{12.5664(\tau_m + T_m) - 4}{a}$.

$$^{23} \chi = 0.79001 + 2.114\left(\frac{\tau_m}{T_m}\right) - 2.5601\left(\frac{\tau_m}{T_m}\right)^2 + 1.8971\left(\frac{\tau_m}{T_m}\right)^3 - 0.86871\left(\frac{\tau_m}{T_m}\right)^4 \\ + 0.25371\left(\frac{\tau_m}{T_m}\right)^5 - 0.048068\left(\frac{\tau_m}{T_m}\right)^6 + 0.0058771\left(\frac{\tau_m}{T_m}\right)^7 - 0.00044707\left(\frac{\tau_m}{T_m}\right)^8 \\ + 1.9231 \cdot 10^{-5}\left(\frac{\tau_m}{T_m}\right)^9 - 3.5718 \cdot 10^{-7}\left(\frac{\tau_m}{T_m}\right)^{10}, \quad 0 < \frac{\tau_m}{T_m} < 4.5; \\ \chi = 1.903177576 + 0.0181584\left(\frac{\tau_m}{T_m} - 4.5\right), \quad \frac{\tau_m}{T_m} \geq 4.5.$$

$$^{24} \text{Performance index} = \int_0^\infty [e^2(t) + K_m^2 u^2(t)] dt.$$

$$^{25} \chi = 0.48124 + 2.5549\left(\frac{\tau_m}{T_m}\right) - 2.896\left(\frac{\tau_m}{T_m}\right)^2 + 2.0495\left(\frac{\tau_m}{T_m}\right)^3 - 0.91115\left(\frac{\tau_m}{T_m}\right)^4 \\ + 0.26094\left(\frac{\tau_m}{T_m}\right)^5 - 0.048775\left(\frac{\tau_m}{T_m}\right)^6 + 0.0059064\left(\frac{\tau_m}{T_m}\right)^7 - 0.00044618\left(\frac{\tau_m}{T_m}\right)^8 \\ + 1.9095 \cdot 10^{-5}\left(\frac{\tau_m}{T_m}\right)^9 - 3.5335 \cdot 10^{-7}\left(\frac{\tau_m}{T_m}\right)^{10}, \quad 0 < \frac{\tau_m}{T_m} < 4.0; \\ \chi = 1.87054524 + 0.021802\left(\frac{\tau_m}{T_m} - 4\right), \quad \frac{\tau_m}{T_m} \geq 4.0.$$

Rule	K_c	T_i	Comment
Arvanitis <i>et al.</i> (2003b). Model: Method 1 (continued)	Minimum performance index ^{26, 27} ,		
Direct synthesis			
Arvanitis <i>et al.</i> (2003b). Model: Method 1	$\frac{4}{(8-\beta)K_m \tau_m}$	$\frac{4\tau_m}{\beta}$	$\beta = \frac{4}{4\xi^2 + 1}$
	$^{28} K_c^{(117)}$	$(1 + 4\xi^2)(\tau_m + T_m)$	

$$^{26} \text{Performance index} = \int_0^\infty e^2(t) + K_m^2 \left(\frac{du}{dt} \right)^2 dt .$$

$$^{27} \chi = 0.31373 + 3.7796 \left(\frac{\tau_m}{T_m} \right) - 5.0294 \left(\frac{\tau_m}{T_m} \right)^2 + 3.8589 \left(\frac{\tau_m}{T_m} \right)^3 - 1.7956 \left(\frac{\tau_m}{T_m} \right)^4 \\ + 0.52906 \left(\frac{\tau_m}{T_m} \right)^5 - 0.10079 \left(\frac{\tau_m}{T_m} \right)^6 + 0.012369 \left(\frac{\tau_m}{T_m} \right)^7 - 0.00094357 \left(\frac{\tau_m}{T_m} \right)^8 \\ + 4.0678 \cdot 10^{-5} \left(\frac{\tau_m}{T_m} \right)^9 - 7.5694 \cdot 10^{-7} \left(\frac{\tau_m}{T_m} \right)^{10}, \quad 0 < \frac{\tau_m}{T_m} < 3.5 ;$$

$$\chi = 1.86144723 + 0.021989 \left(\frac{\tau_m}{T_m} - 3.5 \right), \quad \frac{\tau_m}{T_m} \geq 3.5 .$$

$$^{28} K_c^{(117)} = \frac{(1 + 4\xi^2)(\tau_m + T_m)}{K_m [\tau_m^2 (0.5 + 8\xi^2) + \tau_m T_m (1 + 16\xi^2) + 8\xi^2 T_m^2]} .$$

3.6.4 Controller with proportional term acting on the output 2

$$U(s) = K_c \left(1 + \frac{1}{T_i s} \right) E(s) - K_1 Y(s)$$

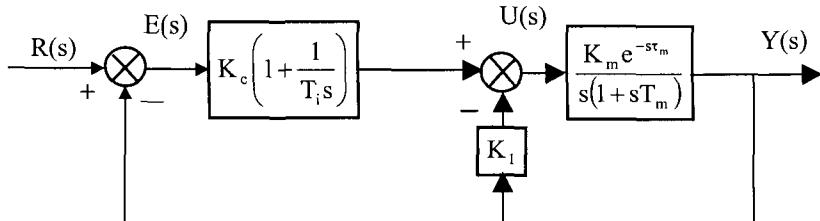


Table 23: PI controller tuning rules – FOLIPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{s(1+sT_m)}$

Rule	K_c	T_i	Comment
Robust			
Lee and Edgar (2002). Model: Method 1	²⁹ $K_c^{(118)}$	$T_i^{(118)}$	$K_1 = 0.25K_u$

²⁹ $K_c^{(118)} = \frac{0.25K_u}{(\lambda + \tau_m)} \left[\frac{4}{K_u K_m} - \tau_m + \frac{\tau_m^2}{2(\lambda + \tau_m)} \right]$, $T_i^{(118)} = \frac{4}{K_u K_m} - \tau_m + \frac{\tau_m^2}{2(\lambda + \tau_m)}$.

3.7 SOSPD model $\frac{K_m e^{-s\tau_m}}{T_{m1}^2 s^2 + 2\xi_m T_{m1}s + 1}$ or $\frac{K_m e^{-s\tau_m}}{(1 + T_{m1}s)(1 + T_{m2}s)}$

3.7.1 Ideal controller $G_c(s) = K_c \left(1 + \frac{1}{T_i s} \right)$

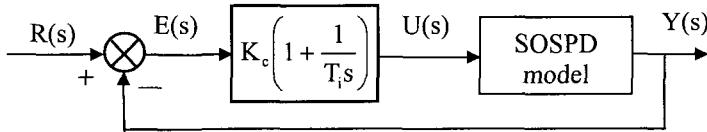


Table 24: PI controller tuning rules – SOSPD model

$$\frac{K_m e^{-s\tau_m}}{T_{m1}^2 s^2 + 2\xi_m T_{m1}s + 1} \text{ or } \frac{K_m e^{-s\tau_m}}{(1 + T_{m1}s)(1 + T_{m2}s)}$$

Rule	K_c	T_i	Comment
Minimum performance index: regulator tuning			
Minimum IAE – Shinskey (1994) – page 158.	${}^1 K_c^{(119)}$	$T_i^{(119)}$	<i>Model: Method 1</i>
<i>Model: Method 1</i> Minimum IAE – Shinskey (1996) – page 48. $\frac{\tau_m}{T_{m1}} = 0.2$	$\frac{0.77T_{m1}}{K_m \tau_m}$	$2.83(\tau_m + T_{m2})$	$\frac{T_{m2}}{T_{m1}} = 0.1$
	$\frac{0.70T_{m1}}{K_m \tau_m}$	$2.65(\tau_m + T_{m2})$	$\frac{T_{m2}}{T_{m1}} = 0.2$
	$\frac{0.80T_{m1}}{K_m \tau_m}$	$2.29(\tau_m + T_{m2})$	$\frac{T_{m2}}{T_{m1}} = 0.5$
	$\frac{0.80T_{m1}}{K_m \tau_m}$	$1.67(\tau_m + T_{m2})$	$\frac{T_{m2}}{T_{m1}} = 1.0$

$${}^1 K_c^{(119)} = \frac{100T_{m1}}{K_m (\tau_m + T_{m2}) \left(50 + 55 \left[1 - e^{-\frac{T_{m1}}{\tau_m + T_{m2}}} \right] \right)},$$

$$T_i^{(119)} = \tau_m \left(0.5 + 3.5 \left[1 - e^{-\frac{3T_{m1}}{(\tau_m + T_{m2})}} \right] \right).$$

Rule	K_c	T_i	Comment
Minimum IAE – Huang <i>et al.</i> (1996). <i>Model: Method 1</i>	$^2 K_c^{(120)}$	$T_i^{(120)}$	$0 < \frac{T_{m2}}{T_{m1}} \leq 1 ;$ $0.1 \leq \frac{\tau_m}{T_{m1}} \leq 1$

² Note: equations continued into the footnote on page 104.

$$\begin{aligned}
K_c^{(120)} = & \frac{1}{K_m} \left[6.4884 + 4.6198 \frac{\tau_m}{T_{m1}} - 3.491 \frac{T_{m2}}{T_{m1}} - 25.3143 \frac{\tau_m T_{m2}}{T_{m1}^2} \right] \\
& + \frac{1}{K_m} \left[0.8196 \left(\frac{\tau_m}{T_{m1}} \right)^{-0.9077} - 5.2132 \left(\frac{\tau_m}{T_{m1}} \right)^{-0.063} - 7.2712 \left(\frac{\tau_m}{T_{m1}} \right)^{0.5961} \right] \\
& + \frac{1}{K_m} \left[-18.0448 \left(\frac{T_{m2}}{T_{m1}} \right)^{0.7204} + 5.3263 \left(\frac{T_{m2}}{T_{m1}} \right)^{1.0049} + 13.9108 \left(\frac{T_{m2}}{T_{m1}} \right)^{1.005} \right] \\
& + \frac{1}{K_m} \left[0.4937 \frac{T_{m2}}{\tau_m} + 19.1783 \frac{T_{m2}}{T_{m1}} \left(\frac{\tau_m}{T_{m1}} \right)^{0.8529} + 12.2494 \frac{T_{m2}}{T_{m1}} \left(\frac{\tau_m}{T_{m1}} \right)^{0.5613} \right] \\
& + \frac{1}{K_m} \left[8.4355 \frac{\tau_m}{T_{m1}} \left(\frac{T_{m2}}{T_{m1}} \right)^{0.557} - 17.6781 \frac{\tau_m}{T_{m1}} \left(\frac{T_{m2}}{T_{m1}} \right)^{1.1818} \right] \\
& + \frac{1}{K_m} \left[-0.7241 e^{\frac{\tau_m}{T_{m1}}} - 2.2525 e^{\frac{T_{m2}}{T_{m1}}} + 5.4959 e^{\frac{\tau_m T_{m2}}{T_{m1}^2}} \right],
\end{aligned}$$

$$\begin{aligned}
T_i^{(120)} = & T_{m1} \left[0.0064 + 3.9574 \frac{\tau_m}{T_{m1}} + 4.4087 \frac{T_{m2}}{T_{m1}} - 6.4789 \left(\frac{\tau_m}{T_{m1}} \right)^2 \right] \\
& + T_{m1} \left[-12.8702 \frac{\tau_m T_{m2}}{T_{m1}^2} - 1.5083 \left(\frac{T_{m2}}{T_{m1}} \right)^2 + 9.4348 \left(\frac{\tau_m}{T_{m1}} \right)^3 \right] \\
& + T_{m1} \left[17.0736 \frac{T_{m2}}{T_{m1}} \left(\frac{\tau_m}{T_{m1}} \right)^2 + 15.9816 \frac{\tau_m}{T_{m1}} \left(\frac{T_{m2}}{T_{m1}} \right)^2 - 3.909 \left(\frac{T_{m2}}{T_{m1}} \right)^3 \right] \\
& + T_{m1} \left[-10.7619 \left(\frac{\tau_m}{T_{m1}} \right)^4 - 10.864 \frac{T_{m2}}{T_{m1}} \left(\frac{\tau_m}{T_{m1}} \right)^3 - 22.3194 \left(\frac{\tau_m}{T_{m1}} \right)^2 \left(\frac{T_{m2}}{T_{m1}} \right)^2 \right] \\
& + T_{m1} \left[-6.6602 \frac{\tau_m}{T_{m1}} \left(\frac{T_{m2}}{T_{m1}} \right)^3 + 6.8122 \left(\frac{T_{m2}}{T_{m1}} \right)^4 + 7.5146 \left(\frac{\tau_m}{T_{m1}} \right)^5 \right]
\end{aligned}$$

Rule	K_c	T_i	Comment
Minimum IAE – Huang <i>et al.</i> (1996). <i>Model: Method 1</i>	$^3 K_c^{(121)}$	$T_i^{(121)}$	$0.4 \leq \xi_m \leq 1$; $0.05 \leq \frac{\tau_m}{T_{m1}} \leq 1$

$$\begin{aligned}
& + T_{m1} \left[2.8724 \frac{T_{m2}}{T_{m1}} \left(\frac{\tau_m}{T_{m1}} \right)^4 + 11.4666 \left(\frac{T_{m2}}{T_{m1}} \right)^2 \left(\frac{\tau_m}{T_{m1}} \right)^3 + 11.1207 \left(\frac{\tau_m}{T_{m1}} \right)^2 \left(\frac{T_{m2}}{T_{m1}} \right)^3 \right] \\
& + T_{m1} \left[-1.2174 \left(\frac{\tau_m}{T_{m1}} \right) \left(\frac{T_{m2}}{T_{m1}} \right)^4 - 4.3675 \left(\frac{T_{m2}}{T_{m1}} \right)^5 - 2.2236 \left(\frac{\tau_m}{T_{m1}} \right)^6 \right] \\
& + T_{m1} \left[-0.112 \frac{T_{m2}}{T_{m1}} \left(\frac{\tau_m}{T_{m1}} \right)^5 + 1.0308 \left(\frac{T_{m2}}{T_{m1}} \right)^6 - 1.9136 \left(\frac{\tau_m}{T_{m1}} \right)^4 \left(\frac{T_{m2}}{T_{m1}} \right)^2 \right] \\
& + T_{m1} \left[-3.4994 \left(\frac{\tau_m}{T_{m1}} \right)^3 \left(\frac{T_{m2}}{T_{m1}} \right)^3 - 1.5777 \left(\frac{\tau_m}{T_{m1}} \right)^2 \left(\frac{T_{m2}}{T_{m1}} \right)^4 + 1.1408 \frac{\tau_m}{T_{m1}} \left(\frac{T_{m2}}{T_{m1}} \right)^5 \right].
\end{aligned}$$

³ Note: equations continued into the footnote on page 105.

$$\begin{aligned}
K_c^{(121)} = & \frac{1}{K_m} \left[-10.4183 - 20.9497 \frac{\tau_m}{T_{m1}} - 5.5175 \xi_m - 26.5149 \xi_m \frac{\tau_m}{T_{m1}} \right] \\
& + \frac{1}{K_m} \left[42.7745 \left(\frac{\tau_m}{T_{m1}} \right)^{1.4439} + 10.5069 \left(\frac{\tau_m}{T_{m1}} \right)^{0.1456} + 15.4103 \left(\frac{\tau_m}{T_{m1}} \right)^{0.3157} \right] \\
& + \frac{1}{K_m} \left[34.3236 \xi_m^{3.7057} - 17.8860 \xi_m^{4.5359} - 54.0584 \xi_m^{1.9593} \right] \\
& + \frac{1}{K_m} \left[22.4263 \xi_m \left(\frac{\tau_m}{T_{m1}} \right)^{-0.0541} + 2.7497 \xi_m \left(\frac{\tau_m}{T_{m1}} \right)^{4.7426} \right] \\
& + \frac{1}{K_m} \left[50.2197 \xi_m^{1.8288} \left(\frac{\tau_m}{T_{m1}} \right) - 17.1968 \frac{\tau_m}{T_{m1}} \xi_m^{2.7227} + 1.0293 \xi_m \frac{T_{m1}}{\tau_m} \right] \\
& + \frac{1}{K_m} \left[-16.7667 e^{\frac{\tau_m}{T_{m1}}} + 14.5737 e^{\xi_m} - 7.3025 e^{\xi_m \frac{\tau_m}{T_{m1}}} \right],
\end{aligned}$$

$$\begin{aligned}
T_i^{(121)} = & T_{m1} \left[11.447 + 4.5128 \frac{\tau_m}{T_{m1}} - 75.2486 \xi_m - 110.807 \left(\frac{\tau_m}{T_{m1}} \right)^2 - 12.282 \xi_m \frac{\tau_m}{T_{m1}} \right] \\
& + T_{m1} \left[345.3228 \xi_m^2 + 191.9539 \left(\frac{\tau_m}{T_{m1}} \right)^3 + 359.3345 \xi_m \left(\frac{\tau_m}{T_{m1}} \right)^2 \right]
\end{aligned}$$

Rule	K_c	T_i	Comment									
Minimum IAE – Shinskey (1996) – page 121. Model: Method 1	$0.48K_u$	$0.83T_u$	$\frac{\tau_m}{T_{m1}} = 0.2$, $\frac{T_{m2}}{T_{m1}} = 0.2$									
Minimum ISE – McAvoy and Johnson (1967). Model: Method 1; Coefficients of K_c and T_i deduced from graphs	x_1/K_m	$x_2 \tau_m$	Coefficient values									
	ξ_m	$\frac{T_{m1}}{\tau_m}$	x_1	x_2	ξ_m	$\frac{T_{m1}}{\tau_m}$	x_1	x_2	ξ_m	$\frac{T_{m1}}{\tau_m}$	x_1	x_2
	1	0.5	0.8	1.82	4	0.5	4.3	3.45	7	0.5	7.8	3.85
	1	4.0	5.7	12.5	4	4.0	27.1	6.67	7	4.0	51.2	5.88
	1	10.0	13.6	25.0								

$$\begin{aligned}
& + T_{m1} \left[-158.7611 \frac{\tau_m}{T_{m1}} \xi_m^2 - 770.2897 \xi_m^3 - 153.633 \left(\frac{\tau_m}{T_{m1}} \right)^4 \right] \\
& + T_{m1} \left[-412.5409 \xi_m \left(\frac{\tau_m}{T_{m1}} \right)^3 - 414.7786 \xi_m^2 \left(\frac{\tau_m}{T_{m1}} \right)^2 + 485.0976 \frac{\tau_m}{T_{m1}} \xi_m^3 \right] \\
& + T_{m1} \left[864.5195 \xi_m^4 + 55.4366 \left(\frac{\tau_m}{T_{m1}} \right)^5 + 222.2685 \xi_m \left(\frac{\tau_m}{T_{m1}} \right)^4 \right] \\
& + T_{m1} \left[275.116 \xi_m^2 \left(\frac{\tau_m}{T_{m1}} \right)^3 + 205.2493 \left(\frac{\tau_m}{T_{m1}} \right)^2 \xi_m^3 - 479.5627 \left(\frac{\tau_m}{T_{m1}} \right) \xi_m^4 \right] \\
& + T_{m1} \left[-473.1346 \xi_m^5 - 6.547 \left(\frac{\tau_m}{T_{m1}} \right)^6 - 43.2822 \xi_m \left(\frac{\tau_m}{T_{m1}} \right)^5 + 99.8717 \xi_m^6 \right] \\
& + T_{m1} \left[-73.5666 \left(\frac{\tau_m}{T_{m1}} \right)^4 \xi_m^2 - 56.4418 \left(\frac{\tau_m}{T_{m1}} \right)^3 \xi_m^3 - 37.497 \left(\frac{\tau_m}{T_{m1}} \right)^2 \xi_m^4 \right] \\
& + T_{m1} \left[160.7714 \frac{\tau_m}{T_{m1}} \xi_m^5 \right].
\end{aligned}$$

Rule	K _c				T _i				Comment			
Minimum ITAE – Lopez <i>et al.</i> (1969).	x_1/K_m				$x_2 T_m$							
Coefficient values												
<i>Model: Method 7; Coefficients of K_c and T_i deduced from graphs</i>	ξ_m	$\frac{\tau_m}{T_{m1}}$	x_1	x_2	ξ_m	$\frac{\tau_m}{T_{m1}}$	x_1	x_2	ξ_m	$\frac{\tau_m}{T_{m1}}$	x_1	x_2
	0.5	0.1	3.0	2.86	1	0.1	7.0	2.00	4	0.1	40.0	0.83
	0.5	1.0	0.2	0.83	1	1.0	0.95	2.22	4	1.0	6.0	3.33
	0.5	10.0	0.3	4.0	1	10.0	0.35	5.00	4	10.0	0.75	10.0
Minimum ITAE – Chao <i>et al.</i> (1989).	${}^4 K_c^{(122)}$				$T_i^{(122)}$							
<i>Model: Method 1</i>	$0.7 \leq \xi_m \leq 3 ; 0.05 \leq \frac{\tau_m}{2\xi_m T_{m1}} \leq 0.5$											
Minimum ITAE – Hassan (1993). <i>Model: Method 7</i>	${}^5 K_c^{(123)}$				$T_i^{(123)}$				$0.5 \leq \xi_m \leq 2 ,$ $0.1 \leq \frac{\tau_m}{T_{m1}} \leq 4$			

$${}^4 K_c^{(122)} = \frac{0.09578 + 0.6206\xi_m - 0.1069\xi_m^2}{K_m} \left(\frac{\tau_m}{2\xi_m T_{m1}} \right)^{-1.108+0.2558\xi_m-0.0579\xi_m^2},$$

$$T_i^{(122)} = 2\xi_m T_{m1} \left(0.31 - 0.0914\xi_m + 0.07464\xi_m^2 \left(\frac{\tau_m}{2\xi_m T_{m1}} \right) \right)^{-0.6441+0.7626\xi_m-0.1193\xi_m^2}.$$

⁵ Formulae correspond to graphs of Lopez *et al.* (1969). $K_c^{(123)}$ is obtained as follows:

$$\log[K_m K_c^{(123)}] = -0.0099458 + 1.9743700\xi_m - 3.8984310 \frac{\tau_m}{T_{m1}} - 1.1225270\xi_m^2 + 2.3493280 \left(\frac{\tau_m}{T_{m1}} \right)^2 + 1.9126490\xi_m \frac{\tau_m}{T_{m1}} + 0.0754568\xi_m^2 \left(\frac{\tau_m}{T_{m1}} \right)^2 - 0.5594610\xi_m \left(\frac{\tau_m}{T_{m1}} \right)^2 - 0.7930846\xi_m^2 \frac{\tau_m}{T_{m1}} + 0.2412770\xi_m^3 - 0.3567954 \left(\frac{\tau_m}{T_{m1}} \right)^3 - 0.0024730\xi_m \left(\frac{\tau_m}{T_{m1}} \right)^3 + 0.0478593\xi_m^2 \left(\frac{\tau_m}{T_{m1}} \right)^2 - 0.1354322\xi_m^3 \frac{\tau_m}{T_{m1}} - 1.1756470\xi_m^3 \left(\frac{\tau_m}{T_{m1}} \right)^2 - 0.0096974\xi_m^3 \left(\frac{\tau_m}{T_{m1}} \right)^3;$$

$T_i^{(123)}$ is obtained as follows (equation continued into the footnote on page 107):

$$\log \left[\frac{T_i^{(123)}}{T_{m1}} \right] = 0.9321658 - 0.7200651\xi_m - 3.4227010 \frac{\tau_m}{T_{m1}} + 0.0432084\xi_m^2$$

Rule	K_c	T_i	Comment
Minimum ITSE – Chao <i>et al.</i> (1989). <i>Model: Method 1</i>	$^6 K_c^{(124)}$	$T_i^{(124)}$	
Nearly minimum IAE, ISE, ITAE – Hwang (1995). <i>Model: Method 10</i>	$^7 K_c^{(125)}$	$T_i^{(125)}$	Decay ratio = 0.2

$$\begin{aligned}
 & + 1.4965440 \left(\frac{\tau_m}{T_{m1}} \right)^2 + 5.3636520 \xi_m \frac{\tau_m}{T_{m1}} - 0.0848431 \xi_m^2 \left(\frac{\tau_m}{T_{m1}} \right)^2 \\
 & - 1.6011510 \xi_m \left(\frac{\tau_m}{T_{m1}} \right)^2 - 2.1777800 \xi_m^2 \frac{\tau_m}{T_{m1}} + 0.0404586 \xi_m^3 \\
 & - 0.1769621 \left(\frac{\tau_m}{T_{m1}} \right)^3 + 0.0912008 \xi_m \left(\frac{\tau_m}{T_{m1}} \right)^3 + 0.1501474 \xi_m^2 \left(\frac{\tau_m}{T_{m1}} \right)^2 \\
 & + 0.3033341 \xi_m^3 \frac{\tau_m}{T_{m1}} + 0.1753407 \xi_m^3 \left(\frac{\tau_m}{T_{m1}} \right)^2 - 0.0622614 \xi_m^3 \left(\frac{\tau_m}{T_{m1}} \right)^3.
 \end{aligned}$$

$$^6 K_c^{(124)} = \frac{0.3366 + 0.6355 \xi_m - 0.1066 \xi_m^2}{K_m} \left(\frac{\tau_m}{2 \xi_m T_{m1}} \right)^{-1.0167+0.1743 \xi_m - 0.03946 \xi_m^2},$$

$$T_i^{(124)} = 2 \xi_m T_{m1} \left(0.9255 - 0.3607 \xi_m + 0.1299 \xi_m^2 \left(\frac{\tau_m}{2 \xi_m T_{m1}} \right) \right)^{-0.5848+0.7294 \xi_m - 0.1136 \xi_m^2}.$$

⁷ $0.2 \leq \frac{\tau_m}{T_{m1}} \leq 2.0$, $0.6 \leq \xi_m \leq 4.2$. Equations continued into the footnote on page 108.

$$K_c^{(125)} = \left(1 - \frac{0.622 [1 - 0.435 \omega_H \tau_m + 0.052 (\omega_H \tau_m)^2]}{K_H K_m / (1 + K_H K_m)} \right) K_H,$$

$$T_i^{(125)} = \frac{K_c^{(125)} (1 + K_H K_m)}{0.0697 \omega_H K_m (1 + 0.752 \omega_H \tau_m - 0.145 \omega_H^2 \tau_m^2)}, \quad \varepsilon < 2.4;$$

$$K_c^{(125)} = \left(1 - \frac{0.724 [1 - 0.469 \omega_H \tau_m + 0.0609 (\omega_H \tau_m)^2]}{K_H K_m / (1 + K_H K_m)} \right) K_H,$$

$$T_i^{(125)} = \frac{K_c^{(125)} (1 + K_H K_m)}{0.0405 \omega_H K_m (1 + 1.93 \omega_H \tau_m - 0.363 \omega_H^2 \tau_m^2)}, \quad 2.4 \leq \varepsilon < 3;$$

$$K_c^{(125)} = \left(1 - \frac{1.26 (0.506)^{\omega_H \tau_m} [1 - 1.07/\varepsilon + 0.616/\varepsilon^2]}{K_H K_m / (1 + K_H K_m)} \right) K_H,$$

Rule	K_c	T_i	Comment			
Minimum performance index: servo tuning						
x_1/K_m		$x_2(T_{m1} + T_{m2} + \tau_m)$				
Representative coefficient values – deduced from graphs						
	$\frac{\tau_m}{2T_{m1}}$	x_1	x_2	$\frac{\tau_m}{2T_{m1}}$	x_1	x_2
Minimum IAE – Gallier and Otto (1968). <i>Model: Method 19.</i>	0.053	3.6	1.35	0.67	0.76	0.72
	0.11	2.3	1.17	1.50	0.53	0.60
	0.25	1.35	0.93	4.0	0.43	0.51
Minimum IAE - Huang <i>et al.</i> (1996). <i>Model: Method 1</i>	$^8 K_c^{(126)}$		$T_i^{(126)}$		$0 < \frac{T_{m2}}{T_{m1}} \leq 1$; $0.1 \leq \frac{\tau_m}{T_{m1}} \leq 1$	

$$T_i^{(125)} = \frac{K_c^{(125)}(1 + K_H K_m)}{0.0661\omega_H K_m (1 + 0.824 \ln[\omega_H \tau_m]) (1 + 1.71/\varepsilon - 1.17/\varepsilon^2)}, \quad 3 \leq \varepsilon < 20;$$

$$K_c^{(125)} = \left(1 - \frac{1.09[1 - 0.497\omega_H \tau_m + 0.0724(\omega_H \tau_m)^2]}{K_H K_m / (1 + K_H K_m)} \right) K_H,$$

$$T_i^{(125)} = \frac{K_c^{(125)}(1 + K_H K_m)}{0.054\omega_H K_m (-1 + 2.54\omega_H \tau_m - 0.457\omega_H^2 \tau_m^2)}, \quad \varepsilon > 20.$$

⁸ Note: equations continued into the footnote on page 109.

$$\begin{aligned}
 K_c^{(126)} = & \frac{1}{K_m} \left[-13.0454 - 9.0916 \frac{\tau_m}{T_{m1}} + 2.6647 \frac{T_{m2}}{T_{m1}} + 9.162 \frac{\tau_m T_{m2}}{T_{m1}^2} \right] \\
 & + \frac{1}{K_m} \left[0.3053 \left(\frac{\tau_m}{T_{m1}} \right)^{-1.0169} + 1.1075 \left(\frac{\tau_m}{T_{m1}} \right)^{3.5959} - 2.2927 \left(\frac{\tau_m}{T_{m1}} \right)^{3.6843} \right] \\
 & + \frac{1}{K_m} \left[-31.0306 \left(\frac{T_{m2}}{T_{m1}} \right)^{0.8476} - 13.0155 \left(\frac{T_{m2}}{T_{m1}} \right)^{2.6083} + 9.6899 \left(\frac{T_{m2}}{T_{m1}} \right)^{2.9049} \right] \\
 & + \frac{1}{K_m} \left[-0.6418 \frac{T_{m2}}{\tau_m} + 18.9643 \frac{T_{m2}}{T_{m1}} \left(\frac{\tau_m}{T_{m1}} \right)^{-0.2016} - 39.7340 \frac{T_{m2}}{T_{m1}} \left(\frac{\tau_m}{T_{m1}} \right)^{1.3293} \right] \\
 & + \frac{1}{K_m} \left[28.155 \frac{\tau_m}{T_{m1}} \left(\frac{T_{m2}}{T_{m1}} \right)^{0.801} - 2.0067 \frac{\tau_m}{T_{m1}} \left(\frac{T_{m2}}{T_{m1}} \right)^{3.956} \right] \\
 & + \frac{1}{K_m} \left[4.8259e^{\frac{\tau_m}{T_{m1}}} + 2.1137e^{\frac{T_{m2}}{T_{m1}}} + 8.4511e^{-\frac{\tau_m T_{m2}}{T_{m1}^2}} \right],
 \end{aligned}$$

Rule	K_c	T_i	Comment
Minimum IAE - Huang <i>et al.</i> (1996). Model: Method 1	⁹ $K_c^{(127)}$	$T_i^{(127)}$	$0.4 \leq \xi_m \leq 1$; $0.05 \leq \frac{\tau_m}{T_{m1}} \leq 1$

$$\begin{aligned}
T_i^{(126)} = & T_{m1} \left[0.9771 - 0.2492 \frac{\tau_m}{T_{m1}} + 0.8753 \frac{T_{m2}}{T_{m1}} + 3.4651 \left(\frac{\tau_m}{T_{m1}} \right)^2 - 3.8516 \frac{\tau_m T_{m2}}{T_{m1}^2} \right] \\
& + T_{m1} \left[7.5106 \left(\frac{T_{m2}}{T_{m1}} \right)^2 - 7.4538 \left(\frac{\tau_m}{T_{m1}} \right)^3 + 11.6768 \frac{T_{m2}}{T_{m1}} \left(\frac{\tau_m}{T_{m1}} \right)^2 \right] \\
& + T_{m1} \left[-10.9909 \frac{\tau_m}{T_{m1}} \left(\frac{T_{m2}}{T_{m1}} \right)^2 - 16.1461 \left(\frac{T_{m2}}{T_{m1}} \right)^3 + 8.2567 \left(\frac{\tau_m}{T_{m1}} \right)^4 \right] \\
& + T_{m1} \left[-18.1011 \frac{T_{m2}}{T_{m1}} \left(\frac{\tau_m}{T_{m1}} \right)^3 + 6.2208 \left(\frac{\tau_m}{T_{m1}} \right)^2 \left(\frac{T_{m2}}{T_{m1}} \right)^2 + 21.9893 \frac{\tau_m}{T_{m1}} \left(\frac{T_{m2}}{T_{m1}} \right)^3 \right] \\
& + T_{m1} \left[15.8538 \left(\frac{T_{m2}}{T_{m1}} \right)^4 - 4.7536 \left(\frac{\tau_m}{T_{m1}} \right)^5 + 14.5405 \frac{T_{m2}}{T_{m1}} \left(\frac{\tau_m}{T_{m1}} \right)^4 \right] \\
& + T_{m1} \left[-2.2691 \left(\frac{T_{m2}}{T_{m1}} \right)^2 \left(\frac{\tau_m}{T_{m1}} \right)^3 - 8.387 \left(\frac{\tau_m}{T_{m1}} \right)^2 \left(\frac{T_{m2}}{T_{m1}} \right)^3 \right] \\
& + T_{m1} \left[-16.651 \left(\frac{\tau_m}{T_{m1}} \right) \left(\frac{T_{m2}}{T_{m1}} \right)^4 - 7.1990 \left(\frac{T_{m2}}{T_{m1}} \right)^5 + 1.1496 \left(\frac{\tau_m}{T_{m1}} \right)^6 \right] \\
& + T_{m1} \left[-4.728 \frac{T_{m2}}{T_{m1}} \left(\frac{\tau_m}{T_{m1}} \right)^5 + 1.1395 \left(\frac{T_{m2}}{T_{m1}} \right)^6 + 0.6385 \left(\frac{\tau_m}{T_{m1}} \right)^4 \left(\frac{T_{m2}}{T_{m1}} \right)^2 \right] \\
& + T_{m1} \left[1.0885 \left(\frac{\tau_m}{T_{m1}} \right)^3 \left(\frac{T_{m2}}{T_{m1}} \right)^3 + 3.1615 \left(\frac{\tau_m}{T_{m1}} \right)^2 \left(\frac{T_{m2}}{T_{m1}} \right)^4 + 4.5398 \frac{\tau_m}{T_{m1}} \left(\frac{T_{m2}}{T_{m1}} \right)^5 \right].
\end{aligned}$$

⁹ Note: equations continued into the footnote on page 110.

$$\begin{aligned}
K_c^{(127)} = & \frac{1}{K_m} \left[-10.95 - 1.8845 \frac{\tau_m}{T_{m1}} - 3.4123 \xi_m + 4.5954 \xi_m \frac{\tau_m}{T_{m1}} - 1.7002 \left(\frac{\tau_m}{T_{m1}} \right)^3 \right] \\
& + \frac{1}{K_m} \left[-2.1324 \xi_m \left(\frac{\tau_m}{T_{m1}} \right)^2 - 14.4149 \xi_m^2 \left(\frac{\tau_m}{T_{m1}} \right) - 0.7683 \xi_m^3 \right]
\end{aligned}$$

$$\begin{aligned}
& + \frac{1}{K_m} \left[7.5142 \left(\frac{\tau_m}{T_{m1}} \right)^{0.421} + 3.7291 \left(\frac{\tau_m}{T_{m1}} \right)^{0.1984} + 5.3444 \left(\frac{\tau_m}{T_{m1}} \right)^{1.8033} \right] \\
& + \frac{1}{K_m} \left[-0.0819 \xi_m^{19.5419} - 3.603 \xi_m^{1.0749} + 7.1163 \xi_m^{1.1006} \right] \\
& + \frac{1}{K_m} \left[3.206 \xi_m \left(\frac{\tau_m}{T_{m1}} \right)^{-0.6753} - 7.8480 \xi_m \left(\frac{\tau_m}{T_{m1}} \right)^{-0.1642} + 11.3222 \xi_m^{1.9948} \left(\frac{\tau_m}{T_{m1}} \right) \right] \\
& + \frac{1}{K_m} \left[2.4239 e^{\frac{\tau_m}{T_{m1}}} + 3.4137 e^{\xi_m} + 1.0251 e^{\frac{\xi_m \tau_m}{T_{m1}}} - 0.5593 \xi_m \frac{T_{m1}}{\tau_m} \right], \\
T_i^{(127)} = & T_{m1} \left[2.4866 - 23.3234 \frac{\tau_m}{T_{m1}} + 5.3662 \xi_m + 65.6053 \left(\frac{\tau_m}{T_{m1}} \right)^2 \right] \\
& + T_{m1} \left[29.0062 \xi_m \frac{\tau_m}{T_{m1}} - 24.1648 \xi_m^2 - 83.6796 \left(\frac{\tau_m}{T_{m1}} \right)^3 \right] \\
& + T_{m1} \left[-135.9699 \xi_m \left(\frac{\tau_m}{T_{m1}} \right)^2 + 43.1477 \frac{\tau_m}{T_{m1}} \xi_m^2 + 51.9749 \xi_m^3 \right] \\
& + T_{m1} \left[86.0228 \left(\frac{\tau_m}{T_{m1}} \right)^4 + 70.4553 \xi_m \left(\frac{\tau_m}{T_{m1}} \right)^3 + 153.4877 \xi_m^2 \left(\frac{\tau_m}{T_{m1}} \right)^2 \right] \\
& + T_{m1} \left[-125.0112 \frac{\tau_m}{T_{m1}} \xi_m^3 - 68.5893 \xi_m^4 - 62.7517 \left(\frac{\tau_m}{T_{m1}} \right)^5 \right] \\
& + T_{m1} \left[27.6178 \xi_m \left(\frac{\tau_m}{T_{m1}} \right)^4 - 152.7422 \xi_m^2 \left(\frac{\tau_m}{T_{m1}} \right)^3 + 20.8705 \left(\frac{\tau_m}{T_{m1}} \right)^2 \xi_m^3 \right] \\
& + T_{m1} \left[54.0012 \left(\frac{\tau_m}{T_{m1}} \right) \xi_m^4 + 58.7376 \xi_m^5 + 13.1193 \left(\frac{\tau_m}{T_{m1}} \right)^6 \right] \\
& + T_{m1} \left[20.2645 \xi_m \left(\frac{\tau_m}{T_{m1}} \right)^5 - 23.2064 \xi_m^6 - 61.6742 \left(\frac{\tau_m}{T_{m1}} \right)^4 \xi_m^2 \right] \\
& + T_{m1} \left[136.2439 \left(\frac{\tau_m}{T_{m1}} \right)^3 \xi_m^3 - 95.4092 \left(\frac{\tau_m}{T_{m1}} \right)^2 \xi_m^4 + 20.4168 \frac{\tau_m}{T_{m1}} \xi_m^5 \right].
\end{aligned}$$

Rule	K _c		T _i		Comment			
Minimum ISE – Keviczky and Csáki (1973). Model: Method 1 <i>Representative K_c, T_i coefficients estimated from graph; K_m = 1</i>	x ₁		x ₂ τ _m		Coefficient values			
ξ								
τ _m /T _{ml}	x ₁	x ₂	x ₁	x ₂	x ₁	x ₂	x ₁	x ₂
0.1	0.26	1.5	0.55	2	2.2	3.5	6	6
0.2	0.21	1.3	0.4	1.5	1.5	2.8	3	4.5
0.5	0.15	1.2	0.25	1.3	0.8	2	2	3.5
1.0	0.12	1.1	0.2	1.2	0.65	1.8	1.2	3.3
2.0	0.1	1.0	0.16	1.2	0.4	1.9	0.9	3.5
5.0	0.1	1.2	0.16	1.4	0.4	3.0	0.65	5.0
10.0	0.15	1.6	0.2	2.0	0.45	5.5	0.6	7.5
Other coefficient values								
τ _m /T _{ml}	x ₁	x ₂	ξ	τ _m /T _{ml}	x ₁	x ₂	ξ	
0.2	30	15	5.0	2.0	4	15	5.0	
0.5	13	14	5.0	2.0	8	30	10.0	
0.5	30	30	10.0	5.0	1.9	17	5.0	
1.0	7	14	5.0	5.0	3.4	32	10.0	
1.0	14	30	10.0	10.0	1.1	19	5.0	
				10.0	2.0	34	10.0	
Minimum ITAE – Chao <i>et al.</i> (1989). Model: Method 1	¹⁰ K _c ⁽¹²⁸⁾		T _i ⁽¹²⁸⁾					
$0.7 \leq \xi_m \leq 3 ; 0.05 \leq \frac{\tau_m}{2\xi_m T_{ml}} \leq 0.5$								
Minimum ITSE – Chao <i>et al.</i> (1989). Model: Method 1	¹¹ K _c ⁽¹²⁹⁾		T _i ⁽¹²⁹⁾					
$0.7 \leq \xi_m \leq 3 ; 0.05 \leq \frac{\tau_m}{2\xi_m T_{ml}} \leq 0.5$								

$$^{10} K_c^{(128)} = \frac{0.2763 + 0.3532\xi_m - 0.06955\xi_m^2}{K_m} \left(\frac{\tau_m}{2\xi_m T_{ml}} \right)^{-0.3772 - 0.2414\xi_m + 0.03342\xi_m^2},$$

$$T_i^{(128)} = 2\xi_m T_{ml} \left(0.9953 + 0.07857\xi_m + 0.005317\xi_m^2 \right) \left(\frac{\tau_m}{2\xi_m T_{ml}} \right)^{-0.3276 + 0.3226\xi_m - 0.05929\xi_m^2}.$$

$$^{11} K_c^{(129)} = \frac{0.3970 + 0.4376\xi_m - 0.08168\xi_m^2}{K_m} \left(\frac{\tau_m}{2\xi_m T_{ml}} \right)^{-0.5629 - 0.1187\xi_m + 0.01328\xi_m^2},$$

$$T_i^{(129)} = 2\xi_m T_{ml} \left(1.3414 - 0.1487\xi_m + 0.07685\xi_m^2 \right) \left(\frac{\tau_m}{2\xi_m T_{ml}} \right)^{-0.4287 + 0.1981\xi_m - 0.01978\xi_m^2}.$$

Rule	K_c	T_i	Comment			
Nearly minimum IAE, ISE, ITAE – Hwang (1995). Model: Method 10	$^{12} K_c^{(130)}$	$T_i^{(130)}$				
For $\xi_m \leq 0.776 + 0.0568 \frac{\tau_m}{T_{m1}} + 0.18 \left(\frac{\tau_m}{T_{m1}} \right)^2$:						
Coefficient values						
x_1	x_2	x_3	x_4	x_5	x_6	x_7
0.822	-0.549	0.122	0.0142	6.96	-1.77	0.786
x_8	x_9	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}
-0.441	0.0569	0.0172	4.62	-0.823	1.28	0.542
x_{15}	x_{16}	x_{17}	x_{18}	x_{19}	x_{20}	x_{21}
-0.986	0.558	0.0476	0.996	2.13	-1.13	1.14
x_{22}	x_{23}	x_{24}	x_{25}	x_{26}		
-0.466	0.0647	0.0609	1.97	-0.323		

12 Decay ratio = 0.1; $0.2 \leq \frac{\tau_m}{T_{m1}} \leq 2.0$; $0.6 \leq \xi_m \leq 4.2$.

$$K_c^{(130)} = \left(1 - \frac{x_1 [1 + x_2 \omega_H \tau_m + x_3 (\omega_H \tau_m)^2]}{K_H K_m / (1 + K_H K_m)} \right) K_H,$$

$$T_i^{(130)} = \frac{K_c^{(130)} (1 + K_H K_m)}{x_4 \omega_H K_m (1 + x_5 \omega_H \tau_m + x_6 \omega_H^2 \tau_m^2)}, \quad \varepsilon < 2.4;$$

$$K_c^{(130)} = \left(1 - \frac{x_7 [1 + x_8 \omega_H \tau_m + x_9 (\omega_H \tau_m)^2]}{K_H K_m / (1 + K_H K_m)} \right) K_H,$$

$$T_i^{(130)} = \frac{K_c^{(130)} (1 + K_H K_m)}{x_{10} \omega_H K_m (1 + x_{11} \omega_H \tau_m + x_{12} \omega_H^2 \tau_m^2)}, \quad 2.4 \leq \varepsilon < 3;$$

$$K_c^{(130)} = \left(1 - \frac{x_{13} x_{14} \omega_H \tau_m [1 + x_{15}/s + x_{16}/\varepsilon^2]}{K_H K_m / (1 + K_H K_m)} \right) K_H,$$

$$T_i^{(130)} = \frac{K_c^{(130)} (1 + K_H K_m)}{x_{17} \omega_H K_m (1 + x_{18} \ln[\omega_H \tau_m]) (1 + x_{19}/s + x_{20}/s^2)}, \quad 3 \leq \varepsilon < 20;$$

$$K_c^{(130)} = \left(1 - \frac{x_{21} [1 + x_{22} \omega_H \tau_m + x_{23} (\omega_H \tau_m)^2]}{K_H K_m / (1 + K_H K_m)} \right) K_H,$$

$$T_i^{(130)} = \frac{K_c^{(130)} (1 + K_H K_m)}{x_{24} \omega_H K_m (-1 + x_{25} \omega_H \tau_m + x_{26} \omega_H^2 \tau_m^2)}, \quad \varepsilon > 20.$$

Rule	K_c	T_i	Comment
Nearly minimum IAE, ISE, ITAE – Hwang (1995) - continued	For $\xi_m > 0.889 + 0.496 \frac{\tau_m}{T_{m1}} + 0.26 \left(\frac{\tau_m}{T_{m1}} \right)^2$;		
Coefficient values			
x_1	x_2	x_3	x_4
0.794	-0.541	0.126	0.0078
x_8	x_9	x_{10}	x_{11}
-0.415	0.0575	0.0124	4.05
x_{15}	x_{16}	x_{17}	x_{18}
-0.959	0.773	0.0335	0.947
x_{22}	x_{23}	x_{24}	x_{25}
-0.466	0.0667	0.0328	2.21
x_5	x_6	x_{26}	
For $\xi_m > 0.776 + 0.0568 \frac{\tau_m}{T_{m1}} + 0.18 \left(\frac{\tau_m}{T_{m1}} \right)^2$ and			
$\xi_m \leq 0.889 + 0.496 \frac{\tau_m}{T_{m1}} + 0.26 \left(\frac{\tau_m}{T_{m1}} \right)^2$:			
Coefficient values			
x_1	x_2	x_3	x_4
0.789	-0.527	0.11	0.009
x_8	x_9	x_{10}	x_{11}
-0.426	0.0551	0.0153	4.37
x_{15}	x_{16}	x_{17}	x_{18}
-0.978	0.659	0.0421	0.969
x_{22}	x_{23}	x_{24}	x_{25}
-0.467	0.0657	0.0477	2.07
x_5	x_6	x_{26}	

Rule	K_c	T_i	Comment		
Direct synthesis					
Bryant <i>et al.</i> (1973). <i>Model: Method 1</i>	0	$^{13}x_2 K_m T_{m1}$	$T_{m1} > T_{m2}$		
	$\frac{x_1}{K_m \tau_m}$	T_{m1}	$T_{m1} > T_{m2}$		
Coefficient values – deduced from graph					
τ_m / T_{m1}	x_1	τ_m / T_{m1}	x_1		
0.33	0.08	1.0	0.15	Critically damped dominant pole	
0.40	0.09	2.0	0.24		
0.50	0.09	10.0	0.33		
0.67	0.11				
0.33	0.14	1.0	0.31	Damping ratio (dominant pole) = 0.6	
0.40	0.16	2.0	0.47		
0.50	0.18	10.0	0.64		
0.67	0.23				

¹³ Representative x_2 values, summarized as follows, are deduced from graphs:

x_2 value	τ_m / T_{m1}						
	ξ_m	0.5	1.0	2.0	3.0	4.0	5.0
0.4	-	-	5.81	7.46	9.17	10.9	12.7
0.6	-	5.13	6.94	8.40	10.0	11.8	14.1
0.8	-	5.99	7.30	9.17	11.9	15.4	20.0
0.9	-	5.81	7.30	10.0	14.1	19.6	28.6
1.0	7.94	9.01	11.5	13.9	16.4	18.9	-
1.1	8.93	9.90	12.3	14.5	16.9	19.6	-
1.2	9.80	10.8	13.2	15.4	17.9	20.4	-
1.5	12.2	13.3	15.4	18.2	20.0	22.7	-
1.9	15.4	16.7	18.9	21.3	23.8	25.6	-

Rule	K_c	T_i	Comment
Bryant <i>et al.</i> (1973) - continued	$^{14} x_1 / K_m$	$x_2 K_m T_{m1}$	$T_{m1} > T_{m2}$
Hougen (1979) – pages 345-346. Model: Method 1	$^{15} K_c^{(131)}$	$T_i^{(131)}$	Maximise crossover frequency

¹⁴ Representative x_1 values, summarised as follows, are deduced from graphs:

x_1 value	τ_m / T_{m1}					
ξ_m	1.0	2.0	3.0	4.0	5.0	6.0
0.4	-	0.006	0.076	0.103	0.115	0.122
0.6	-	0.039	0.093	0.114	0.122	0.120
0.8	0.055	0.110	0.127	0.114	0.093	0.075
0.9	0.139	0.147	0.122	0.093	0.070	0.052

Representative x_2 values, summarised as follows, are deduced from graphs:

x_2 value	τ_m / T_{m1}					
ξ_m	1.0	2.0	3.0	4.0	5.0	6.0
0.4	-	5.81	7.46	9.17	10.9	12.7
0.6	5.13	6.94	8.40	10.0	11.8	14.1
0.8	5.99	7.30	9.17	11.9	15.4	20.0
0.9	5.81	7.30	10.0	14.1	19.6	28.6

$$^{15} K_c^{(131)} = \frac{x_1}{K_m} \left[1 + \frac{1}{T_{m1} T_{m2}} \left(\frac{0.7 T_{m2} (T_{m1} + T_{m2}) + T_{m1}^2}{T_{m1} + 0.7 T_{m2}} \right)^2 \right],$$

$$T_i^{(131)} = (T_{m1} + 0.7 T_{m2}) \left(1 + 0.1 \frac{\tau_m}{T_{m1}} \right);$$

x_1 values are obtained as follows:

x_1 value	τ_m / T_{m1}				
T_{m2} / T_{m1}	0.1	0.2	0.3	0.4	0.5
0.1	0.31	0.19	0.15	0.11	0.09
0.3	0.6	0.42	0.35	0.28	0.23
0.5	0.65	0.5	0.45	0.46	0.32
1.0	0.7	0.6	0.5	0.45	0.38
T_{m2} / T_{m1}	0.6	0.7	0.8	0.9	1.0
0.1	0.085	0.075	0.07	0.065	0.06
0.3	0.2	0.18	0.16	0.155	0.15
0.5	0.29	0.26	0.24	0.23	0.22
1.0	0.34	0.32	0.31	0.29	0.28

Rule	K_c	T_i	Comment
Hougen (1988). Model: Method 1	$^{16} K_c^{(132)}$	$T_i^{(132)}$	Five criteria are fulfilled
Somani <i>et al.</i> (1992). ¹⁷	$x_1 / K_m A_m$	$x_2 \tau_m$	
	Coefficient values – see page 117		

¹⁶ Equations for $K_c^{(132)}$ deduced from graph;

$$K_c^{(132)} = \frac{1}{K_m} 10^{\left[0.73 \log_{10} \left(\frac{T_{m2}}{\tau_m} \right) + 0.65 \right]}, \quad \frac{T_{m2}}{T_{m1}} = 0.1 ;$$

$$K_c^{(132)} = \frac{1}{K_m} 10^{\left[0.61 \log_{10} \left(\frac{T_{m2}}{\tau_m} \right) + 0.29 \right]}, \quad \frac{T_{m2}}{T_{m1}} = 0.2 ;$$

$$K_c^{(132)} = \frac{1}{K_m} 10^{\left[0.48 \log_{10} \left(\frac{T_{m2}}{\tau_m} \right) + 0.05 \right]}, \quad \frac{T_{m2}}{T_{m1}} = 0.5 ;$$

$$K_c^{(132)} = \frac{1}{K_m} 10^{\left[0.41 \log_{10} \left(\frac{T_{m2}}{\tau_m} \right) - 0.06 \right]}, \quad \frac{T_{m2}}{T_{m1}} = 1 . \quad T_i^{(132)} = T_{m1} + 0.7T_{m2} + 0.12\tau_m .$$

¹⁷ For $\frac{25T_{m1}T_{m2}}{T_i} \ll 1$,

$$K_c \approx \frac{0.9806}{K_m A_m} \sqrt{1 + \left(\frac{T_{m1}}{\tau_m} \right)^2} \left[2.944 - \tan^{-1} \left(\frac{T_{m1}}{\tau_m} + \frac{T_{m2}}{\tau_m} \right) \right]^2$$

$$\sqrt{1 + \left(\frac{T_{m2}}{\tau_m} \right)^2} \left[2.944 - \tan^{-1} \left(\frac{T_{m1}}{\tau_m} + \frac{T_{m2}}{\tau_m} \right) \right]^2 \text{ or}$$

$$K_c \approx \frac{0.9806}{K_m A_m} \sqrt{1 + \left(\frac{T_{m1}}{\tau_m} \right)^2} \left[2.944 - \frac{T_{m1}}{\tau_m} - \frac{T_{m2}}{\tau_m} \right]^2 \sqrt{1 + \left(\frac{T_{m2}}{\tau_m} \right)^2} \left[2.944 - \frac{T_{m1}}{\tau_m} - \frac{T_{m2}}{\tau_m} \right]^2 ;$$

For $\frac{25T_{m1}T_{m2}}{T_i} \gg 1$, $K_c \approx \frac{0.9806}{K_m A_m} \sqrt{1 + \left(\frac{T_{m1}}{\tau_m} \right)^2} x_1^2 \sqrt{1 + \left(\frac{T_{m2}}{\tau_m} \right)^2} x_1^2$, with

$$x_1 = 0.981 + \left[0.5 \left(\frac{\tau_m}{T_{m1}} + \frac{\tau_m}{T_{m2}} \right) + 0.945 + \sqrt{0.25 \left(\frac{\tau_m}{T_{m1}} + \frac{\tau_m}{T_{m2}} \right)^2 + 0.945 \left(\frac{\tau_m}{T_{m1}} + \frac{\tau_m}{T_{m2}} \right)} \right]^{0.333}$$

$$+ \left[0.5 \left(\frac{\tau_m}{T_{m1}} + \frac{\tau_m}{T_{m2}} \right) + 0.945 - \sqrt{0.25 \left(\frac{\tau_m}{T_{m1}} + \frac{\tau_m}{T_{m2}} \right)^2 + 0.945 \left(\frac{\tau_m}{T_{m1}} + \frac{\tau_m}{T_{m2}} \right)} \right]^{0.333} .$$

Rule	K_c		T_i		Comment			
Somani <i>et al.</i> (1992) – continued. <i>Model: Method 1;</i> Suggested A_m : $1.75 \leq A_m \leq 3.0$	Coefficient values							
	$\frac{\tau_m}{T_{m2}}$	$\frac{\tau_m}{T_{m1}}$	x_1	x_2	$\frac{\tau_m}{T_{m2}}$	$\frac{\tau_m}{T_{m1}}$	x_1	x_2
0.1	0.147	11.707	12.5	1.00	0.082	12.389	6.25	
0.1	0.440	10.091	8.33	1.00	0.437	3.462	5.00	
0.1	0.954	10.315	6.25	1.00	1.016	2.370	4.17	
0.1	1.953	11.070	5.00	1.00	2.075	2.087	3.57	
0.1	4.587	12.207	4.17	1.67	0.169	6.871	5.00	
0.1	13.89	13.337	3.70	1.67	0.581	2.773	4.17	
0.125	0.121	11.318	12.5	1.67	1.242	1.930	3.57	
0.125	0.404	8.613	8.33	1.67	2.445	1.624	3.13	
0.125	0.897	8.510	6.25	1.67	4.651	1.957	3.13	
0.125	1.842	8.997	5.00	2.5	0.156	7.607	4.55	
0.125	4.219	9.840	4.17	2.5	0.338	4.015	4.17	
0.125	11.628	10.707	3.70	2.5	0.559	2.797	3.85	
0.25	0.064	14.494	11.11	2.5	1.182	1.848	3.33	
0.25	0.256	6.504	8.33	5.0	0.075	16.187	4.17	
0.25	0.667	5.136	6.25	5.0	0.238	5.623	3.85	
0.25	1.406	4.961	5.00	5.0	0.433	3.448	3.57	
0.25	3.021	5.174	4.17	5.0	0.667	2.52	3.33	
0.25	6.452	5.502	3.70	5.0	0.951	2.014	3.13	
0.5	0.062	14.918	8.33	10.0	0.073	17.650	3.85	
0.5	0.376	4.353	6.25	10.0	0.235	5.996	3.57	
0.5	0.909	3.274	5.00	10.0	0.424	4.524	3.33	
0.5	1.880	3.025	4.17	10.0	0.649	2.641	3.13	
0.5	3.401	3.038	3.70	10.0	0.917	2.093	2.94	
Tan <i>et al.</i> (1996). <i>Model: Method 2</i>	$^{18}K_c^{(133)}$		$T_i^{(133)}$		$T_{m1} = T_{m2}$			
Poulin <i>et al.</i> (1996). <i>Model: Method 1</i>	$\frac{T_{m1}}{K_m(T_{m1} + \tau_m)}$		T_{m1}		$T_{m1} > 5T_{m2}$; Minimum $\phi_m = 58^\circ$			

$$^{18}K_c^{(133)} = \frac{\beta T_i^{(133)} \omega_\phi \sqrt{1 + (\beta T_{m1} \omega_\phi)^2}}{A_m \sqrt{1 + (\beta T_i^{(133)} \omega_\phi)^2}}, \quad \beta = 0.8, \frac{\tau_m}{T_{m1}} < 0.5; \beta = 0.5, \frac{\tau_m}{T_{m1}} > 0.5; \omega_\phi < \omega_u;$$

$$T_i^{(133)} = \frac{1}{\beta \omega_\phi \tan[-2 \tan^{-1} \beta T_m \omega_\phi - \beta \tau_m \omega_\phi - \phi_m]}.$$

Rule	K_c	T_i	Comment
Schaedel (1997). <i>Model: Method 1</i>	¹⁹ $K_c^{(134)}$	$T_i^{(134)}$	“Normal” design
	²⁰ $K_c^{(135)}$	$T_i^{(135)}$	“Sharp” design
Pomerleau and Poulin (2004). <i>Model: Method 5</i>	$\frac{T_{ml}}{K_m(T_{ml} + \tau_m)}$	$1.5T_{ml}$	$T_{ml} = T_{m2}$; OS<10%
Leva <i>et al.</i> (2003). <i>Model: Method 1</i>	$\frac{T_{ml}}{2K_m(T_{m2} + \tau_m)}$	$4(T_{m2} + \tau_m)$	$T_{ml} \gg T_{m2} + \tau_m$; Symmetrical optimum principle
Robust			
Brambilla <i>et al.</i> (1990). <i>Model: Method 1</i>	$\frac{T_{ml} + T_{m2} + 0.5\tau_m}{K_m\tau_m(2\lambda + 1)}$	$T_{ml} + T_{m2} + 0.5\tau_m$	$0.1 \leq \frac{\tau_m}{T_{ml} + T_{m2}} \leq 10$
Coefficient values			
λ values obtained from graph	$\frac{\tau_m}{T_{ml} + T_{m2}}$	λ	$\frac{\tau_m}{T_{ml} + T_{m2}}$
	0.1	3.0	2.0
	0.2	1.8	5.0
	0.5	1.0	10.0
	1.0	0.6	

$$\begin{aligned}
 ^{19} K_c^{(134)} &= \frac{0.5T_i^{(134)}}{K_m(2\xi_m T_{ml} + \tau_m - T_i^{(134)})}, \\
 T_i^{(134)} &= \sqrt{(2\xi_m T_{ml} + \tau_m)^2 - 2(T_{ml}^2 + 2\xi_m T_{ml}\tau_m + 0.5\tau_m^2)}. \\
 ^{20} K_c^{(135)} &= \frac{0.375 \frac{4\xi_m^2 T_{ml}^2 + 2\xi_m T_{ml}\tau_m + 0.5\tau_m^2 - T_{ml}^2}{K_m}}{\frac{T_{ml}^2 + 2\xi_m T_{ml}\tau_m + 0.5\tau_m^2}{K_m}}, \\
 T_i^{(135)} &= \frac{4\xi_m^2 T_{ml}^2 + 2\xi_m T_{ml}\tau_m + 0.5\tau_m^2 - T_{ml}^2}{2\xi_m T_{ml} + \tau_m}.
 \end{aligned}$$

Rule	K_c	T_i	Comment
Kristiansson (2003). Model: Method 1	$^{21} K_c^{(136)}$	$2\xi_m T_{m1}$	Suggested $M_{max} = 1.7, \beta = 10$
	$^{22} K_c^{(137)}$		
Panda et al. (2004). Model: Method 1	$^{23} K_c^{(138)}$	$T_i^{(138)}$	24
	$^{25} K_c^{(139)}$	$T_i^{(139)}$	

$$^{21} K_c^{(136)} = \frac{2\xi_m T_{m1}}{K_m \tau_m} \left[\alpha - \sqrt{\alpha^2 - 1 + \frac{1}{M_{max}^2}} \right], \quad \alpha = 1 + \frac{T_{m2}}{\tau_m} \left[1 - \sqrt{1 - \frac{1}{M_{max}^2}} \right].$$

$$^{22} K_c^{(137)} = \frac{2\xi_m T_{m1}}{K_m \tau_m} \left[1 + 0.1913 \frac{T_{m2}}{\tau_m} - \sqrt{0.346 + 0.3826 \frac{T_{m2}}{\tau_m} + 0.0366 \frac{T_{m2}^2}{\tau_m^2}} \right].$$

$$^{23} K_c^{(138)} = \frac{8\xi_m^2 T_{m1} + 2\xi_m \tau_m + T_{m1}}{2\lambda K_m}, \quad T_i^{(138)} = \frac{4\xi_m^2 T_{m1} + \xi_m \tau_m + 0.5 T_{m1}}{2\xi_m}.$$

$$^{24} \lambda \geq \max \text{imum} \left[0.828 + 0.812 \frac{T_{m2}}{T_{m1}} + 0.172 T_{m1} e^{-(6.9 T_{m2}/T_{m1})}, 1.7 \tau_m \right].$$

$$^{25} K_c^{(139)} = \frac{0.828 + 0.812 \frac{T_{m2}}{T_{m1}} + 0.172 T_{m1} e^{-(6.9 T_{m2}/T_{m1})} + \frac{0.558 T_{m2} T_{m1}}{T_{m1} + 1.208 T_{m2}} + \frac{0.5 \tau_m}{\lambda K_m}}{\lambda K_m},$$

$$T_i^{(139)} = 0.828 + 0.812 \frac{T_{m2}}{T_{m1}} + 0.172 T_{m1} e^{-(6.9 T_{m2}/T_{m1})} + \frac{0.558 T_{m2} T_{m1}}{T_{m1} + 1.208 T_{m2}} + 0.5 \tau_m.$$

3.7.2 Controller with set-point weighting

$$U(s) = K_c \left(1 + \frac{1}{T_i s} \right) E(s) - \alpha K_c R(s)$$

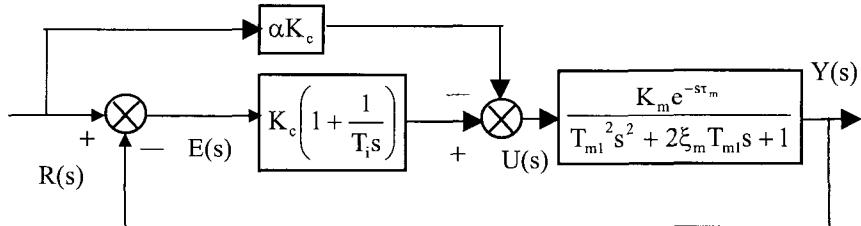


Table 25: PI controller tuning rules – SOSP model $\frac{K_m e^{-s\tau_m}}{T_{m1}^2 s^2 + 2\xi_m T_{m1} s + 1}$

Rule	K_c	T_i	Comment
Minimum performance index: servo/regulator tuning			
Taguchi and Araki (2000). Model: Method 1	${}^1 K_c^{(140)}$	$T_i^{(140)}$	$\xi_m = 1$; $\tau_m / T_m \leq 1.0$; Overshoot (servo step) $\leq 20\%$

$${}^1 K_c^{(140)} = \frac{1}{K_m} \left(0.3717 + \frac{0.5316}{\frac{\tau_m}{T_m} + 0.0003414} \right),$$

$$T_i^{(140)} = T_m \left(2.069 - 0.3692 \frac{\tau_m}{T_m} + 1.081 \left[\frac{\tau_m}{T_m} \right]^2 - 0.5524 \left[\frac{\tau_m}{T_m} \right]^3 \right),$$

$$\alpha = 0.6438 - 0.5056 \frac{\tau_m}{T_m} + 0.3087 \left(\frac{\tau_m}{T_m} \right)^2 - 0.1201 \left(\frac{\tau_m}{T_m} \right)^3.$$

Rule	K_c	T_i	Comment					
Taguchi and Araki (2000) - continued	$^2 K_c^{(141)}$	$T_i^{(141)}$	$\xi_m = 0.5$					
	$x_1 K_u$	$x_2 T_u$						
	Coefficient values							
	x_1	x_2	α	ϕ_c	x_1	x_2	α	ϕ_c
Minimum ITAE - Pecharromán (2000), Pecharromán and Pagola (2000). <i>Model: Method 11</i> $K_m = 1$; $T_{m1} = 1$; $\xi_m = 1$	0.147	1.150	0.411	-146°	0.280	0.512	0.281	-73°
	0.170	1.013	0.401	-140°	0.291	0.503	0.270	-63°
	0.1713	1.0059	0.4002	-139.7°	0.297	0.483	0.260	-52°
	0.195	0.880	0.386	-133°	0.303	0.462	0.246	-41°
	0.210	0.720	0.342	-125°	0.307	0.431	0.229	-30°
	0.234	0.672	0.345	-115°	0.317	0.386	0.171	-19°
	0.249	0.610	0.323	-105°	0.324	0.302	0.004	-10°
	0.262	0.568	0.308	-94°	0.320	0.223	-0.204	-6°
	0.274	0.545	0.291	-84°				

$$^2 K_c^{(141)} = \frac{1}{K_m} \left(0.1000 + \frac{0.05627}{\left[\frac{\tau_m}{T_m} + 0.06041 \right]^2} \right),$$

$$T_i^{(141)} = T_m \left(4.340 - 16.39 \frac{\tau_m}{T_m} + 30.04 \left[\frac{\tau_m}{T_m} \right]^2 - 25.85 \left[\frac{\tau_m}{T_m} \right]^3 + 8.567 \left[\frac{\tau_m}{T_m} \right]^4 \right),$$

$$\alpha = 0.6178 - 0.4439 \frac{\tau_m}{T_m} - 7.575 \left(\frac{\tau_m}{T_m} \right)^2 + 9.317 \left(\frac{\tau_m}{T_m} \right)^3 - 3.182 \left(\frac{\tau_m}{T_m} \right)^4.$$

3.8 SOSIPD Model – Repeated Pole $\frac{K_m e^{-s\tau_m}}{s(1+T_{m1}s)^2}$

3.8.1 Controller with set-point weighting

$$U(s) = K_c \left(1 + \frac{1}{T_i s} \right) E(s) - \alpha K_c R(s)$$

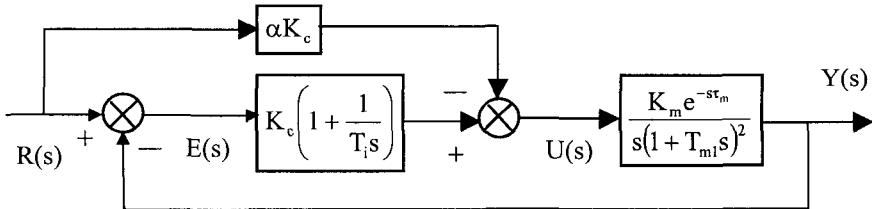


Table 26: PI controller tuning rules – SOSIPD model. $G_m(s) = \frac{K_m e^{-s\tau_m}}{s(1+T_{m1}s)^2}$

Rule	K_c	T_i	Comment
Minimum performance index: servo/regulator tuning			
Taguchi and Araki (2000). <i>Model: Method 1</i>	${}^4 K_c^{(142)}$	$8.549 + 4.029 \frac{\tau_m}{T_{m1}}$	$\tau_m/T_{m1} \leq 1.0$; Overshoot (servo step) $\leq 20\%$
Minimum ITAE - Pecharromán (2000). <i>Model: Method 2</i> $K_m = 1$; $T_{m1} = 1$ $0.1 < \tau_m < 10$	$x_1 K_u$	$x_2 T_u$	
	Coefficient values		
	x_1	x_2	α
	0.049	2.826	0.506
	0.066	2.402	0.512
	0.099	1.962	0.522
	0.129	1.716	0.532
	0.159	1.506	0.544
	0.189	1.392	0.555
			ϕ_c
			-164°
			-160°
			-155°
			-150°
			-145°
			-140°
			α
			0.218
			0.250
			0.286
			0.330
			0.351
			ϕ_c
			1.279
			1.216
			1.127
			1.114
			1.093
			0.577
			-135°
			-130°
			-125°
			-120°
			-118°

$${}^4 K_c^{(142)} = \frac{1}{K_m} \left(0.07368 + \frac{0.3840}{\frac{\tau_m}{T_{m1}} + 0.7640} \right), \quad \alpha = 0.6691 + 0.006606 \frac{\tau_m}{T_{m1}}.$$

3.9 SOSPD Model with a Positive Zero $G_m(s) = \frac{K_m(1 - T_{m3}s)e^{-s\tau_m}}{(1 + sT_{m1})(1 + sT_{m2})}$

3.9.1 Ideal controller $G_c(s) = K_c \left(1 + \frac{1}{T_i s} \right)$

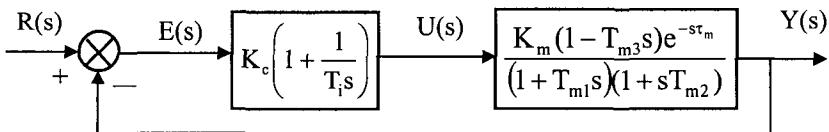


Table 27: PI controller tuning rules – SOSPD model with a positive zero

$$G_m(s) = \frac{K_m(1 - T_{m3}s)e^{-s\tau_m}}{(1 + sT_{m1})(1 + sT_{m2})}$$

Rule	K_c	T_i	Comment
Direct synthesis: time domain criteria			
Pomerleau and Poulin (2004). Model: Method 3	$\frac{1}{K_m} \frac{T_{m1}}{T_{m1} + T_{m3} + \tau_m}$	$1.5T_{m1}$	OS<10%; $T_{m1} = T_{m2}$

Rule	K_c	T_i	Comment		
Direct synthesis: frequency domain criteria					
Khodabakhshian and Golbon (2004). <i>Model: Method 1</i>	$^5 K_c^{(143)}$	$T_i^{(143)}$	Recommended $M_{max} = 0\text{dB}$; $T_{m1} > T_{m2}$		
Ultimate cycle					
Luyben (2000). <i>Model: Method 1</i>	$\frac{K_u}{x_1 K_m}$	$^6 T_u^{(1)}$	$T_{m1} = T_{m2}$		
T_{m3}/T_{m1} values	0.2	0.4	0.8		
x_1 values	3.9	3.3	2.8	3.0	$\tau_m/T_{m3} = 0.2$
	3.8	3.4	3.0	2.9	$\tau_m/T_{m3} = 0.4$
	4.1	3.6	3.2	3.1	$\tau_m/T_{m3} = 0.6$
	4.3	3.8	3.4	3.2	$\tau_m/T_{m3} = 0.8$
	4.8	4.3	3.7	3.3	$\tau_m/T_{m3} = 1.0$
	5.2	4.6	3.9	3.4	$\tau_m/T_{m3} = 1.2$
	5.5	5.0	4.4	3.7	$\tau_m/T_{m3} = 1.4$
	6.0	5.4	4.7	3.8	$\tau_m/T_{m3} = 1.6$

$$^5 K_c^{(143)} = \frac{T_i^{(143)}}{K_m} \sqrt{\frac{(T_{m1}T_{m2})^2 \omega^6 + (T_{m1}^2 + T_{m2}^2)\omega^4 + \omega^2}{(T_{m1}T_{m3})^2 \omega^4 + (T_{m1}^2 + T_{m3}^2)\omega^2 + 1}}, \text{ with } \omega \text{ given by solving}$$

$$\cos^{-1}\left(\frac{1 - 10^{-0.1M_{max}}}{2}\right) = 0.5\pi + \tan^{-1}(T_i^{(143)}\omega) - \tan^{-1}(T_{m3}\omega) - \tan^{-1}(T_{m2}\omega)$$

$$- \tan^{-1}(T_{m1}\omega) - \tau_m\omega ;$$

$$T_i^{(143)} = \frac{1}{T_{m1}} \left(1 + 0.175 \frac{\tau_m}{T_{m1}} + 0.3 \left(\frac{T_{m2}}{T_{m1}} \right)^2 + 0.2 \frac{T_{m2}}{T_{m1}} \right), \frac{\tau_m}{T_{m1}} < 2 ;$$

$$T_i^{(143)} = \frac{1}{T_{m1}} \left(0.65 + 0.35 \frac{\tau_m}{T_{m1}} + 0.3 \left(\frac{T_{m2}}{T_{m1}} \right)^2 + 0.2 \frac{T_{m2}}{T_{m1}} \right), \frac{\tau_m}{T_{m1}} > 2 .$$

$$^6 T^1 = \left(0.5 + 1.56 \frac{T_{m3}}{T_{m1}} \right) + \left(3.44 - 1.56 \frac{T_{m3}}{T_{m1}} \right) \frac{\tau_m}{T_{m3}} .$$

3.10 SOSPD Model (repeated pole) with a Negative Zero

$$G_m(s) = \frac{K_m(1 + T_{m3}s)e^{-s\tau_m}}{(1 + sT_{m1})^2}$$

3.10.1 Ideal controller in series with a first order lag

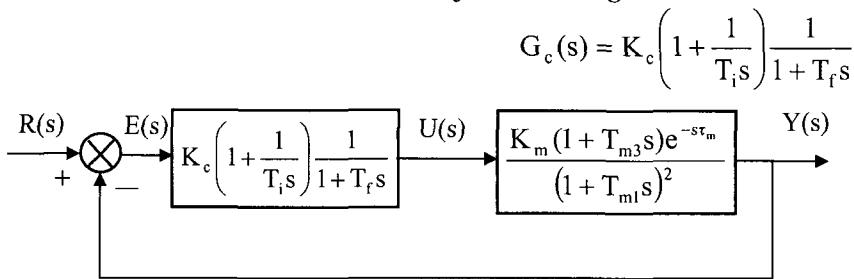


Table 28: PI controller tuning rules – SOSPD model with a negative zero

$$G_m(s) = \frac{K_m(1 + T_{m3}s)e^{-s\tau_m}}{(1 + sT_{m1})^2}$$

Rule	K_c	T_i	Comment
Direct synthesis: time domain criteria			
Pomerleau and Poulin (2004). <i>Model: Method 3</i>	$\frac{1}{K_m} \frac{T_{m1}}{T_{m1} + \tau_m}$	$1.5T_{m1}$	$T_f = T_{m3}; OS < 10\%$

3.11 Third Order System plus Time Delay Model

$$G_m(s) = K_m \frac{1 + b_1 s + b_2 s^2 + b_3 s^3}{1 + a_1 s + a_2 s^2 + a_3 s^3} e^{-s\tau_m} \text{ or}$$

$$G_m(s) = \frac{K_m e^{-s\tau_m}}{(1 + sT_{m1})(1 + sT_{m2})(1 + sT_{m3})}$$

3.11.1 Ideal controller $G_c(s) = K_c \left(1 + \frac{1}{T_i s} \right)$

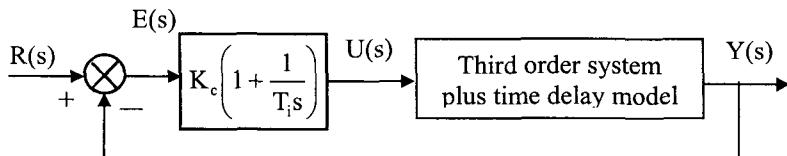


Table 29: PI controller tuning rules – Third order system plus time delay model

$$G_m(s) = K_m \frac{1 + b_1 s + b_2 s^2 + b_3 s^3}{1 + a_1 s + a_2 s^2 + a_3 s^3} e^{-s\tau_m} \text{ or } G_m(s) = \frac{K_m e^{-s\tau_m}}{(1 + sT_{m1})(1 + sT_{m2})(1 + sT_{m3})}.$$

Rule	K_c	T_i	Comment
Direct synthesis			
Hougen (1979) - pages 350-351. Model: Method 1	$\frac{0.7}{K_m} \left(\frac{T_{m1}}{\tau_m} \right)^{0.333}$	$T_i^{(144)}$	$\frac{\tau_m}{T_{m1}} > 0.04 ;$ $T_{m1} \geq T_{m2} \geq T_{m3}$
	${}^l K_c^{(144)}$		$\frac{\tau_m}{T_{m1}} \leq 0.04 ;$ $T_{m1} \geq T_{m2} \geq T_{m3}$

$${}^l K_c^{(144)} = \frac{1}{2K_m} \left[0.7 \left(\frac{T_{m1}}{\tau_m} \right)^{0.333} + 0.8 \frac{T_{m1} + T_{m2} + T_{m3}}{(T_{m1} T_{m2} T_{m3})^{0.333}} \right],$$

$$T_i^{(144)} = 1.5 \tau_m^{0.08} \sqrt{T_{m1}(T_{m2} + T_{m3})}.$$

Rule	K_c	T_i	Comment
Vrančić <i>et al.</i> (1996), Vrančić (1996) – page 121. Model: Method 1	$\frac{0.5A_3}{(A_1A_2 - K_m A_3)}^2$	$\frac{A_3}{A_2}$	$A_m \geq 2, \phi_m \geq 60^\circ$ (Vrančić <i>et al.</i> (1996))
Vrančić <i>et al.</i> (2004a). Model: Method 1	$\frac{0.5A_3}{(A_1A_2 - K_m A_3)}$	${}^3 T_i^{(145)}$	
	0	$\frac{A_1 A_3}{A_1 A_2 - K_m A_3}$	
Vrančić <i>et al.</i> (2004b).	${}^4 K_c^{(146)}$	${}^4 T_i^{(146)}$	Model: Method 1

$${}^2 A_1 = K_m (a_1 - b_1 + \tau_m), \quad A_2 = K_m (b_2 - a_2 + A_1 a_1 - b_1 \tau_m + 0.5 \tau_m^2), \\ A_3 = K_m (a_3 - b_3 + A_2 a_1 - A_1 a_2 + b_2 \tau_m - 0.5 b_1 \tau_m^2 + 0.167 \tau_m^3).$$

$${}^3 T_i^{(145)} = \frac{A_1 A_3 (A_1 A_2 - K_m A_3)}{(A_1 A_2 - K_m A_3)^2 + K_m A_3 (A_1 A_2 - K_m A_3) + 0.25 K_m^2 A_3^2}.$$

$${}^4 K_c^{(146)} = \frac{A_1 A_2 - A_3 K_m - [\text{sign}(A_1 A_2 - A_3 K_m)] A_1 \sqrt{A_2^2 - A_1 A_3}}{A_3 K_m^2 - 2 A_1 A_2 K_m + A_1^3},$$

$$T_i^{(146)} = 2 A_1 \frac{A_1 A_2 - A_3 K_m - [\text{sign}(A_1 A_2 - A_3 K_m)] A_1 \sqrt{A_2^2 - A_1 A_3}}{(A_3 K_m^2 - 2 A_1 A_2 K_m + A_1^3)(1 + K_c^{(146)} K_m)^2}.$$

3.11.2 Controller with set-point weighting

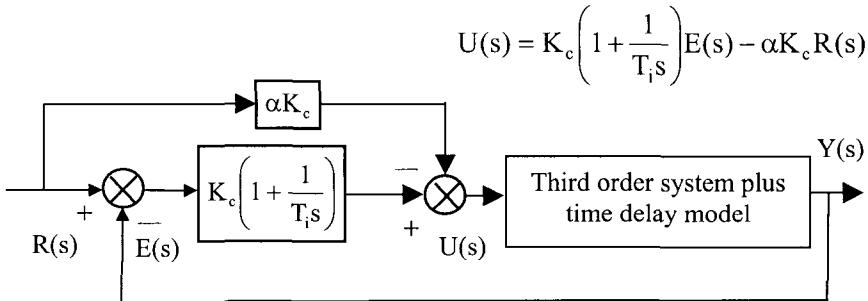


Table 30: PI controller tuning rules – Third order system plus time delay model

$$G_m(s) = K_m \frac{1 + b_1 s + b_2 s^2 + b_3 s^3}{1 + a_1 s + a_2 s^2 + a_3 s^3} e^{-s\tau_m} \quad \text{or} \quad G_m(s) = \frac{K_m e^{-s\tau_m}}{(1 + sT_{m1})(1 + sT_{m2})(1 + sT_{m3})}.$$

Rule	K_c	T_i	Comment
Minimum performance index: servo/regulator tuning			
Taguchi and Araki (2000). <i>Model: Method 1</i>	${}^5 K_c^{(147)}$	$T_i^{(147)}$	$\frac{\tau_m}{T_m} \leq 1.0$
Overshoot (servo step) $\leq 20\% ; T_{m1} = T_{m2} = T_{m3}$			

$${}^5 K_c^{(147)} = \frac{1}{K_m} \left(0.2713 + \frac{0.7399}{\frac{\tau_m}{T_m} + 0.5009} \right), \quad \alpha = 0.4908 - 0.2648 \frac{\tau_m}{T_m} + 0.05159 \left(\frac{\tau_m}{T_m} \right)^2,$$

$$T_i^{(147)} = T_m \left(2.759 - 0.003899 \frac{\tau_m}{T_m} + 0.1354 \left[\frac{\tau_m}{T_m} \right]^2 \right).$$

Rule	K_c	T_i	Comment
Direct synthesis			
Vrančić (1996) – page 136-137. Model: Method 1	${}^6 K_c^{(148)}$	$T_i^{(148)}$	$b = [0.5, 0.8]$ - good servo and regulator response (page 139); $b = 1 - \alpha$

$${}^6 K_c^{(148)} = \frac{A_1 A_2 - K_m A_3 - \chi}{(1 - b^2)(K_m^2 A_3 + A_1^3 - 2K_m A_1 A_2)}, \quad K_m A_3 - A_1 A_2 < 0;$$

$$\chi = \sqrt{(K_m A_3 - A_1 A_2)^2 - (1 - b^2) A_3 (K_m^2 A_3 + A_1^3 - 2K_m A_1 A_2)};$$

$$K_c^{(148)} = \frac{A_1 A_2 - K_m A_3 + \chi}{(1 - b^2)(K_m^2 A_3 + A_1^3 - 2K_m A_1 A_2)}, \quad K_m A_3 - A_1 A_2 > 0.$$

$$T_i^{(148)} = \frac{A_1}{K_m + \frac{1}{2K_c^{(148)}} + \frac{K_c^{(148)} K_m^2}{2} (1 - b^2)} \quad \text{with}$$

$$A_1 = K_m (a_1 - b_1 + \tau_m), \quad A_2 = K_m (b_2 - a_2 + A_1 a_1 - b_1 \tau_m + 0.5 \tau_m^2),$$

$$A_3 = K_m (a_3 - b_3 + A_2 a_1 - A_1 a_2 + b_2 \tau_m - 0.5 b_1 \tau_m^2 + 0.167 \tau_m^3).$$

3.12 Unstable FOLPD Model $G_m(s) = \frac{K_m e^{-s\tau_m}}{T_m s - 1}$

3.12.1 Ideal controller $G_c(s) = K_c \left(1 + \frac{1}{T_i s} \right)$

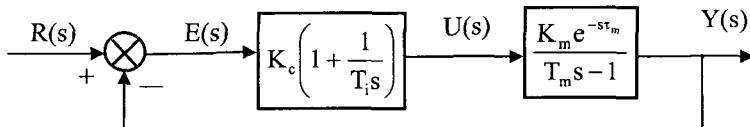


Table 31: PI controller tuning rules – unstable FOLPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{T_m s - 1}$

Rule	K_c	T_i	Comment
Minimum performance index: servo tuning			
Minimum ITSE - Majhi and Atherton (2000). <i>Model: Method 1</i>	¹ $K_c^{(149)}$	$T_i^{(149)}$	$0 < \frac{\tau_m}{T_m} < 0.693$
Minimum ITSE - Majhi and Atherton (2000). <i>Model: Method 3</i>	² $K_c^{(150)}$	$T_i^{(150)}$	$0 < \frac{\tau_m}{T_m} < 0.693 ;$ $K = -A_p/K_m h$

$$^1 K_c^{(149)} = \frac{1}{K_m} \left(0.889 + \frac{e^{-\tau_m/T_m} - 0.064}{e^{\tau_m/T_m} - 0.990} \right), \quad T_i^{(149)} = \frac{2.6316 T_m (e^{\tau_m/T_m} - 0.966)}{(e^{-\tau_m/T_m} - 0.377)}.$$

$$^2 K_c^{(150)} = \frac{1}{K_m} \left(0.889 + \frac{1}{K(1+K)} \right), \quad T_i^{(150)} = \frac{T_m K(1+K)}{0.5(1-0.6382K)\tanh^{-1} K}.$$

Rule	K_c	T_i	Comment
Direct synthesis			
De Paor and O'Malley (1989). <i>Model: Method 1</i>	${}^3 K_c^{(151)}$	$T_i^{(151)}$	$A_m = 2; \frac{\tau_m}{T_m} < 1$
Venkatashankar and Chidambaram (1994). <i>Model: Method 1</i>	${}^4 K_c^{(152)}$	$25(T_m - \tau_m)$	$\frac{\tau_m}{T_m} < 0.67$
Chidambaram (1995b). <i>Model: Method 1</i>	$\frac{1}{K_m} \left(1 + 0.26 \frac{T_m}{\tau_m} \right)$	$25T_m - 27\tau_m$	$\frac{\tau_m}{T_m} < 0.6$
Chidambaram (1997). <i>Model: Method 1</i>	$\frac{1.678}{K_m} \ln \left(\frac{T_m}{\tau_m} \right)$	$0.4015 T_m e^{5.8 \tau_m / T_m}$	<i>Model: Method 1</i>
Valentine and Chidambaram (1998). <i>Model: Method 1</i>	$\frac{2.695}{K_m} e^{-1.277 \tau_m / T_m}$	$0.4015 T_m e^{5.8 \tau_m / T_m}$	${}^5 \xi = 0.333; \frac{\tau_m}{T_m} < 0.7$

$$\begin{aligned}
 {}^3 K_c^{(151)} &= \frac{1}{K_m} \left[\cos \sqrt{\left(1 - \frac{\tau_m}{T_m} \right) \frac{\tau_m}{T_m}} + \sqrt{\frac{T_m}{\tau_m} \left(1 - \frac{\tau_m}{T_m} \right)} \sin \sqrt{\left(1 - \frac{\tau_m}{T_m} \right) \frac{\tau_m}{T_m}} \right], \\
 T_i^{(151)} &= \frac{T_m}{\left[\sqrt{\frac{T_m}{\tau_m} \left(1 - \frac{\tau_m}{T_m} \right)} \tan(0.5\phi) \right]} , \quad \phi = \tan^{-1} \sqrt{\frac{\frac{T_m}{\tau_m} \left(1 - \frac{\tau_m}{T_m} \right)}{\left(1 - \frac{\tau_m}{T_m} \right) \frac{\tau_m}{T_m}}} . \\
 {}^4 K_c^{(152)} &= \frac{1}{K_m} \sqrt{\left(0.98 \sqrt{1 + \frac{0.04 T_m^2}{(T_m - \tau_m)^2}} \left(\frac{25}{\tau_m} \right) \beta (T_m - \tau_m) \right) \sqrt{\frac{1 + \frac{\beta^2 T_m^2}{\tau_m^2}}{1 + \beta^2 \frac{625}{\tau_m^2} (T_m - \tau_m)^2}}} ,
 \end{aligned}$$

$$\beta = 1.373, \frac{\tau_m}{T_m} < 0.25; \beta = 0.953, 0.25 \leq \frac{\tau_m}{T_m} < 0.67.$$

$$\begin{aligned}
 {}^5 \pm 1\% T_s &= 7.5T_m, \quad \tau_m/T_m = 0.1; \quad \pm 1\% T_s = 8.75T_m, \quad \tau_m/T_m = 0.2; \\
 \pm 1\% T_s &= 11.25T_m, \quad \tau_m/T_m = 0.3; \quad \pm 1\% T_s = 15.0T_m, \quad \tau_m/T_m = 0.4; \\
 \pm 1\% T_s &= 17.5T_m, \quad \tau_m/T_m = 0.5; \quad \pm 1\% T_s = 20.0T_m, \quad \tau_m/T_m = 0.6 .
 \end{aligned}$$

Rule	K_c	T_i	Comment
Ho and Xu (1998). Model: Method 1	$\frac{\omega_p T_m}{A_m K_m}$	${}^6 T_i^{(153)}$	$\frac{\tau_m}{T_m} < 0.62$
	Representative results		
	$\frac{0.5775 T_m}{K_m \tau_m}$	$\frac{\tau_m T_m}{0.3212 T_m - \tau_m}$	Good servo response. $A_m = 2.3$, $\phi_m = 25^\circ$
	$\frac{0.6918 T_m}{K_m \tau_m}$	$\frac{\tau_m T_m}{0.3359 T_m - \tau_m}$	Good regulator response. $A_m = 1.9$, $\phi_m = 22.5^\circ$
	$x_1 T_m / K_m \tau_m$	$-T_m$	
	Other representative results: coefficient values		
	x_1	A_m	ϕ_m
	1.0472	1.5	30°
	0.7854	2	45°
	x_1	A_m	ϕ_m
Luyben (1998). Model: Method 1	$0.31 K_u$	$2.2 T_u$	$T_{CL} = 3.16 \tau_m$
Maximum closed loop log modulus = 2 dB			
Gaikwad and Chidambaram (2000). Model: Method 1	${}^7 K_c^{(154)}$	$T_i^{(154)}$	$0.1 \leq \frac{\tau_m}{T_m} \leq 0.6$
Chidambaram (2000c). Model: Method 1	${}^8 K_c^{(155)}$	$T_i^{(155)}$	'small' $\frac{\tau_m}{T_m}$

$${}^6 T_i^{(153)} = \frac{1}{1.57 \omega_p - \omega_p^2 \tau_m - \frac{1}{T_m}}.$$

$${}^7 K_c^{(154)} = \frac{1}{K_m} \left[0.2619 + 0.7266 \frac{T_m}{\tau_m} \right], \quad T_i^{(154)} = T_m \left[0.193 + 4.19 \frac{\tau_m}{T_m} + 8.214 \left(\frac{\tau_m}{T_m} \right)^2 \right].$$

$${}^8 K_c^{(155)} = \frac{T_m T_i^{(155)}}{0.04 T_S^2 \xi^2 + T_i^{(155)} \tau_m}, \quad T_i^{(155)} = \frac{0.04 T_S^2 \xi^2 + T_m \tau_m + 0.4 T_S \xi^2}{T_m - \tau_m}.$$

Rule	K_c	T_i	Comment
⁹ Chandrashekhar <i>et al.</i> (2002). Model: Method 1 Coefficients of K_c and T_i recorded in tables; T_{CL2} as in footnote	$\frac{49.535}{K_m} e^{-21.644\tau_m/T_m}$	$0.1523T_m e^{7.9425\tau_m/T_m}$	$\frac{\tau_m}{T_m} < 0.1$
	$\frac{0.8668}{K_m} \left(\frac{\tau_m}{T_m} \right)^{-0.8288}$		$0.1 \leq \frac{\tau_m}{T_m} \leq 0.7$
	Alternative tuning rules		
	$6.0000/K_m$	$0.6000T_m$	$\tau_m/T_m = 0.1$
	$3.2222/K_m$	$1.4500T_m$	$\tau_m/T_m = 0.2$
	$2.2963/K_m$	$2.6571T_m$	$\tau_m/T_m = 0.3$
	$1.8333/K_m$	$4.4000T_m$	$\tau_m/T_m = 0.4$
	$1.5556/K_m$	$7.0000T_m$	$\tau_m/T_m = 0.5$
	$1.3265/K_m$	$14.6250T_m$	$\tau_m/T_m = 0.6$
	$1.1875/K_m$	$31.0330T_m$	$\tau_m/T_m = 0.7$
Sree <i>et al.</i> (2004). Model: Method 1	$\frac{0.8624}{K_m} \left(\frac{\tau_m}{T_m} \right)^{-0.9744}$	¹⁰ $T_i^{(156)}$	$0.01 \leq \frac{\tau_m}{T_m} \leq 0.6$
Robust			
Rotstein and Lewin (1991). Model: Method 1	$\frac{T_m \lambda \left(\frac{\lambda}{T_m} + 2 \right)}{\lambda^2 K_m}$	$\lambda \left(\frac{\lambda}{T_m} + 2 \right)$	λ obtained graphically – sample values below
	K_m uncertainty = 50%	$\tau_m/T_m = 0.2$	$\lambda \in [0.6T_m, 1.9T_m]$

⁹ Desired closed loop transfer function = $\frac{(\eta s + 1)e^{-s\tau_m}}{(T_{CL2}s + 1)^2};$

$$T_{CL2} = \left(0.025 + 1.75 \frac{\tau_m}{T_m} \right) T_m, \frac{\tau_m}{T_m} \leq 0.1; T_{CL2} = 2\tau_m, 0.1 \leq \frac{\tau_m}{T_m} \leq 0.5;$$

$$T_{CL2} = 2\tau_m + 5\tau_m \left(\frac{\tau_m}{T_m} - 0.5 \right), 0.5 \leq \frac{\tau_m}{T_m} \leq 0.7.$$

$${}^{10} T_i^{(156)} = T_m \left[143.34 \left(\frac{\tau_m}{T_m} \right)^3 - 73.912 \left(\frac{\tau_m}{T_m} \right)^2 + 19.039 \frac{\tau_m}{T_m} - 0.2276 \right].$$

Rule	K_c	T_i	Comment
Rotstein and Lewin (1991) - continued	K_m uncertainty = 30%	$\tau_m/T_m = 0.2$	$\lambda \in [0.5T_m, 4.5T_m]$
		$\tau_m/T_m = 0.4$	$\lambda \in [1.5T_m, 4.5T_m]$
		$\tau_m/T_m = 0.6$	$\lambda \in [3.9T_m, 4.1T_m]$
Tan <i>et al.</i> (1999c). Model: Method I	¹¹ $K_c^{(157)}$	$T_i^{(157)}$	$\frac{\tau_m}{T_m} < 0.5$

$$^{11} K_c^{(157)} = \frac{1}{K_m \left(0.1486 \frac{\tau_m}{T_m} + 0.6564 \right)}, \quad T_i^{(157)} = \frac{T_m \left(12.7708 \frac{\tau_m}{T_m} + 3.8019 \right)}{0.1486 \frac{\tau_m}{T_m} + 0.6564}.$$

3.12.2 Controller with set-point weighting

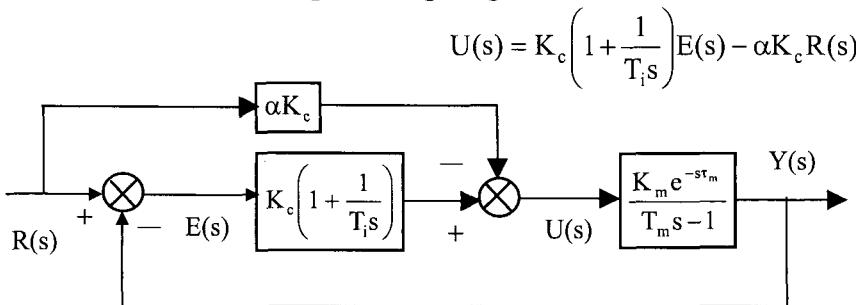


Table 32: PI controller tuning rules – unstable FOLPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{T_m s - 1}$

Rule	K_c	T_i	Comment
Direct synthesis			
Chidambaram (2000c). <i>Model: Method 1</i>	$^{12} K_c^{(158)}$	$T_i^{(158)}$	'small' τ_m/T_m
	$^{13} \frac{2.695}{K_m} e^{-1.277\tau_m/T_m}$	$0.4015 T_m e^{5.8\tau_m/T_m}$	'Dominant pole placement' method; $\xi = 0.333$
Jhunjhunwala and Chidambaram (2001). <i>Model: Method 1</i>	$^{14} K_c^{(159)}$	$T_i^{(159)} = \frac{6.461\tau_m}{0.324 T_m e^{-T_m}}$	$\frac{\tau_m}{T_m} \leq 0.2$
	$0.876 \left(\frac{\tau_m}{T_m} \right)^{-0.898}$		$0.2 \leq \frac{\tau_m}{T_m} \leq 0.6$

$$^{12} K_c^{(158)} = \frac{T_m T_i^{(158)}}{0.04 T_S^2 \xi^2 + T_i^{(158)} \tau_m}, \quad T_i^{(158)} = \frac{0.04 T_S^2 \xi^2 + T_m \tau_m + 0.4 T_S \xi^2}{T_m - \tau_m},$$

$$\alpha = 1 - \frac{0.2 T_S \xi^2 (T_m - \tau_m)}{0.04 T_S^2 \xi^2 + T_m \tau_m + 0.4 T_S \xi^2}.$$

$$^{13} \alpha = 0.733 + 0.6 \frac{\tau_m}{T_m} - 0.36 \left(\frac{\tau_m}{T_m} \right)^2; \quad 0.1 \leq \frac{\tau_m}{T_m} \leq 0.6.$$

$$^{14} K_c^{(159)} = \frac{1}{K_m} \left[10.7572 - 35.1 \frac{\tau_m}{T_m} \right], \quad \alpha = \frac{K_c^{(159)} K_m (2 - T_m) + (K_c^{(159)} K_m - 1) T_i^{(159)}}{2 K_c^{(159)} K_m}.$$

Rule	K_c	T_i	Comment
¹⁵ Chandrashekhar <i>et al.</i> (2002). Model: Method 1 Coefficients of K_c and T_i recorded in tables; T_{CL2} as in footnote	$\frac{49.535}{K_m} e^{-21.644\tau_m/T_m}$	0.1523 $T_m e^{7.9425\tau_m/T_m}$	$\frac{\tau_m}{T_m} < 0.1$
	$\frac{0.8668 \left(\frac{\tau_m}{T_m} \right)^{-0.8288}}{K_m}$		$0.1 \leq \frac{\tau_m}{T_m} \leq 0.7$
Alternative tuning rules			
 Coefficients of K_c and T_i recorded in tables; T_{CL2} as in footnote	K_c	T_i	α
	6.0000/ K_m	0.6000 T_m	0.6667
	3.2222/ K_m	1.4500 T_m	0.7246
	2.2963/ K_m	2.6571 T_m	0.7742
	1.8333/ K_m	4.4000 T_m	0.8182
	1.5556/ K_m	7.0000 T_m	0.8571
	1.3265/ K_m	14.6250 T_m	0.8974
	1.1875/ K_m	31.0330 T_m	0.9323
 Hägglund and Åström (2002). Model: Method 1; $M_s = 1.4$; $0.3 < \alpha < 0.5$	$0.28/K_m T_m$	$8\tau_m$	$\frac{\tau_m}{T_m} = 0.02$
		$7.8\tau_m$	
	$0.2/K_m T_m$	$11\tau_m$	$\frac{\tau_m}{T_m} = 0.05$
	$0.28/K_m T_m$	$10\tau_m$	
	$0.13/K_m T_m$	$7\tau_m$	$\frac{\tau_m}{T_m} = 0.10$
	$0.28/K_m T_m$	$19\tau_m$	
	$0.28/K_m T_m$	$135\tau_m$	$\tau_m/T_m = 0.15$

$$^{15} \text{ Desired closed loop transfer function} = \frac{(\eta s + 1)e^{-s\tau_m}}{(T_{CL2}s + 1)^2};$$

$$T_{CL2} = \left(0.025 + 1.75 \frac{\tau_m}{T_m} \right) T_m, \quad \frac{\tau_m}{T_m} \leq 0.1; \quad T_{CL2} = 2\tau_m, \quad 0.1 \leq \frac{\tau_m}{T_m} \leq 0.5;$$

$$T_{CL2} = 2\tau_m + 5\tau_m \left(\frac{\tau_m}{T_m} - 0.5 \right), \quad 0.5 \leq \frac{\tau_m}{T_m} \leq 0.7.$$

$$\alpha = 0.505 + 3.426 \frac{\tau_m}{T_m} - 10.57 \left(\frac{\tau_m}{T_m} \right)^2, \quad \frac{\tau_m}{T_m} \leq 0.2;$$

$$\alpha = 0.713 + 0.232 \frac{\tau_m}{T_m}, \quad 0.2 \leq \frac{\tau_m}{T_m} \leq 0.7.$$

3.12.3 Controller with proportional term acting on the output 1

$$U(s) = \frac{K_c}{T_i s} E(s) - K_c Y(s)$$

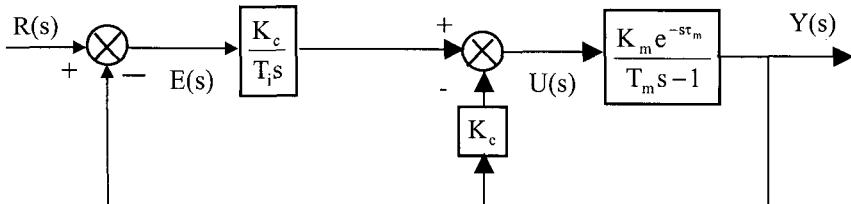


Table 33: PI controller tuning rules – unstable FOLPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{T_m s - 1}$

Rule	K_c	T_i	Comment
Minimum performance index			
Minimum ISE – Paraskevopoulos <i>et al.</i> (2004). Model: Method I	¹ $K_c^{(160)}$	$\frac{1.916\tau_m + \frac{4.78\tau_m^2}{T_m}}{0.88 - \frac{\tau_m}{T_m}}$	$0.05 \leq \frac{\tau_m}{T_m} \leq 0.85$; regulator tuning
		$\frac{2.725\tau_m + \frac{1.71\tau_m^2}{T_m}}{0.88 - \frac{\tau_m}{T_m}}$	$0.05 \leq \frac{\tau_m}{T_m} \leq 0.85$; servo tuning

$${}^1 K_c^{(160)} = \sqrt{K_c^{(\min)} K_c^{(\max)}}$$

$$\text{with } K_c^{(\min)} = \frac{\omega_{\min} T_i}{K_m} \sqrt{\frac{1 + \omega_{\min}^2 T_m^2}{1 + \omega_{\min}^2 T_i^2}}, \quad K_c^{(\max)} = \frac{\omega_{\max} T_i}{K_m} \sqrt{\frac{1 + \omega_{\max}^2 T_m^2}{1 + \omega_{\max}^2 T_i^2}} \text{ and}$$

$$\omega_{\min} = \frac{1}{T_m} \left[\frac{T_i}{T_m} - \frac{\tau_m}{T_m + T_i} \right], \quad \omega_{\max} = \frac{\frac{\pi^2}{2\tau_m} \left[\frac{T_i}{T_i + \tau_m} - \frac{\tau_m}{T_m} \right]}{\frac{\tau_m}{T_m} + \pi \left[\frac{T_i}{T_i + \tau_m} - \frac{\tau_m}{T_m} \right]}, \quad T_i \omega_{\max}^2 \gg 1.$$

Rule	K_c	T_i	Comment		
Minimum performance index: other tuning					
² Paraskevopoulos <i>et al.</i> (2004). <i>Model: Method 1</i>	¹ $K_c^{(160)}$ (page 137)	${}^3 T_i^{(160)}$	$0.01 \leq \frac{\tau_m}{T_m} \leq 0.3$		
		${}^3 T_i^{(161)}$	$0.3 \leq \frac{\tau_m}{T_m} \leq 0.85$		
Minimise $\int_0^\infty [e^2(t) + \left(\frac{du}{dt}\right)^2] dt$ - Paraskevopoulos <i>et al.</i> (2004).	¹ $K_c^{(160)}$ (page 137)	$\frac{2.58\tau_m + \frac{3.60\tau_m^3}{T_m^2}}{0.88 - \frac{\tau_m}{T_m}}$	$0.03 \leq \frac{\tau_m}{T_m} \leq 0.85$; <i>Model: Method 1</i>		
Direct synthesis					
Paraskevopoulos <i>et al.</i> (2004). <i>Model: Method 1</i>	¹ $K_c^{(160)}$ (page 137)	$> T_i^{(\min) 4}$	$0.02 < \frac{\tau_m}{T_m} < 0.7$		
		$> \frac{1.35\tau_m}{\left(1 - \frac{\tau_m}{T_m}\right)^2}$	$0.1 < \frac{\tau_m}{T_m} < 0.9$		
		$x_1 \tau_m + x_2 \sqrt{\tau_m T_m}$	$\frac{\tau_m}{T_m} < 0.3$		
Coefficient values					
x_1	x_2	ξ	x_1	x_2	ξ
0.7269	2.5362	0.5	6.975	5.7053	1.5
1.7016	3.0585	0.75	12.452	8.4358	2
3.0696	3.7509	1			

$${}^2 \text{ Minimise performance index} = \int_0^\infty [e^2(t) + (u(t) - u_\infty)^2] dt$$

$${}^3 T_i^{(160)} = T_m \left[0.747 + 3.66 \frac{\tau_m}{T_m} + 18.64 \left(\frac{\tau_m}{T_m} \right)^3 \right]; {}^3 T_i^{(161)} = T_m \left[\frac{4.52 \frac{\tau_m}{T_m} + 0.476 \left(\frac{\tau_m}{T_m} \right)^3}{0.88 - \frac{\tau_m}{T_m}} \right].$$

$${}^4 T_i^{(\min)} = T_m [1.3146 \frac{\tau_m}{T_m} - 2.3805 \left(\frac{\tau_m}{T_m} \right)^2 + 57.622 \left(\frac{\tau_m}{T_m} \right)^3 - 158.99 \left(\frac{\tau_m}{T_m} \right)^4 + 173.41 \left(\frac{\tau_m}{T_m} \right)^5].$$

Rule	K_c	T_i	Comment					
Paraskevopoulos <i>et al.</i> (2004) – continued.	$^1 K_c^{(160)}$ (page 137)	$x_1 \tau_m + x_2 T_m +$ $x_3 \tau_m^6 T_m^{-5}$	$0.2 < \frac{\tau_m}{T_m} < 0.7$					
Coefficient values								
	x_1	x_2	x_3	ξ	x_1	x_2	x_3	ξ
	4.59	-0.19	8.28	0.5	12.41	1.89	51.95	1.5
	6.16	0.028	9.90	0.75	19.91	3.44	95	2
	8.21	0.37	16.88	1				
Paraskevopoulos <i>et al.</i> (2004). Model: Method 1	$^1 K_c^{(160)}$ (page 137)	$^5 T_i^{(162)}$	$\frac{\tau_m}{T_m} < 0.2$					
Coefficient values								
	x_1	x_2	T_{CL2}	x_1	x_2	T_{CL2}		
	0.409	0.2817	0.5	2.353	2.67	1.5		
	0.717	0.6542	0.75	4.07	4.76	2		
	1.142	1.177	1	8.88	10.73	3		
	$^1 K_c^{(160)}$ (page 137)	$T_m \left[x_1 + x_3 \left(\frac{\tau_m}{T_m} \right)^3 \right]$	$0.2 < \frac{\tau_m}{T_m} < 0.7$					
Coefficient values								
	x_1	x_2	T_{CL2}	x_1	x_2	T_{CL2}		
	1.518	10.54	0.5	13	49.66	1.5		
	3.34	16.89	0.75	23.08	82.87	2		
	5.81	29.17	1	51.94	176.8	3		
Robust								
Zhang and Xu (2002). Model: Method 1	$^6 K_c^{(163)}$	$T_i^{(163)}$	$\tau_m < T_{m1}$; for $\frac{\tau_m}{T_m} \leq 0.5$, $\lambda = 2 - 5\tau_m$					

$$^5 T_i^{(162)} = T_m \left[x_1 - \frac{x_2}{\frac{\tau_m}{T_m} - \sqrt{\frac{\tau_m}{T_m}}} \right]$$

$$^6 K_c^{(163)} = \frac{\lambda^2 + 2\lambda T_m + T_m \tau_m}{K_m (\lambda + \tau_m)^2}, \quad T_i^{(163)} = \frac{\lambda^2 + 2\lambda T_m + T_m \tau_m}{T_m - \tau_m}.$$

3.12.4 Controller with proportional term acting on the output 2

$$U(s) = K_c \left(1 + \frac{1}{T_i s} \right) E(s) - K_1 Y(s)$$

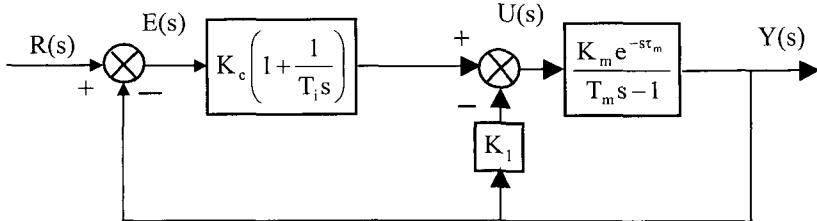


Table 34: PI controller tuning rules – unstable FOLPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{T_m s - 1}$

Rule	K_c	T_i	Comment
Robust			
Chidambaram (1997). <i>Model: Method 1</i>	$\frac{T_m - \tau_m}{\lambda K_m}$	${}^7 T_i^{(164)}$	$\lambda > 1.7 ; \frac{\tau_m}{T_m} < 1$
Lee and Edgar (2002). <i>Model: Method 1</i>	${}^8 K_c^{(165)}$	$T_i^{(165)}$	

$${}^7 T_i^{(164)} = \frac{T_m - \left(\frac{T_m}{\tau_m} \right)^{0.5} \tau_m}{\left(\frac{T_m}{\tau_m} \right)^{0.5} - 1} + 0.5\tau_m, \quad K_1 = \frac{1}{K_m} \left(\frac{T_m}{\tau_m} \right)^{0.5}.$$

$${}^8 K_c^{(165)} = \frac{K_1^{(165)} K_m - 1}{K_m (\lambda + \tau_m)} \left[\frac{T_m - K_1^{(165)} K_m \tau_m}{K_1^{(165)} K_m - 1} + \frac{\tau_m^2}{2(\lambda + \tau_m)} \right],$$

$$T_i^{(165)} = \frac{T_m - K_1^{(165)} K_m \tau_m}{K_1^{(165)} K_m - 1} + \frac{\tau_m^2}{2(\lambda + \tau_m)},$$

$$K_1^{(165)} = \frac{1}{K_m} \left[\cos \sqrt{\left(1 - \frac{\tau_m}{T_m} \right) \frac{\tau_m}{T_m}} + \sqrt{\frac{T_m}{\tau_m} \left(1 - \frac{\tau_m}{T_m} \right)} \sin \sqrt{\left(1 - \frac{\tau_m}{T_m} \right) \frac{\tau_m}{T_m}} \right].$$

3.13 Unstable FOLPD Model with a Positive Zero

$$G_m(s) = \frac{K_m(1-sT_{m3})e^{-s\tau_m}}{T_{m1}s-1}$$

3.13.1 Ideal controller $G_c(s) = K_c \left(1 + \frac{1}{T_i s} \right)$

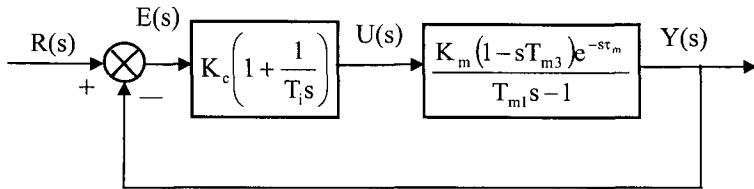


Table 35: PI controller tuning rules – unstable FOLPD model with a positive zero -

$$G_m(s) = \frac{K_m(1-sT_{m3})e^{-s\tau_m}}{T_{m1}s-1}$$

Rule	K_c	T_i	Comment
Direct synthesis: time domain criteria			
Sree and Chidambaram (2003c). <i>Model: Method 1</i>	$1 \frac{\gamma T_{m1}}{K_m T_{m3}}$	$T_i^{(166)}$	

¹ γ = the negative of the value of the inverse jump of the closed loop system; $\gamma \geq \frac{T_{m3}}{T_{m1}}$.

$$T_i^{(166)} = \frac{\frac{\gamma T_{m1}}{T_{m3}} [T_{m3}(1-x_1) - 0.5\tau_m(1+x_1)]}{\frac{\gamma T_{m1}}{T_{m3}} (1-x_1) + x_1}, \quad x_1 > \frac{\gamma T_{m1}}{\gamma T_{m1} - T_{m3}}.$$

3.13.2 Ideal controller in series with a first order lag

$$G_c(s) = K_c \left(1 + \frac{1}{T_i s} \right) \frac{1}{1 + T_f s}$$

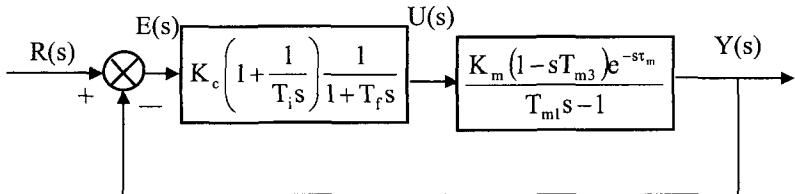


Table 36: PI controller tuning rules – unstable FOLPD model with a positive zero -

$$G_m(s) = \frac{K_m (1 - sT_{m3}) e^{-s\tau_m}}{T_{m1}s - 1}$$

Rule	K_c	T_i	Comment
Direct synthesis: time domain criteria			
Sree and Chidambaram (2003b). Model: Method 1	$^2 K_c^{(167)}$	$T_i^{(167)}$	$\frac{\tau_m}{T_{m1}} - \frac{\tau_m}{T_{m3}} < 0.62$

² Desired closed loop transfer function = $\frac{(1 - sT_{m3})(1 + T_i^{(167)}s)e^{-s\tau_m}}{(1 + T_{CL2}^{(1)}s)(1 + T_{CL2}^{(2)}s)}.$

$$K_c^{(167)} = -\frac{T_i^{(167)}}{K_m(T_{CL2}^{(1)} + T_{CL2}^{(2)} + T_{m3} + \tau_m - T_i^{(167)})},$$

$$T_i^{(167)} = \frac{T_{m1}(T_{CL2}^{(1)}T_{CL2}^{(2)} - T_{m3}\tau_m + [T_{CL2}^{(1)} + T_{CL2}^{(2)} + T_{m3} + \tau_m]T_{m1})}{T_{m1}^2 + T_{m3}\tau_m - (\tau_m + T_{m3})T_{m1}},$$

$$T_f = \frac{T_{m3}\tau_m T_i^{(167)}}{T_{m1}(T_{CL2}^{(1)} + T_{CL2}^{(2)} + T_{m3} + \tau_m - T_i^{(167)})}.$$

3.13.3 Controller with set-point weighting

$$U(s) = K_c \left(1 + \frac{1}{T_i s} \right) E(s) - \alpha K_c R(s)$$

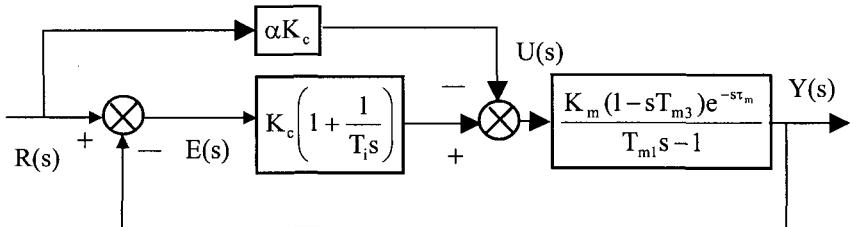


Table 37: PI controller tuning rules – unstable FOLPD model with a positive zero -

$$G_m(s) = \frac{K_m (1 - sT_{m3}) e^{-s\tau_m}}{T_{m1}s - 1}$$

Rule	K_c	T_i	Comment
Direct synthesis: time domain criteria			
Sree and Chidambaram (2003b). Model: Method 1	³ $K_c^{(168)}$	$T_i^{(168)}$	$\frac{\tau_m}{T_{m1}} - \frac{\tau_m}{T_{m3}} < 0.62$

$$\text{Desired closed loop transfer function} = \frac{(1 - sT_{m3})(1 + T_i^{(168)}s)e^{-s\tau_m}}{(1 + T_{CL2}^{(1)}s)(1 + T_{CL2}^{(2)}s)}.$$

$$K_c^{(168)} = -\frac{T_i^{(168)}}{K_m (T_{CL2}^{(1)} + T_{CL2}^{(2)} + T_{m3} + \tau_m - T_i^{(168)})},$$

$$T_i^{(168)} = \frac{T_{m1} (T_{CL2}^{(1)} T_{CL2}^{(2)} - T_{m3} \tau_m + [T_{CL2}^{(1)} + T_{CL2}^{(2)} + T_{m3} + \tau_m] T_{m1})}{T_{m1}^2 + T_{m3} \tau_m - (\tau_m + T_{m3}) T_{m1}},$$

$$T_f = \frac{T_{m3} \tau_m T_i^{(168)}}{T_{m1} (T_{CL2}^{(1)} + T_{CL2}^{(2)} + T_{m3} + \tau_m - T_i^{(168)})}, \quad \alpha = 1 - 0.5 \left(\frac{T_{m3} + T_{CL2}^{(1)} + T_{CL2}^{(2)}}{T_i^{(168)}} \right).$$

Rule	K_c	T_i	Comment
Sree and Chidambaram (2003c). <i>Model: Method 1</i>	$^4 \frac{\gamma T_{m1}}{K_m T_{m3}}$	$T_i^{(169)}$	

⁴ γ = the negative of the value of the inverse jump of the closed loop system;

$$\gamma \geq \frac{T_{m3}}{T_{m1}} . \quad T_i^{(169)} = \frac{\frac{\gamma T_{m1}}{T_{m3}} [T_{m3}(1 - x_1) - 0.5\tau_m(1 + x_1)]}{\frac{\gamma T_{m1}}{T_{m3}}(1 - x_1) + x_1}, \quad x_1 > \frac{\gamma T_{m1}}{\gamma T_{m1} - T_{m3}} ;$$

$$\alpha = 1 - 0.5 \left(\frac{T_{m3} + T_{CL2}^{(1)} + T_{CL2}^{(2)}}{T_i^{(169)}} \right).$$

Desired closed loop transfer function = $\frac{(1 - sT_{m3})(1 + T_i^{(169)}s)e^{-s\tau_m}}{(1 + T_{CL2}^{(1)}s)(1 + T_{CL2}^{(2)}s)}$
 (Sree and Chidambaram (2003b)).

3.14 Unstable SOSPD Model (one unstable pole)

$$G_m(s) = \frac{K_m e^{-s\tau_m}}{(T_{m1}s - 1)(1 + sT_{m2})}$$

3.14.1 Ideal controller $G_c(s) = K_c \left(1 + \frac{1}{T_i s} \right)$

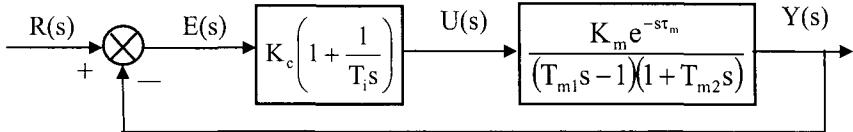


Table 38: PI controller tuning rules – unstable SOSPD model (one unstable pole)

$$G_m(s) = \frac{K_m e^{-s\tau_m}}{(T_{m1}s - 1)(1 + sT_{m2})}$$

Rule	K_c	T_i	Comment			
Minimum performance index: regulator tuning						
Minimum ITAE – Poulin and Pomerleau (1996).		$\frac{4T_{m1}(\tau_m + T_{m2})}{x_1 T_{m1} - 4(\tau_m + T_{m2})}$	Coefficients of K_c, T_i deduced from graph			
<i>Model: Method 1</i> Process output step load disturbance	$^1 K_c^{(170)}$					
	τ_m / T_m	x_1	x_2			
	0.05	0.9479	2.3546			
	0.10	1.0799	2.4111			
	0.15	1.2013	2.4646			
	0.20	1.3485	2.5318			
 Process input step load disturbance	0.25	1.4905	2.5992			
	x_1	τ_m / T_m	x_1	x_2		
	0.05	0.30	1.6163	2.6612		
	0.10	0.35	1.7650	2.7368		
	0.15	0.40	1.9139	2.8161		
	0.20	0.45	2.0658	2.9004		
	0.25	0.50	2.2080	2.9826		
	$x_2 T_{m1}$	$\sqrt{1 + \frac{x_1 T_{m2}^2}{4(\tau_m + T_{m2})^2}}$				
	$^1 K_c^{(170)}$					
	$K_m (x_1 T_{m1} - 4[\tau_m + T_{m2}])$					
	0.05	1.1075	2.4230	0.25	1.6943	2.7007
	0.10	1.2013	2.4646	0.35	1.8161	2.7637
	0.15	1.3132	2.5154	0.40	1.9658	2.8445
	0.20	1.4384	2.5742	0.45	2.1022	2.9210
	0.25	1.5698	2.6381	0.50	2.2379	3.0003

$$^1 K_c^{(170)} = \frac{x_2 T_{m1} \sqrt{1 + \frac{x_1 T_{m2}^2}{4(\tau_m + T_{m2})^2}}}{K_m (x_1 T_{m1} - 4[\tau_m + T_{m2}])}.$$

Rule	K_c	T_i	Comment
Direct synthesis			
Poulin et al. (1996). <i>Model: Method 1</i>	$-\frac{\omega T_{m1}}{\tau_m}$	$^2 T_i^{(171)}$	minimum $\phi_m = 25^0$
	$-\frac{3.53}{K_m}$	T_{m1}	$T_{m2} + \tau_m < 0.16T_{m1};$ $\phi_m \in [25^0, 90^0]$
Ultimate cycle			
McMillan (1984). <i>Model: Method 1</i>	$^3 K_c^{(172)}$	$T_i^{(172)}$	Tuning rules developed from K_u, T_u

$$^2 T_i^{(171)} = \frac{4T_{m1}(\tau_m + T_{m2})}{1.3T_{m1} - 4(\tau_m + T_{m2})}, \quad 0.16T_{m1} \leq T_{m2} + \tau_m \leq 0.3T_{m1}; \quad \omega = \text{frequency}$$

where phase of $G_m(j\omega)G_c(j\omega)$ is maximum.

$$^3 K_c^{(172)} = \frac{1.477}{K_m} \frac{T_{m1} T_{m2}}{\tau_m^2} \left\{ \frac{1}{1 + \left[\frac{(T_{m1} + T_{m2})T_{m1} T_{m2}}{(T_{m1} - T_{m2})(T_{m1} - \tau_m)\tau_m} \right]^{0.65}} \right\}^2,$$

$$T_i^{(172)} = 3.33\tau_m \left\{ 1 + \left[\frac{(T_{m1} + T_{m2})T_{m1} T_{m2}}{(T_{m1} - T_{m2})(T_{m1} - \tau_m)\tau_m} \right]^{0.65} \right\}.$$

3.15 Unstable SOSPD Model with a Positive Zero

$$G_m(s) = \frac{K_m(1 - T_{m3}s)e^{-s\tau_m}}{T_{m1}^2 s^2 - 2\xi_m T_{m1}s + 1}$$

3.15.1 Ideal controller $G_c(s) = K_c \left(1 + \frac{1}{T_i s} \right)$

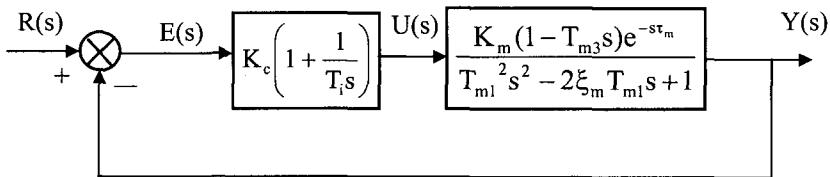


Table 39: PI controller tuning rules – unstable SOSPD model with a positive zero

$$G_m(s) = \frac{K_m(1 - T_{m3}s)e^{-s\tau_m}}{T_{m1}^2 s^2 - 2\xi_m T_{m1}s + 1}$$

Rule	K_c	T_i	Comment
Direct synthesis: time domain criteria			
Sree and Chidambaram (2004). Model: Method 1	⁴ $K_c^{(173)}$	$T_i^{(173)}$	

⁴ Desired closed loop transfer function = $\frac{(1 - sT_{m3})(1 + T_i^{(173)}s)e^{-s\tau_m}}{(1 + sT_{CL}^{(1)})(1 + sT_{CL}^{(2)})(1 + sx_1)}.$

$$K_c^{(173)} = \frac{T_i^{(173)}}{K_m(T_{CL}^{(1)} + T_{CL}^{(2)} + x_1 + T_{m3} + \tau_m - T_i^{(173)})}$$

$$T_i^{(173)} = -\frac{T_{m1}^2(T_{CL}^{(1)} + T_{CL}^{(2)} + x_1 + T_{m3} + \tau_m) - T_{CL}^{(1)}T_{CL}^{(2)}x_1}{T_{m3}\tau_m - T_{m1}^2}, \text{ with}$$

$$x_1 = \frac{T_{m1}(T_{CL}^{(1)} + T_{CL}^{(2)} + T_{m3} + \tau_m)(-2\xi_m T_{m3}\tau_m + T_{m1}(T_{m3} + \tau_m)) - x_2}{(-2\xi_m T_{m1} + T_{m3} + \tau_m)T_{CL}^{(1)}T_{CL}^{(2)} - T_{m1}^2 + (T_{m3}\tau_m - T_{m1}^2)(T_{CL}^{(1)} + T_{CL}^{(2)} + 2\xi_m T_{m1})},$$

and with $x_2 = (T_{m3}\tau_m - T_{m1}^2)(T_{CL}^{(1)}T_{CL}^{(2)} - \tau_m T_{m3})$; $-\tau_m \left(\frac{2\xi_m}{T_{m1}} + \frac{1}{T_{m3}} \right) < 0.62$.

3.15.2 Controller with set-point weighting

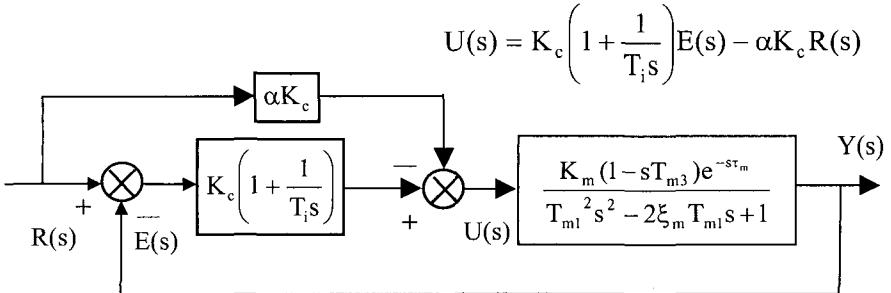


Table 40: PI controller tuning rules — unstable SOSPD model with a positive zero

$$G_m(s) = \frac{K_m(1 - T_{m3}s)e^{-s\tau_m}}{T_{m1}^2 s^2 - 2\xi_m T_{m1}s + 1}$$

Rule	K_c	T_i	Comment
Direct synthesis: time domain criteria			
Sree and Chidambaram (2004). Model: Method 1	⁵ $K_c^{(174)}$	$T_i^{(174)}$	

⁵ Desired closed loop transfer function = $\frac{(1 - s T_{m3})(1 + T_i^{(174)}s)e^{-s\tau_m}}{(1 + s T_{CL}^{(1)})(1 + s T_{CL}^{(2)})(1 + s x_1)}$.

$$K_c^{(174)} = \frac{T_i^{(174)}}{K_m(T_{CL}^{(1)} + T_{CL}^{(2)} + x_1 + T_{m3} + \tau_m - T_i^{(174)})}, \quad -\tau_m \left(\frac{2\xi_m}{T_{m1}} + \frac{1}{T_{m3}} \right) < 0.62 ;$$

$$T_i^{(174)} = -\frac{T_{m1}^2 (T_{CL}^{(1)} + T_{CL}^{(2)} + x_1 + T_{m3} + \tau_m) - T_{CL}^{(1)} T_{CL}^{(2)} x_1}{T_{m3} \tau_m - T_{m1}^2}, \text{ with}$$

$$x_1 = \frac{T_{m1} (T_{CL}^{(1)} + T_{CL}^{(2)} + T_{m3} + \tau_m) (-2\xi_m T_{m3} \tau_m + T_{m1} (T_{m3} + \tau_m)) - x_2}{(-2\xi_m T_{m1} + T_{m3} + \tau_m) T_{CL}^{(1)} T_{CL}^{(2)} - T_{m1}^2 + (T_{m3} \tau_m - T_{m1}^2) (T_{CL}^{(1)} + T_{CL}^{(2)} + 2\xi_m T_{m1})},$$

$$\text{and with } x_2 = (T_{m3} \tau_m - T_{m1}^2) (T_{CL}^{(1)} T_{CL}^{(2)} - \tau_m T_{m3}), \quad \alpha = 1 - \frac{T_{m3} + 0.5(T_{CL}^{(1)} + T_{CL}^{(2)})}{T_i^{(174)}}.$$

3.16 Delay Model $G_m(s) = K_m e^{-s\tau_m}$

3.16.1 Ideal controller $G_c(s) = K_c \left(1 + \frac{1}{T_i s} \right)$

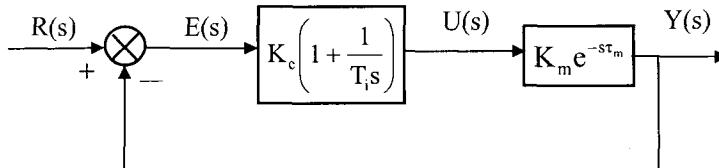


Table 41: PI controller tuning rules – $G_m(s) = K_m e^{-s\tau_m}$

Rule	K_c	T_i	Comment
Process reaction			
Callender <i>et al.</i> (1935/6). <i>Model: Method 1</i>	¹ $0.568/K_m \tau_m$	$3.64\tau_m$	<i>Representative results deduced from graphs</i>
	² $0.65/K_m \tau_m$	$2.6\tau_m$	
	³ $0.79/K_m \tau_m$	$3.95\tau_m$	
	⁴ $0.95/K_m \tau_m$	$3.3\tau_m$	
Two constraints method ~ Wolfe (1951). <i>Model: Method 1</i>	$\frac{0.2}{K_m \tau_m}$	$0.3\tau_m$	Decay ratio is as small as possible
	Minimum error integral (regulator mode)		
Minimum performance index: regulator tuning			
Haalman (1965) – minimum ISE. <i>Model: Method 1</i>	0	$1.5\tau_m$	
	$M_s = 1.9$; $A_m = 2.36$; $\phi_m = 50^0$		
Gerry (1998) – minimum error: step load change.	$\frac{0.3}{K_m}$	$0.42\tau_m$	<i>Model: Method 1</i>
Gerry and Hansen (1987) – minimum IAE.	$\frac{0.345}{K_m}$	$0.455\tau_m$	<i>Model: Method 1</i>

¹ Decay ratio = 0.015; Period of decaying oscillation = $10.5\tau_m$

² Decay ratio = 0.1; Period of decaying oscillation = $8\tau_m$

³ Decay ratio = 0.12; Period of decaying oscillation = $6.28\tau_m$

⁴ Decay ratio = 0.33; Period of decaying oscillation = $5.56\tau_m$

Rule	K_c	T_i	Comment		
Shinskey (1988) – minimum IAE - <i>page 123.</i>	$\frac{0.43}{K_m}$	$0.5\tau_m$	<i>Model: Method 1</i>		
Shinskey (1994) – minimum IAE. <i>Model: Method 1</i>	$0.4/K_m$	$0.5\tau_m$	<i>page 67</i>		
	0	$1.6K_m \tau_m$	<i>page 63</i>		
Shinskey (1988) – minimum IAE – <i>page 148.</i>	$\frac{0.4255}{K_m}$	$0.25T_u$	<i>Model: Method 1</i>		
Shinskey (1996) – minimum IAE – <i>page 167.</i>	$0.4K_u$	$0.25T_u$	<i>Model: Method 1</i>		
Minimum performance index: servo tuning					
Minimum IAE – Huang and Jeng (2002). <i>Model: Method 1</i>	$\frac{0.36}{K_m}$	$0.47\tau_m$	Minimum IAE = $1.377\tau_m \cdot A_m = 2 ; \phi_m = 60^0 .$		
Minimum ISE – Keviczky and Csáki (1973) <i>Model: Method 1</i>	0	$1.33\tau_m$	Minimum ISE = $1.53\tau_m ; K_m = 1$		
	0.5	$0.635\tau_m$	$K_m = 1$		
Minimum performance index: other tuning					
Fertik (1975). <i>Model: Method 1</i>	$0.35/K_m$	$0.4\tau_m$	K_c, T_i values deduced from graph		
Åström and Hägglund (2000). <i>Model: Method 1</i>	$\frac{0.25}{K_m}$	$0.35\tau_m$	Maximise K_c/T_i subject to $M_{max} = 2$		
Skogestad (2001). <i>Model: Method 1</i>	$0.16/K_m$	$0.340\tau_m$	$M_{max} = 1.4$		
Skogestad (2003). <i>Model: Method 1</i>	$0.200/K_m$	$0.318\tau_m$	$M_{max} = 1.6$		
Skogestad (2001). <i>Model: Method 1</i>	$0.26/K_m$	$0.306\tau_m$	$M_{max} = 2.0$		
Åström and Hägglund (2004). <i>Model: Method 1</i>	x_1/K_m	$x_2\tau_m$	Coefficient values		
	x_1	M_{max}	x_1	x_2	M_{max}
	0.057	0.400	1.1	0.211	0.342
	0.103	0.389	1.2	0.227	0.334
	0.139	0.376	1.3	0.241	0.326
	0.168	0.363	1.4	0.254	0.320
	0.191	0.352	1.5	0.264	0.314

Rule	K_c	T_i	Comment					
Direct synthesis								
Van der Grinten (1963). <i>Model: Method 1</i>	$0.5/K_m$	$0.5\tau_m$	Step disturbance					
	$^5 K_c^{(175)}$	$T_i^{(175)}$	Stochastic disturbance					
Buckley (1964). <i>Model: Method 1</i>	0	$1.45K_m\tau_m$	$M_{max} = 2dB$					
	$0.075/K_m$	$0.1\tau_m$	Representative results; $M_{max} = 2dB$					
	$0.53/K_m$	τ_m						
	$0.55/K_m$	$1.59\tau_m$						
Bryant <i>et al.</i> (1973). <i>Model: Method 1</i>	0	$2.70K_m\tau_m$	Critically damped dominant pole					
	0	$2.50K_m\tau_m$	Damping factor (dominant pole) = 0.6					
Keviczky and Csáki (1973). <i>Model: Method 1; Representative K_c, T_i values estimated from graphs; K_m = 1</i>	0	$x_2\tau_m$						
	Coefficient values							
	x_2	OS	x_2	OS	x_2	OS	x_2	OS
	0.64	105%	1.00	50%	1.5	17%	2	4%
	x_1		$2\tau_m$					
	Coefficient values							
	x_1	OS	x_1	OS	x_1	OS	x_1	OS
	0.68	5%	0.72	10%	0.75	15%	0.78	20%
	0		$2K_m\tau_m$		OS = 10%			
Kamimura <i>et al.</i> (1994). <i>Model: Method 1</i>	0		$2.6667K_m\tau_m$		OS = 0%			
	0		$2.3529K_m\tau_m$		“Quick” servo response; “negligible” overshoot			
Hansen (2000). <i>Model: Method 1</i>	$0.2/K_m$		$0.3\tau_m$					
Robust								
Bequette (2003) – page 300.	$\frac{\tau_m}{K_m(2\lambda + \tau_m)}$		$0.5\tau_m$		<i>Model: Method 1</i>			
Skogestad (2003). <i>Model: Method 1</i>	$0.294/K_m$		$0.5\tau_m$		IMC based rule			
Other tuning								
Skogestad (2003).	$0.25/K_m$		$0.333\tau_m$		<i>Model: Method 1</i>			

$$^5 K_c^{(175)} = \frac{e^{-\omega_d \tau_m}}{K_m} \left(1 - 0.5 e^{-\omega_d \tau_m} \right), \quad T_i^{(175)} = \frac{\tau_m}{e^{-\omega_d \tau_m}} \left(1 - 0.5 e^{-\omega_d \tau_m} \right).$$

3.17 General Model with a Repeated Pole

3.17.1 Ideal controller $G_c(s) = K_c \left(1 + \frac{1}{T_i s} \right)$

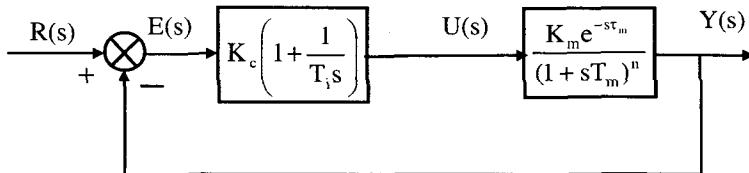


Table 42: PI controller tuning rules – general model with a repeated pole

$$G_m(s) = \frac{K_m e^{-s\tau_m}}{(1 + sT_m)^n}$$

Rule	K_c	T_i	Comment
Direct synthesis: frequency domain criteria			
Schaedel (1997). <i>Model: Method 2</i>	$\frac{0.5}{K_m [n^{0.5} - 1]}$	$\frac{\tau_m + T_{ar}}{\sqrt{n}}$	'Normal' design
	$\frac{0.375}{K_m} \left[\frac{n+1}{n-1} \right]$	$\frac{(n+1)(\tau_m + T_{ar})}{2n}$	'Sharp' design

3.18 General Model with Integrator

$$G_m(s) = \frac{K_m}{s} \frac{\prod_i^i (T_{m_i} s + 1) \prod_i^i (T_{m_2 i}^2 s^2 + 2\xi_{m_n i} T_{m_2 i} s + 1)}{\prod_i^i (T_{m_3 i} s + 1) \prod_i^i (T_{m_4 i}^2 s^2 + 2\xi_{m_d i} T_{m_4 i} s + 1)} e^{-s\tau_m}$$

3.18.1 Ideal controller $G_c(s) = K_c \left(1 + \frac{1}{T_i s} \right)$

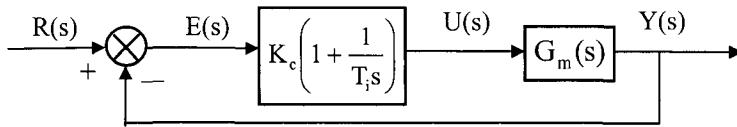


Table 43: PI controller tuning rules – general model with integrator

$$G_m(s) = \frac{K_m}{s} \frac{\prod_i^i (T_{m_i} s + 1) \prod_i^i (T_{m_2 i}^2 s^2 + 2\xi_{m_n i} T_{m_2 i} s + 1)}{\prod_i^i (T_{m_3 i} s + 1) \prod_i^i (T_{m_4 i}^2 s^2 + 2\xi_{m_d i} T_{m_4 i} s + 1)} e^{-s\tau_m}$$

Rule	K_c	T_i	Comment
Direct synthesis			
Loron (1997). Model: Method 1	${}^6 K_c^{(176)}$	$\alpha_1 T_Q$	Symmetrical optimum method
	$K_c^{(177)}$		Non-symmetrical optimum method

$${}^6 K_c^{(176)} = \frac{1}{K_m \sqrt{\alpha_1 T_Q}}, \quad \alpha_1 = \left(\frac{1 + \sin \phi_m}{\cos \phi_m} \right)^2$$

$$T_Q = \left[\sum_i T_{m_3 i} + 2 \sum_i \xi_{m_d i} T_{m_4 i} \right] - \left[\sum_i T_{m_i} + 2 \sum_i \xi_{m_n i} T_{m_2 i} \right] + \tau_m.$$

$$K_c^{(177)} = \frac{\lambda}{K_m \sqrt{\alpha_2 T_Q}}, \quad \lambda = \sqrt{\frac{M_c^2}{M_c^2 - 1}}, \quad M_c = \text{desired closed loop resonant peak},$$

$$\alpha_2 = \left(\frac{1 + \sin \Delta\phi}{\cos \Delta\phi} \right)^2, \quad \Delta\phi = \cos^{-1}(1/\lambda).$$

Chapter 4

Tuning Rules for PID Controllers

4.1 FOLPD Model $G_m(s) = \frac{K_m e^{-s\tau_m}}{1 + sT_m}$

4.1.1 Ideal controller $G_c(s) = K_c \left(1 + \frac{1}{T_i s} + T_d s \right)$

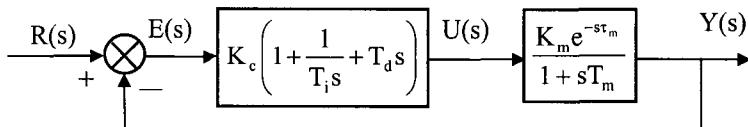


Table 44: PID controller tuning rules – FOLPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{1 + sT_m}$

Rule	K_c	T_i	T_d	Comment
Process reaction				
Callender <i>et al.</i> (1935/6). <i>Model: Method 1</i>	$^1 \frac{1.066}{K_m \tau_m}$	$1.418\tau_m$	$0.353\tau_m$ or $0.47\tau_m$	$\frac{\tau_m}{T_m} = 0.3$
Ziegler and Nichols (1942). <i>Model: Method 2</i>	$\frac{a\tau_m}{K_m \tau_m}$, $a \in [1.2, 2]$	$2\tau_m$	$0.5\tau_m$	Quarter decay ratio

¹ Decay ratio = 0.043; Period of decaying oscillation = $6.28\tau_m$

Rule	K_c	T_i	T_d	Comment
Chien <i>et al.</i> (1952) – regulator. <i>Model: Method 2</i>	$\frac{0.95T_m}{K_m \tau_m}$	$2.38\tau_m$	$0.42\tau_m$	0% overshoot; $0.1 < \frac{\tau_m}{T_m} < 1$
	$\frac{1.2T_m}{K_m \tau_m}$	$2\tau_m$	$0.42\tau_m$	20% overshoot; $0.1 < \frac{\tau_m}{T_m} < 1$
Chien <i>et al.</i> (1952) – servo. <i>Model: Method 2</i>	$\frac{0.6T_m}{K_m \tau_m}$	T_m	$0.5\tau_m$	0% overshoot; $0.1 < \frac{\tau_m}{T_m} < 1$
	$\frac{0.95T_m}{K_m \tau_m}$	$1.36T_m$	$0.47\tau_m$	20% overshoot; $0.1 < \frac{\tau_m}{T_m} < 1$
Cohen and Coon (1953). <i>Model: Method 2</i>	¹ $K_c^{(178)}$	$T_i^{(178)}$	$\frac{0.37\tau_m}{1 + 0.19\frac{\tau_m}{T_m}}$	Quarter decay ratio. $0 < \frac{\tau_m}{T_m} \leq 1$
Three constraints method – Murrill (1967) - page 356. <i>Model: Method 5</i>	$\frac{1.370}{K_m} \left(\frac{T_m}{\tau_m} \right)^{0.950}$	$\frac{T_m}{1.351} \left(\frac{\tau_m}{T_m} \right)^{0.738}$	$^2 T_d^{(179)}$	$\frac{K_c K_m T_d}{T_m} = 0.5$; $0.1 \leq \frac{\tau_m}{T_m} \leq 1$
Quarter decay ratio; minimum integral error (servo mode).				
Åström and Hägglund (1988) - pages 120-126.	$\frac{3}{K_m}$	$T_{90\%}$	$0.5\tau_m$	<i>Model: Method 23</i>
	$T_{90\%} = 90\%$ closed loop step response time (Leeds and Northrup Electromax V controller).			
Parr (1989) – page 194.	$\frac{1.25T_m}{K_m \tau_m}$	$2.5\tau_m$	$0.4\tau_m$	<i>Model: Method 2</i>
Borresen and Grindal (1990).	$\frac{T_m}{K_m \tau_m}$	$3\tau_m$	$0.5\tau_m$	<i>Model: Method 2</i>

$$^1 K_c^{(178)} = \frac{1}{K_m} \left(1.35 \frac{T_m}{\tau_m} + 0.25 \right), T_i^{(178)} = T_m \left(\frac{2.5 \frac{\tau_m}{T_m} + 0.46 \left(\frac{\tau_m}{T_m} \right)^2}{1 + 0.61 \frac{\tau_m}{T_m}} \right).$$

$$^2 T_d^{(179)} = 0.365 T_m \left(\frac{\tau_m}{T_m} \right)^{0.950}.$$

Rule	K_c	T_i	T_d	Comment				
Sain and Özgen (1992).	${}^3 K_c^{(180)}$	$T_i^{(180)}$	$T_d^{(180)}$	<i>Model: Method 24</i>				
Connell (1996) – page 214. <i>Model: Method 2</i>	$\frac{1.6T_m}{K_m \tau_m}$	$1.6667\tau_m$	$0.4\tau_m$	Approximate quarter decay ratio				
Hay (1998) – pages 197,198. <i>Model: Method 1</i>	x_1/K_m	$x_2\tau_m$	$x_3\tau_m$					
Coefficient values - servo tuning								
<i>Coefficients of K_c, T_i, T_d deduced from graphs</i>	$\frac{\tau_m}{T_m}$	x_1	x_2	x_3	$\frac{\tau_m}{T_m}$	x_1	x_2	x_3
	0.125	6.5	9.0	0.47	0.125	5.0	9.8	0.34
	0.167	5.0	6.5	0.45	0.167	3.7	7.0	0.32
	0.25	3.5	4.2	0.40	0.25	2.5	4.8	0.30
	0.5	1.8	2.2	0.35	0.5	1.1	2.5	0.27
Coefficient values - regulator tuning								
	$\frac{\tau_m}{T_m}$	x_1	x_2	x_3	$\frac{\tau_m}{T_m}$	x_1	x_2	x_3
	0.125	9.8	2.0	0.52	0.1	10.0	2.2	0.35
	0.167	7.2	1.8	0.50	0.125	8.0	2.2	0.35
	0.25	4.5	1.6	0.45	0.167	6.0	2	0.34
	0.5	2.2	1.4	0.35	0.25	4.0	1.8	0.32
					0.5	2.0	1.4	0.28
Moros (1999). <i>Model: Method 1</i>	$\frac{1.2T_m}{K_m \tau_m}$	$2\tau_m$	$0.42\tau_m$	attributed to Oppelt				
	$\frac{1.2T_m}{K_m \tau_m}$	$2\tau_m$	$0.44\tau_m$	attributed to Rosenberg				
Lipták (2001).	$\frac{0.85T_m}{K_m \tau_m}$	$1.6\tau_m$	$0.6\tau_m$	<i>Model: Method 2</i>				
ControlSoft Inc. (2005).	$\frac{2}{K_m}$	$\tau_m + T_m$	${}^4 T_d^{(181)}$	<i>Model: Method 10</i>				

$${}^3 K_c^{(180)} = \frac{1}{K_m} \left(0.6939 \frac{T_m}{\tau_m} + 0.1814 \right), \quad T_i^{(180)} = \frac{0.8647 T_m + 0.226 \tau_m}{\frac{T_m}{\tau_m} + 0.8647},$$

$$T_d^{(180)} = \frac{0.0565 T_m}{\frac{0.8647}{T_m} + 0.226}.$$

$$T_d^{(181)} = \max \left[\frac{\tau_m}{3}, \frac{T_m}{6} \right] \text{ (slow loop); } T_d^{(181)} = \min \left[\frac{\tau_m}{3}, \frac{T_m}{6} \right] \text{ (fast loop).}$$

Rule	K_c	T_i	T_d	Comment
Minimum performance index: regulator tuning				
Minimum IAE – Murrill (1967) – pages 358-363.	$\frac{1.435}{K_m} \left(\frac{T_m}{\tau_m} \right)^{0.921}$	$\frac{T_m}{0.878} \left(\frac{\tau_m}{T_m} \right)^{0.749}$	${}^5 T_d^{(182)}$	<i>Model: Method 5;</i> $0.1 \leq \frac{\tau_m}{T_m} \leq 1$
<i>Model: Method 8</i>	$1.3/K_m$	$0.7\tau_m$	$0.01\tau_m$	$\tau_m/T_m = 0.43$
	$1.0/K_m$	$0.68\tau_m$	$0.05\tau_m$	$\tau_m/T_m = 0.67$
	$0.85/K_m$	$0.65\tau_m$	$0.08\tau_m$	$\tau_m/T_m = 1.0$
	$0.65/K_m$	$0.65\tau_m$	$0.10\tau_m$	$\tau_m/T_m = 1.5$
	$0.45/K_m$	$0.55\tau_m$	$0.14\tau_m$	$\tau_m/T_m = 2.33$
	$0.4/K_m$	$0.50\tau_m$	$0.21\tau_m$	$\tau_m/T_m = 4.0$
<i>Coefficients of K_c, T_i, T_d estimated from graphs; robust to $\pm 25\%$ process model parameter changes</i>				
<i>Model: Method 17</i>	$6.558K_m$	$0.925T_m$	$0.061T_m$	$\tau_m/T_m = 0.1$
	$2.311K_m$	$0.677T_m$	$0.189T_m$	$\tau_m/T_m = 0.2$
	$1.479K_m$	$0.633T_m$	$0.233T_m$	$\tau_m/T_m = 0.3$
	$1.117K_m$	$0.647T_m$	$0.250T_m$	$\tau_m/T_m = 0.4$
	$0.947K_m$	$0.705T_m$	$0.268T_m$	$\tau_m/T_m = 0.5$
	$0.825K_m$	$0.752T_m$	$0.289T_m$	$\tau_m/T_m = 0.6$
	$0.731K_m$	$0.792T_m$	$0.312T_m$	$\tau_m/T_m = 0.7$
	$0.695K_m$	$0.871T_m$	$0.316T_m$	$\tau_m/T_m = 0.8$
	$0.630K_m$	$0.895T_m$	$0.326T_m$	$\tau_m/T_m = 0.9$
	$0.651K_m$	$1.030T_m$	$0.306T_m$	$\tau_m/T_m = 1.0$
Minimum IAE – Pessen (1994). <i>Model: Method 1</i>	$0.7K_u$	$0.4T_u$	$0.149T_u$	$0.1 \leq \frac{\tau_m}{T_m} \leq 1.0$
Modified minimum IAE – Cheng and Hung (1985).	$\frac{3}{K_m} \left(\frac{T_m}{\tau_m} \right)^{0.921}$	$\frac{T_m}{0.878} \left(\frac{\tau_m}{T_m} \right)^{0.749}$	${}^6 T_d^{(183)}$	<i>Model: Method 12</i>

$${}^5 T_d^{(182)} = 0.482 T_m \left(\frac{\tau_m}{T_m} \right)^{1.137}.$$

$${}^6 T_d^{(183)} = 0.482 T_m \left(\frac{\tau_m}{T_m} \right)^{1.137}.$$

Rule	K_c	T_i	T_d	Comment
Minimum ISE – Murrill (1967) – pages 358-363.	$\frac{1.495}{K_m} \left(\frac{T_m}{\tau_m} \right)^{0.945}$	$\frac{T_m}{1.101} \left(\frac{\tau_m}{T_m} \right)^{0.771}$	$0.56 T_m \left(\frac{\tau_m}{T_m} \right)^{1.006}$	$0.1 \leq \frac{\tau_m}{T_m} \leq 1$; Model: Method 5
Minimum ISE – Zhuang and Atherton (1993). Model: Method 1	$\frac{1.473}{K_m} \left(\frac{T_m}{\tau_m} \right)^{0.970}$	$\frac{T_m}{1.115} \left(\frac{\tau_m}{T_m} \right)^{0.753}$	${}^7 T_d^{(184)}$	$0.1 \leq \frac{\tau_m}{T_m} \leq 1.0$
	$\frac{1.524}{K_m} \left(\frac{T_m}{\tau_m} \right)^{0.735}$	$\frac{T_m}{1.130} \left(\frac{\tau_m}{T_m} \right)^{0.641}$	${}^7 T_d^{(185)}$	$1.1 \leq \frac{\tau_m}{T_m} \leq 2.0$
Minimum ISE – Ho <i>et al.</i> (1998). Model: Method 1	${}^8 K_c^{(186)}$	$T_i^{(186)}$	$T_d^{(186)}$	$A_m \in [2,5]$, $\phi_m \in [30^0, 60^0]$, $0.1 \leq \frac{\tau_m}{T_m} \leq 1.0$
Minimum ITAE – Murrill (1967) – pages 358-363.	$\frac{1.357}{K_m} \left(\frac{T_m}{\tau_m} \right)^{0.947}$	$\frac{T_m}{0.842} \left(\frac{\tau_m}{T_m} \right)^{0.738}$	${}^9 T_d^{(187)}$	Model: Method 5 $0.1 \leq \frac{\tau_m}{T_m} \leq 1$
Minimum ISTSE - Zhuang and Atherton (1993). Model: Method 1	$\frac{1.468}{K_m} \left(\frac{T_m}{\tau_m} \right)^{0.970}$	$\frac{T_m}{0.942} \left(\frac{\tau_m}{T_m} \right)^{0.725}$	${}^{10} T_d^{(188)}$	$0.1 \leq \frac{\tau_m}{T_m} \leq 1.0$
	$\frac{1.515}{K_m} \left(\frac{T_m}{\tau_m} \right)^{0.730}$	$\frac{T_m}{0.957} \left(\frac{\tau_m}{T_m} \right)^{0.598}$	${}^{10} T_d^{(189)}$	$1.1 \leq \frac{\tau_m}{T_m} \leq 2.0$

$${}^7 T_d^{(184)} = 0.550 T_m \left(\frac{\tau_m}{T_m} \right)^{0.948}, \quad T_d^{(185)} = 0.552 T_m \left(\frac{\tau_m}{T_m} \right)^{0.851}$$

$${}^8 K_c^{(186)} = \frac{1.0722}{K_m} \frac{\phi_m^{-0.116}}{A_m^{0.8432}} \left(\frac{\tau_m}{T_m} \right)^{-0.908}, \quad T_i^{(186)} = \frac{1.2497 T_m \phi_m^{1.0082}}{A_m^{0.2099}} \left(\frac{\tau_m}{T_m} \right)^{0.3678},$$

$$T_d^{(186)} = \frac{0.4763 T_m \phi_m^{-0.328}}{A_m^{0.0961}} \left(\frac{\tau_m}{T_m} \right)^{1.0317};$$

Given A_m , ISE is minimised when $\phi_m = 46.5489 A_m^{0.2035} (\tau_m/T_m)^{0.3693}$
(Ho *et al.* (1999)); $2 \leq A_m \leq 5$; $0.1 \leq \tau_m/T_m \leq 1.0$.

$${}^9 T_d^{(187)} = 0.381 T_m \left(\frac{\tau_m}{T_m} \right)^{0.995}.$$

$${}^{10} T_d^{(188)} = 0.443 T_m \left(\frac{\tau_m}{T_m} \right)^{0.939}, \quad T_d^{(189)} = 0.444 T_m \left(\frac{\tau_m}{T_m} \right)^{0.847}.$$

Rule	K_c	T_i	T_d	Comment
Minimum ISTSE – Zhuang and Atherton (1993).	$^{11} K_c^{(190)}$	$T_i^{(190)}$	$0.144T_u$	$0.1 \leq \frac{\tau_m}{T_m} \leq 2.0$; <i>Model: Method 1</i>
	$^{12} K_c^{(191)}$	$T_i^{(191)}$	$0.15\hat{T}_u$	$0.1 \leq \frac{\tau_m}{T_m} \leq 2.0$; <i>Model:</i> <i>Method 31</i>
Minimum ISTES – Zhuang and Atherton (1993). <i>Model: Method 1</i>	$\frac{1.531}{K_m} \left(\frac{T_m}{\tau_m} \right)^{0.960}$	$\frac{T_m}{0.971} \left(\frac{\tau_m}{T_m} \right)^{0.746}$	$^{15} T_d^{(192)}$	$0.1 \leq \frac{\tau_m}{T_m} \leq 1.0$
	$\frac{1.592}{K_m} \left(\frac{T_m}{\tau_m} \right)^{0.705}$	$\frac{T_m}{0.957} \left(\frac{\tau_m}{T_m} \right)^{0.597}$	$^{15} T_d^{(193)}$	$1.1 \leq \frac{\tau_m}{T_m} \leq 2.0$
Minimum error – step load change – Gerry (1998).	$\frac{0.3}{K_m}$	$0.5\tau_m$	T_m	$\frac{\tau_m}{T_m} > 5$; <i>Model: Method 1</i>
Nearly minimum IAE, ISE, ITAE – Hwang (1995).	$^{16} K_c^{(194)}$	$T_i^{(194)}$	$\frac{0.471K_u}{K_m \omega_u}$	$\varepsilon_1 < 2.4$
	<i>Model: Method 35; Decay ratio = 0.15; $0.1 \leq \frac{\tau_m}{T_m} \leq 2.0$</i>			

$$^{11} K_c^{(190)} = \frac{4.434K_m K_u - 0.966}{5.12K_m K_u + 1.734} K_u, \quad T_i^{(190)} = \frac{1.751K_m K_u - 0.612}{3.776K_m K_u + 1.388} T_u.$$

$$^{12} K_c^{(191)} = \frac{6.068K_m \hat{K}_u - 4.273 \hat{K}_u}{5.758K_m \hat{K}_u - 1.058} \hat{K}_u, \quad T_i^{(191)} = \frac{1.1622K_m \hat{K}_u - 0.748 \hat{T}_u}{2.516K_m \hat{K}_u - 0.505} \hat{T}_u.$$

$$^{15} T_d^{(192)} = 0.413T_m \left(\frac{\tau_m}{T_m} \right)^{0.933}, \quad T_d^{(193)} = 0.414T_m \left(\frac{\tau_m}{T_m} \right)^{0.850}.$$

$$^{16} K_c^{(194)} = \left(K_{H1} - \frac{0.674[1 - 0.447\omega_{H1}\tau_m + 0.0607\omega_{H1}^2\tau_m^2]}{K_m/(1 + K_{H1}K_m)} \right),$$

$$T_i^{(194)} = \frac{K_c^{(194)}(1 + K_{H1}K_m)}{\omega_{H1}K_m 0.0607(1 + 1.05\omega_{H1}\tau_m - 0.233\omega_{H1}^2\tau_m^2)}.$$

Rule	K_c	T_i	T_d	Comment
Nearly minimum IAE, ISE, ITAE – Hwang (1995) – continued.	$^{17} K_c^{(195)}$	$T_i^{(195)}$		$2.4 \leq \varepsilon_1 < 3$
	$^{18} K_c^{(196)}$	$T_i^{(196)}$		$3 \leq \varepsilon_1 < 20$
	$^{19} K_c^{(197)}$	$T_i^{(197)}$		$\varepsilon_1 \geq 20$
Nearly minimum IAE, ITAE – Hwang and Fang (1995). <i>Model:</i> <i>Method 25</i>	$^{20} K_c^{(198)}$	$T_i^{(198)}$	$T_d^{(198)}$	$0.1 \leq \frac{\tau_m}{T_m} \leq 2.0 ;$ decay ratio = 0.12

$$^{17} K_c^{(195)} = \left(K_{H1} - \frac{0.778[1 - 0.467\omega_{H1}\tau_m + 0.0609\omega_{H1}^2\tau_m^2]}{K_m/(1 + K_{H1}K_m)} \right),$$

$$T_i^{(195)} = \frac{K_c^{(195)}(1 + K_{H1}K_m)}{\omega_{H1}K_m 0.0309(1 + 2.84\omega_{H1}\tau_m - 0.532\omega_{H1}^2\tau_m^2)}.$$

$$^{18} K_c^{(196)} = \left(K_{H1} - \frac{1.31(0.519)^{\omega_{H1}\tau_m}[1 - 1.03/\varepsilon_1 + 0.514/\varepsilon_1^2]}{K_m/(1 + K_{H1}K_m)} \right),$$

$$T_i^{(196)} = \frac{K_c^{(196)}(1 + K_{H1}K_m)}{\omega_{H1}K_m 0.0603(1 + 0.929 \ln[\omega_{H1}\tau_m])(1 + 2.01/\varepsilon_1 - 1.2/\varepsilon_1^2)}.$$

$$^{19} K_c^{(197)} = \left(K_{H1} - \frac{1.14[1 - 0.482\omega_{H1}\tau_m + 0.068\omega_{H1}^2\tau_m^2]}{K_m/(1 + K_{H1}K_m)} \right),$$

$$T_i^{(197)} = \frac{K_c^{(197)}(1 + K_{H1}K_m)}{\omega_{H1}K_m 0.0694(-1 + 2.1\omega_{H1}\tau_m - 0.367\omega_{H1}^2\tau_m^2)}.$$

$$^{20} K_c^{(198)} = \left[0.802 - 0.154 \frac{\tau_m}{T_m} + 0.0460 \left(\frac{\tau_m}{T_m} \right)^2 \right] K_u,$$

$$T_i^{(198)} = \frac{K_c^{(198)}}{K_u \omega_u \left[0.190 + 0.0532 \frac{\tau_m}{T_m} - 0.00509 \left(\frac{\tau_m}{T_m} \right)^2 \right]},$$

$$T_d^{(198)} = \left[0.421 + 0.00915 \frac{\tau_m}{T_m} - 0.00152 \left(\frac{\tau_m}{T_m} \right)^2 \right] \frac{K_u}{\omega_u K_c^{(198)}}.$$

Rule	K_c	T_i	T_d	Comment
O'Connor and Denn (1972). <i>Model:</i> Method 1; $K_m = T_m$; $\frac{\tau_m}{T_m} < 0.2$		Minimum $0.5 \int_0^{\infty} a^2 y(t) + \left(\frac{du(t)}{dt} \right)^2 dt$		
	$^{21} \frac{a\tau_m^2 + 2x_1}{(2+x_1)\tau_m}$	$\frac{a\tau_m^2 + 2x_1}{2\tau_m a}$	$\frac{x_1\tau_m}{\tau_m^2 a + 2x_1}$	Recommended $a \in \left[\frac{0.6}{\tau_m^2}, \frac{1}{\tau_m^2} \right]$
Syracos and Kookos (2005). <i>Model: Method 1</i>	$^{22} K_c^{(199)}$	$T_i^{(199)}$	$T_d^{(199)}$	$2 \leq \frac{\tau_m}{T_m} \leq 5$
	Minimum performance index – constraints are placed on the process input and output signals			
Minimum IE – Devanathan (1991). <i>Model:</i> Method 1; Coefficients of K_c, T_i provided are deduced from graphs	$^{23} K_c^{(200)}$	$0.32T_u$	$0.08T_u$	
		$x_2 T_u$	$x_3 T_u$	
	Coefficient values			
	τ_m/T_m	x_2	x_3	τ_m/T_m
	0.2	0.31	0.08	1.2
	0.4	0.29	0.07	1.4
	0.6	0.29	0.07	1.6
	0.8	0.29	0.07	1.8
	1.0	0.28	0.07	
				x_3

$$^{21} x_1 = \frac{a\tau_m^2 - 2\frac{\tau_m}{T_m} + 2\sqrt{\left(\frac{\tau_m}{T_m}\right)^2 + 2\tau_m^2 a}}{2 + \frac{\tau_m}{T_m}}.$$

$$^{22} K_c^{(199)} = \frac{1}{K_m} \left[0.31 + 0.6 \frac{T_m}{\tau_m} \right], \quad T_i^{(199)} = T_m \left[0.777 + 0.45 \frac{\tau_m}{T_m} \right],$$

$$T_d^{(199)} = T_m \left[0.44 - 0.56 \left(\frac{T_m}{\tau_m} \right)^{2.2} \right].$$

$$^{23} K_c^{(200)} = \frac{\xi \cos \left[\tan^{-1} \left(1.57 D_R - \frac{0.16}{D_R} \right) \right]}{K_m \cos \left[\tan^{-1} \left(\frac{6.28 T_m}{T_u} \right) \right]}.$$

Rule	K_c	T_i	T_d	Comment
Minimum performance index: servo tuning				
Minimum IAE – Gallier and Otto (1968). <i>Model: Method 1</i>	x_1/K_m	$x_2(T_m + \tau_m)$	$x_3(T_m + \tau_m)$	
Representative coefficient values – deduced from graphs				
	τ_m/T_m	x_1	x_2	x_3
	0.053	12.3	0.95	0.0215
	0.11	6.8	0.93	0.041
	0.25	3.4	0.88	0.083
Minimum IAE – Rovira <i>et al.</i> (1969).	$\frac{1.086}{K_m} \left(\frac{T_m}{\tau_m} \right)^{0.869}$	T_m	$^{24} T_d^{(201)}$	<i>Model: Method 5</i> $0.1 < \frac{\tau_m}{T_m} \leq 1$
	$0.740 - 0.13 \frac{\tau_m}{T_m}$			
Minimum IAE – Marlin (1995) – pages 301-307. <i>Model: Method 1</i>	$1.3/K_m$	$0.93\tau_m$	$0.02\tau_m$	$\tau_m/T_m = 0.25$
	$1.0/K_m$	$0.80\tau_m$	$0.05\tau_m$	$\tau_m/T_m = 0.43$
	$0.8/K_m$	$0.71\tau_m$	$0.06\tau_m$	$\tau_m/T_m = 0.67$
	$0.7/K_m$	$0.73\tau_m$	$0.09\tau_m$	$\tau_m/T_m = 1.0$
	$0.55/K_m$	$0.64\tau_m$	$0.12\tau_m$	$\tau_m/T_m = 1.5$
	$0.48/K_m$	$0.56\tau_m$	$0.15\tau_m$	$\tau_m/T_m = 2.33$
	$0.4/K_m$	$0.52\tau_m$	$0.21\tau_m$	$\tau_m/T_m = 4.0$
<i>Coefficients of K_c, T_i, T_d estimated from graphs; robust to $\pm 25\%$ process model parameter changes.</i>				
Minimum IAE – Sadeghi and Tych (2003). <i>Model: Method 1</i> $0.05 < \frac{\tau_m}{T_m} < 6$	$^{25} K_c^{(202)}$	$T_i^{(202)}$	$\frac{1.45933\tau_m}{\frac{\tau_m}{T_m} + 3.27873}$	<i>Model: Method 1</i> $0.1 \leq \frac{\tau_m}{T_m} \leq 1.0$
Wang <i>et al.</i> (1995a). <i>Model: Method 1</i>	$^{26} K_c^{(203)}$			Minimum IAE
	$^{26} K_c^{(204)}$	$T_m + 0.5\tau_m$	$\frac{0.5T_m\tau_m}{T_m + 0.5\tau_m}$	Minimum ISE

$$^{24} T_d^{(201)} = 0.348T_m \left(\frac{\tau_m}{T_m} \right)^{0.914}.$$

$$^{25} K_c^{(202)} = \frac{1}{K_m} \left(0.26266 + 0.82714 \frac{T_m}{\tau_m} \right), \quad T_i^{(202)} = T_m \left(0.28743 \frac{\tau_m}{T_m} + 1.35955 \right).$$

$$^{26} K_c^{(203)} = \frac{\left(0.7645 + \frac{0.6032}{\tau_m/T_m} \right) (T_m + 0.5\tau_m)}{K_m(T_m + \tau_m)}, \quad K_c^{(204)} = \frac{\left(0.9155 + \frac{0.7524}{\tau_m/T_m} \right) (T_m + 0.5\tau_m)}{K_m(T_m + \tau_m)}$$

Rule	K_c	T_i	T_d	Comment
Minimum ISE – Zhuang and Atherton (1993). <i>Model: Method 1</i>	$\frac{1.048}{K_m} \left(\frac{T_m}{\tau_m} \right)^{0.897}$	$^{27} T_i^{(205)}$	$T_d^{(205)}$	$0.1 \leq \frac{\tau_m}{T_m} \leq 1.0$
	$\frac{1.154}{K_m} \left(\frac{T_m}{\tau_m} \right)^{0.567}$	$^{28} T_i^{(206)}$	$T_d^{(206)}$	$1.1 \leq \frac{\tau_m}{T_m} \leq 2.0$
Minimum ISE – Ho <i>et al.</i> (1998). <i>Model: Method 1</i>	$^{29} K_c^{(207)}$	$T_i^{(207)}$	$T_d^{(207)}$	$A_m \in [2,5]$, $\phi_m \in [30^0, 60^0]$, $0.1 \leq \frac{\tau_m}{T_m} \leq 1.0$.
Minimum ISE – Sadeghi and Tych (2003).	$^{30} K_c^{(208)}$	$T_i^{(208)}$	$\frac{1.93576\tau_m}{\frac{\tau_m}{T_m} + 3.83528}$	$0.1 \leq \frac{\tau_m}{T_m} \leq 1.0$; <i>Model: Method 1</i>
Minimum ITAE – Rovira <i>et al.</i> (1969).	$\frac{0.965}{K_m} \left(\frac{T_m}{\tau_m} \right)^{0.85}$	$^{31} T_i^{(209)}$	$T_d^{(209)}$	$0.1 \leq \frac{\tau_m}{T_m} \leq 1$; <i>Model: Method 5</i>

$$^{27} T_i^{(205)} = \frac{T_m}{1.195 - 0.368 \frac{\tau_m}{T_m}}, \quad T_d^{(205)} = 0.4897 T_m \left(\frac{\tau_m}{T_m} \right)^{0.888}.$$

$$^{28} T_i^{(206)} = \frac{T_m}{1.047 - 0.220 \frac{\tau_m}{T_m}}, \quad T_d^{(206)} = 0.4907 T_m \left(\frac{\tau_m}{T_m} \right)^{0.708}.$$

$$^{29} K_c^{(207)} = \frac{1.8578}{K_m} \frac{\phi_m^{0.0821}}{A_m^{0.09087}} \left(\frac{\tau_m}{T_m} \right)^{-0.9471}, \quad T_d^{(207)} = \frac{0.4899 T_m \phi_m^{0.1457}}{A_m^{0.0845}} \left(\frac{\tau_m}{T_m} \right)^{1.0264},$$

$$T_i^{(207)} = \frac{0.0211 T_m [1 + 0.3289 A_m + 6.4572 \phi_m + 25.1914 (\tau_m/T_m)]}{1 + 0.0625 A_m - 0.8079 \phi_m + 0.347 (\tau_m/T_m)},$$

Given A_m , with $2 \leq A_m \leq 5$ and $0.1 \leq \tau_m/T_m \leq 1.0$, ISE is minimised when

$$\phi_m = 29.7985 + \frac{62.1189}{A_m} + \frac{40.3182 \tau_m}{T_m} - \frac{76.2833 \tau_m}{A_m T_m} \quad (\text{Ho } et al. (1999)).$$

$$^{30} K_c^{(208)} = \frac{1}{K_m} \left(0.40455 + 0.96441 \frac{T_m}{\tau_m} \right), \quad T_i^{(208)} = T_m \left(0.43770 \frac{\tau_m}{T_m} + 1.39588 \right).$$

$$^{31} T_i^{(209)} = \frac{T_m}{0.796 - 0.1465 \frac{\tau_m}{T_m}}, \quad T_d^{(209)} = 0.308 T_m \left(\frac{\tau_m}{T_m} \right)^{0.929}.$$

Rule	K_c	T_i	T_d	Comment
Minimum ITAE – Wang <i>et al.</i> (1995a). <i>Model: Method 1</i>	$^{32} K_c^{(210)}$	$T_m + 0.5\tau_m$	$\frac{0.5T_m\tau_m}{T_m + 0.5\tau_m}$	$0.05 < \frac{\tau_m}{T_m} < 6$
Minimum ITAE – Sadeghi and Tych (2003). <i>Model: Method 1</i>	$^{33} K_c^{(211)}$	$T_i^{(211)}$	$\frac{1.55237\tau_m}{\tau_m + 4.00000}$	$0.1 \leq \frac{\tau_m}{T_m} \leq 1$; <i>Model: Method 1</i>
Modified minimum ITAE – Cheng and Hung (1985). <i>Model: Method 12</i>	$\frac{1.2}{K_m} \left(\frac{T_m}{\tau_m} \right)^{0.855}$	$^{34} T_i^{(212)}$	$T_d^{(212)}$	$\xi = 0.707$; <i>Model:</i> <i>Method 12</i>
Modified minimum ITAE – Smith (2003). <i>Model: Method 1</i>	$\frac{0.965}{K_m} \left(\frac{T_m}{\tau_m} \right)^{0.855}$	$1.26T_m$	$0.308\tau_m$	<i>Model: Method 1</i>
Minimum ISTSE – Zhuang and Atherton (1993). <i>Model: Method 1</i>	$\frac{1.042}{K_m} \left(\frac{T_m}{\tau_m} \right)^{0.897}$	$^{35} T_i^{(213)}$	$T_d^{(213)}$	$0.1 \leq \frac{\tau_m}{T_m} \leq 1.0$
	$\frac{1.142}{K_m} \left(\frac{T_m}{\tau_m} \right)^{0.579}$	$^{36} T_i^{(214)}$	$T_d^{(214)}$	$1.1 \leq \frac{\tau_m}{T_m} \leq 2.0$

$$^{32} K_c^{(210)} = \frac{\left(0.7303 + \frac{0.5307}{\tau_m/T_m} \right) (T_m + 0.5\tau_m)}{K_m(T_m + \tau_m)}.$$

$$^{33} K_c^{(211)} = \frac{1}{K_m} \left(0.20360 + 0.80451 \frac{T_m}{\tau_m} \right), \quad T_i^{(211)} = T_m \left(0.29349 \frac{\tau_m}{T_m} + 1.29110 \right).$$

$$^{34} T_i^{(212)} = \frac{T_m}{0.796 - 0.147 \frac{\tau_m}{T_m}}, \quad T_d^{(212)} = 0.308T_m \left(\frac{\tau_m}{T_m} \right)^{0.929}.$$

$$^{35} T_i^{(213)} = \frac{T_m}{0.987 - 0.238 \frac{\tau_m}{T_m}}, \quad T_d^{(213)} = 0.385T_m \left(\frac{\tau_m}{T_m} \right)^{0.906}.$$

$$^{36} T_i^{(214)} = \frac{T_m}{0.919 - 0.172 \frac{\tau_m}{T_m}}, \quad T_d^{(214)} = 0.384T_m \left(\frac{\tau_m}{T_m} \right)^{0.839}.$$

Rule	K_c	T_i	T_d	Comment
Minimum ISTSE – Zhuang and Atherton (1993).	$0.509K_u$	$^{37} T_i^{(215)}$	$0.125T_u$	$0.1 \leq \frac{\tau_m}{T_m} \leq 2.0$; <i>Model: Method 1</i>
	$0.604 \hat{K}_u$	$^{37} T_i^{(216)}$	$0.130 \hat{T}_u$	$0.1 \leq \frac{\tau_m}{T_m} \leq 2.0$; <i>Model:</i> <i>Method 31</i>
Minimum ISTES – Zhuang and Atherton (1993). <i>Model: Method 1</i>	$\frac{0.968}{K_m} \left(\frac{T_m}{\tau_m} \right)^{0.904}$	$^{38} T_i^{(217)}$	$T_d^{(217)}$	$0.1 \leq \frac{\tau_m}{T_m} \leq 1.0$
	$\frac{1.061}{K_m} \left(\frac{T_m}{\tau_m} \right)^{0.583}$	$^{39} T_i^{(218)}$	$T_d^{(218)}$	$1.1 \leq \frac{\tau_m}{T_m} \leq 2.0$
Nearly minimum IAE, ISE, ITAE – Hwang (1995). <i>Model:</i> <i>Method 35</i>	$^{40} K_c^{(219)}$	$T_i^{(219)}$	$\frac{0.471K_u}{K_m \omega_u}$	$\varepsilon_1 < 2.4$
	$^{41} K_c^{(220)}$	$T_i^{(220)}$		$2.4 \leq \varepsilon_1 < 3$

$$^{37} T_i^{(215)} = 0.051(3.302K_m K_u + 1)T_u, \quad T_i^{(216)} = 0.04 \left(4.972K_m \hat{K}_u + 1 \right) \hat{T}_u.$$

$$^{38} T_i^{(217)} = \frac{T_m}{0.977 - 0.253 \frac{\tau_m}{T_m}}, \quad T_d^{(217)} = 0.316 T_m \left(\frac{\tau_m}{T_m} \right)^{0.892}.$$

$$^{39} T_i^{(218)} = \frac{T_m}{0.892 - 0.165 \frac{\tau_m}{T_m}}, \quad T_d^{(218)} = 0.315 T_m \left(\frac{\tau_m}{T_m} \right)^{0.832}.$$

$$^{40} K_c^{(219)} = \left(K_{H1} - \frac{0.822[1 - 0.549\omega_{H1}\tau_m + 0.112\omega_{H1}^2\tau_m^2]}{K_m/(1+K_{H1}K_m)} \right),$$

$$T_i^{(219)} = \frac{K_c^{(219)}(1+K_{H1}K_m)}{\omega_{H1}K_m 0.0142(1 + 6.96\omega_{H1}\tau_m - 1.77\omega_{H1}^2\tau_m^2)}.$$

$$^{41} K_c^{(220)} = \left(K_{H1} - \frac{0.786[1 - 0.441\omega_{H1}\tau_m + 0.0569\omega_{H1}^2\tau_m^2]}{K_m/(1+K_{H1}K_m)} \right),$$

$$T_i^{(220)} = \frac{K_c^{(220)}(1+K_{H1}K_m)}{\omega_{H1}K_m 0.0172(1 + 4.62\omega_{H1}\tau_m - 0.823\omega_{H1}^2\tau_m^2)}.$$

Rule	K_c	T_i	T_d	Comment
Nearly minimum IAE, ISE, ITAE – Hwang (1995) – continued	$^{42} K_c^{(221)}$	$T_i^{(221)}$		$3 \leq \varepsilon_1 < 20$
	$^{43} K_c^{(222)}$	$T_i^{(222)}$		$\varepsilon_1 \geq 20$
		Decay ratio = 0.1; $0.1 \leq \frac{\tau_m}{T_m} \leq 2.0$		
Nearly minimum IAE, ITAE – Hwang and Fang (1995).	$^{44} K_c^{(223)}$	$T_i^{(223)}$	$T_d^{(223)}$	<i>Model: Method 25</i>
	$0.1 \leq \frac{\tau_m}{T_m} \leq 2.0$; decay ratio = 0.03			

$$^{42} K_c^{(221)} = \left(K_{H1} - \frac{1.28(0.542)^{\omega_{H1}\tau_m} [1 - 0.986/\varepsilon_1 + 0.558/\varepsilon_1^2]}{K_m/(1 + K_{H1}K_m)} \right),$$

$$T_i^{(221)} = \frac{K_c^{(221)}(1 + K_{H1}K_m)}{\omega_{H1}K_m 0.0476(1 + 0.996 \ln[\omega_{H1}\tau_m])(1 + 2.13/\varepsilon_1 - 1.13/\varepsilon_1^2)}.$$

$$^{43} K_c^{(222)} = \left(K_{H1} - \frac{1.14[1 - 0.466\omega_{H1}\tau_m + 0.0647\omega_{H1}^2\tau_m^2]}{K_m/(1 + K_{H1}K_m)} \right),$$

$$T_i^{(222)} = \frac{K_c^{(222)}(1 + K_{H1}K_m)}{\omega_{H1}K_m 0.0609(-1 + 1.97\omega_{H1}\tau_m - 0.323\omega_{H1}^2\tau_m^2)}.$$

$$^{44} K_c^{(223)} = \left[0.537 - 0.0165 \frac{\tau_m}{T_m} + 0.00173 \left(\frac{\tau_m}{T_m} \right)^2 \right] K_u,$$

$$T_i^{(223)} = \frac{K_c^{(223)}}{K_u \omega_u \left[0.0503 + 0.163 \frac{\tau_m}{T_m} - 0.0389 \left(\frac{\tau_m}{T_m} \right)^2 \right]},$$

$$T_d^{(223)} = \left[0.350 - 0.0344 \frac{\tau_m}{T_m} + 0.00644 \left(\frac{\tau_m}{T_m} \right)^2 \right] \frac{K_u}{\omega_u K_c^{(223)}}.$$

Rule	K_c	T_i	T_d	Comment			
Minimum performance index: other tuning							
Simultaneous Servo/regulator tuning – nearly minimum IAE, ITAE – Hwang and Fang (1995).	⁴⁵ $K_c^{(224)}$	$T_i^{(224)}$	$T_d^{(224)}$	$0.1 \leq \frac{\tau_m}{T_m} \leq 2.0 ;$ decay ratio = 0.05; <i>Model: Method 25</i>			
Servo or regulator tuning – minimum IAE – Huang and Jeng (2003).	$\frac{0.36 + 0.76 \frac{\tau_m}{\tau_m}}{K_m}$	$0.47\tau_m + T_m$	$\frac{0.47T_m\tau_m}{0.47\tau_m + T_m}$	<i>Model: Method 37;</i> $\frac{\tau_m}{T_m} < 0.33$			
	⁴⁶ $K_c^{(225)}$	$T_i^{(225)}$	$T_d^{(225)}$	$\frac{\tau_m}{T_m} \geq 0.33$			
Åström and Hägglund (2004). <i>Model: Method 1</i>	$\frac{x_1\tau_m + x_2T_m}{K_m\tau_m}$	$\frac{x_3\tau_m + x_4T_m}{\tau_m + \alpha_5T_m}\tau_m$	$\frac{x_6T_m\tau_m}{\tau_m + x_7T_m}$				
Coefficient values							
x_1	x_2	x_3	x_4	x_5	x_6	x_7	
0.057	0.139	0.400	0.923	0.012	1.59	4.59	$M_{max} = 1.1$
0.103	0.261	0.389	0.930	0.040	1.62	4.44	$M_{max} = 1.2$
0.139	0.367	0.376	0.900	0.074	1.66	4.39	$M_{max} = 1.3$
0.168	0.460	0.363	0.871	0.111	1.70	4.37	$M_{max} = 1.4$
0.191	0.543	0.352	0.844	0.146	1.74	4.35	$M_{max} = 1.5$

$$\begin{aligned}
 ^{45} K_c^{(224)} &= \left[0.713 - 0.176 \frac{\tau_m}{T_m} + 0.0513 \left(\frac{\tau_m}{T_m} \right)^2 \right] K_u, \\
 T_i^{(224)} &= \frac{K_c^{(224)}}{K_u \omega_u \left[0.149 + 0.0556 \frac{\tau_m}{T_m} - 0.00566 \left(\frac{\tau_m}{T_m} \right)^2 \right]}, \\
 T_d^{(224)} &= \left[0.371 - 0.0274 \frac{\tau_m}{T_m} + 0.00557 \left(\frac{\tau_m}{T_m} \right)^2 \right] \frac{K_u}{\omega_u K_c^{(224)}}. \\
 ^{46} K_c^{(225)} &= \frac{0.8194T_n + 0.2773\tau_m}{K_m \tau_m \frac{0.9738}{0.0262} T_m}, \quad T_i^{(225)} = 1.0297T_m + 0.3484\tau_m, \\
 T_d^{(225)} &= \left(\frac{0.4575T_m + 0.0302\tau_m}{1.0297T_m + 0.3484\tau_m} \right) \tau_m.
 \end{aligned}$$

Rule	K_c	T_i	T_d	Comment
Coefficient values				
Åström and Hägglund (2004) - continued.	x_1	x_2	x_3	x_4
	0.211	0.616	0.342	0.820
	0.227	0.681	0.334	0.799
	0.241	0.740	0.326	0.781
	0.254	0.793	0.320	0.764
	0.264	0.841	0.314	0.751
Direct synthesis: time domain criteria				
Van der Grinten (1963). <i>Model: Method I</i>	$\frac{1}{K_m} \left(0.5 + \frac{T_m}{\tau_m} \right)$	$T_m + 0.5\tau_m$	$\frac{T_m \tau_m}{\tau_m + 2T_m}$	Step disturbance
	$^{47} K_c^{(226)}$	$T_i^{(226)}$	$T_d^{(226)}$	Stochastic disturbance
Regulator – Gorecki <i>et al.</i> (1989). <i>Model: Method I</i>	$^{48} K_c^{(227)}$	$T_i^{(227)}$	$T_d^{(227)}$	Pole is real and has maximum attainable multiplicity; $\tau_m/T_m < 2$

$$^{47} K_c^{(226)} = \frac{e^{-\omega_d \tau_m}}{K_m} \left(1 - 0.5e^{-\omega_d \tau_m} + \frac{T_m}{\tau_m} e^{-\omega_d \tau_m} \right),$$

$$T_i^{(226)} = \frac{\tau_m}{e^{-\omega_d \tau_m}} \left(1 - 0.5e^{-\omega_d \tau_m} + \frac{T_m}{\tau_m} e^{-\omega_d \tau_m} \right), \quad T_d^{(226)} = \frac{(1 - 0.5e^{-\omega_d \tau_m}) T_m}{1 - 0.5e^{-\omega_d \tau_m} + \frac{T_m}{\tau_m} e^{-\omega_d \tau_m}}.$$

$$^{48} K_c^{(227)} = \frac{2}{K_m} \frac{T_m}{\tau_m} \left[6 + \frac{\tau_m}{2T_m} x_1 - 9 - \left(\frac{\tau_m}{2T_m} \right)^2 \right] e^{x_1 - 3 - \frac{\tau_m}{2T_m}},$$

$$T_i^{(227)} = \tau_m \frac{\left[6 + \frac{\tau_m}{2T_m} \right] x_1 - 9 - \frac{\tau_m}{2T_m} - \left(\frac{\tau_m}{2T_m} \right)^2}{\left[21 + 3 \frac{\tau_m}{T_m} + \left(\frac{\tau_m}{2T_m} \right)^2 \right] x_1 - 36 - 4.5 \frac{\tau_m}{T_m} - 6 \left(\frac{\tau_m}{2T_m} \right)^2 - \left(\frac{\tau_m}{2T_m} \right)^3},$$

$$T_d^{(227)} = 0.5 \tau_m \frac{x_1 - 1}{\left[6 + \frac{\tau_m}{2T_m} \right] x_1 - 9 - \frac{\tau_m}{2T_m} - \left(\frac{\tau_m}{2T_m} \right)^2}, \quad x_1 = \sqrt{3 + \left(\frac{\tau_m}{2T_m} \right)^2}.$$

Rule	K_c	T_i	T_d	Comment
Saito <i>et al.</i> (1990). <i>Model: Method 18</i>	$\frac{0.215\tau_m + T_m}{1.37K_m\tau_m}$	$0.315\tau_m + T_m$	$\frac{0.315T_m\tau_m}{0.315\tau_m + T_m}$	$\frac{\tau_m}{T_m} < 1$
			$^{49}T_d^{(228)}$	$1 \leq \frac{\tau_m}{T_m} < 10$
Suyama (1992). <i>Model: Method 1</i>	$^{50}K_c^{(229)}$	$T_m + 0.309\tau_m$	$T_d^{(229)}$	OS=10%
Fruehauf <i>et al.</i> (1993). <i>Model: Method 2</i>	$\frac{0.56T_m}{\tau_m K_m}$	$5\tau_m$	$\leq 0.5\tau_m$	$\frac{\tau_m}{T_m} < 0.33$
	$\frac{0.5T_m}{\tau_m K_m}$	T_m	$\leq 0.5\tau_m$	$\frac{\tau_m}{T_m} \geq 0.33$
Juang and Wang (1995). <i>Model: Method 1</i>	$^{51}K_c^{(230)}$	$T_i^{(230)}$	$T_d^{(230)}$	$T_{CL} = \alpha T_m, 0 < \alpha < 1$
Abbas (1997) <i>Model: Method 1</i>	$^{52}K_c^{(231)}$	$T_m + 0.5\tau_m$	$\frac{T_m\tau_m}{2T_m + \tau_m}$	$0 \leq V \leq 0.2; 0.1 \leq \frac{\tau_m}{T_m} \leq 5.0$
Camacho <i>et al.</i> (1997).	$\frac{1}{K_m} \frac{T_m + \tau_m}{T_m\tau_m}$	$\frac{4T_m\tau_m}{T_m + \tau_m}$	$\frac{T_m\tau_m}{T_m + \tau_m}$	<i>Model: Method 1</i>

$$^{49}T_d^{(228)} = \frac{0.315\tau_m T_m + 0.003\tau_m^2}{0.315\tau_m + T_m}.$$

$$^{50}K_c^{(229)} = \frac{1}{K_m} \left[0.7236 \frac{T_m}{\tau_m} + 0.2236 \right], \quad T_d^{(229)} = \frac{2.236T_m\tau_m}{7.236T_m + 2.236\tau_m}.$$

$$^{51}K_c^{(230)} = \frac{\alpha + \frac{\tau_m}{T_m} + 0.5 \left(\frac{\tau_m}{T_m} \right)^2}{K_m \left(\alpha + \frac{\tau_m}{T_m} \right)^2}, \quad T_i^{(230)} = T_m - \frac{\alpha + \frac{\tau_m}{T_m} + 0.5 \left(\frac{\tau_m}{T_m} \right)^2}{\left(\alpha + \frac{\tau_m}{T_m} \right)},$$

$$T_d^{(230)} = \frac{0.5 \left(\frac{\tau_m}{T_m} \right)^2 T_m \left(\alpha + \frac{\tau_m}{T_m} - 0.5\alpha \frac{\tau_m}{T_m} \right)}{\left(\alpha + \frac{\tau_m}{T_m} \right) \left(\alpha + \frac{\tau_m}{T_m} + 0.5 \left(\frac{\tau_m}{T_m} \right)^2 \right)}.$$

$$^{52}K_c^{(231)} = \frac{0.177 + 0.348 \left(\frac{\tau_m}{T_m} \right)^{-1.002}}{K_m (0.531 - 0.359V^{0.713})}.$$

Rule	K_c	T_i	T_d	Comment
Cluett and Wang (1997), Wang and Cluett (2000). Model: Method 1; $0.01 \leq \frac{\tau_m}{T_m} \leq 10$	$^{53} K_c^{(232)}$	$T_i^{(232)}$	$T_d^{(232)}$	
	$T_{CL} = 4\tau_m, A_m \in [7.97, 8.56], \phi_m \in [79.67^0, 79.70^0]$			
	$^{54} K_c^{(233)}$	$T_i^{(233)}$	$T_d^{(233)}$	
	$T_{CL} = 2\tau_m, A_m \in [4.84, 5.31], \phi_m \in [73.9^0, 74.14^0]$			
	$^{55} K_c^{(234)}$	$T_i^{(234)}$	$T_d^{(234)}$	
	$T_{CL} = 1.33\tau_m, A_m \in [3.81, 4.16], \phi_m \in [69.84^0, 70.70^0]$			
	$^{56} K_c^{(235)}$	$T_i^{(235)}$	$T_d^{(235)}$	
$T_{CL} = \tau_m, A_m \in [3.30, 3.56], \phi_m \in [65.86^0, 68.50^0]$				
$^{57} K_c^{(236)}$			$T_d^{(236)}$	
$T_{CL} = 0.8\tau_m, A_m \in [3.00, 3.19], \phi_m \in [60.60^0, 67.21^0]$				

$$^{53} K_c^{(232)} = \frac{0.019952\tau_m + 0.20042T_m}{K_m \tau_m}, T_i^{(232)} = \frac{0.099508\tau_m + 0.99956T_m}{0.99747\tau_m - 8.7425.10^{-5}T_m} \tau_m,$$

$$T_d^{(232)} = \frac{-0.0069905\tau_m + 0.029480T_m}{0.029773\tau_m + 0.29907T_m} \tau_m.$$

$$^{54} K_c^{(233)} = \frac{0.055548\tau_m + 0.33639T_m}{K_m \tau_m}, T_i^{(233)} = \frac{0.16440\tau_m + 0.99558T_m}{0.98607\tau_m - 1.5032.10^{-4}T_m} \tau_m,$$

$$T_d^{(233)} = \frac{-0.016651\tau_m + 0.093641T_m}{0.093905\tau_m + 0.56867T_m} \tau_m.$$

$$^{55} K_c^{(234)} = \frac{0.092654\tau_m + 0.43620T_m}{K_m \tau_m}, T_i^{(234)} = \frac{0.20926\tau_m + 0.98518T_m}{0.96515\tau_m + 4.2550.10^{-3}T_m} \tau_m,$$

$$T_d^{(234)} = \frac{-0.024442\tau_m + 0.17669T_m}{0.17150\tau_m + 0.80740T_m} \tau_m.$$

$$^{56} K_c^{(235)} = \frac{0.12786\tau_m + 0.51235T_m}{K_m \tau_m}, T_i^{(235)} = \frac{0.24145\tau_m + 0.96751T_m}{0.93566\tau_m + 2.2988.10^{-2}T_m} \tau_m,$$

$$T_d^{(235)} = \frac{-0.030407\tau_m + 0.27480T_m}{0.25285\tau_m + 1.0132T_m} \tau_m.$$

$$^{57} K_c^{(236)} = \frac{0.16051\tau_m + 0.57109T_m}{K_m \tau_m}, T_i^{(236)} = \frac{0.26502\tau_m + 0.94291T_m}{0.89868\tau_m + 6.9355.10^{-2}T_m} \tau_m,$$

$$T_d^{(236)} = \frac{-0.035204\tau_m + 0.38823T_m}{0.33303\tau_m + 1.1849T_m} \tau_m.$$

Rule	K_c	T_i	T_d	Comment
Cluett and Wang (1997), Wang and Cluett (2000) – continued.	$^{58} K_c^{(237)}$	$T_i^{(237)}$	$T_d^{(237)}$	
	$T_{CL} = 0.67\tau_m, A_m \in [2.80, 2.95], \phi_m \in [52.35^\circ, 66.45^\circ]$			
Valentine and Chidambaram (1997a).	$^{59} K_c^{(238)}$	$T_i^{(238)}$	$T_d^{(238)}$	$\xi = 0.707$; Model: Method 1
Morilla <i>et al.</i> (2000). Model: Method 28	$^{60} K_c^{(239)}$	$T_i^{(239)}$	$\alpha T_i^{(239)}$	$\alpha = 0.1$; $\delta_0 = [0.2, 0.5]$

$$^{58} K_c^{(237)} = \frac{0.19067\tau_m + 0.61593T_m}{K_m \tau_m}, \quad T_i^{(237)} = \frac{0.28242\tau_m + 0.91231T_m}{0.85491\tau_m + 0.15937T_m} \tau_m, \\ T_d^{(237)} = \frac{-0.039589\tau_m + 0.51941T_m}{0.40950\tau_m + 1.3228T_m} \tau_m.$$

$$^{59} K_c^{(238)} = \frac{1}{K_m} \left[0.746 - 1.242 \ln \left(\frac{\tau_m}{T_m} \right) \right], \\ T_i^{(238)} = T_m \left[0.3106 + 1.372 \frac{\tau_m}{T_m} - 0.545 \left(\frac{\tau_m}{T_m} \right)^2 \right], \quad T_d^{(238)} = \frac{T_m^2}{16.015T_m - 11.702\tau_m}.$$

$$\pm 1\% T_S = 2.5T_m, \quad 0.1 \leq \frac{\tau_m}{T_m} \leq 0.5; \quad \pm 1\% T_S = 3.0T_m, \quad \frac{\tau_m}{T_m} = 0.6; \quad \pm 1\% T_S = 3.75T_m, \quad \frac{\tau_m}{T_m} = 0.7; \quad \pm 1\% T_S = 4.3T_m, \quad \frac{\tau_m}{T_m} = 0.8; \quad \pm 1\% T_S = 5.0T_m, \quad \frac{\tau_m}{T_m} = 0.9.$$

$$^{60} K_c^{(239)} = \frac{T_i^{(239)} \omega_{n0}}{K_m [2\delta_0 + \omega_{n0}(\tau_m - T_i^{(239)})]}, \quad T_i^{(239)} = \frac{\tau_m}{1 + \sqrt{1 - 4\alpha}}, \\ \omega_{n0} = \frac{-\delta_0 T_m + \sqrt{\delta_0^2 T_m^2 + T_m (\tau_m - T_i^{(239)}) - \alpha [T_i^{(239)}]^2}}{T_m (\tau_m - T_i^{(239)}) - \alpha [T_i^{(239)}]^2}$$

with $\delta_0 = \frac{1}{\sqrt{1 + \left(\frac{2\pi}{\log_e[b/a]} \right)^2}}$, $\frac{b}{a}$ = desired closed loop response decay ratio.

Rule	K_c	T_i	T_d	Comment
Mann <i>et al.</i> (2001). Model: Method 31 or Method 33	$^{61} K_c^{(240)}$	$T_i^{(240)}$	$T_d^{(240)}$	$OS < 5\%;$ $0 < \tau_m/T_m < 1$
	$^{62} K_c^{(241)}$	$T_i^{(241)}$	$T_d^{(241)}$	$OS < 5\%;$ $1 \leq \tau_m/T_m < 2$
Chen and Seborg (2002).	$^{63} K_c^{(242)}$	$T_i^{(242)}$	$T_d^{(242)}$	Model: Method 1

$$\begin{aligned}
 ^{61} K_c^{(240)} &= \frac{\left[0.770 + 0.245 \left(\frac{\tau_m}{T_m} \right)^{0.854} \right] T_m}{K_m \tau_m}, \quad T_i^{(240)} = \left[1.262 + 0.147 \left(\frac{\tau_m}{T_m} \right)^{0.854} \right] T_m, \\
 T_d^{(240)} &= \frac{0.262 + 0.147 \left(\frac{\tau_m}{T_m} \right)^{0.854}}{0.770 + 0.245 \left(\frac{\tau_m}{T_m} \right)^{0.854}} \tau_m. \\
 ^{62} K_c^{(241)} &= \frac{\left[0.603 + 0.275 \left(\frac{\tau_m}{T_m} \right)^{-2.4} \right] T_m}{K_m \tau_m}, \quad T_i^{(241)} = \left[1.1618 + 0.165 \left(\frac{\tau_m}{T_m} \right)^{-2.4} \right] T_m, \\
 T_d^{(241)} &= \frac{0.1618 + 0.165 \left(\frac{\tau_m}{T_m} \right)^{-2.4}}{0.603 + 0.275 \left(\frac{\tau_m}{T_m} \right)^{-2.4}} \tau_m. \\
 ^{63} K_c^{(242)} &= \frac{1}{K_m} \frac{(2T_m \tau_m + 0.5\tau_m^2)(3T_{CL} + 0.5\tau_m) - 2T_{CL}^3 - 3T_{CL}^2\tau_m}{2T_{CL}^3 + 3T_{CL}^2\tau_m + 0.5\tau_m^2(3T_{CL} + 0.5\tau_m)}, \\
 T_i^{(242)} &= \frac{(2T_m \tau_m + 0.5\tau_m^2)(3T_{CL} + 0.5\tau_m) - 2T_{CL}^3 - 3T_{CL}^2\tau_m}{\tau_m(2T_m + \tau_m)}, \\
 T_d^{(242)} &= \frac{3T_{CL}^2\tau_m T_m + 0.5T_m\tau_m^2(3T_{CL} + 0.5\tau_m) - 2(T_m + \tau_m)\tau_{CL}^3}{(2T_m \tau_m + 0.5\tau_m^2)(3T_{CL} + 0.5\tau_m) - 2T_{CL}^3 - 3T_{CL}^2\tau_m}; \\
 \left(\frac{y}{d} \right)_{\text{desired}} &= \frac{T_i^{(242)} s(1 + 0.5\tau_m s) e^{-s\tau_m}}{K_c^{(242)} (T_{CL} s + 1)^3}.
 \end{aligned}$$

Rule	K_c	T_i	T_d	Comment
Vítečková and Víteček (2002). Model: Method 1	$^{64} K_c^{(243)}$	$T_i^{(243)}$	$T_d^{(243)}$	
For $0.05 < \frac{\tau_m}{T_m} < 1.6$, $T_i = 1.2T_m$; overshoot is under 2% (Vítečková and Víteček (2003)).				
Gorez (2003). Model: Method 1	$\frac{(1-v)\tau_m + T_m}{\chi K_m \tau_m}$	$(1-v)\tau_m + T_m$	$\frac{(1-v)\tau_m T_m}{(1-v)\tau_m + T_m}$	65
Sree et al. (2004). Model: Method 1	$\frac{1}{K_m} \left(\frac{T_m}{\tau_m} + 0.5 \right)$	$T_m + 0.5\tau_m$	$^{66} T_d^{(244)}$	
	$\frac{1.377}{K_m} \left(\frac{T_m}{\tau_m} \right)^{0.8422}$	$^{67} T_i^{(245)}$	$T_d^{(245)}$	$0.1 \leq \frac{\tau_m}{T_m} \leq 1$

$$^{64} K_c^{(243)} = \frac{e^{\tau_m x_1}}{K_m} \left[\tau_m^2 T_m x_1^3 + (3\tau_m T_m + \tau_m^2) x_1^2 + \tau_m x_1 - 1 \right],$$

$$x_1 = -\frac{3}{\tau_m} - \frac{0.5}{T_m} + \sqrt{\frac{3}{\tau_m^2} + \frac{0.25}{T_m^2}};$$

$$T_i^{(243)} = -2 \left[\frac{\tau_m^2 T_m x_1^3 + (3\tau_m T_m + \tau_m^2) x_1^2 + \tau_m x_1 - 1}{x_1^3 (\tau_m^2 T_m x_1 + 2\tau_m T_m + \tau_m^2)} \right],$$

$$T_d^{(243)} = -0.5 \left[\frac{\tau_m^2 T_m x_1^2 + (4\tau_m T_m + \tau_m^2) x_1 + 2\tau_m + 2T_m}{\tau_m^2 T_m x_1^3 + (3\tau_m T_m + \tau_m^2) x_1^2 + \tau_m x_1 - 1} \right].$$

$$^{65} \text{ For } 0 \leq \frac{\tau_m}{\tau_m + T_m} \leq \tau_m, v = \frac{\tau_m T_m + T_m^2 + 0.5\tau_m^2}{(\tau_m + T_m)^2},$$

$$\chi = \frac{1}{2M_s(M_s - 1)} + \frac{(1.3M_s^2 - 1)}{2M_s(M_s - 1)} \frac{1.56\tau_m^2 (0.5\tau_m + 3T_m)}{(\tau_m + T_m)^3};$$

$$\text{For } \tau_m \leq \frac{\tau_m}{\tau_m + T_m} \leq 1, v = \frac{\tau_m T_m + T_m^2 + 0.5\tau_m^2}{(\tau_m + T_m)^2}, \chi = \frac{0.65M_s}{M_s - 1}.$$

$$^{66} T_d^{(244)} = \frac{0.5\tau_m (T_m + 0.1667\tau_m)}{T_m + 0.5\tau_m}.$$

$$^{67} T_i^{(245)} = 1.085 T_m \left(\frac{\tau_m}{T_m} \right)^{0.4777}, T_d^{(245)} = T_m \left[0.3899 \frac{\tau_m}{T_m} + 0.0195 \right].$$

Rule	K_c	T_i	T_d	Comment	
Direct synthesis: frequency domain criteria					
Regulator – Gorecki <i>et al.</i> (1989).	$^{68} K_c^{(246)}$	$T_i^{(246)}$	$T_d^{(246)}$	<i>Model: Method 1</i>	
Low frequency part of magnitude Bode diagram is flat.					
Somani <i>et al.</i> (1992). <i>Model: Method 1;</i> Suggested A_m : $1.75 \leq A_m \leq 3.0$	$^{69} K_c^{(247)}$	$x_2 \tau_m$	$x_3 \tau_m$		
Coefficient values					
	τ_m/T_m	x_2	x_3	τ_m/T_m	x_2
	0.0375	3.57	0.71	1.869	2.38
	0.1912	3.33	0.67	2.392	2.27
	0.3693	3.13	0.63	3.067	2.17
	0.5764	2.94	0.59	3.968	2.08
	0.8183	2.78	0.56	5.263	2.00
	1.105	2.63	0.53	7.246	1.92
	1.449	2.50	0.50	10.870	1.85
					0.37

$$^{68} K_c^{(246)} = \frac{1}{K_m} \frac{1}{2 \frac{\tau_m}{T_i^{(246)}} \left(\frac{T_m}{\tau_m} + 1 \right) - 2},$$

$$T_i^{(246)} = \tau_m \frac{7 + 42 \frac{T_m}{\tau_m} + 135 \left(\frac{T_m}{\tau_m} \right)^2 + 240 \left(\frac{T_m}{\tau_m} \right)^3 + 180 \left(\frac{T_m}{\tau_m} \right)^4}{15 \left[2 \frac{T_m}{\tau_m} + 1 \right] \left[1 + 3 \frac{T_m}{\tau_m} + 6 \left(\frac{T_m}{\tau_m} \right)^2 \right]},$$

$$T_d^{(246)} = \tau_m \frac{1 + 7 \frac{T_m}{\tau_m} + 27 \left(\frac{T_m}{\tau_m} \right)^2 + 60 \left(\frac{T_m}{\tau_m} \right)^3 + 60 \left(\frac{T_m}{\tau_m} \right)^4}{7 + 42 \frac{T_m}{\tau_m} + 135 \left(\frac{T_m}{\tau_m} \right)^2 + 240 \left(\frac{T_m}{\tau_m} \right)^3 + 180 \left(\frac{T_m}{\tau_m} \right)^4}.$$

$$^{69} \text{For } \frac{\tau_m}{T_m} < 1, K_c^{(247)} \approx \frac{0.7809}{K_m A_m} \sqrt{\frac{4}{\left(\sqrt{0.803 + 4 \frac{\tau_m}{T_m}} + 0.896 \right)^2}} + 1 \text{ or}$$

$$K_c^{(247)} \approx \frac{0.7809}{K_m A_m} \sqrt{1 + 0.803 \left(\frac{T_m}{\tau_m} \right)^2}; K_c^{(247)} \approx \frac{0.7809}{K_m A_m} \sqrt{1 + 0.455 \left(\frac{T_m}{\tau_m} \right)^2}, \frac{\tau_m}{T_m} > 1.$$

Rule	K_c	T_i	T_d	Comment				
Chidambaram (1995a). Model: Method 1	${}^{70} K_c^{(248)}$	$T_i^{(248)}$	$T_d^{(248)}$					
	$x_1/K_m A_m$	$x_2 \tau_m$	$x_3 \tau_m$	Alternative				
Coefficient values								
	$\frac{\tau_m}{T_m}$	x_1	x_2	x_3	$\frac{\tau_m}{T_m}$	x_1	x_2	x_3
	0.062	29.770	2.34	0.37	0.389	5.104	2.19	0.35
	0.083	22.100	2.33	0.37	0.543	3.781	2.14	0.34
	0.150	12.470	2.29	0.37	1.03	2.262	2.00	0.32
	0.196	9.670	2.27	0.36	2.92	1.209	1.72	0.28
	0.290	6.686	2.23	0.36	3.69	1.105	1.67	0.27
	0.339	5.784	2.21	0.35				
Chidambaram (1995a). Model: Method 1	$\frac{1.20 T_m}{K_m \tau_m}$	$2.4 \tau_m$	$0.38 \tau_m$	$A_m = 1.5$				
	$\frac{1.03 T_m}{K_m \tau_m}$	$2.4 \tau_m$	$0.38 \tau_m$	$A_m = 1.75$				

$${}^{70} K_c^{(248)} \approx \frac{1}{K_m A_m} \left[\frac{3.42}{\left(\sqrt{4.45 + 4 \frac{\tau_m}{T_m}} - 2.11 \right)^2} + 0.85 \right] \text{ or } \frac{0.85}{K_m A_m} \sqrt{1 + 4.45 \left(\frac{T_m}{\tau_m} \right)^2} ;$$

$$T_i^{(248)} \approx \frac{5 T_m}{\sqrt{1.17 \left[\frac{3.42}{\left(\sqrt{4.45 + 4 \frac{\tau_m}{T_m}} - 2.11 \right)^2} + 0.85 \right]^2 - 1}} \text{ or } \frac{5 T_m}{\sqrt{4.45 \left(\frac{T_m}{\tau_m} \right)^2 - 0.15}} ;$$

$$T_d^{(248)} \approx \frac{0.8 T_m}{\sqrt{1.17 \left[\frac{3.42}{\left(\sqrt{4.45 + 4 \frac{\tau_m}{T_m}} - 2.11 \right)^2} + 0.85 \right]^2 - 1}} \text{ or } \frac{0.8 T_m}{\sqrt{4.45 \left(\frac{T_m}{\tau_m} \right)^2 - 0.15}} .$$

Rule	K_c	T_i	T_d	Comment
Chidambaram (1995a) – continued.	$\frac{0.90T_m}{K_m \tau_m}$	$2.4\tau_m$	$0.38\tau_m$	$A_m = 2.0$
Branica <i>et al.</i> (2002). <i>Model:</i> <i>Method I;</i> $M_s = 1.8$	⁷¹ $K_c^{(249)}$	$T_i^{(249)}$	$T_d^{(249)}$	$0.1 \leq \frac{\tau_m}{T_m} \leq 0.5$
	⁷² $K_c^{(250)}$	$T_i^{(250)}$	$T_d^{(250)}$	$0.5 < \frac{\tau_m}{T_m} \leq 1.1$
	⁷³ $K_c^{(251)}$	$T_i^{(251)}$	$T_d^{(251)}$	$1.1 < \frac{\tau_m}{T_m} \leq 2.0$
Minimum ISTSE – Zhuang and Atherton (1993). <i>Model: Method 1</i>	⁷⁴ $m K_u \cos(\phi_m)$	$T_i^{(252)}$	$\frac{T_i^{(252)}}{\alpha}$	$A_m = 2,$ $\phi_m = 60^0$

$$\begin{aligned}
 {}^{71} K_c^{(249)} &= \frac{1}{K_m} \left(0.021 + 0.788 \frac{T_m}{\tau_m} \right), \quad T_i^{(249)} = 1.534 \tau_m \left(\frac{\tau_m}{T_m} \right)^{-0.326}, \\
 &\quad T_d^{(249)} = 0.233 \tau_m \left(\frac{\tau_m}{T_m} \right)^{-0.055}. \\
 {}^{72} K_c^{(250)} &= \frac{1}{K_m} \left(0.292 + 0.671 \frac{T_m}{\tau_m} \right), \quad T_i^{(250)} = 1.239 \tau_m \left(\frac{\tau_m}{T_m} \right)^{-0.554}, \\
 &\quad T_d^{(250)} = 0.230 \tau_m \left(\frac{\tau_m}{T_m} \right)^{-0.069}. \\
 {}^{73} K_c^{(251)} &= \frac{1}{K_m} \left(0.338 + 0.623 \frac{T_m}{\tau_m} \right), \quad T_i^{(251)} = 1.228 \tau_m \left(\frac{\tau_m}{T_m} \right)^{-0.480}, \\
 &\quad T_d^{(251)} = 0.231 \tau_m \left(\frac{\tau_m}{T_m} \right)^{-0.120}.
 \end{aligned}$$

$${}^{74} m = 0.614(1 - 0.233 e^{-0.347 K_m K_u}), \quad \phi_m = 33.8^0 \left(1 - 0.97 e^{-0.45 K_m K_u} \right), \quad 0.1 \leq \frac{\tau_m}{T_m} \leq 2.0,$$

$$\alpha = 0.413(3.302 K_m K_u + 1), \quad T_i^{(252)} = \alpha \frac{\tan(\phi_m) + \sqrt{\frac{4}{\alpha} + \tan^2(\phi_m)}}{2\omega_u}.$$

Rule	K_c	T_i	T_d	Comment
Minimum ISTSE – Zhuang and Atherton (1993) – continued.	⁷⁵ $m K_u \cos(\phi_m)$	$T_i^{(252)}$ (page 176)	$\frac{T_i^{(252)}}{\alpha}$	$A_m = 2$, $\phi_m = 60^0$
Li <i>et al.</i> (1994).	⁷⁶ $K_c^{(253)}$	$4T_d^{(253)}$	$T_d^{(253)}$	
<i>Model: Method 1</i>	ϕ_m A_m η	ϕ_m A_m η	ϕ_m A_m η	ϕ_m A_m η
	15° 2 2.8	30° 1.67 3.8	45° 1.25 4.6	60° 1.11 5.4
	20° 1.67 3.2	35° 1.43 4.0	50° 1.25 4.9	65° 1.11 5.5
	25° 1.67 3.5	40° 1.43 4.2	55° 1.25 5.2	
	$\frac{4h}{\pi A_p A_m}$	0.3183T	0.0796T	simplified algorithm
Tan <i>et al.</i> (1996). <i>Model:</i> <i>Method 30</i>	$\frac{K_u}{A_m} \cos \phi_m$	$\alpha T_d^{(254)}$	⁷⁷ $T_d^{(254)}$	α chosen arbitrarily
	$\frac{K_\phi}{A_m}$	⁷⁸ $T_i^{(255)}$	$T_d^{(255)}$	Arbitrary A_m , ϕ_m at ω_ϕ

$$^{75} \text{Model: Method 31; } m = 0.613(1 - 0.262e^{-0.44K_m \hat{K}_u}), \phi_m = 33.2^0 \left(1 - 1.38e^{-0.68K_m \hat{K}_u}\right),$$

$$\alpha = 1.687K_m \hat{K}_u, 0.1 \leq \frac{\tau_m}{T_m} \leq 2.0.$$

$$^{76} K_c^{(253)} = \frac{4h \cos \phi_m}{\pi A_m \left[\sqrt{A_p^2 - b^2} + \frac{6\pi b}{T} T_d^{(253)} \right]}, T_d^{(253)} = \frac{T}{4\pi} \left[\eta + \sqrt{\eta^2 + \frac{4}{\eta}} \right],$$

$$\eta = \frac{\left[\tan \phi_m - \left(b / \sqrt{A_p^2 - b^2} \right) \right]}{\left[1 + \left(b / \sqrt{A_p^2 - b^2} \right) \right]} \text{ with } \pm b = \text{deadband of relay}, T = \text{limit cycle period}.$$

$$^{77} T_d^{(254)} = T_u \frac{\tan \phi_m + \sqrt{\frac{4}{\alpha} + \tan^2 \phi_m}}{4\pi}; \text{ recommended } A_m = 2, \phi_m = 45^0.$$

$$^{78} T_i^{(255)} = \frac{r_2 K_\phi (\omega_u^2 - \omega_\phi^2)}{\omega_u \omega_\phi^2 \sqrt{K_u^2 - r_2^2 K_\phi^2}}, T_d^{(255)} = \frac{\omega_u \sqrt{K_u^2 - r_2^2 K_\phi^2}}{r_2 K_\phi (\omega_u^2 - \omega_\phi^2)}, r_2 = 0.1 + 0.9 \frac{K_u}{K_\phi}.$$

Rule	K_c	T_i	T_d	Comment
Friman and Waller (1997). <i>Model:</i> Method 1; $A_m > 2$	$0.25K_u$	$1.1879T_u$	$0.2970T_u$	$\tau_m/T_m < 0.25$, $\phi_m > 60^\circ$
	$\frac{0.4830}{ G_p(j\omega_{150^\circ}) }$	$\frac{2.6064}{\omega_{150^\circ}}$	$\frac{0.6516}{\omega_{150^\circ}}$	$0.25 \leq \frac{\tau_m}{T_m} \leq 2.0$, $\phi_m > 45^\circ$
Robust				
Rivera <i>et al.</i> (1986). <i>Model: Method 1</i>	$\frac{T_m + 0.5\tau_m}{K_m(\lambda + 0.5\tau_m)}$	$T_m + 0.5\tau_m$	$\frac{T_m\tau_m}{2T_m + \tau_m}$	$\lambda > 0.1T_m$, $\lambda \geq 0.8\tau_m$.
	$0.5\tau_m \leq \lambda \leq 3\tau_m$ (Zou <i>et al.</i> (1997))			
Brambilla <i>et al.</i> (1990). <i>Model: Method 1</i>	$^{79}\frac{T_m + 0.5\tau_m}{K_m(\lambda + \tau_m)}$	$T_m + 0.5\tau_m$	$\frac{T_m\tau_m}{2T_m + \tau_m}$	$0.1 \leq \frac{\tau_m}{T_m} \leq 10$
	No model uncertainty - $\lambda \approx 0.35\tau_m$			
Lee <i>et al.</i> (1998) <i>Model: Method 1</i>	$^{80}\frac{T_i^{(256)}}{K_m(\lambda + \tau_m)}$	$^{80}T_i^{(256)}$	$T_d^{(256)}$	$T_{CL} = \lambda$
	$^{81}\frac{K_c^{(257)}}{K_c}$	$T_i^{(257)}$	$T_d^{(257)}$	
Gerry (1999). <i>Model:</i> Method 10	$\frac{T_m}{K_m(\lambda + 0.5\tau_m)}$	T_m	$0.5\tau_m$	
	$\lambda = 2\tau_m$ (aggressive, less robust tuning); $\lambda = 2(T_m + \tau_m)$ (more robust tuning)			

$$^{79} \lambda \in [0.1T_m, 0.5T_m] \frac{\tau_m}{T_m} \leq 0.25 ;$$

$$\lambda = 1.5(\tau_m + T_m), 0.25 < \frac{\tau_m}{T_m} \leq 0.75 ; \lambda = 3(\tau_m + T_m), \frac{\tau_m}{T_m} > 0.75 \text{ (Leva (2001))}.$$

$$^{80} T_i^{(256)} = T_m + \frac{\tau_m^2}{2(\lambda + \tau_m)}, T_d^{(256)} = \frac{\tau_m^2}{2(\lambda + \tau_m)} \left(1 - \frac{\tau_m}{3T_i^{(256)}} \right).$$

$$^{81} K_c^{(257)} = \frac{T_i^{(257)}}{K_m(2T_{CL} + \tau_m - \alpha)}, T_i^{(257)} = T_m + \alpha - \frac{T_{CL}^2 + \alpha\tau_m - 0.5\tau_m^2}{2T_{CL} + \tau_m - \alpha},$$

$$T_d^{(257)} = \frac{T_m \alpha - \frac{0.167\tau_m^3 - 0.5\alpha\tau_m^2}{2T_{CL} + \tau_m - \alpha}}{T_i^{(257)}} - \frac{T_{CL}^2 + \alpha\tau_m - 0.5\tau_m^2}{2T_{CL} + \tau_m - \alpha},$$

$$\alpha = T_m - T_m \left(1 - \frac{T_{CL}}{T_m} \right)^2 e^{-\frac{\tau_m}{T_m}}.$$

Rule	K_c	T_i	T_d	Comment
Ultimate cycle				
McMillan (1984). Model: Method 2 or Method 44	⁸² $K_c^{(258)}$	$T_i^{(258)}$	$T_d^{(258)}$	Tuning rules developed from K_u, T_u
Geng and Geary (1993). Model: Method 1	$\frac{0.94T_m}{K_m \tau_m}$	$2\tau_m$	$0.5\tau_m$	$\frac{\tau_m}{T_m} < 0.167$
Wojciszni and Blevins (1995). Model: Method 36	$\frac{T_m + 0.5\tau_m}{aK_m \tau_m}$	$T_m + 0.5\tau_m$	$\frac{T_m \tau_m}{2T_m + \tau_m}$	$1.3 \leq a \leq 1.6$; Model: Method 36
Perić et al. (1997). Model: Method 40	$\hat{K}_u \cos \phi_m$	$\alpha T_d^{(259)}$	⁸³ $T_d^{(259)}$	Recommended $\alpha = 4$
Matsuba et al. (1998). Model: Method 1	⁸⁴ $K_c^{(260)}$	$T_i^{(260)}$	$T_d^{(260)}$	$0.1 \leq \frac{\tau_m}{T_m} \leq 1.0$
Wojciszni et al. (1999). Model: Method 1	$\frac{K_u \cos \phi_m}{A_m}$	⁸⁵ $T_i^{(261)}$	$\alpha T_i^{(261)}$	

$$\begin{aligned}
 & ^{82} K_c^{(258)} = \frac{1.415}{K_m} \frac{T_m}{\tau_m} \left\{ \frac{1}{1 + \left(\frac{T_m}{T_m + \tau_m} \right)^{0.65}} \right\}, \quad T_i^{(258)} = \tau_m \left\{ 1 + \left(\frac{T_m}{T_m + \tau_m} \right)^{0.65} \right\}, \\
 & \quad T_d^{(258)} = 0.25 \tau_m \left\{ 1 + \left(\frac{T_m}{T_m + \tau_m} \right)^{0.65} \right\}. \\
 & ^{83} T_d^{(259)} = \left(\tan \phi_m + \sqrt{\frac{4}{\alpha} + \tan^2 \phi_m} \right) \frac{\hat{T}_u}{4\pi}. \\
 & ^{84} K_c^{(260)} = \left[\frac{1.31}{K_m} \left(\frac{T_m}{\tau_m} \right)^{0.9}, \frac{2.19}{K_m} \left(\frac{T_m}{\tau_m} \right)^{0.9} \right], \quad T_i^{(260)} = 1.59 \tau_m \left(\frac{T_m}{\tau_m} \right)^{0.097}, \\
 & \quad T_d^{(260)} = 0.40 \tau_m \left(\frac{T_m}{\tau_m} \right)^{0.097}. \\
 & ^{85} T_i^{(261)} = \left(\tan \phi_m + \sqrt{4\alpha + \tan^2 \phi_m} \right) \frac{\hat{T}_u}{4\pi\alpha}.
 \end{aligned}$$

Rule	K_c	T_i	T_d	Comment
Wojsznis <i>et al.</i> (1999) – continued.	$0.5K_u \cos \phi_m$	$^{86} T_i^{(262)}$	$0.15T_i^{(262)}$	Default values
	$0.38K_u$	$1.2T_u$	$0.18T_u$	$\phi_m = 45^\circ$; $\tau_m/T_m < 0.2$
	$0.27K_u$	$0.87T_u$	$0.13T_u$	$A_m = 3$, $\phi_m = 33^\circ$; $\tau_m/T_m \approx 0.25$
Wojsznis <i>et al.</i> (1999). Model: Method 36	$^{87} K_c^{(263)}$	$T_i^{(263)}$	$0.125T_i^{(263)}$	$a_1 \in [0.3, 0.4]$; $a_2 \in [0.25, 0.4]$; $0.2 \leq \frac{\tau_m}{T_m} \leq 0.7$

$$^{86} T_i^{(262)} = 0.53T_u \left(\tan \phi_m + \sqrt{0.6 + \tan^2 \phi_m} \right).$$

$$^{87} K_c^{(263)} = K_u \left(a_2 + 0.25 \left[1.0 - a_1 - \frac{0.6}{1 + e^{-\left(\frac{T_u}{\tau_m} - 7.0\right)}} \right] \right), \quad T_i^{(263)} = T_u \left(a_1 + \frac{0.6}{1 + e^{-\left(\frac{T_u}{\tau_m} - 7.0\right)}} \right).$$

4.1.2 Ideal controller in series with a first order lag

$$G_c(s) = K_c \left(1 + \frac{1}{T_i s} + T_d s \right) \frac{1}{T_f s + 1}$$

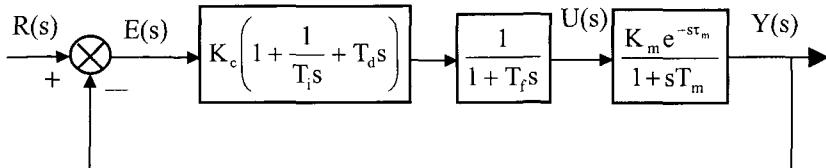


Table 45: PID controller tuning rules – FOLPD model $G_m(s) = \frac{K_m e^{-st_m}}{1 + sT_m}$

Rule	K_c	T_i	T_d	Comment
Direct synthesis: frequency domain criteria				
Schaedel (1997). <i>Model:</i> <i>Method 21</i>	$^1 K_c^{(264)}$	$\frac{T_1^2 - T_2^2}{T_1 - T_d^{(264)}}$	$T_d^{(264)}$	$T_f = T_d^{(264)} / N$ $5 \leq N \leq 20$
Robust				
Horn <i>et al.</i> (1996). <i>Model: Method 1</i>	$\frac{T_m + 0.5\tau_m}{K_m(\lambda + \tau_m)}$	$T_m + 0.5\tau_m$	$\frac{T_m\tau_m}{2T_m + \tau_m}$	$T_f = \frac{\lambda\tau_m}{2(\lambda + \tau_m)},$ $\lambda \in [\tau_m, T_m]$
	$\lambda = \max(0.25\tau_m, 0.2T_m)$ (Luyben (2001)); $\lambda > 0.25\tau_m$ (Bequette, (2003)).			

$$^1 K_c^{(264)} = \frac{0.375 T_i^{(264)}}{K_m \left(T_1 + \frac{T_d^{(264)}}{N} - T_i^{(264)} \right)}, \quad T_d^{(264)} = \frac{T_2^2}{T_1} - \frac{T_3^3}{T_2^2};$$

$$T_1 = \frac{T_m}{1 + 3.45 \frac{\tau_m^a}{T_m}} + \tau_m, \quad T_2^2 = 1.25 T_m \tau_m^a + \frac{T_m}{1 + 3.45 \frac{\tau_m^a}{T_m}} \tau_m + 0.5 \tau_m^2, \quad T_3 = 0,$$

$$0 < \frac{\tau_m^a}{T_m} \leq 0.104;$$

$$T_1 = \frac{0.7 T_m^2}{T_m - 0.7 \tau_m^a} + \tau_m, \quad T_2^2 = T_m \tau_m^a + \frac{0.7 T_m^2}{T_m - 0.7 \tau_m^a} + 0.5 \tau_m^2,$$

$$T_3^3 = \frac{0.952 T_m^2 (\tau_m^a - 0.1)}{T_m - 0.7 \tau_m^a} + T_m \tau_m \tau_m^a + \frac{0.35 T_m^2 \tau_m^2}{T_m - 0.7 \tau_m^a} + 0.167 \tau_m^3, \quad \frac{\tau_m^a}{T_m} > 0.104.$$

Rule	K_c	T_i	T_d	Comment
H _∞ optimal – Tan <i>et al.</i> (1998b). <i>Model: Method 1</i>	$^2 K_c^{(265)}$	$T_m + 0.5\tau_m$	$\frac{\tau_m T_m}{\tau_m + 2T_m}$	
$\lambda = 2$: ‘fast’ response; $\lambda = 1$: ‘robust’ tuning; $\lambda = 1.5$: recommended				
Lee and Edgar (2002). <i>Model: Method 1</i>	$^3 K_c^{(266)}$	$T_i^{(266)}$	$T_d^{(266)}$	$T_f = T_d^{(266)}$
Huang <i>et al.</i> (2002). <i>Model:</i> <i>Method 1</i>	$\frac{T_m + 0.5\tau_m}{K_m(\lambda + \tau_m)}$	$T_m + 0.5\tau_m$	$\frac{T_m}{2T_m + \tau_m}$	$T_f = \frac{\lambda\tau_m}{2(\lambda + \tau_m)}$
$\lambda = 0.65\tau_m$ - minimum ISE; $\lambda = 0.5\tau_m$ - overshoot to a step input $\leq 5\%$, $A_m = 3$. In general, $A_m = 2(\lambda/\tau_m + 1)$.				
Zhang <i>et al.</i> (2002).	$^4 K_c^{(267)}$	$T_m + 0.5\tau_m$	$\frac{T_m\tau_m}{2T_m + \tau_m}$	<i>Model: Method 1</i>

$$^2 K_c^{(265)} = \frac{0.265\lambda + 0.307}{K_m} \left(\frac{T_m}{\tau_m} + 0.5 \right), \quad T_f = \frac{\tau_m}{5.314\lambda + 0.951}.$$

$$^3 K_c^{(266)} = \frac{1}{K_m(\lambda + \tau_m)} \left[T_m + \frac{\tau_m^2}{2(\lambda + \tau_m)} + \tau_m \sqrt{\frac{3T_m - \tau_m}{6(\lambda + \tau_m)} + \frac{\tau_m^2}{4(\lambda + \tau_m)^2}} \right],$$

$$T_i^{(266)} = T_m + \frac{\tau_m^2}{2(\lambda + \tau_m)} + \tau_m \sqrt{\frac{3T_m - \tau_m}{6(\lambda + \tau_m)} + \frac{\tau_m^2}{4(\lambda + \tau_m)^2}},$$

$$T_d^{(266)} = \tau_m \sqrt{\frac{3T_m - \tau_m}{6(\lambda + \tau_m)} + \frac{\tau_m^2}{4(\lambda + \tau_m)^2}}.$$

$$^4 K_c^{(267)} = \frac{1}{K_m} \left(\frac{T_m + 0.5\tau_m}{2\lambda + 0.5\tau_m} \right), \quad T_f = \frac{\lambda^2}{2\lambda + 0.5\tau_m} \text{ or } T_f = \frac{0.1T_m\tau_m}{\tau_m + 2T_m}.$$

4.1.3 Ideal controller in series with a second order filter

$$G_c(s) = K_c \left(1 + \frac{1}{T_i s} + T_d s \right) \frac{1 + b_{f1}s}{1 + a_{f1}s + a_{f2}s^2}$$

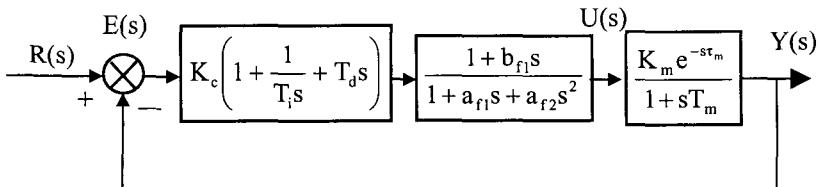


Table 46: PID controller tuning rules – FOLPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{1 + sT_m}$

Rule	K_c	T_i	T_d	Comment
Direct synthesis: frequency domain criteria				
Kristiansson (2003). <i>Model.</i> <i>Method 11</i>	⁵ $K_c^{(268)}$	$T_i^{(268)}$	$T_d^{(268)}$	$0.05 \leq \frac{\tau_m}{T_m} \leq 2 ;$ $1.6 \leq M_s \leq 1.9$

⁵ Equations continued into the footnote on page 184;

$$K_c^{(268)} = \frac{2 \left[1.1 \frac{T_m}{\tau_m} - 0.1 \right] \left[0.06 + 0.68 \frac{\tau_m}{T_m} - 0.12 \left(\frac{\tau_m}{T_m} \right)^2 \right]}{K_m \left[0.68 + 0.84 \frac{\tau_m}{T_m} - 0.25 \left(\frac{\tau_m}{T_m} \right)^2 \right]},$$

$$T_i^{(268)} = \frac{2T_m \left[0.06 + 0.68 \frac{\tau_m}{T_m} - 0.12 \left(\frac{\tau_m}{T_m} \right)^2 \right]}{\left[0.68 + 0.84 \frac{\tau_m}{T_m} - 0.25 \left(\frac{\tau_m}{T_m} \right)^2 \right]},$$

Rule	K_c	T_i	T_d	Comment
Kristiansson (2003) – continued.	$^6 K_c^{(269)}$	$T_i^{(269)}$	$T_d^{(269)}$	$0.7 < \frac{\tau_m}{T_m} < 2 ;$ $1.6 \leq M_s \leq 1.9$
Robust				
Horn <i>et al.</i> (1996). Model: Method I	$^7 K_c^{(270)}$	$T_m + 0.5\tau_m$	$\frac{\tau_m T_m}{\tau_m + 2T_m}$	$\lambda > \tau_m , \lambda < T_m$

$$T_d^{(268)} = \frac{0.5T_m \left[0.06 + 0.68 \frac{\tau_m}{T_m} - 0.12 \left(\frac{\tau_m}{T_m} \right)^2 \right]}{\left[0.68 + 0.84 \frac{\tau_m}{T_m} - 0.25 \left(\frac{\tau_m}{T_m} \right)^2 \right]}, \quad b_{f1} = 0, \quad a_{1f} = 0.8T_f, \quad a_{2f} = T_f^2 \quad \text{with}$$

$$T_f = \frac{T_m \left[0.06 + 0.68 \frac{\tau_m}{T_m} - 0.12 \left(\frac{\tau_m}{T_m} \right)^2 \right]^2 \left[1.1 \frac{T_m}{\tau_m} - 0.1 \right]}{\min \left\{ 5 + 2 \frac{T_m}{\tau_m}, 25 \right\}}.$$

$$^6 K_c^{(269)} = \frac{0.6667(T_m + \tau_m) \left[1.1 \frac{T_m}{\tau_m} - 0.1 \right]}{K_m \tau_m \left[0.68 + 0.84 \frac{\tau_m}{T_m} - 0.25 \left(\frac{\tau_m}{T_m} \right)^2 \right]},$$

$$T_i^{(269)} = \frac{0.6667(T_m + \tau_m)}{\left[0.68 + 0.84 \frac{\tau_m}{T_m} - 0.25 \left(\frac{\tau_m}{T_m} \right)^2 \right]}, \quad T_d^{(269)} = \frac{0.1667(T_m + \tau_m)}{\left[0.68 + 0.84 \frac{\tau_m}{T_m} - 0.25 \left(\frac{\tau_m}{T_m} \right)^2 \right]},$$

$$b_{f1} = 0, \quad a_{1f} = 0.8T_f, \quad a_{2f} = T_f^2 \quad \text{with} \quad T_f = \frac{(T_m + \tau_m)^2 \left[1.1 \frac{T_m}{\tau_m} - 0.1 \right]}{9T_m \left(5 + 2 \frac{T_m}{\tau_m} \right)}.$$

$$^7 K_c^{(270)} = \frac{2T_m + \tau_m}{2(2\lambda + \tau_m - b_{f1})K_m}, \quad b_{f1} = \frac{\lambda^2 \tau_m + 2T_m \tau_m (\tau_m - \lambda)}{T_m (\tau_m + 2\lambda)} + \frac{2\lambda(2T_m - \lambda)}{(\tau_m + 2\lambda)},$$

$$a_{f1} = \frac{2\lambda \tau_m + 2\lambda^2 + b_{f1} \tau_m}{2(2\lambda + \tau_m - b_{f1})}, \quad a_{f2} = \frac{\lambda^2 \tau_m}{2(2\lambda + \tau_m - b_{f1})}.$$

4.1.4 Ideal controller with weighted proportional term

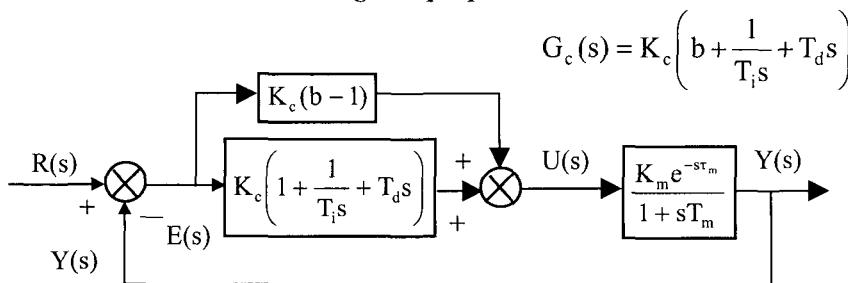


Table 47: PID controller tuning rules – FOLPD model $G_m(s) = \frac{K_m e^{-st_m}}{1 + sT_m}$

Rule	K_c	T_i	T_d	Comment
Direct synthesis				
Åström and Hägglund (1995) – pages 208-210.	$\frac{3.8e^{-8.4\tau+7.3\tau^2}T_m}{K_m\tau_m}$ or $\frac{0.46T_m e^{2.8\tau-2.1\tau^2}}{K_m\tau_m}$	$5.2\tau_m e^{-2.5\tau-1.4\tau^2}$ or $0.467T_m e^{5.0\tau-4.8\tau^2}$	$0.89\tau_m e^{-0.37\tau-4.1\tau^2}$ or $0.077T_m e^{1.9\tau-0.44\tau^2}$	$M_s = 1.4 ;$ $0.14 \leq \frac{\tau_m}{T_m} \leq 5.5$
Model: Method 5 or Method 14	$b = 0.40e^{0.18\tau+2.8\tau^2}$			
	$\frac{8.4e^{-9.6\tau+9.8\tau^2}T_m}{K_m\tau_m}$ or $0.28T_m e^{3.8\tau-1.6\tau^2}$	$3.2\tau_m e^{-1.5\tau-0.93\tau^2}$ or $0.076T_m e^{3.4\tau-1.1\tau^2}$	$0.86\tau_m e^{-1.9\tau-0.44\tau^2}$ or $0.076T_m e^{1.9\tau-0.44\tau^2}$	$M_s = 2.0 ;$ $0.14 \leq \frac{\tau_m}{T_m} \leq 5.5$
$b = 0.22e^{0.65\tau+0.051\tau^2}$				
Gorez (2003). Model: Method 1	$\frac{(1-v)\tau_m + T_m}{\chi K_m \tau_m}$	$(1-v)\tau_m + T_m$	$\frac{(1-v)\tau_m T_m}{(1-v)\tau_m + T_m}$	⁸
	$b = \frac{(\chi-1)\tau_m - T_m}{(1-v)\tau_m + T_m} + 1$			

⁸ For $0 \leq \frac{\tau_m}{\tau_m + T_m} \leq \tau_m$, $v = \frac{\tau_m T_m + T_m^2 + 0.5\tau_m^2}{(\tau_m + T_m)^2}$

$$\chi = \frac{1}{2M_s(M_s - 1)} + \frac{(1.3M_s^2 - 1)}{2M_s(M_s - 1)} \frac{1.56\tau_m^2(0.5\tau_m + 3T_m)}{(\tau_m + T_m)^3},$$

For $\tau_m \leq \frac{\tau_m}{\tau_m + T_m} \leq 1$, $v = \frac{\tau_m T_m + T_m^2 + 0.5\tau_m^2}{(\tau_m + T_m)^2}$, $\chi = \frac{0.65M_s}{M_s - 1}$.

4.1.5 Ideal controller with first order filter and set-point weighting 1

$$U(s) = K_c \left(1 + \frac{1}{T_i s} + T_d s \right) \frac{1}{T_f s + 1} \left[R(s) \frac{1 + 0.4 T_r s}{1 + s T_r} - Y(s) \right]$$

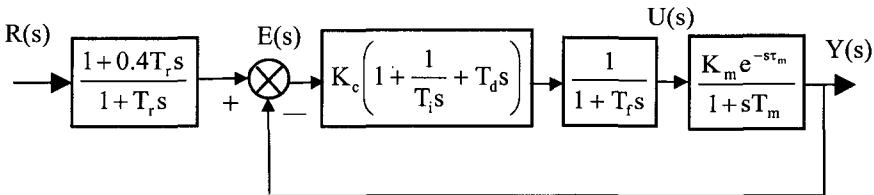


Table 48: PID controller tuning rules – FOLPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{1 + s T_m}$

Rule	K_c	T_i	T_d	Comment
Direct synthesis				
Normey-Rico <i>et al.</i> (2000). <i>Model:</i> <i>Method 1</i>	$\frac{0.375(\tau_m + 2T_m)}{K_m \tau_m}$	$T_m + 0.5\tau_m$	$\frac{T_m \tau_m}{2T_m + \tau_m}$	$T_f = 0.13\tau_m$ $T_r = 0.5\tau_m$

4.1.6 Controller with filtered derivative $G_c(s) = K_c \left(1 + \frac{1}{T_i s} + \frac{T_d s}{1 + s \frac{T_d}{N}} \right)$

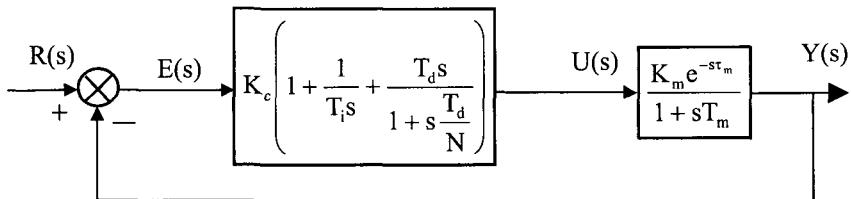


Table 49: PID controller tuning rules – FOLPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{1 + s T_m}$

Rule	K_c	T_i	T_d	Comment
Minimum performance index: servo tuning				
Tavakoli and Tavakoli (2003). <i>Model:</i> <i>Method 1;</i>	¹ $K_c^{(271)}$	$T_i^{(271)}$	$0.0111 T_m$	Minimum IAE
	² $K_c^{(272)}$	$T_i^{(272)}$		Minimum ISE
0.1 $\leq \frac{\tau_m}{T_m} \leq 2$; $N = 10$	$\frac{1}{K_m} \left(\frac{0.8}{\frac{\tau_m}{T_m} + 0.1} \right)$	$\tau_m \left(0.3 + \frac{T_m}{\tau_m} \right)$	$\tau_m \left(\frac{0.06}{\frac{\tau_m}{T_m} + 0.04} \right)$	Minimum ITAE

$$^1 K_c^{(271)} = \frac{1}{K_m} \left(\frac{1}{\frac{\tau_m}{T_m} + 0.2} \right), \quad T_i^{(271)} = \tau_m \left(\frac{0.3 \frac{\tau_m}{T_m} + 1.2}{\frac{\tau_m}{T_m} + 0.08} \right) \tau_m \left(\frac{0.06}{\frac{\tau_m}{T_m} + 0.04} \right).$$

$$^2 K_c^{(272)} = \frac{1}{K_m} \left(\frac{0.3 \frac{\tau_m}{T_m} + 0.75}{\frac{\tau_m}{T_m} + 0.05} \right), \quad T_i^{(272)} = 2.4 \tau_m \left(\frac{1}{\frac{\tau_m}{T_m} + 0.4} \right).$$

Rule	K_c	T_i	T_d	Comment
Direct synthesis				
Davydov <i>et al.</i> (1995). <i>Model: Method 6</i>	${}^3 K_c^{(273)}$	$T_i^{(273)}$	$T_d^{(273)}$	$\xi = 0.9;$ $0.2 \leq \tau_m / T_m \leq 1$ $N = K_m$
	${}^4 K_c^{(274)}$	$T_i^{(274)}$	$T_d^{(274)}$	
Kuhn (1995). <i>Model:</i> <i>Method 14;</i> $N = 5$	$1/K_m$	$0.66(\tau_m + T_m)$	$0.167(\tau_m + T_m)$	“Normale Einstellung”
	$2/K_m$	$0.8(\tau_m + T_m)$	$0.194(\tau_m + T_m)$	“Schnelle Einstellung”

$${}^3 K_c^{(273)} = \frac{1}{K_m \left(1.552 \frac{\tau_m}{T_m} + 0.078 \right)}, \quad T_i^{(273)} = \left(0.186 \frac{\tau_m}{T_m} + 0.532 \right) T_m, \\ T_d^{(273)} = 0.25 \left(0.186 \frac{\tau_m}{T_m} + 0.532 \right) T_m.$$

$${}^4 K_c^{(274)} = \frac{1}{K_m \left(1.209 \frac{\tau_m}{T_m} + 0.103 \right)}, \quad T_i^{(274)} = \left(0.382 \frac{\tau_m}{T_m} + 0.338 \right) T_m, \\ T_d^{(274)} = 0.4 \left(0.382 \frac{\tau_m}{T_m} + 0.338 \right) T_m.$$

Rule	K_c	T_i	T_d	Comment
Schaedel (1997)	$^5 K_c^{(275)}$	$T_i^{(275)}$	$T_d^{(275)}$	<i>Model: Method 21</i>
Goodwin <i>et al.</i> (2001) – page 426.	$^6 K_c^{(276)}$	$T_i^{(276)}$	$T_d^{(276)}$	<i>Model: Method 1</i>

$$^5 K_c^{(275)} = \frac{0.375 T_i^{(275)}}{K_m \left(T_1 + \frac{T_d^{(275)}}{N} - T_i^{(275)} \right)}, \quad T_i^{(275)} = \frac{T_1^2 - T_2^2}{T_1 - T_d^{(275)}}, \quad T_d^{(275)} = \frac{T_2^2}{T_1} - \frac{T_3^3}{T_2^2};$$

$$T_1 = \frac{T_m}{1 + 3.45 \frac{\tau_m^a}{T_m}} + \tau_m, \quad T_2^2 = 1.25 T_m \tau_m^a + \frac{T_m}{1 + 3.45 \frac{\tau_m^a}{T_m}} \tau_m + 0.5 \tau_m^2, \quad T_3 = 0,$$

$$0 < \frac{\tau_m^a}{T_m} \leq 0.104;$$

$$T_1 = \frac{0.7 T_m^2}{T_m - 0.7 \tau_m^a} + \tau_m, \quad T_2^2 = T_m \tau_m^a + \frac{0.7 T_m^2}{T_m - 0.7 \tau_m^a} + 0.5 \tau_m^2,$$

$$T_3^3 = \frac{0.952 T_m^2 (\tau_m^a - 0.1)}{T_m - 0.7 \tau_m^a} + T_m \tau_m \tau_m^a + \frac{0.35 T_m^2 \tau_m^2}{T_m - 0.7 \tau_m^a} + 0.167 \tau_m^3, \quad \frac{\tau_m^a}{T_m} > 0.104.$$

$$N = \frac{T_d^{(275)}}{- \left(\frac{a + \tau_m - T_i^{(275)}}{2} \right) + \sqrt{\frac{(a + \tau_m - T_i^{(275)})^2}{4} + 0.375 \frac{T_i^{(275)} T_d^{(275)}}{K_v}}},$$

$0.05 \leq a \leq 0.2$, K_v = prescribed overshoot in the manipulated variable for a step response in the command variable.

$$^6 K_c^{(276)} = \frac{(4\xi T_{CL2} + \tau_m)(\tau_m + 2T_m) - 4T_{CL2}^2}{K_m (16\xi^2 T_{CL2}^2 + 8\xi T_{CL2} \tau_m + \tau_m^2)},$$

$$T_i^{(276)} = \frac{(4\xi T_{CL2} + \tau_m)(\tau_m + 2T_m) - 4T_{CL2}^2}{2(16\xi^2 T_{CL2}^2 + 8\xi T_{CL2} \tau_m + \tau_m^2)} (4\xi T_{CL2} + \tau_m),$$

$$T_d^{(276)} = (16\xi^2 T_{CL2}^2 + 8\xi T_{CL2} \tau_m + \tau_m^2) \frac{[(4\xi T_{CL2} + \tau_m)^2 \tau_m T_m - 2T_{CL2}^2 (4\xi T_{CL2} + \tau_m)(\tau_m + 2T_m) + 8T_{CL2}^4]}{(4\xi T_{CL2} + \tau_m)^3 [4\xi T_{CL2} + \tau_m](\tau_m + 2T_m) - 4T_{CL2}^2},$$

$$N = \frac{4\xi T_{CL2} + \tau_m}{2T_{CL2}^2} T_d^{(276)} \text{ with } G_{CL}(s) = \frac{1}{T_{CL2}^2 s^2 + 2\xi T_{CL2} s + 1}.$$

Rule	K_c	T_i	T_d	Comment
Robust				
Chien (1988). Model: Method 1	$\frac{T_m + 0.5\tau_m}{K_m(\lambda + 0.5\tau_m)}$	$T_m + 0.5\tau_m$	$\frac{T_m\tau_m}{2T_m + \tau_m}$	$\lambda = [\tau_m, T_m]$ (Chien and Fruehauf (1990)); N=10
Morari and Zafiriou (1989). Model: Method 1	${}^7 K_c^{(277)}$	$T_m + 0.5\tau_m$	$\frac{T_m\tau_m}{2T_m + \tau_m}$	$\lambda > 0.25\tau_m$, $\lambda > 0.2T_m$
Gong <i>et al.</i> (1996). Model: Method 1	${}^8 K_c^{(278)}$	$T_m + 0.3866\tau_m$	$\frac{0.3866T_m\tau_m}{T_m + 0.3866\tau_m}$	N = [3,10]
Maffezzoni and Rocco (1997).	${}^9 K_c^{(279)}$	$T_i^{(279)}$	$T_d^{(279)}$	Model: Method 12
Kasahara <i>et al.</i> (1999).	${}^{10} K_c^{(280)}$	$T_i^{(280)}$	$T_d^{(280)}$	Robust to 20% change in plant parameters
Model: Method 1; N = 10; $0 < \frac{\tau_m}{T_m} < 1$				

$${}^7 K_c^{(277)} = \frac{1}{K_m} \left(\frac{T_m + 0.5\tau_m}{\lambda + \tau_m} \right), \quad N = \frac{2T_m(\lambda + \tau_m)}{\lambda(2T_m + \tau_m)}.$$

$${}^8 K_c^{(278)} = \frac{T_m + 0.3866\tau_m}{K_m(\lambda + 1.0009\tau_m)}, \quad \lambda = \frac{(0.1388 + 0.1247N)T_m + 0.0482N\tau_m}{0.3866(N-1)T_m + 0.1495N\tau_m} \tau_m.$$

$${}^9 K_c^{(279)} = \frac{2T_m(\tau_m + x_1) + \tau_m^2}{2K_m(\tau_m + x_1)^2}, \quad T_i^{(279)} = T_m + \frac{\tau_m^2}{2(\tau_m + x_1)},$$

$$T_d^{(279)} = \frac{\tau_m^2 [2T_m(\tau_m + x_1) - \tau_m x_1]}{2(\tau_m + x_1) [2T_m(\tau_m + x_1) + \tau_m^2]}, \quad N = \frac{2T_m(\tau_m + x_1) - \tau_m x_1}{x_1 [2T_m(\tau_m + x_1) + \tau_m^2]}$$

$x_1 = 0.25\tau_m$ for uncertainty on τ_m only; $x_1 > 0.1T_m, \frac{\tau_m}{T_m} < 0.4$ for uncertainty on the minimum phase dynamics within the control band.

$${}^{10} K_c^{(280)} = 0.6K_u \left[0.718 - 0.057 \frac{\tau_m}{T_m} \right], \quad T_i^{(280)} = 0.5T_u \left[0.296 + 0.838 \frac{\tau_m}{T_m} \right],$$

$$T_d^{(280)} = 0.125T_u \left[0.635 - 1.091 \frac{\tau_m}{T_m} \right].$$

Rule	K_c	T_i	T_d	Comment
Kasahara <i>et al.</i> (1999) – continued.	$^{11} K_c^{(281)}$	$T_i^{(281)}$	$T_d^{(281)}$	Robust to 30% change in plant parameters
	$^{12} K_c^{(282)}$	$T_i^{(282)}$	$T_d^{(282)}$	Robust to 40% change in plant parameters
	$^{13} K_c^{(283)}$	$T_i^{(283)}$	$T_d^{(283)}$	Robust to 50% change in plant parameters
Leva and Colombo (2000).	$^{14} K_c^{(284)}$	$T_m + \frac{\tau_m^2}{2(\lambda + \tau_m)}$	$T_d^{(284)}$	λ not specified; <i>Model: Method 1</i>

$$^{11} K_c^{(281)} = 0.6 K_u \left[0.690 - 0.076 \frac{\tau_m}{T_m} \right], \quad T_i^{(281)} = 0.5 T_u \left[0.274 + 0.788 \frac{\tau_m}{T_m} \right],$$

$$T_d^{(281)} = 0.125 T_u \left[0.526 - 0.057 \frac{\tau_m}{T_m} \right].$$

$$^{12} K_c^{(282)} = 0.6 K_u \left[0.331 + 0.149 \frac{\tau_m}{T_m} \right], \quad T_i^{(282)} = 0.5 T_u \left[0.041 + 0.857 \frac{\tau_m}{T_m} \right],$$

$$T_d^{(282)} = 0.125 T_u \left[0.495 - 0.064 \frac{\tau_m}{T_m} \right].$$

$$^{13} K_c^{(283)} = 0.6 K_u \left[0.195 + 0.266 \frac{\tau_m}{T_m} \right], \quad T_i^{(283)} = 0.5 T_u \left[-0.013 + 0.731 \frac{\tau_m}{T_m} \right],$$

$$T_d^{(283)} = 0.125 T_u \left[0.489 - 0.065 \frac{\tau_m}{T_m} \right].$$

$$^{14} K_c^{(284)} = \frac{1}{K_m} \frac{2(\lambda + \tau_m)T_m + \tau_m^2}{2(\lambda + \tau_m)^2}, \quad T_d^{(284)} = \left[\frac{T_m(\lambda + \tau_m)}{2(\lambda + \tau_m)T_m + \tau_m^2} \right],$$

$$N = \frac{2T_m(\lambda + \tau_m)^2}{\lambda [2(\lambda + \tau_m)T_m + \tau_m^2]}.$$

Rule	K_c	T_i	T_d	Comment
Leva (2001). Model: Method 1	$^{15} K_c^{(285)}$	$T_m + \frac{\tau_m^2}{2(\lambda + \tau_m)}$	$T_d^{(285)}$	
Leva <i>et al.</i> (2003).	$^{16} K_c^{(286)}$	$T_m + \frac{\tau_m^2}{2(\lambda + \tau_m)}$	$T_d^{(286)}$	Model: Method 1
Zhang <i>et al.</i> (2002).	$^{17} K_c^{(287)}$	$T_i^{(287)}$	$T_d^{(287)}$	Model: Method 1

$^{15} K_c^{(285)} = \frac{1}{K_m} \frac{2(\lambda + \tau_m)T_m + \tau_m^2}{2(\lambda + \tau_m)^2}, N = \frac{2T_m(\lambda + \tau_m)^2}{\lambda[2(\lambda + \tau_m)T_m + \tau_m^2]} - 1,$
 $T_d^{(285)} = \tau_m^2 \frac{2\lambda T_m + (2T_m - \lambda)\tau_m}{2(\lambda + \tau_m)[2T_m(\lambda + \tau_m) + \tau_m^2]} . \lambda \in [0.1T_m, 0.5T_m], \frac{\tau_m}{T_m} \leq 0.25;$
 $\lambda = 1.5(\tau_m + T_m), 0.25 < \frac{\tau_m}{T_m} \leq 0.75; \lambda = 3(\tau_m + T_m), \frac{\tau_m}{T_m} > 0.75.$

$^{16} K_c^{(286)} = \frac{1}{K_m} \frac{2(\lambda + \tau_m)T_m + \tau_m^2}{2(\lambda + \tau_m)^2}, T_d^{(286)} = \frac{\tau_m T_m (\lambda + \tau_m)}{[2(\lambda + \tau_m)T_m + \tau_m^2]},$
 $N = \frac{2T_m(\lambda + \tau_m)^2}{\lambda[2(\lambda + \tau_m)T_m + \tau_m^2]} . \lambda \in [0.1T_m, 0.5T_m], \frac{\tau_m}{T_m} \leq 0.33;$
 $\lambda = 1.5(\tau_m + T_m), 0.33 < \frac{\tau_m}{T_m} \leq 3; \lambda = 3(\tau_m + T_m), \frac{\tau_m}{T_m} > 3.$

$^{17} K_c^{(287)} = \frac{0.5\tau_m + T_m - \frac{T_d^{(287)}}{N}}{K_m(2\lambda + 0.5\tau_m)}, T_i^{(287)} = 0.5\tau_m + T_m - \frac{T_d^{(287)}}{N},$
 $T_d^{(287)} = \frac{\tau_m T_m}{2T_i^{(287)}} - \frac{T_d^{(287)}}{N}, \frac{T_d^{(287)}}{N} = \frac{\lambda^2}{2\lambda + 0.5\tau_m} \text{ or } N = 10.$

4.1.7 Blending controller $G_c(s) = K_c \left(1 + \frac{1}{T_i s} + T_d s \right) \frac{1}{s}$

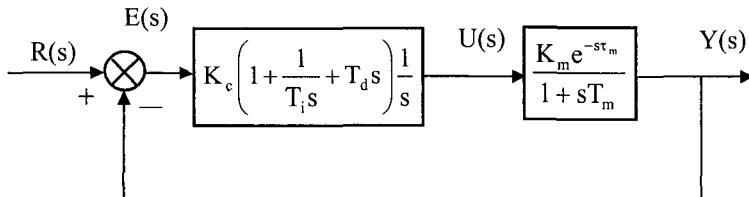


Table 50: PID controller tuning rules – FOLPD model $G_m(s) = \frac{K_m e^{-st_m}}{1 + sT_m}$

Rule	K_c	T_i	T_d	Comment
Direct synthesis: frequency domain criteria				
Chen <i>et al.</i> (2001).	$^{18} K_c^{(288)}$	$T_i^{(288)}$	$T_d^{(288)}$	
Representative λ values				
<i>Model: Method I</i>	$\lambda = 9.56\tau_m$, $M_{max} = 1.20$; $\lambda = 4.68\tau_m$, $M_{max} = 1.26$; $\lambda = 2.19\tau_m$, $M_{max} = 1.50$; $\lambda = 1.40\tau_m$, $M_{max} = 2.00$; $\lambda = 1.11\tau_m$, $M_{max} = 2.50$; $\lambda = 0.95\tau_m$, $M_{max} = 3.00$.			

¹⁸ $K_c^{(288)} = \frac{1}{K_m} \frac{T_m + \tau_m + 2\lambda}{(\lambda + \tau_m)^2}$, $T_i^{(288)} = T_m + \tau_m + 2\lambda$, $T_d^{(288)} = \frac{T_m(2\lambda + \tau_m)}{T_m + \tau_m + 2\lambda}$.

4.1.8 Classical controller 1 $G_c(s) = K_c \left(1 + \frac{1}{T_i s} \right) \left(\frac{1 + T_d s}{1 + \frac{T_d}{N} s} \right)$

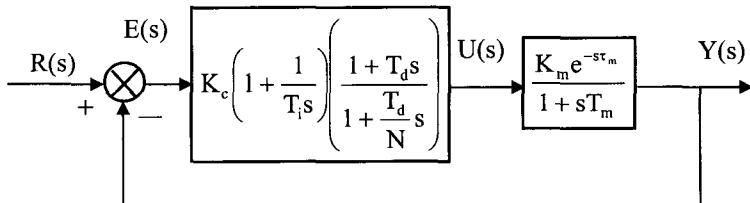


Table 51: PID controller tuning rules – FOLPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{1 + sT_m}$

Rule	K_c	T_i	T_d	Comment
Process reaction				
Witt and Waggoner (1990). Model: Method 2	$\frac{x_1 T_m}{K_m \tau_m},$ $x_1 \in [0.6, 1.0]$	τ_m	τ_m	Equivalent to Ziegler and Nichols (1942); $N \in [10, 20]$
Witt and Waggoner (1990). Model: Method 2	¹ $K_c^{(289)}$	$T_i^{(289)}$	$T_d^{(289)}$	Preferred tuning; $N \in [10, 20]$

¹ Tuning equivalent to Cohen and Coon (1953).

$$K_c^{(289)} = \frac{1.350 \frac{T_m}{\tau_m} + 0.25 + \frac{T_m}{\tau_m} \sqrt{0.7425 + 0.0150 \frac{\tau_m}{T_m} + 0.0625 \left(\frac{\tau_m}{T_m} \right)^2}}{2K_m},$$

$$T_i^{(289)} = \frac{T_m}{1.350 \frac{T_m}{\tau_m} + 0.25 - \frac{T_m}{\tau_m} \sqrt{0.7425 + 0.0150 \frac{\tau_m}{T_m} + 0.0625 \left(\frac{\tau_m}{T_m} \right)^2}},$$

$$T_d^{(289)} = \frac{T_m}{1.350 \frac{T_m}{\tau_m} + 0.25 + \frac{T_m}{\tau_m} \sqrt{0.7425 + 0.0150 \frac{\tau_m}{T_m} + 0.0625 \left(\frac{\tau_m}{T_m} \right)^2}}.$$

Rule	K_c	T_i	T_d	Comment
Witt and Waggoner (1990) (continued).	$^2 K_c^{(290)}$	$T_i^{(290)}$	$T_d^{(290)}$	Alternate tuning; $N \in [10,20]$
St. Clair (1997) – page 21. Model: Method 2	$\frac{T_m}{K_m \tau_m}$	$5\tau_m$	$0.5\tau_m$	'aggressive' tuning; $N \geq 1$
	$\frac{0.5T_m}{K_m \tau_m}$	$5\tau_m$	$0.5\tau_m$	'conservative' tuning; $N \geq 1$
Harrold (1999). Model: Method 10; N not specified	$1/K_m$	T_m	$\leq 0.25T_m$	$\tau_m/T_m \leq 0.25$
	$0.5/K_m$			$\tau_m/T_m \approx 0.5$
	$0.25/K_m$			$\tau_m/T_m \geq 1$
Shinskey (2000). Model: Method 2	$\frac{0.889T_m}{K_m \tau_m}$	$1.75\tau_m$	$0.70\tau_m$	$\frac{\tau_m}{T_m} = 0.167$; N not specified
Minimum performance index: regulator tuning				
Minimum IAE – Huang and Chao (1982).	$0.817 \left(\frac{T_m}{K_m \tau_m} \right)^{0.982}$	$^3 T_i^{(291)}$	$T_d^{(291)}$	$N = 8$; Model: Method 1

$$\begin{aligned}
 ^2 K_c^{(290)} &= \frac{1.350 \frac{T_m}{\tau_m} + 0.25 - \frac{T_m}{\tau_m} \sqrt{0.7425 + 0.0150 \frac{\tau_m}{T_m} + 0.0625 \left(\frac{\tau_m}{T_m} \right)^2}}{2K_m}, \\
 T_i^{(290)} &= \frac{T_m}{1.350 \frac{T_m}{\tau_m} + 0.25 + \frac{T_m}{\tau_m} \sqrt{0.7425 + 0.0150 \frac{\tau_m}{T_m} + 0.0625 \left(\frac{\tau_m}{T_m} \right)^2}}, \\
 T_d^{(290)} &= \frac{T_m}{1.350 \frac{T_m}{\tau_m} + 0.25 - \frac{T_m}{\tau_m} \sqrt{0.7425 + 0.0150 \frac{\tau_m}{T_m} + 0.0625 \left(\frac{\tau_m}{T_m} \right)^2}}. \\
 ^3 T_i^{(291)} &= 0.903 T_m \left(\frac{\tau_m}{T_m} \right)^{0.780}, \quad T_d^{(291)} = 0.602 T_m \left(\frac{\tau_m}{T_m} \right)^{0.954}.
 \end{aligned}$$

Rule	K_c	T_i	T_d	Comment
Minimum IAE - Kaya and Scheib (1988). Model: Method 5	$^4 K_c^{(292)}$	$T_i^{(292)}$	$T_d^{(292)}$	$0 < \frac{\tau_m}{T_m} \leq 1 ; N=10$
Minimum IAE - Witt and Waggoner (1990). Model: Method 2	$^5 K_c^{(293)}$	$T_i^{(293)}$	$T_d^{(293)}$	Preferred tuning
	$^6 K_c^{(294)}$	$T_i^{(294)}$	$T_d^{(294)}$	Alternate tuning
		$0.1 < \frac{\tau_m}{T_m} \leq 0.258 ; N \in [10,20]$		
Minimum IAE - Shinskey (1988) - page 143. Model: Method 1	$0.95T_m/K_m\tau_m$	$1.43\tau_m$	$0.52\tau_m$	$\tau_m/T_m = 0.2$
	$0.95T_m/K_m\tau_m$	$1.17\tau_m$	$0.48\tau_m$	$\tau_m/T_m = 0.5$
	$1.14T_m/K_m\tau_m$	$1.03\tau_m$	$0.40\tau_m$	$\tau_m/T_m = 1$
	$1.39T_m/K_m\tau_m$	$0.77\tau_m$	$0.35\tau_m$	$\tau_m/T_m = 2$

$$^4 K_c^{(292)} = \frac{0.98089}{K_m} \left(\frac{T_m}{\tau_m} \right)^{0.76167}, \quad T_i^{(292)} = \frac{T_m}{0.91032} \left(\frac{\tau_m}{T_m} \right)^{1.05211},$$

$$T_d^{(292)} = 0.59974 T_m \left(\frac{\tau_m}{T_m} \right)^{0.89819}.$$

$$^5 K_c^{(293)} = \frac{0.718}{K_m} \left(\frac{\tau_m}{T_m} \right)^{-0.921} \left[1 + \sqrt{1 - 1.693 \left(\frac{\tau_m}{T_m} \right)^{-1.886}} \right],$$

$$T_i^{(293)} = \frac{0.964 T_m \left(\frac{\tau_m}{T_m} \right)^{1.137}}{1 - \sqrt{1 - 1.693 \left(\frac{\tau_m}{T_m} \right)^{1.886}}}, \quad T_d^{(293)} = \frac{0.964 T_m \left(\frac{\tau_m}{T_m} \right)^{1.137}}{1 + \sqrt{1 + 1.693 \left(\frac{\tau_m}{T_m} \right)^{1.886}}}.$$

$$^6 K_c^{(294)} = \frac{0.718}{K_m} \left(\frac{\tau_m}{T_m} \right)^{-0.921} \left[1 - \sqrt{1 - 1.693 \left(\frac{\tau_m}{T_m} \right)^{-1.886}} \right],$$

$$T_i^{(294)} = \frac{0.964 T_m \left(\frac{\tau_m}{T_m} \right)^{1.137}}{1 - \sqrt{1 + 1.693 \left(\frac{\tau_m}{T_m} \right)^{1.886}}}, \quad T_d^{(294)} = \frac{0.964 T_m \left(\frac{\tau_m}{T_m} \right)^{1.137}}{1 + \sqrt{1 - 1.693 \left(\frac{\tau_m}{T_m} \right)^{1.886}}}.$$

Rule	K_c	T_i	T_d	Comment
Minimum IAE – Shinskey (1996) – page 119.	$0.96T_m/K_m\tau_m$	$1.43\tau_m$	$0.52\tau_m$	Model: Method 1 $\tau_m/T_m = 0.2$
Minimum IAE – Edgar et al. (1997) – pages 8-14, 8-15.	$0.94T_m/K_m\tau_m$	$1.5\tau_m$	$0.55\tau_m$	Model: Method 1 Time constant dominant; N = 10
Minimum IAE – Edgar et al. (1997) – page 8-15.	$0.88T_m/K_m\tau_m$	$1.8\tau_m$	$0.70\tau_m$	Model: Method 2; N = 10
Minimum IAE – Smith and Corripio (1997) – page 345.	$\frac{T_m}{K_m\tau_m}$	T_m	$0.5\tau_m$	Model: Method 1; $0.1 \leq \frac{\tau_m}{T_m} \leq 1.5$
Minimum IAE – Shinskey (1988) – page 148. Model: Method 1	$0.56K_u$	$0.39T_u$	$0.14T_u$	$\tau_m/T_m = 0.2$
	$0.49K_u$	$0.34T_u$	$0.14T_u$	$\tau_m/T_m = 0.5$
	$0.51K_u$	$0.33T_u$	$0.13T_u$	$\tau_m/T_m = 1$
	$0.46K_u$	$0.28T_u$	$0.13T_u$	$\tau_m/T_m = 2$
Minimum IAE – Shinskey (1994) – page 167.	$\frac{K_u\tau_m}{3\tau_m - 0.32T_u}$	${}^7 T_i^{(295)}$	$0.14T_u$	Model: Method 1; N not specified
Minimum ISE – Huang and Chao (1982).	$\frac{1.101}{K_m} \left(\frac{T_m}{\tau_m} \right)^{0.881}$	${}^8 T_i^{(296)}$	$T_d^{(296)}$	Model: Method 1; N = 8
Minimum ISE – Kaya and Scheib (1988). Model: Method 5	${}^9 K_c^{(297)}$	$T_i^{(297)}$	$T_i^{(297)}$	$0 < \frac{\tau_m}{T_m} \leq 1$; N=10

$${}^7 T_i^{(295)} = T_u \left(0.15 \frac{T_u}{\tau_m} - 0.05 \right).$$

$${}^8 T_i^{(296)} = 1.134 T_m \left(\frac{\tau_m}{T_m} \right)^{0.883}, \quad T_d^{(296)} = 0.563 T_m \left(\frac{\tau_m}{T_m} \right)^{0.881}.$$

$${}^9 K_c^{(297)} = \frac{1.11907}{K_m} \left(\frac{T_m}{\tau_m} \right)^{0.89711}, \quad T_i^{(297)} = \frac{T_m}{0.7987} \left(\frac{\tau_m}{T_m} \right)^{0.9548},$$

$$T_d^{(297)} = 0.54766 T_m \left(\frac{\tau_m}{T_m} \right)^{0.87798}.$$

Rule	K_c	T_i	T_d	Comment
Minimum ITAE – Huang and Chao (1982).	$\frac{0.745}{K_m} \left(\frac{T_m}{\tau_m} \right)^{1.036}$	$^{10} T_i^{(298)}$	$T_d^{(298)}$	<i>Model: Method 1; $N = 8$</i>
Minimum ITAE – Kaya and Scheib (1988).	$^{11} K_c^{(299)}$	$T_i^{(299)}$	$T_d^{(299)}$	$0 < \frac{\tau_m}{T_m} \leq 1$
<i>N=10; Model: Method 5</i>				
Minimum ITAE – Witt and Waggoner (1990).	$^{12} K_c^{(300)}$	$T_i^{(300)}$	$T_d^{(300)}$	Preferred tuning
	$^{13} K_c^{(301)}$	$T_i^{(301)}$	$T_d^{(301)}$	Alternate tuning
<i>Model: Method 2; N ∈ [10,20]</i>				

$$^{10} T_i^{(298)} = 0.771 T_m \left(\frac{\tau_m}{T_m} \right)^{0.595}, \quad T_d^{(298)} = 0.597 T_m \left(\frac{\tau_m}{T_m} \right)^{1.006}.$$

$$^{11} K_c^{(299)} = \frac{0.77902}{K_m} \left(\frac{T_m}{\tau_m} \right)^{1.06401}, \quad T_i^{(299)} = \frac{T_m}{1.14311} \left(\frac{\tau_m}{T_m} \right)^{0.70949},$$

$$T_d^{(299)} = 0.57137 T_m \left(\frac{\tau_m}{T_m} \right)^{1.03826}.$$

$$^{12} K_c^{(300)} = \frac{0.679}{K_m} \left(\frac{\tau_m}{T_m} \right)^{-0.947} \left[1 + \sqrt{1 - 1.283 \left(\frac{\tau_m}{T_m} \right)^{-1.733}} \right], \quad 0.1 < \frac{\tau_m}{T_m} < 0.379;$$

$$T_i^{(300)} = \frac{0.762 T_m \left(\frac{\tau_m}{T_m} \right)^{0.995}}{1 - \sqrt{1 - 1.283 \left(\frac{\tau_m}{T_m} \right)^{1.733}}}, \quad T_d^{(300)} = \frac{0.762 T_m \left(\frac{\tau_m}{T_m} \right)^{0.995}}{1 + \sqrt{1 + 1.283 \left(\frac{\tau_m}{T_m} \right)^{1.733}}}.$$

$$^{13} K_c^{(301)} = \frac{0.679}{K_m} \left(\frac{\tau_m}{T_m} \right)^{-0.947} \left[1 - \sqrt{1 - 1.283 \left(\frac{\tau_m}{T_m} \right)^{-1.733}} \right],$$

$$T_i^{(301)} = \frac{0.762 T_m \left(\frac{\tau_m}{T_m} \right)^{0.995}}{1 - \sqrt{1 + 1.283 \left(\frac{\tau_m}{T_m} \right)^{1.733}}}, \quad T_d^{(301)} = \frac{0.762 T_m \left(\frac{\tau_m}{T_m} \right)^{0.995}}{1 + \sqrt{1 - 1.283 \left(\frac{\tau_m}{T_m} \right)^{1.733}}}.$$

Rule	K_c	T_i	T_d	Comment
Minimum ITAE - Ou and Chen (1995).	$K_c^{(302)} = \frac{1.357}{K_m} \left(\frac{T_m}{\tau_m} \right)^{0.947}$	$T_i^{(302)} = \frac{T_m}{0.842} \left(\frac{\tau_m}{T_m} \right)^{0.738}$	$^{14} T_d^{(302)}$	<i>Model: Method 46</i>
Minimum ITSE - Huang and Chao (1982).	$\frac{0.994}{K_m} \left(\frac{T_m}{\tau_m} \right)^{0.907}$	$^{15} T_i^{(303)}$	$T_d^{(303)}$	<i>Model: Method 1; N = 8</i>
Minimum performance index: servo tuning				
Minimum IAE - Huang and Chao (1982).	$\frac{0.673}{K_m} \left(\frac{\tau_m}{T_m} \right)^{-1.00}$	$^{16} T_i^{(304)}$	$T_d^{(304)}$	<i>Model: Method 1; N = 8</i>
Minimum IAE - Kaya and Scheib (1988). <i>Model: Method 5</i>	$\frac{0.65}{K_m} \left(\frac{T_m}{\tau_m} \right)^{1.04432}$	$^{17} T_i^{(305)}$	$T_d^{(305)}$	$0 < \frac{\tau_m}{T_m} \leq 1 ; N=10$

$$^{14} T_d^{(302)} = 0.381 T_m \left(\frac{\tau_m}{T_m} \right)^{0.995},$$

$$N = \pm \frac{0.75 K_m K_c^{(302)} T_d^{(302)}}{T_i^{(302)} + K_m K_c^{(302)} (T_d^{(302)} + T_i^{(302)} - \tau_m - T_m)}, \quad N > 0.$$

$$^{15} T_i^{(303)} = 1.032 T_m \left(\frac{\tau_m}{T_m} \right)^{0.869}, \quad T_d^{(303)} = 0.570 T_m \left(\frac{\tau_m}{T_m} \right)^{0.884}.$$

$$^{16} T_i^{(304)} = \frac{T_m}{0.148 \left(\frac{\tau_m}{T_m} \right) + 0.979}, \quad T_d^{(304)} = 0.502 T_m \left(\frac{\tau_m}{T_m} \right)^{1.032}.$$

$$^{17} T_i^{(305)} = \frac{T_m}{0.9895 + 0.09539 \frac{\tau_m}{T_m}}, \quad T_d^{(305)} = 0.50814 T_m \left(\frac{\tau_m}{T_m} \right)^{1.08433}$$

Rule	K_c	T_i	T_d	Comment
Minimum IAE – Witt and Waggoner (1990). <i>Model: Method 2</i>	$^{18} K_c^{(306)}$	$T_i^{(306)}$	$T_d^{(306)}$	Preferred tuning; $0.1 \leq \frac{\tau_m}{T_m} \leq 1$; $N \in [10, 20]$
	$^{19} K_c^{(307)}$	$T_i^{(307)}$	$T_d^{(307)}$	Alternate tuning
Minimum IAE – Smith and Corripio (1997) – page 345. <i>Model: Method 1</i>	$\frac{0.83T_m}{K_m\tau_m}$	T_m	$0.5\tau_m$	$0.1 \leq \frac{\tau_m}{T_m} \leq 1.5$; N not specified

$$\begin{aligned}
 ^{18} K_c^{(306)} &= \frac{1.086}{K_m} \left(\frac{\tau_m}{T_m} \right)^{-0.869} \left[1 + \sqrt{1 - 1.392 \left(\frac{\tau_m}{T_m} \right)^{0.914} \left\{ 0.74 - 0.13 \frac{\tau_m}{T_m} \right\}} \right], \\
 T_i^{(306)} &= \frac{0.696 T_m \left(\frac{\tau_m}{T_m} \right)^{0.914}}{1 - \sqrt{1 - 1.392 \left(\frac{\tau_m}{T_m} \right)^{0.869} \left\{ 0.74 - 0.13 \frac{\tau_m}{T_m} \right\}}}, \\
 T_d^{(306)} &= \frac{0.696 T_m \left(\frac{\tau_m}{T_m} \right)^{0.914}}{1 + \sqrt{1 - 1.392 \left(\frac{\tau_m}{T_m} \right)^{0.869} \left\{ 0.74 - 0.13 \frac{\tau_m}{T_m} \right\}}}. \\
 ^{19} K_c^{(307)} &= \frac{1.086}{K_m} \left(\frac{\tau_m}{T_m} \right)^{-0.869} \left[1 - \sqrt{1 - 1.392 \left(\frac{\tau_m}{T_m} \right)^{0.914} \left\{ 0.74 - 0.13 \frac{\tau_m}{T_m} \right\}} \right], \\
 T_i^{(307)} &= \frac{0.696 T_m \left(\frac{\tau_m}{T_m} \right)^{0.914}}{1 + \sqrt{1 - 1.392 \left(\frac{\tau_m}{T_m} \right)^{0.869} \left\{ 0.74 - 0.13 \frac{\tau_m}{T_m} \right\}}}, \\
 T_d^{(307)} &= \frac{0.696 T_m \left(\frac{\tau_m}{T_m} \right)^{0.914}}{1 - \sqrt{1 - 1.392 \left(\frac{\tau_m}{T_m} \right)^{0.869} \left\{ 0.74 - 0.13 \frac{\tau_m}{T_m} \right\}}}.
 \end{aligned}$$

Rule	K_c	T_i	T_d	Comment
Minimum ISE – Huang and Chao (1982).	$\frac{0.787}{K_m} \left(\frac{T_m}{\tau_m} \right)^{0.964}$	$^{20} T_i^{(308)}$	$T_d^{(308)}$	<i>Model:</i> <i>Method 1;</i> $N = 8$
Minimum ISE – Kaya and Scheib (1988). <i>Model: Method 5</i>	$^{21} K_c^{(309)}$	$T_i^{(309)}$	$T_d^{(309)}$	$0 < \frac{\tau_m}{T_m} \leq 1$; $N=10$
Minimum ITAE – Huang and Chao (1982).	$\frac{0.684}{K_m} \left(\frac{T_m}{\tau_m} \right)^{0.995}$	$^{22} T_i^{(310)}$	$T_d^{(310)}$	<i>Model:</i> <i>Method 1;</i> $N = 8$
Minimum ITAE – Kaya and Scheib (1988). <i>Model: Method 5</i>	$^{23} K_c^{(311)}$	$T_i^{(311)}$	$T_d^{(311)}$	$0 < \frac{\tau_m}{T_m} \leq 1$; $N=10$

$$^{20} T_i^{(308)} = 1.678 T_m \left(\frac{\tau_m}{T_m} \right)^{-1.028}, \quad T_d^{(308)} = 0.594 T_m \left(\frac{\tau_m}{T_m} \right)^{0.971}.$$

$$^{21} K_c^{(309)} = \frac{0.71959}{K_m} \left(\frac{T_m}{\tau_m} \right)^{1.03092}, \quad T_i^{(309)} = \frac{T_m}{1.12666 - 0.18145 \frac{\tau_m}{T_m}},$$

$$T_d^{(309)} = 0.54568 T_m \left(\frac{\tau_m}{T_m} \right)^{0.86411}.$$

$$^{22} T_i^{(310)} = \frac{T_m}{0.114 \left(\frac{\tau_m}{T_m} \right) + 0.986}, \quad T_d^{(310)} = 0.491 T_m \left(\frac{\tau_m}{T_m} \right)^{1.049}.$$

$$^{23} K_c^{(311)} = \frac{1.12762}{K_m} \left(\frac{T_m}{\tau_m} \right)^{0.80368}, \quad T_i^{(311)} = \frac{T_m}{0.99783 + 0.02860 \frac{\tau_m}{T_m}},$$

$$T_d^{(311)} = 0.42844 T_m \left(\frac{\tau_m}{T_m} \right)^{1.0081}.$$

Rule	K_c	T_i	T_d	Comment
Minimum ITAE - Witt and Waggoner (1990).	$^{24} K_c^{(312)}$	$T_i^{(312)}$	$T_d^{(312)}$	Preferred tuning; $0.1 \leq \frac{\tau_m}{T_m} \leq 1$; $N \in [10,20]$
Model: Method 2	$^{25} K_c^{(313)}$	$T_i^{(313)}$	$T_d^{(313)}$	Alternate tuning

$$\begin{aligned}
^{24} K_c^{(312)} &= \frac{0.965}{K_m} \left(\frac{\tau_m}{T_m} \right)^{-0.85} \left[1 + \sqrt{1 - 1.232 \left(\frac{\tau_m}{T_m} \right)^{0.929} \left\{ 0.796 - 0.1465 \frac{\tau_m}{T_m} \right\}} \right], \\
T_i^{(312)} &= \frac{0.616 T_m \left(\frac{\tau_m}{T_m} \right)^{0.929}}{1 - \sqrt{1 - 1.232 \left(\frac{\tau_m}{T_m} \right)^{0.85} \left\{ 0.796 - 0.1465 \frac{\tau_m}{T_m} \right\}}}, \\
T_d^{(312)} &= \frac{0.616 T_m \left(\frac{\tau_m}{T_m} \right)^{0.929}}{1 + \sqrt{1 - 1.232 \left(\frac{\tau_m}{T_m} \right)^{0.85} \left\{ 0.796 - 0.1465 \frac{\tau_m}{T_m} \right\}}}. \\
^{25} K_c^{(313)} &= \frac{0.965}{K_m} \left(\frac{\tau_m}{T_m} \right)^{-0.85} \left[1 - \sqrt{1 - 1.232 \left(\frac{\tau_m}{T_m} \right)^{0.929} \left\{ 0.796 - 0.1465 \frac{\tau_m}{T_m} \right\}} \right], \\
T_i^{(313)} &= \frac{0.616 T_m \left(\frac{\tau_m}{T_m} \right)^{0.929}}{1 + \sqrt{1 - 1.232 \left(\frac{\tau_m}{T_m} \right)^{0.85} \left\{ 0.796 - 0.1465 \frac{\tau_m}{T_m} \right\}}}, \\
T_d^{(313)} &= \frac{0.616 T_m \left(\frac{\tau_m}{T_m} \right)^{0.929}}{1 - \sqrt{1 - 1.232 \left(\frac{\tau_m}{T_m} \right)^{0.85} \left\{ 0.796 - 0.1465 \frac{\tau_m}{T_m} \right\}}}.
\end{aligned}$$

Rule	K_c	T_i	T_d	Comment				
Minimum ITAE – Ou and Chen (1995).	$K_c^{(314)} = \frac{0.965}{K_m} \left(\frac{T_m}{\tau_m} \right)^{0.85}$	$^{26} T_i^{(314)}$	$T_d^{(314)}$	<i>Model: Method 46</i>				
Minimum ITSE – Huang and Chao (1982).	$\frac{0.718}{K_m} \left(\frac{T_m}{\tau_m} \right)^{1.00}$	$^{27} T_i^{(315)}$	$T_d^{(315)}$	<i>Model: Method 1; N = 8</i>				
Direct synthesis								
Tsang <i>et al.</i> (1993). <i>Model: Method 15</i>	$\frac{x_1 T_m}{K_m \tau_m}$	T_m	$0.25 \tau_m$	$N = 2.5$				
Coefficient values								
	x_1	ξ	x_1	ξ	x_1	ξ	x_1	ξ
	1.6818	0.0	0.9916	0.3	0.6693	0.6	0.4957	0.9
	1.3829	0.1	0.8594	0.4	0.6000	0.7	0.4569	1.0
	1.1610	0.2	0.7542	0.5	0.5429	0.8		
Tsang and Rad (1995). <i>Model: Method 15</i>	$\frac{0.809 T_m}{K_m \tau_m}$	T_m	$0.5 \tau_m$	Overshoot = 16%; N=5				
Smith and Corripio (1997) page 346. <i>Model: Method 1</i>	$\frac{0.5 T_m}{K_m \tau_m}$	T_m	$0.5 \tau_m$	Servo – 5% overshoot; N not specified				

$$^{26} T_i^{(314)} = \frac{T_m}{0.796 - 0.1465 \frac{\tau_m}{T_m}}, \quad T_d^{(314)} = 0.308 T_m \left(\frac{\tau_m}{T_m} \right)^{0.929},$$

$$N = \pm \frac{0.75 K_m K_c^{(314)} T_d^{(314)}}{T_i^{(314)} + K_m K_c^{(314)} (T_d^{(314)} + T_i^{(314)} - \tau_m - T_m)}, \quad N > 0.$$

$$^{27} T_i^{(315)} = \frac{T_m}{0.063 \left(\frac{\tau_m}{T_m} \right) + 1.061}, \quad T_d^{(315)} = 0.596 T_m \left(\frac{\tau_m}{T_m} \right)^{1.028}.$$

Rule	K_c	T_i	T_d	Comment
Schaedel (1997) <i>Model: Method 2I</i>	$^{28} K_c^{(316)}$	$T_1 - \frac{T_2^2}{T_1}$	$\sqrt{T_1^2 - 2T_2^2}$	N not defined
O'Dwyer (2001a). <i>Model: Method 1</i>	$\frac{x_1 T_m}{K_m \tau_m}$	$\frac{T_m}{N}$	T_m	$A_m = \pi/2x_1 N ;$ $\phi_m =$ $0.5\pi - x_1 N$
	$\frac{0.079 T_m}{K_m \tau_m}$	$0.1 T_m$	T_m	$A_m = 2.0 ;$ $\phi_m = 45^\circ$
	$\frac{1}{K_m \left(1 + \frac{\tau_m}{2T_m}\right)}$	$\frac{T_m}{K_m \left(1 + \frac{\tau_m}{2T_m}\right)}$	$0.5\tau_m$	Labeled IMC; $N = 10$
Wang <i>et al.</i> (2002). <i>Model: Method 1</i>	$\frac{0.65 \left(\frac{T_m}{\tau_m}\right)^{1.04432}}{K_m}$	$^{29} T_i^{(317)}$	$T_d^{(317)}$	Labeled IAE setpoint; $N = 10$
	$^{30} K_c^{(318)}$	$T_i^{(318)}$	$T_d^{(318)}$	Labeled IAE load; $N = 10$

$$^{28} K_c^{(316)} = \frac{0.375 \left(T_1 - \frac{T_2^2}{T_1} \right)}{K_m \left(\frac{\sqrt{T_1^2 - 2T_2^2}}{N} + \frac{T_2^2}{T_1} - \sqrt{T_1^2 - 2T_2^2} \right)},$$

$$T_1 = \frac{T_m}{1 + 3.45 \frac{\tau_m^a}{T_m}} + \tau_m, T_2^2 = 1.25 T_m \tau_m^a + \frac{T_m}{1 + 3.45 \frac{\tau_m^a}{T_m}} \tau_m + 0.5 \tau_m^2, 0 < \frac{\tau_m^a}{T_m} \leq 0.104;$$

$$T_1 = \frac{0.7 T_m^2}{T_m - 0.7 \tau_m^a} + \tau_m, T_2^2 = T_m \tau_m^a + \frac{0.7 T_m^2}{T_m - 0.7 \tau_m^a} + 0.5 \tau_m^2, \frac{\tau_m^a}{T_m} > 0.104.$$

$$^{29} T_i^{(317)} = \frac{T_m}{0.9895 + 0.09539 \frac{\tau_m}{T_m}}, T_d^{(317)} = 0.50814 \left(\frac{\tau_m}{T_m} \right)^{1.08433}.$$

$$^{30} K_c^{(318)} = \frac{0.98089}{K_m} \left(\frac{T_m}{\tau_m} \right)^{0.76167}, T_i^{(318)} = 1.09851 T_m \left(\frac{\tau_m}{T_m} \right)^{1.05211},$$

$$T_d^{(318)} = 0.59974 T_m \left(\frac{\tau_m}{T_m} \right)^{0.89819}.$$

Rule	K_c	T_i	T_d	Comment
Wang <i>et al.</i> (2002) – continued.	$^{31} K_c^{(319)}$	$T_i^{(319)}$	$T_d^{(319)}$	Labeled ISE setpoint; $N = 10$
	$^{32} K_c^{(320)}$	$T_i^{(320)}$	$T_d^{(320)}$	Labeled ISE load; $N = 10$
	$^{33} K_c^{(321)}$	$T_i^{(321)}$	$T_d^{(321)}$	Labeled ITAE setpoint; $N = 10$
	$^{34} K_c^{(322)}$	$T_i^{(322)}$	$T_d^{(322)}$	Labeled ITAE load; $N = 10$
Huang <i>et al.</i> (2005). Model: Method 42	$\frac{0.65T_m}{K_m \tau_m}$	T_m	$0.4\tau_m$	$N=20$ $A_m = 2.7$, $\phi_m = 65^\circ$
	The above is a representative, default, tuning rule; other such rules may be deduced from a graph provided by the authors for other A_m, ϕ_m values.			

$$^{31} K_c^{(319)} = \frac{0.71959}{K_m} \left(\frac{T_m}{\tau_m} \right)^{1.03092}, \quad T_i^{(319)} = \frac{T_m}{1.12666 - 0.18145 \frac{\tau_m}{T_m}}, \\ T_d^{(319)} = 0.54568 \left(\frac{\tau_m}{T_m} \right)^{0.86411}.$$

$$^{32} K_c^{(320)} = \frac{1.11907}{K_m} \left(\frac{T_m}{\tau_m} \right)^{0.89711}, \quad T_i^{(320)} = 1.2520 T_m \left(\frac{\tau_m}{T_m} \right)^{0.95480}, \\ T_d^{(320)} = 0.54766 T_m \left(\frac{\tau_m}{T_m} \right)^{0.87798}.$$

$$^{33} K_c^{(321)} = \frac{1.12762}{K_m} \left(\frac{T_m}{\tau_m} \right)^{0.80368}, \quad T_i^{(321)} = \frac{T_m}{0.9978 + 0.02860 \frac{\tau_m}{T_m}}, \\ T_d^{(321)} = 0.42844 \left(\frac{\tau_m}{T_m} \right)^{1.0081}.$$

$$^{34} K_c^{(322)} = \frac{0.77902}{K_m} \left(\frac{T_m}{\tau_m} \right)^{1.06401}, \quad T_i^{(322)} = 0.87481 T_m \left(\frac{\tau_m}{T_m} \right)^{0.70949}, \\ T_d^{(322)} = 0.57137 T_m \left(\frac{\tau_m}{T_m} \right)^{1.03826}.$$

Rule	K_c	T_i	T_d	Comment
Robust				
Harris and Tyreus (1987). Model: Method 1	$\frac{T_m}{K_m(\tau_m + \lambda)}$	T_m	$0.5\tau_m$	$N = (\tau_m + \lambda)/\lambda$; $\lambda \geq \max \text{imum}$ $[0.45\tau_m, 0.1T_m]$
Chien (1988). Model: Method 1	$\frac{T_m}{K_m(\lambda + 0.5\tau_m)}$	T_m	$0.5\tau_m$	$\lambda \in [\tau_m, T_m]$ (Chien and Fruehauf (1990)); $N=10$
	$\frac{0.5\tau_m}{K_m(\lambda + 0.5\tau_m)}$	$0.5\tau_m$	T_m	
Zhang et al. (1996). Model: Method 38	³⁵ $K_c^{(323)}$	$0.5\tau_m$	T_m	$\lambda \in [0.2\tau_m, 1.2\tau_m]$
	³⁶ $K_c^{(324)}$	T_m	$0.5\tau_m$	
Zhang et al. (2002).	³⁷ $K_c^{(325)}$	T_m	$0.5\tau_m$	Model: Method 1
Shi and Lee (2002). Model: Method 1	³⁸ $K_c^{(326)}$	T_m	$\frac{1}{\omega_g} \tan\left(\frac{\omega_g \tau_m}{2}\right)$	Suggested $\omega_g = \frac{0.6}{\tau_m}$
Lee and Shi (2002). Model: Method 1	³⁹ $K_c^{(327)}$	T_m	$0.5\tau_m$	Suggested $\omega_g = \frac{0.6}{\tau_m}$
		⁴⁰ $T_i^{(328)}$		

$$^{35} K_c^{(323)} = \frac{0.5\tau_m}{K_m(0.5\tau_m + 2\lambda)}, \quad N = \frac{T_m(2\lambda + 0.5\tau_m)}{\lambda^2}.$$

$$^{36} K_c^{(324)} = \frac{T_m}{K_m(0.5\tau_m + 2\lambda)}, \quad N = \frac{0.5\tau_m(2\lambda + 0.5\tau_m)}{\lambda^2}.$$

$$^{37} K_c^{(325)} = \frac{T_m}{K_m(2\lambda + 0.5\tau_m)}, \quad N = 10 \text{ or } N = \frac{\lambda^2}{0.5\tau_m(2\lambda + 0.5\tau_m)}.$$

$$^{38} K_c^{(326)} = \frac{\omega_{bw} T_m}{K_m \left[1 + \frac{2\omega_{bw}}{\omega_g} \tan\left(\frac{\omega_g \tau_m}{2}\right) \right]}, \quad N = 1 + \frac{2\omega_{bw}}{\omega_g} \tan\left(\frac{\omega_g \tau_m}{2}\right).$$

$$^{39} K_c^{(327)} = \frac{\omega_{bw} T_m}{K_m \left[1 + \frac{2\omega_{bw}}{\omega_g} \tan\left(\frac{\omega_g \tau_m}{2}\right) \right]}, \quad N = 1 + \frac{2\omega_{bw}}{\omega_g} \tan\left(\frac{\omega_g \tau_m}{2}\right).$$

$$^{40} T_i^{(328)} = \frac{T_m}{\gamma}, \quad 1 < \gamma < 1 + \frac{T_m}{\tau_m} + \frac{1}{\omega_{bw} \tau_m}.$$

Rule	K_c	T_i	T_d	Comment
Ultimate cycle				
Huang <i>et al.</i> (2005). <i>Model:</i> Method 42	$^{41} K_c^{(329)}$	$T_i^{(329)}$	$T_d^{(329)}$	$N=20$ $A_m = 2.7$, $\phi_m = 65^0$
The above is a representative, default, tuning rule; other such rules may be deduced from a graph provided by the authors for other A_m, ϕ_m values.				

$$^{41} K_c^{(329)} = \frac{0.65}{K_m} \left[\frac{\sqrt{\left(\hat{K}_u\right)^2 - 1}}{\pi - \tan^{-1} \left[\sqrt{\left(\hat{K}_u\right)^2 - 1} \right]} \right], \quad T_i^{(329)} = 0.159 \hat{T}_u \sqrt{\left(\hat{K}_u\right)^2 - 1},$$

$$T_d^{(329)} = 0.064 \hat{T}_u \left[\pi - \tan^{-1} \sqrt{\left(\hat{K}_u\right)^2 - 1} \right].$$

4.1.9 Classical controller 2 $G_c(s) = K_c \left(1 + \frac{1}{T_i s} \right) \left(\frac{1 + NT_d s}{1 + T_d s} \right)$

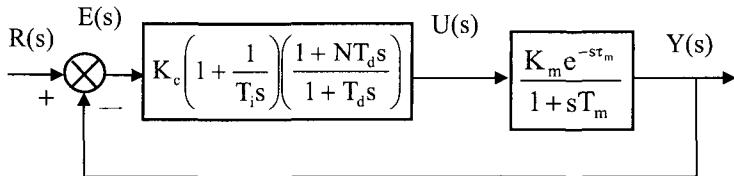


Table 52: PID controller tuning rules – FOLPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{1 + sT_m}$

Rule	K_c	T_i	T_d	Comment
Direct synthesis: frequency domain criteria				
Hougen (1979) – page 335. <i>Model: Method 1</i>	¹ $K_c^{(330)}$	T_m	$0.45 \frac{\tau_m}{N}$	Maximise crossover frequency; $N \in [10,30]$

¹ Values deduced from graph.

$$K_c^{(330)} = 9.0/K_m, \tau_m/T_m = 0.1; K_c^{(330)} = 4.2/K_m, \tau_m/T_m = 0.2;$$

$$K_c^{(330)} = 2.8/K_m, \tau_m/T_m = 0.3; K_c^{(330)} = 2.1/K_m, \tau_m/T_m = 0.4;$$

$$K_c^{(330)} = 1.7/K_m, \tau_m/T_m = 0.5; K_c^{(330)} = 1.45/K_m, \tau_m/T_m = 0.6;$$

$$K_c^{(330)} = 1.2/K_m, \tau_m/T_m = 0.7; K_c^{(330)} = 1.1/K_m, \tau_m/T_m = 0.8;$$

$$K_c^{(330)} = 0.95/K_m, \tau_m/T_m = 0.9; K_c^{(330)} = 0.85/K_m, \tau_m/T_m = 1.0.$$

4.1.10 Series controller (classical controller 3)

$$G_c(s) = K_c \left(1 + \frac{1}{T_i s} \right) (1 + s T_d)$$

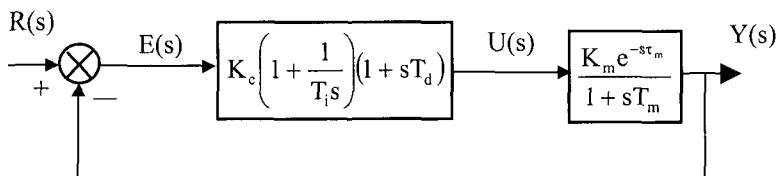


Table 53: PID controller tuning rules – FOLPD model $G_m(s) = \frac{K_m e^{-st_m}}{1 + sT_m}$

Rule	K_c	T_i	T_d	Comment
Process reaction				
Kraus (1986). <i>Model:</i> <i>Method 19</i>	$\frac{0.833T_m}{K_m \tau_m}$	$1.5\tau_m$	$0.25\tau_m$	Foxboro EXACT controller pre-tuning
Tan <i>et al.</i> (1999a) – page 25.	$\frac{0.6T_m}{K_m \tau_m}$	τ_m	τ_m	<i>Model: Method 2</i>
O'Dwyer (2001b).	$\frac{x_1 T_m}{K_m \tau_m}$	$x_2 \tau_m$	$x_3 \tau_m$	<i>Model: Method 2</i>
Representative results				
<i>Chien et al.</i> (1952) equivalent – regulator	x_1	x_2	x_3	.
	0.7236	1.8353	0.5447	0% overshoot; $0.1 < \tau_m/T_m < 1$
	0.84	1.4	0.6	20% overshoot; $0.1 < \tau_m/T_m < 1$
O'Dwyer (2001b). <i>Model: Method 2</i>	Representative results - Chien <i>et al.</i> (1952) equivalent – servo			
	$^2 K_c^{(331)}$	$T_i^{(331)}$	$T_d^{(331)}$	0% overshoot; $\tau_m/T_m < 0.5$

$$^2 K_c^{(331)} = \frac{0.3T_m}{K_m \tau_m} \left[1 - \sqrt{1 - \frac{2\tau_m}{T_m}} \right], \quad T_i^{(331)} = \frac{T_m}{2} \left[1 + \sqrt{1 - \frac{2\tau_m}{T_m}} \right],$$

$$T_d^{(331)} = \frac{T_m}{2} \left[1 - \sqrt{1 - \frac{2\tau_m}{T_m}} \right]$$

Rule	K_c	T_i	T_d	Comment
O'Dwyer (2001b) – continued.	${}^3 K_c^{(332)}$	$T_i^{(332)}$	$T_d^{(332)}$	20% overshoot; $\tau_m/T_m < 0.7234$
Direct synthesis				
Tsang <i>et al.</i> (1993). <i>Model:</i> <i>Method 15</i>	$\frac{x_1 T_m}{K_m \tau_m}$		T_m	
	x_1	ξ	x_1	ξ
	1.8194	0.0	1.0894	0.3
	1.5039	0.1	0.9492	0.4
	1.2690	0.2	0.8378	0.5
Ultimate cycle				
Pessen (1994). <i>Model: Method 1</i>	$0.35 K_u$		$0.25 T_u$	$0.1 \leq \frac{\tau_m}{T_m} \leq 1$

$${}^3 K_c^{(332)} = \frac{0.475 T_m}{K_m \tau_m} \left[1 + \sqrt{1 - 1.3824 \frac{\tau_m}{T_m}} \right], \quad T_i^{(332)} = 0.68 T_m \left[1 + \sqrt{1 - 1.3824 \frac{\tau_m}{T_m}} \right],$$

$$T_d^{(332)} = 0.68 T_m \left[1 - \sqrt{1 - 1.3824 \frac{\tau_m}{T_m}} \right]$$

4.1.11 Classical controller 4 $G_c(s) = K_c \left(1 + \frac{1}{T_i s} \right) \left(1 + \frac{s T_d}{1 + \frac{s T_d}{N}} \right)$

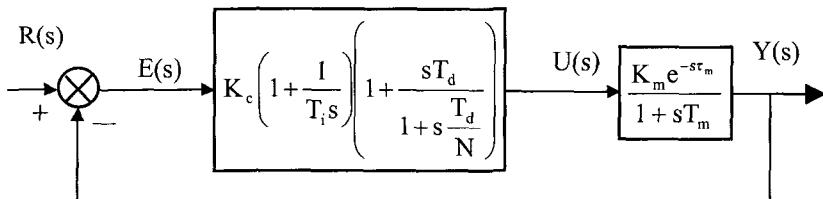


Table 54: PID controller tuning rules – FOLPD model $\frac{K_m e^{-s\tau_m}}{1 + sT_m}$

Rule	K_c	T_i	T_d	Comment
Process reaction				
Hang <i>et al.</i> (1993b) – page 76. <i>Model: Method 2</i>	$\frac{0.83T_m}{K_m \tau_m}$	$1.5\tau_m$	$0.25\tau_m$	Foxboro EXACT controller ‘pretune’; N not specified
Robust				
Chien (1988). <i>Model: Method 1</i>	$\frac{T_m}{K_m (\lambda + 0.5\tau_m)}$	T_m	$0.5\tau_m$	$\lambda \in [\tau_m, T_m]$ (Chien and Fruehauf (1990)); N=10
	$\frac{0.5\tau_m}{K_m (\lambda + 0.5\tau_m)}$	$0.5\tau_m$	T_m	

4.1.12 Non-interacting controller 1

$$U(s) = K_c \left(1 + \frac{1}{T_i s} \right) \begin{pmatrix} E(s) - \frac{T_d s}{1 + \frac{T_d s}{N}} Y(s) \\ 1 + \frac{T_d s}{N} \end{pmatrix}$$

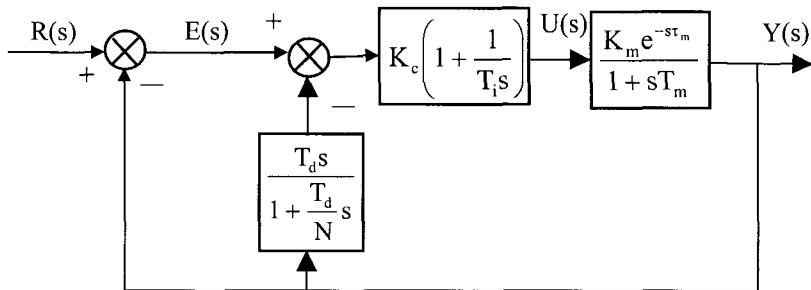


Table 55: PID controller tuning rules – FOLPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{1 + sT_m}$

Rule	K_c	T_i	T_d	Comment
Direct synthesis				
Huang <i>et al.</i> (2000). <i>Model:</i> <i>Method 1</i>	$\frac{0.8(T_m + 0.4\tau_m)}{K_m \tau_m}$	$T_m + 0.4\tau_m$	$\frac{0.4T_m \tau_m}{T_m + 0.4\tau_m}$	$N = \min[20, 20T_d]$; regulator tuning
	$\frac{0.6(T_m + 0.4\tau_m)}{K_m \tau_m}$			$N = \min[20, 20T_d]$; servo tuning

4.1.13 Non-interacting controller 2a

$$U(s) = K_c \left(1 + \frac{1}{T_i s} \right) E(s) - \frac{T_d s}{1 + \frac{s T_d}{N}} Y(s)$$

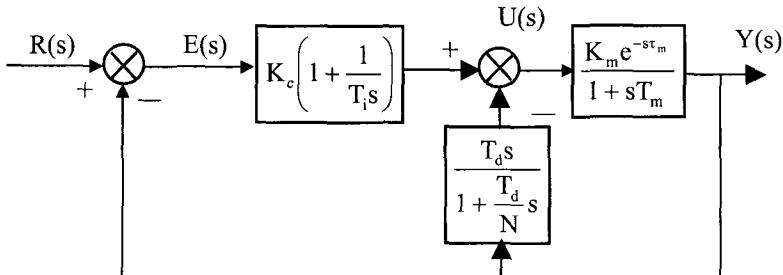


Table 56: PID controller tuning rules – FOLPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{1+sT_m}$

Rule	K_c	T_i	T_d	Comment
Minimum performance index: servo tuning				
Minimum ISE - Zhuang and Atherton (1993). <i>Model: Method 1</i>	$\frac{1.260}{K_m} \left(\frac{T_m}{\tau_m} \right)^{0.887}$	${}^4 T_i^{(333)}$	$T_d^{(333)}$	$0.1 \leq \frac{\tau_m}{T_m} \leq 1.0$
	$\frac{1.295}{K_m} \left(\frac{T_m}{\tau_m} \right)^{0.619}$	${}^5 T_i^{(334)}$	$T_d^{(334)}$	$1.1 \leq \frac{\tau_m}{T_m} \leq 2.0$
Minimum ISTSE - Zhuang and Atherton (1993). <i>Model: Method 1</i>	$\frac{1.053}{K_m} \left(\frac{T_m}{\tau_m} \right)^{0.930}$	${}^6 T_i^{(335)}$	$T_d^{(335)}$	$0.1 \leq \frac{\tau_m}{T_m} \leq 1.0$;

$${}^4 T_i^{(333)} = \frac{T_m}{0.701 - 0.147 \frac{\tau_m}{T_m}}, \quad T_d^{(333)} = 0.375 T_m \left(\frac{\tau_m}{T_m} \right)^{0.886}, \quad N=10.$$

$${}^5 T_i^{(334)} = \frac{T_m}{0.661 - 0.110 \frac{\tau_m}{T_m}}, \quad T_d^{(334)} = 0.378 T_m \left(\frac{\tau_m}{T_m} \right)^{0.756}, \quad N=10.$$

$${}^6 T_i^{(335)} = \frac{T_m}{0.736 - 0.126 \frac{\tau_m}{T_m}}, \quad T_d^{(335)} = 0.349 T_m \left(\frac{\tau_m}{T_m} \right)^{0.907}, \quad N=10.$$

Rule	K_c	T_i	T_d	Comment
Minimum ISTSE – Zhuang and Atherton (1993) (continued)	$\frac{1.120}{K_m} \left(\frac{T_m}{\tau_m} \right)^{0.625}$	${}^7 T_i^{(336)}$	$T_d^{(336)}$	$1.1 \leq \frac{\tau_m}{T_m} \leq 2.0 ; N=10$
Minimum ISTES – Zhuang and Atherton (1993). <i>Model: Method 1</i>	$\frac{0.942}{K_m} \left(\frac{T_m}{\tau_m} \right)^{0.933}$	${}^8 T_i^{(337)}$	$T_d^{(337)}$	$0.1 \leq \frac{\tau_m}{T_m} \leq 1.0$
	$\frac{1.001}{K_m} \left(\frac{T_m}{\tau_m} \right)^{0.624}$	${}^9 T_i^{(338)}$	$T_d^{(338)}$	$1.1 \leq \frac{\tau_m}{T_m} \leq 2.0$
Minimum ISTSE – Zhuang and Atherton (1993).	${}^{10} K_c^{(339)}$	$T_i^{(339)}$	$0.112 T_u$	$0.1 \leq \frac{\tau_m}{T_m} \leq 2.0 ;$ <i>Model: Method 1</i>
	${}^{11} K_c^{(340)}$	$0.271 \hat{K}_u \hat{T}_u K_m$	$0.116 \hat{T}_u$	$0.1 \leq \frac{\tau_m}{T_m} \leq 2.0 ;$ <i>Model:</i> <i>Method 31</i>

$${}^7 T_i^{(336)} = \frac{T_m}{0.720 - 0.114 \frac{\tau_m}{T_m}}, \quad T_d^{(336)} = 0.350 T_m \left(\frac{\tau_m}{T_m} \right)^{0.811}.$$

$${}^8 T_i^{(337)} = \frac{T_m}{0.770 - 0.130 \frac{\tau_m}{T_m}}, \quad T_d^{(337)} = 0.308 T_m \left(\frac{\tau_m}{T_m} \right)^{0.897}, \quad N=10.$$

$${}^9 T_i^{(338)} = \frac{T_m}{0.754 - 0.116 \frac{\tau_m}{T_m}}, \quad T_d^{(338)} = 0.308 T_m \left(\frac{\tau_m}{T_m} \right)^{0.813}, \quad N=10.$$

$${}^{10} K_c^{(339)} = \frac{4.437 K_m K_u - 1.587}{8.024 K_m K_u - 1.435} K_u, \quad T_i^{(339)} = 0.037 (5.89 K_m K_u + 1) T_u, \quad N=10.$$

$${}^{11} K_c^{(340)} = \frac{2.354 K_m K_u - 0.696 \hat{K}_u}{3.363 K_m \hat{K}_u + 0.517} \hat{K}_u, \quad N=10.$$

4.1.14 Non-interacting controller 2b

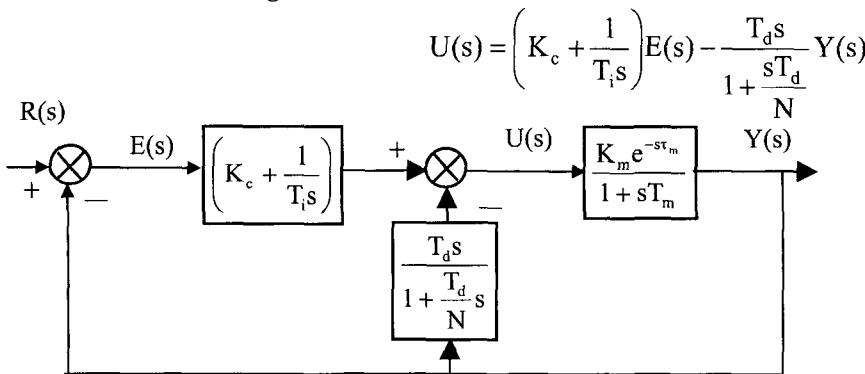


Table 57: PID controller tuning rules – FOLPD model $\frac{K_m e^{-st_m}}{1 + sT_m}$

Rule	K_c	T_i	T_d	Comment
Minimum performance index: regulator tuning				
Minimum IAE - Kaya and Scheib (1988).	$^{12} K_c^{(341)}$	$T_i^{(341)}$	$T_d^{(341)}$	<i>Model: Method 5</i>
Minimum ISE - Kaya and Scheib (1988).	$^{13} K_c^{(342)}$	$T_i^{(342)}$	$T_d^{(342)}$	

$$\begin{aligned}
 ^{12} K_c^{(341)} &= \frac{1.31509}{K_m} \left(\frac{T_m}{\tau_m} \right)^{0.8826}, \quad T_i^{(341)} = \frac{T_m}{1.2587} \left(\frac{\tau_m}{T_m} \right)^{1.3756}, \\
 T_d^{(341)} &= 0.5655 T_m \left(\frac{\tau_m}{T_m} \right)^{0.4576}, \quad 0 < \frac{\tau_m}{T_m} \leq 1; N=10. \\
 ^{13} K_c^{(342)} &= \frac{1.34466}{K_m} \left(\frac{T_m}{\tau_m} \right)^{0.9308}, \quad T_i^{(342)} = \frac{T_m}{1.6585} \left(\frac{\tau_m}{T_m} \right)^{1.25738}, \\
 T_d^{(342)} &= 0.79715 T_m \left(\frac{\tau_m}{T_m} \right)^{0.41941}, \quad 0 < \frac{\tau_m}{T_m} \leq 1; N=10.
 \end{aligned}$$

Rule	K_c	T_i	T_d	Comment
Minimum ITAE - Kaya and Scheib (1988).	¹⁴ $K_c^{(343)}$	$T_i^{(343)}$	$T_d^{(343)}$	<i>Model: Method 5</i>
Minimum performance index: servo tuning				
Minimum IAE - Kaya and Scheib (1988).	¹⁵ $K_c^{(344)}$	$T_i^{(344)}$	$T_d^{(344)}$	
Minimum ISE - Kaya and Scheib (1988).	¹⁶ $K_c^{(345)}$	$T_i^{(345)}$	$T_d^{(345)}$	<i>Model: Method 5</i>
Minimum ITAE - Kaya and Scheib (1988).	¹⁷ $K_c^{(346)}$	$T_i^{(346)}$	$T_d^{(346)}$	

$$^{14} K_c^{(343)} = \frac{1.3176}{K_m} \left(\frac{T_m}{\tau_m} \right)^{0.7937}, \quad T_i^{(343)} = \frac{T_m}{1.12499} \left(\frac{\tau_m}{T_m} \right)^{1.42603},$$

$$T_d^{(343)} = 0.49547 T_m \left(\frac{\tau_m}{T_m} \right)^{0.41932}, \quad 0 < \frac{\tau_m}{T_m} \leq 1; N=10.$$

$$^{15} K_c^{(344)} = \frac{1.13031}{K_m} \left(\frac{T_m}{\tau_m} \right)^{0.81314}, \quad T_i^{(344)} = \frac{T_m}{5.7527 - 5.7241 \frac{\tau_m}{T_m}},$$

$$T_d^{(344)} = 0.32175 T_m \left(\frac{\tau_m}{T_m} \right)^{0.17707}, \quad 0 < \frac{\tau_m}{T_m} \leq 1; N=10.$$

$$^{16} K_c^{(345)} = \frac{1.26239}{K_m} \left(\frac{T_m}{\tau_m} \right)^{0.8388}, \quad T_i^{(345)} = \frac{T_m}{6.0356 - 6.0191 \frac{\tau_m}{T_m}},$$

$$T_d^{(345)} = 0.47617 T_m \left(\frac{\tau_m}{T_m} \right)^{0.24572}, \quad 0 < \frac{\tau_m}{T_m} \leq 1; N=10.$$

$$^{17} K_c^{(346)} = \frac{0.98384}{K_m} \left(\frac{T_m}{\tau_m} \right)^{0.49851}, \quad T_i^{(346)} = \frac{T_m}{2.71348 - 2.29778 \frac{\tau_m}{T_m}},$$

$$T_d^{(346)} = 0.21443 T_m \left(\frac{\tau_m}{T_m} \right)^{0.16768}, \quad 0 < \frac{\tau_m}{T_m} \leq 1; N=10.$$

4.1.15 Non-interacting controller based on the two degree of freedom structure 1

$$U(s) = K_c \left(1 + \frac{1}{T_i s} + \frac{T_d s}{1 + \frac{T_d}{N} s} \right) E(s) - K_c \left(\alpha + \frac{\beta T_d s}{1 + \frac{T_d}{N} s} \right) R(s)$$

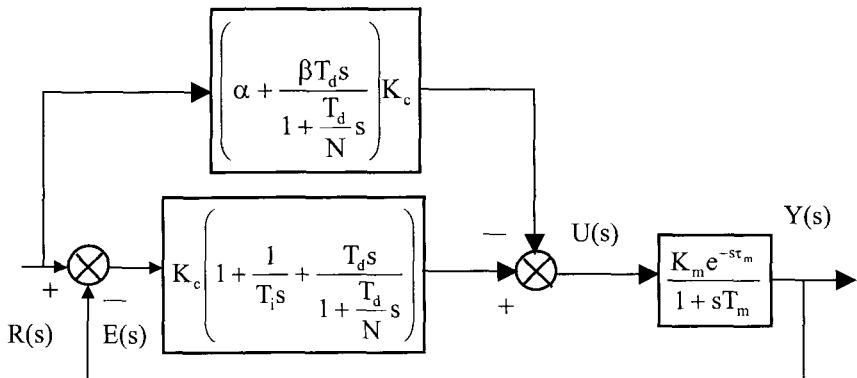


Table 58: PID controller tuning rules – FOLPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{1 + sT_m}$

Rule	K_c	T_i	T_d	Comment
Process reaction				
Hiroi and Terauchi (1986) – regulator. <i>Model:</i> <i>Method 2;</i> $\alpha = 0.6$; $\beta = 1$	$\frac{0.95T_m}{K_m \tau_m}$	$2.38\tau_m$	$0.42\tau_m$	0% overshoot; $0.1 < \frac{\tau_m}{T_m} < 1$
	$\frac{1.2T_m}{K_m \tau_m}$	$2\tau_m$	$0.42\tau_m$	20% overshoot; $0.1 < \frac{\tau_m}{T_m} < 1$

Rule	K_c	T_i	T_d	Comment
Minimum performance index: Servo/regulator tuning				
Taguchi and Araki (2000).	$^1 K_c^{(347)}$	$T_i^{(347)}$	$T_d^{(347)}$	Overshoot (servo step) $\leq 20\%$
Minimum performance index: other tuning				
Shen (2002). <i>Model:</i> <i>Method 5;</i> $N=0$, $\beta = 0$, $M_s \leq 2$		Minimum $\int_0^\infty r(t) - y(t) dt + \int_0^\infty d(t) - y(t) dt$		
	$^2 K_c^{(348)}$	$T_i^{(348)}$	$T_d^{(348)}$	$0 < \frac{\tau_m}{T_m} < \infty$

$$\begin{aligned}
 ^1 K_c^{(347)} &= \frac{1}{K_m} \left(0.1415 + \frac{1.224}{\frac{\tau_m}{T_m} - 0.001582} \right), \\
 T_i^{(347)} &= T_m \left(0.01353 + 2.200 \frac{\tau_m}{T_m} - 1.452 \left[\frac{\tau_m}{T_m} \right]^2 + 0.4824 \left[\frac{\tau_m}{T_m} \right]^3 \right), \\
 T_d^{(347)} &= T_m \left(0.0002783 + 0.4119 \frac{\tau_m}{T_m} - 0.04943 \left[\frac{\tau_m}{T_m} \right]^2 \right), \\
 \alpha &= 0.6656 - 0.2786 \frac{\tau_m}{T_m} + 0.03966 \left[\frac{\tau_m}{T_m} \right]^2, \quad N \text{ fixed (but not specified)}; \\
 \beta &= 0.6816 - 0.2054 \frac{\tau_m}{T_m} + 0.03936 \left[\frac{\tau_m}{T_m} \right]^2, \quad \frac{\tau_m}{T_m} \leq 1.0. \quad \text*Model: Method 1.*
 \end{aligned}$$

$$\begin{aligned}
 ^2 K_c^{(348)} &= \frac{T_m}{K_m \tau_m} \exp \left[2.94 - 11.63 \left(\frac{\tau_m}{\tau_m + T_m} \right) + 11.15 \left(\frac{\tau_m}{\tau_m + T_m} \right)^2 \right], \\
 T_i^{(348)} &= \tau_m \exp \left[1.88 - 3.63 \left(\frac{\tau_m}{\tau_m + T_m} \right) + 0.86 \left(\frac{\tau_m}{\tau_m + T_m} \right)^2 \right], \\
 T_d^{(348)} &= \tau_m \exp \left[-0.25 - 0.06 \left(\frac{\tau_m}{\tau_m + T_m} \right) - 1.99 \left(\frac{\tau_m}{\tau_m + T_m} \right)^2 \right], \\
 \alpha &= 1 - \exp \left[-0.22 - 0.90 \left(\frac{\tau_m}{\tau_m + T_m} \right) + 1.45 \left(\frac{\tau_m}{\tau_m + T_m} \right)^2 \right].
 \end{aligned}$$

Rule	K_c	T_i	T_d	Comment	
Direct synthesis					
Kasahara <i>et al.</i> (1997). <i>Model: Method 1</i>	${}^3 K_c^{(349)}$	$T_i^{(349)}$	$T_d^{(349)}$	Closed loop transfer function is 4 th order; $N = 0$	
Huang <i>et al.</i> (2000). <i>Model: Method 1</i>	$\frac{\lambda(T_m + 0.4\tau_m)}{K_m \tau_m}$	$T_m + 0.4\tau_m$	$\frac{0.4T_m \tau_m}{T_m + 0.4\tau_m}$	For $\lambda = 0.80$, $A_m = 2.13$	
Srinivas and Chidambaran (2001).	${}^4 K_c^{(350)}$	$T_i^{(350)}$	$T_d^{(350)}$	<i>Model: Method 1; N = ∞</i>	
Kim <i>et al.</i> (2004). <i>Model: Method 1</i>	x_1/K_m	$x_2 T_m$	$x_3 T_m$		
	Coefficient values				
	τ_m/T_m	x_1	x_2	x_3	α
	0.1	12.5	0.22	0.04	0.68
	0.2	6.1	0.41	0.08	0.63
	0.3	4.1	0.57	0.11	0.62
					β
					0.75
					0.70
					0.70

$$\begin{aligned}
 {}^3 K_c^{(349)} &= \frac{1}{K_m} \left[\frac{0.2\tau_m^3 + 1.2\tau_m^2 T_m + 5.4\tau_m T_m^2 + 9.6T_m^3}{\tau_m(\tau_m + 3T_m)^2} \right], \\
 T_i^{(349)} &= 1.667\tau_m \frac{\tau_m + 3T_m}{\tau_m + 2T_m} - 1.389\tau_m^2 \frac{(\tau_m + 3T_m)^3}{(\tau_m + 2T_m)^4}, \\
 T_d^{(349)} &= \frac{T_m^2 \tau_m (\tau_m + 3T_m)}{1.2(\tau_m + 2T_m)^3 - \tau_m(\tau_m + 3T_m)^2}, \\
 \alpha &= 1 - 0.723\tau_m \left(\frac{\tau_m + 3T_m}{\tau_m + 2T_m} \right) \left[1.667\tau_m \left(\frac{\tau_m + 3T_m}{\tau_m + 2T_m} \right) - 1.389 \frac{\tau_m^2 (\tau_m + 3T_m)^3}{(\tau_m + 2T_m)^4} \right]^{-1}, \\
 \beta &= 1 - 0.261\tau_m^2 \left(\frac{\tau_m + 3T_m}{\tau_m + 2T_m} \right)^2 \frac{1}{T_i^{(349)} T_d^{(349)}}.
 \end{aligned}$$

⁴ Sample $K_c^{(350)}$, $T_i^{(350)}$, $T_d^{(350)}$ taken by the authors correspond to the process reaction tuning rules of Zeigler and Nichols (1942); $\alpha = \frac{\tau_m}{T_i^{(350)}} - \frac{1}{K_c^{(350)} K_m} = 0.5 - 0.83 \frac{\tau_m}{T_m}$

and $\beta = \frac{1}{T_d^{(350)}} \left[\tau_m - \frac{T_m}{K_c^{(350)} K_m} - \frac{\tau_m^2}{2T_i^{(350)}} \right] = -0.16$ for this tuning rule.

Rule	K_c	T_i	T_d	Comment	
Coefficient values					
Kim <i>et al.</i> (2004) - continued	τ_m/T_m	x_1	x_2	x_3	α
	0.4	3.1	0.71	0.15	0.59
	0.5	2.5	0.83	0.18	0.58
	0.6	2.11	0.94	0.21	0.56
	0.7	1.82	1.05	0.24	0.54
	0.8	1.61	1.13	0.28	0.51
	0.9	1.44	1.22	0.31	0.48
	1.0	1.33	1.26	0.34	0.49
	OS < 20% (servo step); settling time \leq that of corresponding rule of Chien <i>et al.</i> (1952); a cost function is minimized.				
Robust					
Leva and Colombo (2004).	${}^5 K_c^{(351)}$	$T_i^{(351)}$	$T_d^{(351)}$	<i>Model: Method 41</i>	
Ultimate cycle					
Hang and Åström (1988a).	${}^6 0.6K_u$	$0.5T_u$	$0.125T_u$	<i>Model: Method 2</i>	
	${}^7 0.6K_u$	$T_i^{(352)}$	$0.125T_u$		

$${}^5 K_c^{(351)} = \frac{1}{K_m(\tau_m + \lambda)} \left[T_m + \frac{\tau_m^2}{2(\tau_m + \lambda)} \right], \quad T_i^{(351)} = T_m + \frac{\tau_m^2}{2(\tau_m + \lambda)},$$

$$T_d^{(351)} = \frac{\tau_m T_m (\tau_m + \lambda)}{2T_m(\tau_m + \lambda) + \tau_m^2} - \frac{\lambda \tau_m}{2(\tau_m + \lambda)}, \quad N = \frac{2T_m(\tau_m + \lambda)^2}{\lambda [2T_m(\tau_m + \lambda) + \tau_m^2]} - 1.$$

α not specified; $\beta = 1 \cdot \lambda \geq \frac{3\Delta\tau_m}{\pi}$, $\Delta\tau_m$ = maximum variation in the time delay, with only variation in the time delay considered (Leva and Colombo (2004)); λ chosen so that nominal cut-off frequency = ω_u^* (Leva, 2005).

$${}^6 \alpha = 1 - 2(x - 0.1) - \frac{1.66\tau_m}{T_m}, \quad \frac{\tau_m}{T_m} < 0.3; \quad \alpha = 1 - 2x - \frac{\tau_m}{T_m}, \quad 0.3 \leq \frac{\tau_m}{T_m} < 0.6;$$

x = % overshoot. $N=10$, $\beta = 1$.

$${}^7 T_i^{(352)} = 0.5 \left(1.5 - 0.83 \frac{\tau_m}{T_m} \right) T_u. \quad \alpha = \frac{\tau_m}{T_m} - 0.6, \quad 0.6 \leq \frac{\tau_m}{T_m} < 0.8;$$

$$\alpha = 0.2, \quad 0.8 \leq \frac{\tau_m}{T_m} < 1.0.$$

Rule	K_c	T_i	T_d	Comment
Hang and Åström (1988a) - continued	$0.6K_u$	$0.335T_u$	$0.125T_u$	$\frac{\tau_m}{T_m} > 1.0 ;$ $\alpha = 0.2$
Hang <i>et al.</i> (1991) - servo.	⁸ $0.6K_u$	$0.5T_u$	$0.125T_u$	<i>Model:</i> <i>Method 2;</i> $\beta = 1$
	⁹ $0.6K_u$	$0.222K^{'T_u}$	$0.125T_u$	
Hang and Cao (1996). <i>Model:</i> <i>Method 31</i>	$0.6K_u$	¹⁰ $T_i^{(353)}$	$T_d^{(353)}$	$0.1 \leq \frac{\tau_m}{T_m} < 0.5 ;$ $N=10 ; \beta = 1$

$$^8 0.16 \leq \frac{\tau_m}{T_m} < 0.57 ; N = 10. \alpha = 1 - \frac{15 - K^{'}}{15 + K^{'}} , 10\% \text{ overshoot};$$

$$\alpha = 1 - \frac{36}{27 + 5K^{'}} , 20\% \text{ overshoot with } K^{' } = 2 \left(\frac{11[\tau_m/T_m] + 13}{37[\tau_m/T_m] - 4} \right).$$

$$^9 0.57 \leq \frac{\tau_m}{T_m} < 0.96 ; N = 10. \alpha = 1 - \frac{8}{17} \left(\frac{4}{9} K^{' } + 1 \right) , 20\% \text{ overshoot, } 10\% \text{ undershoot.}$$

$$^{10} T_i^{(353)} = \left(0.53 - 0.22 \frac{\tau_m}{T_m} \right) T_u , \quad T_d^{(353)} = \left(0.53 - 0.22 \frac{\tau_m}{T_m} \right) \frac{T_u}{4} . \quad \alpha \text{ takes on values of}$$

1.0, 0.2 and $0.40(\tau_m/T_m)^2 - 0.05(\tau_m/T_m) + 0.58$ at different points on the open loop process step response.

4.1.16 Non-interacting controller based on the two degree of freedom structure 2

$$U(s) = K_c \left(1 + \frac{1}{T_i s} + \frac{T_d s}{1 + \frac{T_d}{N} s} \right) E(s) - K_c \left(\alpha + \frac{\beta T_d s}{1 + \frac{T_d}{N} s} - \frac{\chi}{1 + T_i s} \right) R(s)$$

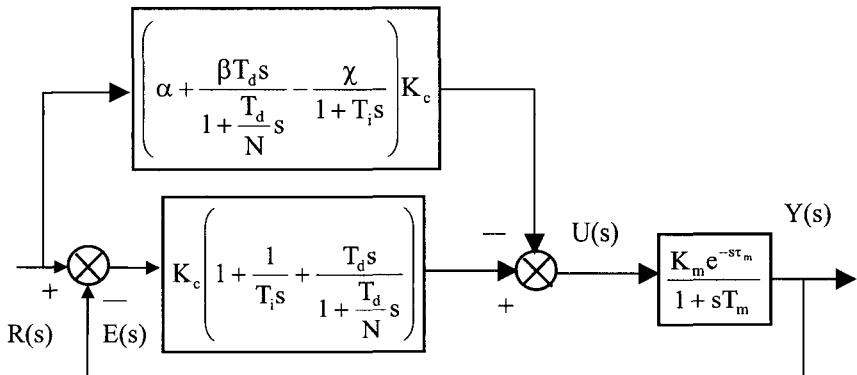


Table 59: PID tuning rules – FOLPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{1+sT_m}$

Rule	K_c	T_i	T_d	Comment
Process reaction				
Hiroi and Terauchi (1986) - regulator. <i>Model: Method 2</i>	$\frac{0.95T_m}{K_m \tau_m}$	$2.38\tau_m$	$0.42\tau_m$	0% overshoot; $0.1 < \frac{\tau_m}{T_m} < 1$
	$\frac{1.2T_m}{K_m \tau_m}$	$2\tau_m$	$0.42\tau_m$	20% overshoot; $0.1 < \frac{\tau_m}{T_m} < 1$
	$\alpha = 0.6 ; \beta = 1 \text{ or } \beta = -1.48 ; \chi = 0.15$			

4.1.17 Non-interacting controller based on the two degree of freedom structure 3

$$U(s) = K_c [(b-1) + (c-1)T_d s] R(s) + K_c \left(1 + \frac{1}{T_i s} + T_d s \right) \left[R(s) - \frac{1}{(1+sT_f)^2} Y(s) \right]$$

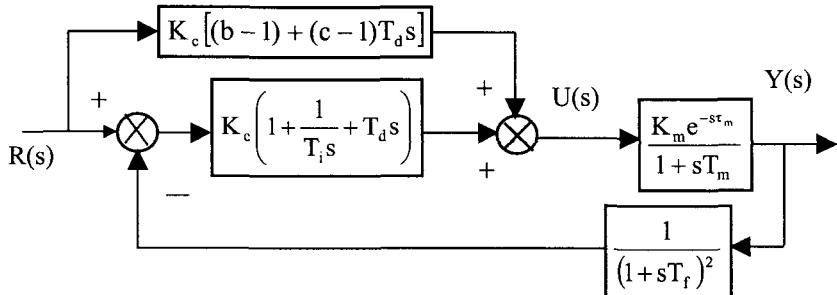


Table 60: PID controller tuning rules – FOLPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{1 + sT_m}$

Rule	K_c	T_i	T_d	Comment
Minimum performance index: other tuning				
Åström and Hägglund (2004).	¹¹ $K_c^{(354)}$	$T_i^{(354)}$	$\frac{0.5\tau_m T_m}{0.3\tau_m + T_m}$	Model: Method 1
No precisely defined performance specification				
Åström and Hägglund (2004).	¹² $K_c^{(355)}$	$T_i^{(355)}$	$T_d^{(355)}$	Model: Method 45
No precisely defined performance specification				

$$^{11} K_c^{(354)} = \frac{1}{K_m} \left[0.2 + 0.45 \frac{T_m}{\tau_m} \right], \quad T_i^{(354)} = \frac{0.4\tau_m + 0.8T_m}{\tau_m + 0.1T_m} \tau_m; \quad b = 0, \quad \frac{\tau_m}{T_m} \leq 1;$$

$$b = 1, \quad \frac{\tau_m}{T_m} > 1; \quad c = 0; \quad T_f = 0.1\tau_m.$$

$$^{12} K_c^{(355)} = \frac{1}{K_m} \left[0.2 + 0.45 \frac{T_m + 0.5T_f}{\tau_m + 0.5T_f} \right], \quad b = 1, \quad c = 0. \quad T_f = \frac{T_d^{(355)}}{N}, \quad N \in [4, 10];$$

$$T_i^{(355)} = \frac{0.4\tau_m + 0.8T_m + 0.6T_f}{\tau_m + 0.1T_m + 0.55T_f} (\tau_m + 0.5T_f),$$

$$T_d^{(355)} = \frac{0.5(\tau_m + 0.5T_f)(T_m + 0.5T_f)}{0.3\tau_m + T_m + 0.65T_f}; \quad \frac{\tau_m}{T_m} > 1.$$

4.1.18 Non-interacting controller 4

$$U(s) = K_c \left(1 + \frac{1}{T_i s} \right) E(s) - K_c T_d s Y(s)$$

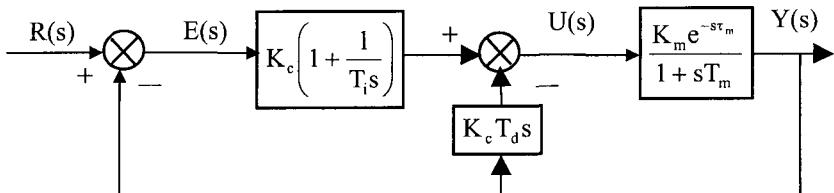


Table 61: PID controller tuning rules – FOLPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{1+sT_m}$

Rule	K_c	T_i	T_d	Comment
Process reaction				
VanDoren (1998).	$\frac{1.5T_m}{K_m \tau_m}$	$2.5\tau_m$	$0.4\tau_m$	<i>Model: Method 2</i>
ABB (2001). <i>Model: Method 2</i>	$\frac{1.25T_m}{K_m \tau_m}$	$2\tau_m$	$0.5\tau_m$	
Minimum performance index: regulator tuning				
Minimum IAE - Shinskey (1996) – page 117.	$1.32T_m/K_m \tau_m$	$1.80\tau_m$	$0.44\tau_m$	$\tau_m/T_m = 0.1$; <i>Model: Method 1</i>
Minimum IAE – Shinskey (1988) – page 143. <i>Model: Method 1</i>	$1.32T_m/K_m \tau_m$	$1.77\tau_m$	$0.41\tau_m$	$\tau_m/T_m = 0.2$
	$1.35T_m/K_m \tau_m$	$1.43\tau_m$	$0.41\tau_m$	$\tau_m/T_m = 0.5$
	$1.49T_m/K_m \tau_m$	$1.17\tau_m$	$0.37\tau_m$	$\tau_m/T_m = 1$
	$1.82T_m/K_m \tau_m$	$0.92\tau_m$	$0.32\tau_m$	$\tau_m/T_m = 2$
Minimum IAE – Edgar et al. (1997) – pages 8-14, 8-15.	$1.30T_m/K_m \tau_m$	$1.8\tau_m$	$0.45\tau_m$	<i>Model: Method 1</i>
	$1.33T_m/K_m \tau_m$	$2.1\tau_m$	$0.63\tau_m$	<i>Model: Method 2</i>
Minimum performance index: other tuning				
Minimum IAE – Shinskey (1988) – page 148. <i>Model: Method 1</i>	$0.77K_u$	$0.48T_u$	$0.11T_u$	$\tau_m/T_m = 0.2$
	$0.70K_u$	$0.42T_u$	$0.12T_u$	$\tau_m/T_m = 0.5$
	$0.66K_u$	$0.38T_u$	$0.12T_u$	$\tau_m/T_m = 1$
	$0.60K_u$	$0.34T_u$	$0.12T_u$	$\tau_m/T_m = 2$

Rule	K_c	T_i	T_d	Comment
Shinskey (1994) – page 167.	$^1 K_c^{(356)}$	$0.125 \frac{T_u^2}{\tau_m}$	$0.12T_u$	Minimum IAE; <i>Model: Method 1</i>
ABB (2001).	$^2 K_c^{(357)}$	$T_i^{(357)}$	$T_d^{(357)}$	<i>Model: Method 7</i>
Direct synthesis				
Haeri (2002)	$^3 K_c^{(358)}$	$T_i^{(358)}$	$T_d^{(357)}$	<i>Model: Method 1</i>

$$^1 K_c^{(356)} = \frac{K_u}{3.73 - 0.69 \frac{T_u}{\tau_m}}, \quad \frac{T_u}{\tau_m} < 2.7; \quad K_c^{(356)} = \frac{K_u}{2.62 - 0.35 \frac{T_u}{\tau_m}}, \quad \frac{T_u}{\tau_m} \geq 2.7.$$

$$^2 K_c^{(357)} = \frac{1.3570}{K_m} \left(\frac{T_m}{\tau_m} \right)^{0.947}, \quad T_i^{(357)} = 1.1760 T_m \left(\frac{\tau_m}{T_m} \right)^{0.738},$$

$$T_d^{(357)} = 0.3810 T_m \left(\frac{\tau_m}{T_m} \right)^{0.995}, \quad \frac{\tau_m}{T_m} \geq 0.5.$$

$$^3 K_c^{(358)} = \frac{1}{K_m} \left[\frac{6.84}{1 + 2.33 \left(\frac{\tau_m}{T_m} \right)^{0.7} + 7.82 \left(\frac{\tau_m}{T_m} \right)^{3.5}} + 0.64 \right],$$

$$T_i^{(358)} = \tau_m \left[0.95 + 2.58 \frac{\tau_m}{T_m} + 3.57 \left(\frac{\tau_m}{T_m} \right)^2 \right], \quad 0 < \frac{\tau_m}{T_m} \leq 0.29,$$

$$T_i^{(358)} = \tau_m \left[2.15 - 0.76 \frac{\tau_m}{T_m} + 0.33 \left(\frac{\tau_m}{T_m} \right)^2 \right], \quad 0.29 < \frac{\tau_m}{T_m} < 4;$$

$$T_d^{(358)} = \tau_m \left[0.29 \left(\frac{\tau_m}{T_m + \tau_m} \right) + 3.94 \left(\frac{\tau_m}{T_m + \tau_m} \right)^2 - 4.65 \left(\frac{\tau_m}{T_m + \tau_m} \right)^{2.8} \right], \quad 0 < \frac{\tau_m}{T_m} \leq 1.79;$$

$$T_d^{(358)} = \tau_m \left[0.87 \left(\frac{\tau_m}{T_m} \right) - 0.49 \left(\frac{\tau_m}{T_m} \right)^2 + 0.09 \left(\frac{\tau_m}{T_m} \right)^{2.8} \right], \quad 1.79 < \frac{\tau_m}{T_m} < 4.$$

4.1.19 Non-interacting controller 5

$$U(s) = K_c \left(b + \frac{1}{T_i s} \right) E(s) - (c + T_d s) Y(s)$$

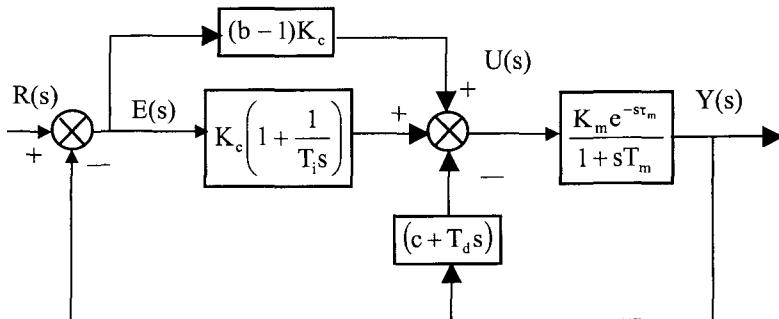


Table 62: PID controller tuning rules – FOLPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{1 + sT_m}$

Rule	K_c	T_i	T_d	Comment
Direct synthesis				
Chidambaran (2000a).	${}^4 K_c^{(359)}$	$T_i^{(359)}$	$T_d^{(359)}$	$c = K_c^{(359)}$
Model: Method I	${}^5 \frac{1.2\tau_m}{K_m T_m}$	$0.5\tau_m$	$\frac{0.15\tau_m^2}{K_m T_m}$	$c = \frac{1.2\tau_m}{K_m T_m}$

$${}^4 K_c^{(359)} = \frac{1}{K_m} \left[4.114 - 6.575 \frac{\tau_m}{T_m} + 3.342 \left(\frac{\tau_m}{T_m} \right)^2 \right],$$

$$T_i^{(359)} = T_m \left[0.311 + 1.372 \frac{\tau_m}{T_m} - 0.545 \left(\frac{\tau_m}{T_m} \right)^2 \right], \quad T_d^{(359)} = \frac{K_c^{(359)} T_i^{(359)}}{\left[16.016 - 11.7 \left(\frac{\tau_m}{T_m} \right) \right]},$$

$$b = 0.793 - 3.149 \frac{\tau_m}{T_m} + 8.405 \left(\frac{\tau_m}{T_m} \right)^2 - 8.431 \left(\frac{\tau_m}{T_m} \right)^3, \quad \frac{\tau_m}{T_m} \leq 0.5,$$

$$b = 0.0584 + 0.44 \frac{\tau_m}{T_m}, \quad 0.5 \leq \frac{\tau_m}{T_m} \leq 0.9.$$

$${}^5 b = 0.3238 + 0.4332 \frac{\tau_m}{T_m}, \quad \frac{\tau_m}{T_m} \leq 0.8; \quad b = 0.8507, \quad \frac{\tau_m}{T_m} \approx 0.9.$$

4.1.20 Non-interacting controller 6 (I-PD controller)

$$U(s) = \frac{K_c}{T_i s} E(s) - K_c (1 + T_d s) Y(s)$$

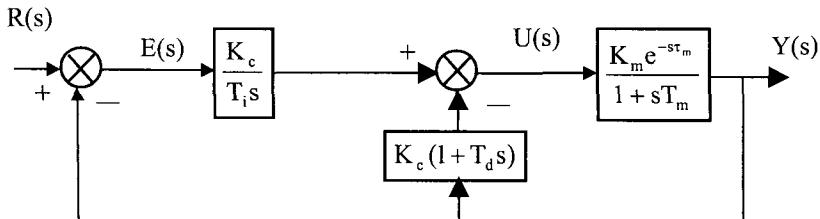


Table 63: PID controller tuning rules – FOLPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{1+sT_m}$

Rule	K_c	T_i	T_d	Comment
Direct synthesis: time domain criteria				
Suyama (1993).	${}^6 K_c^{(360)}$	$T_i^{(360)}$	$T_d^{(360)}$	<i>Model: Method 1</i>
Closed loop transfer function is 4 th order				
Shigemasa <i>et al.</i> (1987), Suyama (1993).	${}^7 K_c^{(361)}$	$T_i^{(361)}$	$T_d^{(361)}$	<i>Model: Method 1</i>
	Closed loop transfer function is 4 th order Butterworth			

$$\begin{aligned}
 {}^6 K_c^{(360)} &= \frac{1}{K_m} \left[\frac{0.2\tau_m^3 + 1.2\tau_m^2 T_m + 5.4\tau_m T_m^2 + 9.6T_m^3}{\tau_m(\tau_m + 3T_m)^2} \right], \\
 T_i^{(360)} &= 1.667\tau_m \frac{\tau_m + 3T_m}{\tau_m + 2T_m} - 1.389\tau_m^2 \frac{(\tau_m + 3T_m)^3}{(\tau_m + 2T_m)^4}, \\
 T_d^{(360)} &= \frac{T_m^2 \tau_m (\tau_m + 3T_m)}{1.2(\tau_m + 2T_m)^3 - \tau_m(\tau_m + 3T_m)^2}. \\
 {}^7 K_c^{(361)} &= \frac{1.3319(\tau_m + 2T_m)^3}{K_m \tau_m (\tau_m + 3T_m)^2} - \frac{1}{K_m}, \\
 T_i^{(361)} &= 2.2532\tau_m \frac{\tau_m + 3T_m}{\tau_m + 2T_m} - 1.692\tau_m^2 \frac{(\tau_m + 3T_m)^3}{(\tau_m + 2T_m)^4}, \\
 T_d^{(361)} &= \tau_m(\tau_m + 3T_m) \frac{1.5095(\tau_m + 2T_m)^2 - (\tau_m + T_m)(\tau_m + 3T_m)}{1.3319(\tau_m + 2T_m)^3 - \tau_m(\tau_m + 3T_m)^2}.
 \end{aligned}$$

Rule	K_c	T_i	T_d	Comment	
Shigemasa <i>et al.</i> (1987), Suyama (1993) (continued). <i>Model: Method 1</i>	${}^8 K_c^{(362)}$	$T_i^{(362)}$	$T_d^{(362)}$	Toshiba AdTune TOSDIC 211D8 product	
	Closed loop transfer function is 4 th order ITAE				
	${}^9 K_c^{(363)}$	$T_i^{(363)}$	$T_d^{(363)}$		
	Closed loop transfer function is 4 th order Bessel				
	${}^{10} K_c^{(364)}$	$T_i^{(364)}$	$T_d^{(364)}$		
Closed loop transfer function is 4 th order Binomial					

$${}^8 K_c^{(362)} = \frac{2.6242(\tau_m + 2T_m)^3}{K_m \tau_m (\tau_m + 3T_m)^2} - \frac{1}{K_m},$$

$$T_i^{(362)} = 1.8898\tau_m \frac{\tau_m + 3T_m}{\tau_m + 2T_m} - 0.720\tau_m^2 \frac{(\tau_m + 3T_m)^3}{(\tau_m + 2T_m)^4},$$

$$T_d^{(362)} = \tau_m (\tau_m + 3T_m) \frac{2.3130(\tau_m + 2T_m)^2 - (\tau_m + T_m)(\tau_m + 3T_m)}{2.6242(\tau_m + 2T_m)^3 - \tau_m (\tau_m + 3T_m)^2}.$$

$${}^9 K_c^{(363)} = \frac{0.9450(\tau_m + 2T_m)^3}{K_m \tau_m (\tau_m + 3T_m)^2} - \frac{1}{K_m},$$

$$T_i^{(363)} = 3.3333\tau_m \frac{\tau_m + 3T_m}{\tau_m + 2T_m} - 3.527\tau_m^2 \frac{(\tau_m + 3T_m)^3}{(\tau_m + 2T_m)^4},$$

$$T_d^{(363)} = \tau_m (\tau_m + 3T_m) \frac{1.3444(\tau_m + 2T_m)^2 - (\tau_m + T_m)(\tau_m + 3T_m)}{1.9450(\tau_m + 2T_m)^3 - \tau_m (\tau_m + 3T_m)^2}.$$

$${}^{10} K_c^{(364)} = \frac{0.5622(\tau_m + 2T_m)^3}{K_m \tau_m (\tau_m + 3T_m)^2} - \frac{1}{K_m},$$

$$T_i^{(364)} = 5.3337\tau_m \frac{\tau_m + 3T_m}{\tau_m + 2T_m} - 9.488\tau_m^2 \frac{(\tau_m + 3T_m)^3}{(\tau_m + 2T_m)^4},$$

$$T_d^{(364)} = \tau_m (\tau_m + 3T_m) \frac{1.1244(\tau_m + 2T_m)^2 - (\tau_m + T_m)(\tau_m + 3T_m)}{0.5622(\tau_m + 2T_m)^3 - \tau_m (\tau_m + 3T_m)^2}.$$

Rule	K_c	T_i	T_d	Comment
Chien <i>et al.</i> (1999).	$^{11} K_c^{(365)}$	$T_i^{(365)}$	$T_d^{(365)}$	<i>Model: Method 1</i>
Kasahara <i>et al.</i> (2001). <i>Model: Method 1</i>	$^{12} K_c^{(366)}$	$T_i^{(366)}$	$T_d^{(366)}$	OS = 0%
	$^{13} K_c^{(367)}$	$T_i^{(367)}$	$T_d^{(367)}$	OS = 10%
Ozawa <i>et al.</i> (2003).	$^{14} K_c^{(368)}$	$T_i^{(368)}$	$T_d^{(368)}$	<i>Model: Method 1</i> OS $\in [0\%, 10\%]$
Minimum performance index: servo tuning				
Minimum ISE – Argelaguet <i>et al.</i> (2000).	$\frac{2T_m + \tau_m}{2K_m \tau_m}$	$T_m + 0.5\tau_m$	$\frac{T_m \tau_m}{2T_m + \tau_m}$	<i>Model: Method 1</i>

¹¹ Underdamped system response - $\xi = 0.707$. $\tau_m > 0.2T_m$.

$$K_c^{(365)} = \frac{1.414T_{CL2}T_m + \tau_m T_m + 0.25\tau_m^2 - T_{CL2}^2}{K_m(T_{CL2}^2 + 0.707T_{CL2}\tau_m + 0.25\tau_m^2)},$$

$$T_i^{(365)} = \frac{1.414T_{CL2}T_m + \tau_m T_m + 0.25\tau_m^2 - T_{CL2}^2}{T_m + 0.5\tau_m},$$

$$T_d^{(365)} = \frac{0.707T_m T_{CL2}\tau_m + 0.25T_m \tau_m^2 - 0.5\tau_m T_{CL2}^2}{T_m \tau_m + 0.25\tau_m^2 + 1.414T_{CL2}T_m - T_{CL2}^2}.$$

$$^{12} K_c^{(366)} = \frac{0.281(\tau_m + 2T_m)^3}{K_m \tau_m (\tau_m + 3T_m)^2} - \frac{1}{K_m},$$

$$T_i^{(366)} = 5.33\tau_m \frac{\tau_m + 3T_m}{\tau_m + 2T_m} - 18.98\tau_m^2 \frac{(\tau_m + 3T_m)^3}{(\tau_m + 2T_m)^4},$$

$$T_d^{(366)} = \tau_m (\tau_m + 3T_m) \frac{0.562(\tau_m + 2T_m)^2 - (\tau_m + T_m)(\tau_m + 3T_m)}{0.281(\tau_m + 2T_m)^3 - \tau_m (\tau_m + 3T_m)^2}.$$

$$^{13} K_c^{(367)} = \frac{1.066T_m - 0.467\tau_m}{K_m \tau_m}, \quad T_i^{(367)} = \frac{4.695\tau_m (1.066T_m - 0.467\tau_m)}{\tau_m + 2T_m},$$

$$T_d^{(367)} = \tau_m \left(\frac{0.331T_m - 0.334\tau_m}{1.066T_m - 0.467\tau_m} \right).$$

$$^{14} K_c^{(368)} = \frac{1.6193(\tau_m + 2T_m)^3}{K_m \tau_m (\tau_m + 3T_m)^2} - \frac{1}{K_m},$$

$$T_i^{(368)} = 2.7051\tau_m \frac{\tau_m + 3T_m}{\tau_m + 2T_m} - 1.6706\tau_m^2 \frac{(\tau_m + 3T_m)^3}{(\tau_m + 2T_m)^4},$$

$$T_d^{(368)} = \tau_m (\tau_m + 3T_m) \frac{1.7793(\tau_m + 2T_m)^2 - (\tau_m + T_m)(\tau_m + 3T_m)}{1.6193(\tau_m + 2T_m)^3 - \tau_m (\tau_m + 3T_m)^2}.$$

4.1.21 Non-interacting controller 7

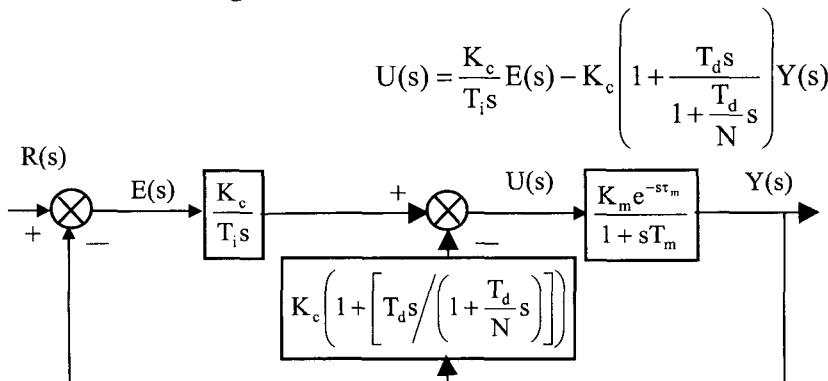


Table 64: PID controller tuning rules – FOLPD model $\frac{K_m e^{-st_m}}{1 + sT_m}$

Rule	K_c	T_i	T_d	Comment
Direct synthesis: time domain criteria				
Ogawa and Katayama (2001).	$^{15} K_c^{(369)}$	$T_i^{(369)}$	$T_d^{(369)}$	$N=10$
Minimum ISE - servo. Model: Method 1				

$$^{15} \text{Desired closed loop response} = \frac{e^{-s\tau_{CL}}}{(1+sT_{CL})^2} \cdot K_c^{(369)} = \frac{1}{K_m} \left(\frac{\frac{\tau_{CL}}{T_m} - 2 \frac{T_{CL}}{T_m} + 4}{\frac{\tau_{CL}}{T_m} + 2 \frac{T_{CL}}{T_m}} \right),$$

$$T_i^{(369)} = T_m \left(\frac{\left(\frac{\tau_{CL}}{T_m} + 2 \frac{T_{CL}}{T_m} \right) \left(\frac{\tau_{CL}}{T_m} - 2 \frac{T_{CL}}{T_m} + 4 \right)}{2 \frac{\tau_{CL}}{T_m} + 4} \right),$$

$$T_d^{(369)} = T_m \left(\frac{\frac{\tau_{CL}}{T_m} \left(\frac{\tau_{CL}}{T_m} + 4 \frac{T_{CL}}{T_m} - 2 \left(\frac{\tau_{CL}}{T_m} \right)^2 \right)}{\left(\frac{\tau_{CL}}{T_m} + 2 \frac{T_{CL}}{T_m} \right) \left(\frac{\tau_{CL}}{T_m} - 2 \frac{T_{CL}}{T_m} + 4 \right)} \right); \frac{T_{CL}}{T_m} \in [0.01, 1.00];$$

$$\frac{T_{CL}}{T_m} = -0.1902 \left(\frac{\tau_{CL}}{T_m} \right)^2 + 0.6974 \left(\frac{\tau_{CL}}{T_m} \right) + 0.007393, \frac{\tau_{CL}}{T_m} \in [0.05, 1.00].$$

4.1.22 Non-interacting controller 11

$$U(s) = K_c \left(1 + \frac{1}{T_i s} + T_d s \right) E(s) - K_i (1 + T_{di} s) Y(s)$$

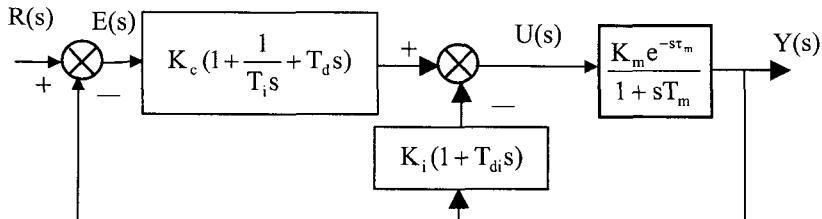


Table 65: PID controller tuning rules – FOLPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{1 + sT_m}$

Rule	K_c	T_i	T_d	Comment
Robust				
Lee and Edgar (2002). <i>Model: Method 1</i>	$^{16} K_c^{(370)}$	$T_i^{(370)}$	$T_d^{(370)}$	$K_i = 0.25K_u$, $T_{di} = 0$

$$\begin{aligned}
 ^{16} K_c^{(370)} &= \frac{1 + 0.25K_u K_m}{K_m(\lambda + \tau_m)} \left[\frac{T_m - 0.25K_u K_m \tau_m}{1 + 0.25K_u K_m} + \frac{\tau_m^2}{2(\lambda + \tau_m)} \right], \\
 T_i^{(370)} &= \frac{T_m - 0.25K_u K_m \tau_m}{1 + 0.25K_u K_m} + \frac{\tau_m^2}{2(\lambda + \tau_m)}, \\
 T_d^{(370)} &= \frac{1 + 0.25K_u K_m}{K_m(\lambda + \tau_m)} \left\{ \frac{0.25K_u K_m \tau_m^2}{2(1 + 0.25K_u K_m)} - \frac{\tau_m^3}{6(\lambda + \tau_m)} + \frac{\tau_m^4}{4(\lambda + \tau_m)^2} \right. \\
 &\quad \left. + \frac{\tau_m^2(T_m - 0.25K_u K_m \tau_m)}{2(1 + 0.25K_u K_m)(\lambda + \tau_m)} \right\}.
 \end{aligned}$$

4.1.23 Non-interacting controller 12

$$U(s) = K_c \left(1 + \frac{1}{T_i s} + T_d s \right) \frac{1}{T_d s + 1} E(s) - K_1 Y(s)$$

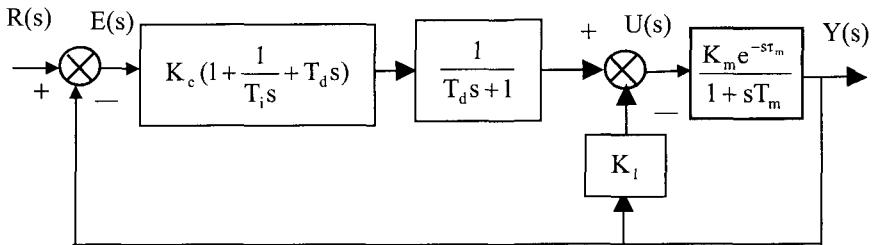


Table 66: PID tuning rules – FOLPD model $G_m(s) = \frac{K_m e^{-st_m}}{1+sT_m}$

Rule	K_c	T_i	T_d	Comment
Robust				
Lee and Edgar (2002). Model: Method 1	$^{17} K_c^{(371)}$	$T_i^{(371)}$	$T_d^{(371)}$	$K_1 = 0.25K_u$

$$\begin{aligned}
 ^{17} K_c^{(371)} &= \frac{1 + 0.25K_u K_m}{K_m(\lambda + \tau_m)} \left\{ \frac{T_m - 0.25K_u K_m \tau_m}{1 + 0.25K_u K_m} + \frac{\tau_m^2}{2(\lambda + \tau_m)} + \right. \\
 &\quad \left. \tau_m \left[\frac{0.25K_u K_m}{2(1 + 0.25K_u K_m)} - \frac{\tau_m}{6(\lambda + \tau_m)} + \frac{\tau_m^2}{4(\lambda + \tau_m)^2} + \frac{(T_m - 0.25K_u K_m \tau_m)\tau_m^2}{2(1 + 0.25K_u K_m)(\lambda + \tau_m)} \right]^{0.5} \right\}, \\
 T_i^{(371)} &= \frac{T_m - 0.25K_u K_m \tau_m}{1 + 0.25K_u K_m} + \frac{\tau_m^2}{2(\lambda + \tau_m)} + \\
 &\quad \tau_m \left[\frac{0.25K_u K_m}{2(1 + 0.25K_u K_m)} - \frac{\tau_m}{6(\lambda + \tau_m)} + \frac{\tau_m^2}{4(\lambda + \tau_m)^2} + \frac{(T_m - 0.25K_u K_m \tau_m)\tau_m^2}{2(1 + 0.25K_u K_m)(\lambda + \tau_m)} \right]^{0.5}, \\
 T_d^{(371)} &= \\
 &\quad \tau_m \left[\frac{0.25K_u K_m}{2(1 + 0.25K_u K_m)} - \frac{\tau_m}{6(\lambda + \tau_m)} + \frac{\tau_m^2}{4(\lambda + \tau_m)^2} + \frac{(T_m - 0.25K_u K_m \tau_m)\tau_m^2}{2(1 + 0.25K_u K_m)(\lambda + \tau_m)} \right]^{0.5}.
 \end{aligned}$$

4.1.24 Industrial controller $U(s) = K_c \left(1 + \frac{1}{T_i s} \right) \left(R(s) - \frac{1 + T_d s}{1 + \frac{T_d s}{N}} Y(s) \right)$

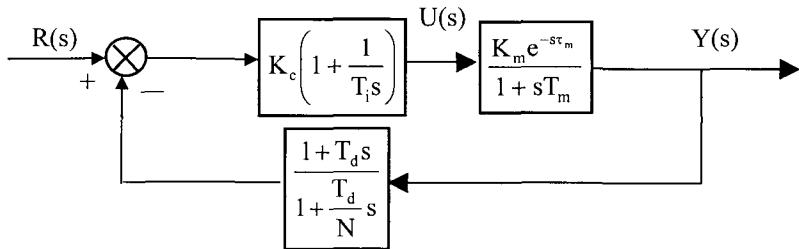


Table 67: PID controller tuning rules – FOLPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{1+sT_m}$

Rule	K_c	T_i	T_d	Comment
Minimum performance index: regulator tuning				
Kaya and Scheib (1988). Model: Method 5	$\frac{x_1}{K_m} \left(\frac{T_m}{\tau_m} \right)^{x_2}$	$\frac{T_m}{x_3} \left(\frac{\tau_m}{T_m} \right)^{x_4}$	$x_5 T_m \left(\frac{\tau_m}{T_m} \right)^{x_6}$	$0 < \frac{\tau_m}{T_m} \leq 1$; $N=10$
Coefficient values				
Minimum IAE	0.91	0.7938	1.01495	1.00403
Minimum ISE	1.1147	0.8992	0.9324	0.8753
Minimum ITAE	0.7058	0.8872	1.03326	0.99138
Minimum performance index: servo tuning				
Kaya and Scheib (1988). Model: Method 5	$\frac{x_1}{K_m} \left(\frac{T_m}{\tau_m} \right)^{x_2}$	$\frac{T_m}{x_3 - x_4 \frac{\tau_m}{T_m}}$	$x_5 T_m \left(\frac{\tau_m}{T_m} \right)^{x_6}$	$0 < \frac{\tau_m}{T_m} \leq 1$; $N=10$
Coefficient values				
Minimum IAE	0.81699	1.004	1.09112	0.22387
Minimum ISE	1.1427	0.9365	0.99223	0.35269
Minimum ITAE	0.8326	0.7607	1.00268	0.00854

Rule	K_c	T_i	T_d	Comment
Robust				
Harris and Tyreus (1987). Model: Method 1	$\frac{T_m}{K_m(0.5\tau_m + \lambda)}$	T_m	$0.5\tau_m$	$\lambda \geq \max \text{imum } [0.8\tau_m, T_m]; N$ not specified
Ho <i>et al.</i> (2001). Model: Method 1	$^{18} K_c^{(372)}$	T_m	$0.5\tau_m$	N not specified
	$\frac{1.551T_m}{K_m A_m \tau_m}$	T_m	$0.5\tau_m$	

$$^{18} K_c^{(372)} = \frac{T_m}{K_m(0.5\tau_m + T_{CL})}, T_{CL} = -0.5\tau_m + \sqrt{0.25\tau_m^2 + \omega_g^{-2}},$$

ω_g = frequency at which the Nyquist curve has a magnitude of unity.

4.2 Non-Model Specific

4.2.1 Ideal controller $G_c(s) = K_c \left(1 + \frac{1}{T_i s} + T_d s \right)$

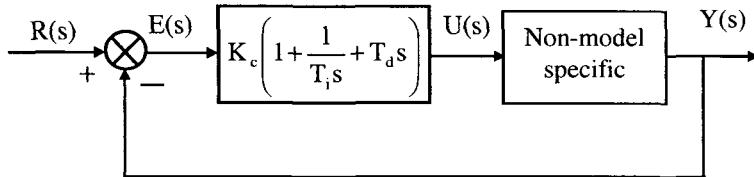


Table 68: PID controller tuning rules – non-model specific

Rule	K_c	T_i	T_d	Comment
Ultimate cycle				
Ziegler and Nichols (1942).	$[0.6K_u, K_u]$	$0.5T_u$	$0.125T_u$	Quarter decay ratio
Farrington (1950).	$[0.33K_u, 0.5K_u]$	T_u	$[0.1T_u, 0.25T_u]$	
McAvoy and Johnson (1967).	$0.54K_u$	T_u	$0.2T_u$	
Atkinson and Davey (1968).	$0.25K_u$	$0.75T_u$	$0.25T_u$	
20% overshoot – servo response				
Carr (1986); Pettit and Carr (1987).	K_u	$0.5T_u$	$0.125T_u$	Underdamped
	$0.6667K_u$	T_u	$0.167T_u$	Critically damped
	$0.5K_u$	$1.5T_u$	$0.167T_u$	Overdamped
Parr (1989) – pages 190, 191, 193.	$0.5K_u$	T_u	$0.2T_u$	$\xi \approx 0.45$
	$0.5K_u$	T_u	$0.25T_u$	
	$0.5K_u$	$0.34T_u$	$0.08T_u$	
Tinham (1989).	$0.4444K_u$	$0.6T_u$	$0.19T_u$	Less than quarter decay ratio response
Blickley (1990).	$0.5K_u$	T_u	$[0.125T_u, 0.167T_u]$	Quarter decay ratio

Rule	K_c	T_i	T_d	Comment
Corripio (1990) – page 27.	$0.75K_u$	$0.63T_u$	$0.1T_u$	Quarter decay ratio
Åström (1982).	$K_u \cos \phi_m$	$^1 T_i^{(373)}$	$\alpha T_i^{(373)}$	
		Representative results		
	$0.87K_u$	$0.55T_u$	$0.14T_u$	$\phi_m = 30^\circ$
	$0.71K_u$	$0.77T_u$	$0.19T_u$	$\phi_m = 45^\circ$
	$0.50K_u$	$1.19T_u$	$0.30T_u$	$\phi_m = 60^\circ$
Åström and Hägglund (1984).	$\frac{K_u}{A_m}$	arbitrary	$\frac{T_u}{4\pi^2 T_i}$	Specify A_m
	$K_u \cos \phi_m$	$\alpha T_d^{(374)}$	$^2 T_d^{(374)}$	Specify ϕ_m
Hang and Åström (1988b).	$K_u \sin \phi_m$	$\frac{T_u(1 - \cos \phi_m)}{\pi \sin \phi_m}$	$\frac{T_u(1 - \cos \phi_m)}{4\pi \sin \phi_m}$	
Åström and Hägglund (1988) – page 60.	$0.35K_u$	$0.77T_u$	$0.19T_u$	$A_m \geq 2$, $\phi_m \geq 45^\circ$
Åström and Hägglund (1995) – page 141-142.	$0.4698K_u$	$0.4546T_u$	$0.1136T_u$	$A_m = 2$, $\phi_m = 20^\circ$
	$0.1988K_u$	$1.2308T_u$	$0.3077T_u$	$A_m = 2.44$, $\phi_m = 61^\circ$
	$0.2015K_u$	$0.7878T_u$	$0.1970T_u$	$A_m = 3.45$, $\phi_m = 46^\circ$
De Paor (1993).	$0.906K_u$	$0.5T_u$	$0.125T_u$	$\phi_m = 25^\circ$
	$0.866K_u$	$0.5T_u$	$0.125T_u$	$\phi_m = 30^\circ$
McMillan (1994) – page 90.	$0.5K_u$	$0.5T_u$	$0.125T_u$	
Calcev and Gerez (1995).	$0.3536K_u$	$0.1592T_u$	$0.0398T_u$	
		$\phi_m = 45^\circ$, small τ_m ; $\phi_m = 15^\circ$, large τ_m		

$$^1 T_i^{(373)} = \left(\tan \phi_m + \sqrt{4\alpha + \tan^2 \phi_m} \right) \frac{T_u}{4\alpha\pi} . \quad \alpha = 0.25 \text{ (Åström, 1982);}$$

$\alpha = 0.5$ (Seki et al., 2000); $0.1 \leq \alpha \leq 0.3$ (Seki et al., 2000).

$$^2 T_d^{(374)} = \left(\tan \phi_m + \sqrt{1 + \tan^2 \phi_m} \right) \frac{T_u}{\pi} .$$

Rule	K_c	T_i	T_d	Comment
ABB Commander 300/310 (1996).	$0.625K_u$	$0.5T_u$	$0.083T_u$	'P+D' tuning rule
Luo <i>et al.</i> (1996).	$0.48K_u$	$0.5T_u$	$0.125T_u$	
Karaboga and Kalinli (1996).	$[0.32K_u, 0.6K_u]$	$[0.213T_u, 1.406T_u]$	$[0.133T_u, 0.469T_u]$	
Luyben and Luyben (1997) – page 97.	$0.46K_u$	$2.20T_u$	$0.16T_u$	Based on work by Tyreus and Luyben (1992)
Tan <i>et al.</i> (1999a) – page 26.	$0.4K_u$	$0.5T_u$	$0.125T_u$	
Tan <i>et al.</i> (1999a) – page 27.	$0.5K_u$	T_u	$0.125T_u$	Tighter damping than quarter decay ratio tuning
Wojsznis <i>et al.</i> (1999).	$0.4K_u$	$0.333T_u$	$0.083T_u$	
Yu (1999) – page 11.	$0.33K_u$	$0.5T_u$	$0.125T_u$	Some overshoot
	$0.2K_u$	$0.5T_u$	$0.125T_u$	No overshoot
Tan <i>et al.</i> (2001).	$K_c^{(375)}$	${}^3 T_i^{(375)}$	$0.25T_i^{(375)}$	
Chau (2002) – page 115.	$0.33K_u$	$0.5T_u$	$0.333T_u$	"Just a bit of" overshoot
	$0.2K_u$	$0.5T_u$	$0.333T_u$	No overshoot
Robbins (2002).	$0.45K_u$	${}^4 T_i^{(376)}$	$0.25T_u$	Minimum IAE – step change in setpoint
	$0.45K_u$	${}^4 T_i^{(377)}$	$0.25T_u$	"Good" response – step change in load
Smith (2003).	$0.75K_u$	$0.625T_u$	$0.1T_u$	
Other rules				
Rutherford (1950).	$1.25K_{37\%}$	$T_{37\%}$	$0.125T_{37\%}$	37% decay ratio
	$1.12K_{37\%}$	$T_{37\%}$	$0.25T_{37\%}$	

$${}^3 K_c^{(375)} = \frac{T_i^{(375)} \omega_u |G_c(j\omega_u)|}{0.25[T_i^{(375)}]^2 \omega_u^2 + 1}, \quad T_i^{(375)} = 0.3183T_u \tan \frac{0.5\pi + \angle G_c(j\omega_u)}{2},$$

$G_c(j\omega_u)$ = desired frequency response of controller $G_c(j\omega)$ at ω_u .

$${}^4 T_i^{(376)} = (0.24 + 0.111K_m K_u)T_u, \quad T_i^{(377)} = (0.27 + 0.054K_m K_u)T_u$$

Rule	K_c	T_i	T_d	Comment
MacLellan (1950).	1.77 $K_{37\%}$	1.29 $T_{37\%}$	0.16 $T_{37\%}$	37% decay ratio
	1.67 $K_{37\%}$	1.29 $T_{37\%}$	0.16 $T_{37\%}$	
	1.49 $K_{37\%}$	0.98 $T_{37\%}$	0.24 $T_{37\%}$	
Harriott (1964) – pages 179-180.	$K_{25\%}$	0.167 $T_{25\%}$	0.667 $T_{25\%}$	Quarter decay ratio
Lipták (1970) – pages 846-847.	$K_{25\%}$	0.667 $T_{25\%}$	0.167 $T_{25\%}$	Quarter decay ratio
Chesmond (1982) – page 400.	⁵ $K_c^{(378)}$	$T_i^{(378)}$	0.25 $T_i^{(378)}$	$x = 100$ (decay ratio)
	0.999 $K_{25\%}$	0.488 $T_{25\%}$	0.122 $T_{25\%}$	Example: $x = 25$ i.e. quarter decay ratio
Parr (1989) – page 191.	$K_{25\%}$	0.67 $T_{25\%}$	0.17 $T_{25\%}$	Quarter decay ratio
McMillan (1994) – page 43	0.83 $K_{25\%}$	0.5 $T_{25\%}$	0.1 $T_{25\%}$	'Fast' tuning
	0.67 $K_{25\%}$	0.5 $T_{25\%}$	0.1 $T_{25\%}$	'Slow' tuning
Wade (1994) – page 114.	[1.002 $K_{25\%}$, 1.67 $K_{25\%}$]	0.45 $T_{25\%}$	0.1125 $T_{25\%}$	
Hay (1998) – page 189.	$K_{25\%}$	0.667 $T_{25\%}$	0.167 $T_{25\%}$	
ECOSSE team (1996b).	0.8497 $K_{50\%}$	0.475 $T_{50\%}$	0.1188 $T_{50\%}$	
Bateson (2002) – page 616.	$K_{25\%}$	0.6667 T_u	0.1667 T_u	'modified' ultimate cycle
Rotach (1994).	⁶ $K_c^{(379)}$	$T_i^{(379)}$	$-0.5 \frac{d\phi_{\omega_{M_{max}}}}{d\omega_{M_{max}}}$	

$${}^5 K_c^{(378)} = \frac{K_x \%}{0.5 + 0.361 \ln x}, \quad T_i^{(378)} = \frac{T_x \%}{2\sqrt{1 + 0.025 \ln^2 x}}.$$

$${}^6 K_c^{(379)} = \frac{M_{max}}{\sqrt{M_{max}^2 - 1}} |G_p(j\omega_{M_{max}})|, \quad T_i^{(379)} = -\frac{2}{\omega_{M_{max}}^2 \left[\frac{d\phi_{\omega_{M_{max}}}}{d\omega_{M_{max}}} \right]}.$$

Rule	K_c	T_i	T_d	Comment
Rotach (1994) -- continued.	${}^7 K_c^{(380)}$	$T_i^{(380)}$	$T_d^{(380)}$	
García and Castelo (2000).	${}^8 K_c^{(381)}$	$T_i^{(381)}$	$0.25T_i^{(381)}$	$\omega_1 < \omega_u$
Lloyd (1994). Model: Method 3	$0.4 \hat{K}_u$	$0.3333 \hat{T}_u$	$0.083 \hat{T}_u$	Self-regulating processes
	$0.4 \hat{K}_u$	\hat{T}_u	$0.159 \hat{T}_u$	Non self-regulating processes
Jones <i>et al.</i> (1997).	$0.7490 \frac{h}{A_p}$	$a \hat{T}_u$,	$0.125 \hat{T}_u$	Model: Method 2
		$a \in [0.2526, 0.5053]$		

$${}^7 K_c^{(380)} = \frac{M_{\max}}{\sqrt{M_{\max}^2 - 1}} |G_p(j\omega_r)|,$$

$$T_i^{(380)} = - \frac{2}{\omega_r \left[\omega_r \frac{d\phi_{\omega_r}}{d\omega_r} \left\{ 1 + \tan^2 \left(\pi - \sin^{-1} \frac{1}{M_{\max}} + \phi_{\omega_r} \right) \right\} - \tan \left(\pi - \sin^{-1} \frac{1}{M_{\max}} + \phi_{\omega_r} \right) \right]},$$

$$T_d^{(380)} = - \frac{0.5}{\omega_r} \left[\omega_r \frac{d\phi_{\omega_r}}{d\omega_r} \left\{ 1 + \tan^2 \left(\pi - \sin^{-1} \frac{1}{M_{\max}} + \phi_{\omega_r} \right) \right\} + \tan \left(\pi - \sin^{-1} \frac{1}{M_{\max}} + \phi_{\omega_r} \right) \right].$$

$${}^8 K_c^{(381)} = \frac{\cos(180^\circ + \phi_m - \angle G_p(j\omega_1))}{|G_p(j\omega_1)|}, T_i^{(381)} = \frac{2[\sin(180^\circ + \phi_m - \angle G_p(j\omega_1)) + 1]}{\omega_1 \cos(180^\circ + \phi_m - \angle G_p(j\omega_1))}.$$

Rule	K_c	T_i	T_d	Comment
Shin <i>et al.</i> (1997).	${}^9 K_c^{(382)}$	$T_i^{(382)}$	$\alpha T_i^{(382)}$	<i>Model: Method 2</i>
NI Labview (2001). <i>Model: Method 2</i>	$0.6 \hat{K}_u$	$0.5 \hat{T}_u$	$0.12 \hat{T}_u$	Quarter decay ratio
	$0.25 \hat{K}_u$	$0.5 \hat{T}_u$	$0.12 \hat{T}_u$	'Some' overshoot
	$0.15 \hat{K}_u$	$0.5 \hat{T}_u$	$0.12 \hat{T}_u$	'Little' overshoot
Tang <i>et al.</i> (2002).	$\frac{2.5465h \sin \phi_m}{A_p x_1 \omega_{90^\circ} A_m}$	$\delta T_d^{(383)}$, $\delta \in [1.5, 4]$	${}^{10} T_d^{(383)}$	$0.81 \leq x_1 \leq 1$
<i>Model: Method 4</i>				
Dutton <i>et al.</i> (1997).	${}^{11} 0.5 K_u^e$	T_u^e	$0.25 T_u^e$	pages 576-8.

$${}^9 K_c^{(382)} = \frac{\rho}{x_1 \left(\alpha \omega_u T_i^{(382)} - \frac{1}{\omega_u T_i^{(382)}} \right) - x_2 \left(\alpha \hat{\omega}_u \hat{T}_i^{(382)} - \frac{1}{\hat{\omega}_u \hat{T}_i^{(382)}} \right) - y_2 - \rho x_2},$$

$$T_i^{(382)} = \frac{(x_2 - x_1) + \sqrt{(x_1 - x_2)^2 + 4(\alpha \rho x_1 \omega_u + \alpha y_2 \hat{\omega}_u) \left(\frac{\rho x_1}{\omega_u} + \frac{y_2}{\hat{\omega}_u} \right)}}{2 \left(\alpha \rho x_1 \omega_u + \alpha y_2 \hat{\omega}_u \right)},$$

$$x_1 = -\frac{1}{K_u}, \quad x_2 = \frac{1}{\hat{K}_u} \cos(\angle G_p(j\hat{\omega}_u)), \quad \rho = \frac{(\omega_u - \hat{\omega}_u)}{\omega_u} \frac{\sqrt{1 - \xi^2}}{\xi},$$

$$y_2 = \frac{1}{\hat{K}_u} \sin(\angle G_p(j\hat{\omega}_u)). \text{ Typical } \alpha : 0.1; \text{ typical } \xi : [0.3, 0.7].$$

$${}^{10} T_d^{(383)} = \frac{-\cot \phi_m + \sqrt{\cot^2 \phi_m + 4/\delta}}{2 \omega_{90^\circ}}.$$

$${}^{11} K_u^e = \frac{K_c^*}{\sqrt{1 - \frac{(8-r)^2}{55}}}, \quad T_u^e = \frac{6.283}{\omega_d} \left[1 - \frac{(8-r)^{3.5}}{1110} \right]^{0.286} \quad \text{with } r = \text{ratio of the height of the}$$

first peak to the height of the first trough of the closed loop response, when controller gain K_c^* is applied; ω_d = damped natural frequency.

4.2.2 Ideal controller in series with a first order lag

$$G_c(s) = K_c \left(1 + \frac{1}{T_i s} + T_d s \right) \frac{1}{T_f s + 1}$$

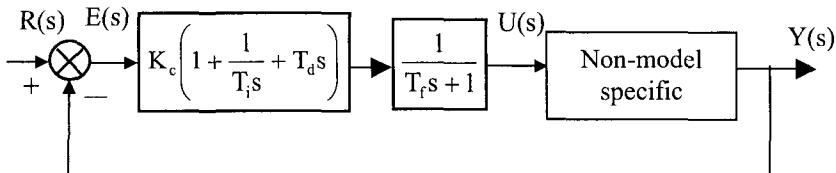


Table 69: PID controller tuning rules – non-model specific

Rule	K_c	T_i	T_d	Comment
Other tuning: minimum performance index				
Kristiansson (2003).	$^{12} K_c^{(384)}$	$T_i^{(384)}$	$T_d^{(384)}$	
“Almost” optimal tuning rule; $K_{150^0} \geq 0.03 ; 1.6 \leq M_s \leq 1.9$				

$$^{12} K_c^{(384)} = \frac{0.60}{K_m \left[\frac{K_{150^0}}{K_m} + 0.07 \right] \left[0.32 + 1.6 \frac{K_{150^0}}{K_m} - 0.8 \left(\frac{K_{150^0}}{K_m} \right)^2 \right]},$$

$$T_i^{(384)} = \frac{1.5}{\omega_{150^0} \left[0.32 + 1.6 \frac{K_{150^0}}{K_m} - 0.8 \left(\frac{K_{150^0}}{K_m} \right)^2 \right]},$$

$$T_d^{(384)} = \frac{0.6667}{\omega_{150^0} \left[0.32 + 1.6 \frac{K_{150^0}}{K_m} - 0.8 \left(\frac{K_{150^0}}{K_m} \right)^2 \right]},$$

$$T_f^{(384)} = \frac{0.4}{\omega_{150^0} \left[\frac{K_{150^0}}{K_m} + 0.07 \right] \left[0.32 + 1.6 \frac{K_{150^0}}{K_m} - 0.8 \left(\frac{K_{150^0}}{K_m} \right)^2 \right]^2 \min \left\{ 6 + \frac{K_m}{K_{150^0}}, 25 \right\} \quad \text{or}$$

$$T_f^{(384)} = \frac{1}{\omega_{150^0} \left[20 \frac{K_{150^0}}{K_m} + 0.5 \right] \left[0.32 + 1.6 \frac{K_{150^0}}{K_m} - 0.8 \left(\frac{K_{150^0}}{K_m} \right)^2 \right]}.$$

Rule	K_c	T_i	T_d	Comment
Direct synthesis				
Wang <i>et al</i> (1995b).	$^{13} K_c^{(385)}$	$T_i^{(385)}$	$T_d^{(385)}$	

¹³ For a stable process, K_m is assumed known; for an integrating process, K_m and τ_m

$$\text{are assumed known. } K_c^{(385)} = \frac{1}{\omega_{CL}} \text{Im} \left[\frac{j\omega_{CL} G_{CL}(j\omega_{CL})}{G_p(j\omega_{CL})(1 - G_{CL}(j\omega_{CL}))} \right],$$

$$T_i^{(385)} = \frac{1}{\omega_{CL}} \frac{\text{Im} \left[\frac{j\omega_{CL} G_{CL}(j\omega_{CL})}{G_p(j\omega_{CL})(1 - G_{CL}(j\omega_{CL}))} \right]}{\text{Re} \left[\frac{j\omega_{CL} G_{CL}(j\omega_{CL})}{G_p(j\omega_{CL})(1 - G_{CL}(j\omega_{CL}))} \right]} + \frac{1}{3} \left[\text{Re} \left\{ \frac{j\omega_{CL} G_{CL}(j\omega_{CL})}{G_p(j\omega_{CL})(1 - G_{CL}(j\omega_{CL}))} \right\} - \text{Re} \left\{ \frac{j2\omega_{CL} G_{CL}(j2\omega_{CL})}{G_p(j2\omega_{CL})(1 - G_{CL}(j2\omega_{CL}))} \right\} \right],$$

$$T_d^{(385)} = \frac{1}{3} \left[\frac{\text{Re} \left\{ \frac{j\omega_{CL} G_{CL}(j\omega_{CL})}{G_p(j\omega_{CL})(1 - G_{CL}(j\omega_{CL}))} \right\} - \text{Re} \left\{ \frac{j2\omega_{CL} G_{CL}(j2\omega_{CL})}{G_p(j2\omega_{CL})(1 - G_{CL}(j2\omega_{CL}))} \right\}}{\text{Im} \left\{ \frac{j\omega_{CL} G_{CL}(j\omega_{CL})}{G_p(j\omega_{CL})(1 - G_{CL}(j\omega_{CL}))} \right\}} \right].$$

4.2.3 Ideal controller in series with a second order filter

$$G_c(s) = K_c \left(1 + \frac{1}{T_i s} + T_d s \right) \frac{1 + b_{f1}s}{1 + a_{f1}s + a_{f2}s^2}$$

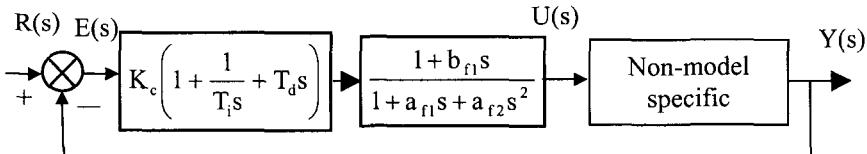


Table 70: PID controller tuning rules – non-model specific

Rule	K_c	T_i	T_d	Comment
Minimum performance index: other tuning				
Kristiansson <i>et al.</i> (2000).	${}^1 K_c^{(386)}$	$T_i^{(386)}$	$T_d^{(386)}$	$M_s \approx 1.7$
“Almost” optimal tuning rule				

$${}^1 K_c^{(386)} = \frac{1.6 \left[1.6 \frac{|G_p(j\omega_u)|^2}{K_m^2} - 2.3 \frac{|G_p(j\omega_u)|}{K_m} + 1.1 \right]}{K_m \left[0.37 + \frac{|G_p(j\omega_u)|}{K_m} \right]}, \quad b_{f1} = 0, \quad a_{1f} = T_f, \quad a_{2f} = T_f^2,$$

$$T_i^{(386)} = \frac{1.6}{\omega_u \left[0.37 + \frac{|G_p(j\omega_u)|}{K_m} \right]}, \quad T_d^{(386)} = \frac{0.625}{\omega_u \left[0.37 + \frac{|G_p(j\omega_u)|}{K_m} \right]},$$

$$T_f = \frac{\left[1.6 \frac{|G_p(j\omega_u)|^2}{K_m^2} - 2.3 \frac{|G_p(j\omega_u)|}{K_m} + 1.1 \right]}{\omega_u \left[0.37 + \frac{|G_p(j\omega_u)|}{K_m} \right]^2 \left[13 - 20 \frac{|G_p(j\omega_u)|}{K_m} \right]}, \quad \frac{|G_p(j\omega_u)|}{K_m} \leq 0.5 \text{ or}$$

$$T_f = \frac{\left[1.6 \frac{|G_p(j\omega_u)|^2}{K_m^2} - 2.3 \frac{|G_p(j\omega_u)|}{K_m} + 1.1 \right]}{3\omega_u \left[0.37 + \frac{|G_p(j\omega_u)|}{K_m} \right]^2 \left[13 - 20 \frac{|G_p(j\omega_u)|}{K_m} \right]}, \quad \frac{|G_p(j\omega_u)|}{K_m} > 0.5.$$

Rule	K_c	T_i	T_d	Comment
Kristiansson (2003).	$^2K_c^{(387)}$	$T_i^{(387)}$	$T_d^{(387)}$	“Almost” optimal tuning rule; $K_{150^\circ} \geq 0.03$; $1.6 \leq M_s \leq 1.9$

$$^2K_c^{(387)} = \frac{0.60}{K_m \left[\frac{K_{150^\circ}}{K_m} + 0.07 \right] \left[0.32 + 1.6 \frac{K_{150^\circ}}{K_m} - 0.8 \left(\frac{K_{150^\circ}}{K_m} \right)^2 \right]},$$

$$T_i^{(387)} = \frac{1.5}{\omega_{150^\circ} \left[0.32 + 1.6 \frac{K_{150^\circ}}{K_m} - 0.8 \left(\frac{K_{150^\circ}}{K_m} \right)^2 \right]},$$

$$T_d^{(387)} = \frac{0.6667}{\omega_{150^\circ} \left[0.32 + 1.6 \frac{K_{150^\circ}}{K_m} - 0.8 \left(\frac{K_{150^\circ}}{K_m} \right)^2 \right]}, b_{1f} = 0,$$

$$a_{1f} = \frac{0.8}{\omega_{150^\circ} \left[20 \frac{K_{150^\circ}}{K_m} + 0.5 \right] \left[0.32 + 1.6 \frac{K_{150^\circ}}{K_m} - 0.8 \left(\frac{K_{150^\circ}}{K_m} \right)^2 \right]},$$

$$a_{2f} = \frac{1}{\omega_{150^\circ}^2 \left[20 \frac{K_{150^\circ}}{K_m} + 0.5 \right]^2 \left[0.32 + 1.6 \frac{K_{150^\circ}}{K_m} - 0.8 \left(\frac{K_{150^\circ}}{K_m} \right)^2 \right]^2}.$$

4.2.4 Ideal controller with weighted proportional term

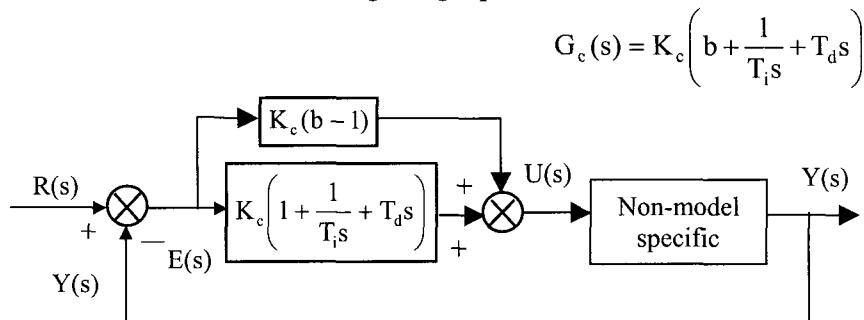


Table 71: PID controller tuning rules – non-model specific

Rule	K_c	T_i	T_d	Comment
Direct synthesis				
Åström and Hägglund (1995) – page 217.	³ $K_c^{(388)}$	$T_i^{(388)}$	$T_d^{(388)}$	$0 < K_m K_u < \infty$ $M_{max} = 1.4$
	⁴ $K_c^{(389)}$	$T_i^{(389)}$	$T_d^{(389)}$	$0 < K_m K_u < \infty$ $M_{max} = 2.0$
Streeter <i>et al.</i> (2003).	⁵ $K_c^{(390)}$	$T_i^{(390)}$	$T_d^{(390)}$	$0 < K_m K_u < \infty$ $M_{max} = 2.0$

$${}^3 K_c^{(388)} = (0.33e^{-0.31\kappa-\kappa^2})K_u, \quad T_i^{(388)} = (0.76e^{-1.6\kappa-0.36\kappa^2})T_u, \quad (0.33e^{-0.31\kappa-\kappa^2})K_u \\ T_d^{(388)} = (0.17e^{-0.46\kappa-2.1\kappa^2})T_u, \quad b = 0.58e^{-1.3\kappa+3.5\kappa^2}.$$

$${}^4 K_c^{(389)} = (0.72e^{-1.6\kappa+1.2\kappa^2})K_u, \quad T_i^{(389)} = (0.59e^{-1.3\kappa+0.38\kappa^2})T_u, \\ T_d^{(389)} = (0.15e^{-1.4\kappa+0.56\kappa^2})T_u, \quad b = 0.25e^{0.56\kappa-0.12\kappa^2}.$$

$${}^5 K_c^{(390)} = (0.72e^{-1.6\kappa+1.2\kappa^2})K_u - 0.0012340T_u - 6.1173 \cdot 10^{-6}, \\ T_i^{(390)} = \frac{(0.59e^{-1.3\kappa+0.38\kappa^2})(0.72e^{-1.6\kappa+1.2\kappa^2})K_u - 0.0012340T_u - 6.1173 \cdot 10^{-6}}{(0.72e^{-1.6\kappa+1.2\kappa^2} - 0.040430e^{-1.3\kappa+0.38\kappa^2})K_u},$$

$$T_d^{(390)} = \frac{0.108e^{-3\kappa+1.76\kappa^2}K_uT_u - 0.0026640(e^{T_u})^{\log(1.6342 \log K_u)}}{0.72K_ue^{-1.6\kappa+1.2\kappa^2} - 0.0012340T_u - 6.1173 \cdot 10^{-6}},$$

$$b = 0.25e^{0.56\kappa-0.12\kappa^2} + \frac{K_u}{e^{K_u}}.$$

4.2.5 Controller with filtered derivative $G_c(s) = K_c \left(1 + \frac{1}{T_i s} + \frac{T_d s}{1 + \frac{T_d s}{N}} \right)$

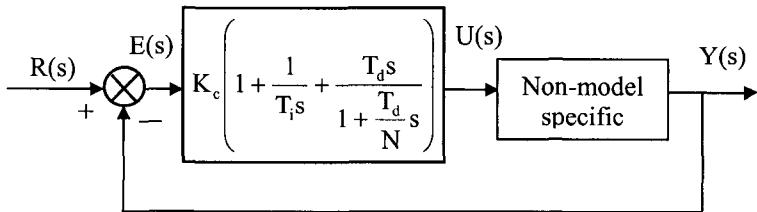


Table 72: PID controller tuning rules – non-model specific

Rule	K_c	T_i	T_d	Comment
Direct synthesis				
Leva (1993). N not specified	⁶ $K_c^{(391)}$	$T_i^{(391)}$	$6.283/\beta\omega$	$\beta = 10$
	⁷ $K_c^{(392)}$	$\alpha T_d^{(392)}$	$T_d^{(392)}$	$6 \leq \alpha \leq 10$
Yi and De Moor (1994).	⁸ $K_c^{(393)}$	$T_i^{(393)}$	$T_d^{(393)}$	N not defined

$$6 \quad K_c^{(391)} = \frac{\omega T_i^{(391)}}{|G_p(j\omega)| \sqrt{\left(1 - \omega T_i^{(391)} \frac{2\pi}{\beta}\right)^2 + \omega^2 [T_i^{(391)}]^2}},$$

$$T_i^{(391)} = \frac{\tan(\phi_m - \phi_\omega - 0.5\pi)}{\omega \sqrt{1 + \frac{2\pi}{\beta} \tan(\phi_m - \phi_\omega - 0.5\pi)}}, \quad \phi_\omega > \phi_m - \pi.$$

$$7 \quad K_c^{(392)} = \frac{\omega T_i^{(392)}}{|G_p(j\omega)| \sqrt{\left(1 - \omega^2 T_i^{(392)} T_d^{(392)}\right)^2 + \omega^2 [T_i^{(392)}]^2}},$$

$$T_d^{(392)} = \frac{-\alpha\omega + \sqrt{\alpha^2\omega^2 + 4\alpha\omega^2 \tan^2(\phi_m - \phi_\omega - 0.5\pi)}}{2\alpha\omega^2 \tan(\phi_m - \phi_\omega - 0.5\pi)}, \quad \phi_\omega < \phi_m - \pi.$$

$$8 \quad K_c^{(393)} = \frac{0.5 \cos[225^\circ - \angle G_p(j\omega_\phi)]}{|G_p(j\omega_\phi)|}, \quad T_i^{(393)} = 4T_d^{(393)},$$

$$T_d^{(393)} = \frac{\tan[225^\circ - \angle G_p(j\omega_\phi)] + \sqrt{4 + \tan^2[225^\circ - \angle G_p(j\omega_\phi)]}}{2\omega_\phi}.$$

Rule	K_c	T_i	T_d	Comment
Vrančić (1996) – page 130.	$\frac{T_i^{(394)}}{2(A_1 - T_i^{(394)})}$	${}^9 T_i^{(394)}$	$T_d^{(394)}$	$N < 10$
Vrančić (1996) – pages 120-121.	${}^{10} K_c^{(395)}$	$T_i^{(395)}$	$\frac{A_3 A_4 - A_2 A_5}{A_3^2 - A_1 A_5}$	$N \geq 10$
Vrančić (1996) – page 134.	${}^{11} K_c^{(396)}$	$T_i^{(396)}$	$\chi T_i^{(396)}$	$N = 10 ; \chi = [0.2, 0.25]$
Vrančić et al. (1999).	${}^{12} K_c^{(397)}$	$T_i^{(397)}$	$T_d^{(397)}$	$8 \leq N \leq 20$

$${}^9 T_i^{(394)} = \frac{A_3}{A_2 - T_d^{(394)} A_1 - \frac{[T_d^{(394)}]^2}{N}},$$

$$T_d^{(394)} = \frac{-(A_3^2 - A_5 A_1) + \sqrt{(A_3^2 - A_5 A_1)^2 - \frac{4}{N}(A_3 A_2 - A_5)(A_5 A_2 - A_4 A_3)}}{\frac{2}{N}(A_3 A_2 - A_5)}.$$

$${}^{10} K_c^{(395)} = \frac{0.5 A_3 (A_3^2 - A_1 A_5)}{(A_3^2 - A_1 A_5)(A_1 A_2 - A_3 K_m) - (A_3 A_4 - A_2 A_5)A_1^2},$$

$$T_i^{(395)} = \frac{A_3 (A_3^2 - A_1 A_5)}{(A_3^2 - A_1 A_5)A_2 - (A_3 A_4 - A_2 A_5)A_1}.$$

$${}^{11} K_c^{(396)} = \frac{0.5 T_i^{(396)}}{A_1 - K_m T_i^{(396)}}, T_i^{(396)} = \frac{A_2 - \sqrt{A_2^2 - 4\chi A_1 A_3}}{2\chi A_1}.$$

$${}^{12} K_c^{(397)} = \frac{A_3}{2 \left(A_1 A_2 - A_0 A_3 - T_d^{(397)} A_1^2 - \frac{[T_d^{(397)}]^2}{N} A_0 A_1 \right)},$$

$$T_i^{(397)} = \frac{A_3}{A_2 - T_d^{(397)} A_1 - \frac{[T_d^{(397)}]^2}{N} K_m}, T_d^{(397)} \text{ is obtained by solving:}$$

$$\frac{A_0 A_3}{N^3} [T_d^{(397)}]^4 + \frac{A_1 A_3}{N^2} [T_d^{(397)}]^3 - \frac{A_0 A_5 - A_2 A_3}{N} [T_d^{(397)}]^2 + (A_3^2 - A_1 A_5) [T_d^{(397)}] + (A_2 A_5 - A_3 A_4) = 0.$$

Rule	K_c	T_i	T_d	Comment
Lennartson and Kristiansson (1997).	$^{13} K_c^{(398)}$	$T_i^{(398)}$	$0.4T_i^{(398)}$	$K_u K_m \geq 1.67$
Kristiansson and Lennartson (1998a).	$^{14} K_c^{(399)}$	$T_i^{(399)}$	$0.4T_i^{(399)}$	
		$T_i^{(400)}$	$0.4T_i^{(400)}$	
	$^{15} K_c^{(401)}$	$T_i^{(401)}$	$0.4T_i^{(401)}$	$N = 2.5$
Kristiansson and Lennartson (1998b).	$^{16} K_c^{(402)}$	$T_i^{(402)}$	$0.333T_i^{(402)}$	

$$^{13} K_c^{(398)} = K_u K_m \frac{12K_u^2 K_m^2 - 35K_u K_m + 30}{K_u K_m + 2.5[12K_u^2 K_m^2 - 35K_u K_m + 30]},$$

$$T_i^{(398)} = \frac{K_c^{(398)}}{-0.053\omega_u^3 + 0.47\omega_u^2 - 0.14\omega_u + 0.11},$$

$$N = \frac{2.5}{K_m K_u} \left[12 - \frac{35}{K_m K_u} + \frac{30}{K_m^2 K_u^2} \right].$$

$$^{14} K_c^{(399)} = K_u K_m \frac{12K_u^2 K_m^2 - 35K_u K_m + 30}{K_u K_m + x_1[12K_u^2 K_m^2 - 35K_u K_m + 30]},$$

$$T_i^{(399)} = \frac{K_c^{(399)}}{-0.525\omega_u^3 + 0.473\omega_u^2 - 0.143\omega_u + 0.113}, \frac{\omega_u}{K_u K_m} \leq 0.4;$$

$$T_i^{(400)} = \frac{K_c^{(400)}}{-0.185\omega_u^3 + 1.052\omega_u^2 - 0.854\omega_u + 0.309}, \frac{\omega_u}{K_u K_m} \geq 0.4;$$

$$N = \frac{x_1}{K_m K_u} \left[12 - \frac{35}{K_m K_u} + \frac{30}{K_m^2 K_u^2} \right]; x_1 = 3, K_u K_m > 10; x_1 = 2.5, K_u K_m < 10.$$

$$^{15} K_c^{(401)} = \frac{7.71}{K_m^2 K_u^2} - \frac{9.14}{K_m K_u} + 3.14, K_u K_m > 1.67, \frac{\omega_u}{K_u K_m} > 0.45;$$

$$T_i^{(401)} = \frac{K_c^{(401)}}{-0.63\omega_u^3 + 0.39\omega_u^2 + 0.15\omega_u + 0.0082}.$$

$$^{16} K_c^{(402)} = \frac{12K_u^3 K_p^2 - 35K_u^2 K_m + 30K_u}{K_m^3 K_u^3 + 2.5[12K_u^2 K_m^2 - 35K_u K_m + 30]},$$

$$T_i^{(402)} = \frac{K_c^{(402)} K_m^3 K_u^2}{\omega_u [0.95K_m^2 K_u^2 - 2K_m K_u + 1.4]}, N = \frac{2.5}{K_m K_u} \left[12 - \frac{35}{K_m K_u} + \frac{30}{K_m^2 K_u^2} \right].$$

Rule	K_c	T_i	T_d	Comment
Kristiansson and Lennartson (2000).	$^{17} K_c^{(403)}$	$T_i^{(403)}$	$T_d^{(403)}$	$0.1 \leq K_u K_m \leq 0.5$
	$^{18} K_c^{(404)}$	$T_i^{(404)}$	$T_d^{(404)}$	$K_u K_m > 0.5$

$$^{17} K_c^{(403)} = \frac{(1.1K_u^2 K_m^2 - 2.3K_u K_m + 1.6)^2}{K_m^2 K_u (-20 + 13K_m K_u)(1 + 0.37K_m K_u)^2} \left[\frac{1.6(-20 + 13K_m K_u)(1 + 0.37K_m K_u)}{1.1K_m^2 K_u^2 - 2.3K_m K_u + 1.6} - 1 \right],$$

$$T_i^{(403)} = \frac{K_m K_u (1.1K_u^2 K_m^2 - 2.3K_u K_m + 1.6)}{\omega_u (-20 + 13K_m K_u)(1 + 0.37K_m K_u)^2} \left[\frac{1.6(-20 + 13K_m K_u)(1 + 0.37K_m K_u)}{1.1K_m^2 K_u^2 - 2.3K_m K_u + 1.6} - 1 \right],$$

$$T_d^{(403)} = \frac{K_m K_u (1.1K_u^2 K_m^2 - 2.3K_u K_m + 1.6)}{\omega_u (-20 + 13K_m K_u)(1 + 0.37K_m K_u)^2} \left[\frac{\frac{(-20 + 13K_m K_u)^2 (1 + 0.37K_m K_u)^2}{(1.1K_m^2 K_u^2 - 2.3K_m K_u + 1.6)^2}}{\frac{1.6(-20 + 13K_m K_u)(1 + 0.37K_m K_u)}{1.1K_m^2 K_u^2 - 2.3K_m K_u + 1.6} - 1} \right], N = \frac{T_d^{(403)}}{T_f^{(403)}},$$

$$T_f^{(403)} = \frac{K_m K_u (1.1K_u^2 K_m^2 - 2.3K_u K_m + 1.6)}{\omega_u (-20 + 13K_m K_u)(1 + 0.37K_m K_u)^2}.$$

$$^{18} K_c^{(404)} = \frac{(1.1K_u^2 K_m^2 - 2.3K_u K_m + 1.6)^2}{3\omega_u K_m^2 K_u^2 (1 + 0.37K_m K_u)^2} \left[\frac{4.8K_m K_u (1 + 0.37K_m K_u)}{1.1K_m^2 K_u^2 - 2.3K_m K_u + 1.6} - 1 \right],$$

$$T_i^{(404)} = \frac{(1.1K_u^2 K_m^2 - 2.3K_u K_m + 1.6)}{3\omega_u (1 + 0.37K_m K_u)^2} \left[\frac{4.8K_m K_u (1 + 0.37K_m K_u)}{1.1K_m^2 K_u^2 - 2.3K_m K_u + 1.6} - 1 \right],$$

$$T_d^{(404)} = \frac{(1.1K_u^2 K_m^2 - 2.3K_u K_m + 1.6)}{3\omega_u (1 + 0.37K_m K_u)^2} \left[\frac{\frac{3K_m^2 K_u^2 (1 + 0.37K_m K_u)^2}{(1.1K_m^2 K_u^2 - 2.3K_m K_u + 1.6)^2}}{\frac{4.8K_m K_u (1 + 0.37K_m K_u)}{1.1K_m^2 K_u^2 - 2.3K_m K_u + 1.6} - 1} \right],$$

$$N = \frac{T_d^{(404)}}{T_f^{(404)}}, T_f^{(404)} = \frac{1.1K_u^2 K_m^2 - 2.3K_u K_m + 1.6}{3\omega_u (1 + 0.37K_m K_u)^2}.$$

Rule	K_c	T_i	T_d	Comment
Kristiansson and Lennartson (2000) – continued.	$^{19} K_c^{(405)}$	$T_i^{(405)}$	$T_d^{(405)}$	$K_u K_m < 0.1$
Kristiansson and Lennartson (2002).	$^{20} K_c^{(406)}$	$T_i^{(406)}$	$T_d^{(406)}$	$K_u K_m \leq 10$

$$^{19} K_c^{(405)} = \frac{(-6 + 3.7K_{135^\circ} K_m)^2}{13K_m (1.8 + 0.3K_m K_{135^\circ})^2} \left[\frac{20.8(1.8 + 0.3K_m K_{135^\circ})}{(-6 + 3.7K_m K_{135^\circ})} - 1 \right],$$

$$T_i^{(405)} = \frac{(-6 + 3.7K_{135^\circ} K_m)K_{135^\circ} K_m}{13\omega_{135^\circ} (1.8 + 0.3K_m K_{135^\circ})^2} \left[\frac{20.8(1.8 + 0.3K_m K_{135^\circ})}{(-6 + 3.7K_m K_{135^\circ})} - 1 \right],$$

$$T_d^{(405)} = \frac{(-6 + 3.7K_m K_{135^\circ})K_m K_{135^\circ}}{13\omega_{135^\circ} (1.8 + 0.3K_m K_{135^\circ})^2} \left[\frac{\frac{169(1.8 + 0.3K_m K_{135^\circ})^2}{(-6 + 3.7K_m K_{135^\circ})^2} - 1}{\frac{20.8(1.8 + 0.3K_m K_{135^\circ})}{(-6 + 3.7K_m K_{135^\circ})} - 1} \right],$$

$$N = \frac{T_d^{(405)}}{T_f^{(405)}}, T_f^{(405)} = \frac{(-6 + 3.7K_m K_{135^\circ})K_m K_{135^\circ}}{13\omega_{135^\circ} (1.8 + 0.3K_m K_{135^\circ})^2}.$$

$$^{20} K_c^{(406)} = \frac{K_u}{K_m} \frac{[1.5(K_m K_u + 4)(0.4K_m K_u + 0.75) - K_m K_u x_1]x_1}{(0.4K_m K_u + 0.75)^2 (4 + K_m K_u)},$$

$$T_i^{(406)} = \frac{K_m K_u}{\omega_u} \frac{[1.5(K_m K_u + 4)(0.4K_m K_u + 0.75) - K_m K_u x_1]}{(0.4K_m K_u + 0.75)^2 (4 + K_m K_u)},$$

$$T_d^{(406)} = \frac{K_m K_u}{\omega_u} \frac{(4 + K_m K_u)}{[1.5(4 + K_m K_u)(0.4K_m K_u + 0.75) - K_m K_u x_1](1 + N^{-1})},$$

$$N = \frac{T_d^{(406)}}{T_f^{(406)}}, T_f^{(406)} = \frac{K_m^2 K_u^2}{\omega_u} \frac{x_1}{(0.4K_m K_u + 0.75)^2 (4 + K_m K_u)},$$

$$x_1 = 0.13 + 0.16K_m K_u - 0.007K_m^2 K_u^2.$$

Rule	K_c	T_i	T_d	Comment
Kristiansson and Lennartson (2002) – continued.	$^{21} K_c^{(407)}$	$T_i^{(407)}$	$T_d^{(407)}$	$K_u K_m > 10$

$$^{21} K_c^{(407)} = \frac{K_{135^0}}{K_m} \frac{|1.5(0.44K_m K_{135^0} + 1.4)x_2 - K_m K_{135^0} x_3| x_3}{(0.44K_m K_{135^0} + 1.4)^2 x_2},$$

$$T_i^{(407)} = \frac{K_m K_{135^0}}{\omega_{135^0}} \frac{|1.5(0.44K_m K_{135^0} + 1.4)x_2 - K_m K_u x_3|}{(0.44K_m K_{135^0} + 1.4)^2 x_2},$$

$$T_d^{(407)} = \frac{K_m K_{135^0}}{\omega_{135^0}} \frac{x_2}{[1.5x_2(0.44K_m K_{135^0} + 1.4) - K_m K_{135^0} x_3](1 + N^{-1})},$$

$$N = \frac{T_d^{(407)}}{T_f^{(407)}}, \quad T_f^{(407)} = \frac{K_m^2 K_{135^0}^2}{\omega_{135^0}} \frac{x_3}{(0.44K_m K_{135^0} + 1.4)^2 x_2},$$

$$x_2 = \min(14 + 0.5K_m K_{135^0}, 25), \quad x_3 = 1.35 + 0.35K_m K_{135^0} - 0.006K_m^2 K_{135^0}^2.$$

4.2.6 Ideal controller with set-point weighting 1

$$U(s) = K_c \left(F_p R(s) - Y(s) \right) + \frac{K_c}{T_i s} \left(F_i R(s) - Y(s) \right) + K_c T_d s \left(F_d R(s) - Y(s) \right)$$

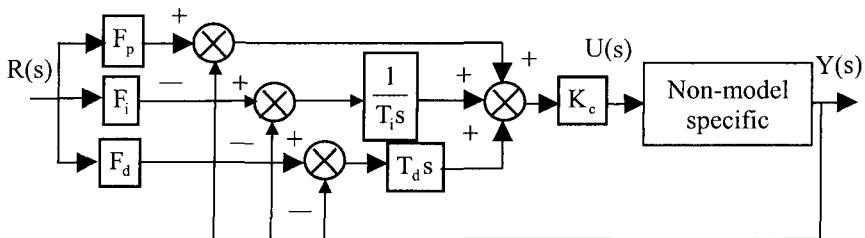


Table 73: PID controller tuning rules – non-model specific

Rule	K_c	T_i	T_d	Comment
Ultimate cycle				
Mantz and Tacconi (1989).	$0.6K_u$; $F_p = 0.17$	$0.5T_u$; $F_i = 1$	$0.125T_u$; $F_d = 0.654$	Quarter decay ratio

4.2.7 Classical controller 1 $G_c(s) = K_c \left(1 + \frac{1}{T_i s} \right) \frac{1 + s T_d}{1 + s \frac{T_d}{N}}$

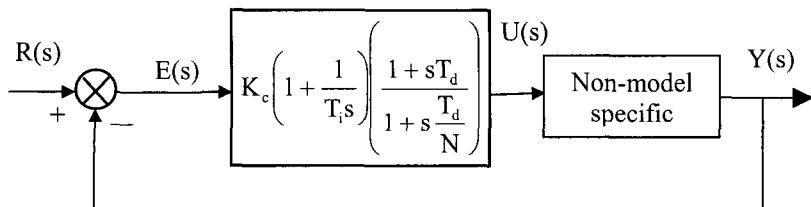


Table 74: PID controller tuning rules – non-model specific

Rule	K_c	T_i	T_d	Comment
Minimum performance index: regulator tuning				
Minimum IAE - Edgar <i>et al.</i> (1997) –page 8- 15.	$0.56K_u$	$0.39T_u$	$0.14T_u$	$N = 10$
Ultimate cycle				
Kinney (1983).	$0.25K_u$	$0.5T_u$	$0.12T_u$	N not defined
Corripio (1990) – page 27.	$0.6K_u$	$0.5T_u$	$0.125T_u$	$10 \leq N \leq 20$
St. Clair (1997) – page 17. $N \geq 1$	$0.5K_u$	$1.2T_u$	$0.125T_u$	'aggressive' tuning
	$0.25K_u$	$1.2T_u$	$0.125T_u$	'conservative' tuning
Harrold (1999).	$0.67K_u$	$0.5T_u$	$0.125T_u$	N not defined

4.2.8 Series controller (classical controller 3)

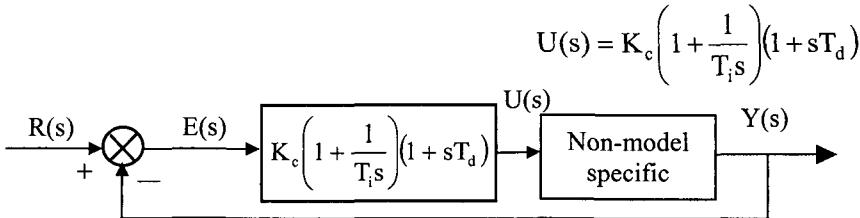


Table 75: PID controller tuning rules – non-model specific

Rule	K_c	T_i	T_d	Comment
Ultimate cycle				
Pessen (1953).	$0.25K_u$	$0.33T_u$	$0.5T_u$	'optimum' servo response
	$0.2K_u$	$0.25T_u$	$0.5T_u$	'optimum' regulator response - step changes
	$0.33K_u$	$0.5T_u$	$0.33T_u$	
Pessen (1954).	$0.33K_u$	$0.5T_u$	$0.33T_u$	'Some' overshoot
Tan <i>et al.</i> (1999a) – page 26.	$0.3K_u$	$0.15T_u$	$0.25T_u$	Ziegler-Nichols ultimate cycle equivalent
O'Dwyer (2001b) – representative results - Åström and Hägglund (1995) equivalent – page 142.	$0.2349K_u$	$0.2273T_u$	$0.2273T_u$	$A_m = 2$, $\phi_m = 20^0$
	$0.0944K_u$	$0.6154T_u$	$0.6154T_u$	$A_m = 2.44$, $\phi_m = 61^0$
	$0.1008K_u$	$0.3939T_u$	$0.3939T_u$	$A_m = 3.45$, $\phi_m = 46^0$
Tan <i>et al.</i> (2001).	¹ $K_c^{(408)}$	$T_i^{(408)}$	$0.25T_i^{(408)}$	

$$K_c^{(408)} = \frac{T_i^{(408)} \omega_u |G_c'(j\omega_u)|}{\sqrt{[T_i^{(408)}]^2 \omega_u^2 + 1} \sqrt{0.0625[T_i^{(408)}]^2 \omega_u^2 + 1}},$$

$$T_i^{(408)} = \frac{\sqrt{6.25\omega_u^2 + 4\omega_u^2 \tan^2 \left[\frac{2\angle G_c'(j\omega_u) + \pi}{2} \right]} - 2.5\omega_u}{\omega_u^2 \tan \left[\frac{2\angle G_c'(j\omega_u) + \pi}{2} \right]},$$

$G_c'(j\omega_u)$ = desired controller frequency response at $\omega = \omega_u$.

4.2.9 Classical controller 4 $G_c(s) = K_c \left(1 + \frac{1}{T_i s} \right) \left(1 + \frac{s T_d}{1 + \frac{s T_d}{N}} \right)$

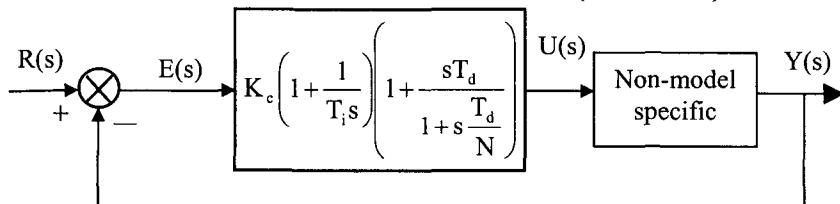


Table 76: PID controller tuning rules – non-model specific

Rule	K_c	T_i	T_d	Comment
Ultimate cycle				
Hang <i>et al.</i> (1993b) - page 58.	$0.35K_u$	$1.13T_u$	$0.20T_u$	N not specified

4.2.10 Non-interacting controller based on the two degree of freedom structure 1

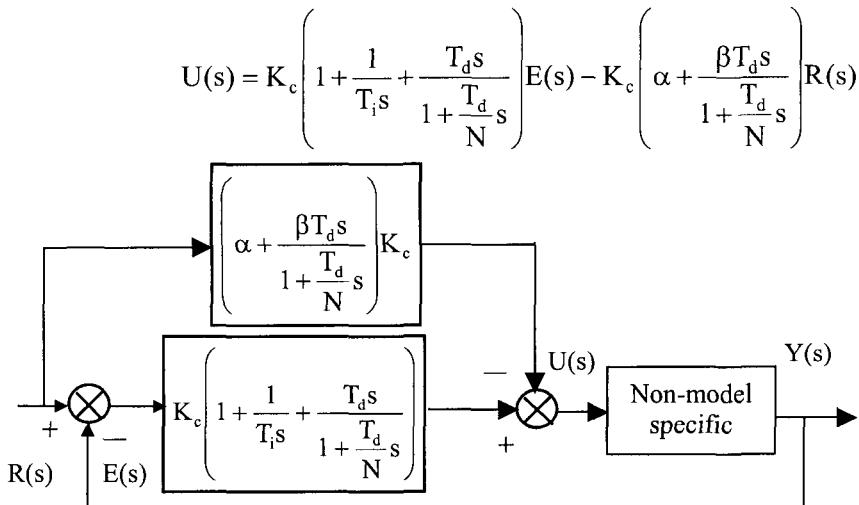


Table 77: PID controller tuning rules – non-model specific

Rule	K_c	T_i	T_d	Comment
Minimum performance index: other tuning				
Shen (2002).	$^2 K_c^{(409)}$	$T_i^{(409)}$	$T_d^{(409)}$	$\hat{K}_u > 1/K_m$

2 Minimise $\int_0^\infty |r(t) - y(t)| dt + \int_0^\infty |d(t) - y(t)| dt ; N=0, \beta = 0, M_s \leq 2 ;$

$$K_c^{(409)} = \hat{K}_u \exp \left[0.17 - 2.62 \left/ \left(K_m \hat{K}_u \right) \right. + 1.79 \left/ \left(K_m^2 \hat{K}_u^2 \right) \right],$$

$$T_i^{(409)} = \hat{T}_u \exp \left[-0.02 - 2.62 \left/ \left(K_m \hat{K}_u \right) \right. + 1.34 \left/ \left(K_m^2 \hat{K}_u^2 \right) \right],$$

$$T_d^{(409)} = \hat{T}_u \exp \left[-1.70 - 0.59 \left/ \left(K_m \hat{K}_u \right) \right. - 0.25 \left/ \left(K_m^2 \hat{K}_u^2 \right) \right],$$

$$\alpha = 1 - \exp \left[-0.30 - 0.48 \left/ \left(K_m \hat{K}_u \right) \right. + 0.93 \left/ \left(K_m^2 \hat{K}_u^2 \right) \right].$$

4.2.11 Non-interacting controller 4

$$U(s) = K_c \left(1 + \frac{1}{T_i s} \right) E(s) - K_c T_d s Y(s)$$

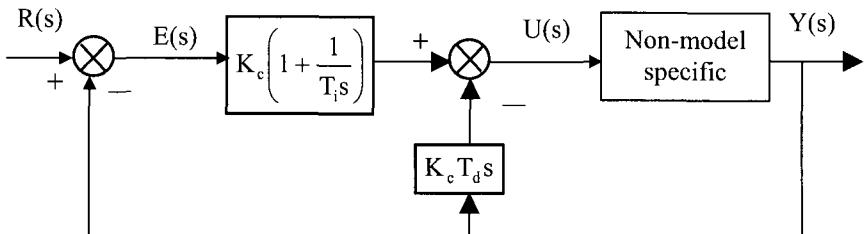


Table 78: PID controller tuning rules – non-model specific

Rule	K_c	T_i	T_d	Comment
Minimum performance index: regulator tuning				
Minimum IAE - Edgar <i>et al.</i> (1997) – page 8-15.	$0.77K_u$	$0.48T_u$	$0.11T_u$	
Ultimate cycle				
VanDoren (1998).	$0.75K_u$	$0.625T_u$	$0.1T_u$	

4.2.12 Non-interacting controller 9

$$U(s) = K_c \left(1 + \frac{1}{T_i s} + T_d s \right) E(s) - \frac{T_d}{N} s Y(s)$$

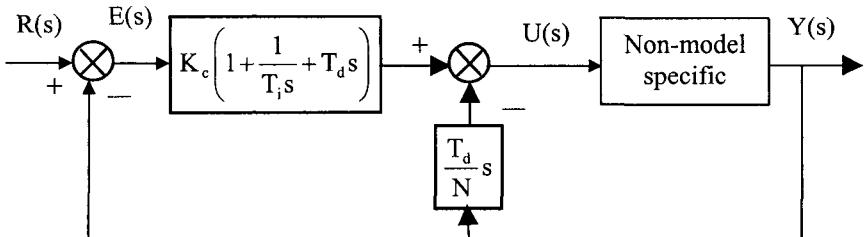


Table 79: PID controller tuning rules – non-model specific

Rule	K_c	T_i	T_d	Comment
Other tuning				
Bateson (2002) - pages 629-637. <i>Model: Method 1</i>	³ $K_c^{(410)}$	$T_i^{(410)}$	$\frac{2}{\omega_{180^\circ}}$	$N = 10$

³ $K_c^{(410)} = 10^{\min[\{-|G_p(j\omega_{140^\circ})|\} \text{ or } \{-|G_p(j\omega_{180^\circ})|-6\}]}$, $|G_p(j\omega_{140^\circ})|$ and $|G_p(j\omega_{180^\circ})|$ are given in dB; $T_i^{(410)} = \min[\omega_{82^\circ}, 0.2\omega_{170^\circ}]$.

4.3 IPD Model $G_m(s) = \frac{K_m e^{-s\tau_m}}{s}$

4.3.1 Ideal controller $G_c(s) = K_c \left(1 + \frac{1}{T_i s} + T_d s \right)$

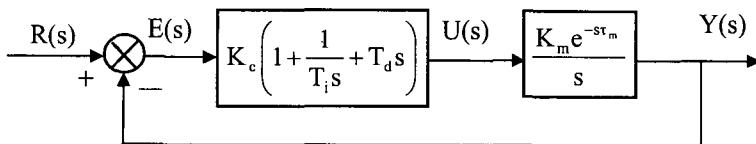


Table 80: PID controller tuning rules – IPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{s}$

Rule	K_c	T_i	T_d	Comment
Process reaction				
Ford (1953). <i>Model: Method 3</i>	$\frac{1.48}{K_m \tau_m}$	$2\tau_m$	$0.37\tau_m$	Decay ratio 2.7:1
Åström and Hägglund (1995) – page 139.	$\frac{0.94}{K_m \tau_m}$	$2\tau_m$	$0.5\tau_m$	<i>Model: Method 1</i>
Ultimate cycle Ziegler-Nichols equivalent				
Hay (1998) – page 188.	$\frac{0.4}{K_m \tau_m}$	$3.2\tau_m$	$0.8\tau_m$	<i>Model: Method 3</i>
Hay (1998) – page 199. <i>Model: Method 1; K_c, T_d deduced from graphs</i>	10.0 4.0 2.5 2.0 1.8	$3.2K_m \tau_m^2$	0.55 τ_m 0.30 τ_m 0.25 τ_m 0.25 τ_m 0.25 τ_m	$K_m \tau_m = 0.1$ $K_m \tau_m = 0.2$ $K_m \tau_m = 0.3$ $K_m \tau_m = 0.4$ $K_m \tau_m = 0.5, 0.6$
Minimum performance index: regulator tuning				
Visioli (2001). <i>Model: Method 1</i>	$x_1/K_m \tau_m$	$x_2 \tau_m$	$x_3 \tau_m$	
	Coefficient values			
	1.37	1.49	0.59	Minimum ISE
	1.36	1.66	0.53	Minimum ITSE
	1.34	1.83	0.49	Minimum ISTSE

Rule	K_c	T_i	T_d	Comment
Minimum performance index: servo tuning				
Visioli (2001). <i>Model: Method I</i>	$x_1/K_m \tau_m$	0	$x_3 \tau_m$	
	Coefficient values			
	1.03		0.49	Minimum ISE
	0.96		0.45	Minimum ITSE
	0.90		0.45	Minimum ISTSE
Minimum performance index: other tuning				
Åström and Hägglund (2004). <i>Model: Method I</i>	$x_1/K_m \tau_m$	$x_2 \tau_m$	$x_3 \tau_m$	
	Coefficient values			
	x_1	x_2	x_3	M_{\max}
	0.139	76.9	0.346	1.1
	0.261	23.3	0.365	1.2
	0.367	12.2	0.378	1.3
	0.460	7.85	0.389	1.4
	0.543	5.78	0.400	1.5
Direct synthesis				
Leonard (1994). <i>Model: Method I</i>	$\frac{0.74}{K_m \tau_m}$	$12.2 \tau_m$	$0.41 \tau_m$	OS (step input) < 10%; Minimum IAE (disturbance ramp).
	$0.47 K_u$	$3.05 T_u$	$0.10 T_u$	
Wang and Cluett (1997).	${}^1 K_c^{(411)}$	$T_i^{(411)}$	$T_d^{(411)}$	<i>Model: Method I</i>
	${}^2 K_c^{(412)}$	$T_i^{(412)}$	$T_d^{(412)}$	
<i>K_u, T_u deduced from graph</i>				
Cluett and Wang (1997). <i>Model: Method I</i>	$x_1/K_m \tau_m$	$x_2 \tau_m$	$x_3 \tau_m$	$T_{CL} = x_4 \tau_m$
	Coefficient values			
	x_1	x_2	x_3	x_4
	0.9588	3.0425	0.3912	1
	0.6232	5.2586	0.2632	2
	0.4668	7.2291	0.2058	3

$${}^1 K_c^{(411)} = \frac{1}{K_m \tau_m (0.7138 T_{CL} + 0.3904)}, \quad T_i^{(411)} = (1.4020 T_{CL} + 1.2076) \tau_m,$$

$$T_d^{(411)} = \frac{\tau_m}{1.4167 T_{CL} + 1.6999}; \quad T_{CL} \in [\tau_m, 16\tau_m], \quad \xi = 0.707.$$

$${}^2 K_c^{(412)} = \frac{1}{K_m \tau_m (0.5080 T_{CL} + 0.6208)}, \quad T_i^{(412)} = (1.9885 T_{CL} + 1.2235) \tau_m,$$

$$T_d^{(412)} = \frac{\tau_m}{1.0043 T_{CL} + 1.8194}; \quad T_{CL} \in [\tau_m, 16\tau_m], \quad \xi = 1.$$

Rule	K_c	T_i	T_d	Comment
Rotach (1995). Model: Method 5	$\frac{1.21}{K_m \tau_m}$	$1.60\tau_m$	$0.48\tau_m$	
Damping factor for oscillations to a disturbance input = 0.75.				
Chen <i>et al.</i> (1999a).	$\frac{1.661}{A_m K_m \tau_m}$	0	${}^3 T_d^{(413)}$	Model: Method 1
			${}^4 T_d^{(414)}$	
Chidambaran and Sree (2003).	$\frac{1.2346}{K_m \tau_m}$	$4.5\tau_m$	$0.45\tau_m$	Model: Method 1
Sree and Chidambaran (2005b).	$\frac{0.896}{K_m \tau_m}$	$2.5\tau_m$	$0.55\tau_m$	Model: Method 1
Chen and Seborg (2002).	${}^5 K_c^{(415)}$	$T_i^{(415)}$	$T_d^{(415)}$	Model: Method 1
Robust				
Zou <i>et al.</i> (1997), Zou and Brigham (1998). Model: Method 7 or Method 8	$\frac{2}{K_m (\lambda + 0.5\tau_m)}$	$2\lambda + \tau_m$	$\frac{\lambda + 0.25\tau_m}{2\lambda + \tau_m} \tau_m$	$0.5\tau_m \leq \lambda \leq 3\tau_m$

$${}^3 T_d^{(413)} = 1.273\tau_m [1 - 0.6002A_m (1.571 - \phi_m)].$$

$${}^4 T_d^{(414)} = \tau_m \left(1.2732 - \frac{r_1 A_m}{1.6661} \right).$$

$${}^5 \left(\frac{y}{d} \right)_{desired} = \frac{T_i^{(415)} s (1 + 0.5\tau_m s) e^{-s\tau_m}}{K_c^{(415)} (T_{CL}s + 1)^3}, \quad K_c^{(415)} = \frac{\tau_m (3T_{CL} + 0.5\tau_m)}{K_m (T_{CL} + 0.5\tau_m)^3},$$

$$T_i^{(415)} = 3T_{CL} + 0.5\tau_m, \quad T_d^{(415)} = \frac{1.5T_{CL}^2 \tau_m + 0.75T_{CL}\tau_m^2 + 0.125\tau_m^3 - T_{CL}^3}{\tau_m (3T_{CL} + 0.5\tau_m)}.$$

4.3.2 Ideal controller in series with a first order lag

$$G_c(s) = K_c \left(1 + \frac{1}{T_i s} + T_d s \right) \frac{1}{T_f s + 1}$$

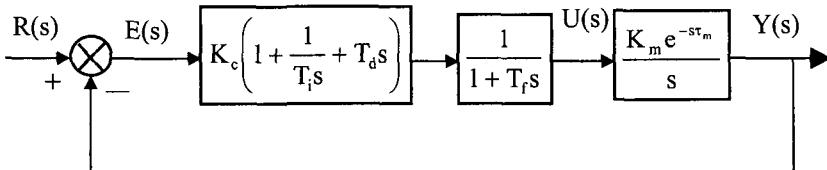


Table 81: PID controller tuning rules – IPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{s}$

Rule	K_c	T_i	T_d	Comment
Robust				
Rice and Cooper (2002). Model: Method 1	${}^6 K_c^{(416)}$	$2\lambda + 1.5\tau_m$	$\frac{0.5\tau_m^2 + \lambda\tau_m}{2\lambda + 1.5\tau_m}$	Suggested $\lambda = 3.162\tau_m$

$${}^6 K_c^{(416)} = \frac{2\lambda + 1.5\tau_m}{K_m (\lambda^2 + 2\lambda\tau_m + 0.5\tau_m^2)}, \quad T_f = \frac{0.5\lambda^2\tau_m}{\lambda^2 + 2\lambda\tau_m + 0.5\tau_m^2}.$$

4.3.3 Ideal controller with first order filter and set-point weighting 2

$$U(s) = K_c \left(1 + \frac{1}{T_i s} + T_d s \right) \frac{1}{T_f s + 1} \left[R(s) \frac{1 + 0.4 T_r s}{1 + s T_r} - Y(s) \right] - K_0 Y(s)$$

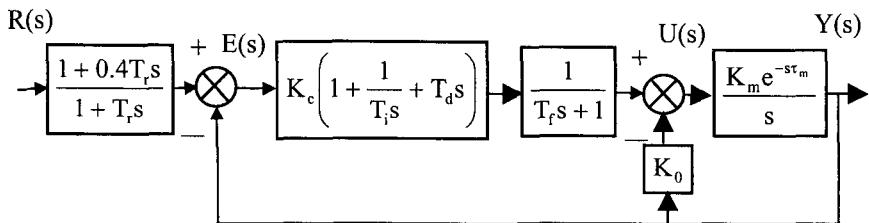


Table 82: PID controller tuning rules – IPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{s}$

Rule	K_c	T_i	T_d	Comment
Direct synthesis				
Normey-Rico <i>et al.</i> (2000). <i>Model: Method 1</i>	$\frac{0.563}{K_m \tau_m}$	$1.5 \tau_m$	$0.667 \tau_m$	$T_f = 0.13 \tau_m$ $K_0 = \frac{1}{2 K_m \tau_m}$ $T_r = 0.75 \tau_m$

4.3.4 Controller with filtered derivative $G_c(s) = K_c \left(1 + \frac{1}{T_i s} + \frac{T_d s}{1 + \frac{T_d s}{N}} \right)$

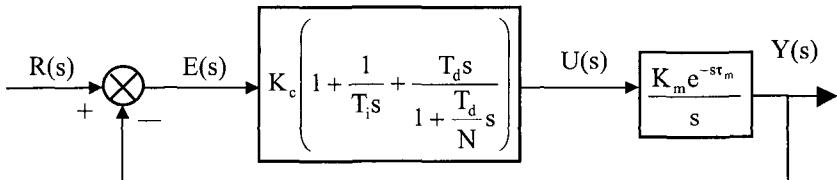


Table 83: PID controller tuning rules – IPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{s}$

Rule	K_c	T_i	T_d	Comment
Direct synthesis				
Kristiansson and Lennartson (2002).	${}^7 K_c^{(417)}$	$T_i^{(417)}$	$T_d^{(417)}$	<i>Model: Method 1</i>
Robust				
Chien (1988). <i>Model: Method 1</i>	${}^8 K_c^{(418)}$	$2\lambda + \tau_m$	$\frac{\tau_m(\lambda + 0.25\tau_m)}{2\lambda + \tau_m}$	N=10

$${}^7 K_c^{(417)} = \frac{2.5\omega_u}{K_m} \left(-0.08 + \frac{0.055}{K_1} + \frac{0.059}{K_1^2} \right) \left(\frac{2}{2.3 - 2K_1} - \frac{(0.059/K_1^2) + (0.055/K_1) - 0.08}{0.4(10 + 1/K_1)} \right),$$

$$T_i^{(417)} = \frac{2.5}{\omega_u} \left(\frac{2}{2.3 - 2K_1} - \frac{(0.059/K_1^2) + (0.055/K_1) - 0.08}{0.4(10 + 1/K_1)} \right),$$

$$T_d^{(417)} = \frac{2.5}{\omega_u} \left(\frac{2}{2.3 - 2K_1} - \frac{(0.059/K_1^2) + (0.055/K_1) - 0.08}{0.4(10 + 1/K_1)} \right)^{-1} \left(\frac{1}{N} + 1 \right)^{-1},$$

$$N = \frac{T_d^{(417)}}{T_f^{(417)}}, \quad T_f^{(417)} = \frac{(0.059/K_1^2) + (0.055/K_1) - 0.08}{0.16\omega_u(10 + 1/K_1)}, \quad K_1 \in [0.5, 1].$$

$${}^8 K_c^{(418)} = \frac{2\lambda + \tau_m}{K_m (0.5\lambda + \tau_m)^2}; \quad \lambda \in [1/K_m, \tau_m] \quad (\text{Chien and Fruehauf (1990)}).$$

4.3.5 Controller with filtered derivative and dynamics on the controlled variable

$$G_c(s) = K_c \left(1 + \frac{1}{T_i s} + \frac{T_d s}{1 + \frac{s T_d}{N}} \right) E(s) - K_0 Y(s)$$

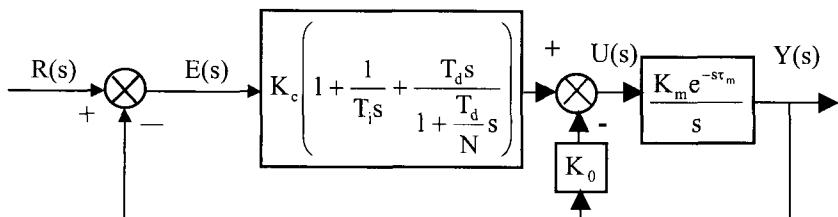


Table 84: PID controller tuning rules – IPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{s}$

Rule	K_c	T_i	T_d	Comment
Direct synthesis – time domain criteria				
Normey-Rico <i>et al.</i> (2001). <i>Model: Method 1</i>	$^9 K_c^{(419)}$	$(1.5 - \gamma)\tau_m$	$T_d^{(419)}$	$K_0 = \frac{1}{2K_m \tau_m}$
	$\frac{0.1716}{K_m \tau_m}$	τ_m	$0.5\tau_m$	$N = 1$; $K_0 = \frac{1}{2K_m \tau_m}$
	Recommended $\gamma = 0.5$; non-oscillatory response			

$$^9 K_c^{(419)} = \frac{1.5 - \gamma}{K_m \tau_m} \left[4\gamma + 1 - 4\sqrt{\gamma^2 + 0.5\gamma} \right], \quad T_d^{(419)} = \frac{\tau_m}{1.5 - \gamma} - \gamma\tau_m, \quad N = \frac{1 - \gamma(1.5 - \gamma)}{(1.5 - \gamma)\gamma}.$$

4.3.6 Classical controller 1 $G_c(s) = K_c \left(1 + \frac{1}{T_i s} \right) \left(\frac{1 + T_d s}{1 + \frac{T_d s}{N}} \right)$

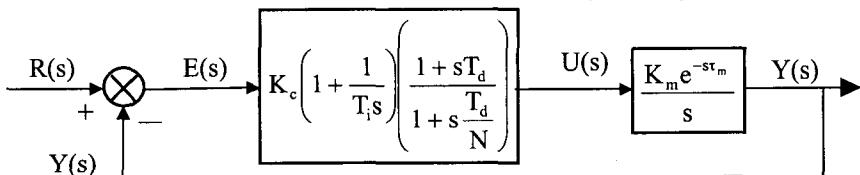


Table 85: PID controller tuning rules – IPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{s}$

Rule	K_c	T_i	T_d	Comment
Process reaction				
Bunzemeier (1998). <i>Model:</i> Method 4; $G_p =$ K_p $s(1 + sT_p)^n$	$\frac{x_1}{K_m \tau_m}$	$x_2 \tau_m$	$x_3 \tau_m$	0% overshoot (servo)
	$\frac{x_4}{K_m \tau_m}$			10% overshoot (servo)
Coefficient values				
x_1	x_2	x_3	x_4	N
0.21	0.730	0.280	0.24	5.60
0.68	1.570	0.796	0.94	5.99
0.47	1.200	0.669	0.62	6.03
0.35	0.981	0.587	0.46	5.99
0.30	0.866	0.530	0.38	6.02
0.27	0.828	0.487	0.34	6.01
0.25	0.801	0.452	0.32	6.03
0.23	0.782	0.424	0.30	5.97
0.22	0.767	0.401	0.29	5.99
0.22	0.756	0.381	0.27	5.95
Minimum performance index: regulator tuning				
Minimum IAE – Shinskey (1988) – page 143.	$\frac{0.93}{K_m \tau_m}$	$1.57 \tau_m$	$0.58 \tau_m$	<i>Model:</i> Method 2; N not specified
Minimum IAE – Shinskey (1994) – page 74. <i>Model:</i> Method 1	$\frac{0.93}{K_m \tau_m}$	$1.60 \tau_m$	$0.58 \tau_m$	$N=10$
	$\frac{0.93}{K_m \tau_m}$	$1.48 \tau_m$	$0.63 \tau_m$	$N=20$

Rule	K_c	T_i	T_d	Comment
Minimum IAE – Shinskey (1996) – page 117.	$\frac{0.93}{K_m \tau_m}$	$1.57\tau_m$	$0.56\tau_m$	<i>Model: Method 2; N not specified</i>
Minimum IAE – Shinskey (1996) – page 121.	$0.56K_u$	$0.39T_u$	$0.15T_u$	<i>Model: Method 1; N not specified</i>
Robust				
Chien (1988). <i>Model: Method 1</i>	$^{10} K_c^{(420)}$	$0.5\tau_m$	$2\lambda + 0.5\tau_m$	$\lambda \in [1/K_m, \tau_m]$ (Chien and Fruehauf (1990)); N=10
	$^{10} K_c^{(421)}$	$2\lambda + 0.5\tau_m$	$0.5\tau_m$	
Rice and Cooper (2002). <i>Model: Method 1</i>	$^{11} K_c^{(422)}$	$2\lambda + \tau_m$	$0.5\tau_m$	Suggested $\lambda = 3.162\tau_m$
Ultimate cycle				
Luyben (1996). <i>Model: Method 9</i>	$0.46K_u$	$2.2T_u$	$0.16T_u$	Maximum closed loop log modulus of +2dB ; N=10
Belanger and Luyben (1997). <i>Model: Method 1</i>	$3.11K_u$	$2.2T_u$	$3.64T_u$	N=0.1

$$^{10} K_c^{(420)} = \frac{1}{K_m} \left(\frac{0.5\tau_m}{[\lambda + 0.5\tau_m]^2} \right), \quad K_c^{(421)} = \frac{1}{K_m} \left(\frac{2\lambda + 0.5\tau_m}{[\lambda + 0.5\tau_m]^2} \right).$$

$$^{11} K_c^{(422)} = \frac{2\lambda + \tau_m}{K_m (\lambda^2 + 2\lambda\tau_m + 0.5\tau_m^2)}, \quad N = \frac{\lambda^2 + 2\lambda\tau_m + 0.5\tau_m^2}{\lambda^2}.$$

4.3.7 Classical controller 2 $G_c(s) = K_c \left(1 + \frac{1}{T_i s} \right) \left(\frac{1 + NT_d s}{1 + T_d s} \right)$

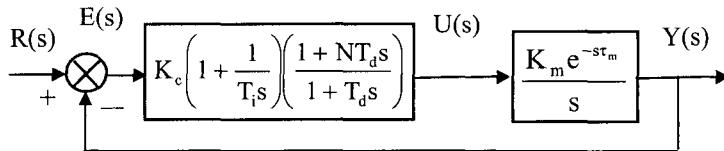


Table 86: PID controller tuning rules – IPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{s}$

Rule	K_c	T_i	T_d	Comment
Direct synthesis – frequency domain criteria				
Hougen (1979) – page 333-334. Model: Method 1	¹² $K_c^{(423)}$	∞	$0.45 \frac{\tau_m}{N}$	Maximise crossover frequency $N \in [10,30]$
Hougen (1988). Model: Method 1	¹³ $K_c^{(424)}$	∞	$10^{\lceil \log_{10} \tau_m + 0.65 \rceil}$	Five criteria are fulfilled; $N = 10$.

¹² Values deduced from graph:

$K_m \tau_m$	0.1	0.2	0.3	0.4	0.5
$K_c^{(423)}$	9	4.2	2.8	2.1	1.7
$K_m \tau_m$	0.6	0.7	0.8	0.9	1.0
$K_c^{(423)}$	1.45	1.2	1.1	0.95	0.85

¹³ Equations deduced from graph; $K_c^{(424)} \approx \frac{1}{K_m} 10^{\left\lceil \log_{10} \left(\frac{\tau_m}{T_m} \right) \right\rceil}$, $K_m = \frac{K_c}{T_m}$.

4.3.8 Classical controller 4 $G_c(s) = K_c \left(1 + \frac{1}{T_i s} \right) \left(1 + \frac{T_d s}{1 + \frac{T_d s}{N}} \right)$

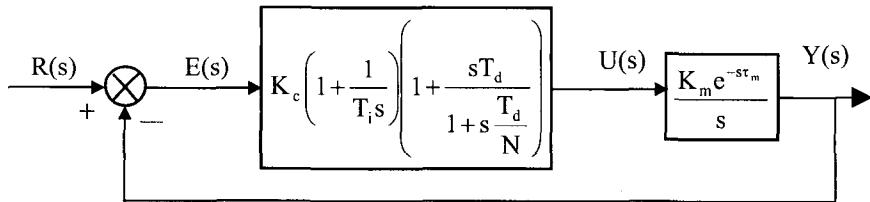


Table 87: PID controller tuning rules – IPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{s}$

Rule	K_c	T_i	T_d	Comment
Robust				
Chien (1988). Model: Method 1	¹ $K_c^{(425)}$	$2\lambda + 0.5\tau_m$	$0.5\tau_m$	$\lambda \in [1/K_m, \tau_m]$
	² $K_c^{(426)}$	$0.5\tau_m$	$2\lambda + 0.5\tau_m$	(Chien and Fruehauf (1990)); N=10

$$^1 K_c^{(425)} = \frac{1}{K_m} \left(\frac{2\lambda + 0.5\tau_m}{[\lambda + 0.5\tau_m]^2} \right)$$

$$^2 K_c^{(426)} = \frac{1}{K_m} \left(\frac{0.5\tau_m}{[\lambda + 0.5\tau_m]^2} \right)$$

4.3.9 Non-interacting controller based on the two degree of freedom structure 1

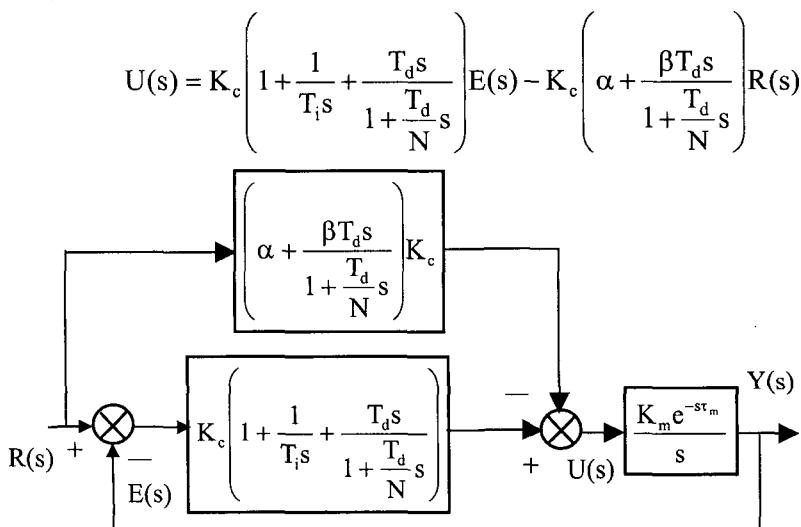


Table 88: PID controller tuning rules – IPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{s}$

Rule	K_c	T_i	T_d	Comment
Minimum performance index: servo/regulator tuning				
Minimum ITAE – Pecharromán and Pagola (2000). <i>Model:</i> <i>Method 10</i>	$x_1 K_u$	$x_2 T_u$	$x_3 T_u$	
	Coefficient values			
	x_1	x_2	x_3	α
	1.672	0.366	0.136	0.601
	1.236	0.427	0.149	0.607
	0.994	0.486	0.155	0.610
	0.842	0.538	0.154	0.616
	0.752	0.567	0.157	0.605
	0.679	0.610	0.149	0.610
	0.635	0.637	0.142	0.612

Rule	K_c	T_i	T_d	Comment
Taguchi and Araki (2000). <i>Model: Method 1</i>	$\frac{1}{K_m} \left(\frac{1.253}{\tau_m} \right)$	$2.388\tau_m$	$0.4137\tau_m$	$\frac{\tau_m}{T_m} \leq 1.0$
$\alpha = 0.6642, \beta = 0.6797, \text{Overshoot (servo step)} \leq 20\%;$ N fixed but not specified				
Direct synthesis				
Hansen (2000). <i>Model: Method 1</i>	$0.938K_m\tau_m$	$2.7\tau_m$	$0.313\tau_m$	$N = 0; \beta = 1;$ $\alpha = 0.833$
Chidambaram (2000b). <i>Model: Method 1</i>	$\frac{0.9425}{K_m\tau_m}$	$2\tau_m$	$0.5\tau_m$	$N = \infty; \beta = 1;$ $\alpha = 0.707 \text{ or}$ $\alpha = 0.65$
Chidambaram and Sree (2003). <i>Model: Method 1</i>	$\frac{1.2346}{K_m\tau_m}$	$4.5\tau_m$	$0.45\tau_m$	$N = 0; \beta = 0;$ $\alpha = 0.6$

4.3.10 Non-interacting controller based on the two degree of freedom structure 3

$$U(s) = K_c [(b-1) + (c-1)T_d s] R(s) + K_c \left(1 + \frac{1}{T_i s} + T_d s \right) \left[R(s) - \frac{1}{(1+sT_f)^2} Y(s) \right]$$

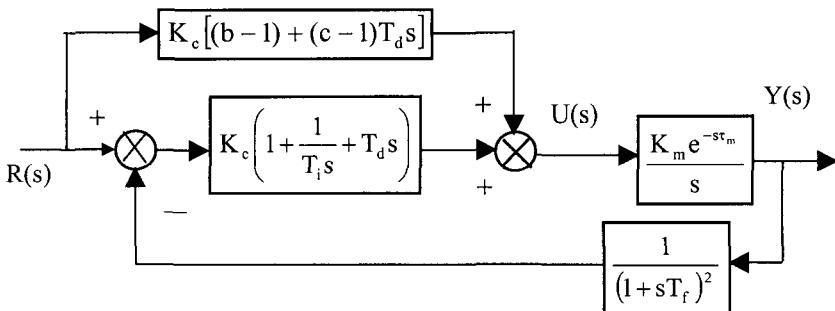


Table 89: PID controller tuning rules – IPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{s}$

Rule	K_c	T_i	T_d	Comment
Minimum performance index: other tuning				
Åström and Hägglund (2004). Model: Method 1	$\frac{0.45}{K_m}$	$8\tau_m$	$0.5\tau_m$	No precisely defined performance specification
	$b = 0, \frac{\tau_m}{T_m} \leq 1$	$b = 1, \frac{\tau_m}{T_m} > 1$	$c = 0$	$T_f = 0.1\tau_m$

4.3.11 Non-interacting controller 4

$$U(s) = K_c \left(1 + \frac{1}{T_i s} \right) E(s) - K_c T_d s Y(s)$$

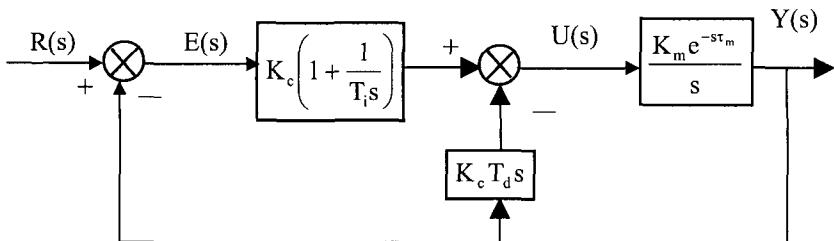


Table 90: PID controller tuning rules – IPD model $\frac{K_m e^{-st_m}}{s}$

Rule	K_c	T_i	T_d	Comment
Minimum performance index: regulator tuning				
Minimum IAE – Shinskey (1988) – page 143. <i>Model: Method 2</i>	$\frac{1.28}{K_m \tau_m}$	$1.90\tau_m$	$0.48\tau_m$	Also defined by Shinskey (1996), page 117.
Minimum IAE - Shinskey (1994) – page 74.	$\frac{1.28}{K_m \tau_m}$	$1.90\tau_m$	$0.46\tau_m$	<i>Model: Method 1</i>
Minimum IAE – Shinskey (1988) – page 148.	$0.82K_u$	$0.48T_u$	$0.12T_u$	<i>Model: Method 1</i>
Minimum IAE – Shinskey (1996) – page 121.	$0.77K_u$	$0.48T_u$	$0.12T_u$	<i>Model: Method 1</i>

4.3.12 Non-interacting controller 6 (I-PD controller)

$$U(s) = \frac{K_c}{T_i s} E(s) - K_c (1 + T_d s) Y(s)$$

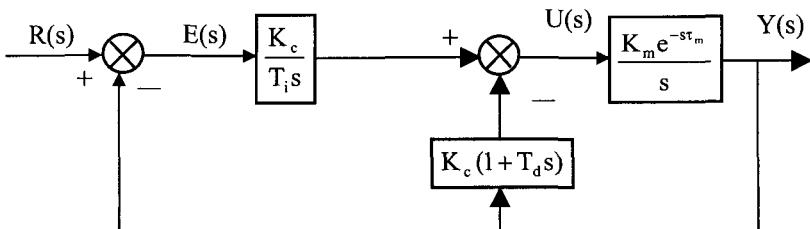


Table 91: PID controller tuning rules – IPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{s}$

Rule	K_c	T_i	T_d	Comment
Minimum performance index: regulator tuning				
Minimum ISE– Arvanitis <i>et al.</i> (2003a).	$\frac{1.4394}{K_m \tau_m}$	$2.4569\tau_m$	$0.3982\tau_m$	<i>Model: Method 1</i>
Arvanitis <i>et al.</i> (2003a).	$\frac{1.2986}{K_m \tau_m}$	$3.2616\tau_m$	$0.4234\tau_m$	<i>Model: Method 1</i>
Minimise performance index $\int_0^\infty [e^2(t) + K_m^2 u^2(t)] dt$				
Arvanitis <i>et al.</i> (2003a).	$\frac{1.1259}{K_m \tau_m}$	$6.7092\tau_m$	$0.4627\tau_m$	<i>Model: Method 1</i>
Minimise performance index $\int_0^\infty [e^2(t) + K_m^2 \left(\frac{du}{dt} \right)^2] dt$				
Minimum performance index: servo tuning				
Minimum ISE– Arvanitis <i>et al.</i> (2003a).	$\frac{1.5521}{K_m \tau_m}$	$2.1084\tau_m$	$0.3814\tau_m$	<i>Model: Method 1</i>
Arvanitis <i>et al.</i> (2003a).	$\frac{1.4057}{K_m \tau_m}$	$2.5986\tau_m$	$0.4038\tau_m$	<i>Model: Method 1</i>
Minimise performance index $\int_0^\infty [e^2(t) + K_m^2 u^2(t)] dt$				

Rule	K_c	T_i	T_d	Comment
Arvanitis <i>et al.</i> (2003a).	$\frac{1.3332}{K_m \tau_m}$	$3.0010\tau_m$	$0.4167\tau_m$	<i>Model: Method 1</i>
	Minimise performance index $\int_0^\infty e^2(t) + K_m^2 \left(\frac{du}{dt} \right)^2 dt$			
Direct synthesis				
Chien <i>et al.</i> (1999). <i>Model: Method 2</i>	${}^3 K_c^{(427)}$	$1.414T_{CL2} + \tau_m$	$T_d^{(427)}$	Underdamped system response - $\xi = 0.707$
Arvanitis <i>et al.</i> (2003a).	$\frac{16(2\xi^2 + 1)}{(32\xi^2 + 4)K_m \tau_m}$	$(2\xi^2 + 1)\tau_m$	$\frac{16\xi^2 + 4}{16(2\xi^2 + 1)}\tau_m$	<i>Model: Method 1</i>

$${}^3 K_c^{(427)} = \frac{1.414T_{CL2} + \tau_m}{K_m \left(T_{CL2}^2 + 0.707T_{CL2}\tau_m + 0.25\tau_m^2 \right)}, \quad T_d^{(427)} = \frac{0.25\tau_m^2 + 0.707T_{CL2}\tau_m}{1.414T_{CL2} + \tau_m}.$$

4.3.13 Non-interacting controller 8

$$U(s) = K_c \left(1 + \frac{1}{T_i s} + T_d s \right) E(s) - K_i (1 + T_{di} s) Y(s)$$

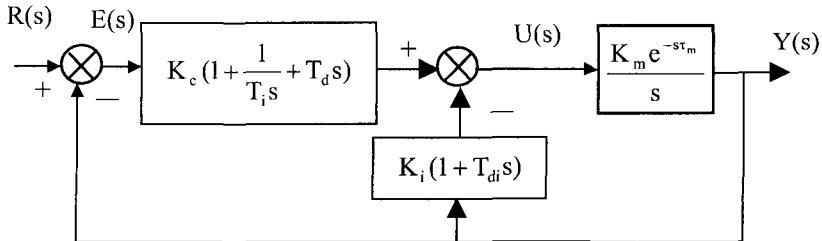


Table 92: PID controller tuning rules – IPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{s}$

Rule	K_c	T_i	T_d	Comment
Robust				
Lee and Edgar (2002). Model: Method 1	${}^4 K_c^{(428)}$	$T_i^{(428)}$	$T_d^{(428)}$	$K_i = 0.25K_u$, $T_{di} = 0$

$$\begin{aligned} {}^4 K_c^{(428)} &= \frac{0.25K_u}{(\lambda + \tau_m)} \left[\frac{4}{K_u K_m} - \tau_m + \frac{\tau_m^2}{2(\lambda + \tau_m)} \right], \quad T_i^{(428)} = \frac{4}{K_u K_m} - \tau_m + \frac{\tau_m^2}{2(\lambda + \tau_m)} \\ T_d^{(428)} &= \frac{0.25K_u}{(\lambda + \tau_m)} \left\{ 0.25\tau_m^2 - \frac{\tau_m^3}{6(\lambda + \tau_m)} + \frac{\tau_m^4}{4(\lambda + \tau_m)^2} + \frac{\tau_m^2(1 - 0.25K_u K_m \tau_m)}{0.5K_u K_m (\lambda + \tau_m)} \right\}. \end{aligned}$$

4.3.14 Non-interacting controller 10

$$U(s) = K_c \left(1 + \frac{1}{T_i s} \right) E(s) - K_f \left(\frac{T_d}{T_m} s + 1 \right) Y(s)$$

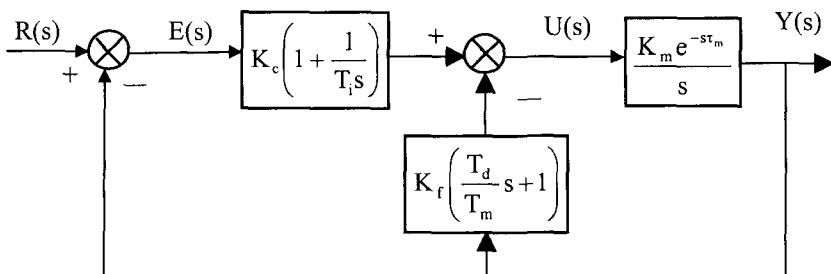


Table 93: PID tuning rules – IPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{s}$

Rule	K_c	T_i	T_d	Comment
Direct synthesis				
Kaya and Atherton (1999), Kaya (2003).	$0.313 \hat{K}_u$	$0.733 \hat{T}_u$	$^s T_d^{(429)}$	<i>Model: Method 9</i>

$$^s T_d^{(429)} = \frac{0.753 \left[\left(\frac{K_m \hat{K}_u}{\tau_m \hat{T}_u} \right)^{0.333} - \frac{1}{\tau_m} \right] \tau_m T_m}{\frac{0.567 \left(\frac{K_m \hat{K}_u}{\tau_m \hat{T}_u} \right)^{0.667} - 0.313 \hat{K}_u}{K_m}}.$$

4.3.15 Non-interacting controller 12

$$U(s) = K_c \left(1 + \frac{1}{T_i s} + T_d s \right) \frac{1}{T_d s + 1} E(s) - K_l Y(s)$$

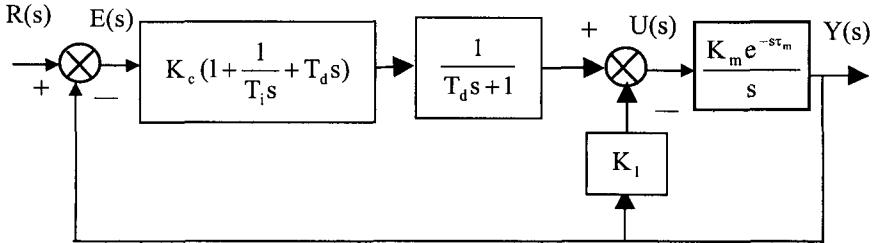


Table 94: PID controller tuning rules – IPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{s}$

Rule	K_c	T_i	T_d	Comment
Robust				
Lee and Edgar (2002). Model: Method 1	${}^6 K_c^{(430)}$	$T_i^{(430)}$	$T_d^{(430)}$	$K_l = 0.25K_u$

$$\begin{aligned} {}^6 K_c^{(430)} &= \frac{0.25K_u}{(\lambda + \tau_m)} \left\{ \frac{4}{K_u K_m} - \tau_m + \frac{\tau_m^2}{2(\lambda + \tau_m)} + \right. \\ &\quad \left. \tau_m \left[0.5 - \frac{\tau_m}{6(\lambda + \tau_m)} + \frac{\tau_m^2}{4(\lambda + \tau_m)^2} + \frac{(1 - 0.25K_u K_m \tau_m)}{0.5K_u K_m (\lambda + \tau_m)} \right]^{0.5} \right\}, \\ T_i^{(430)} &= \frac{4}{K_u K_m} - \tau_m + \frac{\tau_m^2}{2(\lambda + \tau_m)} + \\ &\quad \tau_m \left[0.5 - \frac{\tau_m}{6(\lambda + \tau_m)} + \frac{\tau_m^2}{4(\lambda + \tau_m)^2} + \frac{(1 - 0.25K_u K_m \tau_m)}{0.5K_u K_m (\lambda + \tau_m)} \right]^{0.5}, \\ T_d^{(430)} &= \tau_m \left[0.5 - \frac{\tau_m}{6(\lambda + \tau_m)} + \frac{\tau_m^2}{4(\lambda + \tau_m)^2} + \frac{(1 - 0.25K_u K_m \tau_m)}{0.5K_u K_m (\lambda + \tau_m)} \right]^{0.5}. \end{aligned}$$

4.4 FOLIPD Model $G_m(s) = \frac{K_m e^{-s\tau_m}}{s(1 + sT_m)}$

4.4.1 Ideal controller $G_c(s) = K_c \left(1 + \frac{1}{T_i s} + T_d s \right)$

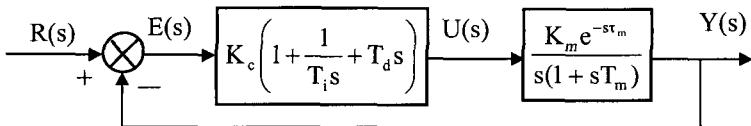


Table 95: PID controller tuning rules – FOLIPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{s(1 + sT_m)}$

Rule	K_c	T_i	T_d	Comment
Minimum performance index: regulator tuning				
Minimum ISE – Haalman (1965).	$\frac{2}{3K_m \tau_m}$	0	T_m	<i>Model: Method 1</i>
	$A_m = 2.36 ; \phi_m = 50^0 ; M_s = 1.9 .$			
Direct synthesis				
Van der Grinten (1963). <i>Model: Method 1</i>	$\frac{1}{K_m \tau_m}$	0	$T_m + 0.5\tau_m$	Step disturbance
	$\frac{e^{-2\omega_d \tau_m}}{K_m \tau_m}$	0	${}^1 T_d^{(431)}$	Stochastic disturbance
Tachibana (1984).	$\frac{\lambda}{K_m (1 + \lambda \tau_m)}$	0	T_m	<i>Model: Method 5</i>

$${}^1 T_d^{(431)} = \left(1 - 0.5 e^{-\omega_d \tau_m} + \frac{T_m}{\tau_m} e^{-\omega_d \tau_m} \right) \tau_m e^{-\omega_d \tau_m} .$$

Rule	K_c	T_i	T_d	Comment		
Chen <i>et al.</i> (1999a).	$\frac{\omega_p T_m}{K_m A_m T_d^{(432)}}$	0	$^2 T_d^{(432)}$	<i>Model: Method 1</i>		
	$\frac{r_l \omega_r T_m}{K_m T_d^{(433)}}$	0	$^2 T_d^{(433)}$	<i>Model: Method 1</i>		
Vitečková (1999), Vitečková <i>et al.</i> (2000a). <i>Model: Method 1</i>	$x_1 / K_m \tau_m$	0	T_m			
	Coefficient values					
	x_1	OS	x_1	OS	x_1	OS
	0.368	0%	0.641	15%	0.801	30%
	0.514	5%	0.696	20%	0.853	35%
	0.581	10%	0.748	25%	0.906	40%
O'Dwyer (2001a). <i>Model: Method 1</i>	$\frac{x_1 T_m}{K_m \tau_m}$	0	T_m	$A_m = 0.5/x_1$; $\phi_m = 0.5\pi - x_1$		
	Representative coefficient values					
	x_1	A_m	ϕ_m	x_1	A_m	ϕ_m
	0.785	2	45°	0.524	3	60°
Chen and Seborg (2002).	$^3 K_c^{(434)}$	$3T_{CL} + \tau_m$	$T_d^{(434)}$	<i>Model: Method 1</i>		
Robust						
Rivera and Jun (2000). <i>Model: Method 1</i>	$\frac{\tau_m + T_m + 2\lambda}{K_m(\tau_m + \lambda)^2}$	$\tau_m + T_m + 2\lambda$	$\frac{T_m(\tau_m + 2\lambda)}{\tau_m + T_m + 2\lambda}$	λ not specified		

$$^2 T_d^{(432)} = \frac{1}{2\omega_p - \frac{4\omega_p^2 \tau_m}{\pi} + \frac{1}{T_m}}, \quad T_d^{(433)} = \frac{1}{\omega_r - \frac{4\omega_r^2 \tau_m}{\pi} + \frac{1}{T_m}},$$

$$\omega_r = \frac{0.25\pi(2r_l A_m - 1)}{(r_l^2 A_m^2 - 1)\tau_m}.$$

$$^3 \left(\frac{y}{d} \right)_{desired} = \frac{(3T_{CL} + \tau_m)se^{-s\tau_m}}{K_c^{(434)}(T_{CL}s + 1)^3}, \quad K_c^{(434)} = \frac{(3T_{CL} + \tau_m)(T_m + \tau_m)}{(T_{CL} + \tau_m)^3},$$

$$T_d^{(434)} = \frac{3T_{CL}^2 T_m + 3T_{CL} T_m \tau_m - T_{CL}^3 + T_m \tau_m^2}{(3T_{CL} + \tau_m)(T_m + \tau_m)}.$$

Rule	K_c	T_i	T_d	Comment
Ultimate cycle				
McMillan (1984). Model: Method 1	${}^4 K_c^{(435)}$	$T_i^{(435)}$	$0.25T_i^{(435)}$	Tuning rules developed from K_u, T_u
Perić <i>et al.</i> (1997). Model: Method 8	$\hat{K}_u \cos \phi_m$	$\alpha T_d^{(436)}$	${}^5 T_d^{(436)}$	Recommended $\alpha = 4$

$${}^4 K_c^{(435)} = \frac{1.111}{K_m} \frac{T_m}{\tau_m^2} \left\{ \frac{1}{1 + \left(\frac{T_m}{\tau_m} \right)^{0.65}} \right\}^2, \quad T_i^{(435)} = 2\tau_m \left\{ 1 + \left(\frac{T_m}{\tau_m} \right)^{0.65} \right\}.$$

$${}^5 T_d^{(436)} = \left(\tan \phi_m + \sqrt{\frac{4}{\alpha} + \tan^2 \phi_m} \right) \frac{\hat{T}_u}{4\pi}.$$

4.4.2 Ideal controller in series with a first order lag

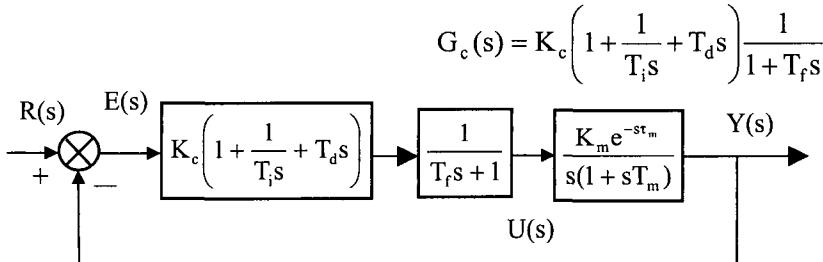


Table 96: PID controller tuning rules – FOLIPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{s(1+sT_m)}$

Rule	K_c	T_i	T_d	Comment
Robust				
H _∞ optimal – Tan <i>et al.</i> (1998b).	⁶ $K_c^{(437)}$	$T_i^{(437)}$	$T_d^{(437)}$	<i>Model: Method I</i>
$\lambda = 0.5$ (performance), $\lambda = 0.1$ (robustness), $\lambda = 0.25$ (acceptable)				
Zhang <i>et al.</i> (1999). <i>Model: Method I</i>	⁷ $K_c^{(438)}$	$3\lambda + T_m + \tau_m$	$\frac{(3\lambda + \tau_m)T_m}{3\lambda + \tau_m + T_m}$	$1.5\tau_m \leq \lambda \leq 4.5\tau_m$
$\lambda = 1.5\tau_m$, OS = 58%, $T_S = 6\tau_m$; $\lambda = 2.5\tau_m$, OS = 35%, $T_S = 11\tau_m$; $\lambda = 3.5\tau_m$, OS = 26%, $T_S = 16\tau_m$; $\lambda = 4.5\tau_m$, OS = 22%, $T_S = 20\tau_m$.				
Tan <i>et al.</i> (1998a). <i>Model: Method I; λ not specified</i>	⁸ $K_c^{(439)}$	$T_m + 8.1633\tau_m$	$\frac{T_m\tau_m}{0.1225T_m + \tau_m}$	$T_f = 0.5549\tau_m$
	⁸ $K_c^{(440)}$	$T_m + 5.3677\tau_m$	$\frac{T_m\tau_m}{0.1863T_m + \tau_m}$	$T_f = 0.4482\tau_m$

$$^6 K_c^{(437)} = \frac{(0.463\lambda + 0.277)([0.238\lambda + 0.123]T_m + \tau_m)}{K_m \tau_m^2}, \quad T_f = \frac{\tau_m}{5.750\lambda + 0.590},$$

$$T_i^{(437)} = T_m + \frac{\tau_m}{0.238\lambda + 0.123}, \quad T_d^{(437)} = \frac{T_m \tau_m}{(0.238\lambda + 0.123)T_m + \tau_m}.$$

$$^7 K_c^{(438)} = \frac{3\lambda + T_m + \tau_m}{K_m (3\lambda^2 + 3\lambda\tau_m + \tau_m^2)}, \quad T_f = \frac{\lambda^3}{3\lambda^2 + 3\lambda\tau_m + \tau_m^2}.$$

$$^8 K_c^{(439)} = \frac{0.0337T_m}{K_m \tau_m^2} \left(1 + \frac{\tau_m}{0.1225T_m} \right), \quad K_c^{(440)} = \frac{0.0754T_m}{K_m \tau_m^2} \left(1 + \frac{\tau_m}{0.1863T_m} \right).$$

Rule	K_c	T_i	T_d	Comment
Tan <i>et al.</i> , (1998a) – continued.	${}^9 K_c^{(441)}$	$T_m + 3.9635\tau_m$	$\frac{T_m \tau_m}{0.2523T_m + \tau_m}$	$T_f = 0.2863\tau_m$
Rivera and Jun (2000). <i>Model: Method 1</i>	${}^{10} K_c^{(442)}$	$2(\tau_m + \lambda) + T_m$	$\frac{2T_m(\tau_m + \lambda)}{2(\tau_m + \lambda) + T_m}$	λ not specified

$${}^9 K_c^{(441)} = \frac{0.1344T_m}{K_m \tau_m^2} \left(1 + \frac{\tau_m}{0.2523T_m} \right).$$

$${}^{10} K_c^{(442)} = \frac{2(\tau_m + \lambda) + T_m}{K_m (2\tau_m^2 + 4\tau_m \lambda + \lambda^2)}, \quad T_f = \frac{\tau_m \lambda^2}{2\tau_m^2 + 4\tau_m \lambda + \lambda^2}.$$

4.4.3 Ideal controller with weighted proportional term

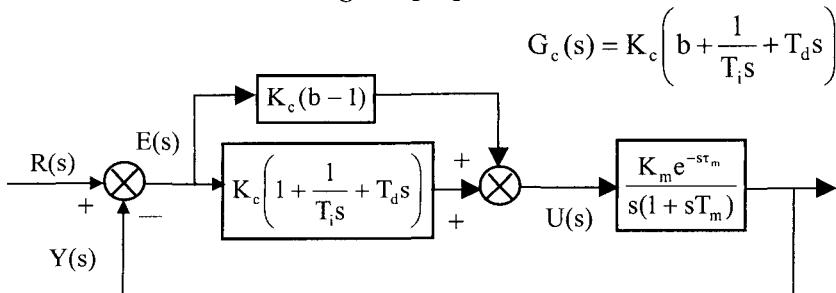


Table 97: PID controller tuning rules – FOLIPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{s(1+sT_m)}$

Rule	K_c	T_i	T_d	Comment
Direct synthesis				
Åström and Hägglund (1995) – pages 212-213. Model: Method 1 or Method 3	$\frac{5.6e^{-8.8\tau+6.8\tau^2}}{K_m(T_m + \tau_m)}$	$1.1\tau_m e^{6.7\tau-4.4\tau^2}$	$1.7\tau_m e^{-6.4\tau+2.0\tau^2}$	$M_{max} = 1.4$
		$b = 0.12e^{6.9\tau-6.6\tau^2}$		
	$\frac{8.6e^{-7.1\tau+5.4\tau^2}}{K_m(T_m + \tau_m)}$	$1.0\tau_m e^{3.3\tau-2.3\tau^2}$	$^{11}T_d^{(443)}$	$M_{max} = 2.0$

¹¹ $T_d^{(443)} = 0.38\tau_m e^{0.056\tau-0.60\tau^2}$.

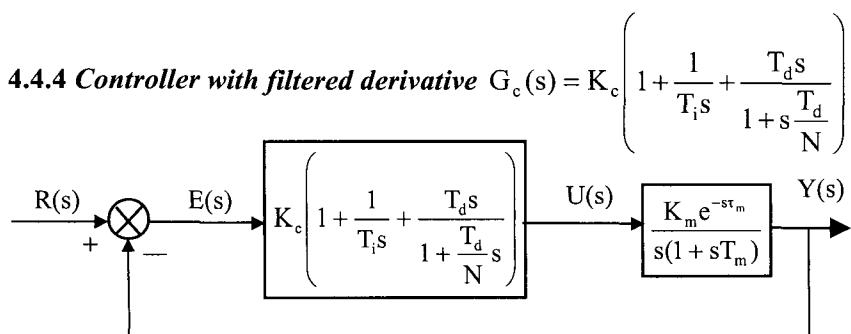


Table 98: PID controller tuning rules – FOLIPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{s(1 + sT_m)}$

Rule	K_c	T_i	T_d	Comment
Direct synthesis				
Kristiansson and Lennartson (2002).	$^{12} K_c^{(444)}$	$T_i^{(444)}$	$T_d^{(444)}$	<i>Model: Method 1; $K_1 \in [0.5, 1]$</i>
Robust				
Chien (1988). <i>Model: Method 1</i>	$\frac{2\lambda + \tau_m + T_m}{K_m(\lambda + \tau_m)^2}$	$2\lambda + T_m + \tau_m$	$\frac{T_m(2\lambda + \tau_m)}{2\lambda + T_m + \tau_m}$	$\lambda \in [\tau_m, T_m]$ (Chien and Fruehauf (1990)); $N = 10$

$$^{12} K_c^{(444)} = \frac{2.5 \omega_u}{K_m} \left(-0.08 + \frac{0.055}{K_1} + \frac{0.059}{K_1^2} \right) \\ \left(\frac{2}{2.3 - 2K_1} - \frac{(0.059/K_1^2) + (0.055/K_1) - 0.08}{0.4(10 + 1/K_1)} \right),$$

$$T_i^{(444)} = \frac{2.5}{\omega_u} \left(\frac{2}{2.3 - 2K_1} - \frac{(0.059/K_1^2) + (0.055/K_1) - 0.08}{0.4(10 + 1/K_1)} \right),$$

$$T_d^{(444)} = \frac{2.5}{\omega_u} \left(\frac{2}{2.3 - 2K_1} - \frac{(0.059/K_1^2) + (0.055/K_1) - 0.08}{0.4(10 + 1/K_1)} \right)^{-1} \left(\frac{1}{N} + 1 \right)^{-1},$$

$$N = \frac{T_d^{(444)}}{T_f^{(444)}}, \quad T_f^{(444)} = \frac{(0.059/K_1^2) + (0.055/K_1) - 0.08}{0.16\omega_u(10 + 1/K_1)}.$$

4.4.5 Controller with filtered derivative with set-point weighting 1

$$G_c(s) = K_c \left(1 + \frac{1}{T_i s} + \frac{T_d s}{1 + \frac{T_d}{N} s} \right) \left(\frac{1 + b_{f1} s}{1 + a_{f1} s} R(s) - Y(s) \right)$$

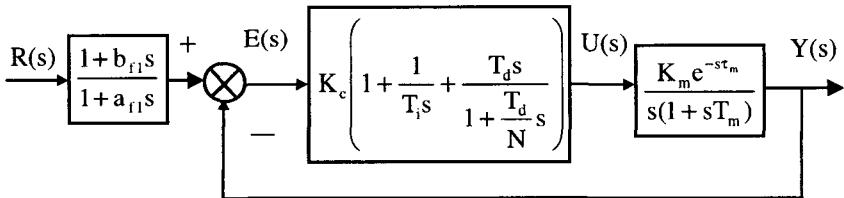


Table 99: PID controller tuning rules – FOLIPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{s(1+sT_m)}$

Rule	K_c	T_i	T_d	Comment		
Robust						
Liu <i>et al.</i> (2003). <i>Model:</i> Method I; recommended $\lambda = \tau_m$	$^{13} K_c^{(445)}$	$T_i^{(445)}$	$T_d^{(445)}$			
Representative λ values deduced from a graph						
	λ	OS	Rise time	λ	OS	Rise time
	$0.3\tau_m$	$\approx 57\%$	$\approx 3\tau_m$	$2\tau_m$	$\approx 0\%$	$\approx 8\tau_m$
	$0.5\tau_m$	$\approx 33\%$	$\approx 3.5\tau_m$	$2.5\tau_m$	$\approx 0\%$	$\approx 10.5\tau_m$
	τ_m	$\approx 7\%$	$\approx 5\tau_m$	$3\tau_m$	$\approx 0\%$	$\approx 12.5\tau_m$
	$1.5\tau_m$	$\approx 0\%$	$\approx 8.5\tau_m$	$3.5\tau_m$	$\approx 0\%$	$\approx 14\tau_m$

$$\begin{aligned}
 ^{13} K_c^{(445)} &= \frac{(3\lambda + \tau_m + T_m)(3\lambda^2 + 3\lambda\tau_m + \tau_m^2) - \lambda^3}{K_m (3\lambda^2 + 3\lambda\tau_m + \tau_m^2)^2}, \\
 T_i^{(445)} &= \frac{(3\lambda + \tau_m + T_m)(3\lambda^2 + 3\lambda\tau_m + \tau_m^2) - \lambda^3}{(3\lambda^2 + 3\lambda\tau_m + \tau_m^2)}, \\
 T_d^{(445)} &= \frac{T_m (3\lambda + \tau_m)(3\lambda^2 + 3\lambda\tau_m + \tau_m^2)}{(3\lambda + \tau_m + T_m)(3\lambda^2 + 3\lambda\tau_m + \tau_m^2) - \lambda^3} - \frac{\lambda^3}{3\lambda^2 + 3\lambda\tau_m + \tau_m^2}, \\
 N &= \frac{T_m (3\lambda + \tau_m)}{\lambda^3 [(3\lambda + \tau_m + T_m)(3\lambda^2 + 3\lambda\tau_m + \tau_m^2) - \lambda^3]} - 1, \quad b_{f1} = \lambda, \quad a_{f1} = 3\lambda + \tau_m.
 \end{aligned}$$

4.4.6 Controller with filtered derivative with set-point weighting 3

$$G_c(s) = K_c \left(1 + \frac{1}{T_i s} + \frac{T_d s}{1 + \frac{T_d}{N} s} \right) \frac{1 + b_{f1}s + b_{f2}s^2 + b_{f3}s^3}{1 + a_{f1}s + a_{f2}s^2 + a_{f3}s^3} R(s) - Y(s)$$

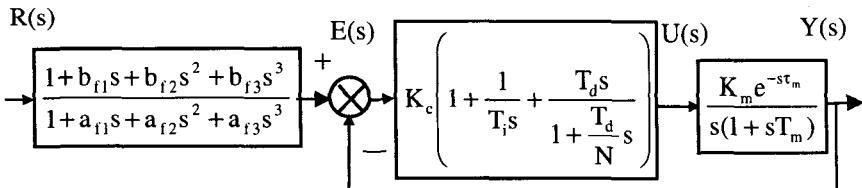


Table 100: PID controller tuning rules – FOLIPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{s(1 + sT_m)}$

Rule	K_c	T_i	T_d	Comment		
Robust						
Liu et al. (2003). Model: Method 1	${}^1 K_c^{(446)}$	$T_i^{(446)}$	$T_d^{(446)}$			
	$a_{f1} = 7\lambda + \tau_m ; a_{f2} = \lambda(16\lambda + 4\tau_m) , a_{f3} = 4\lambda^2(3\lambda + \tau_m)$					
Representative λ values (deduced from a graph)	λ	OS	Rise time	λ	OS	Rise time
	$0.3\tau_m$	$\approx 43\%$	$\approx 3.5\tau_m$	$2\tau_m$	$\approx 0\%$	$\approx 16.5\tau_m$
	$0.5\tau_m$	$\approx 15\%$	$\approx 4.5\tau_m$	$2.5\tau_m$	$\approx 0\%$	$\approx 20\tau_m$
	τ_m	$\approx 0\%$	$\approx 8\tau_m$	$3\tau_m$	$\approx 0\%$	$\approx 24\tau_m$
	$1.5\tau_m$	$\approx 0\%$	$\approx 12.5\tau_m$	$3.5\tau_m$	$\approx 0\%$	$\approx 28\tau_m$

$$\begin{aligned} {}^1 K_c^{(446)} &= \frac{(3\lambda + \tau_m + T_m)(3\lambda^2 + 3\lambda\tau_m + \tau_m^2) - \lambda^3}{K_m (3\lambda^2 + 3\lambda\tau_m + \tau_m^2)^2}, \\ T_i^{(446)} &= \frac{(3\lambda + \tau_m + T_m)(3\lambda^2 + 3\lambda\tau_m + \tau_m^2) - \lambda^3}{(3\lambda^2 + 3\lambda\tau_m + \tau_m^2)}, \quad b_{f1} = 3\lambda, \quad b_{f2} = 3\lambda^2, \quad b_{f3} = \lambda^3, \\ T_d^{(446)} &= \frac{T_m (3\lambda + \tau_m)(3\lambda^2 + 3\lambda\tau_m + \tau_m^2)}{(3\lambda + \tau_m + T_m)(3\lambda^2 + 3\lambda\tau_m + \tau_m^2) - \lambda^3} - \frac{\lambda^3}{3\lambda^2 + 3\lambda\tau_m + \tau_m^2}, \\ N &= \frac{T_m (3\lambda + \tau_m)}{\lambda^3 [(3\lambda + \tau_m + T_m)(3\lambda^2 + 3\lambda\tau_m + \tau_m^2) - \lambda^3]} - 1. \end{aligned}$$

Rule	K _c	T _i	T _d	Comment			
Liu <i>et al.</i> (2003) – continued.	$a_{f1} = 4\lambda + \tau_m ; a_{f2} = \lambda(3.25\lambda + \tau_m) , a_{f3} = 0.25\lambda^2(3\lambda + \tau_m)$						
	Representative λ values (deduced from a graph)						
	λ	OS	Rise time	λ	OS		
	$0.3\tau_m$	$\approx 61\%$	$\approx 3\tau_m$	$2\tau_m$	$\approx 4\%$		
	$0.5\tau_m$	$\approx 40\%$	$\approx 3\tau_m$	$2.5\tau_m$	$\approx 4\%$		
	τ_m	$\approx 15\%$	$\approx 3.5\tau_m$	$3\tau_m$	$\approx 1\%$		
	$1.5\tau_m$	$\approx 7\%$	$\approx 5\tau_m$	$3.5\tau_m$	$\approx 0\%$		
					$\approx 12\tau_m$		

4.4.7 Ideal controller with set-point weighting 1

$$U(s) = K_c \left(F_p R(s) - Y(s) \right) + \frac{K_c}{T_i s} \left(F_i R(s) - Y(s) \right) + K_c T_d s \left(F_d R(s) - Y(s) \right)$$

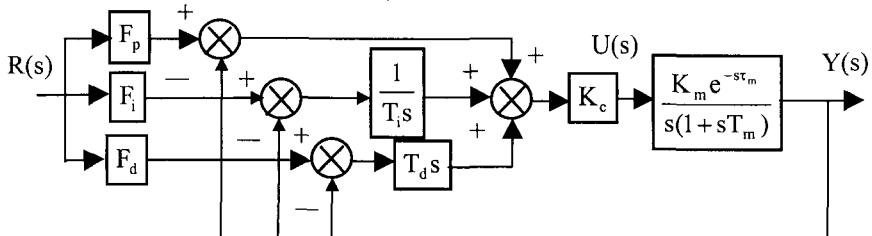


Table 101: PID controller tuning rules – FOLIPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{s(1+sT_m)}$

Rule	K_c	T_i	T_d	Comment
Ultimate cycle				
Oubrahim and Leonard (1998). Model: Method 1	$0.6K_u$, $F_p = 0.1$	$0.5T_u$, $F_i = 1$	$0.125T_u$, $F_d = 0.01$	$0.05 < \frac{\tau_m}{T_m} < 0.8$ 20% overshoot

4.4.8 Classical controller 1 $G_c(s) = K_c \left(1 + \frac{1}{T_i s}\right) \left(\frac{1 + T_d s}{1 + \frac{T_d s}{N}} \right)$

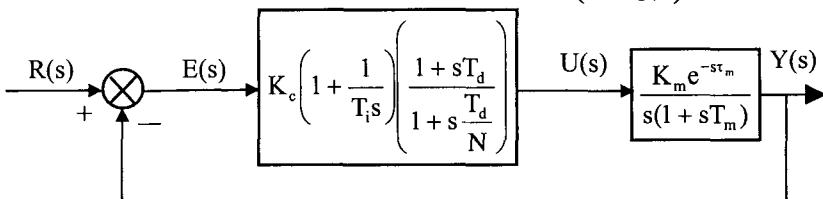


Table 102: PID controller tuning rules – FOLIPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{s(1+sT_m)}$

Rule	K_c	T_i	T_d	Comment
Minimum performance index: regulator tuning				
Minimum IAE – Shinskey (1994) – page 75.	$\frac{0.78}{K_m(\tau_m + T_m)}$	$1.38(\tau_m + T_m)$	$0.66(\tau_m + T_m)$	$\tau_m = T_m$; $N=10$. Model: Method 1
Minimum IAE – Shinskey (1994) – pages 158-159. Model: Method 1; N not specified	${}^2 K_c^{(447)}$ ${}^3 K_c^{(448)}$	$T_i^{(447)}$	$0.56\tau_m + 0.75T_m$	$\frac{\tau_m}{T_m} < 2$ $\frac{\tau_m}{T_m} \geq 2$

$${}^2 K_c^{(447)} = \frac{100}{108K_m \tau_m \left(1.22 - 0.03 \frac{T_m}{\tau_m}\right)}, \quad T_i^{(447)} = 1.57\tau_m \left(1 + 1.2 \left[1 - e^{-\frac{T_m}{\tau_m}}\right]\right).$$

$${}^3 K_c^{(448)} = \frac{100}{108K_m \tau_m \left(1 + 0.4 \frac{T_m}{\tau_m}\right)}.$$

Rule	K_c	T_i	T_d	Comment
Minimum ITAE – Poulin and Pomerleau (1996), (1997). <i>Model:</i> <i>Method 1</i>	${}^4 K_c^{(449)}$	$x_2(\tau_m + T_m)$	T_m	$0 \leq \frac{\tau_m}{(T_m/N)} \leq 2$ $3.33 \leq N \leq 10$
Coefficient values (deduced from graphs)				
	$\frac{\tau_m}{T_m/N}$	x_1	x_2	$\frac{\tau_m}{T_m/N}$
Process output step load disturbance	0.2	5.0728	0.5231	1.2
	0.4	4.9688	0.5237	1.4
	0.6	4.8983	0.5241	1.6
	0.8	4.8218	0.5245	1.8
	1.0	4.7839	0.5249	2.0
Process input step load disturbance	0.2	3.9465	0.5320	1.2
	0.4	3.9981	0.5315	1.4
	0.6	4.0397	0.5311	1.6
	0.8	4.0397	0.5311	1.8
	1.0	4.0397	0.5311	2.0
Direct synthesis				
Poulin et al. (1996). <i>Model: Method 1</i>	${}^5 K_c^{(450)}$	$21(0.2T_m + \tau_m)$	T_m	$N = 5$. $M_{max} = 1 \text{ dB}$
Robust				
Chien (1988). <i>Model: Method 1</i>	$\frac{T_m}{K_m(\lambda + \tau_m)^2}$	T_m	$2\lambda + \tau_m$	$\lambda \in [\tau_m, T_m]$ (Chien and Fruehauf (1990)); $N=10$
	$\frac{2\lambda + \tau_m}{K_m(\lambda + \tau_m)^2}$	$2\lambda + \tau_m$	T_m	

$${}^4 K_c^{(449)} = \frac{x_2}{K_m(\tau_m + T_m)} \sqrt{\frac{T_m^2}{x_1(\tau_m + T_m)^2} + 1}.$$

$${}^5 K_c^{(450)} = \frac{0.5455}{K_m(\tau_m + 0.2T_m)}.$$

4.4.9 Classical controller 2 $G_c(s) = K_c \left(1 + \frac{1}{T_i s} \right) \left(\frac{1 + NT_d s}{1 + T_d s} \right)$

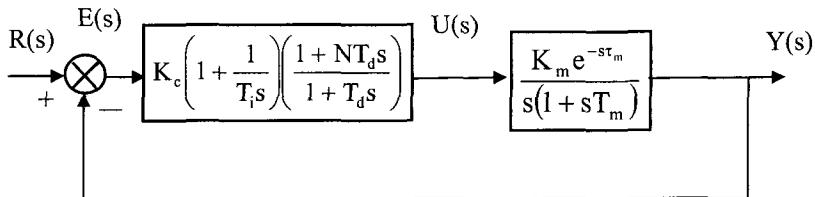


Table 103: PID controller tuning rules – FOLIPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{s(1 + sT_m)}$

Rule	K_c	T_i	T_d	Comment
Direct synthesis: frequency domain criteria				
Hougen (1979) – page 338. Model: Method 1	$^6 \frac{0.106x_1}{K_m \tau_m}$	∞	$0.135T_m^{0.7} \tau_m^{0.3}$	Maximise crossover frequency; $N \in [10,30]$

⁶ x_1 values deduced from graph:

τ_m/T_m	1	2	3	4	5	6
x_1	6	3	2.1	1.8	1.6	1.3
τ_m/T_m	7	8	9	10	11	12
x_1	1.1	1.0	0.9	0.8	0.7	0.7

4.4.10 Series controller (classical controller 3)

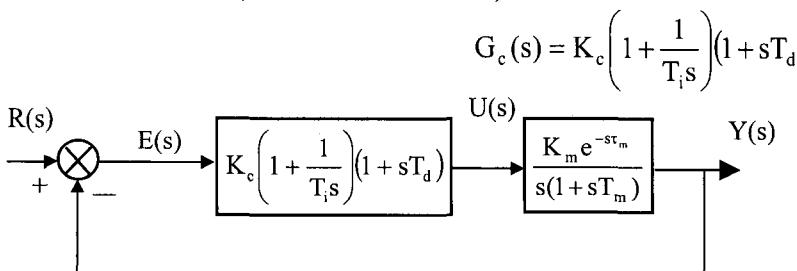


Table 104: PID tuning rules – FOLIPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{s(1 + sT_m)}$

Rule	K_c	T_i	T_d	Comment
Direct synthesis				
Skogestad (2003). <i>Model: Method 1</i>	$\frac{0.5}{K_m \tau_m}$	$8\tau_m$	T_m	

4.4.11 Classical controller 4 $G_c(s) = K_c \left(1 + \frac{1}{T_i s} \right) \left(1 + \frac{T_d s}{1 + \frac{T_d s}{N}} \right)$

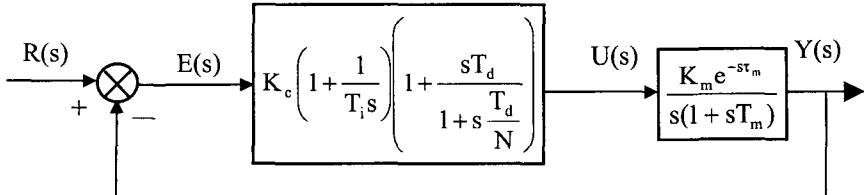


Table 105: PID controller tuning rules – FOLIPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{s(1+sT_m)}$

Rule	K_c	T_i	T_d	Comment
Robust				
Chien (1988). <i>Model: Method 1</i>	$\frac{2\lambda + \tau_m}{K_m (\lambda + \tau_m)^2}$	$2\lambda + \tau_m$	T_m	$\lambda \in [\tau_m, T_m]$ (Chien and Fruehauf (1990)); N=10
	$\frac{T_m}{K_m (\lambda + \tau_m)^2}$	T_m	$2\lambda + \tau_m$	

4.4.12 Non-interacting controller based on the two degree of freedom structure 1

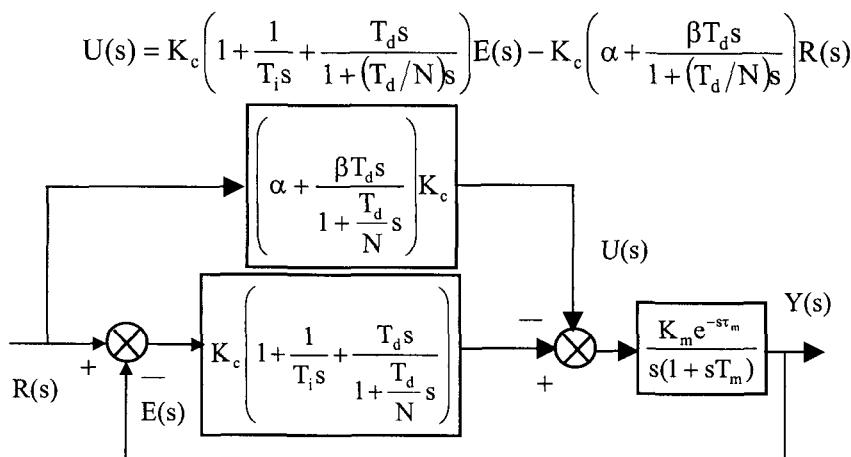


Table 106: PID controller tuning rules – FOLIPD model $G_m(s) = \frac{K_m e^{-st_m}}{s(1+sT_m)}$

Rule	K_c	T_i	T_d	Comment
Minimum performance index: servo/regulator tuning				
$x_1 K_u$		$x_2 T_u$		$x_3 T_u$
Coefficient values				
Minimum ITAE – Pecharromán and Pagola (2000). <i>Model:</i> <i>Method 4;</i> $K_m = 1$; $T_m = 1$; $\beta = 1$, $N = 10$; $0.05 < \tau_m < 0.8$	x_1	x_2	x_3	α
	1.672	0.366	0.136	0.601
	1.236	0.427	0.149	0.607
	0.994	0.486	0.155	0.610
	0.842	0.538	0.154	0.616
	0.752	0.567	0.157	0.605
	0.679	0.610	0.149	0.610
	0.635	0.637	0.142	0.612
	0.590	0.669	0.133	0.610
	0.551	0.690	0.114	0.616
	0.520	0.776	0.087	0.609
	0.509	0.810	0.068	0.611
				ϕ_c
				– 164°
				– 160°
				– 155°
				– 150°
				– 145°
				– 140°
				– 135°
				– 130°
				– 125°
				– 120°
				– 118°

Rule	K_c	T_i	T_d	Comment
Taguchi and Araki (2000). Model: Method 1	¹ $K_c^{(451)}$	$T_i^{(451)}$	$T_d^{(451)}$	$\frac{\tau_m}{T_m} \leq 1.0$
Direct synthesis				
Wang and Cai (2001). Model: Method 1	² $K_c^{(452)}$	$T_i^{(452)}$	$T_d^{(452)}$	$N=0; \beta = 0$.
	$\frac{0.6068}{K_m \tau_m}$	$5.9784 \tau_m$	${}^3 T_d^{(453)}$	
Wang and Cai (2002).	$\frac{0.6189}{K_m \tau_m}$	$5.9112 \tau_m$	${}^4 T_d^{(454)}$	Model: Method 1
Xu and Shao (2003a).	⁵ $K_c^{(455)}$	$2(T_m + 1.5\tau_m)$	$\frac{(T_m + 0.5\tau_m)^2}{(T_m + 1.5\tau_m)}$	Model: Method 1
Xu and Shao (2003c).	$\frac{1.5\tau_m + 0.5T_m}{2K_m \tau_m^2}$	$T_m + \tau_m$	$\frac{1.5T_m \tau_m}{1.5\tau_m + 0.5T_m}$	⁶ Model: Method 1

$${}^1 K_c^{(451)} = \frac{1}{K_m} \left(0.7608 + \frac{0.5184}{[\frac{\tau_m}{T_m} + 0.01308]^2} \right),$$

$$T_i^{(451)} = T_m \left(0.03330 + 3.997 \frac{\tau_m}{T_m} - 0.5517 \left[\frac{\tau_m}{T_m} \right]^2 \right),$$

$$T_d^{(451)} = T_m \left(0.03432 + 2.058 \frac{\tau_m}{T_m} - 1.774 \left[\frac{\tau_m}{T_m} \right]^2 + 0.6878 \left[\frac{\tau_m}{T_m} \right]^3 \right),$$

$$\alpha = 0.6647, \beta = 0.8653 - 0.1277(\tau_m/T_m) + 0.03330(\tau_m/T_m)^2,$$

N fixed but not specified; OS $\leq 20\%$.

$${}^2 K_c^{(452)} = \frac{1}{K_m \tau_m} \left[1.3608 - \frac{1.2064}{M_s} \right], T_i^{(452)} = \frac{\tau_m (1.3608 M_s - 1.2064)}{0.29 M_s - 0.3016},$$

$$T_d^{(452)} = \frac{(T_m + 0.1\tau_m)(1.450 M_s - 1.508)}{1.3608 M_s - 1.2064}, \alpha = \frac{0.2 M_s}{1.3608 M_s - 1.2064}, 1.3 \leq M_s \leq 2.$$

$${}^3 T_d^{(453)} = 0.8363(T_m + 0.1\tau_m), \alpha = 0.3296, M_s = 1.6 (A_m > 2.66, \phi_m > 36.4^\circ).$$

$${}^4 T_d^{(454)} = 0.8460(T_m + 0.1\tau_m), N=0, \beta = 0, \alpha = 0.3232, A_m = 3, \phi_m = 60^\circ.$$

$${}^5 K_c^{(455)} = \frac{0.5(T_m + 1.5\tau_m)}{K_m \tau_m (T_m + \tau_m)}, \alpha = \frac{\tau_m}{T_m + 1.5\tau_m}, N=\infty, \beta = 0, A_m > 2, \phi_m > 60^\circ.$$

$${}^6 \alpha = \frac{\tau_m}{1.5T_m + 0.5\tau_m}, \beta = 0.6667, N=\infty, A_m > 2, \phi_m > 60^\circ.$$

4.4.13 Non-interacting controller 4

$$U(s) = K_c \left(1 + \frac{1}{T_i s} \right) E(s) - K_c T_d s Y(s)$$

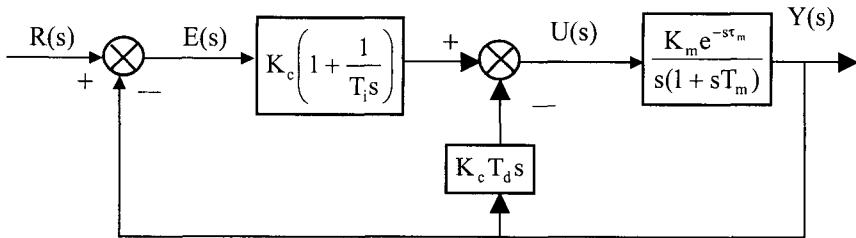


Table 107: PID controller tuning rules – FOLIPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{s(1+sT_m)}$

Rule	K_c	T_i	T_d	Comment
Minimum performance index: regulator tuning				
Minimum IAE - Shinskey (1994) – page 75.	$\frac{1.16}{K_m(\tau_m + T_m)}$	$1.38(\tau_m + T_m)$	$0.55(\tau_m + T_m)$	<i>Model: Method 1</i>
Minimum IAE - Shinskey (1994) – page 159.	$^7 K_c^{(456)}$	$T_i^{(456)}$	$0.48\tau_m + 0.7T_m$	<i>Model: Method 1</i>

⁷ $K_c^{(456)} = \frac{1.28}{K_m \tau_m (1 + 0.24 \frac{T_m}{\tau_m} - 0.14 \left[\frac{T_m}{\tau_m} \right]^2)}, T_i^{(456)} = 1.9 \tau_m \left(1 + 0.75 [1 - e^{-\frac{T_m}{\tau_m}}] \right).$

4.4.14 Non-interacting controller 6 (I-PD controller)

$$U(s) = \frac{K_c}{T_i s} E(s) - K_c (1 + T_d s) Y(s)$$

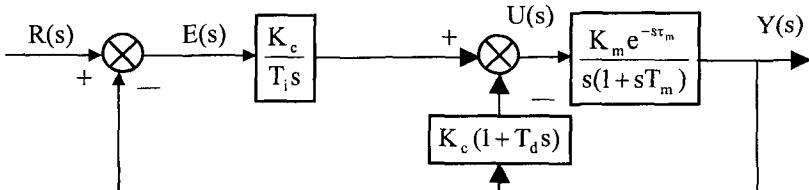


Table 108: PID tuning rules ~FOLIPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{s(1+sT_m)}$

Rule	K_c	T_i	T_d	Comment
Minimum performance index: regulator tuning				
Arvanitis <i>et al.</i> (2003b).	$\frac{16}{(16 - 3\beta)K_m \tau_m}$	$\frac{4}{\beta} \tau_m$	$\frac{(8 - \beta)}{16} \tau_m$	<i>Model: Method 1</i>
Minimum ISE ⁸	Minimum $\int_0^\infty [e^2(t) + K_m^2 u^2(t)] dt$ ⁹			

$${}^8 \beta = 1.5841 + 0.3101 \left(\frac{\tau_m}{T_m} \right) + 0.38841 \left(\frac{\tau_m}{T_m} \right)^2 - 0.4622 \left(\frac{\tau_m}{T_m} \right)^3 + 0.21918 \left(\frac{\tau_m}{T_m} \right)^4 \\ - 0.060691 \left(\frac{\tau_m}{T_m} \right)^5 + 0.010771 \left(\frac{\tau_m}{T_m} \right)^6 - 0.0012466 \left(\frac{\tau_m}{T_m} \right)^7 + 9.1201 \cdot 10^{-5} \left(\frac{\tau_m}{T_m} \right)^8 \\ - 3.8302 \cdot 10^{-6} \left(\frac{\tau_m}{T_m} \right)^9 + 7.0316 \cdot 10^{-8} \left(\frac{\tau_m}{T_m} \right)^{10}.$$

$${}^9 \beta = 0.066485 + 2.0757 \left(\frac{\tau_m}{T_m} \right) + 1.0972 \left(\frac{\tau_m}{T_m} \right)^2 - 2.7957 \left(\frac{\tau_m}{T_m} \right)^3 + 1.9019 \left(\frac{\tau_m}{T_m} \right)^4 \\ - 0.68418 \left(\frac{\tau_m}{T_m} \right)^5 + 0.14763 \left(\frac{\tau_m}{T_m} \right)^6 - 0.019733 \left(\frac{\tau_m}{T_m} \right)^7 + 0.0016016 \left(\frac{\tau_m}{T_m} \right)^8 \\ - 7.2368 \cdot 10^{-5} \left(\frac{\tau_m}{T_m} \right)^9 + 1.397 \cdot 10^{-6} \left(\frac{\tau_m}{T_m} \right)^{10}, \quad 0 < \frac{\tau_m}{T_m} < 4.0;$$

$$\beta = 2.128638 - 0.00560812 \left(\frac{\tau_m}{T_m} - 4.0 \right), \quad \frac{\tau_m}{T_m} \geq 4.0.$$

Rule	K_c	T_i	T_d	Comment
Arvanitis <i>et al.</i> (2003b). Model: Method 1	$^{10} K_c^{(457)}$	$T_i^{(457)}$	$T_d^{(457)}$	
				Minimum ISE ¹¹
				Minimum performance index ^{12 13} ,

$$^{10} K_c^{(457)} = \frac{12.5664(\tau_m + T_m) - 4}{K_m \tau_m [25.1328(\tau_m + T_m) - 8 - (\tau_m + 2T_m)a]} \quad \text{with } a = [0.5708 + \sqrt{\frac{3.4674\tau_m + 3.1416T_m - 1}{\tau_m}}]\chi,$$

$$T_i^{(457)} = \frac{12.5664(\tau_m + T_m) - 4}{a}, \quad T_d^{(457)} = T_m - T_m^2 \frac{a}{12.5664(\tau_m + T_m) - 4}.$$

$$\begin{aligned} ^{11} \chi = & 1.178 + 4.5663\left(\frac{\tau_m}{T_m}\right) - 7.0371\left(\frac{\tau_m}{T_m}\right)^2 + 5.5784\left(\frac{\tau_m}{T_m}\right)^3 - 2.6103\left(\frac{\tau_m}{T_m}\right)^4 \\ & + 0.7679\left(\frac{\tau_m}{T_m}\right)^5 - 0.1458\left(\frac{\tau_m}{T_m}\right)^6 + 0.017833\left(\frac{\tau_m}{T_m}\right)^7 - 0.0013561\left(\frac{\tau_m}{T_m}\right)^8 \\ & + 5.83 \cdot 10^{-5}\left(\frac{\tau_m}{T_m}\right)^9 - 1.0823 \cdot 10^{-6}\left(\frac{\tau_m}{T_m}\right)^{10}, \quad 0 < \frac{\tau_m}{T_m} < 3.5, \\ \chi = & 2.2036 - 0.0033077\left(\frac{\tau_m}{T_m} - 6.5\right), \quad \frac{\tau_m}{T_m} \geq 3.5. \end{aligned}$$

$$^{12} \text{Performance index} = \int_0^\infty [e^2(t) + K_m^2 u^2(t)] dt.$$

$$\begin{aligned} ^{13} \chi = & -0.1185 + 4.781\left(\frac{\tau_m}{T_m}\right) - 3.9977\left(\frac{\tau_m}{T_m}\right)^2 + 1.733\left(\frac{\tau_m}{T_m}\right)^3 - 0.42245\left(\frac{\tau_m}{T_m}\right)^4 \\ & + 0.057708\left(\frac{\tau_m}{T_m}\right)^5 - 0.0038421\left(\frac{\tau_m}{T_m}\right)^6 + 2.6386 \cdot 10^{-5}\left(\frac{\tau_m}{T_m}\right)^7 + 1.078 \cdot 10^{-5}\left(\frac{\tau_m}{T_m}\right)^8 \\ & - 4.2486 \cdot 10^{-7}\left(\frac{\tau_m}{T_m}\right)^9. \end{aligned}$$

Rule	K_c	T_i	T_d	Comment	
Minimum performance index: servo tuning					
Arvanitis <i>et al.</i> (2003b).	$\frac{16}{(16 - 3\beta)K_m \tau_m}$	$\frac{4}{\beta} \tau_m$	$\frac{(8 - \beta)}{16} \tau_m$	<i>Model: Method 1</i>	
	Minimum ISE ¹⁴				
	Minimum performance index ^{15, 16}				

$$\begin{aligned}
 ^{14} \beta = & 1.2171 + 0.7339 \left(\frac{\tau_m}{T_m} \right) + 0.083449 \left(\frac{\tau_m}{T_m} \right)^2 - 0.14905 \left(\frac{\tau_m}{T_m} \right)^3 + 0.030257 \left(\frac{\tau_m}{T_m} \right)^4 \\
 & + 0.0045567 \left(\frac{\tau_m}{T_m} \right)^5 - 0.0029943 \left(\frac{\tau_m}{T_m} \right)^6 + 0.00056821 \left(\frac{\tau_m}{T_m} \right)^7 - 5.4957 \cdot 10^{-5} \left(\frac{\tau_m}{T_m} \right)^8 \\
 & + 2.7505 \cdot 10^{-6} \left(\frac{\tau_m}{T_m} \right)^9 - 5.6642 \cdot 10^{-8} \left(\frac{\tau_m}{T_m} \right)^{10}.
 \end{aligned}$$

$$^{15} \text{Performance index} = \int_0^{\infty} [e^2(t) + K_m^2 u^2(t)] dt$$

$$\begin{aligned}
 ^{16} \beta = & 0.083709 + 2.2875 \left(\frac{\tau_m}{T_m} \right) - 0.18739 \left(\frac{\tau_m}{T_m} \right)^2 - 1.0059 \left(\frac{\tau_m}{T_m} \right)^3 + 0.79718 \left(\frac{\tau_m}{T_m} \right)^4 \\
 & - 0.3036 \left(\frac{\tau_m}{T_m} \right)^5 + 0.067859 \left(\frac{\tau_m}{T_m} \right)^6 - 0.0093104 \left(\frac{\tau_m}{T_m} \right)^7 + 0.00077161 \left(\frac{\tau_m}{T_m} \right)^8 \\
 & - 3.5473 \cdot 10^{-5} \left(\frac{\tau_m}{T_m} \right)^9 + 6.9487 \cdot 10^{-7} \left(\frac{\tau_m}{T_m} \right)^{10}, \quad 0 < \frac{\tau_m}{T_m} < 4.5
 \end{aligned}$$

$$\beta = 2.444547 - 0.0146274714 \left(\frac{\tau_m}{T_m} - 4.5 \right), \quad \frac{\tau_m}{T_m} \geq 4.5.$$

Rule	K_c	T_i	T_d	Comment
Arvanitis <i>et al.</i> (2003b).	$^{17} K_c^{(458)}$	$T_i^{(458)}$	$T_d^{(458)}$	
<i>Model: Method 1</i>	Minimum ISE ¹⁸			
	Minimum performance index ^{19 20} ,			

$$^{17} K_c^{(458)} = \frac{12.5664(\tau_m + T_m) - 4}{K_m \tau_m [25.1328(\tau_m + T_m) - 8 - (\tau_m + 2T_m)a]} \\ \text{with } a = [0.5708 + \sqrt{\frac{3.4674\tau_m + 3.1416T_m - 1}{\tau_m}}]\chi$$

$$T_i^{(458)} = \frac{12.5664(\tau_m + T_m) - 4}{a}, \quad T_d^{(458)} = T_m - T_m^2 \frac{a}{12.5664(\tau_m + T_m) - 4}.$$

$$^{18} \chi = 1.1571 + 5.0775 \left(\frac{\tau_m}{T_m} \right) - 8.2021 \left(\frac{\tau_m}{T_m} \right)^2 + 6.6353 \left(\frac{\tau_m}{T_m} \right)^3 - 3.1355 \left(\frac{\tau_m}{T_m} \right)^4 \\ + 0.92683 \left(\frac{\tau_m}{T_m} \right)^5 - 0.17635 \left(\frac{\tau_m}{T_m} \right)^6 + 0.02158 \left(\frac{\tau_m}{T_m} \right)^7 - 0.0016402 \left(\frac{\tau_m}{T_m} \right)^8 \\ + 7.0439 \cdot 10^{-5} \left(\frac{\tau_m}{T_m} \right)^9 - 1.3056 \cdot 10^{-6} \left(\frac{\tau_m}{T_m} \right)^{10}, \quad 0 < \frac{\tau_m}{T_m} < 4.5 \\ \chi = 2.124307 - 0.004844182 \left(\frac{\tau_m}{T_m} - 4.5 \right), \quad \frac{\tau_m}{T_m} \geq 4.5.$$

$$^{19} \text{Performance index} = \int_0^\infty [e^2(t) + K_m^2 u^2(t)] dt.$$

$$^{20} \chi = -0.70861 + 7.4212 \left(\frac{\tau_m}{T_m} \right) - 8.1214 \left(\frac{\tau_m}{T_m} \right)^2 + 4.8168 \left(\frac{\tau_m}{T_m} \right)^3 - 1.7217 \left(\frac{\tau_m}{T_m} \right)^4 \\ + 0.38998 \left(\frac{\tau_m}{T_m} \right)^5 - 0.056975 \left(\frac{\tau_m}{T_m} \right)^6 + 0.0053141 \left(\frac{\tau_m}{T_m} \right)^7 - 0.00030233 \left(\frac{\tau_m}{T_m} \right)^8 \\ + 9.3969 \cdot 10^{-6} \left(\frac{\tau_m}{T_m} \right)^9 - 1.1861 \cdot 10^{-7} \left(\frac{\tau_m}{T_m} \right)^{10}, \quad 0 < \frac{\tau_m}{T_m} < 4.5 \\ \chi = 2.1069751341 - 0.0017098 \left(\frac{\tau_m}{T_m} - 4.5 \right), \quad \frac{\tau_m}{T_m} \geq 4.5.$$

Rule	K_c	T_i	T_d	Comment
Arvanitis <i>et al.</i> (2003b) (continued).	Minimum performance index ^{21, 22} ,			
Direct synthesis				
Arvanitis <i>et al.</i> (2003b). Model: Method 1	²³ $K_c^{(459)}$	$(2\xi^2 + 1)\tau_m$	$\frac{16\xi^2 + 4}{16(2\xi^2 + 1)}\tau_m$	
	²⁴ $K_c^{(460)}$	$T_i^{(460)}$	$T_d^{(460)}$	

$$^{21} \text{Performance index} = \int_0^\infty e^2(t) + K_m^2 \left(\frac{du}{dt} \right)^2 dt .$$

$$^{22} \chi = 0.05486 + 0.81662 \left(\frac{\tau_m}{T_m} \right) - 1.4989 \left(\frac{\tau_m}{T_m} \right)^2 + 1.1955 \left(\frac{\tau_m}{T_m} \right)^3 - 0.48464 \left(\frac{\tau_m}{T_m} \right)^4 \\ + 0.11011 \left(\frac{\tau_m}{T_m} \right)^5 - 0.013386 \left(\frac{\tau_m}{T_m} \right)^6 + 0.00059665 \left(\frac{\tau_m}{T_m} \right)^7 + 3.8633 \cdot 10^{-5} \left(\frac{\tau_m}{T_m} \right)^8 \\ - 5.307 \cdot 10^{-6} \left(\frac{\tau_m}{T_m} \right)^9 + 1.6437 \cdot 10^{-7} \left(\frac{\tau_m}{T_m} \right)^{10}, \quad 0 < \frac{\tau_m}{T_m} < 6.0 ;$$

$$\chi = 2.1111 - 0.001025 \left(\frac{\tau_m}{T_m} - 6 \right), \quad \frac{\tau_m}{T_m} \geq 6 .$$

$$^{23} K_c^{(459)} = \frac{16(2\xi^2 + 1)}{(32\xi^2 + 4)K_m\tau_m} .$$

$$^{24} K_c^{(460)} = \frac{\tau_m + T_m + 4\tau_m\xi^2}{K_m\tau_m^2(1 + 8\xi^2)}, \quad T_i^{(460)} = \tau_m + T_m + 4\tau_m\xi^2, \quad T_d^{(460)} = \frac{\tau_m T_m (1 + 4\xi^2)}{\tau_m + T_m + 4\tau_m\xi^2} .$$

4.4.15 Non-interacting controller 8

$$U(s) = K_c \left(1 + \frac{1}{T_i s} + T_d s \right) E(s) - K_i (1 + T_{di} s) Y(s)$$

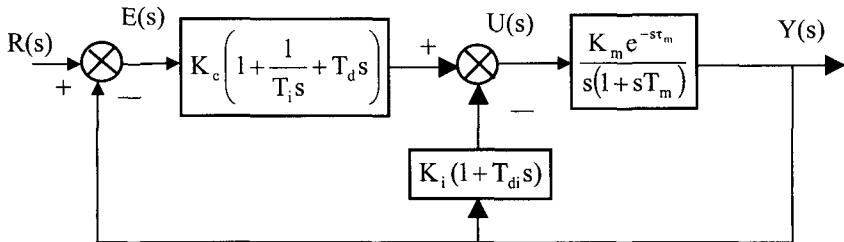


Table 109: PID controller tuning rules – FOLIPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{s(1+sT_m)}$

Rule	K_c	T_i	T_d	Comment
Direct synthesis				
Wang <i>et al.</i> (2001b). <i>Model: Method 7</i>	$\frac{0.4189}{K_m \tau_m}$	$4\tau_m$	$1.25(T_m + 0.1\tau_m)$	$K_i = \frac{0.2}{K_m \tau_m}$
$T_{di} = 0; A_m = 3; \phi_m = 60^\circ$				
Robust				
Lee and Edgar (2002). <i>Model: Method 1</i>	$^1 K_c^{(461)}$	$T_i^{(461)}$	$T_d^{(461)}$	$K_i = 0.25K_u, T_{di} = 0$

$$\begin{aligned}
 ^1 K_c^{(461)} &= \frac{0.25K_u}{(\lambda + \tau_m)} \left[\frac{4}{K_u K_m} - \tau_m + \frac{\tau_m^2}{2(\lambda + \tau_m)} \right], \quad T_i^{(461)} = \frac{4}{K_u K_m} - \tau_m + \frac{\tau_m^2}{2(\lambda + \tau_m)}, \\
 T_d^{(461)} &= \frac{0.25K_u}{(\lambda + \tau_m)} \left\{ \frac{4T_m}{K_u K_m} + 0.5\tau_m^2 - \frac{\tau_m^3}{6(\lambda + \tau_m)} + \frac{\tau_m^4}{4(\lambda + \tau_m)^2} \right. \\
 &\quad \left. + \frac{\tau_m^2(1 - 0.25K_u K_m \tau_m)}{0.5K_u K_m (\lambda + \tau_m)} \right\}.
 \end{aligned}$$

4.4.16 Non-interacting controller 11

$$U(s) = K_c \left(1 + \frac{1}{T_i s} \right) (1 + T_d s) E(s) - K_i (1 + T_{di} s) Y(s)$$

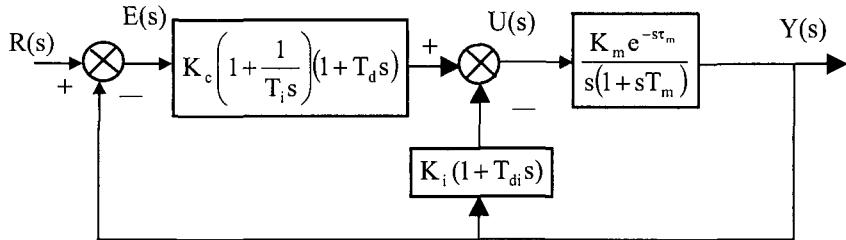


Table 110: PID controller tuning rules – FOLIPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{s(1 + sT_m)}$

Rule	K_c	T_i	T_d	Comment
Direct synthesis				
Majhi and Mahanta (2001). Model: Method 6	$\frac{0.5236}{K_m \tau_m}$	$2\tau_m e^{\tau_m s}$	T_m	$T_{di} = T_m$; $K_i = \frac{0.5}{K_m \tau_m}$; $A_m = 3$; $\phi_m = 60^\circ$

4.4.17 Non-interacting controller 12

$$U(s) = K_c \left(1 + \frac{1}{T_i s} + T_d s \right) \frac{1}{T_d s + 1} E(s) - K_I Y(s)$$

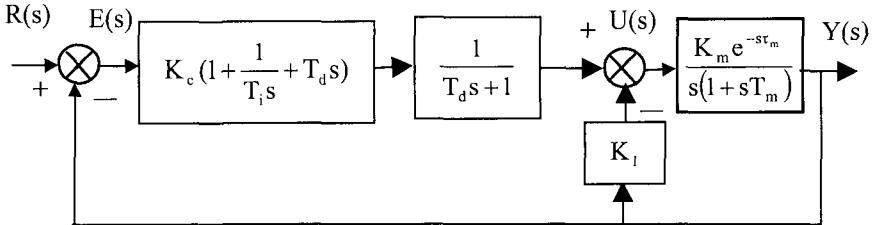


Table 111: PID controller tuning rules – FOLIPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{s(1 + sT_m)}$

Rule	K_c	T_i	T_d	Comment
Robust				
Lee and Edgar (2002). Model: Method 1	$^2 K_c^{(462)}$	$T_i^{(462)}$	$T_d^{(462)}$	$K_I = 0.25K_u$

$$\begin{aligned}
 ^2 K_c^{(462)} &= \frac{0.25K_u}{\lambda + \tau_m} \left\{ \frac{4}{K_u K_m} - \tau_m + \frac{\tau_m^2}{2(\lambda + \tau_m)} + \right. \\
 &\quad \left. \left[\frac{4T_m}{K_u K_m} + 0.5\tau_m^2 - \frac{\tau_m^3}{6(\lambda + \tau_m)} + \frac{\tau_m^4}{4(\lambda + \tau_m)^2} + \frac{(1 - 0.25K_u K_m \tau_m)\tau_m^2}{0.5K_u K_m (\lambda + \tau_m)} \right]^{0.5} \right\}, \\
 T_i^{(462)} &= \frac{4}{K_u K_m} - \tau_m + \frac{\tau_m^2}{2(\lambda + \tau_m)} + \\
 &\quad \left[\frac{4T_m}{K_u K_m} + 0.5\tau_m^2 - \frac{\tau_m^3}{6(\lambda + \tau_m)} + \frac{\tau_m^4}{4(\lambda + \tau_m)^2} + \frac{(1 - 0.25K_u K_m \tau_m)\tau_m^2}{0.5K_u K_m (\lambda + \tau_m)} \right]^{0.5}, \\
 T_d^{(462)} &= \left[\frac{4T_m}{K_u K_m} + 0.5\tau_m^2 - \frac{\tau_m^3}{6(\lambda + \tau_m)} + \frac{\tau_m^4}{4(\lambda + \tau_m)^2} + \frac{(1 - 0.25K_u K_m \tau_m)\tau_m^2}{0.5K_u K_m (\lambda + \tau_m)} \right]^{0.5}.
 \end{aligned}$$

4.4.18 Industrial controller $U(s) = K_c \left(1 + \frac{1}{T_i s} \right) \left(R(s) - \frac{1 + T_d s}{1 + \frac{T_d s}{N}} Y(s) \right)$

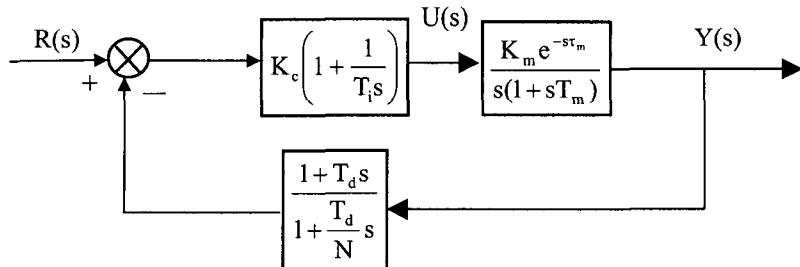


Table 112: PID controller tuning rules – FOLIPD model $G_m(s) = \frac{K_m e^{-st_m}}{s(1+sT_m)}$

Rule	K_c	T_i	T_d	Comment
Direct synthesis: time domain criteria				
Skogestad (2004b). <i>Model:</i> <i>Method 1</i>	$\frac{1}{K_m (T_{CL} + \tau_m)}$	$4\xi^2 (T_{CL} + \tau_m)$	T_m	'good' robustness - $T_{CL} = \tau_m$; $\xi = 0.7$ or 1

$N \in [5,10]$; $N = 2$ if measurement noise is a serious problem

4.4.19 Alternative controller I $G_c(s) = K_c \left(\frac{1 + T_i s}{1 + \frac{T_d s}{N}} \right) \left(\frac{1 + T_d s}{1 + \frac{T_d s}{N}} \right)$

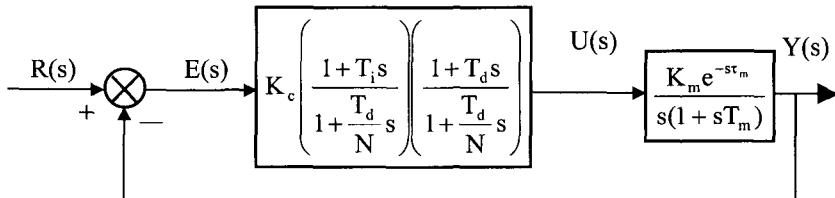


Table 113: PID controller tuning rules – FOLIPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{s(1 + sT_m)}$

Rule	K_c	T_i	T_d	Comment
Direct synthesis				
Tsang <i>et al.</i> (1993).	$\frac{x_1}{K_m \tau_m}$	T_m	$0.25\tau_m$	$N = 2.5$
Coefficient values				
Model: Method 2	x_1	ξ	x_1	ξ
	1.6818	0.0	0.9916	0.3
	1.3829	0.1	0.8594	0.4
	1.1610	0.2	0.7542	0.5
Tsang and Rad (1995). Model: Method 2	$\frac{0.809}{K_m \tau_m}$	T_m	$0.5\tau_m$	Maximum overshoot = 16%; $N = 8.33$

4.4.20 Alternative controller 2

$$G_c(s) = K_c \left(\frac{1 + T_i s}{1 + \frac{T_d s}{N}} \right) \left(\frac{1 + 0.5\tau_m s + 0.0833\tau_m^2 s^2}{1 + \frac{T_d s}{N}} \right)$$

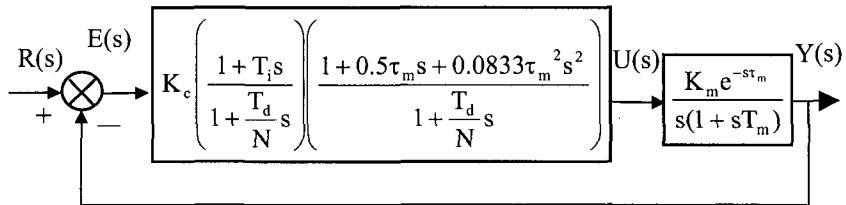


Table 114: PID controller tuning rules – FOLIPD model $G_m(s) = \frac{K_m e^{-st_m}}{s(1+sT_m)}$

Rule	K_c	T_i	T_d	Comment
Direct synthesis				
Tsang <i>et al.</i> (1993). Model: Method 2	$\frac{x_1}{K_m \tau_m}$	T_m	$0.25\tau_m$	$N = 2.5$
Coefficient values				
x_1	ξ	x_1	ξ	x_1
1.8512	0.0	1.1595	0.3	0.8411
1.5520	0.1	1.0280	0.4	0.7680
1.3293	0.2	0.9246	0.5	0.6953
				0.6219 0.9
				0.5527 1.0

4.5 SOSPD Model

$$G_m(s) = \frac{K_m e^{-sT_m}}{(1+sT_{m1})(1+sT_{m2})} \text{ or } G_m(s) = \frac{K_m e^{-sT_m}}{T_{m1}^2 s^2 + 2\xi_m T_{m1}s + 1}$$

4.5.1 Ideal controller $G_c(s) = K_c \left(1 + \frac{1}{T_i s} + T_d s \right)$

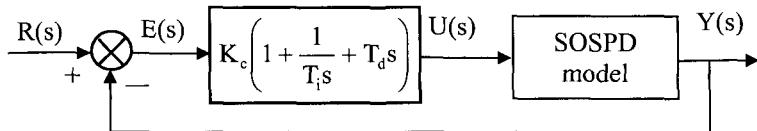


Table 115: PID tuning rules – SOSPD model $G_m(s) = \frac{K_m e^{-sT_m}}{(1+sT_{m1})(1+sT_{m2})}$ or

$$G_m(s) = \frac{K_m e^{-sT_m}}{T_{m1}^2 s^2 + 2\xi_m T_{m1}s + 1}$$

Rule	K_c	T_i	T_d	Comment
Process reaction				
Auslander <i>et al.</i> (1975).	¹ $K_c^{(463)}$	$T_i^{(463)}$	$0.25T_i^{(463)}$	<i>Model: Method 3</i>

¹ Note: equations continued into the footnote on page 310.

$$\begin{aligned}
 K_c^{(463)} &= \frac{1.2}{K_m(T_{m1} - T_{m2})} \left\{ \left[\tau_m + \frac{T_{m1}T_{m2}}{T_{m1} - T_{m2}} \ln \frac{T_{m1}}{T_{m2}} \right] \left[\left(\frac{T_{m2}}{T_{m1}} \right)^{\frac{T_{m2}}{T_{m1} - T_{m2}}} - \left(\frac{T_{m2}}{T_{m1}} \right)^{\frac{T_{m1}}{T_{m1} - T_{m2}}} \right] \right. \\
 &\quad \left. - \frac{1}{(T_{m1} - T_{m2})^2} \left[\left(\frac{T_{m2}}{T_{m1}} \right)^{\frac{T_{m2}}{T_{m1} - T_{m2}}} - \left(\frac{T_{m2}}{T_{m1}} \right)^{\frac{T_{m1}}{T_{m1} - T_{m2}}} \right]^2 \right\}^{-1}, \\
 &\quad 1 + \frac{T_{m2}}{T_{m1} - T_{m2}} \left(\frac{T_{m2}}{T_{m1}} \right)^{\frac{T_{m1}}{T_{m1} - T_{m2}}} - \frac{T_{m1}}{T_{m1} - T_{m2}} \left(\frac{T_{m2}}{T_{m1}} \right)^{\frac{T_{m2}}{T_{m1} - T_{m2}}}
 \end{aligned}$$

Rule	K_c	T_i	T_d	Comment
Minimum performance index: regulator tuning				
Minimum IAE – Wills (1962b) - deduced from graph. <i>Model: Method 1</i>	$\frac{5.8}{K_m}$	$0.36T_u$	$0.21T_u$	Representative tuning values; $T_{m2} = \tau_m$ $= 0.1T_{m1}$
	$\frac{5}{K_m}$	$0.29T_u$	$0.29T_u$	
Minimum IAE – Shinskey (1988) – page 151. <i>Model: Method 2</i>	$0.62K_u$	$0.38T_u$	$0.15T_u$	$\frac{T_{m2}}{T_{m2} + \tau_m} = 0.25$
	$0.68K_u$	$0.33T_u$	$0.19T_u$	$\frac{T_{m2}}{T_{m2} + \tau_m} = 0.5$
	$0.79K_u$	$0.26T_u$	$0.21T_u$	$\frac{T_{m2}}{T_{m2} + \tau_m} = 0.75$
Minimum ITAE – Lopez et al. (1969). <i>Model:</i> <i>Method 17</i>	x_1/K_m	$x_2 T_{m1}$	$x_3 T_{m1}$	
	Representative coefficient values (from graphs)			
	x_1	x_2	x_3	ξ_m
	25	0.5	0.25	0.5
	0.7	1.3	1.2	0.5
	0.35	5	1.0	0.5
	25	0.5	0.2	1.0
	1.8	1.7	0.7	1.0
	9.0	2	0.45	4.0
τ_m/T_{m1}				
0.1				
1				
10				
0.1				
1				

$$T_i^{(463)} =$$

$$\frac{\frac{2T_{m1}T_{m2}}{T_{m1}-T_{m2}} \ln\left(\frac{T_{m1}}{T_{m2}}\right) + 2\tau_m}{1 + \frac{T_{m2}}{T_{m1}-T_{m2}} \left(\frac{T_{m2}}{T_{m1}}\right)^{\frac{T_{m1}}{T_{m1}-T_{m2}}}} - \frac{\frac{2}{(T_{m1}-T_{m2})} \left[\left(\frac{T_{m2}}{T_{m1}}\right)^{\frac{T_{m2}}{T_{m1}-T_{m2}}} - \left(\frac{T_{m2}}{T_{m1}}\right)^{\frac{T_{m1}}{T_{m1}-T_{m2}}} \right]}{1 + \frac{T_{m2}}{T_{m1}-T_{m2}} \left(\frac{T_{m2}}{T_{m1}}\right)^{\frac{T_{m1}}{T_{m1}-T_{m2}}}} - \frac{T_{m1}}{T_{m1}-T_{m2}} \left(\frac{T_{m2}}{T_{m1}}\right)^{\frac{T_{m2}}{T_{m1}-T_{m2}}}}.$$

Rule	K_c	T_i	T_d	Comment
Minimum ITAE - Bohl and McAvoy (1976b).	$^2 K_c^{(464)}$	$T_i^{(464)}$	$T_d^{(464)}$	<i>Model: Method 1</i>

$$^2 \frac{T_{m2}}{T_{m1}} = 0.12, 0.3, 0.5, 0.7, 0.9 ; \frac{\tau_m}{T_{m1}} = 0.1, 0.2, 0.3, 0.4, 0.5$$

$$K_c^{(464)} = \frac{10.9507}{K_m} \left[\left(10 \frac{\tau_m}{T_{m1}} \right)^{-1.2096 + 0.1760 \ln \left(10 \frac{\tau_m}{T_{m1}} \right)} \right] \\ \left[\left(10 \frac{T_{m2}}{T_{m1}} \right)^{0.1044 + 0.1806 \ln \left(10 \frac{T_{m2}}{T_{m1}} \right) - 0.2071 \ln \left(10 \frac{\tau_m}{T_{m1}} \right)} \right],$$

$$T_i^{(464)} = 0.2979 T_{m1} \left[\left(10 \frac{\tau_m}{T_{m1}} \right)^{0.7750 - 0.1026 \ln \left(10 \frac{\tau_m}{T_{m1}} \right)} \right] \\ \left[\left(10 \frac{T_{m2}}{T_{m1}} \right)^{0.1701 + 0.0092 \ln \left(10 \frac{T_{m2}}{T_{m1}} \right) + 0.0081 \ln \left(10 \frac{\tau_m}{T_{m1}} \right)} \right]$$

$$T_d^{(464)} = 0.1075 T_{m1} \left[\left(10 \frac{\tau_m}{T_{m1}} \right)^{0.6025 - 0.0624 \ln \left(10 \frac{\tau_m}{T_{m1}} \right)} \right] \\ \left[\left(10 \frac{T_{m2}}{T_{m1}} \right)^{0.4531 - 0.0479 \ln \left(10 \frac{T_{m2}}{T_{m1}} \right) + 0.0128 \ln \left(10 \frac{\tau_m}{T_{m1}} \right)} \right]$$

Rule	K_c	T_i	T_d	Comment
Minimum ITAE - Hassan (1993).	$^3 K_c^{(465)}$	$T_i^{(465)}$	$T_d^{(465)}$	Model: Method 7

³ $0.5 \leq \xi_m \leq 2$; $0.1 \leq \frac{\tau_m}{T_{m1}} \leq 4$. $K_c^{(465)}$ is obtained as follows:

$$\begin{aligned} \log \left[K_m K_c^{(465)} \right] = & 1.9763370 - 0.6436825 \xi_m - 5.1887660 \frac{\tau_m}{T_{m1}} + 0.4375558 \xi_m^2 \\ & + 2.9005550 \left(\frac{\tau_m}{T_{m1}} \right)^2 + 3.1468010 \xi_m \frac{\tau_m}{T_{m1}} - 0.1697221 \xi_m^2 \left(\frac{\tau_m}{T_{m1}} \right)^2 \\ & - 0.8161808 \xi_m \left(\frac{\tau_m}{T_{m1}} \right)^2 - 1.2048220 \xi_m^2 \frac{\tau_m}{T_{m1}} - 0.0810373 \xi_m^3 \\ & - 0.4444091 \left(\frac{\tau_m}{T_{m1}} \right)^3 + 0.0319431 \xi_m \left(\frac{\tau_m}{T_{m1}} \right)^3 + 0.1054399 \xi_m^2 \left(\frac{\tau_m}{T_{m1}} \right)^2 \\ & + 0.1652807 \xi_m^3 \frac{\tau_m}{T_{m1}} + 0.1175991 \xi_m^3 \left(\frac{\tau_m}{T_{m1}} \right)^2 - 0.0375245 \xi_m^3 \left(\frac{\tau_m}{T_{m1}} \right)^3; \end{aligned}$$

$T_i^{(465)}$ is obtained as follows:

$$\begin{aligned} \log \left[\frac{T_i^{(465)}}{T_{m1}} \right] = & -0.7865873 + 0.6796885 \xi_m + 2.1891540 \frac{\tau_m}{T_{m1}} - 0.3471095 \xi_m^2 \\ & - 1.9003610 \left(\frac{\tau_m}{T_{m1}} \right)^2 - 0.7007801 \xi_m \frac{\tau_m}{T_{m1}} + 0.3077857 \xi_m^2 \left(\frac{\tau_m}{T_{m1}} \right)^2 \\ & + 0.8566974 \xi_m \left(\frac{\tau_m}{T_{m1}} \right)^2 - 0.2535062 \xi_m^2 \frac{\tau_m}{T_{m1}} + 0.0412943 \xi_m^3 \\ & + 0.3484161 \left(\frac{\tau_m}{T_{m1}} \right)^3 - 0.1626562 \xi_m \left(\frac{\tau_m}{T_{m1}} \right)^3 - 0.0661899 \xi_m^2 \left(\frac{\tau_m}{T_{m1}} \right)^2 \\ & + 0.2247806 \xi_m^3 \frac{\tau_m}{T_{m1}} - 0.2470783 \xi_m^3 \left(\frac{\tau_m}{T_{m1}} \right)^2 + 0.0493011 \xi_m^3 \left(\frac{\tau_m}{T_{m1}} \right)^3, \end{aligned}$$

$T_d^{(465)}$ is obtained as follows (continued into footnote on page 313):

$$\begin{aligned} \log \left[\frac{T_d^{(465)}}{T_{m1}} \right] = & -0.6726798 - 0.2072477 \xi_m + 2.6826330 \frac{\tau_m}{T_{m1}} + 0.0807474 \xi_m^2 \\ & - 1.7707830 \left(\frac{\tau_m}{T_{m1}} \right)^2 - 1.6685140 \xi_m \frac{\tau_m}{T_{m1}} + 0.0845958 \xi_m^2 \left(\frac{\tau_m}{T_{m1}} \right)^2 \end{aligned}$$

Rule	K_c	T_i	T_d	Comment
Minimum ITAE – Sung <i>et al.</i> (1996). <i>Model: Method 9</i>	$^4 K_c^{(466)}$	$T_i^{(466)}$	$T_d^{(466)}$	$0.05 < \frac{\tau_m}{T_{ml}} \leq 2$

$$\begin{aligned}
& + 0.7159307 \xi_m^3 \left(\frac{\tau_m}{T_{ml}} \right)^2 + 0.5631447 \xi_m^2 \frac{\tau_m}{T_{ml}} - 0.0225269 \xi_m^3 \\
& + 0.2821874 \left(\frac{\tau_m}{T_{ml}} \right)^3 - 0.0616288 \xi_m \left(\frac{\tau_m}{T_{ml}} \right)^3 - 0.0626626 \xi_m^2 \left(\frac{\tau_m}{T_{ml}} \right)^2 \\
& - 0.0372784 \xi_m^3 \frac{\tau_m}{T_{ml}} - 0.0948097 \xi_m^3 \left(\frac{\tau_m}{T_{ml}} \right)^2 + 0.0272541 \xi_m^3 \left(\frac{\tau_m}{T_{ml}} \right)^3.
\end{aligned}$$

⁴ Formulae continued into footnote on page 314.

$$K_c^{(466)} = \frac{1}{K_m} \left[-0.67 + 0.297 \left(\frac{\tau_m}{T_{ml}} \right)^{-2.001} + 2.189 \left(\frac{\tau_m}{T_{ml}} \right)^{-0.766} \xi_m \right], \quad \frac{\tau_m}{T_{ml}} < 0.9 \text{ or}$$

$$K_c^{(466)} = \frac{1}{K_m} \left[-0.365 + 0.260 \left(\frac{\tau_m}{T_{ml}} - 1.4 \right)^2 + 2.189 \left(\frac{\tau_m}{T_{ml}} \right)^{-0.766} \xi_m \right], \quad \frac{\tau_m}{T_{ml}} \geq 0.9;$$

$$T_i^{(466)} = T_{ml} \left[2.212 \left(\frac{\tau_m}{T_{ml}} \right)^{0.520} - 0.3 \right], \quad \frac{\tau_m}{T_{ml}} < 0.4 \text{ or}$$

$$T_i^{(466)} = T_{ml} \left\{ -0.975 + 0.910 \left(\frac{\tau_m}{T_{ml}} - 1.845 \right)^2 + \alpha \right\}, \quad \frac{\tau_m}{T_{ml}} \geq 0.4 \text{ with}$$

$$\alpha = \left[1 - e^{-\frac{\xi_m}{0.15+0.33\frac{\tau_m}{T_{ml}}}} \right] \left[5.25 - 0.88 \left(\frac{\tau_m}{T_{ml}} - 2.8 \right)^2 \right];$$

$$T_d^{(466)} = \frac{T_{ml}}{\left[1 - e^{-\frac{\xi_m}{-0.15+0.939\left(\frac{\tau_m}{T_{ml}}\right)^{-1.121}}} \right] \left[1.45 + 0.969 \left(\frac{T_{ml}}{\tau_m} \right)^{1.171} \right] - 1.9 + 1.576 \left(\frac{\tau_m}{T_{ml}} \right)^{-0.530}}.$$

Rule	K_c	T_i	T_d	Comment
Nearly minimum IAE, ISE, ITAE – Hwang (1995). <i>Model:</i> <i>Method 10</i>	$^5 K_c^{(467)}$	$T_i^{(467)}$	$T_d^{(467)}$	$\varepsilon_2 < 2.4$
	$^6 K_c^{(468)}$	$T_i^{(468)}$		$2.4 \leq \varepsilon_2 < 3$
	$^7 K_c^{(469)}$	$T_i^{(469)}$		$3 \leq \varepsilon_2 < 20$

$$^5 K_c^{(467)} = \left(K_{H2} - \frac{0.622[1 - 0.435\omega_{H2}\tau_m + 0.052\omega_{H2}^2\tau_m^2]}{K_m/(1 + K_{H2}K_m)} \right),$$

$$T_i^{(467)} = \frac{K_c^{(467)}(1 + K_{H2}K_m)}{\omega_{H2}K_m 0.0697[1 + 0.752\omega_{H2}\tau_m - 0.145\omega_{H2}^2\tau_m^2]},$$

$$T_d^{(467)} = \frac{1.45(1 + K_u K_m)}{K_m^2 \omega_u} \left(1 - \frac{1.16}{\varepsilon_0} \right) \left(1 - 0.612\omega_u \tau_m + 0.103\omega_u^2 \tau_m^2 \right).$$

$$^6 K_c^{(468)} = \left(K_{H2} - \frac{0.724[1 - 0.469\omega_{H2}\tau_m + 0.0609\omega_{H2}^2\tau_m^2]}{K_m/(1 + K_{H2}K_m)} \right),$$

$$T_i^{(468)} = \frac{K_c^{(468)}(1 + K_{H2}K_m)}{\omega_{H2}K_m 0.0405[1 + 1.93\omega_{H2}\tau_m - 0.363\omega_{H2}^2\tau_m^2]}.$$

$$^7 K_c^{(469)} = \left(K_{H2} - \frac{1.26(0.506)^{\omega_{H2}\tau_m}[1 - 1.07/\varepsilon_2 + 0.616/\varepsilon_2^2]}{K_m/(1 + K_{H2}K_m)} \right),$$

$$T_i^{(469)} = \frac{K_c^{(469)}(1 + K_{H2}K_m)}{\omega_{H2}K_m 0.0661(1 + 0.824 \ln[\omega_{H2}\tau_m])(1 + 1.71/\varepsilon_2 - 1.17/\varepsilon_2^2)}.$$

Rule	K_c	T_i	T_d	Comment
Hwang (1995) - continued	$^8 K_c^{(470)}$	$T_i^{(470)}$	$T_d^{(467)}$ (page 314)	$\varepsilon_2 \geq 20$
	Decay ratio = 0.2; $0.2 \leq \tau_m/T_{m1} \leq 2.0$ and $0.6 \leq \xi_m \leq 4.2$			
Minimum performance index: servo tuning				
Minimum IAE - Wills (1962b) - deduced from graph. Model: Method 1	$6.5/K_m$	$1.45T_u$	$0.14T_u$	Representative tuning values; $T_{m2} = \tau_m = 0.1T_{m1}$
	$7/K_m$	$0.95T_u$	$0.22T_u$	
Minimum IAE - Gallier and Otto (1968). Model: Method 1	x_1/K_m	$^9 T_i^{(471)}$	$T_d^{(471)}$	$T_{m1} = T_{m2}$
	Representative coefficient values - deduced from graphs			
	$\frac{\tau_m}{2T_{m1}}$	x_1	x_2	x_3
	0.053	6.7	0.89	0.24
	0.11	4.15	0.84	0.24
	0.25	2.35	0.77	0.24
Minimum ITAE - Wills (1962a) - deduced from graph.	$\frac{18}{K_m}$	$\approx T_u$		$T_{m1} = T_{m2};$ $\tau_m = 0.1T_{m1};$ Model: Method 1

$$^8 K_c^{(470)} = \left(K_{H2} - \frac{1.09 \left[1 - 0.497\omega_{H2}\tau_m + 0.0724\omega_{H2}^2\tau_m^2 \right]}{K_m / (1 + K_{H2}K_m)} \right),$$

$$T_i^{(470)} = \frac{K_c^{(470)}(1 + K_{H2}K_m)}{\omega_{H2}K_m 0.054 \left(1 + 2.54\omega_{H2}\tau_m - 0.457\omega_{H2}^2\tau_m^2 \right)}.$$

$$^9 T_i^{(471)} = x_2(T_{m1} + T_{m2} + \tau_m), \quad T_d^{(471)} = x_3(T_{m1} + T_{m2} + \tau_m).$$

Rule	K_c	T_i	T_d	Comment
Minimum ITAE – Sung <i>et al.</i> (1996). <i>Model: Method 9</i>	$^{10} K_c^{(472)}$	$T_i^{(472)}$	$T_d^{(472)}$	$0.05 < \frac{\tau_m}{T_{ml}} \leq 2$
Nearly minimum IAE, ISE, ITAE – Hwang (1995). <i>Model:</i> <i>Method 10</i>	$^{11} K_c^{(473)}$	$T_i^{(473)}$	$\frac{0.471K_u}{K_m\omega_u}$	Decay ratio = 0.1; $0.2 \leq \frac{\tau_m}{T_{ml}} \leq 2.0$; $0.6 \leq \xi_m \leq 4.2$ ¹²

$$^{10} K_c^{(472)} = \frac{1}{K_m} \left[-0.04 + \left| 0.333 + 0.949 \left(\frac{\tau_m}{T_{ml}} \right)^{-0.983} \right| \xi_m \right], \quad \xi_m \leq 0.9 \text{ or}$$

$$K_c^{(472)} = \frac{1}{K_m} \left[-0.544 + 0.308 \frac{\tau_m}{T_{ml}} + 1.408 \left(\frac{\tau_m}{T_{ml}} \right)^{-0.832} \right] \xi_m, \quad \xi_m > 0.9.$$

$$T_i^{(472)} = T_{ml} [2.055 + 0.072(\tau_m/T_{ml})\xi_m], \quad \tau_m/T_{ml} \leq 1 \text{ or}$$

$$T_i^{(472)} = T_{ml} [1.768 + 0.329(\tau_m/T_{ml})\xi_m], \quad \tau_m/T_{ml} > 1.$$

$$T_d^{(472)} = \frac{T_{ml}}{\left[1 - e^{-\frac{\left(\frac{\tau_m}{T_{ml}} \right)^{1.060}}{0.870} \xi_m} \right] \left[0.55 + 1.683 \left(\frac{T_{ml}}{\tau_m} \right)^{1.090} \right]}.$$

$$^{11} K_c^{(473)} = \left(K_{H2} - \frac{0.822 [1 - 0.549 \omega_{H2} \tau_m + 0.112 \omega_{H2}^2 \tau_m^2]}{K_m / (1 + K_{H2} K_m)} \right),$$

$$T_i^{(473)} = \frac{K_c^{(473)} (1 + K_{H2} K_m)}{\omega_{H2} K_m 0.0142 (1 + 6.96 \omega_{H2} \tau_m - 1.77 \omega_{H2}^2 \tau_m^2)}.$$

$$^{12} \xi \leq 0.613 + 0.613 \frac{\tau_m}{T_{ml}} + 0.117 \left(\frac{\tau_m}{T_{ml}} \right)^2.$$

Rule	K_c	T_i	T_d	Comment
Hwang (1995) – continued.	$^{13} K_c^{(474)}$	$T_i^{(474)}$	$\frac{0.471K_u}{K_m \omega_u}$	Decay ratio = 0.1; $0.2 \leq \frac{\tau_m}{T_{ml}} \leq 2.0$; $0.6 \leq \xi_m \leq 4.2$ ¹⁴
	$^{15} K_c^{(475)}$	$T_i^{(475)}$		
	$^{16} K_c^{(476)}$	$T_i^{(476)}$		
Hwang (1995) – continued.	$^{17} K_c^{(477)}$	$T_i^{(477)}$	$\frac{0.471K_u}{K_m \omega_u}$	Decay ratio = 0.1; $0.2 \leq \frac{\tau_m}{T_{ml}} \leq 2.0$; $0.6 \leq \xi_m \leq 4.2$ ¹⁸

$$^{13} K_c^{(474)} = \left(K_{H2} - \frac{0.786[1 - 0.441\omega_{H2}\tau_m + 0.0569\omega_{H2}^2\tau_m^2]}{K_m/(1 + K_{H2}K_m)} \right),$$

$$T_i^{(474)} = \frac{K_c^{(474)}(1 + K_{H2}K_m)}{\omega_{H2}K_m 0.0172(1 + 4.62\omega_{H2}\tau_m - 0.823\omega_{H2}^2\tau_m^2)}.$$

$$^{14} \xi \leq 0.613 + 0.613 \frac{\tau_m}{T_{ml}} + 0.117 \left(\frac{\tau_m}{T_{ml}} \right)^2.$$

$$^{15} K_c^{(475)} = \left(K_{H2} - \frac{1.28(0.542)^{\omega_{H2}\tau_m}[1 - 0.986/\varepsilon_2 + 0.558/\varepsilon_2^2]}{K_m/(1 + K_{H2}K_m)} \right),$$

$$T_i^{(475)} = \frac{K_c^{(475)}(1 + K_{H2}K_m)}{\omega_{H2}K_m 0.0476(1 + 0.996 \ln[\omega_{H2}\tau_m])(1 + 2.13/\varepsilon_2 - 1.13/\varepsilon_2^2)}.$$

$$^{16} K_c^{(476)} = \left(K_{H2} - \frac{1.14[1 - 0.466\omega_{H2}\tau_m + 0.0647\omega_{H2}^2\tau_m^2]}{K_m/(1 + K_{H2}K_m)} \right),$$

$$T_i^{(476)} = \frac{K_c^{(476)}(1 + K_{H2}K_m)}{\omega_{H2}K_m 0.0609(-1 + 1.97\omega_{H2}\tau_m - 0.323\omega_{H2}^2\tau_m^2)}.$$

$$^{17} K_c^{(477)} = \left(K_{H2} - \frac{0.794[1 - 0.541\omega_{H2}\tau_m + 0.126\omega_{H2}^2\tau_m^2]}{K_m/(1 + K_{H2}K_m)} \right),$$

$$T_i^{(477)} = \frac{K_c^{(477)}(1 + K_{H2}K_m)}{\omega_{H2}K_m 0.0078(1 + 8.38\omega_{H2}\tau_m - 1.97\omega_{H2}^2\tau_m^2)},$$

$$^{18} \xi > 0.649 + 0.58 \frac{\tau_m}{T_{ml}} - 0.005 \left(\frac{\tau_m}{T_{ml}} \right)^2.$$

Rule	K_c	T_i	T_d	Comment
Hwang (1995) – continued.	$^{19} K_c^{(478)}$	$T_i^{(478)}$	$\frac{0.471K_u}{K_m \omega_u}$	Decay ratio = 0.1;
	$^{21} K_c^{(479)}$	$T_i^{(479)}$		$0.2 \leq \frac{\tau_m}{T_{ml}} \leq 2.0$;
	$^{22} K_c^{(480)}$	$T_i^{(480)}$		$0.6 \leq \xi_m \leq 4.2$ ²⁰
Hwang (1995) – continued.	$^{23} K_c^{(481)}$	$T_i^{(481)}$	$\frac{0.471K_u}{K_m \omega_u}$	Decay ratio = 0.1; $0.2 \leq \frac{\tau_m}{T_{ml}} \leq 2.0$; $0.6 \leq \xi_m \leq 4.2$ ²⁴

$$^{19} K_c^{(478)} = \left(K_{H2} - \frac{0.738[1 - 0.415\omega_{H2}\tau_m + 0.0575\omega_{H2}^2\tau_m^2]}{K_m/(1 + K_{H2}K_m)} \right),$$

$$T_i^{(478)} = \frac{K_c^{(478)}(1 + K_{H2}K_m)}{\omega_{H2}K_m 0.0124(1 + 4.05\omega_{H2}\tau_m - 0.63\omega_{H2}^2\tau_m^2)}.$$

$$^{20} \xi > 0.649 + 0.58 \frac{\tau_m}{T_{ml}} - 0.005 \left(\frac{\tau_m}{T_{ml}} \right)^2.$$

$$^{21} K_c^{(479)} = \left(K_{H2} - \frac{1.15(0.564)^{\omega_{H2}\tau_m}[1 - 0.959/\varepsilon_2 + 0.773/\varepsilon_2^2]}{K_m/(1 + K_{H2}K_m)} \right),$$

$$T_i^{(479)} = \frac{K_c^{(479)}(1 + K_{H2}K_m)}{\omega_{H2}K_m 0.0335(1 + 0.947 \ln[\omega_{H2}\tau_m])(1 + 1.9/\varepsilon_2 - 1.07/\varepsilon_2^2)}.$$

$$^{22} K_c^{(480)} = \left(K_{H2} - \frac{1.07[1 - 0.466\omega_{H2}\tau_m + 0.0667\omega_{H2}^2\tau_m^2]}{K_m/(1 + K_{H2}K_m)} \right),$$

$$T_i^{(480)} = \frac{K_c^{(480)}(1 + K_{H2}K_m)}{\omega_{H2}K_m 0.0328(-1 + 2.21\omega_{H2}\tau_m - 0.338\omega_{H2}^2\tau_m^2)}.$$

$$^{23} K_c^{(481)} = \left(K_{H2} - \frac{0.789[1 - 0.527\omega_{H2}\tau_m + 0.11\omega_{H2}^2\tau_m^2]}{K_m/(1 + K_{H2}K_m)} \right),$$

$$T_i^{(481)} = \frac{K_c^{(481)}(1 + K_{H2}K_m)}{\omega_{H2}K_m 0.009(1 + 9.7\omega_{H2}\tau_m - 2.4\omega_{H2}^2\tau_m^2)}.$$

$$^{24} \xi \leq 0.649 + 0.58 \frac{\tau_m}{T_{ml}} - 0.005 \left(\frac{\tau_m}{T_{ml}} \right)^2, \quad \xi > 0.613 + 0.613 \frac{\tau_m}{T_{ml}} + 0.117 \left(\frac{\tau_m}{T_{ml}} \right)^2.$$

Rule	K_c	T_i	T_d	Comment	
Hwang (1995) – continued.	$^{25} K_c^{(482)}$	$T_i^{(482)}$	$\frac{0.471K_u}{K_m \omega_u}$	Decay ratio = 0.1;	
	$^{27} K_c^{(483)}$	$T_i^{(483)}$		$0.2 \leq \frac{\tau_m}{T_{m1}} \leq 2.0$;	
	$^{28} K_c^{(484)}$	$T_i^{(484)}$		$0.6 \leq \xi_m \leq 4.2$ ²⁶	
Minimum performance index: other tuning					
Wilton (1999). Model: Method 1	Minimum $\int_0^\infty [e^2(t) + x_2^2 K_m^2 (y(t) - y_\infty)^2] dt$; $T_{m1} > T_{m2}$				
	$\frac{x_1}{K_m}$	$T_{m1} + T_{m2}$	$\frac{T_{m2}}{1 + T_{m2}/T_{m1}}$		
	Coefficient values (obtained from graphs)				
	$T_{m2}/T_{m1} = 0.25$	$T_{m2}/T_{m1} = 0.5$	$T_{m2}/T_{m1} = 0.75$		
	x_1	x_2	x_1	x_2	
	1250.1	0.0001			
	22.946	0.04	20.396	0.04	
	4.8990	0.2	4.8990	0.2	
	2.1213	1	2.1213	1	

$$^{25} K_c^{(482)} = \left(K_{H2} - \frac{0.76 \left[1 - 0.426 \omega_{H2} \tau_m + 0.0551 \omega_{H2}^2 \tau_m^2 \right]}{K_m / (1 + K_{H2} K_m)} \right),$$

$$T_i^{(482)} = \frac{K_c^{(482)} (1 + K_{H2} K_m)}{\omega_{H2} K_m 0.0153 \left(1 + 4.37 \omega_{H2} \tau_m - 0.743 \omega_{H2}^2 \tau_m^2 \right)}.$$

$$^{26} \xi \leq 0.649 + 0.58 \frac{\tau_m}{T_{m1}} - 0.005 \left(\frac{\tau_m}{T_{m1}} \right)^2, \quad \xi > 0.613 + 0.613 \frac{\tau_m}{T_{m1}} + 0.117 \left(\frac{\tau_m}{T_{m1}} \right)^2.$$

$$^{27} K_c^{(483)} = \left(K_{H2} - \frac{1.22(0.55)^{\omega_{H2} \tau_m} \left[1 - 0.978/\varepsilon_2 + 0.659/\varepsilon_2^2 \right]}{K_m / (1 + K_{H2} K_m)} \right),$$

$$T_i^{(483)} = \frac{K_c^{(483)} (1 + K_{H2} K_m)}{\omega_{H2} K_m 0.0421 (1 + 0.969 \ln[\omega_{H2} \tau_m]) (1 + 2.02/\varepsilon_2 - 1.11/\varepsilon_2^2)}.$$

$$^{28} K_c^{(484)} = \left(K_{H2} - \frac{1.11 \left[1 - 0.467 \omega_{H2} \tau_m + 0.0657 \omega_{H2}^2 \tau_m^2 \right]}{K_m / (1 + K_{H2} K_m)} \right),$$

$$T_i^{(484)} = \frac{K_c^{(484)} (1 + K_{H2} K_m)}{\omega_{H2} K_m 0.0477 (-1 + 2.07 \omega_{H2} \tau_m - 0.333 \omega_{H2}^2 \tau_m^2)}.$$

Rule	K_c	T_i	T_d	Comment
Wilton (1999) – continued.	$\frac{x_1}{K_m}$	$T_{m1} + T_{m2}$	$\frac{T_{m2}}{1 + T_{m2}/T_{m1}}$	$T_{m2}/T_{m1} = 0.25$ $T_{m2}/T_{m1} = 0.5$ $T_{m2}/T_{m1} = 0.75$ $x_1 \quad x_2 \quad x_1 \quad x_2 \quad x_1 \quad x_2$ 180.01 0.0001 230.01 0.0001 300.01 0.0001 8.1584 0.04 9.1782 0.04 10.708 0.04 2.9394 0.2 3.1843 0.2 3.4293 0.2 1.2728 1 1.3435 1 1.4142 1 90.004 0.0001 100.00 0.0001 112.01 0.0001 4.5891 0.04 4.6911 0.04 5.354 0.04 2.0331 0.2 2.0821 0.2 2.1311 0.2 0.8768 1 1.0324 1 1.0748 1
Keane <i>et al.</i> (2000).	²⁹ $K_c^{(485)}$	$T_i^{(485)}$	$T_d^{(485)}$	
Model: Method I	Minimum ITAE servo plus ITAE regulator, with minimum M_{max} and with good noise attenuation; $T_{m1} = T_{m2}$			
Huang and Jeng (2003). Model: Method 13	³⁰ $K_c^{(486)}$	$2\xi_m T_{m1}$	$T_d^{(486)}$	$\xi_m \leq 1.1$

$$29 \quad K_c^{(485)} = \frac{(\ln K_u)[T_u + T_{m1}(1 + \ln K_u)]}{T_u K_m}, \quad T_d^{(485)} = \frac{T_u T_{m1}}{T_u + T_{m1}(1 + \ln K_u)},$$

$$T_i^{(485)} = \frac{(\ln K_u)[T_u + T_{m1}(1 + \ln K_u)]}{(\ln K_u)(1 + \ln K_u) + [\ln K_u + \ln(1 + e^{T_m - K_u})][\ln(K_u + 1.33419\tau_m) - 1]}.$$

$$30 \quad K_c^{(486)} = \frac{1.2858 T_m \xi_m}{K_m \tau_m} \left(\frac{T_m}{\tau_m} \right)^{-0.0544},$$

$$T_d^{(486)} = \frac{[(-0.0349 \xi_m + 1.0064)T_m + (0.4196 \xi_m - 0.1100)\tau_m]^2}{2T_m \xi_m}.$$

Rule	K_c	T_i	T_d	Comment
Huang and Jeng (2003) – continued.	$^{31} K_c^{(487)}$	$T_i^{(487)}$	$T_d^{(487)}$	$\xi_m > 1.1$
Servo or regulator tuning – Minimum IAE				
Direct synthesis: Frequency domain criteria				
Hang <i>et al.</i> (1993a). <i>Model:</i> <i>Method 8;</i> $\frac{\tau_m}{T_{m1}} > 0.3$	$\frac{\pi T_{m1}}{A_m K_m \tau_m}$	$2T_{m1}$	$0.5T_{m1}$	$T_{m1} = T_{m2}$
Sample results				
(Yang and Clarke (1996)).	$\frac{1.5708T_{m1}}{K_m \tau_m}$	$2T_{m1}$	$0.5T_{m1}$	$A_m = 2.0,$ $\phi_m = 45^\circ$
	$\frac{1.0472T_{m1}}{K_m \tau_m}$	$2T_{m1}$	$0.5T_{m1}$	$A_m = 3.0,$ $\phi_m = 60^\circ$
Ho <i>et al.</i> (1994). <i>Model: Method 1</i>	$^{32} K_c^{(488)}$	$T_i^{(488)}$	$T_d^{(488)}$	$\tau_m / T_m < 2\xi_m$
Ho <i>et al.</i> (1995a). <i>Model: Method 1</i>	$\frac{\omega_p T_{m1}}{A_m K_m}$	$^{33} T_i^{(489)}$	T_{m2}	$T_{m1} > T_{m2}.$

$$^{31} K_c^{(487)} = \frac{0.5890}{K_m \tau_m} \left(\frac{T_{m2}}{\tau_m} \right)^{-0.0030} \left(0.0052 \frac{T_{m2}^2}{\tau_m} + 0.8980 T_{m2} + 0.4877 \tau_m + T_{m1} \right),$$

$$T_i^{(487)} = 0.0052 \frac{T_{m2}^2}{\tau_m} + 0.8980 T_{m2} + 0.4877 \tau_m + T_{m1},$$

$$T_d^{(487)} = T_{m1} \left(\frac{0.0052 \frac{T_{m2}^2}{\tau_m} + 0.8980 T_{m2} + 0.4877 \tau_m}{0.0052 \frac{T_{m2}^2}{\tau_m} + 0.8980 T_{m2} + 0.4877 \tau_m + T_{m1}} \right).$$

$$^{32} K_c^{(488)} = \frac{2\omega_p T_{m1}^2}{\pi A_m} \left(\frac{\pi \xi_m}{\omega_p T_{m1}} + \pi - 2\omega_p \tau_m \right), T_i^{(488)} = \frac{2}{\pi} T_{m1}^2 \left(\frac{\pi \xi_m}{T_{m1} \omega_p} + \pi - 2\omega_p \tau_m \right),$$

$$T_d^{(488)} = - \frac{\pi T_{m1}}{2 \left(\frac{\pi \xi_m}{\omega_p} + \pi T_{m1} - 2T_{m1} \omega_p \tau_m \right)}.$$

$$^{33} T_i^{(489)} = \frac{1}{2\omega_p - \frac{4\omega_p^2 \tau_m}{\pi} + \frac{1}{T_{m1}}}.$$

Rule	K_c	T_i	T_d	Comment
Sample results				
Ho <i>et al.</i> (1995a) – continued.	$\frac{0.7854T_{m1}}{K_m \tau_m}$	T_{m1}	T_{m2}	$A_m = 2.0$, $\phi_m = 45^\circ$
	$\frac{0.5236T_{m1}}{K_m \tau_m}$	T_{m1}	T_{m2}	$A_m = 3.0$, $\phi_m = 60^\circ$
Ho <i>et al.</i> (1997). Model: Method 1	³⁴ $K_c^{(490)}$	$T_i^{(490)}$	$T_d^{(490)}$	³⁵
Leva <i>et al.</i> (1994). Model: Method 15	³⁶ $K_c^{(491)}$	$T_i^{(491)}$	$0.25T_i^{(491)}$	$\phi_m = 70^\circ$ (at least).

$$\begin{aligned}
{}^{34} K_c^{(490)} &= \frac{2}{\pi A_m K_m} \left(\pi \xi_m + \pi - 2 \frac{\tau_m}{T_{m1}} \right), \quad T_i^{(490)} = \frac{2}{\pi} T_{m1} (\pi \xi_m + \pi - 2 \tau_m), \\
T_d^{(490)} &= \frac{\pi T_{m1}^2}{2(\pi \xi_m T_{m1} + \pi T_{m1} - 2 \tau_m)}, \quad \frac{\tau_m}{T_m} \leq 1, \quad 2 \xi_m \geq \frac{\tau_m}{T_m}. \\
{}^{35} \phi_m &< \frac{\left(\pi + \sqrt{\pi^2 + \frac{8\pi\tau_m\xi_m}{T_{m1}}} \right) (A_m^2 - 1) - 2\pi A_m (A_m - 1)}{4A_m}.
\end{aligned}$$

$$\begin{aligned}
{}^{36} K_c^{(491)} &= \frac{\omega_{cn} T_i^{(491)}}{K_m} \sqrt{\frac{(1 + \omega_{cn}^2 T_{m1}^2)^2 + 4\xi_m^2 \omega_{cn}^2 T_{m1}^2}{(1 - T_i^{(491)} T_d^{(491)} \omega_{cn}^2)^2 + [T_i^{(491)}]^2 \omega_{cn}^2}}, \\
\omega_{cn} &= \frac{1}{\tau_m} \left[4.07 - \phi_m + \tan^{-1} \left\{ \frac{2\xi_m \tau_m T_{m1} (0.5\pi - \phi_m)}{(0.5\pi - \phi_m)^2 T_{m1}^2 - \tau_m^2} \right\} \right], \\
T_i^{(491)} &= \frac{2}{\omega_{cn}} \tan \left[0.5 \left\{ \omega_{cn} \tau_m + \phi_m - 0.5\pi - \tan^{-1} \left(\frac{2\xi_m \omega_{cn} T_{m1}}{\omega_{cn}^2 T_{m1}^2 - 1} \right) \right\} \right], \\
\xi_m &\leq 1 \text{ or } \xi_m > 1 \text{ (with } 0.5\pi - \phi_m > \frac{3\tau_m}{T_{m1}} \left[\xi_m + \sqrt{\xi_m^2 - 1} \right] \text{)}.
\end{aligned}$$

Rule	K_c	T_i	T_d	Comment
Leva <i>et al.</i> (1994) – continued.	$^{37} K_c^{(492)}$	$T_i^{(492)}$	$T_d^{(492)}$	
Wang <i>et al.</i> (1999).	$\frac{\xi_m T_{ml}}{K_m \tau_m}$	$2\xi_m T_{ml}$	$\frac{0.5T_{ml}}{\xi_m}$	<i>Model:</i> <i>Method 12.</i> ³⁸
Wang and Shao (1999). <i>Model:</i> <i>Method 12</i>	$x_1 \frac{\xi_m T_{ml}}{K_m \tau_m}$	$2\xi_m T_{ml}$	$0.5 \frac{T_{ml}}{\xi_m}$	
	Some coefficient values			
	x_1	Comment	x_1	Comment
	1.571	$A_m = 2, \phi_m = 45^\circ$	0.785	$A_m = 4, \phi_m = 67.5^\circ$
	1.047	$A_m = 3, \phi_m = 60^\circ$	0.628	$A_m = 5, \phi_m = 72^\circ$

$$^{37} K_c^{(492)} = \frac{\omega_{cn} T_{ml}^2}{K_m T_d^{(492)}} \sqrt{\frac{\omega_{cn}^2 + \frac{1}{T_{ml}^2} \left(2\xi_m \sqrt{\xi_m^2 - 1} - 1 \right)}{\omega_{cn}^2 + z^2}},$$

$$\omega_{cn} = \frac{1}{\tau_m} \left[4.07 - \phi_m + \tan^{-1} \left\{ \frac{2\xi_m \tau_m T_{ml} (0.5\pi - \phi_m)}{(0.5\pi - \phi_m)^2 T_{ml}^2 - \tau_m^2} \right\} \right],$$

$$z = \frac{\omega_{cn}}{\tan \left[\phi_m - 0.5\pi + \omega_{cn} \tau_m + \tan^{-1} \left\{ \frac{\omega_{cn} T_{ml}}{\left(\xi_m + \sqrt{\xi_m^2 - 1} \right)} \right\} \right]},$$

$$T_i^{(492)} = \frac{T_{ml} z + \left(\xi_m + \sqrt{\xi_m^2 - 1} \right)}{z \left(\xi_m + \sqrt{\xi_m^2 - 1} \right)}, \quad T_d^{(492)} = \frac{T_{ml}}{T_{ml} z + \left(\xi_m + \sqrt{\xi_m^2 - 1} \right)}, \quad \xi_m > 1$$

with $\frac{3\tau_m}{T_{ml}} \left[\xi_m - \sqrt{\xi_m^2 - 1} \right] < 0.5\pi - \phi_m \leq \frac{3\tau_m}{T_{ml}} \left[\xi_m + \sqrt{\xi_m^2 - 1} \right]$.

³⁸ $\xi_m > 0.7071$

$$\text{or } 0.05 < \frac{0.7071\tau_m}{T_{ml}\sqrt{2\xi_m^2 - 1}} < 0.15, \quad \frac{0.7071\tau_m}{T_{ml}\sqrt{2\xi_m^2 - 1}} > 1, \quad \xi_m \geq 1$$

$$\text{or } 0.05 < \frac{\xi_m \tau_m}{T_{ml}} < 0.15, \quad \frac{\xi_m \tau_m}{T_{ml}} > 1, \quad \xi_m < 1.$$

Rule	K_c	T_i	T_d	Comment
Wang and Shao (2000b). <i>Model:</i> Method 12	$^{39} K_c^{(493)}$	$2\xi_m T_{m1}$	$\frac{T_{m1}}{2\xi_m}$	Suggested $M_{\max} = 1.6$
Chen <i>et al.</i> (1999b). <i>Model: Method 1</i>	$\frac{x_1 \xi_m T_{m1}}{\tau_m K_m}$	$2\xi_m \tau_m$	$\frac{\tau_m}{2\xi_m}$	
	x_1	Comment		
	1.00	$A_m = 3.14, \phi_m = 61.4^0, M_s = 1.00$		
	1.22	$A_m = 2.58, \phi_m = 55.0^0, M_s = 1.10$		
	1.34	$A_m = 2.34, \phi_m = 51.6^0, M_s = 1.20$		
	1.40	$A_m = 2.24, \phi_m = 50.0^0, M_s = 1.26$		
	1.44	$A_m = 2.18, \phi_m = 48.7^0, M_s = 1.30$		
	1.52	$A_m = 2.07, \phi_m = 46.5^0, M_s = 1.40$		
	1.60	$A_m = 1.96, \phi_m = 44.1^0, M_s = 1.50$		
Leva <i>et al.</i> (2003). <i>Model: Method 1</i>	$^{40} K_c^{(494)}$	$16(T_{m2} + \tau_m)$	$4(T_{m2} + \tau_m)$	
	$^{41} K_c^{(495)}$	$5T_{m2} + 4\tau_m$	$T_d^{(495)}$	

$$^{39} K_c^{(493)} = \frac{2\xi_m T_{m1}}{K_m \tau_m} \left(1.451 - \frac{1.508}{M_{\max}} \right).$$

$$^{40} K_c^{(494)} = \frac{T_{m1} T_{m2}}{8K_m (T_{m2} + \tau_m)^2}, \quad T_{m1} \geq 4(T_{m2} + \tau_m).$$

$$^{41} K_c^{(495)} = \frac{T_{m1} (5T_{m2} + 4\tau_m)}{8K_m (T_{m2} + \tau_m)^2}, \quad T_d^{(495)} = \frac{4T_{m2} (T_{m2} + \tau_m)}{5T_{m2} + 4\tau_m}, \quad T_{m1} \geq 8(T_{m2} + \tau_m).$$

Rule	K_c	T_i	T_d	Comment
Direct synthesis: time domain criteria				
Wills (1962a). Model: Method 1 Servo response; $\xi = 1$. $T_{m1} = T_{m2}$; $\tau_m = 0.1T_{m1}$	x_1/K_m	$x_2 T_u$	$x_3 T_u$	
Representative coefficient values – deduced from graphs				
	x_1	x_2	x_3	x_1
	2.5	2.9	0.035	1.6
	2.75	2.9	0.073	16
	3.2	2.9	0.143	10
	20	2.9	0.285	0.42
	12	2.9	0.57	0.4
	1.4	1.4	0.035	10
	1.5	1.4	0.073	2
				x_2
				0.48
				0.143
				0.48
				0.36
				0.28
				0.073
				0.28
				0.143
				0.28
				0.28
Van der Grinten (1963). Model: Method 1	$^{42} K_c^{(496)}$	$T_{m1} + T_{m2}$ $+ 0.5\tau_m$	$T_d^{(496)}$	Step disturbance
Pemberton (1972b).	$\frac{2(T_{m1} + T_{m2})}{3K_m \tau_m}$	$T_{m1} + T_{m2}$	$\frac{T_{m1} T_{m2}}{T_{m1} + T_{m2}}$	Model: Method 1
Pemberton (1972a), (1972b). Model: Method 1	$\frac{(T_{m1} + T_{m2})}{K_m \tau_m}$	$T_{m1} + T_{m2}$	$\frac{T_{m1} T_{m2}}{T_{m1} + T_{m2}}$	$0.1 \leq \frac{T_{m1}}{T_{m2}} \leq 1.0$ $0.2 \leq \frac{\tau_m}{T_{m2}} \leq 1.0$

$$^{42} K_c^{(496)} = \frac{1}{K_m} \left(0.5 + \frac{T_{m1} + T_{m2}}{\tau_m} \right), \quad T_d^{(496)} = \frac{(T_{m1} + T_{m2})\tau_m + 2T_{m1}T_{m2}}{\tau_m + 2(T_{m1} + T_{m2})}.$$

$$^{43} K_c^{(497)} = \frac{e^{-\omega_d \tau_m}}{K_m} \left(1 - 0.5e^{-\omega_d \tau_m} + \frac{T_{m1} + T_{m2}}{\tau_m} e^{-\omega_d \tau_m} \right),$$

$$T_i^{(497)} = \frac{\tau_m}{e^{-\omega_d \tau_m}} \left(1 - 0.5e^{-\omega_d \tau_m} + \frac{T_{m1} + T_{m2}}{\tau_m} e^{-\omega_d \tau_m} \right),$$

$$T_d^{(497)} = \frac{\left(1 - 0.5e^{-\omega_d \tau_m} + \frac{T_{m1} T_{m2} e^{-\omega_d \tau_m}}{(T_{m1} + T_{m2})\tau_m} \right) (T_{m1} + T_{m2})}{1 - 0.5e^{-\omega_d \tau_m} + \frac{T_{m1} + T_{m2}}{\tau_m} e^{-\omega_d \tau_m}}.$$

Rule	K_c	T_i	T_d	Comment																												
Pemberton (1972b). Model: Method 1	$\frac{2(T_{m1} + T_{m2})}{3K_m \tau_m}$	$T_{m1} + T_{m2}$	$\frac{T_{m1} + T_{m2}}{4}$	$0.1 \leq \frac{T_{m1}}{T_{m2}} \leq 1.0$ $0.2 \leq \frac{\tau_m}{T_{m2}} \leq 1.0$																												
Chiu et al. (1973). Model: Method 1	$\frac{\lambda T_{m1} + T_{m2}}{K_m (1 + \lambda \tau_m)}$	$T_{m1} + T_{m2}$	$\frac{T_{m1} T_{m2}}{T_{m1} + T_{m2}}$	λ variable; suggested values are 0.2, 0.4, 0.6 and 1.0.																												
Mollenkamp et al. (1973). Model: Method 1	$\frac{2\xi_m T_{m1}}{K_m (T_{CL} + \tau_m)}$	$2\xi_m T_{m1}$	$\frac{T_{m1}}{2\xi_m}$	Appropriate for “small τ_m ”																												
Smith et al. (1975). Model: Method 1	$\frac{\lambda T_{m1}}{K_m (1 + \lambda \tau_m)}$	T_{m1}	T_{m2}	$T_{m1} > T_{m2}$																												
$\lambda = \text{pole of specified FOLPD closed loop response.}$																																
Suyama (1992). Model: Method 1	$\frac{T_{m1} + T_{m2}}{2K_m \tau_m}$	$T_{m1} + T_{m2}$	$\frac{T_{m1} T_{m2}}{T_{m1} + T_{m2}}$	OS=10%																												
Wang and Clements (1995).	$\frac{2\lambda \xi_m T_{m1}}{K_m (1 + \lambda \tau_m)}$	$2\xi_m T_{m1}$	$\frac{T_{m1}}{2\xi_m}$	Model: Method 16; Underdamped response																												
Gorez and Klán (2000). Model: Method 1	⁴⁴ $K_c^{(498)}$	$2\xi_m T_{m1}$	$\frac{T_{m1}}{2\xi_m}$	Non-dominant time delay																												
Vítecková (1999), Vítecková et al. (2000a). Model: Method 1	$\frac{x_1 (T_{m1} + T_{m2})}{K_m \tau_m}$ <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>x_1</td> <td>OS (%)</td> </tr> <tr> <td>0.368</td> <td>0</td> </tr> <tr> <td>0.514</td> <td>5</td> </tr> <tr> <td>0.581</td> <td>10</td> </tr> <tr> <td>0.641</td> <td>15</td> </tr> </table>	x_1	OS (%)	0.368	0	0.514	5	0.581	10	0.641	15	$T_{m1} + T_{m2}$ <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>x_1</td> <td>OS (%)</td> </tr> <tr> <td>0.696</td> <td>20</td> </tr> <tr> <td>0.748</td> <td>25</td> </tr> <tr> <td>0.801</td> <td>30</td> </tr> <tr> <td>0.853</td> <td>35</td> </tr> </table>	x_1	OS (%)	0.696	20	0.748	25	0.801	30	0.853	35	$\frac{T_{m1} T_{m2}}{T_{m1} + T_{m2}}$ <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>x_1</td> <td>OS (%)</td> </tr> <tr> <td>0.906</td> <td>40</td> </tr> <tr> <td>0.957</td> <td>45</td> </tr> <tr> <td>1.008</td> <td>50</td> </tr> </table>	x_1	OS (%)	0.906	40	0.957	45	1.008	50	Closed loop overshoot (OS) defined Overdamped process; $T_{m1} > T_{m2}$
x_1	OS (%)																															
0.368	0																															
0.514	5																															
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x_1	OS (%)																															
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1.008	50																															

⁴⁴ $K_c^{(498)} = \frac{2\xi_m T_{m1}}{K_m (2\xi_m T_{m1} + \tau_m)}$.

Rule	K _c		T _i		T _d		Comment
Vítečková (1999), Vítečková et al. (2000a) – continued.	$\frac{x_1 \xi_m T_{m1}}{K_m \tau_m}$		$2\xi_m T_{m1}$		$\frac{T_{m1}}{2\xi_m}$		Underdamped process; $0.5 < \xi_m \leq 1$
	x ₁	OS (%)	x ₁	OS (%)	x ₁	OS (%)	
	0.736	0	1.392	20	1.812	40	
	1.028	5	1.496	25	1.914	45	
	1.162	10	1.602	30	2.016	50	
	1.282	15	1.706	35			
Skogestad (2004b). Model: Method 1	$\frac{2\xi_m T_{m1}}{K_m (T_{CL} + \tau_m)}$		$2\xi_m T_{m1}$		$\frac{T_{m1}}{2\xi_m}$		Recommended $T_{CL} = \tau_m$
$A_m = 3.14, \phi_m = 61.4^0, M_{max} = 1.59$							
Vítečková et al. (2000b). Model: Method 4	$\frac{0.74 T_{m1}}{K_m \tau_m}$		$2T_{m1}$		$0.5T_{m1}$		$T_{m1} = T_{m2}$
Arvanitis et al. (2000).	⁴⁵ $K_c^{(499)}$		$2\xi_m^{des} T_{m1}$		$\frac{T_{m1}}{2\xi_m^{des}}$		Model: Method 1
Bi et al. (2000). Model: Method 8	$\frac{1.0128 \xi_m T_{m1}}{K_m \tau_m}$		$1.9747 K_m \tau_m$		$\frac{0.5064 T_{m1}^2}{K_m \tau_m}$		
Chen and Seborg (2002).	⁴⁶ $K_c^{(500)}$		$T_i^{(500)}$		$T_d^{(500)}$		Model: Method 1

$$^{45} K_c^{(499)} = \frac{2\xi_m^{des} T_{m1}}{K_m \tau_m (\sqrt{2\xi_m^{des}} + 1)}.$$

$$^{46} K_c^{(500)} = \frac{1}{K_m} \frac{[(T_{m1} + T_{m2})\tau_m + T_{m1}T_{m2}](3T_{CL} + \tau_m) - T_{CL}^3 - 3T_{CL}^2\tau_m}{(T_{CL} + \tau_m)^3},$$

$$T_i^{(500)} = \frac{[(T_{m1} + T_{m2})\tau_m + T_{m1}T_{m2}](3T_{CL} + \tau_m) - T_{CL}^3 - 3T_{CL}^2\tau_m}{T_{m1}T_{m2} + (T_{m1} + T_{m2} + \tau_m)\tau_m},$$

$$T_d^{(500)} = \frac{3T_{CL}^2 T_{m1} T_{m2} + T_{m1} T_{m2} \tau_m (3T_{CL} + \tau_m) - (T_{m1} + T_{m2} + \tau_m) T_{CL}^3}{[(T_{m1} + T_{m2})\tau_m + T_{m1}T_{m2}](3T_{CL} + \tau_m) - T_{CL}^3 - 3T_{CL}^2\tau_m},$$

$$\left(\frac{y}{d}\right)_{desired} = \frac{T_i^{(500)} e^{-s\tau_m}}{K_c^{(500)} (T_{CL}s + 1)^3}.$$

Rule	K_c	T_i	T_d	Comment
Chen and Seborg (2002) – continued.	⁴⁷ $K_c^{(501)}$	$T_i^{(501)}$	$T_d^{(501)}$	
Robust				
Brambilla <i>et al.</i> (1990). <i>Model:</i> <i>Method 1</i>	⁴⁸ $K_c^{(502)}$	$T_i^{(502)}$	$T_d^{(502)}$	
	Representative coefficient values (deduced from graph)			
	$\frac{\tau_m}{T_{m1} + T_{m2}}$	λ	$\frac{\tau_m}{T_{m1} + T_{m2}}$	λ
	0.1	0.50	1.0	0.29
	0.2	0.47	2.0	0.22
	0.5	0.39	5.0	0.16
Lee <i>et al.</i> (1998). <i>Model: Method 1</i>	$\frac{T_i^{(503)}}{K_m(2\lambda + \tau_m)}$	⁴⁹ $T_i^{(503)}$	$T_d^{(503)}$	

$$^{47} K_c^{(501)} = \frac{1}{K_m} \frac{(2\xi_m T_{m1} \tau_m + T_{m1}^2)(3T_{CL} + \tau_m) - T_{CL}^3 - 3T_{CL}^2 \tau_m}{(T_{CL} + \tau_m)^3},$$

$$T_i^{(501)} = \frac{(2\xi_m T_{m1} \tau_m + T_{m1}^2)(3T_{CL} + \tau_m) - T_{CL}^3 - 3T_{CL}^2 \tau_m}{T_{m1}^2 + (2\xi_m T_{m1} + \tau_m) \tau_m},$$

$$T_d^{(501)} = \frac{3T_{CL}^2 T_{m1}^2 + T_{m1}^2 \tau_m (3T_{CL} + \tau_m) - (2T_{m1} \xi_m + \tau_m) T_{CL}^3}{(2\xi_m T_{m1} \tau_m + T_{m1}^2)(3T_{CL} + \tau_m) - T_{CL}^3 - 3T_{CL}^2 \tau_m},$$

$$\left(\frac{y}{d}\right)_{\text{desired}} = \frac{T_i^{(501)} s e^{-s\tau_m}}{K_c^{(501)} (T_{CL}s + 1)^3}.$$

$$^{48} K_c^{(502)} = \frac{T_{m1} + T_{m2} + 0.5\tau_m}{K_m \tau_m (2\lambda + 1)}, \quad T_i^{(502)} = T_{m1} + T_{m2} + 0.5\tau_m,$$

$$T_d^{(502)} = \frac{T_{m1} T_{m2} + 0.5(T_{m1} + T_{m2})\tau_m}{T_{m1} + T_{m2} + 0.5\tau_m}, \quad 0.1 \leq \frac{\tau_m}{T_{m1} + T_{m2}} \leq 10.$$

$$^{49} T_i^{(503)} = 2\xi_m T_{m1} - \frac{2\lambda^2 - \tau_m^2}{2(2\lambda + \tau_m)}, \quad T_d^{(503)} = T_i^{(503)} - 2\xi_m T_{m1} + \frac{T_{m1}^2}{T_i^{(503)}} -$$

$$\frac{\tau_m^3}{6T_i^{(503)}(2\lambda + \tau_m)}; \quad \text{desired closed loop response} = \frac{e^{-\tau_m s}}{(\lambda s + 1)^2}.$$

Rule	K_c	T_i	T_d	Comment
Lee <i>et al.</i> (1998). <i>Model: Method 1</i>	$\frac{T_i^{(504)}}{K_m(2\lambda + \tau_m)}$	${}^{50}T_i^{(504)}$	$T_d^{(504)}$	
	$\lambda = \max \text{imum}[0.25\tau_m, 0.2T_{ml}]$ (Panda <i>et al.</i> , 2004)			
	$\frac{T_i^{(505)}}{K_m(\lambda + \tau_m)}$	${}^{51}T_i^{(505)}$	$T_d^{(505)}$	Desired closed loop response = $\frac{e^{-\tau_m s}}{(\lambda s + 1)}$
Huang <i>et al.</i> (1998), Rivera and Jun (2000). <i>Model: Method 1</i>	$\frac{T_i^{(506)}}{K_m(\lambda + \tau_m)}$	${}^{52}T_i^{(506)}$	$T_d^{(506)}$	
	$\frac{2\xi_m T_{ml}}{K_m(\tau_m + \lambda)}$	$2\xi_m T_{ml}$	$\frac{T_{ml}}{2\xi_m}$	
λ specified graphically for different OS and T_R values (Huang <i>et al.</i> (1998))				
Marchetti and Scali (2000).	${}^{53}K_c^{(507)}$	$T_i^{(507)}$	$T_d^{(507)}$	<i>Model:</i> <i>Method 1</i>

$${}^{50}T_i^{(504)} = T_{ml} + T_{m2} - \frac{2\lambda^2 - \tau_m^2}{2(2\lambda + \tau_m)}, \quad T_d^{(504)} = T_i^{(504)} - T_{ml} - T_{m2} + \frac{T_{ml}T_{m2}}{T_i^{(504)}} - \frac{\tau_m^3}{6T_i^{(504)}(2\lambda + \tau_m)}; \text{ desired closed loop response} = \frac{e^{-\tau_m s}}{(\lambda s + 1)^2}.$$

$${}^{51}T_i^{(505)} = 2\xi_m T_{ml} + \frac{\tau_m^2}{2(\lambda + \tau_m)}, \quad T_d^{(505)} = T_i^{(505)} - 2\xi_m T_{ml} \left(\frac{T_{ml}^2 - \frac{\tau_m^3}{6(\lambda + \tau_m)}}{T_i^{(505)}} \right).$$

$${}^{52}T_i^{(506)} = T_{ml} + T_{m2} + \frac{\tau_m^2}{2(\lambda + \tau_m)}, \quad T_d^{(506)} = T_i^{(506)} - (T_{ml} + T_{m2}) \left(\frac{T_{ml}T_{m2} - \frac{\tau_m^3}{6(\lambda + \tau_m)}}{T_i^{(506)}} \right).$$

$${}^{53}K_c^{(507)} = \frac{2\xi_m T_{ml} + 0.5\tau_m}{K_m(\lambda + \tau_m)}, \quad T_i^{(507)} = 2\xi_m T_{ml} + 0.5\tau_m, \quad T_d^{(507)} = \frac{T_{ml}(T_{ml} + \xi_m \tau_m)}{2\xi_m T_{ml} + 0.5\tau_m}.$$

Rule	K_c	T_i	T_d	Comment					
Smith (2003). Model: Method 1	$\frac{T_{m1}}{K_m(\lambda + \tau_m)}$	T_{m1}	T_{m2}	$\lambda \in [\tau_m, T_{m2}]$, $T_{m1} \geq T_{m2}$					
Kristiansson (2003).	$^{54} K_c^{(508)}$	$2\xi_m T_{m1}$	$\frac{T_{m1}}{2\xi_m}$	Model: Method 1					
Ultimate cycle									
Wills (1962a). Model: Method 1 Quarter decay ratio tuning. $T_{m1} = T_{m2}$; $\tau_m = 0.1T_{m1}$	x_1/K_m		$x_2 T_u$						
	Coefficient values (deduced from graph)								
	x_1	x_2	x_3	x_1	x_2	x_3	x_1	x_2	x_3
	10	2.9	0.036	28	1.4	0.143	18	0.48	0.143
	8	1.4	0.036	6	0.67	0.068	9	0.28	0.143
	14	2.9	0.068	2	0.48	0.068	18	0.67	0.24
	33	2.9	0.143	0.6	0.28	0.068	18	0.48	0.24
	18	1.4	0.068	30	0.67	0.143	13	0.28	0.28
	$^{55} K_c^{(509)}$		$T_i^{(509)}$		$T_d^{(509)}$		Model: Method 1		
Landau and Voda (1992). Model: Method 1	$^{56} K_c^{(510)}$		$\frac{4+\beta}{\beta} \frac{1}{\omega_{135^\circ}}$	$\frac{4}{4+\beta} \frac{1}{\omega_{135^\circ}}$	$1 \leq \beta \leq 2$				
	$3K_u$		$0.51T_u$	$0.13T_u$	$\omega_u \tau_m \leq 0.16$				

$$^{54} K_c^{(508)} = \frac{2\xi_m T_{m1}}{K_m \tau_m} \left[1 - \frac{1}{M_{\max}} \right]. \text{ Recommended values of } M_{\max} : 1.4, 1.7, 2.0.$$

$$^{55} K_c^{(509)} = \frac{2.2689}{K_m} \left[\left(\frac{\tau_m}{T_u} \right)^{2.0302+0.9553 \ln \left(\frac{\tau_m}{T_u} \right)} \right] \left[(K_u K_m)^{1.3135+0.1809 \ln(K_u K_m)+0.5219 \ln \left(\frac{\tau_m}{T_u} \right)} \right],$$

$$T_i^{(509)} = 0.2693 T_u \left[\left(\frac{\tau_m}{T_u} \right)^{-0.0517-0.0643 \ln \left(\frac{\tau_m}{T_u} \right)} \right] \left[(K_u K_m)^{0.5914-0.0832 \ln(K_u K_m)+0.0687 \ln \left(\frac{\tau_m}{T_u} \right)} \right],$$

$$T_d^{(509)} = 0.0068 T_u \left[\left(\frac{\tau_m}{T_u} \right)^{-4.2613-1.5855 \ln \left(\frac{\tau_m}{T_u} \right)} \right] \left[(K_u K_m)^{-1.1436-0.2658 \ln(K_u K_m)-1.1100 \ln \left(\frac{\tau_m}{T_u} \right)} \right].$$

$$^{56} K_c^{(510)} = \frac{4+\beta}{4} \frac{\beta}{2\sqrt{2}|G_p(j\omega_{135^\circ})|}.$$

Rule	K_c	T_i	T_d	Comment
Landau and Voda (1992) – continued.	$1.9K_u$	$0.64T_u$	$0.16T_u$	$0.16 < \omega_u \tau_m \leq 0.2$

4.5.2 Ideal controller in series with a first order lag

$$G_c(s) = K_c \left(1 + \frac{1}{T_i s} + T_d s \right) \frac{1}{T_f s + 1}$$

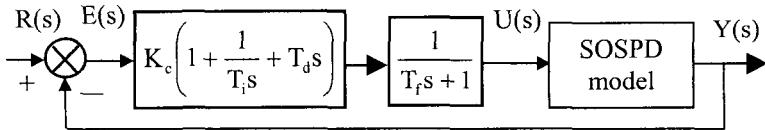


Table 116: PID controller tuning rules – SOSPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{(1+sT_{m1})(1+sT_{m2})}$ or

$$G_m(s) = \frac{K_m e^{-s\tau_m}}{T_{m1}^2 s^2 + 2\xi_m T_{m1} s + 1}$$

Rule	K_c	T_i	T_d	Comment
Direct synthesis: time domain criteria				
Lim <i>et al.</i> (1985).	¹ $K_c^{(511)}$	$T_{m1} + T_{m2}$	$\frac{T_{m1} T_{m2}}{T_{m1} + T_{m2}}$	<i>Model: Method 1</i>
Arvanitis <i>et al.</i> (2000). <i>Model: Method 1</i>	² $K_c^{(512)}$	$2\xi_m^{\text{des}} T_{m1}$	$\frac{T_{m1}}{2\xi_m^{\text{des}}}$	$T_{CL2} > \frac{\tau_m}{\sqrt{2}}$
Panda <i>et al.</i> (2004). <i>Model: Method 1</i>	$\frac{\xi_m T_{m1}}{K_m \tau_m}$	$2\xi_m T_{m1}$	$\frac{T_{m1}}{2\xi_m}$	$T_f = \frac{T_{m1}}{2N\xi_m}$, N not defined
Direct synthesis: frequency domain criteria				
Huang <i>et al.</i> (2005). <i>Model: Method 14</i>	$\frac{\xi_m T_{m1}}{K_m \tau_m}$	$2\xi_m T_{m1}$	$\frac{T_{m1}}{2\xi_m}$	$T_f = \frac{0.025 T_{m1}}{\xi_m}$
$A_m \approx 3, \phi_m \approx 60^\circ$				
Robust				
Hang <i>et al.</i> (1993a). <i>Model: Method 8</i>	$\frac{2T_{m1} + \tau_m}{2(\lambda + \tau_m)K_m}$	$T_{m1} + 0.5\tau_m$	$\frac{T_{m1}\tau_m}{2T_{m1} + \tau_m}$	$\lambda > 0.25\tau_m$. $T_f = \frac{\lambda\tau_m}{2(\lambda + \tau_m)}$
	Model has a repeated pole (T_{m1}) .			

$$^1 K_c^{(511)} = \frac{T_{m1} + T_{m2}}{K_m (2\xi T_{CL2} + \tau_m)}, \quad T_f = \frac{T_{CL2}^2}{2\xi T_{CL2} + \tau_m}.$$

$$^2 K_c^{(512)} = \frac{2\xi_m^{\text{des}} T_{m1}}{K_m (2\xi_m^{\text{des}} T_{CL2} + \tau_m)}, \quad T_f = \frac{2(2T_{CL2}^2 - \tau_m^2)}{2\xi_m^{\text{des}} T_{CL2} + \tau_m}.$$

Rule	K_c	T_i	T_d	Comment
Rivera and Jun (2000). Model: Method I; λ not specified	$\frac{2\xi_m T_{m1}}{\lambda}$	$2\xi_m T_{m1}$	$\frac{T_{m1}}{2\xi_m}$	$T_f = \tau_m$
	$\frac{2\xi_m T_{m1}}{K_m(2\tau_m + \lambda)}$	$2\xi_m T_{m1}$	$\frac{T_{m1}}{2\xi_m}$	$T_f = \frac{\tau_m \lambda}{2\tau_m + \lambda}$
Lee and Edgar (2002). Model: Method I	${}^3 K_c^{(513)}$	$T_i^{(513)}$	$T_d^{(513)}$	
	$\frac{T_{m1} + T_{m2}}{K_m(\tau_m + 2\lambda)}$	$T_{m1} + T_{m2}$	$\frac{T_{m1} T_{m2}}{T_{m1} + T_{m2}}$	$T_f = \frac{\lambda^2}{2\lambda + \tau_m};$ H_∞ method
Zhang et al. (2002). Model: Method I 5% overshoot: $\lambda = 0.5\tau_m$	${}^4 K_c^{(514)}$	$\frac{T_{m1} T_{m2}}{T_{m1} + T_{m2}}$	$T_{m1} + T_{m2}$	$T_f = \frac{\lambda \tau_m}{\lambda + 2\tau_m};$ H_2 method
	$\frac{T_i^{(515)}}{K_m(2\lambda + \tau_m)}$	${}^5 T_i^{(515)}$	$T_d^{(515)}$	$T_f = 0;$ Maclaurin method

$${}^3 K_c^{(513)} = \frac{1}{K_m(2\lambda + \tau_m)} \left(2\xi_m T_{m1} + \frac{0.5\tau_m^2 - \lambda^2}{2\lambda + \tau_m} + \sqrt{T_m^2 - \frac{\tau_m^3}{6(2\lambda + \tau_m)} + \left(2\xi_m T_{m1} + \frac{0.5\tau_m^2 - \lambda^2}{2\lambda + \tau_m} \right) \frac{0.5\tau_m^2 - \lambda^2}{2\lambda + \tau_m}} \right),$$

$$T_i^{(513)} = 2\xi_m T_{m1} + \frac{0.5\tau_m^2 - \lambda^2}{2\lambda + \tau_m} + \sqrt{T_m^2 - \frac{\tau_m^3}{6(2\lambda + \tau_m)} + \left(2\xi_m T_{m1} + \frac{0.5\tau_m^2 - \lambda^2}{2\lambda + \tau_m} \right) \frac{0.5\tau_m^2 - \lambda^2}{2\lambda + \tau_m}},$$

$$T_d^{(513)} = \sqrt{T_m^2 - \frac{\tau_m^3}{6(2\lambda + \tau_m)} + \left(2\xi_m T_{m1} + \frac{0.5\tau_m^2 - \lambda^2}{2\lambda + \tau_m} \right) \frac{0.5\tau_m^2 - \lambda^2}{2\lambda + \tau_m}}.$$

$${}^4 K_c^{(514)} = \frac{T_{m1} T_{m2}}{K_m(T_{m1} + T_{m2})(\lambda + 2\tau_m)}.$$

$${}^5 T_i^{(515)} = T_{m1} + T_{m2} - \frac{2\lambda^2 - \tau_m^2}{2(2\lambda + \tau_m)}, \quad T_d^{(515)} = \frac{T_{m1} T_{m2} - \frac{\tau_m^3}{12\lambda + 6\tau_m}}{T_i^{(515)}} - \frac{2\lambda^2 - \tau_m^2}{2(2\lambda + \tau_m)}.$$

Rule	K_c	T_i	T_d	Comment
Kristiansson (2003). <i>Model: Method I</i>	${}^6 K_c^{(516)}$	$2\xi_m T_{m1}$	$\frac{T_{m1}}{2\xi_m}$	
	${}^7 K_c^{(517)}$			
Panda <i>et al.</i> (2004). <i>Model: Method I</i>	${}^8 K_c^{(518)}$	$T_i^{(518)}$	$T_d^{(518)}$	
	${}^9 K_c^{(519)}$	$T_i^{(519)}$	$T_d^{(519)}$	

$${}^6 K_c^{(516)} = \frac{2\xi_m T_{m1}}{K_m \tau_m} \left[\alpha - \sqrt{\alpha^2 - 1 + \frac{1}{M_{max}^2}} \right], \quad \alpha = 1 + \frac{T_f}{\tau_m} \left[1 - \sqrt{1 - \frac{1}{M_{max}^2}} \right], \quad T_f = \frac{T_{m1}}{\beta}.$$

$${}^7 K_c^{(517)} = \frac{2\xi_m T_{m1}}{K_m \tau_m} \left[1 + 0.019 \frac{T_{m1}}{\tau_m} - \sqrt{0.346 + 0.038 \frac{T_{m1}}{\tau_m} + 0.00036 \frac{T_{m1}^2}{\tau_m^2}} \right];$$

Suggested $M_{max} = 1.7, \beta = 10$.

$${}^8 K_c^{(518)} = \frac{2\xi_m^2 T_{m1} + 0.25 T_{m1} + 0.5 \xi_m \tau_m}{K_m (\lambda + 2\xi_m T_{m1})},$$

$$T_i^{(518)} = \frac{4\xi_m^2 T_{m1} + 0.5 T_{m1} + \xi_m \tau_m}{2\xi_m}, \quad T_d^{(518)} = \frac{(T_{m1} + 2\xi_m \tau_m) T_{m1} \xi_m}{4\xi_m^2 T_{m1} + \xi_m \tau_m + 0.5 T_{m1}},$$

$$T_f = \frac{\lambda(T_{m1} + 2\xi_m \tau_m)}{4\xi_m \lambda + 4\xi_m \tau_m + 2T_{m1}}, \quad \lambda = \max \text{imum} \left[0.25 \left(\frac{T_{m1}}{2\xi_m} + \tau_m \right), 0.4 \xi_m T_{m1} \right].$$

$${}^9 K_c^{(519)} = \frac{1}{K_m} \frac{0.828 + 0.812 \frac{T_{m2}}{T_{m1}} + 0.172 T_{m1} e^{-(6.9 T_{m2}/T_{m1})} + \frac{0.558 T_{m2} T_{m1}}{T_{m1} + 1.208 T_{m2}} + 0.5 \tau_m}{\lambda + 0.828 + 0.812 \frac{T_{m2}}{T_{m1}} + 0.172 T_{m1} e^{-(6.9 T_{m2}/T_{m1})} + \frac{0.558 T_{m2} T_{m1}}{T_{m1} + 1.208 T_{m2}}},$$

$$T_i^{(519)} = 0.828 + 0.812 \frac{T_{m2}}{T_{m1}} + 0.172 T_{m1} e^{-(6.9 T_{m2}/T_{m1})} + \frac{0.558 T_{m2} T_{m1}}{T_{m1} + 1.208 T_{m2}} + 0.5 \tau_m,$$

$$T_d^{(519)} = \frac{\left[0.828 + 0.812 \frac{T_{m2}}{T_{m1}} + 0.172 T_{m1} e^{-(6.9 T_{m2}/T_{m1})} \right] \left[\frac{1.116 T_{m2} T_{m1}}{T_{m1} + 1.208 T_{m2}} + \tau_m \right]}{1.656 + 1.624 \frac{T_{m2}}{T_{m1}} + 0.344 T_{m1} e^{-(6.9 T_{m2}/T_{m1})} + \frac{1.116 T_{m2} T_{m1}}{T_{m1} + 1.208 T_{m2}} + \tau_m},$$

$$T_f = \frac{\lambda [1.116 T_{m1} T_{m2} + \tau_m (T_{m1} + 1.208 T_{m2})]}{2(\lambda + \tau_m)(T_{m1} + 1.208 T_{m2}) + 2.232 T_{m2} T_{m1}},$$

$$\lambda = \max \text{imum} \left[\frac{0.279 T_{m2} T_{m1}}{T_{m1} + 1.208 T_{m2}} + 0.25 \tau_m, 0.1656 + 0.1624 \frac{T_{m2}}{T_{m1}} + 0.0344 T_{m1} e^{-(6.9 T_{m2}/T_{m1})} \right]$$

Rule	K_c	T_i	T_d	Comment
Ultimate cycle				
Huang <i>et al.</i> (2005). <i>Model:</i> <i>Method 14</i>	$^{10} K_c^{(520)}$	$T_i^{(520)}$	$T_d^{(520)}$	$T_f = \frac{T_d^{(520)}}{20}$
$A_m \approx 3, \phi_m \approx 60^\circ$				

$$^{10} K_c^{(520)} = 0.080 \frac{\hat{K}_u \hat{T}_u}{\hat{K}_m \tau_m} \sin \left(\frac{6.283 \tau_m}{\hat{T}_u} \right), \quad T_i^{(520)} = 0.159 \hat{K}_u \hat{T}_u \sin \left(\frac{6.283 \tau_m}{\hat{T}_u} \right),$$

$$T_d^{(520)} = 0.159 \frac{\hat{T}_u}{\hat{K}_u} \left[\frac{1 + \hat{K}_u \cos \left(\frac{6.283 \tau_m}{\hat{T}_u} \right)}{\sin \left(\frac{6.283 \tau_m}{\hat{T}_u} \right)} \right].$$

4.5.3 Ideal controller in series with a first order filter

$$G_c(s) = K_c \left(1 + \frac{1}{T_i s} + T_d s \right) \frac{b_{f1}s + 1}{a_{f1}s + 1}$$

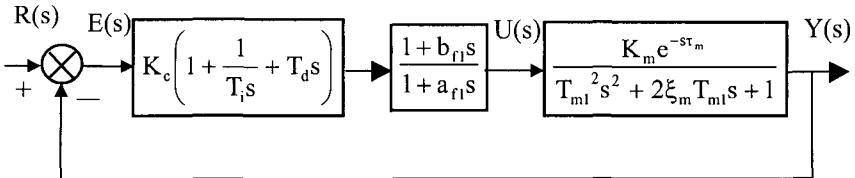


Table 117: PID controller tuning rules – SOSPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{T_{m1}^2 s^2 + 2\xi_m T_{m1} s + 1}$

Rule	K_c	T_i	T_d	Comment
Robust				
Jahanmiri and Fallahi (1997). Model: Method 6	$\frac{2\xi_m T_{m1}}{K_m(\tau_m + \lambda)}$	$2\xi_m T_{m1}$	$\frac{T_{m1}}{2\xi_m}$	$b_{f1} = 0.5\tau_m$, $a_{f1} = \frac{\lambda\tau_m}{2(\tau_m + \lambda)}$
$\lambda = 0.25\tau_m + 0.1\xi_m T_{m1}$				

4.5.4 Ideal controller in series with a second order filter

$$G_c(s) = K_c \left(1 + \frac{1}{T_i s} + T_d s \right) \frac{1 + b_{f1}s}{1 + a_{f1}s + a_{f2}s^2}$$

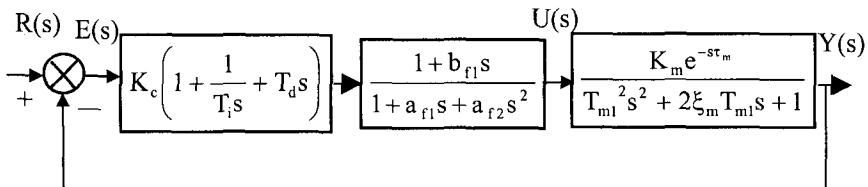


Table 118: PID controller tuning rules – SOSPD model $G_m(s) = \frac{K_m e^{-st_m}}{T_{m1}^2 s^2 + 2\xi_m T_{m1} s + 1}$

Rule	K_c	T_i	T_d	Comment
Robust				
Kristiansson (2003). <i>Model: Method 1</i>	¹¹ $K_c^{(521)}$	$2\xi_m T_{m1}$	$\frac{T_{m1}}{2\xi_m}$	$M_{\max} = 1.7$, $\beta = 10$.
$b_{ff} = 0, a_{ff} = 0.08T_{m1}, a_{2f} = 0.01T_{m1}^2$				

$$\stackrel{11}{K}_c^{(521)} = \frac{2\xi_m T_{m1}}{K_m \tau_m} \left[1 + 0.019 \frac{T_{m1}}{\tau_m} - \sqrt{0.346 + 0.038 \frac{T_{m1}}{\tau_m} + 0.00036 \frac{T_{m1}^2}{\tau_m^2}} \right].$$

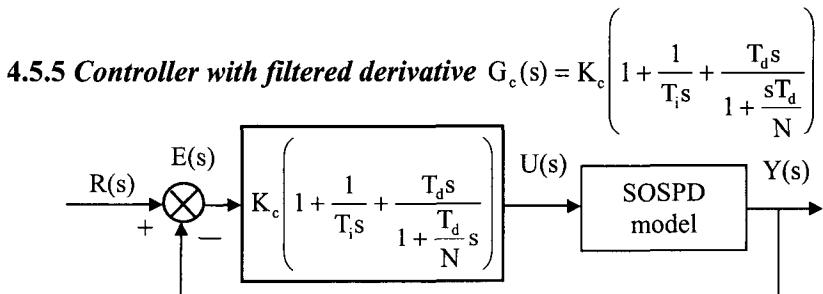


Table 119: PID controller tuning rules – SOSPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{(1+sT_{m1})(1+sT_{m2})}$ or

$$G_m(s) = \frac{K_m e^{-s\tau_m}}{T_{m1}^2 s^2 + 2\xi_m T_{m1} s + 1}$$

Rule	K_c	T_i	T_d	Comment
Direct synthesis				
Hang <i>et al.</i> (1994). <i>Model: Method 1</i>	$\frac{\omega_p T_{m1}}{A_m K_m}$	$^{12} T_i^{(522)}$	T_{m2}	$N = 20$. $T_{m1} > T_{m2}$
	$\frac{0.7854 T_{m1}}{K_m \tau_m}$	T_{m1}	T_{m2}	$A_m = 2.0$, $\phi_m = 45^\circ$
Robust				
Hang <i>et al.</i> (1994). <i>Model: Method 1</i>	$\frac{T_{m1}}{K_m (T_{CL} + \tau_m)}$	T_{m1}	T_{m2}	$N = 20$. $T_{m1} > T_{m2}$
Zhong and Li (2002).	$^{13} K_c^{(523)}$	$T_i^{(523)}$	$T_d^{(523)}$	<i>Model: Method 1</i>

$$^{12} T_i^{(522)} = \frac{1}{2\omega_p - \frac{4\omega_p^2 \tau_m}{\pi} + \frac{1}{T_{m1}}}.$$

$$^{13} K_c^{(523)} = \frac{2\xi_m T_{m1} (2\alpha + \tau_m) - \alpha^2}{K_m (2\alpha + \tau_m)^2}, \quad T_i^{(523)} = \frac{2\alpha + \tau_m}{2\xi_m T_{m1} (2\alpha + \tau_m) - \alpha^2},$$

$$T_d^{(523)} = T_i^{(523)} \left[T_{m1}^2 - \frac{2\xi_m T_{m1} \alpha^2 (2\alpha + \tau_m) - \alpha^4}{(2\alpha + \tau_m)^2} \right], \quad N = \frac{2\alpha + \tau_m}{\alpha^2} T_d^{(523)}.$$

4.5.6 Controller with filtered derivative in series with a second order filter

$$\text{filter } G_c(s) = K_c \left(1 + \frac{1}{T_i s} + \frac{T_d s}{1 + \frac{T_d}{N} s} \right) \left(\frac{1 + b_{f1}s + b_{f2}s^2}{1 + a_{f1}s + a_{f2}s^2} \right)$$

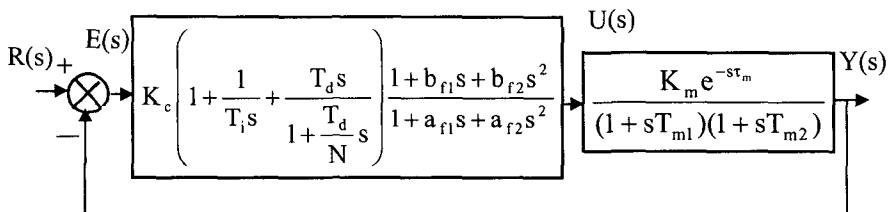


Table 120: PID controller tuning rules – SOSPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{(1+sT_{m1})(1+sT_{m2})}$

Rule	K_c	T_i	T_d	Comment
Robust				
Shi and Lee (2004). Model: Method 1	$^{14} K_c^{(524)}$	$T_i^{(524)}$	$T_d^{(524)}$	

$$\begin{aligned}
 &^{14} K_c^{(524)} = \frac{\omega_{-6dB}}{K_m (2 + \omega_{-6dB} \tau_m)} \left(T_{m1} + T_{m2} - \frac{0.5}{\omega_{-6dB}} \right), \quad b_{f1} = \frac{1 + \omega_{-6dB} \tau_m}{2 \omega_{-6dB}}, \\
 & b_{f2} = \frac{\tau_m}{4 \omega_{-6dB}}, \quad a_{f1} = \frac{1}{2 \omega_{-6dB}}, \quad a_{f2} = \frac{\tau_m}{2 \omega_{-6dB} (1 + \omega_{-6dB} \tau_m)}, \\
 & T_i^{(524)} = T_{m1} + T_{m2} - \frac{0.5}{\omega_{-6dB}}, \quad T_d^{(524)} = \frac{T_{m1} T_{m2} \omega_{-6dB}^2 - 0.5 [(T_{m1} + T_{m2}) \omega_{-6dB} - 0.5]}{\omega_{-6dB} [(T_{m1} + T_{m2}) \omega_{-6dB} - 0.5]}, \\
 & N = \frac{0.5 [(T_{m1} + T_{m2}) \omega_{-6dB} - 0.5]}{T_{m1} T_{m2} \omega_{-6dB}^2 - 0.5 [(T_{m1} + T_{m2}) \omega_{-6dB} - 0.5]}.
 \end{aligned}$$

4.5.7 Ideal controller with set-point weighting 1

$$U(s) = K_c \left(F_p R(s) - Y(s) \right) + \frac{K_c}{T_i s} \left(F_i R(s) - Y(s) \right) + K_c T_d s \left(F_d R(s) - Y(s) \right)$$

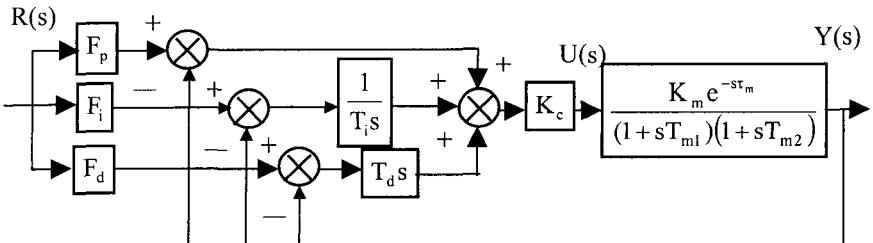


Table 121: PID tuning rules – SOSPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{(1+sT_{m1})(1+sT_{m2})}$

Rule	K_c	T_i	T_d	Comment
Ultimate cycle				
Oubrahim and Leonard (1998). <i>Model:</i> <i>Method 1;</i> $T_{m1} = T_{m2}$; $0.1 < \frac{\tau_m}{T_{m1}} < 3$	¹ $0.6K_u$	$0.5T_u$ $F_i = 1$	$0.125T_u$	10% overshoot
	² $0.6K_u$	$0.5T_u$ $F_i = 1$	$0.125T_u$	20% overshoot

$$^1 F_p = 1.3 \frac{16 - K_m K_u}{17 + K_m K_u}, \quad F_d = 1.69 \left(\frac{16 - K_m K_u}{17 + K_m K_u} \right)^2.$$

$$^2 F_p = \frac{38}{29 + 3.5K_m K_u}, \quad F_d = \frac{1.717}{(1 + 0.121K_m K_u)^2}.$$

4.5.8 Ideal controller with set-point weighting 2

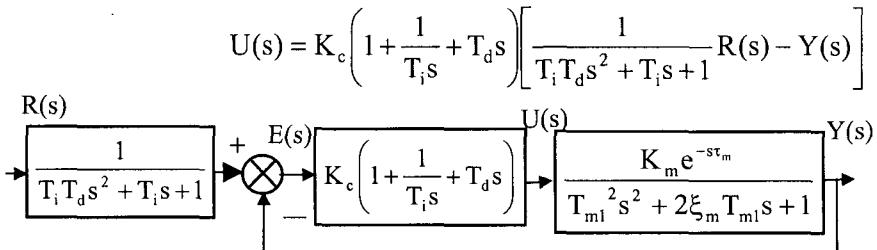


Table 122: PID controller tuning rules – SOSPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{T_{m1}^2 s^2 + 2\xi_m T_{m1} s + 1}$

Rule	K_c	T_i	T_d	Comment
Direct synthesis				
Arvanitis <i>et al.</i> (2000).	${}^3 K_c^{(525)}$	$T_i^{(525)}$	$T_d^{(525)}$	<i>Model: Method 1</i>
	${}^4 K_c^{(526)}$	$T_i^{(526)}$	$T_d^{(526)}$	

$${}^3 K_c^{(525)} = \frac{\chi T_{m1}}{K_m \tau_m} \left(2\xi_m + \frac{T_{m1}}{\tau_m} \right), \quad 0 < \chi < 1, \text{ with suggested } \chi = 0.4;$$

$$T_i^{(525)} = \frac{\tau_m}{a^2} \left[a + 2\xi^2 \left(\frac{1}{\chi} - 1 \right) + 2\xi \sqrt{\left(\frac{1}{\chi} - 1 \right) \left(a + \xi^2 \left(\frac{1}{\chi} - 1 \right) \right)} \right], \text{ with}$$

$$1 + \frac{\tau_m^2}{\chi T_{m1} (2\xi_m \tau_m + T_{m1})} + \xi^2 \left(\frac{1}{\chi} - 1 \right) \geq 0 \text{ and with } a = 1 + \frac{\tau_m^2}{\chi T_{m1} (2\xi_m \tau_m + T_{m1})};$$

$$T_d^{(525)} = \frac{1}{\chi} \frac{T_{m1} \tau_m}{2\xi_m \tau_m + T_{m1}}.$$

$${}^4 K_c^{(526)} = \frac{1}{K_m \tau_m} \frac{(2\xi T_{CL2} + \tau_m)(2\xi_m \tau_m + T_{m1})T_{m1} - \tau_m T_{CL2}^2}{T_{CL2}^2 + \tau_m (2\xi T_{CL2} + \tau_m)},$$

$$T_i^{(526)} = \frac{(2\xi T_{CL2} + \tau_m)(2\xi_m \tau_m + T_{m1})T_{m1} - \tau_m T_{CL2}^2}{\tau_m^2 + \tau_m^2 T_{m1} (2\xi_m \tau_m + T_{m1})},$$

$$T_d^{(526)} = \frac{T_{m1}^2 [T_{CL2}^2 + \tau_m (2\xi T_{CL2} + \tau_m)]}{(2\xi T_{CL2} + \tau_m)(2\xi_m \tau_m + T_{m1})T_{m1} - \tau_m T_{CL2}^2}.$$

4.5.9 Classical controller 1 $G_c(s) = K_c \left(1 + \frac{1}{T_i s}\right) \left(\frac{1 + T_d s}{1 + \frac{T_d s}{N}} \right)$

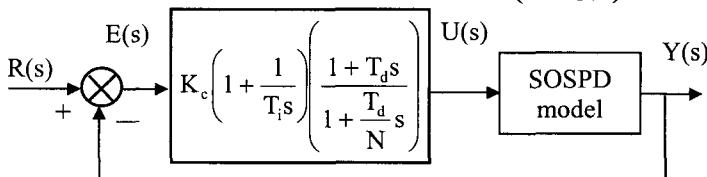


Table 123: PID controller tuning rules – SOSPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{(1+sT_{m1})(1+sT_{m2})}$ or

$$G_m(s) = \frac{K_m e^{-s\tau_m}}{T_{m1}^2 s^2 + 2\xi_m T_{m1}s + 1}$$

Rule	K_c	T_i	T_d	Comment
Minimum performance index: regulator tuning				
Minimum IAE - Shinskey (1988) – page 149. Model: Method 1; N not specified	$\frac{0.80T_{m1}}{K_m \tau_m}$	$1.5(T_{m2} + \tau_m)$	$0.60(T_{m2} + \tau_m)$	$\frac{T_{m2}}{T_{m2} + \tau_m} = 0.25$
	$\frac{0.77T_{m1}}{K_m \tau_m}$	$1.2(T_{m2} + \tau_m)$	$0.70(T_{m2} + \tau_m)$	$\frac{T_{m2}}{T_{m2} + \tau_m} = 0.5$
	$\frac{0.83T_{m1}}{K_m \tau_m}$	$0.75(T_{m2} + \tau_m)$	$0.60(T_{m2} + \tau_m)$	$\frac{T_{m2}}{T_{m2} + \tau_m} = 0.75$
Minimum IAE – Chao et al. (1989). Model: Method 1	${}^5 K_c^{(527)}$	$T_i^{(527)}$	$T_d^{(527)}$	$N = 8$
	$0.8 \leq \xi_m \leq 3; 0.05 \leq \frac{\tau_m}{2\xi_m T_{m1}} \leq 0.5$			

$${}^5 K_c^{(527)} = \frac{-0.01133 + 0.5017\xi_m - 0.07813\xi_m^2}{K_m} \left(\frac{\tau_m}{2\xi_m T_{m1}} \right)^{-1.890 + 0.7902\xi_m - 0.1554\xi_m^2},$$

$$T_i^{(527)} = 2\xi_m T_{m1} \left(1.235 - 0.4126\xi_m + 0.09873\xi_m^2 \left(\frac{\tau_m}{2\xi_m T_{m1}} \right) \right)^{-0.03793 + 0.3975\xi_m - 0.05354\xi_m^2},$$

$$T_d^{(527)} = 2\xi_m T_{m1} \left(1.214 - 0.6250\xi_m + 0.1358\xi_m^2 \left(\frac{\tau_m}{2\xi_m T_{m1}} \right) \right)^{0.5696 - 0.8484\xi_m + 0.0505\xi_m^2}.$$

Rule	K_c	T_i	T_d	Comment							
Minimum IAE - Shinskey (1994) - page 159. Model: Method 1 N not specified	${}^6 K_c^{(528)}$	$T_i^{(528)}$	$T_d^{(528)}$	$\frac{T_{m2}}{\tau_m} \leq 3$							
	$\frac{2.5T_{m1}}{K_m \tau_m}$	$\tau_m + 0.2T_{m2}$	$\tau_m + 0.2T_{m2}$	$\frac{T_{m2}}{\tau_m} > 3$							
Minimum IAE - Shinskey (1996) - page 121. Model: Method 1 N not specified	$0.59K_u$	$0.36T_u$	$0.26T_u$	$\frac{\tau_m}{T_{m1}} = 0.2$, $\frac{T_{m2}}{T_{m1}} = 0.2$							
Minimum IAE Shinskey (1996) - page 119. Model: Method 1 $\frac{\tau_m}{T_{m1}} = 0.2$; N not specified	$\frac{0.85T_{m1}}{K_m \tau_m}$	$1.98\tau_m$	$0.86\tau_m$	$\frac{T_{m2}}{T_{m1}} = 0.1$							
	$\frac{0.87T_{m1}}{K_m \tau_m}$	$2.30\tau_m$	$1.65\tau_m$	$\frac{T_{m2}}{T_{m1}} = 0.2$							
	$\frac{1.00T_{m1}}{K_m \tau_m}$	$2.50\tau_m$	$2.00\tau_m$	$\frac{T_{m2}}{T_{m1}} = 0.5$							
	$\frac{1.25T_{m1}}{K_m \tau_m}$	$2.75\tau_m$	$2.75\tau_m$	$\frac{T_{m2}}{T_{m1}} = 1.0$							
	$\frac{x_1}{K_m}$	$x_2 \tau_m$	$x_3 \tau_m$	Model: Method 1							
Minimum ISE - McAvoy and Johnson (1967).	Representative coefficient values (deduced from graph)										
	ξ_m	$\frac{T_{m1}}{\tau_m}$	x_1	x_2	x_3	ξ_m	$\frac{T_{m1}}{\tau_m}$	x_1	x_2	x_3	
	N = 20	1	0.5	0.7	0.97	0.75	4	0.5	3.0	1.16	0.85
		1	4.0	7.6	3.33	2.03	4	4.0	22.7	1.89	1.28
		1	10.0	34.3	5.00	2.7	7	0.5	5.4	1.19	0.85
							7	4.0	40.0	1.64	1.14
	N = 10	1	0.5	0.9	1.10	0.64	4	0.5	3.2	1.33	0.78
		1	4.0	8.0	4.00	1.83	4	4.0	23.9	2.17	1.17
		1	10.0	33.5	6.25	2.43	7	0.5	5.9	1.39	0.78
							7	4.0	42.9	1.89	1.04

$${}^6 K_c^{(528)} = \frac{100}{\left(48 + 57 \left[1 - e^{-\frac{1.2T_{m1}}{\tau_m}} \right] \right) \frac{K_m \tau_m}{T_{m1}} \left(1 + 0.34 \frac{T_{m2}}{\tau_m} - 0.2 \left[\frac{T_{m2}}{\tau_m} \right]^2 \right)},$$

$$T_i^{(528)} = \tau_m \left(1.5 - e^{-\frac{T_{m1}}{1.5\tau_m}} \right) \left(1 + 0.9 \left[1 - e^{-\frac{T_{m2}}{\tau_m}} \right] \right), T_d^{(528)} = 0.56\tau_m \left(1 - e^{-\frac{1.2T_{m1}}{\tau_m}} \right) + 0.6T_{m2}.$$

Rule	K_c	T_i	T_d	Comment
Minimum ISE – Chao <i>et al.</i> (1989). <i>Model: Method 1</i>	${}^7 K_c^{(529)}$	$T_i^{(529)}$	$T_d^{(529)}$	$N = 8$
		$0.8 \leq \xi_m \leq 3; 0.05 \leq \frac{\tau_m}{2\xi_m T_{m1}} \leq 0.5$		
Minimum ITAE – Chao <i>et al.</i> (1989). <i>Model: Method 1</i>	${}^8 K_c^{(530)}$	$T_i^{(530)}$	$T_d^{(530)}$	$0.8 \leq \xi_m \leq 3$

$${}^7 K_c^{(529)} = \frac{0.2538 + 0.3875\xi_m - 0.04283\xi_m^2}{K_m} \left(\frac{\tau_m}{2\xi_m T_{m1}} \right)^{-1.652+0.5416\xi_m-0.09544\xi_m^2},$$

$$T_i^{(529)} = 2\xi_m T_{m1} \left(1.8283 - 0.8083\xi_m + 0.1852\xi_m^2 \right) \left(\frac{\tau_m}{2\xi_m T_{m1}} \right)^{0.1363+0.2342\xi_m-0.01445\xi_m^2},$$

$$T_d^{(529)} = 2\xi_m T_{m1} \left(1.077 - 0.4952\xi_m + 0.1062\xi_m^2 \right) \left(\frac{\tau_m}{2\xi_m T_{m1}} \right)^{0.4772+0.0442\xi_m+0.01694\xi_m^2}.$$

$${}^8 K_c^{(530)} = \frac{-0.1033 + 0.5347\xi_m - 0.07995\xi_m^2}{K_m} \left(\frac{\tau_m}{2\xi_m T_{m1}} \right)^{-1.798+0.7191\xi_m-0.1416\xi_m^2},$$

$$T_i^{(530)} = 2\xi_m T_{m1} \left(0.9096 - 0.1299\xi_m + 0.04179\xi_m^2 \right) \left(\frac{\tau_m}{2\xi_m T_{m1}} \right)^{-0.1277+0.5220\xi_m-0.08629\xi_m^2},$$

$$T_d^{(530)} = 2\xi_m T_{m1} \left(1.2226 - 0.6456\xi_m + 0.1373\xi_m^2 \right) \left(\frac{\tau_m}{2\xi_m T_{m1}} \right)^{0.4780-0.03653\xi_m+0.04523\xi_m^2}.$$

Rule	K_c	T_i	T_d	Comment
Minimum ITAE – Chao <i>et al.</i> (1989) – continued.	$^9 K_c^{(531)}$	$T_i^{(531)}$	$T_d^{(531)}$	$0.3 < \xi_m < 0.8$
				$N = 8; 0.05 \leq \frac{\tau_m}{2\xi_m T_{m1}} \leq 0.5$
Minimum ITSE – Chao <i>et al.</i> (1989). <i>Model: Method I</i>	$^{10} K_c^{(532)}$	$T_i^{(532)}$	$T_d^{(532)}$	$0.8 \leq \xi_m \leq 3$

$$\begin{aligned}
^9 K_c^{(531)} &= \frac{1.3292 - 2.8527\xi_m + 2.0365\xi_m^2}{K_m} \left(\frac{\tau_m}{2\xi_m T_{m1}} \right)^{-1.6159 - 0.06616\xi_m + 0.5351\xi_m^2}, \\
T_i^{(531)} &= 2\xi_m T_{m1} \exp(x_1), \quad x_1 = \left(3.3368 - 7.7919\xi_m + 4.3556\xi_m^2 \right) \\
&\quad + \left(1.4094 - 4.4581\xi_m + 3.4857\xi_m^2 \right) \ln \left(\frac{\tau_m}{2\xi_m T_{m1}} \right) \\
&\quad + \left(0.1160 - 0.8142\xi_m + 0.7402\xi_m^2 \right) \left[\ln \left(\frac{\tau_m}{2\xi_m T_{m1}} \right) \right]^2, \\
T_d^{(531)} &= 2\xi_m T_{m1} \exp(x_2), \quad x_2 = \left(2.3054 - 6.4328\xi_m + 3.9743\xi_m^2 \right) \\
&\quad + \left(1.8243 - 5.6458\xi_m + 4.6238\xi_m^2 \right) \ln \left(\frac{\tau_m}{2\xi_m T_{m1}} \right) \\
&\quad + \left(0.2866 - 1.4727\xi_m + 1.2893\xi_m^2 \right) \left[\ln \left(\frac{\tau_m}{2\xi_m T_{m1}} \right) \right]^2. \\
^{10} K_c^{(532)} &= \frac{0.1034 + 0.4857\xi_m - 0.0709\xi_m^2}{K_m} \left(\frac{\tau_m}{2\xi_m T_{m1}} \right)^{-1.713 + 0.6238\xi_m - 0.1178\xi_m^2}, \\
T_i^{(532)} &= 2\xi_m T_{m1} \left(1.5144 - 0.6060\xi_m + 0.1414\xi_m^2 \right) \left(\frac{\tau_m}{2\xi_m T_{m1}} \right)^{0.1008 + 0.2625\xi_m - 0.02203\xi_m^2}, \\
T_d^{(532)} &= 2\xi_m T_{m1} \left(1.0783 - 0.4886\xi_m + 0.1038\xi_m^2 \right) \left(\frac{\tau_m}{2\xi_m T_{m1}} \right)^{0.4316 + 0.08163\xi_m + 0.009173\xi_m^2}.
\end{aligned}$$

Rule	K_c	T_i	T_d	Comment
Minimum ITSE – Chao <i>et al.</i> (1989) – continued.	$^{11} K_c^{(533)}$	$T_i^{(533)}$	$T_d^{(533)}$	$0.3 < \xi_m < 0.8$
	$N = 8; 0.05 \leq \frac{\tau_m}{2\xi_m T_{ml}} \leq 0.5$			
Minimum performance index: servo tuning				
Minimum IAE – Chao <i>et al.</i> (1989). <i>Model: Method 1</i>	$^{12} K_c^{(534)}$	$T_i^{(534)}$	$T_d^{(534)}$	$N = 8$
	$0.8 \leq \xi_m \leq 3; 0.05 \leq \frac{\tau_m}{2\xi_m T_{ml}} \leq 0.5$			

$$\begin{aligned}
^{11} K_c^{(533)} &= \frac{1.8342 - 3.8984\xi_m + 2.7315\xi_m^2}{K_m} \left(\frac{\tau_m}{2\xi_m T_{ml}} \right)^{-1.5641+0.1628\xi_m+0.09337\xi_m^2}, \\
T_i^{(533)} &= 2\xi_m T_{ml} \exp(x_1), \quad x_1 = \left(3.2026 - 7.4812\xi_m + 4.3686\xi_m^2 \right) \\
&\quad + \left(0.9475 - 3.6318\xi_m + 2.9969\xi_m^2 \right) \ln \left(\frac{\tau_m}{2\xi_m T_{ml}} \right) \\
&\quad + \left(-0.06927 - 0.3875\xi_m + 0.4220\xi_m^2 \right) \ln \left(\frac{\tau_m}{2\xi_m T_{ml}} \right)^2; \\
T_d^{(533)} &= 2\xi_m T_{ml} \exp(x_2), \quad x_2 = \left(1.7393 - 3.7971\xi_m + 1.7608\xi_m^2 \right) \\
&\quad + \left(1.0983 - 1.5794\xi_m + 1.0744\xi_m^2 \right) \ln \left(\frac{\tau_m}{2\xi_m T_{ml}} \right) \\
&\quad + \left(0.01553 - 0.01414\xi_m - 0.009083\xi_m^2 \right) \ln \left(\frac{\tau_m}{2\xi_m T_{ml}} \right)^2.
\end{aligned}$$

$$\begin{aligned}
^{12} K_c^{(534)} &= \frac{-0.1779 + 0.6763\xi_m - 0.1264\xi_m^2}{K_m} \left(\frac{\tau_m}{2\xi_m T_{ml}} \right)^{-0.9941+0.1669\xi_m-0.04381\xi_m^2}, \\
T_i^{(534)} &= 2\xi_m T_{ml} \left(0.2857 + 0.4671\xi_m - 0.08353\xi_m^2 \right) \left(\frac{\tau_m}{2\xi_m T_{ml}} \right)^{x_1}, \\
x_1 &= 0.1665 - 0.3324\xi_m + 0.1776\xi_m^2 - 0.02897\xi_m^3; \\
T_d^{(534)} &= 2\xi_m T_{ml} \exp(x_2), \quad x_2 = \left(-0.8776 + 0.4353\xi_m - 0.1237\xi_m^2 \right) \\
&\quad + \left(-2.0552 + 2.786\xi_m - 0.5687\xi_m^2 \right) \ln \left(\frac{\tau_m}{2\xi_m T_{ml}} \right) \\
&\quad + \left(-0.5008 + 0.6232\xi_m - 0.1430\xi_m^2 \right) \ln \left(\frac{\tau_m}{2\xi_m T_{ml}} \right)^2.
\end{aligned}$$

Rule	K_c	T_i	T_d	Comment
Minimum ISE – Chao <i>et al.</i> (1989). Model: Method 1	$^{13} K_c^{(535)}$	$T_i^{(535)}$	$T_d^{(535)}$	$N = 8$
$0.8 \leq \xi_m \leq 3; 0.05 \leq \frac{\tau_m}{2\xi_m T_{ml}} \leq 0.5$				
Minimum ITAE – Chao <i>et al.</i> (1989). Model: Method 1	$^{14} K_c^{(536)}$	$T_i^{(536)}$	$T_d^{(536)}$	$0.8 \leq \xi_m \leq 3$

$$\begin{aligned}
^{13} K_c^{(535)} &= \frac{0.1446 + 0.4302\xi_m - 0.07501\xi_m^2}{K_m} \left(\frac{\tau_m}{2\xi_m T_{ml}} \right)^{-1.214+0.3123\xi_m-0.06889\xi_m^2}; \\
T_i^{(535)} &= 2\xi_m T_{ml} \exp(x_1), \quad x_1 = \left(-0.1256 + 0.1434\xi_m - 0.03277\xi_m^2 \right) \\
&\quad + \left(-2.2502 + 3.7015\xi_m - 1.7699\xi_m^2 + 0.2680\xi_m^3 \right) \ln \left(\frac{\tau_m}{2\xi_m T_{ml}} \right) \\
&\quad + \left(-0.4823 + 0.9479\xi_m - 0.4913\xi_m^2 + 0.07792\xi_m^3 \right) \left[\ln \left(\frac{\tau_m}{2\xi_m T_{ml}} \right) \right]^2; \\
T_d^{(535)} &= 2\xi_m T_{ml} \left(0.9105 - 0.3874\xi_m + 0.08662\xi_m^2 \left(\frac{\tau_m}{2\xi_m T_{ml}} \right)^{x_2} \right), \\
x_1 &= 0.0779 + 0.2855\xi_m - 0.01985\xi_m^2. \\
^{14} K_c^{(536)} &= \frac{-0.2641 + 0.7736\xi_m - 0.1486\xi_m^2}{K_m} \left(\frac{\tau_m}{2\xi_m T_{ml}} \right)^{-0.7225-0.04344\xi_m-0.001247\xi_m^2}; \\
T_i^{(536)} &= 2\xi_m T_{ml} \left(0.22257 + 0.7452\xi_m - 0.1451\xi_m^2 \left(\frac{\tau_m}{2\xi_m T_{ml}} \right)^{x_1} \right), \\
x_1 &= 0.03138 - 0.04430\xi_m + 0.01120\xi_m^2; \\
T_d^{(536)} &= 2\xi_m T_{ml} \exp(x_2), \quad x_2 = \left(0.05515 - 0.7031\xi_m + 0.1433\xi_m^2 \right) \\
&\quad + \left(-1.2256 + 1.8544\xi_m - 0.3366\xi_m^2 \right) \ln \left(\frac{\tau_m}{2\xi_m T_{ml}} \right) \\
&\quad + \left(-0.2315 + 0.3402\xi_m - 0.06757\xi_m^2 \right) \left[\ln \left(\frac{\tau_m}{2\xi_m T_{ml}} \right) \right]^2.
\end{aligned}$$

Rule	K_c	T_i	T_d	Comment
Minimum ITAE – Chao <i>et al.</i> (1989). <i>Model: Method 1</i>	$^{15} K_c^{(537)}$	$T_i^{(537)}$	$T_d^{(537)}$	$0.3 < \xi_m < 0.8$
				$N = 8; 0.05 \leq \frac{\tau_m}{2\xi_m T_{ml}} \leq 0.5$
Minimum ITSE – Chao <i>et al.</i> (1989). <i>Model: Method 1</i>	$^{16} K_c^{(538)}$	$T_i^{(538)}$	$T_d^{(538)}$	$0.8 \leq \xi_m \leq 3$

$$^{15} K_c^{(537)} = \frac{1.1295 - 2.8584\xi_m + 2.1176\xi_m^2}{K_m} \left(\frac{\tau_m}{2\xi_m T_{ml}} \right)^{-0.9904 - 2.5311\xi_m + 2.8008\xi_m^2};$$

$$T_i^{(537)} = 2\xi_m T_{ml} \left(8.6286 - 20.70\xi_m + 13.203\xi_m^2 \right) \left(\frac{\tau_m}{2\xi_m T_{ml}} \right)^{x_1},$$

$$x_1 = 0.7962 - 2.4809\xi_m + 1.7611\xi_m^2;$$

$$T_d^{(537)} = 2\xi_m T_{ml} \exp(x_2), \quad x_2 = \left(3.9285 - 12.874\xi_m + 8.6434\xi_m^2 \right) + \left(3.2228 - 12.034\xi_m + 9.2547\xi_m^2 \right) \ln \left(\frac{\tau_m}{2\xi_m T_{ml}} \right) + \left(0.6366 - 3.0208\xi_m + 2.4603\xi_m^2 \right) \left[\ln \left(\frac{\tau_m}{2\xi_m T_{ml}} \right) \right]^2.$$

$$^{16} K_c^{(538)} = \frac{0.0820 + 0.4471\xi_m - 0.07797\xi_m^2}{K_m} \left(\frac{\tau_m}{2\xi_m T_{ml}} \right)^{-0.9246 + 0.07955\xi_m - 0.02436\xi_m^2};$$

$$T_i^{(538)} = 2\xi_m T_{ml} \left(0.5930 + 0.1457\xi_m - 0.02062\xi_m^2 \right) \left(\frac{\tau_m}{2\xi_m T_{ml}} \right)^{x_1},$$

$$x_1 = -0.2201 + 0.1576\xi_m - 0.03202\xi_m^2;$$

$$T_d^{(538)} = 2\xi_m T_{ml} \exp(x_2), \quad x_2 = \left(-0.6534 - 0.1770\xi_m - 0.0240\xi_m^2 \right) + \left(-1.0018 + 1.5914\xi_m - 0.2831\xi_m^2 \right) \ln \left(\frac{\tau_m}{2\xi_m T_{ml}} \right) + \left(-0.2291 + 0.3064\xi_m - 0.0634\xi_m^2 \right) \left[\ln \left(\frac{\tau_m}{2\xi_m T_{ml}} \right) \right]^2.$$

Rule	K_c	T_i	T_d	Comment				
Minimum ITSE – Chao <i>et al.</i> (1989) – continued.	$^{17} K_c^{(539)}$	$T_i^{(539)}$	$T_d^{(539)}$	$0.3 < \xi_m < 0.8$				
	$N = 8; 0.05 \leq \frac{\tau_m}{2\xi_m T_{m1}} \leq 0.5$							
Direct synthesis								
Smith <i>et al.</i> (1975). Model: Method 1	$\frac{\lambda T_{m1}}{K_m(\lambda\tau_m + 1)}$	T_{m1}	T_{m2}	$\lambda = \frac{1}{T_{CL}}$; N not specified				
Smith <i>et al.</i> (1975). Model: Method 1	$\frac{x_1 T_{m1}}{K_m \tau_m}$	T_{m1}	T_{m2}	$\rho = \frac{\tau_m}{T_{m1} + T_{m2}}$; $N = 10$				
Coefficient values (deduced from graph)								
ξ_m	ρ	x_1	ξ_m	ρ	x_1	ξ_m	ρ	x_1
6	≥ 0.02	0.51	3	≥ 0.04	0.50	2	≥ 0.06	0.45
1.75	0.27	0.46	1.75	0.13	0.42	1.75	0.09	0.39
1.75	0.07	0.36	1.5	0.33	0.44	1.5	0.17	0.38
1.5	0.11	0.33	1.5	0.08	0.28	1.0	0.50	0.40
1.0	0.25	0.46	1.0	0.17	0.48	1.0	0.13	0.49
Sklaroff (1992).	$^{18} K_c^{(540)}$	$T_{m1} + T_{m2}$	$\frac{T_{m1} T_{m2}}{T_{m1} + T_{m2}}$					
	Also given by Åström and Hägglund (1995) – page 250-251. Model: Method 7. N = 8 – Honeywell UDC6000 controller.							

$$^{17} K_c^{(539)} = \frac{0.9666 - 2.3853\xi_m + 2.4096\xi_m^2}{K_m} \left(\frac{\tau_m}{2\xi_m T_{m1}} \right)^{-1.533 - 0.05442\xi_m + 10.916\xi_m^2};$$

$$T_i^{(539)} = 2\xi_m T_{m1} \left(6.3891 - 15.773\xi_m + 10.916\xi_m^2 \right) \left(\frac{\tau_m}{2\xi_m T_{m1}} \right)^{x_1},$$

$$x_1 = 0.04175 - 1.3860\xi_m + 1.4053\xi_m^2;$$

$$T_d^{(539)} = 2\xi_m T_{m1} \exp(x_2), \quad x_2 = \left(3.2620 - 10.789\xi_m + 7.6420\xi_m^2 \right)$$

$$+ \left(2.277 - 8.9402\xi_m + 7.7176\xi_m^2 \right) \ln \left(\frac{\tau_m}{2\xi_m T_{m1}} \right)$$

$$+ \left(0.3465 - 1.9274\xi_m + 1.8030\xi_m^2 \right) \left[\ln \left(\frac{\tau_m}{2\xi_m T_{m1}} \right) \right]^2.$$

$$^{18} K_c^{(540)} = \frac{3}{K_m \left(1 + \frac{3\tau_m}{T_{m1} + T_{m2}} \right)}.$$

Rule	K_c	T_i	T_d	Comment
Poulin <i>et al.</i> (1996). Model: Method 1	$\frac{T_{m1}}{K_m(T_{m1} + \tau_m)}$	T_{m1}	T_{m2}	$N = 5$; $T_{m1} \leq 5T_{m2}$; Minimum $\phi_m = 55^\circ$
Schaedel (1997). Model: Method 1	¹⁹ $K_c^{(541)}$	$T_i^{(541)}$	$T_d^{(541)}$	N not defined
Panda <i>et al.</i> (2004). Model: Method 7	²⁰ $K_c^{(542)}$	$2\xi_m T_{m1}$	$\frac{T_{m1}}{2\xi_m}$	$N = 8$
Panda <i>et al.</i> (2004). Model: Method 1	$\frac{T_{m1}}{2K_m \tau_m}$	T_{m1}	T_{m2}	N not defined

$$^{19} K_c^{(541)} = \frac{0.375}{K_m} \frac{4\xi_m^2 T_{m1}^2 + 2\xi_m T_{m1} \tau_m + 0.5\tau_m^2 - T_{m1}^2}{(2\xi_m T_{m1} + \tau_m)^2 + \left(\frac{1}{N} - 1\right)(2\xi_m T_{m1} + \tau_m) T_{m1} \sqrt{2(2\xi_m^2 - 1)} - x},$$

$$x = 4\xi_m^2 T_{m1}^2 + 2\xi_m T_{m1} \tau_m + 0.5\tau_m^2 - T_{m1}^2;$$

$$T_i^{(541)} = \frac{4\xi_m^2 T_{m1}^2 + 2\xi_m T_{m1} \tau_m + 0.5\tau_m^2 - T_{m1}^2}{2\xi_m T_{m1} + \tau_m};$$

$$T_d^{(541)} = \sqrt{(2\xi_m T_{m1} + \tau_m)^2 - 2(T_{m1}^2 + 2\xi_m T_{m1} \tau_m + 0.5\tau_m^2)}.$$

$$^{20} K_c^{(542)} = \frac{3}{K_m \left(1 + \frac{1.5\tau_m}{\xi_m T_{m1}} \right)}.$$

4.5.10 Classical controller 2 $G_c(s) = K_c \left(1 + \frac{1}{T_i s} \right) \left(\frac{1 + NT_d s}{1 + T_d s} \right)$

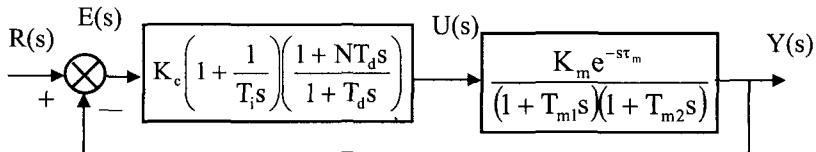


Table 124: PID controller tuning rules – SOSPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{(1+sT_{m1})(1+sT_{m2})}$

Rule	K_c	T_i	T_d	Comment
Direct synthesis				
Hougen (1979) – page 348-349. <i>Model: Method 1</i>	$\frac{0.8T_{m1}^{0.7}T_{m2}^{0.3}}{\tau_m}$	$0.5T_{m1} + T_{m2}$	$0.1 \sqrt[3]{\tau_m T_{m1} T_{m2}}$	$N=10$
	$\frac{0.84T_{m1}^{0.8}T_{m2}^{0.2}}{\tau_m}$	$^{21} T_i^{(543)}$	$T_d^{(543)}$	$N=30$
Hougen (1988). <i>Model: Method 1</i>	$^{22} K_c^{(544)}$	$0.5T_{m1} + T_{m2}$	$T_d^{(544)}$	$N=10; \tau_m < T_{m1}$

$$^{21} T_i^{(543)} = 0.53T_{m1} + 1.3T_{m2}, T_d^{(543)} = 0.08(\tau_m T_{m1} T_{m2})^{0.28}.$$

$$^{22} K_c^{(544)} = \frac{1}{K_m} 10^{\left[x_1 - x_2 \log_{10} \left(\frac{T_{m2}}{T_{m1}} \right) \right]}. \text{ Coefficients of } K_c^{(544)} \text{ deduced from graphs:}$$

τ_m / T_{m2}	0.1	0.3	0.5	1	2
x_1	0.9	0.42	0.24	-0.10	-0.39
x_2	0.7	0.7	0.68	0.70	0.69

$$T_d^{(544)} = (T_{m1} + T_{m2}) 10^{\left[0.33 \log_{10} \left(\frac{\tau_m}{T_{m1}} \right) - 1.40 \right]}, \frac{T_{m2}}{T_{m1}} = 0.1;$$

$$T_d^{(544)} = (T_{m1} + T_{m2}) 10^{\left[0.31 \log_{10} \left(\frac{\tau_m}{T_{m1}} \right) - 1.63 \right]}, 0.2 \leq \frac{T_{m2}}{T_{m1}} \leq 1.$$

4.5.11 Series controller (classical controller 3)

$$G_c(s) = K_c \left(1 + \frac{1}{T_i s} \right) (1 + s T_d)$$

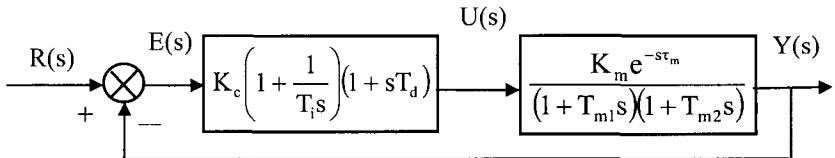


Table 125: PID controller tuning rules – SOSPD model $G_m(s) = \frac{K_m e^{-sT_m}}{(1+sT_{m1})(1+sT_{m2})}$

Rule	K_c	T_i	T_d	Comment
Minimum performance index: regulator tuning				
Minimum ISE - Haalman (1965).	$\frac{2T_{m1}}{3\tau_m K_m}$	T_{m1}	T_{m2}	<i>Model: Method 1</i>
	$T_{m1} > T_{m2}, M_s = 1.9, A_m = 2.36, \phi_m = 50^\circ$.			
Bohl and McAvoy (1976b).	$^1 K_c^{(545)}$	$T_i^{(545)}$	$T_d^{(545)}$	Minimum ITAE; <i>Model: Method 1</i> $T_d^{(545)} = T_i^{(545)}$

$$^1 \frac{\tau_m}{T_{m1}} = 0.1, 0.2, 0.3, 0.4, 0.5; \frac{T_{m2}}{T_{m1}} = 0.12, 0.3, 0.5, 0.7, 0.9.$$

$$K_c^{(545)} = \frac{5.5030}{K_m} \left[\left(10 \frac{\tau_m}{T_{m1}} \right)^{-1.1445+0.1100 \ln \left(10 \frac{\tau_m}{T_{m1}} \right)} \right] \left[\left(10 \frac{T_{m2}}{T_{m1}} \right)^{0.1648+0.1709 \ln \left(10 \frac{T_{m2}}{T_{m1}} \right)-0.2142 \ln \left(10 \frac{\tau_m}{T_{m1}} \right)} \right];$$

$$T_i^{(545)} = 1.8681 T_{m1} \left[\left(10 \frac{\tau_m}{T_{m1}} \right)^{0.6212-0.0760 \ln \left(10 \frac{\tau_m}{T_{m1}} \right)} \right] \left[\left(10 \frac{T_{m2}}{T_{m1}} \right)^{0.3144-0.0062 \ln \left(10 \frac{T_{m2}}{T_{m1}} \right)+0.0085 \ln \left(10 \frac{\tau_m}{T_{m1}} \right)} \right].$$

Rule	K_c	T_i	T_d	Comment
Direct synthesis				
Yang and Clarke (1996). Model: Method 8	$\frac{\omega_p T_{m1}}{K_m A_m}$	${}^2 T_i^{(546)}$	T_{m1}	$T_{m1} = T_{m2}$
O'Dwyer (2001a). Model: Method 1	$\frac{x_1 T_{m1}}{K_m \tau_m}$	T_{m1}	T_{m2}	$A_m = \frac{\pi}{2x_1};$ $\phi_m = 0.5\pi - x_1.$
	$\frac{0.785 T_{m1}}{K_m \tau_m}$	T_{m1}	T_{m2}	$A_m = 2.0;$ $\phi_m = 45^\circ$
Skogestad (2003).	$\frac{0.5 T_{m1}}{K_m \tau_m}$	$\min(T_{m1}, 8\tau_m)$	T_{m2}	Model: Method 18
Majhi and Litz (2003). Model: Method 8	${}^3 K_c^{(547)}$	$T_i^{(547)}$	T_{m1}	$T_{m1} = T_{m2}$
Ultimate cycle				
Bohl and McAvoy (1976b).	${}^4 K_c^{(548)}$	$T_i^{(548)}$	$T_d^{(548)}$	$T_d^{(548)} = T_i^{(548)};$ Model: Method 1

$${}^2 T_i^{(546)} = \frac{1}{2\omega_p - \frac{4\omega_p^2 \tau_m}{\pi} + \frac{1}{T_{m1}}}.$$

$${}^3 K_c^{(547)} = \frac{2\phi_m + \pi(A_m - 1)}{2(A_m^2 - 1)} \frac{T_{m1}}{K_m \tau_m},$$

$$T_i^{(547)} = \frac{T_{m1}}{1 + \frac{T_{m1}}{\tau_m} \left(\frac{A_m}{A_m^2 - 1} \right) [2\phi_m + \pi(A_m - 1)] \left[1 - \frac{A_m}{\pi(A_m^2 - 1)} (2\phi_m + \pi(A_m - 1)) \right]}.$$

$${}^4 K_c^{(548)} = \frac{0.5458}{K_m} \left[\left(\frac{\tau_m}{T_u} \right)^{1.2264 + 0.4568 \ln \left(\frac{\tau_m}{T_u} \right)} \right] \left[(K_u K_m)^{1.2190 - 0.0958 \ln(K_u K_m) - 0.1007 \ln \left(\frac{\tau_m}{T_u} \right)} \right],$$

$$T_i^{(548)} = 0.0393 T_u \left[\left(\frac{\tau_m}{T_u} \right)^{-1.9314 - 0.6221 \ln \left(\frac{\tau_m}{T_u} \right)} \right] \left[(K_u K_m)^{-0.0703 - 0.1125 \ln(K_u K_m) - 0.2944 \ln \left(\frac{\tau_m}{T_u} \right)} \right].$$

4.5.12 Non-interacting controller 1

$$U(s) = K_c \left(1 + \frac{1}{T_i s} \right) \begin{pmatrix} E(s) - \frac{T_d s}{1 + \frac{T_d s}{N}} Y(s) \\ 1 + \frac{T_d s}{N} \end{pmatrix}$$

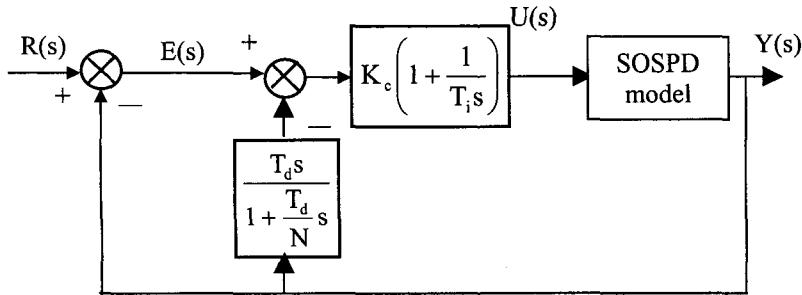


Table 126: PID controller tuning rules – SOSPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{(1+sT_{m1})(1+sT_{m2})}$ or

$$G_m(s) = \frac{K_m e^{-s\tau_m}}{T_{m1}^2 s^2 + 2\xi_m T_{m1} s + 1}$$

Rule	K_c	T_i	T_d	Comment
Direct synthesis				
Huang <i>et al.</i> (2000). <i>Model: Method I</i>	$s K_c^{(549)}$	$T_i^{(549)}$	$T_d^{(549)}$	
Servo tuning				
$\lambda = 0.5, \frac{\tau_m}{T_{m1}} \xi_m \leq 1; \lambda = 0.6, 1 < \frac{\tau_m}{T_{m1}} \xi_m \leq 2; \lambda = 0.7, \frac{\tau_m}{T_{m1}} \xi_m > 2,$				
$\lambda = 0.6, \frac{\tau_m}{T_{m1}} \xi_m \leq 1, \lambda = 0.7, 1 < \frac{\tau_m}{T_{m1}} \xi_m \leq 2,$				
$\lambda = 0.8, \frac{\tau_m}{T_{m1}} \xi_m > 2, \text{Regulator tuning}$				

$${}^5 K_c^{(549)} = \lambda \frac{2T_{m1}\xi_m + 0.4\tau_m}{K_m \tau_m}, \quad T_i^{(549)} = 2T_{m1}\xi_m + 0.4\tau_m,$$

$$T_d^{(549)} = \frac{T_{m1}^2 + 0.8T_{m1}\xi_m\tau_m}{0.8T_{m1}\xi_m\tau_m}.$$

Rule	K_c	T_i	T_d	Comment
Minimum performance index: regulator tuning				
Minimum IAE - Huang <i>et al.</i> (1996).	${}^6 K_c^{(550)}$	$T_i^{(550)}$	$T_d^{(550)}$	Model: Method I; N = 10

⁶ Note: equations continued into the footnote on page 356; $0 < T_{m2}/T_{ml} \leq 1$;
 $0.1 \leq \tau_m/T_{ml} \leq 1$.

$$\begin{aligned}
K_c^{(550)} &= \frac{1}{K_m} \left[0.1098 - 8.6290 \frac{\tau_m}{T_{ml}} + 76.6760 \frac{T_{m2}}{T_{ml}} - 3.3397 \frac{\tau_m T_{m2}}{T_{ml}^2} \right] \\
&\quad + \frac{1}{K_m} \left[1.1863 \left(\frac{\tau_m}{T_{ml}} \right)^{-0.8058} + 23.1098 \left(\frac{\tau_m}{T_{ml}} \right)^{0.6642} + 20.3519 \left(\frac{\tau_m}{T_{ml}} \right)^{2.1482} \right] \\
&\quad + \frac{1}{K_m} \left[-52.0778 \left(\frac{T_{m2}}{T_{ml}} \right)^{0.8405} - 12.1033 \left(\frac{T_{m2}}{T_{ml}} \right)^{2.1123} \right] \\
&\quad + \frac{1}{K_m} \left[9.4709 \frac{\tau_m}{T_{ml}} \left(\frac{T_{m2}}{T_{ml}} \right)^{0.5306} + 13.6581 \frac{T_{m2}}{T_{ml}} \left(\frac{\tau_m}{T_{ml}} \right)^{-1.0781} \right] \\
&\quad + \frac{1}{K_m} \left[-19.4944 \frac{T_{m2}}{T_{ml}} \left(\frac{\tau_m}{T_{ml}} \right)^{-0.4500} - 28.2766 \frac{\tau_m}{T_{ml}} \left(\frac{T_{m2}}{T_{ml}} \right)^{1.1427} \right] \\
&\quad + \frac{1}{K_m} \left[-19.1463 e^{\frac{\tau_m}{T_{ml}}} + 8.8420 e^{\frac{T_{m2}}{T_{ml}}} + 7.4298 e^{\frac{\tau_m T_{m2}}{T_{ml}^2}} - 11.4753 \frac{T_{m2}}{\tau_m} \right]; \\
T_i^{(550)} &= T_{ml} \left[-0.0145 + 2.0555 \frac{\tau_m}{T_{ml}} + 0.7435 \frac{T_{m2}}{T_{ml}} - 4.4805 \left(\frac{\tau_m}{T_{ml}} \right)^2 + 1.2069 \frac{\tau_m T_{m2}}{T_{ml}^2} \right] \\
&\quad + T_{ml} \left[0.2584 \left(\frac{T_{m2}}{T_{ml}} \right)^2 + 7.7916 \left(\frac{\tau_m}{T_{ml}} \right)^3 - 6.0330 \frac{T_{m2}}{T_{ml}} \left(\frac{\tau_m}{T_{ml}} \right)^2 \right] \\
&\quad + T_{ml} \left[3.9585 \frac{\tau_m}{T_{ml}} \left(\frac{T_{m2}}{T_{ml}} \right)^2 - 3.0626 \left(\frac{T_{m2}}{T_{ml}} \right)^3 - 7.0263 \left(\frac{\tau_m}{T_{ml}} \right)^4 \right] \\
&\quad + T_{ml} \left[7.0004 \frac{T_{m2}}{T_{ml}} \left(\frac{\tau_m}{T_{ml}} \right)^3 - 2.7755 \left(\frac{T_{m2}}{T_{ml}} \right)^2 \left(\frac{\tau_m}{T_{ml}} \right)^2 - 1.5769 \left(\frac{\tau_m}{T_{ml}} \right) \left(\frac{T_{m2}}{T_{ml}} \right)^3 \right] \\
&\quad + T_{ml} \left[3.1663 \left(\frac{T_{m2}}{T_{ml}} \right)^4 + 2.4311 \left(\frac{\tau_m}{T_{ml}} \right)^5 - 0.9439 \left(\frac{T_{m2}}{T_{ml}} \right)^5 \right]
\end{aligned}$$

$$\begin{aligned}
& + T_{m1} \left[-2.4506 \frac{T_{m2}}{T_{m1}} \left(\frac{\tau_m}{T_{m1}} \right)^4 - 0.2227 \left(\frac{T_{m2}}{T_{m1}} \right)^2 \left(\frac{\tau_m}{T_{m1}} \right)^3 \right] \\
& + T_{m1} \left[1.9228 \left(\frac{\tau_m}{T_{m1}} \right)^2 \left(\frac{T_{m2}}{T_{m1}} \right)^3 - 0.5494 \frac{\tau_m}{T_{m1}} \left(\frac{T_{m2}}{T_{m1}} \right)^4 \right]; \\
T_d^{(550)} &= T_{m1} \left[-0.0206 + 0.9385 \frac{\tau_m}{T_{m1}} + 0.7759 \frac{T_{m2}}{T_{m1}} - 2.3820 \left(\frac{\tau_m}{T_{m1}} \right)^2 + 2.9230 \frac{\tau_m T_{m2}}{T_{m1}^2} \right] \\
& + T_{m1} \left[-3.2336 \left(\frac{T_{m2}}{T_{m1}} \right)^2 + 7.2774 \left(\frac{\tau_m}{T_{m1}} \right)^3 - 9.9017 \frac{T_{m2}}{T_{m1}} \left(\frac{\tau_m}{T_{m1}} \right)^2 \right] \\
& + T_{m1} \left[2.7095 \frac{\tau_m}{T_{m1}} \left(\frac{T_{m2}}{T_{m1}} \right)^2 + 6.1539 \left(\frac{T_{m2}}{T_{m1}} \right)^3 - 11.1018 \left(\frac{\tau_m}{T_{m1}} \right)^4 \right] \\
& + T_{m1} \left[10.6303 \frac{T_{m2}}{T_{m1}} \left(\frac{\tau_m}{T_{m1}} \right)^3 + 5.7105 \left(\frac{\tau_m}{T_{m1}} \right)^2 \left(\frac{T_{m2}}{T_{m1}} \right)^2 - 2.274 \left(\frac{\tau_m}{T_{m1}} \right)^6 \right] \\
& + T_{m1} \left[-7.9490 \frac{\tau_m}{T_{m1}} \left(\frac{T_{m2}}{T_{m1}} \right)^3 - 6.6597 \left(\frac{T_{m2}}{T_{m1}} \right)^4 + 8.0849 \left(\frac{\tau_m}{T_{m1}} \right)^5 \right] \\
& + T_{m1} \left[-4.4897 \frac{T_{m2}}{T_{m1}} \left(\frac{\tau_m}{T_{m1}} \right)^4 - 7.6469 \left(\frac{T_{m2}}{T_{m1}} \right)^2 \left(\frac{\tau_m}{T_{m1}} \right)^3 \right] \\
& + T_{m1} \left[2.2155 \left(\frac{\tau_m}{T_{m1}} \right)^2 \left(\frac{T_{m2}}{T_{m1}} \right)^3 + 5.0694 \left(\frac{\tau_m}{T_{m1}} \right) \left(\frac{T_{m2}}{T_{m1}} \right)^4 + 4.1225 \left(\frac{T_{m2}}{T_{m1}} \right)^5 \right] \\
& + T_{m1} \left[0.519 \frac{T_{m2}}{T_{m1}} \left(\frac{\tau_m}{T_{m1}} \right)^5 - 1.1295 \left(\frac{T_{m2}}{T_{m1}} \right)^6 - 1.6307 \left(\frac{\tau_m}{T_{m1}} \right)^2 \left(\frac{T_{m2}}{T_{m1}} \right)^4 \right] \\
& + T_{m1} \left[2.2875 \left(\frac{\tau_m}{T_{m1}} \right)^4 \left(\frac{T_{m2}}{T_{m1}} \right)^2 + 0.9524 \left(\frac{\tau_m}{T_{m1}} \right)^3 \left(\frac{T_{m2}}{T_{m1}} \right)^3 - 0.9321 \frac{\tau_m}{T_{m1}} \left(\frac{T_{m2}}{T_{m1}} \right)^5 \right].
\end{aligned}$$

Rule	K_c	T_i	T_d	Comment
Minimum IAE - Huang <i>et al.</i> (1996).	$^7 K_c^{(551)}$	$T_i^{(551)}$	$T_d^{(551)}$	<i>Model: Method 1; N = 10.</i>

⁷ Note: equations continued into the footnote on page 358. $0.4 \leq \xi_m \leq 1$,

$$0.05 \leq \tau_m / T_{ml} \leq 1.$$

$$\begin{aligned}
K_c^{(551)} &= \frac{1}{K_m} \left[-35.7307 + 14.19 \frac{\tau_m}{T_{ml}} + 1.4023 \xi_m + 6.8618 \xi_m \left(\frac{\tau_m}{T_{ml}} \right) \right] \\
&\quad + \frac{1}{K_m} \left[-0.9773 \left(\frac{\tau_m}{T_{ml}} \right)^3 + 55.5898 \xi_m \left(\frac{\tau_m}{T_m} \right)^{0.086} - 3.3093 \xi_m \left(\frac{\tau_m}{T_{ml}} \right)^2 \right] \\
&\quad + \frac{1}{K_m} \left[53.8651 \frac{\tau_m}{T_{ml}} \xi_m^2 + 11.4911 \xi_m^3 + 0.8778 \left(\frac{\tau_m}{T_{ml}} \right)^{-1.6624} \right] \\
&\quad + \frac{1}{K_m} \left[-29.8822 \left(\frac{\tau_m}{T_{ml}} \right)^{-0.6951} + 53.535 \left(\frac{\tau_m}{T_{ml}} \right)^{-0.4762} - 16.9807 \xi_m^{1.1197} \right] \\
&\quad + \frac{1}{K_m} \left[-25.4293 \xi_m^{1.4622} - 0.1671 \xi_m^{58981} + 0.0034 \xi_m \left(\frac{\tau_m}{T_{ml}} \right)^{-2.1208} \right] \\
&\quad + \frac{1}{K_m} \left[-25.0355 \frac{\tau_m}{T_{ml}} \xi_m^{3.0836} - 54.9617 \frac{\tau_m}{T_{ml}} \xi_m^{1.2103} - 0.1398 e^{\frac{\tau_m}{T_{ml}}} \right] \\
&\quad + \frac{1}{K_m} \left[-8.2721 e^{\xi_m} + 6.3542 e^{\frac{\tau_m \xi_m}{T_{ml}}} + 1.0479 \frac{\xi_m T_{ml}}{\tau_m} \right]; \\
T_i^{(551)} &= T_{ml} \left[0.2563 + 11.8737 \frac{\tau_m}{T_{ml}} - 1.6547 \xi_m - 16.1913 \left(\frac{\tau_m}{T_{ml}} \right)^2 - 9.7061 \xi_m \frac{\tau_m}{T_{ml}} \right] \\
&\quad + T_{ml} \left[3.5927 \xi_m^2 + 19.5201 \left(\frac{\tau_m}{T_{ml}} \right)^3 - 14.5581 \xi_m \left(\frac{\tau_m}{T_{ml}} \right)^2 + 2.939 \xi_m^2 \left(\frac{\tau_m}{T_{ml}} \right) \right] \\
&\quad + T_{ml} \left[-0.4592 \xi_m^3 - 34.6273 \left(\frac{\tau_m}{T_{ml}} \right)^4 + 50.5163 \xi_m \left(\frac{\tau_m}{T_{ml}} \right)^3 \right] \\
&\quad + T_{ml} \left[8.9259 \xi_m^2 \left(\frac{\tau_m}{T_{ml}} \right)^2 + 8.6966 \frac{\tau_m}{T_{ml}} \xi_m^3 - 6.9436 \xi_m^4 \right]
\end{aligned}$$

$$\begin{aligned}
& + T_{m1} \left[27.2386 \left(\frac{\tau_m}{T_{m1}} \right)^5 - 20.0697 \xi_m \left(\frac{\tau_m}{T_{m1}} \right)^4 - 42.2833 \xi_m^2 \left(\frac{\tau_m}{T_{m1}} \right)^3 \right] \\
& + T_{m1} \left[8.5019 \left(\frac{\tau_m}{T_{m1}} \right)^2 \xi_m^3 - 12.2957 \left(\frac{\tau_m}{T_{m1}} \right) \xi_m^4 + 8.0694 \xi_m^5 - 2.7691 \xi_m^6 \right] \\
& + T_{m1} \left[-7.7887 \left(\frac{\tau_m}{T_{m1}} \right)^6 + 2.3012 \xi_m \left(\frac{\tau_m}{T_{m1}} \right)^5 + 4.6594 \frac{\tau_m}{T_{m1}} \xi_m^5 \right] \\
& + T_{m1} \left[8.8984 \left(\frac{\tau_m}{T_{m1}} \right)^4 \xi_m^2 + 10.2494 \left(\frac{\tau_m}{T_{m1}} \right)^3 \xi_m^3 - 5.4906 \left(\frac{\tau_m}{T_{m1}} \right)^2 \xi_m^4 \right]; \\
T_d^{(551)} & = T_{m1} \left[-0.021 + 3.3385 \frac{\tau_m}{T_{m1}} + 0.185 \xi_m - 0.5164 \left(\frac{\tau_m}{T_{m1}} \right)^2 - 0.9643 \xi_m \frac{\tau_m}{T_{m1}} \right] \\
& + T_{m1} \left[-0.8815 \xi_m^2 + 0.584 \left(\frac{\tau_m}{T_{m1}} \right)^3 - 12.513 \xi_m \left(\frac{\tau_m}{T_{m1}} \right)^2 + 1.3468 \xi_m^2 \left(\frac{\tau_m}{T_{m1}} \right) \right] \\
& + T_{m1} \left[2.3181 \xi_m^3 - 5.2368 \left(\frac{\tau_m}{T_{m1}} \right)^4 + 15.3014 \xi_m \left(\frac{\tau_m}{T_{m1}} \right)^3 - 3.1988 \xi_m^4 \right] \\
& + T_{m1} \left[11.9607 \xi_m^2 \left(\frac{\tau_m}{T_{m1}} \right)^2 - 2.0411 \frac{\tau_m}{T_{m1}} \xi_m^3 + 3.4675 \left(\frac{\tau_m}{T_{m1}} \right)^5 \right] \\
& + T_{m1} \left[-0.8219 \xi_m \left(\frac{\tau_m}{T_{m1}} \right)^4 - 15.0718 \xi_m^2 \left(\frac{\tau_m}{T_{m1}} \right)^3 - 1.8859 \left(\frac{\tau_m}{T_{m1}} \right)^2 \xi_m^3 \right] \\
& + T_{m1} \left[0.4841 \left(\frac{\tau_m}{T_{m1}} \right) \xi_m^4 + 2.2821 \xi_m^5 - 0.9315 \left(\frac{\tau_m}{T_{m1}} \right)^6 + 0.529 \xi_m \left(\frac{\tau_m}{T_{m1}} \right)^5 \right] \\
& + T_{m1} \left[-0.6772 \xi_m^6 - 1.4212 \left(\frac{\tau_m}{T_{m1}} \right)^4 \xi_m^2 + 7.1176 \left(\frac{\tau_m}{T_{m1}} \right)^3 \xi_m^3 \right] \\
& + T_{m1} \left[-2.3636 \left(\frac{\tau_m}{T_{m1}} \right)^2 \xi_m^4 + 0.5497 \frac{\tau_m}{T_{m1}} \xi_m^5 \right].
\end{aligned}$$

Rule	K_c	T_i	T_d	Comment
Minimum performance index: servo tuning				
Minimum IAE - Huang <i>et al.</i> (1996).	$^8 K_c^{(552)}$	$T_i^{(552)}$	$T_d^{(552)}$	Model: Method 1. N = 10

⁸ Note: equations continued into the footnote on page 360; $0 < T_{m2}/T_{ml} \leq 1$,
 $0.1 \leq \tau_m/T_{ml} \leq 1$.

$$K_c^{(552)} = \frac{1}{K_m} \left[7.0636 + 66.6512 \frac{\tau_m}{T_{ml}} - 137.8937 \frac{T_{m2}}{T_{ml}} - 122.7832 \frac{\tau_m T_{m2}}{T_{ml}^2} \right] \\ + \frac{1}{K_m} \left[26.1928 \left(\frac{\tau_m}{T_{ml}} \right)^{0.0865} + 33.6578 \left(\frac{\tau_m}{T_{ml}} \right)^{2.6405} + 3.0098 \left(\frac{T_{m2}}{T_{ml}} \right)^{1.0309} \right] \\ + \frac{1}{K_m} \left[-10.9347 \left(\frac{T_{m2}}{T_{ml}} \right)^{2.345} + 141.511 \left(\frac{T_{m2}}{T_{ml}} \right)^{1.0570} + 29.4068 \frac{T_{m2}}{\tau_m} \right] \\ + \frac{1}{K_m} \left[34.3156 \frac{T_{m2}}{T_{ml}} \left(\frac{\tau_m}{T_{ml}} \right)^{-0.9450} - 70.1035 \frac{T_{m2}}{T_{ml}} \left(\frac{\tau_m}{T_{ml}} \right)^{-0.9282} \right] \\ + \frac{1}{K_m} \left[152.6392 \frac{\tau_m}{T_{ml}} \left(\frac{T_{m2}}{T_{ml}} \right)^{0.8866} - 47.9791 \frac{\tau_m}{T_{ml}} \left(\frac{T_{m2}}{T_{ml}} \right)^{0.8148} \right] \\ + \frac{1}{K_m} \left[-57.9370 e^{\frac{\tau_m}{T_{ml}}} + 10.4002 e^{\frac{T_{m2}}{T_{ml}}} + 6.7646 e^{\frac{\tau_m T_{m2}}{T_{ml}^2}} + 7.3453 \left(\frac{\tau_m}{T_{ml}} \right)^{-0.4062} \right];$$

$$T_i^{(552)} = T_{ml} \left[0.9923 + 0.2819 \frac{\tau_m}{T_{ml}} - 0.2679 \frac{T_{m2}}{T_{ml}} - 1.4510 \left(\frac{\tau_m}{T_{ml}} \right)^2 - 0.6712 \frac{\tau_m T_{m2}}{T_{ml}^2} \right] \\ + T_{ml} \left[0.6424 \left(\frac{T_{m2}}{T_{ml}} \right)^2 + 2.504 \left(\frac{\tau_m}{T_{ml}} \right)^3 + 2.5324 \frac{T_{m2}}{T_{ml}} \left(\frac{\tau_m}{T_{ml}} \right)^2 \right] \\ + T_{ml} \left[2.3641 \frac{\tau_m}{T_{ml}} \left(\frac{T_{m2}}{T_{ml}} \right)^2 + 2.0500 \left(\frac{T_{m2}}{T_{ml}} \right)^3 - 1.8759 \left(\frac{\tau_m}{T_{ml}} \right)^4 \right] \\ + T_{ml} \left[0.8519 \frac{\tau_m}{T_{ml}} \left(\frac{T_{m2}}{T_{ml}} \right)^4 - 1.3496 \frac{T_{m2}}{T_{ml}} \left(\frac{\tau_m}{T_{ml}} \right)^3 - 3.4972 \left(\frac{\tau_m}{T_{ml}} \right)^2 \left(\frac{T_{m2}}{T_{ml}} \right)^2 \right] \\ + T_{ml} \left[-2.4216 \frac{\tau_m}{T_{ml}} \left(\frac{T_{m2}}{T_{ml}} \right)^3 - 3.1142 \left(\frac{T_{m2}}{T_{ml}} \right)^4 + 0.5862 \left(\frac{\tau_m}{T_{ml}} \right)^5 \right]$$

$$\begin{aligned}
& + T_{m1} \left[0.0797 \frac{T_{m2}}{T_{m1}} \left(\frac{\tau_m}{T_{m1}} \right)^4 + 0.985 \left(\frac{T_{m2}}{T_{m1}} \right)^2 \left(\frac{\tau_m}{T_{m1}} \right)^3 \right] \\
& + T_{m1} \left[1.2892 \left(\frac{\tau_m}{T_{m1}} \right)^2 \left(\frac{T_{m2}}{T_{m1}} \right)^3 + 1.2108 \left(\frac{T_{m2}}{T_{m1}} \right)^5 \right]; \\
T_d^{(552)} & = T_{m1} \left[0.0075 + 0.3449 \frac{\tau_m}{T_{m1}} + 0.3924 \frac{T_{m2}}{T_{m1}} - 0.0793 \left(\frac{\tau_m}{T_{m1}} \right)^2 + 2.7495 \frac{\tau_m T_{m2}}{T_{m1}^2} \right] \\
& + T_{m1} \left[0.6485 \left(\frac{T_{m2}}{T_{m1}} \right)^2 + 0.8089 \left(\frac{\tau_m}{T_{m1}} \right)^3 - 9.7483 \frac{T_{m2}}{T_{m1}} \left(\frac{\tau_m}{T_{m1}} \right)^2 \right] \\
& + T_{m1} \left[3.4679 \frac{\tau_m}{T_{m1}} \left(\frac{T_{m2}}{T_{m1}} \right)^2 - 5.8194 \left(\frac{T_{m2}}{T_{m1}} \right)^3 - 1.0884 \left(\frac{\tau_m}{T_{m1}} \right)^4 \right] \\
& + T_{m1} \left[12.0049 \frac{T_{m2}}{T_{m1}} \left(\frac{\tau_m}{T_{m1}} \right)^3 - 1.4056 \left(\frac{\tau_m}{T_{m1}} \right)^2 \left(\frac{T_{m2}}{T_{m1}} \right)^2 - 3.7055 \frac{\tau_m}{T_{m1}} \left(\frac{T_{m2}}{T_{m1}} \right)^3 \right] \\
& + T_{m1} \left[10.0045 \left(\frac{T_{m2}}{T_{m1}} \right)^4 + 0.3520 \left(\frac{\tau_m}{T_{m1}} \right)^5 - 6.3603 \frac{T_{m2}}{T_{m1}} \left(\frac{\tau_m}{T_{m1}} \right)^4 \right] \\
& + T_{m1} \left[-3.2980 \left(\frac{T_{m2}}{T_{m1}} \right)^2 \left(\frac{\tau_m}{T_{m1}} \right)^3 + 7.0404 \left(\frac{\tau_m}{T_{m1}} \right)^2 \left(\frac{T_{m2}}{T_{m1}} \right)^3 \right] \\
& + T_{m1} \left[1.4294 \left(\frac{\tau_m}{T_{m1}} \right) \left(\frac{T_{m2}}{T_{m1}} \right)^4 - 6.9064 \left(\frac{T_{m2}}{T_{m1}} \right)^5 + 0.0471 \left(\frac{\tau_m}{T_{m1}} \right)^6 \right] \\
& + T_{m1} \left[1.1839 \frac{T_{m2}}{T_{m1}} \left(\frac{\tau_m}{T_{m1}} \right)^5 + 1.7087 \left(\frac{T_{m2}}{T_{m1}} \right)^6 + 1.7444 \left(\frac{\tau_m}{T_{m1}} \right)^4 \left(\frac{T_{m2}}{T_{m1}} \right)^2 \right] \\
& + T_{m1} \left[-1.2817 \left(\frac{\tau_m}{T_{m1}} \right)^3 \left(\frac{T_{m2}}{T_{m1}} \right)^3 - 2.1281 \left(\frac{\tau_m}{T_{m1}} \right)^2 \left(\frac{T_{m2}}{T_{m1}} \right)^4 + 1.5121 \frac{\tau_m}{T_{m1}} \left(\frac{T_{m2}}{T_{m1}} \right)^5 \right].
\end{aligned}$$

Rule	K_c	T_i	T_d	Comment
Minimum IAE - Huang <i>et al.</i> (1996).	$^9 K_c^{(553)}$	$T_i^{(553)}$	$T_d^{(553)}$	Model: Method I. N = 10.

⁹ Note: equations continued into the footnote on page 362. $0.4 \leq \xi_m \leq 1$;

$$0.05 \leq \tau_m/T_{m1} \leq 1.$$

$$\begin{aligned}
K_c^{(553)} &= \frac{1}{K_m} \left[-8.1727 - 32.9042 \frac{\tau_m}{T_{m1}} + 31.9179 \xi_m + 38.3405 \xi_m \left(\frac{\tau_m}{T_{m1}} \right) \right] \\
&\quad + \frac{1}{K_m} \left[+0.2079 \left(\frac{\tau_m}{T_{m1}} \right)^{-1.9009} + 29.3215 \left(\frac{\tau_m}{T_{m1}} \right)^{0.1571} + 35.9456 \left(\frac{\tau_m}{T_{m1}} \right)^{1.2234} \right] \\
&\quad + \frac{1}{K_m} \left[-21.4045 \xi_m^{0.1311} + 5.1159 \xi_m^{1.9814} - 21.9381 \xi_m^{1.737} \right] \\
&\quad + \frac{1}{K_m} \left[-17.7448 \xi_m \left(\frac{\tau_m}{T_{m1}} \right)^{-0.1303} + 26.8655 \xi_m \left(\frac{\tau_m}{T_{m1}} \right)^{1.2008} \right] \\
&\quad + \frac{1}{K_m} \left[-52.9156 \frac{\tau_m}{T_{m1}} \xi_m^{1.1207} - 22.4297 \frac{\tau_m}{T_{m1}} \xi_m^{0.3626} - 3.3331 e^{\frac{\tau_m}{T_{m1}}} \right] \\
&\quad + \frac{1}{K_m} \left[8.5175 e^{\xi_m} - 1.5312 e^{\frac{\tau_m \xi_m}{T_{m1}}} + 0.8906 \frac{\xi_m T_{m1}}{\tau_m} \right] \text{ [Huang (2002)];} \\
T_i^{(553)} &= T_{m1} \left[1.1731 + 6.3082 \frac{\tau_m}{T_{m1}} - 0.6937 \xi_m + 8.5271 \left(\frac{\tau_m}{T_{m1}} \right)^2 - 24.7291 \xi_m \frac{\tau_m}{T_{m1}} \right] \\
&\quad + T_{m1} \left[-6.7123 \xi_m \left(\frac{\tau_m}{T_{m1}} \right)^2 + 7.9559 \xi_m^2 - 32.3937 \left(\frac{\tau_m}{T_{m1}} \right)^3 + 5.5268 \xi_m^6 \right] \\
&\quad + T_{m1} \left[-27.1372 \left(\frac{\tau_m}{T_{m1}} \right)^4 + 166.9272 \xi_m \left(\frac{\tau_m}{T_{m1}} \right)^3 + 36.3954 \xi_m^2 \left(\frac{\tau_m}{T_{m1}} \right) \right] \\
&\quad + T_{m1} \left[-94.8879 \xi_m^2 \left(\frac{\tau_m}{T_{m1}} \right)^2 - 22.6065 \xi_m^3 - 1.6084 \frac{\tau_m}{T_{m1}} \xi_m^3 + 29.9159 \xi_m^4 \right] \\
&\quad + T_{m1} \left[49.6314 \left(\frac{\tau_m}{T_{m1}} \right)^5 - 84.3776 \xi_m \left(\frac{\tau_m}{T_{m1}} \right)^4 - 93.8912 \xi_m^2 \left(\frac{\tau_m}{T_{m1}} \right)^3 \right]
\end{aligned}$$

$$\begin{aligned}
& + T_{m1} \left[110.1706 \left(\frac{\tau_m}{T_{m1}} \right)^2 \xi_m^3 - 25.1896 \left(\frac{\tau_m}{T_{m1}} \right) \xi_m^4 - 19.7569 \xi_m^5 \right] \\
& + T_{m1} \left[-12.4348 \left(\frac{\tau_m}{T_{m1}} \right)^6 - 11.7589 \xi_m \left(\frac{\tau_m}{T_{m1}} \right)^5 + 68.3097 \left(\frac{\tau_m}{T_{m1}} \right)^4 \xi_m^2 \right] \\
& + T_{m1} \left[-17.8663 \left(\frac{\tau_m}{T_{m1}} \right)^3 \xi_m^3 - 22.5926 \left(\frac{\tau_m}{T_{m1}} \right)^2 \xi_m^4 + 9.5061 \frac{\tau_m}{T_{m1}} \xi_m^5 \right]; \\
T_d^{(553)} &= T_{m1} \left[0.0904 + 0.8637 \frac{\tau_m}{T_{m1}} - 0.1301 \xi_m + 4.9601 \left(\frac{\tau_m}{T_{m1}} \right)^2 + 14.3899 \xi_m \frac{\tau_m}{T_{m1}} \right] \\
& + T_{m1} \left[0.7170 \xi_m^2 - 12.5311 \left(\frac{\tau_m}{T_{m1}} \right)^3 - 42.5012 \xi_m \left(\frac{\tau_m}{T_{m1}} \right)^2 + 17.0952 \xi_m^4 \right] \\
& + T_{m1} \left[-21.4907 \xi_m^2 \left(\frac{\tau_m}{T_{m1}} \right) - 6.9555 \xi_m^3 - 12.3016 \left(\frac{\tau_m}{T_{m1}} \right)^4 \right] \\
& + T_{m1} \left[102.9447 \xi_m \left(\frac{\tau_m}{T_{m1}} \right)^3 + 7.5855 \xi_m^2 \left(\frac{\tau_m}{T_{m1}} \right)^2 + 19.1257 \frac{\tau_m}{T_{m1}} \xi_m^3 \right] \\
& + T_{m1} \left[10.8688 \left(\frac{\tau_m}{T_{m1}} \right)^5 - 17.2130 \xi_m \left(\frac{\tau_m}{T_{m1}} \right)^4 - 110.0342 \xi_m^2 \left(\frac{\tau_m}{T_{m1}} \right)^3 \right] \\
& + T_{m1} \left[50.6455 \left(\frac{\tau_m}{T_{m1}} \right)^2 \xi_m^3 - 16.7073 \left(\frac{\tau_m}{T_{m1}} \right) \xi_m^4 - 16.2013 \xi_m^5 + 5.4409 \xi_m^6 \right] \\
& + T_{m1} \left[-0.0979 \left(\frac{\tau_m}{T_{m1}} \right)^6 - 10.9260 \xi_m \left(\frac{\tau_m}{T_{m1}} \right)^5 + 29.4445 \left(\frac{\tau_m}{T_{m1}} \right)^4 \xi_m^2 \right] \\
& + T_{m1} \left[21.6061 \left(\frac{\tau_m}{T_{m1}} \right)^3 \xi_m^3 - 24.1917 \left(\frac{\tau_m}{T_{m1}} \right)^2 \xi_m^4 + 6.2798 \frac{\tau_m}{T_{m1}} \xi_m^5 \right].
\end{aligned}$$

4.5.13 Non-interacting controller based on the two degree of freedom structure I

$$U(s) = K_c \left(1 + \frac{1}{T_i s} + \frac{T_d s}{1 + \frac{T_d}{N} s} \right) E(s) - K_c \left(\alpha + \frac{\beta T_d s}{1 + \frac{T_d}{N} s} \right) R(s)$$

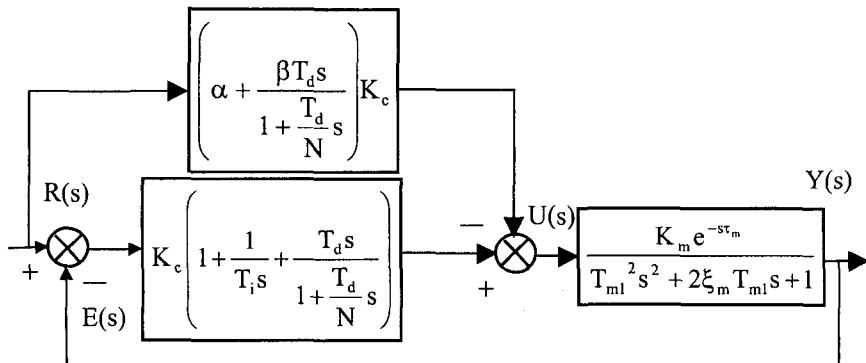


Table 127: PID controller tuning rules – SOSPD model $\frac{K_m e^{-s\tau_m}}{T_{m1}^2 s^2 + 2\xi_m T_{m1} s + 1}$

Rule	K_c	T_i	T_d	Comment
Minimum performance index: regulator tuning				
Minimum IE – Shen (1999).	${}^1 K_c^{(554)}$	$T_i^{(554)}$	$T_d^{(554)}$	Model: Method 1; $K_m = 1$

$${}^1 x_1 = \exp[a + b \left(\frac{\tau_m}{T_{m1}} \right) + c \left(\frac{\tau_m}{T_{m1}} \right)^2 + d \left(\frac{\tau_m}{T_{m1}} \right)^3 + e \frac{\xi_m \tau_m}{T_{m1}} + f \xi_m + g \xi_m^2 + h \frac{\xi_m^2 \tau_m}{T_{m1}} + i \left(\frac{\tau_m}{T_{m1}} \right)^3 \xi_m + j \left(\frac{\tau_m}{T_{m1}} \right)^2 \xi_m^2 + k \left(\frac{\tau_m}{T_{m1}} \right)^3 \xi_m^2];$$

$$K_c^{(554)} = \frac{T_{m1}}{\tau_m} x_1, \quad T_i^{(554)} = \tau_m x_1, \quad T_d^{(554)} = \tau_m x_1, \quad \alpha = 1 - x_1; \text{ coefficients of } x_1 \text{ are}$$

given below (and continued into the footnote on page 364). $1.4 \leq M_s \leq 2; \beta = 0; N = 0.$

	$K_c^{(554)}$	$T_i^{(554)}$	$T_d^{(554)}$	α
a	1.40	2.43	1.32	-1.44
b	-11.60	-2.50	-0.51	1.27
c	14.17	-4.16	0.70	1.34

Rule	K_c	T_i	T_d	Comment
Minimum IE – Shen (2000).	$^2 K_c^{(555)}$	$T_i^{(555)}$	$T_d^{(555)}$	Model: Method I; $K_m = 1$

(continued)	$K_c^{(554)}$	$T_i^{(554)}$	$T_d^{(554)}$	α
d	-6.44	2.85	-1.06	-0.37
e	8.28	-0.30	0.02	1
f	-0.07	0.1	-0.09	-5.97
g	0.13	-1.72	-7.43	-0.78
h	-4.66	15.51	1.77	-10.63
i	-0.99	-7.98	2.64	18.85
j	-3.03	-8.45	4.77	-10.97
k	2.89	4.66	-3.74	6.44

$$^2 x_1 = \exp[a + b\left(\frac{\tau_m}{\tau_m + T_{m1}}\right) + c\left(\frac{\tau_m}{\tau_m + T_{m1}}\right)^2 + d\left(\frac{\tau_m}{\tau_m + T_{m1}}\right)^3 + e\xi_m + f\xi_m^2 + g\left(\frac{\tau_m}{\tau_m + T_{m1}}\right)\xi_m + h\left(\frac{\tau_m}{\tau_m + T_{m1}}\right)^2\xi_m + i\left(\frac{\tau_m}{\tau_m + T_{m1}}\right)^3\xi_m + j\left(\frac{\tau_m}{\tau_m + T_{m1}}\right)\xi_m^2 + k\left(\frac{\tau_m}{\tau_m + T_{m1}}\right)^2\xi_m^2 + l\left(\frac{\tau_m}{\tau_m + T_{m1}}\right)^3\xi_m^2];$$

$$K_c^{(555)} = \frac{T_{m1}}{\tau_m} x_1, \quad T_i^{(555)} = \tau_m x_1, \quad T_d^{(555)} = \tau_m x_1, \quad \alpha = 1 - x_1; \text{ coefficients of } x_1 \text{ are}$$

given below; $M_s = 2$; $\beta = 0$; $N = 0$; $0 < \xi_m < 1$; $0 \leq \frac{\tau_m}{T_{m1}} \leq 1$.

	$K_c^{(555)}$	$T_i^{(555)}$	$T_d^{(555)}$	α
a	1.73	2.28	1.43	-1.60
b	-17.51	0.87	-1.94	4.30
c	30.73	-24.00	14.82	-12.16
d	-24.69	15.09	-21.44	26.54
e	0.15	-0.01	-0.25	1.33
f	-0.12	-0.02	0.16	-1.08
g	3.30	-8.03	-1.06	-17.56
h	33.07	45.21	-43.39	38.13
i	-37.64	-22.48	51.16	-56.45
j	2.63	3.89	-3.25	14.80
k	-34.57	-22.39	36.04	-39.33
l	36.88	8.32	-34.95	51.18

Rule	K_c	T_i	T_d	Comment
Minimum IE – Shen (2000). Model: Method I	$^3 K_c^{(556)}$	$T_i^{(556)}$	$T_d^{(556)}$	$K_m = 1$
Minimum performance index: servo/regulator tuning				
Minimum ITAE – Pecharromán and Pagola (2000).	0.7236 K_u	0.5247 T_u	0.1650 T_u	N = 10; Model: Method II
	$\alpha = 0.5840, \beta = 1, \phi_c = -139.65^\circ, K_m = 1; T_{m1} = 1; \xi_m = 1$			
Minimum ITAE – Pecharromán (2000). Model: Method II	$x_1 K_u$	$x_2 T_u$	$x_3 T_u$	
	Coefficient values and other data			
	x_1	x_2	x_3	α
	0.803	0.509	0.167	0.585
	0.727	0.524	0.165	-146°

$$\begin{aligned}
 ^3 x_1 = & \exp[a + b\left(\frac{\tau_m}{\tau_m + T_{m1}}\right) + c\left(\frac{\tau_m}{\tau_m + T_{m1}}\right)^2 + d\left(\frac{\tau_m}{\tau_m + T_{m1}}\right)^3 + e\xi_m + f\xi_m^2 \\
 & + g\left(\frac{\tau_m}{\tau_m + T_{m1}}\right)\xi_m + h\left(\frac{\tau_m}{\tau_m + T_{m1}}\right)^2\xi_m + i\left(\frac{\tau_m}{\tau_m + T_{m1}}\right)^3\xi_m \\
 & + j\left(\frac{\tau_m}{\tau_m + T_{m1}}\right)\xi_m^2 + k\left(\frac{\tau_m}{\tau_m + T_{m1}}\right)^2\xi_m^2 + l\left(\frac{\tau_m}{\tau_m + T_{m1}}\right)^3\xi_m^2];
 \end{aligned}$$

$$K_c^{(556)} = \frac{T_{m1}}{\tau_m} x_1, \quad T_i^{(556)} = \tau_m x_1, \quad T_d^{(556)} = \tau_m x_1, \quad \alpha = 1 - x_1; \text{ coefficients of } x_1$$

are given below; $M_s = 2; \beta = 0; N = 0; 0.4 \leq \xi_m \leq 1; 1 < \frac{\tau_m}{T_{m1}} \leq 15$.

	$K_c^{(556)}$	$T_i^{(556)}$	$T_d^{(556)}$	α
a	128.9	92.8	-3.1	-119.9
b	-560.3	-391	46.3	539.5
c	769.2	521.6	-107.4	-785.1
d	-338.9	-225.5	63.1	371.8
e	-324.8	-224.5	5.2	233.9
f	194.3	137.5	-2.5	-115.7
g	1421	963.9	-79.6	-1068
h	-1981.6	-1310.6	176.0	1560.5
i	895.0	574.4	-103.6	-741.5
j	-848.1	-590.8	36.2	529.2
k	1186.1	808.0	-78.4	-774.6
l	-538.1	-356.7	45.8	369.2

Rule	K_c	T_i	T_d	Comment
Minimum ITAE - Pecharromán (2000) - continued.				
Coefficient values and other data (continued)				
$K_m = 1$;	x_1	x_2	x_3	α
$T_{ml} = 1$;	0.672	0.532	0.161	0.577
$\beta = 1$; $N = 10$;	0.669	0.486	0.170	-134^0
$\xi_m = 1$	0.600	0.498	0.157	-125^0
$0.1 < \tau_m < 10$	0.578	0.481	0.154	-115^0
	0.557	0.467	0.149	-105^0
	0.544	0.466	0.141	-93^0
	0.537	0.444	0.144	-84^0
	0.527	0.450	0.131	-73^0
	0.521	0.440	0.129	-63^0
	0.515	0.429	0.126	-52^0
	0.509	0.399	0.132	-41^0
	0.496	0.374	0.123	-30^0
	0.480	0.315	0.112	-19^0
	0.430	0.242	0.084	-10^0
				$\xi_m = 6^0$
Taguchi and Araki (2000). <i>Model: Method I</i>	${}^4 K_c^{(557)}$	$T_i^{(557)}$	$T_d^{(557)}$	$\xi_m = 1.0$

$$\begin{aligned}
{}^4 K_c^{(557)} &= \frac{1}{K_m} \left(1.389 + \frac{0.6978}{[\frac{\tau_m}{T_{ml}} + 0.02295]^2} \right), \\
T_i^{(557)} &= T_{ml} \left(0.02453 + 4.104 \frac{\tau_m}{T_{ml}} - 3.434 \left[\frac{\tau_m}{T_{ml}} \right]^2 + 1.231 \left[\frac{\tau_m}{T_{ml}} \right]^3 \right), \\
T_d^{(557)} &= T_{ml} \left(0.03459 + 1.852 \frac{\tau_m}{T_{ml}} - 2.741 \left[\frac{\tau_m}{T_{ml}} \right]^2 + 2.359 \left[\frac{\tau_m}{T_{ml}} \right]^3 - 0.7962 \left[\frac{\tau_m}{T_{ml}} \right]^4 \right), \\
\alpha &= 0.6726 - 0.1285 \frac{\tau_m}{T_{ml}} - 0.1371 \left[\frac{\tau_m}{T_{ml}} \right]^2 + 0.07345 \left[\frac{\tau_m}{T_{ml}} \right]^3, \\
\beta &= 0.8665 - 0.2679 \frac{\tau_m}{T_{ml}} + 0.02724 \left[\frac{\tau_m}{T_{ml}} \right]^2.
\end{aligned}$$

Rule	K_c	T_i	T_d	Comment
Taguchi and Araki (2000) – continued.	${}^5 K_c^{(558)}$	$T_i^{(558)}$	$T_d^{(558)}$	$\xi_m = 0.5$
$\tau_m/T_{m1} \leq 1.0$; Overshoot (servo step) $\leq 20\%$; N not specified				
Direct synthesis				
Huang <i>et al.</i> (2000) - servo tuning. Model: Method 1	${}^6 K_c^{(559)}$	$T_i^{(559)}$	$T_d^{(559)}$	
		$\lambda = 0.6, \frac{\tau_m}{T_{m1}} \xi_m \leq 1, A_m = 2.83;$		
		$\lambda = 0.7, 1 < \frac{\tau_m}{T_{m1}} \xi_m \leq 2, A_m = 2.43;$		
		$\lambda = 0.8, \frac{\tau_m}{T_{m1}} \xi_m > 2, A_m = 2.13.$		
		$\alpha = \frac{2T_{m1}\xi_m}{2T_{m1}\xi_m + 0.4\tau_m} - 1; \beta = \frac{0.5T_{m1}^2}{T_{m1}^2 + 0.8T_{m1}\xi_m\tau_m} - 1;$		
		$N = \min[20, 20T_d]$		

$${}^5 K_c^{(558)} = \frac{1}{K_m} \left(0.3363 + \frac{0.5013}{\left[\frac{\tau_m}{T_{m1}} + 0.01147 \right]^2} \right),$$

$$T_i^{(558)} = T_{m1} \left(-0.02337 + 4.858 \frac{\tau_m}{T_{m1}} - 5.522 \left[\frac{\tau_m}{T_{m1}} \right]^2 + 2.054 \left[\frac{\tau_m}{T_{m1}} \right]^3 \right),$$

$$T_d^{(558)} = T_{m1} \left(0.03392 + 2.023 \frac{\tau_m}{T_{m1}} - 1.161 \left[\frac{\tau_m}{T_{m1}} \right]^2 + 0.2826 \left[\frac{\tau_m}{T_{m1}} \right]^3 \right),$$

$$\alpha = 0.6678 - 0.05413 \frac{\tau_m}{T_{m1}} - 0.5680 \left[\frac{\tau_m}{T_{m1}} \right]^2 + 0.1699 \left[\frac{\tau_m}{T_{m1}} \right]^3,$$

$$\beta = 0.8646 - 0.1205 \frac{\tau_m}{T_{m1}} - 0.1212 \left[\frac{\tau_m}{T_{m1}} \right]^2.$$

$${}^6 K_c^{(559)} = \lambda \frac{2T_{m1}\xi_m + 0.4\tau_m}{K_m \tau_m}, \quad T_i^{(559)} = 2T_{m1}\xi_m + 0.4\tau_m,$$

$$T_d^{(559)} = \frac{T_{m1}^2 + 0.8T_{m1}\xi_m\tau_m}{0.8T_{m1}\xi_m\tau_m}.$$

4.5.14 Non-interacting controller 4

$$U(s) = K_c \left(1 + \frac{1}{T_i s} \right) E(s) - K_c T_d s Y(s)$$

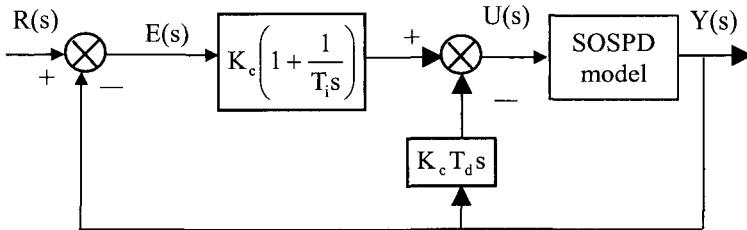


Table 128: PID controller tuning rules – SOSP model $G_m(s) = \frac{K_m e^{-s\tau_m}}{(1+sT_{m1})(1+sT_{m2})}$ or

$$G_m(s) = \frac{K_m e^{-s\tau_m}}{T_{m1}^2 s^2 + 2\xi_m T_{m1} s + 1}$$

Rule	K_c	T_i	T_d	Comment
Minimum performance index: regulator tuning				
Minimum IAE - Shinskey (1996) – page 119. Model: Method 1; $\frac{\tau_m}{T_{m1}} = 0.2$	$\frac{1.18T_{m1}}{K_m \tau_m}$	$2.20\tau_m$	$0.72\tau_m$	$\frac{T_{m2}}{T_{m1}} = 0.1$
	$\frac{1.25T_{m1}}{K_m \tau_m}$	$2.20\tau_m$	$1.10\tau_m$	$\frac{T_{m2}}{T_{m1}} = 0.2$
	$\frac{1.67T_{m1}}{K_m \tau_m}$	$2.40\tau_m$	$1.65\tau_m$	$\frac{T_{m2}}{T_{m1}} = 0.5$
	$\frac{2.5T_{m1}}{K_m \tau_m}$	$2.15\tau_m$	$2.15\tau_m$	$\frac{T_{m2}}{T_{m1}} = 1.0$

Rule	K_c	T_i	T_d	Comment
Minimum IAE - Shinskey (1994) - page 159. <i>Model: Method 1</i>	$^1 K_c^{(560)}$	$T_i^{(560)}$	$T_d^{(560)}$	$\frac{T_{m2}}{\tau_m} \leq 3$
	$\frac{3.33T_{m1}}{K_m \tau_m}$	$\tau_m + 0.2T_{m2}$	$\tau_m + 0.2T_{m2}$	$\frac{T_{m2}}{\tau_m} > 3$
Minimum IAE - Shinskey (1996) - page 121. <i>Model: Method 1</i>	$0.85K_u$	$0.35T_u$	$0.17T_u$	$\frac{\tau_m}{T_{m1}} = 0.2$, $\frac{T_{m2}}{T_{m1}} = 0.2$

$$^1 K_c^{(560)} = \frac{100}{\left(38 + 40 \left[1 - e^{-\frac{1.5T_{m1}}{\tau_m}} \right] \right) \frac{K_m \tau_m}{T_{m1}} \left(1 + 0.34 \frac{T_{m2}}{\tau_m} - 0.2 \left[\frac{T_{m2}}{\tau_m} \right]^2 \right)},$$

$$T_i^{(560)} = \tau_m \left(0.5 + 1.4 \left[1 - e^{-\frac{T_{m1}}{1.5\tau_m}} \right] \right) \left(1 + 0.48 \left[1 - e^{-\frac{T_{m2}}{\tau_m}} \right] \right),$$

$$T_d^{(560)} = 0.42 \tau_m \left(1 - e^{-\frac{1.2T_{m1}}{\tau_m}} \right) + 0.6 T_{m2}.$$

4.5.15 Non-interacting controller 5

$$U(s) = K_c \left(b + \frac{1}{T_i s} \right) E(s) - (c + T_d s) Y(s)$$

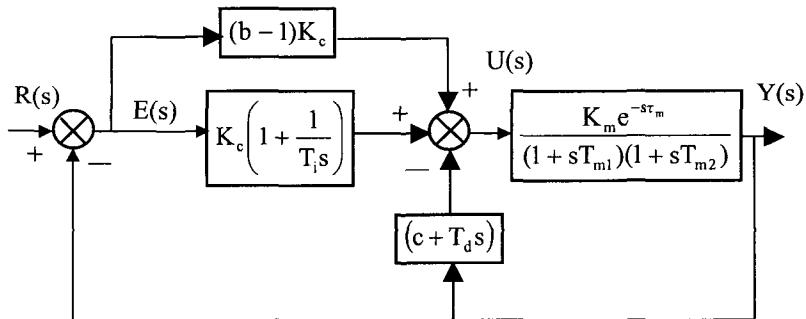


Table 129: PID controller tuning rules – SOSPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{(1+sT_{m1})(1+sT_{m2})}$

Rule	K_c	T_i	T_d	Comment
Direct synthesis				
Hansen (1998). Model: Method 1	$^2 K_c^{(561)}$	$1.54 T_i^{(561)}$	$T_d^{(561)}$	$b = 0.198$; nearly zero overshoot
		$1.27 T_i^{(561)}$		$b = 0.289$; nearly minimum IAE
		$1.75 T_i^{(561)}$		$b = 0.143$; conservative tuning

$$^2 K_c^{(561)} = \frac{2[T_{m1} T_{m2} + (T_{m1} + T_{m2})\tau_m + 0.5\tau_m^2]^3}{9K_m [T_{m1} T_{m2} \tau_m + 0.5(T_{m1} + T_{m2})\tau_m^2 + 0.167\tau_m^3]^2},$$

$$T_i^{(561)} = \frac{3[T_{m1} T_{m2} \tau_m + 0.5(T_{m1} + T_{m2})\tau_m^2 + 0.167\tau_m^3]}{[T_{m1} T_{m2} + (T_{m1} + T_{m2})\tau_m + 0.5\tau_m^2]},$$

$$T_d^{(561)} = \frac{2[T_{m1} T_{m2} + (T_{m1} + T_{m2})\tau_m + 0.5\tau_m^2]^2}{3K_m [T_{m1} T_{m2} \tau_m + 0.5(T_{m1} + T_{m2})\tau_m^2 + 0.167\tau_m^3]} - \frac{T_{m1} + T_{m2} + \tau_m}{K_m},$$

$$c = K_c^{(561)} - \frac{1}{K_m}.$$

4.5.16 Non-interacting controller 6 $U(s) = \frac{K_c}{T_i s} E(s) - K_c (1 + T_d s) Y(s)$

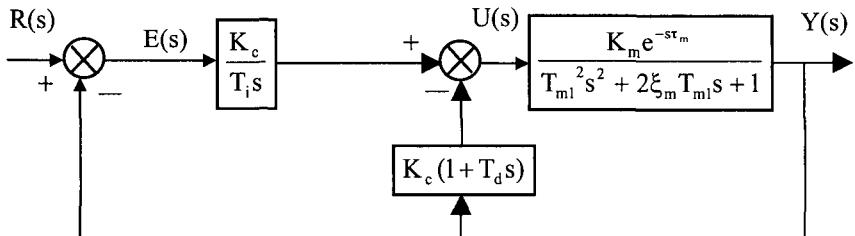


Table 130: PID controller tuning rules – SOSPD model $G_m(s) = \frac{K_m e^{-st_m}}{T_{m1}^2 s^2 + 2\xi_m T_{m1} s + 1}$

Rule	K_c	T_i	T_d	Comment
Direct synthesis				
Arvanitis <i>et al.</i> (2000). Model: Method 1	${}^3 K_c^{(562)}$	$T_i^{(562)}$	$\frac{T_{m1}}{\varepsilon \left(2\xi_m + \frac{T_{m1}}{\tau_m} \right)}$	$0 < \varepsilon < 1$; suggested $\varepsilon = 0.5$

$${}^3 K_c^{(562)} = \frac{\varepsilon T_{m1}}{K_m \tau_m} \left(2\xi_m + \frac{T_{m1}}{\tau_m} \right),$$

$$T_i^{(562)} = \frac{\frac{\varepsilon T_{m1}}{\tau_m} \left(2\xi_m + \frac{T_{m1}}{\tau_m} \right)}{\frac{1}{\tau_m} \left[1 + \frac{T_{m1} (2\xi_m \tau_m + T_{m1}) (4\xi^2 (1 - \varepsilon) + \varepsilon)}{\tau_m^2} \right] - a} \quad \text{with}$$

$$a = \frac{\xi}{\tau_m^2} \sqrt{8(1 - \varepsilon) \frac{T_{m1}}{\tau_m} (2\xi_m \tau_m + T_{m1}) \left[(2\xi^2 (1 - \varepsilon) + \varepsilon) \frac{T_{m1}}{\tau_m} (2\xi_m \tau_m + T_{m1}) + \tau_m \right]},$$

$$\text{where } \frac{T_{m1}}{K_m^2} (2\xi_m \tau_m + T_{m1}) + \frac{1}{\tau_m^2} \left[\varepsilon + 2\xi^2 (1 - \varepsilon)^2 \left(\frac{T_{m1}}{K_m} \right)^2 (2\xi_m \tau_m + T_{m1})^2 \right] \geq 0.$$

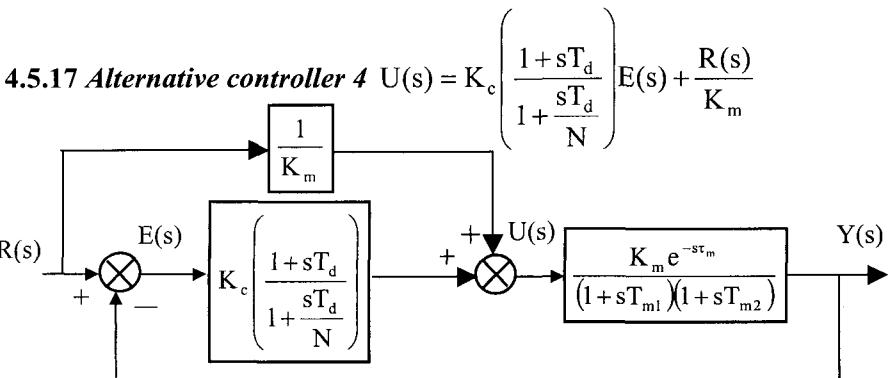


Table 131: PID controller tuning rules – SOSPD model $G_m(s) = \frac{K_m e^{-st_m}}{(1+sT_{m1})(1+sT_{m2})}$

Rule	K_c	T_d	Comment
Direct synthesis: time domain criteria			
Bohl and McAvoy (1976a). <i>Model: Method 1</i>	${}^4 K_c^{(563)}$	$T_d^{(563)}$	$N = 10$

⁴ Response reaches a new steady state in minimum time, to a servo input, with no observable overshoot. Upper and lower limits exist on the manipulated variable.

$$\frac{\tau_m}{T_{m1}} = 0.1, 0.2, 0.3, 0.4, 0.5; \quad \frac{T_{m2}}{T_{m1}} = 0.12, 0.3, 0.5, 0.7, 0.9.$$

$$K_c^{(563)} = \frac{4.486}{K_m} \left[\left(10 \frac{\tau_m}{T_{m1}} \right)^{-1.240 + 0.1409 \ln \left(10 \frac{\tau_m}{T_{m1}} \right)} \right] \left[\left(10 \frac{T_{m2}}{T_{m1}} \right)^{-0.0252 + 0.1542 \ln \left(10 \frac{T_{m2}}{T_{m1}} \right) - 0.1616 \ln \left(10 \frac{\tau_m}{T_{m1}} \right)} \right],$$

$$T_d^{(563)} = 0.104 T_{m1} \left[\left(10 \frac{\tau_m}{T_{m1}} \right)^{0.3664 - 0.0476 \ln \left(10 \frac{\tau_m}{T_{m1}} \right)} \right] \left[\left(10 \frac{T_{m2}}{T_{m1}} \right)^{0.7797 - 0.0773 \ln \left(10 \frac{T_{m2}}{T_{m1}} \right) + 0.0270 \ln \left(10 \frac{\tau_m}{T_{m1}} \right)} \right].$$

Rule	K_c	T_d	Comment
Ultimate cycle			
Bohl and McAvoy (1976a). Model: Method 1	$^5 K_c^{(564)}$	$T_d^{(564)}$	$N = 10$

$$^5 K_c^{(564)} = \frac{0.2455}{K_m} \left[\left(\frac{\tau_m}{T_u} \right)^{-0.4986 - 0.7510 \ln\left(\frac{\tau_m}{T_u}\right)} \right] \left[(K_u K_m)^{0.3875 - 0.2667 \ln(K_u K_m) - 1.1111 \ln\left(\frac{\tau_m}{T_u}\right)} \right],$$

$$T_d^{(564)} = 0.000726 T_u \left[\left(\frac{\tau_m}{T_u} \right)^{-6.1523 - 1.4285 \ln\left(\frac{\tau_m}{T_u}\right)} \right] \left[(K_u K_m)^{-1.4494 + 0.0402 \ln(K_u K_m) - 0.4602 \ln\left(\frac{\tau_m}{T_u}\right)} \right]$$

4.6 I²PD Model $G_m(s) = \frac{K_m e^{-s\tau_m}}{s^2}$

4.6.1 Controller with filtered derivative with set-point weighting 2

$$G_c(s) = K_c \left(1 + \frac{1}{T_i s} + \frac{T_d s}{1 + \frac{T_d}{N} s} \right) \frac{1 + b_{f1}s + b_{f2}s^2}{1 + a_{f1}s + a_{f2}s^2} R(s) - Y(s)$$

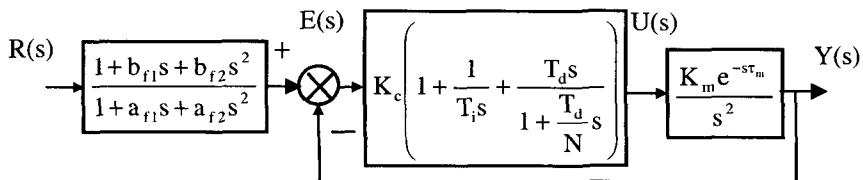


Table 132: PID tuning rules - I²PD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{s^2}$

Rule	K_c	T_i	T_d	Comment
Robust				
Liu <i>et al.</i> (2003), Model: Method 1	¹ $K_c^{(565)}$	$T_i^{(565)}$	$T_d^{(565)}$	

$$\begin{aligned} {}^1 K_c^{(565)} &= \frac{(4\lambda + \tau_m)(4\lambda^3 + 6\lambda^2\tau_m + 4\lambda\tau_m^2 + \tau_m^3) - \lambda^4}{K_m(4\lambda^3 + 6\lambda^2\tau_m + 4\lambda\tau_m^2 + \tau_m^3)^2}, \\ T_i^{(565)} &= \frac{(4\lambda + \tau_m)(4\lambda^3 + 6\lambda^2\tau_m + 4\lambda\tau_m^2 + \tau_m^3) - \lambda^4}{(4\lambda^3 + 6\lambda^2\tau_m + 4\lambda\tau_m^2 + \tau_m^3)}, \\ T_d^{(565)} &= \frac{(6\lambda^2 + 4\lambda\tau_m + \tau_m^2)(4\lambda^3 + 6\lambda^2\tau_m + 4\lambda\tau_m^2 + \tau_m^3)}{(4\lambda + \tau_m)(4\lambda^3 + 6\lambda^2\tau_m + 4\lambda\tau_m^2 + \tau_m^3) - \lambda^4} \\ &\quad - \frac{\lambda^4}{4\lambda^3 + 6\lambda^2\tau_m + 4\lambda\tau_m^2 + \tau_m^3}, \quad a_{f1} = 4\lambda + \tau_m; \quad a_{f2} = 6\lambda^2 + 4\lambda\tau_m + \tau_m^2, \\ N &= \frac{6\lambda^2 + 4\lambda\tau_m + \tau_m^2}{\lambda^3[(4\lambda + \tau_m)(4\lambda^3 + 6\lambda^2\tau_m + 4\lambda\tau_m^2 + \tau_m^3) - \lambda^4]} - 1, \quad b_{f1} = 2\lambda, \quad b_{f2} = \lambda^2. \end{aligned}$$

Rule	K_c	T_i	T_d	Comment	
Sample data (deduced from a graph)					
Liu <i>et al.</i> (2003) – continued.	λ	OS	Rise time	λ	OS
	$0.8\tau_m$	$\approx 33\%$	$\approx 4\tau_m$	$2.5\tau_m$	$\approx 0\%$
	τ_m	$\approx 20\%$	$\approx 5\tau_m$	$3\tau_m$	$\approx 0\%$
	$1.5\tau_m$	$\approx 3\%$	$\approx 7\tau_m$	$3.5\tau_m$	$\approx 0\%$
	$2\tau_m$	$\approx 0\%$	$\approx 8\tau_m$	$4\tau_m$	$\approx 0\%$
					$\approx 10\tau_m$
					$\approx 12.5\tau_m$
					$\approx 14\tau_m$
					$\approx 16\tau_m$

4.6.2 Controller with filtered derivative with set-point weighting 4

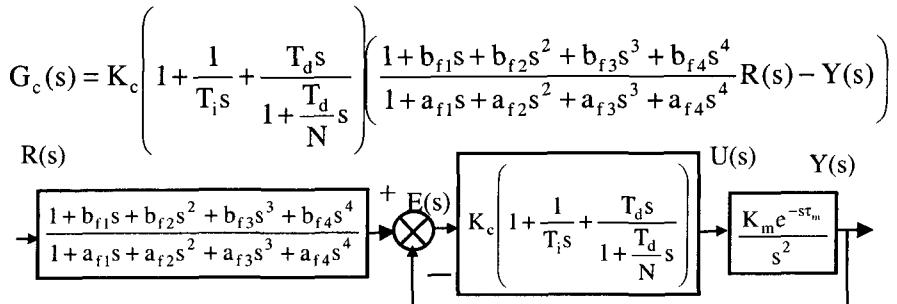


Table 133: PID tuning rules – I²PD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{s^2}$

Rule	K_c	T_i	T_d	Comment
Robust				
Liu et al. (2003). Model: Method 1		$T_i^{(566)}$	$T_d^{(566)}$	
Sample data (deduced from a graph) ³				
λ	OS	Rise time	λ	OS
$0.8\tau_m$	$\approx 14\%$	$\approx 3\tau_m$	$2.5\tau_m$	$\approx 0\%$
τ_m	$\approx 4\%$	$\approx 3.5\tau_m$	$3\tau_m$	$\approx 0\%$
$1.5\tau_m$	$\approx 0\%$	$\approx 4.5\tau_m$	$3.5\tau_m$	$\approx 0\%$
				$\approx 10.5\tau_m$

$$K_c^{(566)} = \frac{(4\lambda + \tau_m)(4\lambda^3 + 6\lambda^2\tau_m + 4\lambda\tau_m^2 + \tau_m^3) - \lambda^4}{K_m(4\lambda^3 + 6\lambda^2\tau_m + 4\lambda\tau_m^2 + \tau_m^3)^2},$$

$$T_i^{(566)} = \frac{(4\lambda + \tau_m)(4\lambda^3 + 6\lambda^2\tau_m + 4\lambda\tau_m^2 + \tau_m^3) - \lambda^4}{(4\lambda^3 + 6\lambda^2\tau_m + 4\lambda\tau_m^2 + \tau_m^3)},$$

$$T_d^{(566)} = \frac{(6\lambda^2 + 4\lambda\tau_m + \tau_m^2)(4\lambda^3 + 6\lambda^2\tau_m + 4\lambda\tau_m^2 + \tau_m^3)}{(4\lambda + \tau_m)(4\lambda^3 + 6\lambda^2\tau_m + 4\lambda\tau_m^2 + \tau_m^3) - \lambda^4} - \frac{\lambda^4}{4\lambda^3 + 6\lambda^2\tau_m + 4\lambda\tau_m^2 + \tau_m^3}, \quad b_{f1} = 4\lambda, b_{f2} = 6\lambda^2, b_{f3} = 4\lambda^3, b_{f4} = \lambda^4,$$

$$N = \frac{6\lambda^2 + 4\lambda\tau_m + \tau_m^2}{\lambda^3[(4\lambda + \tau_m)(4\lambda^3 + 6\lambda^2\tau_m + 4\lambda\tau_m^2 + \tau_m^3) - \lambda^4]} - 1.$$

$$^3 a_{f1} = 8\lambda + \tau_m, a_{f2} = 26\lambda^2 + 8\lambda\tau_m + \tau_m^2, a_{f3} = 4\lambda(10\lambda^2 + 20\lambda\tau_m + 4\tau_m^2), \\ a_{f4} = 4\lambda^2(6\lambda^2 + 4\lambda\tau_m + \tau_m^2).$$

Rule	K_c	T_i	T_d	Comment	
Liu <i>et al.</i> (2003) – continued.	Sample data (deduced from a graph) - continued				
	$2\tau_m$	$\approx 0\%$	$\approx 6\tau_m$	$4\tau_m$	$\approx 0\%$
	Sample data (deduced from a graph) ⁴				
	λ	OS	Rise time	λ	OS
	$0.8\tau_m$	$\approx 46\%$	$\approx 6\tau_m$	$2.5\tau_m$	$\approx 4\%$
	τ_m	$\approx 34\%$	$\approx 13.5\tau_m$	$3\tau_m$	$\approx 2\%$
	$1.5\tau_m$	$\approx 16\%$	$\approx 13\tau_m$	$3.5\tau_m$	$\approx 1\%$
	$2\tau_m$	$\approx 8\%$	$\approx 16.5\tau_m$	$4\tau_m$	$\approx 0\%$

⁴ $a_{f1} = 5\lambda + \tau_m$, $a_{f2} = 10.25\lambda^2 + 5\lambda\tau_m + \tau_m^2$, $a_{f3} = \lambda(7\lambda^2 + 4.25\lambda\tau_m + \tau_m^2)$,
 $a_{f4} = 0.25\lambda^2(6\lambda^2 + 4\lambda\tau_m + \tau_m^2)$.

4.6.3 Series controller (classical controller 3)

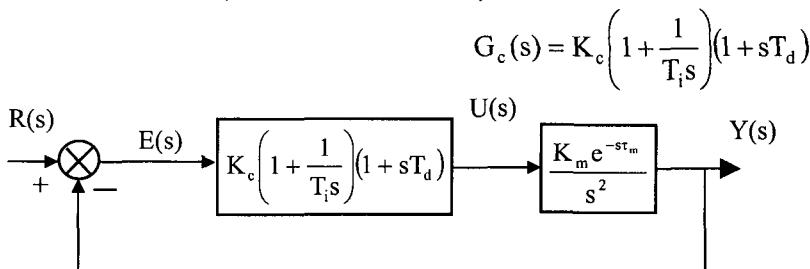


Table 134: PID tuning rules – I²PD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{s^2}$

Rule	K_c	T_i	T_d	Comment
Direct synthesis				
Skogestad (2003). Model: Method 1	$\frac{0.0625}{K_m \tau_m^2}$	$8\tau_m$	$8\tau_m$	

4.6.4 Non-interacting controller based on the two degree of freedom structure 1

$$U(s) = K_c \left(1 + \frac{1}{T_i s} + \frac{T_d s}{1 + \frac{T_d}{N} s} \right) E(s) - K_c \left(\alpha + \frac{\beta T_d s}{1 + \frac{T_d}{N} s} \right) R(s)$$

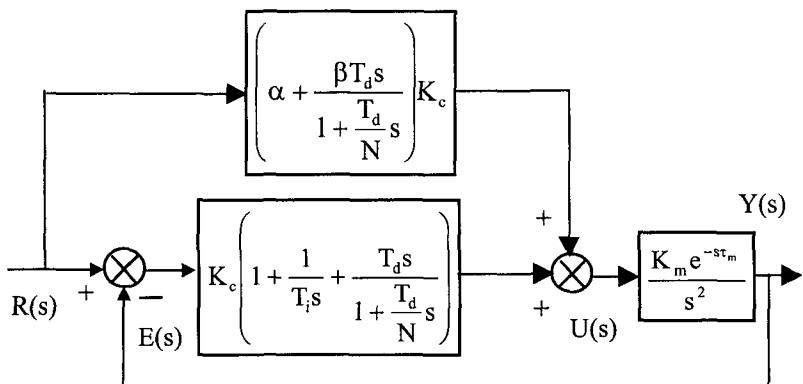


Table 135: PID tuning rules – I²PD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{s^2}$

Rule	K_c	T_i	T_d	Comment
Direct synthesis				
Hansen (2000). Model: Method 1	$3.75K_m \tau_m^2$	$5.4\tau_m$	$2.5\tau_m$	$N = 0; \beta = 1;$ $\alpha = 0.833$

4.6.5 Industrial controller $U(s) = K_c \left(1 + \frac{1}{T_i s} \right) \left(R(s) - \frac{1 + T_d s}{1 + \frac{T_d s}{N}} Y(s) \right)$

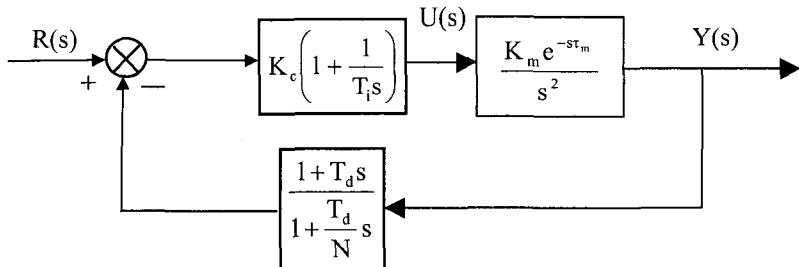


Table 136: PID tuning rules – I²PD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{s^2}$

Rule	K_c	T_i	T_d	Comment
Direct synthesis: time domain criteria				
Skogestad (2004b). Model: Method I	$\frac{0.25}{K_m (T_{CL} + \tau_m)^2}$	$4\xi^2 (T_{CL} + \tau_m)$	$4(T_{CL} + \tau_m)$	'good' robustness - $T_{CL} = \tau_m$; $\xi = 0.7$ or 1
	$N \in [5,10]$; $N = 2$ if measurement noise is a serious problem			

4.7 SOSIPD Model (repeated pole) $G_m(s) = \frac{K_m e^{-s\tau_m}}{s(1 + T_{m1}s)^2}$

4.7.1 Non-interacting controller based on the two degree of freedom structure 1

$$U(s) = K_c \left(1 + \frac{1}{T_i s} + \frac{T_d s}{1 + \frac{T_d}{N} s} \right) E(s) - K_c \left(\alpha + \frac{\beta T_d s}{1 + \frac{T_d}{N} s} \right) R(s)$$

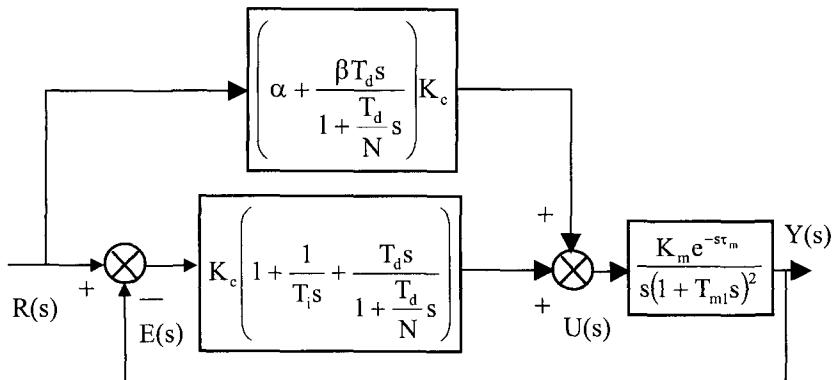


Table 137: PID controller tuning rules – SOSIPD model (repeated pole) $\frac{K_m e^{-s\tau_m}}{s(1 + T_{m1}s)^2}$

Rule	K_c	T_i	T_d	Comment
Minimum performance index: servo/regulator tuning				
Minimum ITAE – Pecharromán and Pagola (2000).	$x_1 K_u$	$x_2 T_u$	$x_3 T_u$	
Coefficient values				
	x_1	x_2	x_3	α
<i>Model: Method 2</i> $K_m = 1$; $T_m = 1$; $0.1 < \tau_m < 10$; $\beta = 1$, $N = 10$.	1.672	0.366	0.136	0.601
	1.236	0.427	0.149	0.607
	0.994	0.486	0.155	0.610
	0.842	0.538	0.154	0.616
	0.752	0.567	0.157	0.605
				-164^0
				-160^0
				-155^0
				-150^0
				-145^0

Rule	K_c	T_i	T_d	Comment
Coefficient values (continued)				
Pecharromán and Pagola (2000) – continued.	x_1	x_2	x_3	α
	0.679	0.610	0.149	0.610
	0.635	0.637	0.142	0.612
	0.590	0.669	0.133	0.610
	0.551	0.690	0.114	0.616
	0.520	0.776	0.087	0.609
	0.509	0.810	0.068	0.611
Taguchi and Araki (2000). Model: Method 1	$^1 K_c^{(567)}$	$T_i^{(567)}$	$T_d^{(567)}$	$\frac{\tau_m}{T_{m1}} \leq 1.0 ;$ Overshoot (servo step) $\leq 20\% ; N$ fixed but not specified

$$^1 K_c^{(567)} = \frac{1}{K_m} \left(0.1778 + \frac{0.5667}{\frac{\tau_m}{T_{m1}} + 0.002325} \right),$$

$$T_i^{(567)} = T_{m1} \left(0.2011 + 11.16 \frac{\tau_m}{T_{m1}} - 14.98 \left[\frac{\tau_m}{T_{m1}} \right]^2 + 13.70 \left[\frac{\tau_m}{T_{m1}} \right]^3 - 4.835 \left[\frac{\tau_m}{T_{m1}} \right]^4 \right),$$

$$T_d^{(567)} = T_{m1} \left(1.262 + 0.3620 \frac{\tau_m}{T_{m1}} \right), \quad \alpha = 0.6666,$$

$$\beta = 0.8206 - 0.09750 \frac{\tau_m}{T_{m1}} + 0.03845 \left[\frac{\tau_m}{T_{m1}} \right]^2.$$

4.8 SOSPD Model with a Positive Zero

$$G_m(s) = \frac{K_m(1-sT_{m3})e^{-s\tau_m}}{T_{m1}^2s^2 + 2\xi_m T_{m1}s + 1} \text{ or } G_m(s) = \frac{K_m(1-sT_{m3})e^{-s\tau_m}}{(1+sT_{m1})(1+sT_{m2})}$$

4.8.1 Ideal controller $G_c(s) = K_c \left(1 + \frac{1}{T_i s} + T_d s \right)$

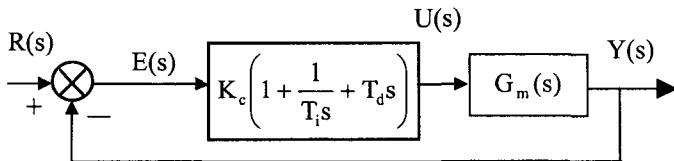


Table 138: PID controller tuning rules – SOSPD model with a positive zero

$$G_m(s) = \frac{K_m(1-sT_{m3})e^{-s\tau_m}}{(1+sT_{m1})(1+sT_{m2})} \text{ or } \frac{K_m(1-sT_{m3})e^{-s\tau_m}}{T_{m1}^2s^2 + 2\xi_m T_{m1}s + 1}$$

Rule	K_c	T_i	T_d	Comment
Minimum performance index: servo tuning				
Minimum IAE – Wang <i>et al.</i> (1995a). <i>Model: Method 1</i>	$\frac{1}{K_m} \frac{p_0 q_1 - p_1 r}{p_0^2}$	$q_1 - \frac{p_1}{p_0}$	$\frac{p_0 q_2 - p_2}{p_0 q_1 - p_1} \frac{p_1}{p_0}$	

¹ $p_0 = T_{m1} + T_{m2} + T_{m3} + \tau_m$, $p_1 = T_{m1} T_{m2} + 0.5\tau_m(T_{m1} + T_{m2}) - 0.5\tau_m T_{m3}$,
 $p_2 = 0.5T_{m1} T_{m2} \tau_m$, $q_1 = T_{m1} + T_{m2} + 0.5\tau_m$, $q_2 = T_{m1} T_{m2} + 0.5\tau_m(T_{m1} + T_{m2})$,
 $r = \delta_0 + \delta_1 \left(\frac{\tau_m}{T_{m1}} \right) + \delta_2 \left(\frac{T_{m2}}{T_{m1}} \right) + \delta_3 \left(\frac{T_{m3}}{T_{m1}} \right) + \left[\delta_4 \left(\frac{\tau_m}{T_{m1}} \right) + \delta_7 \left(\frac{T_{m2}}{T_{m1}} \right) + \delta_9 \left(\frac{T_{m3}}{T_{m1}} \right) \right] \frac{\tau_m}{T_{m1}}$
 $+ \left[\delta_7 \left(\frac{\tau_m}{T_{m1}} \right) + \delta_5 \left(\frac{T_{m2}}{T_{m1}} \right) + \delta_8 \left(\frac{T_{m3}}{T_{m1}} \right) \right] \frac{T_{m2}}{T_{m1}} + \left[\delta_9 \left(\frac{\tau_m}{T_{m1}} \right) + \delta_8 \left(\frac{T_{m2}}{T_{m1}} \right) + \delta_6 \left(\frac{T_{m3}}{T_{m1}} \right) \right] \frac{T_{m3}}{T_{m1}}$;
 $\delta_0 = 3.5550$, $\delta_1 = -3.6167$, $\delta_2 = 2.1781$, $\delta_3 = -5.5203$, $\delta_4 = 1.4704$,
 $\delta_5 = -0.4918$, $\delta_6 = 2.5356$, $\delta_7 = -0.4685$, $\delta_8 = -0.3318$, $\delta_9 = 1.4746$.

Rule	K_c	T_i	T_d	Comment
Minimum ISE – Wang <i>et al.</i> (1995a). Model: Method 1	$\frac{1}{K_m} \frac{p_0 q_1 - p_1}{p_0^2} r^2$	$q_1 - \frac{p_1}{p_0}$	$\frac{p_0 q_2 - p_2}{p_0 q_1 - p_1} - \frac{p_1}{p_0}$	
Minimum ITAE – Wang <i>et al.</i> (1995a). Model: Method 1	$\frac{1}{K_m} \frac{p_0 q_1 - p_1}{p_0^2} r^3$	$q_1 - \frac{p_1}{p_0}$	$\frac{p_0 q_2 - p_2}{p_0 q_1 - p_1} - \frac{p_1}{p_0}$	
Direct synthesis				
Wang <i>et al.</i> (2000b), (2001a). Model: Method 2	${}^4 K_c^{(568)}$	$2\xi_m T_{m1}$	T_{m1}^2	$M_s = 1.5$

² $p_0, p_1, p_2, q_1, q_2, r$ defined as on page 383.

$$\begin{aligned}\delta_0 &= 3.9395, \delta_1 = -3.2164, \delta_2 = 1.6185, \delta_3 = -5.8240, \delta_4 = 1.0933, \\ \delta_5 &= -0.6679, \delta_6 = 2.5648, \delta_7 = -0.2383, \delta_8 = -0.0564, \delta_9 = 1.3508.\end{aligned}$$

³ $p_0, p_1, p_2, q_1, q_2, r$ defined as on page 383.

$$\begin{aligned}\delta_0 &= 3.2950, \delta_1 = -3.4779, \delta_2 = 2.5336, \delta_3 = -5.5929, \delta_4 = 1.4407, \\ \delta_5 &= -0.3268, \delta_6 = 2.7129, \delta_7 = -0.5712, \delta_8 = -0.5790, \delta_9 = 1.5340.\end{aligned}$$

$${}^4 K_c^{(568)} = \frac{1}{M_s} \frac{\cos \theta}{\cos[\omega \tau_m - \tan^{-1}(-T_{m3}\omega)]} \frac{2\xi_m T_{m1}\omega}{K_m \sqrt{T_{m3}^2 \omega^2 + 1}};$$

ω and θ are determined by solving the equations:

$$\tan^{-1} \left(\frac{\cos \theta}{M_s - \sin \theta} \right) = \tan^{-1}(-T_{m3}\omega) - \omega \tau_m - 0.5\pi \quad \text{and}$$

$$\theta = \tan^{-1} \left[\frac{(-T_{m3}\tau_m \omega^2 - 1)\cos(\omega \tau_m) - \omega \tau_m \sin(\omega \tau_m)}{(-T_{m3}\tau_m \omega^2 - 1)\sin(\omega \tau_m) + \omega \tau_m \cos(\omega \tau_m)} \right].$$

4.8.2 Ideal controller in series with a first order lag

$$G_c(s) = K_c \left(1 + \frac{1}{T_i s} + T_d s \right) \frac{1}{T_f s + 1}$$

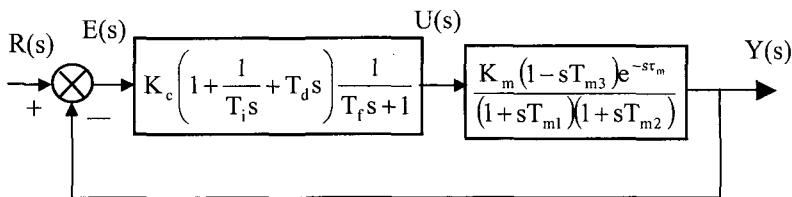


Table 139: PID controller tuning rules – SOSPD model with a positive zero

$$G_m(s) = \frac{K_m (1 - sT_{m3}) e^{-s\tau_m}}{(1 + sT_{m1})(1 + sT_{m2})}$$

Rule	K_c	T_i	T_d	Comment
Direct synthesis: time domain criteria				
Li <i>et al.</i> (2004). Model: Method I	${}^5 K_c^{(569)}$	$T_{m1} + T_{m2}$	$\frac{T_{m1} T_{m2}}{T_{m1} + T_{m2}}$	

$${}^5 K_c^{(569)} = \frac{T_{m1} + T_{m2}}{K_m (2T_{CL2} + T_{m3} + \tau_m)}, \quad T_f = \frac{{T_{CL2}}^2 - T_{m3}\tau_m}{2T_{CL2} + T_{m3} + \tau_m};$$

$$\text{desired closed loop transfer function} = -\frac{(1 - sT_{m3})e^{-s\tau_m}}{({T_{CL2}s + 1})^2}.$$

4.8.3 Controller with filtered derivative $G_c(s) = K_c \left(1 + \frac{1}{T_i s} + \frac{T_d s}{1 + \frac{T_d s}{N}} \right)$

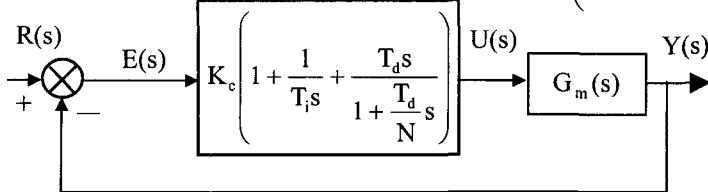


Table 140: PID controller tuning rules – SOSPD model with a positive zero

$$G_m(s) = \frac{K_m(1-sT_{m3})e^{-s\tau_m}}{(1+sT_{m1})(1+sT_{m2})} \text{ or } G_m(s) = \frac{K_m(1-sT_{m3})e^{-s\tau_m}}{T_{m1}^2 s^2 + 2\xi_m T_{m1} s + 1}$$

Rule	K_c	T_i	T_d	Comment
Robust				
Chien (1988). <i>Model: Method 1</i>	${}^6 K_c^{(570)}$	$T_i^{(570)}$	$T_d^{(570)}$	
	${}^7 K_c^{(571)}$	$T_i^{(571)}$	$T_d^{(571)}$	
$T_{m1} > T_{m2} > T_{m3}; N=10; \lambda = [T_{m1}, \tau_m]$ (Chien and Fruehauf (1990)).				

$${}^6 K_c^{(570)} = \frac{T_{m1} + T_{m2} + \frac{T_{m3}\tau_m}{\lambda + T_{m3} + \tau_m}}{K_m(\lambda + T_{m3} + \tau_m)}, \quad T_i^{(570)} = T_{m1} + T_{m2} + \frac{T_{m3}\tau_m}{\lambda + T_{m3} + \tau_m},$$

$$T_d^{(570)} = \frac{T_{m3}\tau_m}{\lambda + T_{m3} + \tau_m} + \frac{T_{m1}T_{m2}}{T_{m1} + T_{m2} + \frac{T_{m3}\tau_m}{\lambda + T_{m3} + \tau_m}}.$$

$${}^7 K_c^{(571)} = \frac{2\xi_m T_{m1} + \frac{T_{m3}\tau_m}{\lambda + T_{m3} + \tau_m}}{K_m(\lambda + T_{m3} + \tau_m)}, \quad T_i^{(571)} = 2\xi_m T_{m1} + \frac{T_{m3}\tau_m}{\lambda + T_{m3} + \tau_m},$$

$$T_d^{(571)} = \frac{T_{m3}\tau_m}{\lambda + T_{m3} + \tau_m} + \frac{T_{m1}^2}{2\xi_m T_{m1} + \frac{T_{m3}\tau_m}{\lambda + T_{m3} + \tau_m}}.$$

4.8.4 Classical controller 1 $G_c(s) = K_c \left(1 + \frac{1}{T_i s}\right) \left(\frac{1 + T_d s}{1 + \frac{T_d s}{N}} \right)$

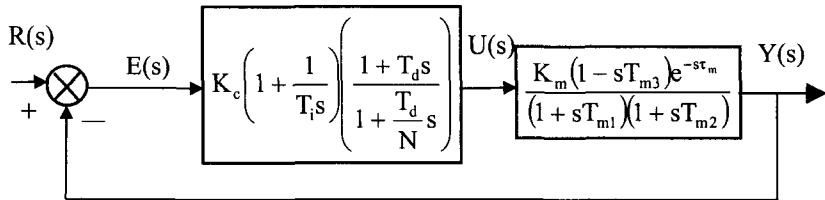


Table 141: PID controller tuning rules – SOSPD model with a positive zero

$$G_m(s) = \frac{K_m (1 - sT_{m3}) e^{-s\tau_m}}{(1 + sT_{m1})(1 + sT_{m2})}$$

Rule	K_c	T_i	T_d	Comment
Direct synthesis				
Poulin <i>et al.</i> (1996). Model: Method 1	$^8 K_c^{(572)}$	T_{m1}	T_{m2}	Minimum $\phi_m = 65^\circ$
O'Dwyer (2001a). Model: Method 1	$\frac{x_1 T_{m1}}{K_m \tau_m}$	T_{m1}	T_{m2}	$A_m = 0.5\pi/x_1$; $\phi_m = 0.5\pi - x_1$; $N = T_{m2}/T_{m3}$
	$\frac{0.785 T_{m1}}{K_m \tau_m}$	T_{m1}	T_{m2}	$A_m = 2.0$; $\phi_m = 45^\circ$
Robust				
Chien (1988). Model: Method 1	$\frac{T_{m2}}{K_m (\lambda + \tau_m)}$	T_{m2}	T_{m1}	$N=10$; $\lambda \in [T_{m1}, \tau_m]$ (Chien and Fruehauf (1990))
	$\frac{T_{m1}}{K_m (\lambda + \tau_m)}$	T_{m1}	T_{m2}	
$T_{m1} > T_{m2} > T_{m3}$				

$$^8 K_c^{(572)} = \frac{T_{m1}}{K_m (T_{m1} + 2T_{m3} + \tau_m)}, \quad N = \frac{T_{m2}(T_{m1} + 2T_{m3})}{T_{m1} T_{m3}},$$

$$\phi_m = 56^\circ \text{ when } T_{m3} = T_{m1} = \tau_m.$$

4.8.5 Series controller (classical controller 3)

$$G_c(s) = K_c \left(1 + \frac{1}{T_i s} \right) (1 + T_d s)$$

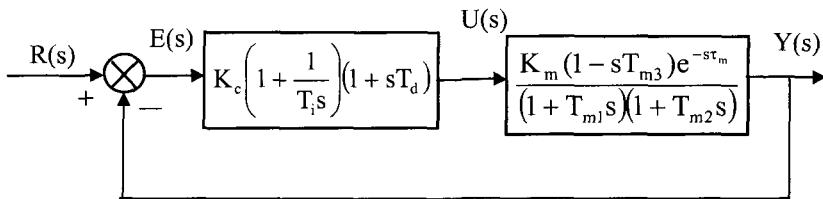


Table 142: PID controller tuning rules – SOSPD model with a positive zero

$$G_m(s) = \frac{K_m (1 - sT_{m3}) e^{-s\tau_m}}{(1 + sT_{m1})(1 + sT_{m2})}$$

Rule	K_c	T_i	T_d	Comment
Direct synthesis				
Zhang (1994). Model: Method 1	$\frac{T_{m1}}{K_m \tau_m}$	T_{m1}	T_{m2}	“Good” servo response; τ_m is “small”.

4.8.6 Classical controller 4 $G_c(s) = K_c \left(1 + \frac{1}{T_i s} \right) \left(1 + \frac{T_d s}{1 + \frac{T_d s}{N}} \right)$

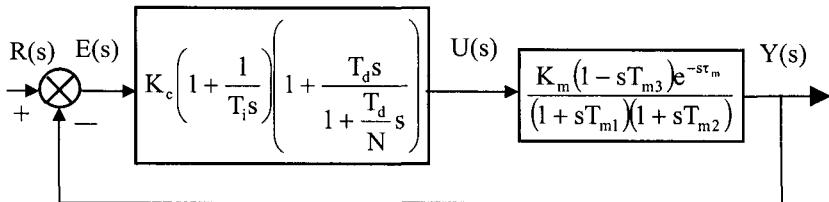


Table 143: PID controller tuning rules – SOSPD model with a positive zero

$$G_m(s) = \frac{K_m(1-sT_{m3})e^{-s\tau_m}}{(1+sT_{m1})(1+sT_{m2})}$$

Rule	K_c	T_i	T_d	Comment
Robust				
Chien (1988). Model: Method 1	$\frac{T_{m2}}{K_m(\lambda + \tau_m)}$	T_{m2}	${}^9 T_d^{(573)}$	
	$\frac{T_{m1}}{K_m(\lambda + \tau_m)}$	T_{m1}	${}^{10} T_d^{(574)}$	
N=10; $\lambda \in [T, \tau_m]$, T = dominant time constant (Chien and Fruehauf (1990)).				

$${}^9 T_d^{(573)} = T_{m1} + \frac{T_{m3}\tau_m}{\lambda + T_{m3} + \tau_m} .$$

$${}^{10} T_d^{(574)} = T_{m2} + \frac{T_{m3}\tau_m}{\lambda + T_{m3} + \tau_m} .$$

4.8.7 Non-interacting controller I

$$U(s) = K_c \left(1 + \frac{1}{T_i s} \right) \begin{pmatrix} E(s) - \frac{T_d s}{1 + \frac{T_d s}{N}} Y(s) \\ 1 + \frac{T_d s}{N} \end{pmatrix}$$

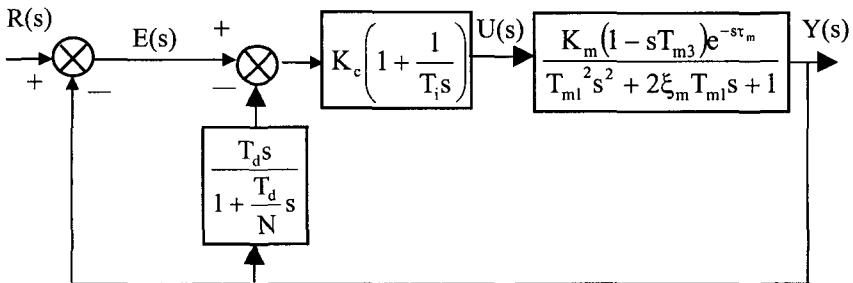


Table 144: PID controller tuning rules – SOSPD model with a positive zero

$$G_m(s) = \frac{K_m(1-sT_{m3})e^{-sT_m}}{T_{m1}^2 s^2 + 2\xi_m T_{m1}s + 1}$$

Rule	K_c	T_i	T_d	Comment
Direct synthesis				
Huang <i>et al.</i> (2000). Model: Method 1	$^{11} K_c^{(575)}$	$2T_{m1}\xi_m + 0.4\tau_m$	$T_d^{(575)}$	
N not specified		$\lambda = 0.5, \frac{\tau_m}{T_{m1}} \xi_m \leq 1 ; \lambda = 0.6, 1 < \frac{\tau_m}{T_{m1}} \xi_m \leq 2 ;$		
		$\lambda = 0.7, \frac{\tau_m}{T_{m1}} \xi_m > 2$, servo tuning		
		$\lambda = 0.6, \frac{\tau_m}{T_{m1}} \xi_m \leq 1 ; \lambda = 0.7, 1 < \frac{\tau_m}{T_{m1}} \xi_m \leq 2 ;$		
		$\lambda = 0.8, \frac{\tau_m}{T_{m1}} \xi_m > 2$, regulator tuning		

$$^{11} K_c^{(575)} = \lambda \frac{2T_{m1}\xi_m + 0.4\tau_m}{K_m(\tau_m + 2T_{m3})}, \quad T_d^{(575)} = \frac{T_{m1}^2 + 0.8T_{m1}\xi_m\tau_m}{0.8T_{m1}\xi_m\tau_m}.$$

4.8.8 Non-interacting controller based on the two degree of freedom structure 1

$$U(s) = K_c \left(1 + \frac{1}{T_i s} + \frac{T_d s}{1 + T_d s/N} \right) E(s) - K_c \left(\alpha + \frac{\beta T_d s}{1 + T_d s/N} \right) R(s)$$

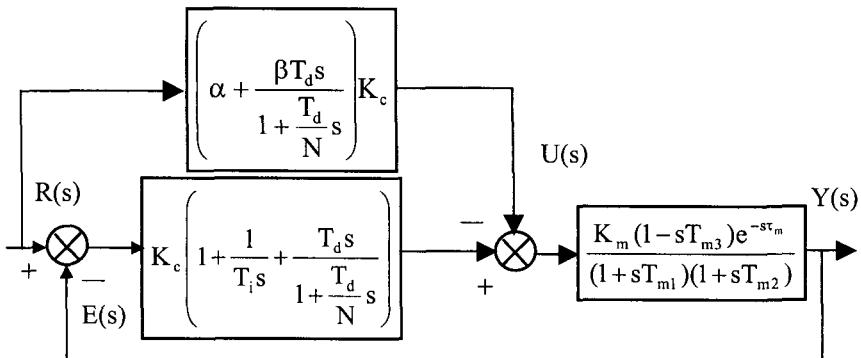


Table 145: PID controller tuning rules – SOSPD model with a positive zero

$$G_m(s) = \frac{K_m(1-sT_{m3})e^{-sT_m}}{(1+sT_{m1})(1+sT_{m2})}$$

Rule	K_c	T_i	T_d	Comment
Direct synthesis				
Huang <i>et al.</i> (2000).	$^{12} K_c^{(576)}$	$T_i^{(576)}$	$T_d^{(576)}$	Servo tuning
Model: Method 1 $N = \min[20,$ $20T_d]$	$\lambda = 0.6, \frac{\tau_m}{T_{m1}} \xi_m \leq 1, A_m = 2.83; \quad \lambda = 0.7, 1 < \frac{\tau_m}{T_{m1}} \xi_m \leq 2,$ $A_m = 2.43; \quad \lambda = 0.8, \frac{\tau_m}{T_{m1}} \xi_m > 2, \quad A_m = 2.13;$ $\alpha = \frac{2T_{m1}\xi_m}{2T_{m1}\xi_m + 0.4\tau_m} - 1, \quad \beta = \frac{0.5T_{m1}^2}{T_{m1}^2 + 0.8T_{m1}\xi_m\tau_m} - 1$			

$$^{12} K_c^{(576)} = \lambda \frac{2T_{m1}\xi_m + 0.4\tau_m}{K_m(\tau_m + 2T_{m3})}, \quad T_i^{(576)} = 2T_{m1}\xi_m + 0.4\tau_m,$$

$$T_d^{(576)} = \frac{T_{m1}^2 + 0.8T_{m1}\xi_m\tau_m}{0.8T_{m1}\xi_m\tau_m}.$$

4.9 SOSPD Model with a Negative Zero

$$G_m(s) = \frac{K_m(1+sT_{m3})e^{-s\tau_m}}{T_{m1}^2 s^2 + 2\xi_m T_{m1}s + 1} \quad \text{or} \quad G_m(s) = \frac{K_m(1+sT_{m3})e^{-s\tau_m}}{(1+sT_{m1})(1+sT_{m2})}$$

4.9.1 Ideal controller $G_c(s) = K_c \left(1 + \frac{1}{T_i s} + T_d s \right)$

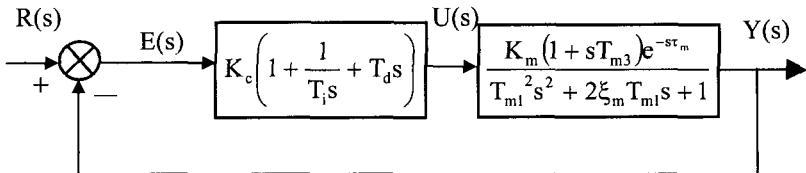


Table 146: PID controller tuning rules – SOSPD model with a negative zero

$$G_m(s) = \frac{K_m(1+sT_{m3})e^{-s\tau_m}}{T_{m1}^2 s^2 + 2\xi_m T_{m1}s + 1}$$

Rule	K_c	T_i	T_d	Comment
Direct synthesis				
Wang <i>et al.</i> (2000b), (2001a). Model: Method 2	¹ $K_c^{(577)}$	$2\xi_m T_{m1}$	T_{m1}^2	$M_s = 1.5$

$$1 \quad K_c^{(577)} = \frac{1}{M_s} \frac{\cos \theta}{\cos[\omega \tau_m - \tan^{-1}(T_{m3}\omega)]} \frac{2\xi_m T_{m1} \omega}{K_m \sqrt{T_{m3}^2 \omega^2 + 1}}; \quad \omega \text{ and } \theta \text{ are determined by}$$

solving the equations:

$$\tan^{-1} \left(\frac{\cos \theta}{M_s - \sin \theta} \right) = \tan^{-1}(T_{m3}\omega) - \omega \tau_m - 0.5\pi \quad \text{and}$$

$$\theta = \tan^{-1} \left[\frac{(T_{m3}\tau_m \omega^2 - 1)\cos(\omega \tau_m) - \omega \tau_m \sin(\omega \tau_m)}{(T_{m3}\tau_m \omega^2 - 1)\sin(\omega \tau_m) + \omega \tau_m \sin(\omega \tau_m)} \right].$$

4.9.2 Controller with filtered derivative $G_c(s) = K_c \left(1 + \frac{1}{T_i s} + \frac{T_d s}{1 + \frac{T_d s}{N}} \right)$

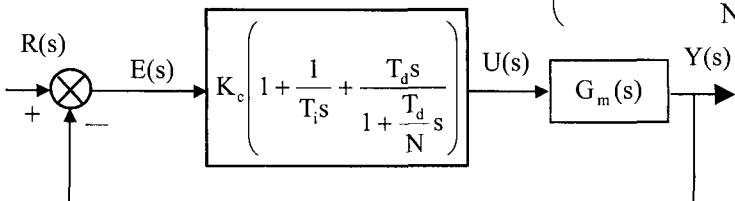


Table 147: PID controller tuning rules – SOSPD model with a negative zero

$$G_m(s) = \frac{K_m (1+sT_{m3}) e^{-s\tau_m}}{(1+sT_{m1})(1+sT_{m2})} \text{ or } G_m(s) = \frac{K_m (1+sT_{m3}) e^{-s\tau_m}}{T_{m1}^2 s^2 + 2\xi_m T_{m1} s + 1}$$

Rule	K_c	T_i	T_d	Comment
Robust				
Chien (1988). Model: Method 1	${}^2 K_c^{(578)}$ $\frac{2\xi_m T_{m1} - T_{m3}}{K_m (\lambda + \tau_m)}$	$T_i^{(578)}$ $2\xi_m T_{m1} - T_{m3}$	$T_d^{(578)}$ ${}^3 T_d^{(579)}$	$N=10;$ $\lambda \in [T_{m1}, \tau_m]$ (Chien and Fruehauf (1990)).

$$\begin{aligned}
 {}^2 K_c^{(578)} &= \frac{T_{m1} + T_{m2} - T_{m3}}{K_m (\lambda + \tau_m)}, \quad T_i^{(578)} = T_{m1} + T_{m2} - T_{m3}, \\
 T_d^{(578)} &= \frac{T_{m1} T_{m2} - (T_{m1} + T_{m2} - T_{m3}) T_{m3}}{T_{m1} + T_{m2} - T_{m3}}, \quad T_{m1} > T_{m2} > T_{m3}. \\
 {}^3 T_d^{(579)} &= \frac{T_{m1}^2 - (2\xi_m T_{m1} - T_{m3}) T_{m3}}{2\xi_m T_{m1} - T_{m3}}.
 \end{aligned}$$

4.9.3 Classical controller 1 $G_c(s) = K_c \left(1 + \frac{1}{T_i s} \right) \left(\frac{1 + T_d s}{1 + \frac{T_d}{N} s} \right)$

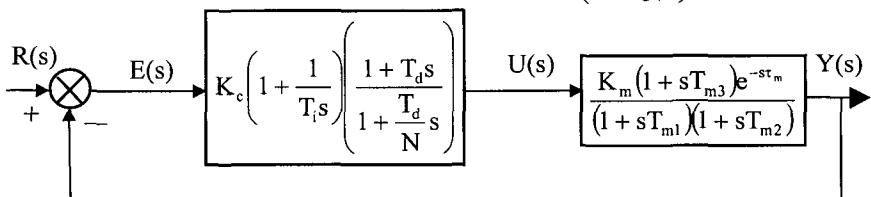


Table 148: PID controller tuning rules – SOSPD model with a negative zero

$$G_m(s) = \frac{K_m (1 + sT_{m3}) e^{-s\tau_m}}{(1 + sT_{m1})(1 + sT_{m2})}$$

Rule	K_c	T_i	T_d	Comment
Direct synthesis				
Zhang (1994). Model: Method 1	$\frac{T_{m1}}{K_m \tau_m}$	T_{m1}	T_{m2}	$N = T_{m2}/T_{m3}$
“small” τ_m ; “good” servo response.				
Poulin et al. (1996). Model: Method 1	$\frac{T_{m1}}{K_m (T_{m1} + \tau_m)}$	T_{m1}	T_{m2}	$N = T_{m2}/T_{m3}$; Minimum $\phi_m = 90^\circ$; $\phi_m = 60^\circ$, $\tau_m = T_{m1}$
Robust				
Chien (1988). Model: Method 1	$\frac{T_{m2}}{K_m (\lambda + \tau_m)}$	T_{m2}	T_{m1}	
	$\frac{T_{m1}}{K_m (\lambda + \tau_m)}$	T_{m1}	T_{m2}	
$T_{m1} > T_{m2}$; $N=10$; $\lambda \in [T_{m1}, \tau_m]$ (Chien and Fruehauf (1990)).				

4.9.4 Classical controller 4 $G_c(s) = K_c \left(1 + \frac{1}{T_i s} \right) \left(1 + \frac{T_d s}{1 + \frac{T_d s}{N}} \right)$

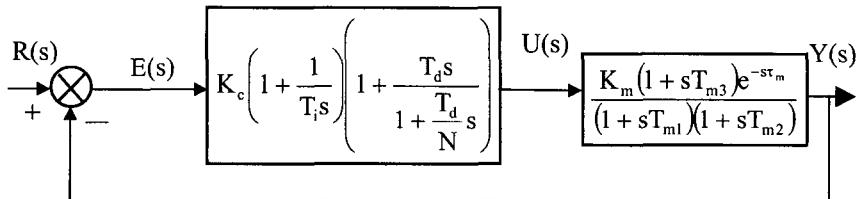


Table 149: PID controller tuning rules – SOSPD model with a negative zero

$$G_m(s) = \frac{K_m (1 + sT_{m3}) e^{-s\tau_m}}{(1 + sT_{m1})(1 + sT_{m2})}$$

Rule	K_c	T_i	T_d	Comment
Robust				
Chien (1988). Model: Method 1	$\frac{T_{m2}}{K_m (\lambda + \tau_m)}$	T_{m2}	$T_{m1} - T_{m3}$	
	$\frac{T_{m1}}{K_m (\lambda + \tau_m)}$	T_{m1}	$T_{m2} - T_{m3}$	
$T_{m1} > T_{m2} > T_{m3}; N=10; \lambda \in [T_{m1}, \tau_m]$ (Chien and Fruehauf (1990)).				

4.9.5 Non-interacting controller I

$$U(s) = K_c \left(1 + \frac{1}{T_i s} \right) \begin{cases} E(s) - \frac{T_d s}{1 + \frac{T_d s}{N}} Y(s) \\ \end{cases}$$

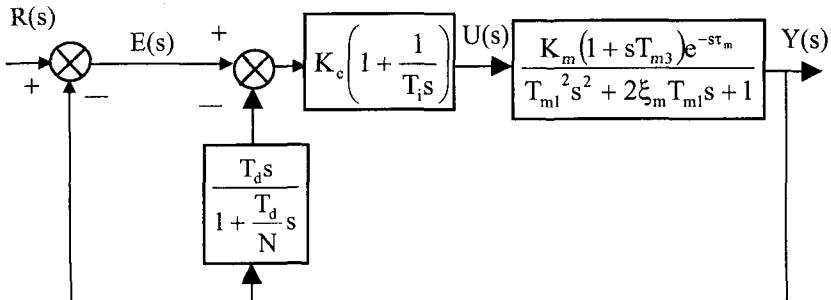


Table 150: PID controller tuning rules – SOSPD model with a negative zero

$$G_m(s) = \frac{K_m(1+sT_{m3})e^{-sT_m}}{T_{m1}^2 s^2 + 2\xi_m T_{m1}s + 1}$$

Rule	K_c	T_i	T_d	Comment
Direct synthesis				
Huang <i>et al.</i> (2000). Model: Method 1	${}^4 K_c^{(580)}$	$2T_{m1}\xi_m + 0.4\tau_m$	$T_d^{(580)}$	
N not specified	$\lambda = 0.5, \frac{\tau_m}{T_{m1}}\xi_m \leq 1 ; \lambda = 0.6, 1 < \frac{\tau_m}{T_{m1}}\xi_m \leq 2 ; \lambda = 0.7, \frac{\tau_m}{T_{m1}}\xi_m > 2 ;$ servo tuning			
	$\lambda = 0.6, \frac{\tau_m}{T_{m1}}\xi_m \leq 1 ; \lambda = 0.7, 1 < \frac{\tau_m}{T_{m1}}\xi_m \leq 2 ; \lambda = 0.8, \frac{\tau_m}{T_{m1}}\xi_m > 2 ;$ regulator tuning			

$${}^4 K_c^{(580)} = \lambda \frac{2T_{m1}\xi_m + 0.4\tau_m}{K_m(\tau_m + 2T_{m3})}, \quad T_d^{(580)} = \frac{T_{m1}^2 + 0.8T_{m1}\xi_m\tau_m}{0.8T_{m1}\xi_m\tau_m}.$$

4.9.6 Non-interacting controller based on the two degree of freedom structure 1

$$U(s) = K_c \left(1 + \frac{1}{T_i s} + \frac{T_d s}{1 + \frac{T_d}{N} s} \right) E(s) - K_c \left(\alpha + \frac{\beta T_d s}{1 + \frac{T_d}{N} s} \right) R(s)$$

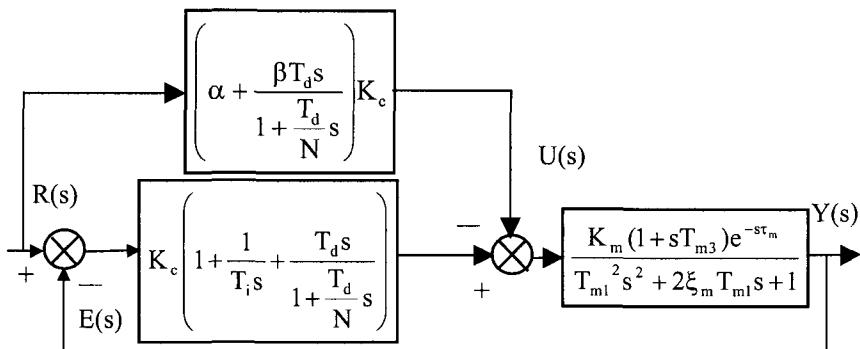


Table 151: PID controller tuning rules – SOSPD model with a negative zero

$$G_m(s) = \frac{K_m(1+sT_{m3})e^{-s\tau_m}}{T_{m1}^2 s^2 + 2\xi_m T_{m1} s + 1}$$

Rule	K_c	T_i	T_d	Comment
Direct synthesis				
Huang <i>et al.</i> (2000) - servo tuning. <i>Model: Method I</i>	$5 K_c^{(581)}$	$2T_{m1}\xi_m + 0.4\tau_m$	$T_d^{(581)}$	$N = \min[20, 20T_d]$
	$\alpha = \frac{2T_{m1}\xi_m}{2T_{m1}\xi_m + 0.4\tau_m} - 1 ; \beta = \frac{0.5T_{m1}^2}{T_{m1}^2 + 0.8T_{m1}\xi_m\tau_m} - 1 ;$			
	$\lambda = 0.6, \frac{\tau_m}{T_{m1}}\xi_m \leq 1 , A_m = 2.83 ; \lambda = 0.7, 1 < \frac{\tau_m}{T_{m1}}\xi_m \leq 2 , A_m = 2.43 ; \lambda = 0.8, \frac{\tau_m}{T_{m1}}\xi_m > 2 , A_m = 2.13 .$			

$$5 K_c^{(581)} = \lambda \frac{2T_{m1}\xi_m + 0.4\tau_m}{K_m(\tau_m + 2T_{m3})}, T_d^{(581)} = \frac{T_{m1}^2 + 0.8T_{m1}\xi_m\tau_m}{0.8T_{m1}\xi_m\tau_m} .$$

4.10 Third Order System plus Time Delay Model

$$G_m(s) = \frac{K_m e^{-s\tau_m}}{1 + a_1 s + a_2 s^2 + a_3 s^3} \quad \text{or} \quad G_m(s) = \frac{K_m e^{-s\tau_m}}{(1 + sT_{m1})(1 + sT_{m2})(1 + sT_{m3})}$$

4.10.1 Ideal controller $G_c(s) = K_c \left(1 + \frac{1}{T_i s} + T_d s \right)$

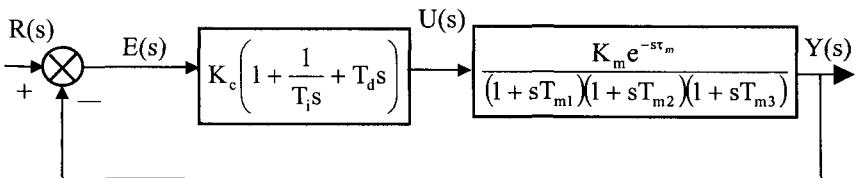


Table 152: PID controller tuning rules – third order system plus time delay model

$$G_m(s) = \frac{K_m e^{-s\tau_m}}{(1 + sT_{m1})(1 + sT_{m2})(1 + sT_{m3})}$$

Rule	K_c	T_i	T_d	Comment
Minimum performance index				
Standard form optimisation - binomial - Polonyi (1989).	¹ $K_c^{(582)}$	$T_i^{(582)}$	τ_m	<i>Model: Method I</i>
Standard form optimisation – minimum ITAE - Polonyi (1989).	² $K_c^{(583)}$	$T_i^{(583)}$	τ_m	$T_{m1} > 10(T_{m2} + \tau_m)$

$$^1 K_c^{(582)} = \left(1 - 4 \sqrt{\frac{T_{m2}}{6\tau_m}} + \frac{T_{m2}}{T_{m3}} \right) \frac{T_{m1}}{\tau_m}, \quad T_i^{(582)} = 4\sqrt{6T_{m2}\tau_m} + \tau_m.$$

$$^2 K_c^{(583)} = \left(1 - 2.1 \sqrt{\frac{T_{m2}}{3.4\tau_m}} + \frac{T_{m2}}{T_{m3}} \right) \frac{T_{m1}}{\tau_m}, \quad T_i^{(583)} = 2.7\sqrt{3.4T_{m2}\tau_m} + \tau_m.$$

4.10.2 Ideal controller in series with a first order lag

$$G_c(s) = K_c \left(1 + \frac{1}{T_i s} + T_d s \right) \frac{1}{T_f s + 1}$$

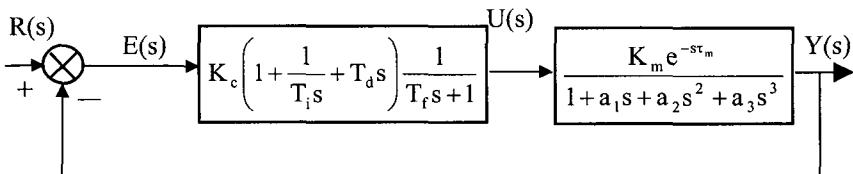


Table 153: PID controller tuning rules –Third order system plus time delay model

$$G_m(s) = \frac{K_m e^{-s\tau_m}}{1 + a_1 s + a_2 s^2 + a_3 s^3}$$

Rule	K_c	T_i	T_d	Comment
Direct synthesis: frequency domain criteria				
Schaedel (1997). Model: Method 1	${}^3 K_c^{(584)}$	$T_i^{(584)}$	$T_d^{(584)}$	$T_f = T_d^{(584)} / N ;$ $5 \leq N \leq 20$

$${}^3 K_c^{(584)} = \frac{0.375 T_i^{(584)}}{K_m \left(a_1 + \tau_m + \frac{T_d^{(584)}}{N} - T_i^{(584)} \right)}, \quad T_i^{(584)} = \frac{a_1^2 - a_2 + a_1 \tau_m + 0.5 \tau_m^2}{a_1 + \tau_m - T_d^{(584)}},$$

$$T_d^{(584)} = \frac{a_2 + a_1 \tau_m + 0.5 \tau_m^2 - a_3 + a_2 \tau_m + 0.5 a_1 \tau_m^2 + 0.167 \tau_m^3}{a_1 + \tau_m - a_2 + a_1 \tau_m + 0.5 \tau_m^2}.$$

4.10.3 Controller with filtered derivative $G_c(s) = K_c \left(1 + \frac{1}{T_i s} + \frac{T_d s}{1 + \frac{T_d}{N} s} \right)$

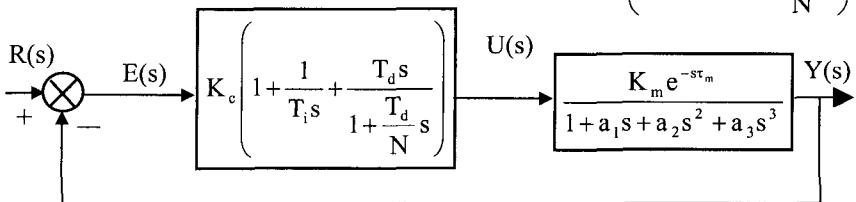


Table 154: PID controller tuning rules – Third order system plus time delay model

$$G_m(s) = \frac{K_m e^{-s\tau_m}}{1 + a_1 s + a_2 s^2 + a_3 s^3}$$

Rule	K_c	T_i	T_d	Comment
Direct synthesis: frequency domain criteria				
Schaedel (1997).	${}^4 K_c^{(585)}$	$T_i^{(585)}$	$T_d^{(585)}$	Model: Method I

$$\begin{aligned} {}^4 K_c^{(585)} &= \frac{0.375 T_i^{(585)}}{K_m \left(a_1 + \tau_m + \frac{T_d^{(585)}}{N} - T_i^{(585)} \right)}, \quad T_i^{(585)} = \frac{a_1^2 - a_2 + a_1 \tau_m + 0.5 \tau_m^2}{a_1 + \tau_m - T_d^{(585)}}, \\ T_d^{(585)} &= \frac{a_2 + a_1 \tau_m + 0.5 \tau_m^2}{a_1 + \tau_m} - \frac{a_3 + a_2 \tau_m + 0.5 a_1 \tau_m^2 + 0.167 \tau_m^3}{a_2 + a_1 \tau_m + 0.5 \tau_m^2}, \\ N &= \frac{T_d^{(585)}}{- \left(\frac{a + \tau_m - T_i^{(585)}}{2} \right) + \sqrt{\frac{(a + \tau_m - T_i^{(585)})^2}{4} + 0.375 \frac{T_i^{(585)} T_d^{(585)}}{K_v}}}, \end{aligned}$$

$0.05 \leq a \leq 0.2$, K_v = prescribed overshoot in the manipulated variable for a step response in the command variable.

4.10.4 Non-interacting controller based on the two degree of freedom structure 1

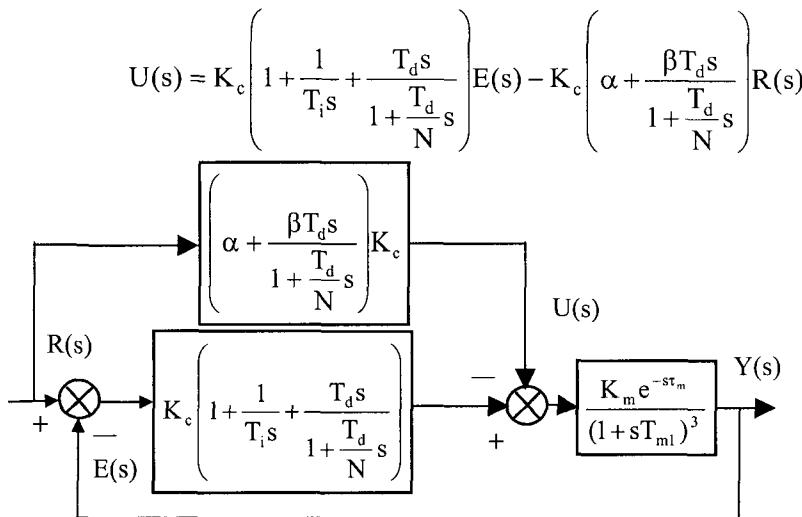


Table 155: PID controller tuning rules – third order system plus time delay model –

$$G_m(s) = \frac{K_m e^{-s\tau_m}}{(1+sT_{ml})^3}$$

Rule	K_c	T_i	T_d	Comment
Minimum performance index: servo/regulator tuning				
Taguchi and Araki (2000). Model: Method 1	⁵ $K_c^{(586)}$	$T_i^{(586)}$	$T_d^{(586)}$	Overshoot (servo step) $\leq 20\%$; N fixed but not specified

⁵ Equations continued into the footnote on page 402; $\tau_m/T_{ml} \leq 1.0$.

$$K_c^{(586)} = \frac{1}{K_m} \left(0.4020 + \frac{1.275}{\frac{\tau_m}{T_{ml}} + 0.003273} \right),$$

$$T_i^{(586)} = T_{ml} \left(0.3572 + 7.467 \frac{\tau_m}{T_{ml}} - 12.86 \left[\frac{\tau_m}{T_{ml}} \right]^2 + 11.77 \left[\frac{\tau_m}{T_{ml}} \right]^3 - 4.146 \left[\frac{\tau_m}{T_{ml}} \right]^4 \right),$$

$$T_d^{(586)} = T_{ml} \left(0.8335 + 0.2910 \frac{\tau_m}{T_{ml}} - 0.04000 \left[\frac{\tau_m}{T_{ml}} \right]^2 \right),$$

$$\alpha = 0.6661 - 0.2509 \frac{\tau_m}{T_{ml}} + 0.04773 \left[\frac{\tau_m}{T_{ml}} \right]^2, \quad \beta = 0.8131 - 0.2303 \frac{\tau_m}{T_{ml}} + 0.03621 \left[\frac{\tau_m}{T_{ml}} \right]^2.$$

4.11 Unstable FOLPD Model $G_m(s) = \frac{K_m e^{-s\tau_m}}{T_m s - 1}$

4.11.1 Ideal controller $G_c(s) = K_c \left(1 + \frac{1}{T_i s} + T_d s \right)$

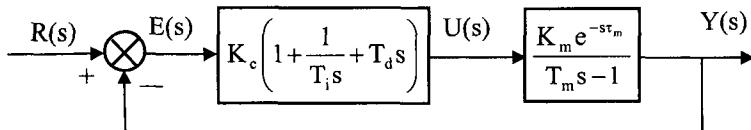


Table 156: PID controller tuning rules – unstable FOLPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{T_m s - 1}$

Rule	K_c	T_i	T_d	Comment
Minimum performance index: regulator tuning				
Minimum ISE – Visioli (2001).	$\frac{1.37}{K_m} \left(\frac{\tau_m}{T_m} \right)$	$2.42T_m \left(\frac{\tau_m}{T_m} \right)^{1.18}$	$0.60\tau_m$	<i>Model: Method 1</i>
Minimum ITSE – Visioli (2001).	$\frac{1.37}{K_m} \left(\frac{\tau_m}{T_m} \right)$	$4.12T_m \left(\frac{\tau_m}{T_m} \right)^{0.90}$	$0.55\tau_m$	<i>Model: Method 1</i>
Minimum ISTSE – Visioli (2001).	$\frac{1.70}{K_m} \left(\frac{\tau_m}{T_m} \right)$	$4.52T_m \left(\frac{\tau_m}{T_m} \right)^{1.13}$	$0.50\tau_m$	<i>Model: Method 1</i>
Minimum performance index: servo tuning				
Minimum ISE – Visioli (2001).	$\frac{1.32}{K_m} \left(\frac{\tau_m}{T_m} \right)^{0.92}$	$4.00T_m \left(\frac{\tau_m}{T_m} \right)^{0.47}$	$^1 T_d^{(587)}$	<i>Model: Method 1</i>
Minimum ITSE – Visioli (2001).	$\frac{1.38}{K_m} \left(\frac{\tau_m}{T_m} \right)^{0.90}$	$4.12T_m \left(\frac{\tau_m}{T_m} \right)^{0.90}$	$^2 T_d^{(588)}$	<i>Model: Method 1</i>

$$^1 T_d^{(587)} = 3.78T_m \left[1 - 0.84 \left(\frac{T_m}{\tau_m} \right)^{0.02} \right] \left(\frac{\tau_m}{T_m} \right)^{0.95}.$$

$$^2 T_d^{(588)} = 3.62T_m \left[1 - 0.85 \left(\frac{T_m}{\tau_m} \right)^{0.02} \right] \left(\frac{\tau_m}{T_m} \right)^{0.93}.$$

Rule	K_c	T_i	T_d	Comment
Minimum ISTSE – Visioli (2001).	$\frac{1.35}{K_m} \left(\frac{\tau_m}{T_m} \right)^{0.95}$	$4.52 T_m \left(\frac{\tau_m}{T_m} \right)^{1.13}$	$^3 T_d^{(589)}$	<i>Model: Method 1</i>
Minimum performance index: servo/regulator tuning				
Jhunjhunwala and Chidambaram (2001).	$^4 K_c^{(590)}$	$T_i^{(590)}$	$T_d^{(590)}$	<i>Model: Method 1</i>
Direct synthesis				
De Paor and O'Malley (1989). <i>Model: Method 1</i>	$^5 K_c^{(591)}$	$T_i^{(591)}$	$T_d^{(591)}$	$\left \frac{\tau_m}{T_m} \right < 1$
Chidambaram (1995b). <i>Model: Method 1</i>	$^6 K_c^{(592)}$	$T_m \left(25 - 27 \frac{\tau_m}{T_m} \right)$	$0.46 \tau_m$	$\frac{\tau_m}{T_m} < 0.6$

$$^3 T_d^{(589)} = 3.70 T_m \left[1 - 0.86 \left(\frac{T_m}{\tau_m} \right)^{0.02} \right] \left(\frac{\tau_m}{T_m} \right)^{0.97}.$$

$$^4 K_c^{(590)} = \frac{1}{K_m} \left(13.0 - 39.712 \frac{\tau_m}{T_m} \right), \quad \frac{\tau_m}{T_m} < 0.2;$$

$$K_c^{(590)} = \frac{1.397}{K_m} \left(\frac{T_m}{\tau_m} \right)^{0.769}, \quad 0.2 \leq \frac{\tau_m}{T_m} \leq 1.4;$$

$$T_i^{(590)} = 0.856 T_m e^{-\frac{2.044 \tau_m}{T_m}}, \quad 0 \leq \frac{\tau_m}{T_m} \leq 1; \quad T_i^{(590)} = 0.3444 T_m e^{-\frac{2.9595 \tau_m}{T_m}}, \quad 1 \leq \frac{\tau_m}{T_m} \leq 1.4;$$

$$T_d^{(590)} = T_m \left(0.5643 \frac{\tau_m}{T_m} + 0.0075 \right), \quad 0 \leq \frac{\tau_m}{T_m} \leq 1.4.$$

$$^5 K_c^{(591)} = \frac{1}{K_m} \left[\cos \sqrt{\left(1 - \frac{\tau_m}{T_m} \right) \frac{\tau_m}{T_m}} + \sqrt{\frac{1 - \tau_m/T_m}{\tau_m/T_m}} \sin \sqrt{\left(1 - \frac{\tau_m}{T_m} \right) \frac{\tau_m}{T_m}} \right],$$

$$T_i^{(591)} = \frac{T_m}{\left[\sqrt{\frac{1 - \tau_m/T_m}{\tau_m/T_m}} \right] \tan(0.75\phi)}, \quad T_d^{(591)} = T_m \sqrt{\frac{\tau_m/T_m}{1 - \tau_m/T_m}} \tan(0.75\phi),$$

$$\phi = \tan^{-1} \sqrt{\left(1 - \frac{\tau_m}{T_m} \right) \frac{T_m}{\tau_m}} - \sqrt{\frac{\tau_m}{T_m} \left(1 - \frac{\tau_m}{T_m} \right)}.$$

$$^6 K_c^{(592)} = \frac{1}{K_m} \left(1.3 + 0.3 \frac{T_m}{\tau_m} \right).$$

Rule	K_c	T_i	T_d	Comment
Chidambaram (1997). Model: Method 1	$\frac{1.165}{K_m} \left(\frac{T_m}{\tau_m} \right)$	${}^7 T_i^{(593)}$	$\frac{{}^7 T_i^{(593)}}{\alpha}$	
Valentine and Chidambaram (1997b) - dominant pole placement. Model: Method 1	$\frac{1.165}{K_m} \left(\frac{T_m}{\tau_m} \right)^{0.245}$	${}^8 T_i^{(594)}$	${}^8 T_d^{(594)}$	$\left \frac{\tau_m}{T_m} \right < 0.6$
		${}^9 T_i^{(595)}$		$0.6 \leq \left \frac{\tau_m}{T_m} \right \leq 0.8$
		${}^{10} T_i^{(596)}$		$0.8 < \left \frac{\tau_m}{T_m} \right \leq 1$

$${}^7 T_i^{(593)} = \left(0.176 + 0.360 \frac{\tau_m}{T_m} \right) T_m ; \alpha = 0.179 - 0.324 \frac{\tau_m}{T_m} + 0.161 \left(\frac{\tau_m}{T_m} \right)^2 , \frac{\tau_m}{T_m} < 0.6 ;$$

$$\alpha = 0.04 , 0.6 < \frac{\tau_m}{T_m} < 0.8 ; \alpha = 0.12 - 0.1 \frac{\tau_m}{T_m} , 0.8 < \frac{\tau_m}{T_m} < 1.0 .$$

$${}^8 T_i^{(594)} = \frac{\left(0.176 + 0.36 \frac{\tau_m}{T_m} \right) T_m}{0.179 - 0.324 \frac{\tau_m}{T_m} + 0.161 \left(\frac{\tau_m}{T_m} \right)^2} , {}^8 T_d^{(594)} = \left(0.176 + 0.36 \frac{\tau_m}{T_m} \right) T_m .$$

$${}^9 T_i^{(595)} = \left(0.176 + 0.36 \frac{\tau_m}{T_m} \right) 25 T_m .$$

$${}^{10} T_i^{(596)} = \frac{0.176 T_m + 0.36 \tau_m}{0.12 - 0.1 \frac{\tau_m}{T_m}} .$$

Rule	K_c	T_i	T_d	Comment
Pramod and Chidambaran (2000). Model: Method 1	$^{11} K_c^{(597)}$	$T_i^{(597)}$	$T_d^{(597)}$	$\left \frac{\tau_m}{T_m} \right < 0.6$
		$^{12} T_i^{(598)}$		$0.6 < \left \frac{\tau_m}{T_m} \right < 0.8$
Gaikwad and Chidambaran (2000). Model: Method 1	$^{13} K_c^{(599)}$	$T_i^{(599)}$	$T_d^{(599)}$	$0.1 \leq \frac{\tau_m}{T_m} \leq 0.6$
Sree et al. (2004) Model: Method 1	$^{14} K_c^{(600)}$	$T_i^{(600)}$	$0.4917\tau_m$	$0.01 \leq \frac{\tau_m}{T_m} \leq 0.9$

$$\begin{aligned}
 ^{11} K_c^{(597)} &= \frac{1}{K_m} \left[2.121 - 1.906 \frac{\tau_m}{T_m} + 0.945 \left(\frac{\tau_m}{T_m} \right)^2 \right], \\
 T_i^{(597)} &= \frac{\left(0.176 + 0.36 \frac{\tau_m}{T_m} \right) T_m}{0.179 - 0.324 \frac{\tau_m}{T_m} + 0.161 \left(\frac{\tau_m}{T_m} \right)^2}, \quad T_d^{(597)} = \left(0.176 + 0.36 \frac{\tau_m}{T_m} \right) T_m. \\
 ^{12} T_i^{(598)} &= \left(0.176 + 0.36 \frac{\tau_m}{T_m} \right) 2.5 T_m. \\
 ^{13} K_c^{(599)} &= \frac{1}{K_m} \left[0.299 + 0.726 \frac{T_m}{\tau_m} \right], \quad T_i^{(599)} = T_m \left[4.104 \left(\frac{\tau_m}{T_m} \right)^2 + 2.099 \frac{\tau_m}{T_m} + 0.096 \right], \\
 T_d^{(599)} &= T_m \left[0.822 \left(\frac{\tau_m}{T_m} \right)^2 + 0.419 \frac{\tau_m}{T_m} + 0.019 \right]. \\
 ^{14} K_c^{(600)} &= \frac{1.4183}{K_m} \left(\frac{\tau_m}{T_m} \right)^{-0.9147}, \\
 T_i^{(600)} &= T_m \left[16.327 \left(\frac{\tau_m}{T_m} \right)^2 + 5.5778 \frac{\tau_m}{T_m} + 0.8158 \right], \quad 0.01 \leq \frac{\tau_m}{T_m} \leq 0.6; \\
 T_i^{(600)} &= T_m \left[196 \left(\frac{\tau_m}{T_m} \right)^2 - 247.28 \frac{\tau_m}{T_m} + 87.72 \right], \quad 0.6 \leq \frac{\tau_m}{T_m} \leq 0.9.
 \end{aligned}$$

Rule	K_c	T_i	T_d	Comment
Sree et al. (2004) – continued.	$^{15} K_c^{(601)}$	$T_i^{(601)}$	$T_d^{(601)}$	$0.01 \leq \frac{\tau_m}{T_m} \leq 1.2$
Robust				
Tan et al. (1999b). <i>Model: Method 1</i>	$^{16} K_c^{(602)}$	$T_i^{(602)}$	$T_d^{(602)}$	$\frac{\tau_m}{T_m} < 1.0$
Lee et al. (2000) <i>Model: Method 1</i>	$^{17} K_c^{(603)}$	$T_i^{(603)}$	$T_d^{(603)}$	$0 \leq \frac{\tau_m}{T_m} < 2$

$$^{15} K_c^{(601)} = \frac{1.2824}{K_m} \left(\frac{\tau_m}{T_m} \right)^{-0.8325},$$

$$T_i^{(601)} = T_m \left[5.5734 \frac{\tau_m}{T_m} - 0.0063 \right], \quad 0.01 \leq \frac{\tau_m}{T_m} \leq 0.5;$$

$$T_i^{(601)} = 0.483 T_m e^{3.3739 \frac{\tau_m}{T_m}}, \quad 0.5 \leq \frac{\tau_m}{T_m} \leq 1.2; \quad T_d^{(601)} = T_m \left[0.507 \frac{\tau_m}{T_m} + 0.0028 \right].$$

$$^{16} K_c^{(602)} = \frac{1}{K_m \left\{ \left[-\frac{0.0373}{\lambda} + 0.1862 \right] \frac{\tau_m}{T_m} + [0.6508 - 0.1498 \log \lambda] \right\}},$$

$$T_i^{(602)} = \frac{\left\{ \left[\frac{10.5045}{\lambda} + 0.8032 \right] \frac{\tau_m}{T_m} + \left[\frac{3.2312}{\lambda} - 0.3445 \right] \right\}}{\left\{ \left[-\frac{0.0373}{\lambda} + 0.1862 \right] \frac{\tau_m}{T_m} + [0.6508 - 0.1498 \log \lambda] \right\}} T_m,$$

$$T_d^{(602)} = T_m \left[0.9065 \lambda^{0.1632} \frac{\tau_m}{T_m} - \frac{1}{0.2075 \lambda + 2.0335} \right]$$

$$\left[\left(\frac{-0.0373}{\lambda} + 0.1862 \right) \frac{\tau_m}{T_m} + (0.6508 - 0.1498 \log \lambda) \right], \quad \lambda \text{ not specified.}$$

$$^{17} K_c^{(603)} = \frac{T_i^{(603)}}{-K_m (2T_{CL} + \tau_m - \alpha)}, \quad \alpha = T_m \left[\left(\frac{T_{CL}}{T_m} + 1 \right)^2 e^{\tau_m/T_m} - 1 \right]$$

$$T_i^{(603)} = -T_m + \alpha - \frac{T_{CL}^2 + \alpha \tau_m - 0.5 \tau_m^2}{2T_{CL} + \tau_m - \alpha},$$

$$T_d^{(603)} = \frac{-T_m \alpha - \frac{\tau_m^2 (0.167 \tau_m - 0.5 \alpha)}{2T_{CL} + \tau_m - \alpha}}{T_i^{(603)}} - \frac{T_{CL}^2 + \alpha \tau_m - 0.5 \tau_m^2}{2T_{CL} + \tau_m - \alpha}.$$

4.11.2 Ideal controller in series with a first order lag

$$G_c(s) = K_c \left(1 + \frac{1}{T_i s} + T_d s \right) \frac{1}{T_f s + 1}$$

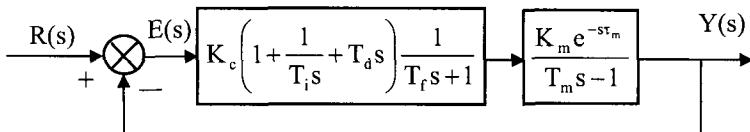


Table 157: PID controller tuning rules – unstable FOLPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{T_m s - 1}$

Rule	K_c	T_i	T_d	Comment
Direct synthesis				
Chandrashekhar <i>et al.</i> (2002).	$^{18} K_c^{(604)}$	$T_i^{(604)}$	$0.5\tau_m$	<i>Model: Method 1</i>

$$^{18} K_c^{(604)} = \frac{1}{K_m} \left[4282 \left(\frac{\tau_m}{T_m} \right)^2 - 1334.6 \frac{\tau_m}{T_m} + 101 \right], \quad 0 < \frac{\tau_m}{T_m} < 0.2;$$

$$K_c^{(604)} = \frac{1.1161}{K_m} \left(\frac{\tau_m}{T_m} \right)^{-0.9427}, \quad 0.2 \leq \frac{\tau_m}{T_m} \leq 1.0.$$

$$T_i^{(604)} = T_m \left[36.842 \left(\frac{\tau_m}{T_m} \right)^2 - 10.3 \left(\frac{\tau_m}{T_m} \right) + 0.8288 \right], \quad 0 \leq \frac{\tau_m}{T_m} \leq 0.8;$$

$$T_i^{(604)} = 76.241 T_m \left(\frac{\tau_m}{T_m} \right)^{6.77}, \quad 0.8 \leq \frac{\tau_m}{T_m} \leq 1.0.$$

$$T_f = \left(0.1233 \frac{\tau_m}{T_m} + 0.0033 \right) T_m, \quad 0 \leq \frac{\tau_m}{T_m} \leq 1.0.$$

$$\text{Desired closed loop transfer function} = \frac{(\eta s + 1)e^{-s\tau_m}}{(T_{CL2}s + 1)^2}; \quad T_{CL2} = 4.5115 T_m \left(\frac{\tau_m}{T_m} \right)^{5.661},$$

$$0.8 \leq \frac{\tau_m}{T_m} \leq 1.0, \quad T_{CL2} = \left(0.0326 + 0.4958 \frac{\tau_m}{T_m} + 2.03 \left(\frac{\tau_m}{T_m} \right)^2 \right) T_m, \quad 0 \leq \frac{\tau_m}{T_m} \leq 0.8.$$

Rule	K_c	T_i	T_d	T_f	T_{CL2}	Comment
Alternative tuning rules						
Chandrashekhar <i>et al.</i> (2002). <i>Model: Method 1</i>	K_c	T_i	T_d	T_f	T_{CL2}	τ_m/T_m
	101.0000 K_m	$0.0404T_m$	0.0	0.0	$0.02T_m$	0.0
	10.3662 K_m	$0.3874T_m$	$0.0435T_m$	$0.0134T_m$	$0.10T_m$	0.1
	5.3750 K_m	$0.8600T_m$	$0.0884T_m$	$0.0250T_m$	$0.20T_m$	0.2
	3.2655 K_m	$1.8018T_m$	$0.1375T_m$	$0.0435T_m$	$0.40T_m$	0.3
	2.4516 K_m	$3.0400T_m$	$0.1868T_m$	$0.0581T_m$	$0.60T_m$	0.4
	2.0217 K_m	$4.6500T_m$	$0.2366T_m$	$0.0696T_m$	$0.80T_m$	0.5
	1.7525 K_m	$6.7286T_m$	$0.2866T_m$	$0.0784T_m$	$1.00T_m$	0.6
	1.5458 K_m	$10.337T_m$	$0.3380T_m$	$0.0885T_m$	$1.30T_m$	0.7
	1.3723 K_m	$17.693T_m$	$0.3910T_m$	$0.1005T_m$	$1.80T_m$	0.8
	1.2420 K_m	$33.610T_m$	$0.4440T_m$	$0.1124T_m$	$2.60T_m$	0.9
	1.1400 K_m	$80.620T_m$	$0.4969T_m$	$0.1247T_m$	$4.20T_m$	1.0

4.11.3 Ideal controller with set-point weighting 1

$$U(s) = K_c (F_p R(s) - Y(s)) + \frac{K_c}{T_i s} (F_i R(s) - Y(s)) + K_c T_d s (F_d R(s) - Y(s))$$

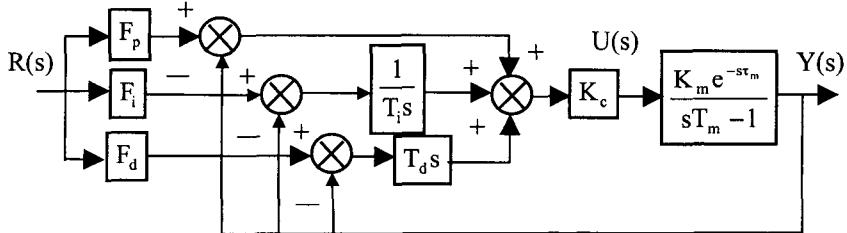


Table 158: PID controller tuning rules – unstable FOLPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{T_m s - 1}$

Rule	K_c	T_i	T_d	Comment
Direct synthesis				
Prashanti and Chidambaram (2000). Model: Method I	${}^1 K_c^{(605)}$ $F_p = 0.17$	$T_i^{(605)}$ $F_i = 1$	$T_d^{(605)}$ $F_d = 0.654$	Modified method of De Paor and O'Malley (1989); $ \tau_m/T_m < 1$

$${}^1 K_c^{(605)} = \frac{1}{K_m} \left[\cos \sqrt{\left(1 - \frac{\tau_m}{T_m}\right) \frac{\tau_m}{T_m}} + \sqrt{\frac{1 - \tau_m/T_m}{\tau_m/T_m}} \sin \sqrt{\left(1 - \frac{\tau_m}{T_m}\right) \frac{\tau_m}{T_m}} \right],$$

$$T_i^{(605)} = \frac{T_m}{\left[\sqrt{\frac{1 - \tau_m/T_m}{\tau_m/T_m}} \tan(0.75\phi) \right]}, \quad T_d^{(605)} = T_m \sqrt{\frac{\tau_m/T_m}{1 - \tau_m/T_m}} \tan(0.75\phi),$$

$$\phi = \tan^{-1} \sqrt{\left(1 - \frac{\tau_m}{T_m}\right) \frac{T_m}{\tau_m}} - \sqrt{\frac{\tau_m}{T_m} \left(1 - \frac{\tau_m}{T_m}\right)}.$$

Rule	K_c	T_i	T_d	Comment
Prashanti and Chidambararam (2000) - Alternative tuning 1. <i>Model:</i> <i>Method I;</i> $K_c^{(605)}$, $T_i^{(605)}$, $T_d^{(605)}$ specified on page 410.	$K_c^{(605)}$	$T_i^{(605)}$	$T_d^{(605)}$	$0.05 \leq \tau_m/T_m \leq 0.90$
	Representative results ²			
	$F_p = 0.309$	$F_i = 1$	$F_d = 2.834$	$\tau_m/T_m = 0.05$
	$F_p = 0.281$	$F_i = 1$	$F_d = 2.605$	$\tau_m/T_m = 0.10$
	$F_p = 0.244$	$F_i = 1$	$F_d = 2.413$	$\tau_m/T_m = 0.20$
	$F_p = 0.213$	$F_i = 1$	$F_d = 2.346$	$\tau_m/T_m = 0.30$
	$F_p = 0.182$	$F_i = 1$	$F_d = 2.336$	$\tau_m/T_m = 0.40$
	$F_p = 0.150$	$F_i = 1$	$F_d = 2.207$	$\tau_m/T_m = 0.50$
	$F_p = 0.116$	$F_i = 1$	$F_d = 2.093$	$\tau_m/T_m = 0.60$
	$F_p = 0.080$	$F_i = 1$	$F_d = 1.884$	$\tau_m/T_m = 0.70$
	$F_p = 0.040$	$F_i = 1$	$F_d = 1.508$	$\tau_m/T_m = 0.80$
	$F_p = 0.0091$	$F_i = 1$	$F_d = 1.159$	$\tau_m/T_m = 0.90$
Prashanti and Chidambararam (2000) - Alternative tuning 2. <i>Model:</i> <i>Method I;</i> $K_c^{(605)}$, $T_i^{(605)}$, $T_d^{(605)}$ specified on page 410.	$K_c^{(605)}$	$T_i^{(605)}$	$T_d^{(605)}$	
	$F_p = 0.9726 e^{\frac{-6.405\tau_m}{T_m}}$, $F_i = 1$, $F_d = 0$; $0.05 \leq \frac{\tau_m}{T_m} \leq 0.90$.			
	Representative results			
	$F_p = 0.777$, $\tau_m/T_m = 0.05$	$F_p = 0.046$, $\tau_m/T_m = 0.50$		
	$F_p = 0.457$, $\tau_m/T_m = 0.10$	$F_p = 0.026$, $\tau_m/T_m = 0.60$		
	$F_p = 0.227$, $\tau_m/T_m = 0.20$	$F_p = 0.013$, $\tau_m/T_m = 0.70$		
	$F_p = 0.121$, $\tau_m/T_m = 0.30$	$F_p = 0.0057$, $\tau_m/T_m = 0.80$		
	$F_p = 0.078$, $\tau_m/T_m = 0.40$	$F_p = 0.0022$, $\tau_m/T_m = 0.90$		

² $F_p = 0.3127 - 0.3369 \frac{\tau_m}{T_m} + 0.0378 \left(\frac{\tau_m}{T_m} \right)^2 - 0.045 \left(\frac{\tau_m}{T_m} \right)^3$, $F_i = 1$,

$$F_d = 2.779 - 2.44 \frac{\tau_m}{T_m} + 5 \left(\frac{\tau_m}{T_m} \right)^2 - 4.81 \left(\frac{\tau_m}{T_m} \right)^3$$
.

Rule	K_c	T_i	T_d	Comment
Prashanti and Chidambaram (2000). Model: Method I	${}^3 K_c^{(606)}$	$T_i^{(606)}$	$\alpha T_i^{(606)}$	
	Representative results			
	$F_p = 0.542$	$F_i = 1$	$F_d = 2.916$	$\tau_m/T_m = 0.05$
	$F_p = 0.437$	$F_i = 1$	$F_d = 2.878$	$\tau_m/T_m = 0.10$
	$F_p = 0.366$	$F_i = 1$	$F_d = 2.525$	$\tau_m/T_m = 0.20$
	$F_p = 0.303$	$F_i = 1$	$F_d = 2.307$	$\tau_m/T_m = 0.30$
	$F_p = 0.263$	$F_i = 1$	$F_d = 1.948$	$\tau_m/T_m = 0.40$
	$F_p = 0.214$	$F_i = 1$	$F_d = 1.729$	$\tau_m/T_m = 0.50$
	$F_p = 0.164$	$F_i = 1$	$F_d = 1.530$	$\tau_m/T_m = 0.60$
	$F_p = 0.151$	$F_i = 1$	$F_d = 1.297$	$\tau_m/T_m = 0.70$
	$F_p = 0.139$	$F_i = 1$	$F_d = 1.093$	$\tau_m/T_m = 0.80$
	$F_p = 0.113$	$F_i = 1$	$F_d = 0.962$	$\tau_m/T_m = 0.90$
Prashanti and Chidambaram (2000) – alternative tuning. Model: Method I	$F_p = 0.7757 - 1.289 \frac{\tau_m}{T_m} + 0.6267 \left(\frac{\tau_m}{T_m} \right)^2, \quad F_i = 1, \quad F_d = 0;$ $0.05 \leq \frac{\tau_m}{T_m} \leq 0.90$			
	Representative results			
	$F_p = 0.605, \tau_m/T_m = 0.05$	$F_p = 0.322, \tau_m/T_m = 0.50$		

$${}^3 K_c^{(606)} = \frac{1}{K_m} [2.121 - 1.906 \frac{\tau_m}{T_m} + 0.945 \left(\frac{\tau_m}{T_m} \right)^2], \quad T_i^{(606)} = \frac{T_m}{\alpha} \left[0.176 + 0.36 \frac{\tau_m}{T_m} \right];$$

$$\alpha = 0.179 - 0.324 \frac{\tau_m}{T_m} + 0.161 \left(\frac{\tau_m}{T_m} \right)^2, \quad \frac{\tau_m}{T_m} < 0.6 \quad \alpha = 0.04, \quad 0.6 < \frac{\tau_m}{T_m} \leq 0.8;$$

$$\alpha = 0.12 - 0.1 \frac{\tau_m}{T_m}, \quad 0.8 < \frac{\tau_m}{T_m} \leq 1.0;$$

$$F_p = 0.5165 - 0.8482 \frac{\tau_m}{T_m} + 0.5133 \left(\frac{\tau_m}{T_m} \right)^2 - 0.071 \left(\frac{\tau_m}{T_m} \right)^3; \quad F_i = 1;$$

$$F_d = 3.206 - 3.461 \frac{\tau_m}{T_m} + 0.984 \left(\frac{\tau_m}{T_m} \right)^2 + 0.089 \left(\frac{\tau_m}{T_m} \right)^3.$$

Rule	K_c	T_i	T_d	Comment
Representative results (continued)				
Prashanti and Chidambaram (2000) - continued.	$F_p = 0.664, \tau_m/T_m = 0.10$	$F_p = 0.248, \tau_m/T_m = 0.60$		
	$F_p = 0.550, \tau_m/T_m = 0.20$	$F_p = 0.228, \tau_m/T_m = 0.70$		
	$F_p = 0.456, \tau_m/T_m = 0.30$	$F_p = 0.209, \tau_m/T_m = 0.80$		
	$F_p = 0.395, \tau_m/T_m = 0.40$	$F_p = 0.169, \tau_m/T_m = 0.90$		
Jhunjhunwala and Chidambaram (2001).	${}^4 K_c^{(607)}$	$T_i^{(607)}$	$T_d^{(607)}$	Model: Method 1
Robust				
Lee et al. (2000). Model: Method 1	${}^5 K_c^{(608)}$	$T_i^{(608)}$	$T_d^{(608)}$	$0 \leq \frac{\tau_m}{T_m} < 2$
	$F_p = F_i = F_d = \frac{1}{\alpha s + 1}, \alpha = T_m \left[\left(\frac{T_{CL}}{T_m} + 1 \right)^2 e^{\tau_m/T_m} - 1 \right]$			

$${}^4 K_c^{(607)} = \frac{1}{K_m} \left(13.0 - 39.712 \frac{\tau_m}{T_m} \right), \frac{\tau_m}{T_m} < 0.2;$$

$$K_c^{(607)} = \frac{1.397}{K_m} \left(\frac{T_m}{\tau_m} \right)^{0.769}, 0.2 \leq \frac{\tau_m}{T_m} \leq 1.4;$$

$$T_i^{(607)} = 0.856 T_m e^{-\frac{\tau_m}{T_m}}, 0 \leq \frac{\tau_m}{T_m} \leq 1; T_i^{(607)} = 0.3444 T_m e^{-\frac{\tau_m}{T_m}}, 1 \leq \frac{\tau_m}{T_m} \leq 1.4;$$

$$T_d^{(607)} = T_m \left(0.5643 \frac{\tau_m}{T_m} + 0.0075 \right), 0 \leq \frac{\tau_m}{T_m} \leq 1.4;$$

$$F_i = 0; F_d = 0; F_p = 0.6863, \frac{\tau_m}{T_m} = 0.1; F_p = 0.4772 e^{-\frac{\tau_m}{T_m}}, 0.2 \leq \frac{\tau_m}{T_m} \leq 1.$$

$${}^5 K_c^{(608)} = \frac{T_i^{(608)}}{-K_m(2T_{CL} + \tau_m - \alpha)}, \alpha = T_m \left[\left(\frac{T_{CL}}{T_m} + 1 \right)^2 e^{\tau_m/T_m} - 1 \right];$$

$$T_i^{(608)} = -T_m + \alpha - \frac{T_{CL}^2 + \alpha \tau_m - 0.5 \tau_m^2}{2T_{CL} + \tau_m - \alpha},$$

$$T_d^{(608)} = \frac{-T_m \alpha - \frac{\tau_m^2 (0.167 \tau_m - 0.5 \alpha)}{2T_{CL} + \tau_m - \alpha}}{T_i^{(608)}} - \frac{T_{CL}^2 + \alpha \tau_m - 0.5 \tau_m^2}{2T_{CL} + \tau_m - \alpha}.$$

4.11.4 Classical controller I $G_c(s) = K_c \left(1 + \frac{1}{T_i s} \right) \left(\frac{1 + T_d s}{1 + \frac{T_d s}{N}} \right)$

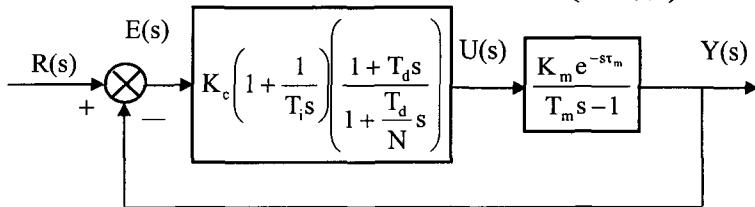


Table 159: PID controller tuning rules – unstable FOLPD model $G_m(s) = \frac{K_m e^{-sr_m}}{T_m s - 1}$

Rule	K_c	T_i	T_d	Comment
Minimum performance index: regulator tuning				
Shinskey (1988) - minimum IAE - page 381. Method: Model I	$\frac{0.91T_m}{K_m \tau_m}$	$1.70\tau_m$	$0.60\tau_m$	$ \tau_m/T_m = 0.1$
	$\frac{1.00T_m}{K_m \tau_m}$	$1.90\tau_m$	$0.60\tau_m$	$ \tau_m/T_m = 0.2$
N not specified	$\frac{0.89T_m}{K_m \tau_m}$	$2.00\tau_m$	$0.80\tau_m$	$ \tau_m/T_m = 0.5$
	$\frac{0.86T_m}{K_m \tau_m}$	$2.25\tau_m$	$0.90\tau_m$	$ \tau_m/T_m = 0.67$
	$\frac{0.83T_m}{K_m \tau_m}$	$2.40\tau_m$	$1.00\tau_m$	$ \tau_m/T_m = 0.8$

4.11.5 Series controller (classical controller 3)

$$G_c(s) = K_c \left(1 + \frac{1}{T_i s} \right) (1 + s T_d)$$

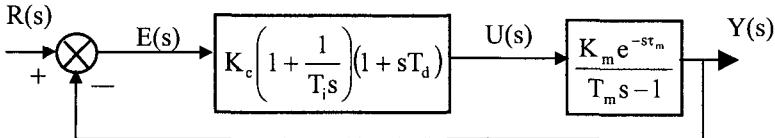


Table 160: PID controller tuning rules – unstable FOLPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{T_m s - 1}$

Rule	K_c	T_i	T_d	Comment
Direct synthesis: time domain criteria				
Huang and Chen (1997), (1999).	⁶ $K_c^{(609)}$	$T_i^{(609)}$	$T_d^{(609)}$	<i>Model: Method 4</i>

⁶ $0 < \frac{\tau_m}{T_m} < 1$; ‘Good’ servo and regulator response.

$$K_c^{(609)} = \frac{0.5}{K_m} \left[1 + 1.61519 \left(\frac{\tau_m}{T_m} \right)^{-1.02379} \right], \quad T_i^{(609)} = \frac{K_c^{(609)} K_m T_m}{K_c^{(609)} K_m - 1} \left(\frac{2x_1}{T_m} + \frac{\tau_m}{T_m} \right),$$

$$\text{with recommended } x_1 = T_m \left[-1.3735 \cdot 10^{-3} + 0.22091 \frac{\tau_m}{T_m} + 1.6653 \left(\frac{\tau_m}{T_m} \right)^2 \right];$$

$$T_d^{(609)} = T_m \left[1.5006 \cdot 10^{-4} + 0.23549 \frac{\tau_m}{T_m} + 0.16970 \left(\frac{\tau_m}{T_m} \right)^2 \right].$$

4.11.6 Non-interacting controller based on the two degree of freedom structure I

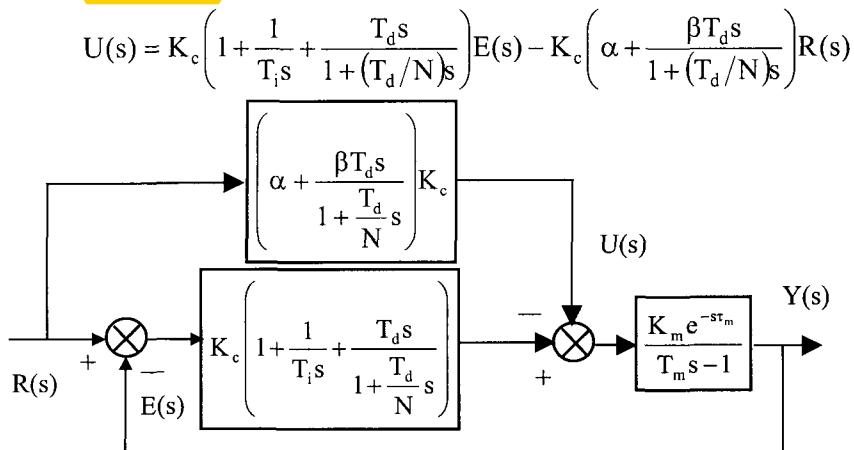


Table 161: PID controller tuning rules – unstable FOLPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{T_m s - 1}$

Rule	K_c	T_i	T_d	Comment
Direct synthesis				
Chidambaram (2000c). Model: Method 1	$\frac{1.165 \left(\frac{T_m}{\tau_m} \right)^{0.245}}{K_m}$	${}^7 T_i^{(610)}$	${}^7 T_d^{(610)}$	$\left \frac{\tau_m}{T_m} \right < 0.6$
		$4.4 T_m + 9 \tau_m$		$0.6 \leq \left \frac{\tau_m}{T_m} \right \leq 0.8$
		${}^8 T_i^{(611)}$		$0.8 < \left \frac{\tau_m}{T_m} \right \leq 0.9$

$${}^7 T_i^{(610)} = \frac{\left(0.176 + 0.36 \frac{\tau_m}{T_m} \right) T_m}{0.179 - 0.324 \frac{\tau_m}{T_m} + 0.161 \left(\frac{\tau_m}{T_m} \right)^2}, \quad {}^7 T_d^{(610)} = \left(0.176 + 0.36 \frac{\tau_m}{T_m} \right) T_m.$$

$${}^8 T_i^{(611)} = \frac{0.176 T_m + 0.36 \tau_m}{0.12 - 0.1 \frac{\tau_m}{T_m}}, \quad \alpha = 0.74 + 0.42 \frac{\tau_m}{T_m} - 0.23 \left(\frac{\tau_m}{T_m} \right)^2.$$

Rule	K_c	T_i	T_d	Comment
Srinivas and Chidambaram (2001). Model: Method 1	⁹ $K_c^{(612)}$	$T_i^{(612)}$	$T_d^{(612)}$	$N = \infty$
Chandrashekhar et al. (2002). Model: Method 1	¹⁰ $K_c^{(613)}$	$T_i^{(613)}$	$0.5\tau_m$	$N = \infty ; \beta = 0$

⁹ Sample $K_c^{(612)}$, $T_i^{(612)}$, $T_d^{(612)}$ taken by the authors correspond to the dominant pole placement tuning rules of Valentine and Chidambaram (1997b) (see page 405);

$$\alpha = \frac{\tau_m}{T_i^{(612)}} + \frac{1}{K_c^{(612)} K_m} ; \quad \beta = \frac{1}{T_d^{(612)}} \left[\tau_m - \frac{T_m}{K_c^{(612)} K_m} - \frac{\tau_m^2}{2T_i^{(612)}} \right].$$

$$\begin{aligned} {}^{10} K_c^{(613)} &= \frac{1}{K_m} \left[4282 \left(\frac{\tau_m}{T_m} \right)^2 - 1334.6 \frac{\tau_m}{T_m} + 101 \right], \quad 0 < \frac{\tau_m}{T_m} < 0.2 ; \\ &\quad K_c^{(613)} = \frac{1.1161}{K_m} \left(\frac{\tau_m}{T_m} \right)^{-0.9427}, \quad 0.2 \leq \frac{\tau_m}{T_m} \leq 1.0 . \end{aligned}$$

$$T_i^{(613)} = T_m \left[36.842 \left(\frac{\tau_m}{T_m} \right)^2 - 10.3 \left(\frac{\tau_m}{T_m} \right) + 0.8288 \right], \quad 0 \leq \frac{\tau_m}{T_m} \leq 0.8 ;$$

$$T_i^{(613)} = 76.241 T_m \left(\frac{\tau_m}{T_m} \right)^{6.77}, \quad 0.8 \leq \frac{\tau_m}{T_m} \leq 1.0 .$$

$$\alpha = 0.83, \quad \frac{\tau_m}{T_m} \leq 0.1 ; \quad \alpha = 0.8053 - 0.1935 \frac{\tau_m}{T_m}, \quad 0.1 \leq \frac{\tau_m}{T_m} \leq 0.7 .$$

$$\text{Desired closed loop transfer function} = \frac{(\eta s + 1)e^{-s\tau_m}}{(T_{CL2}s + 1)^2} ;$$

$$T_{CL2} = \left(0.0326 + 0.4958 \frac{\tau_m}{T_m} + 2.03 \left(\frac{\tau_m}{T_m} \right)^2 \right) T_m, \quad 0 \leq \frac{\tau_m}{T_m} \leq 0.8 ;$$

$$T_{CL2} = 4.5115 T_m \left(\frac{\tau_m}{T_m} \right)^{5.661}, \quad 0.8 \leq \frac{\tau_m}{T_m} \leq 1.0 .$$

Rule	K_c	T_i	T_d	Comment			
Alternative tuning rules							
$K_c = \frac{x_1}{K_m}, T_i = x_2 T_m, T_d = x_3 T_m, T_f = x_4 T_m, T_{CL2} = x_5 T_m$							
Model: Method 1							
Coefficient values							
	x_1	x_2	x_3	x_4	x_5	α	τ_m/T_m
	101.000	0.0404	0.0	0.0	0.02	0.505	0.0
	10.3662	0.3874	0.0435	0.0134	0.10	0.742	0.1
	5.3750	0.8600	0.0884	0.0250	0.20	0.767	0.2
	3.2655	1.8018	0.1375	0.0435	0.40	0.778	0.3
	2.4516	3.0400	0.1868	0.0581	0.60	0.803	0.4
	2.0217	4.6500	0.2366	0.0696	0.80	0.828	0.5
	1.7525	6.7286	0.2866	0.0784	1.00	0.851	0.6
	1.5458	10.337	0.3380	0.0885	1.30	0.874	0.7
	1.3723	17.693	0.3910	0.1005	1.80	0.898	0.8
	1.2420	33.610	0.4440	0.1124	2.60	0.923	0.9
	1.1400	80.620	0.4969	0.1247	4.20	0.948	1.0
Wang and Cai (2002). Model: Method 1	$^{11}K_c^{(614)}$	$T_i^{(614)}$	$T_d^{(614)}$	$N=0; \beta = 0$			

$$^{11}K_c^{(614)} = \frac{0.5236T_m}{K_m\tau_m} - \frac{0.4764}{K_m}\sqrt{\frac{T_m}{\tau_m}}, \quad T_i^{(614)} = \frac{0.5236T_m - 0.4764\sqrt{T_m\tau_m}}{0.5236\left(\sqrt{\frac{T_m}{\tau_m}} - 1\right)},$$

$$T_d^{(614)} = \frac{0.2618\tau_m\sqrt{T_m\tau_m}}{0.5236T_m - 0.4764\sqrt{T_m\tau_m}}, \quad \alpha = \frac{1.9099\sqrt{\frac{\tau_m}{T_m}}}{1 + 0.9099\sqrt{\frac{\tau_m}{T_m}}} \quad (A_m = 3, \phi_m = 60^\circ).$$

Rule	K_c	T_i	T_d	Comment
Xu and Shao (2003b). Model: Method I	$^{12} K_c^{(615)}$	$T_i^{(615)}$	$T_d^{(615)}$	$N=0; \beta = 0$
Sree and Chidambaram (2005a). Model: Method I	$^{13} K_c^{(616)}$	$T_i^{(616)}$	$T_d^{(616)}$	$N=\infty; \beta = 1$

$$^{12} K_c^{(615)} = \frac{0.5}{K_m} \left(\frac{T_m}{\tau_m} + \sqrt{\frac{T_m}{\tau_m}} \right), \quad T_i^{(615)} = \frac{\tau_m \left(\frac{T_m}{\tau_m} + \sqrt{\frac{T_m}{\tau_m}} \right)}{\sqrt{\frac{T_m}{\tau_m}} - 1},$$

$$T_d^{(615)} = \frac{0.5 \sqrt{T_m \tau_m}}{\left(\frac{T_m}{\tau_m} + \sqrt{\frac{T_m}{\tau_m}} \right)}, \quad \alpha = \frac{2 \sqrt{T_m \tau_m}}{T_m + \sqrt{T_m \tau_m}} \quad (A_m > 2, \phi_m > 60^\circ).$$

¹³ Sample $K_c^{(616)}$, $T_i^{(616)}$, $T_d^{(616)}$ taken by the authors correspond to the tuning rules of Chandrashekhar *et al.* (2002) and Huang and Chen (1999); the latter tuning rule is defined for an unstable SOSPD process model.

$$\text{Desired closed loop transfer function} = \frac{(1-\alpha)T_i^{(616)}s + 1}{T_{CL}^2 s^2 + 2\xi T_{CL}s + 1} e^{-s\tau_m}.$$

The optimum value of α is chosen by minimizing the ISE (servo response). Then,

$$\alpha = 1 - \frac{2T_{CL}}{T_i^{(616)}}, \quad \xi = 1; \quad \alpha = 1 - \frac{2T_{CL}\xi}{T_i^{(616)}}, \quad \xi < 1;$$

$$\alpha = 1 - \frac{T_{CL}}{T_i^{(616)}} \left[\frac{\left(-\xi + \sqrt{\xi^2 - 1} \right) \left(-\xi - \sqrt{\xi^2 - 1} \right)^2 - \left(-\xi + \sqrt{\xi^2 - 1} \right)^3 + x_1}{\left(-\xi + \sqrt{\xi^2 - 1} \right) \left(-\xi - \sqrt{\xi^2 - 1} \right)^3 + x_2} \right], \quad \xi > 1$$

$$\text{with } x_1 = -\left(-\xi - \sqrt{\xi^2 - 1} \right)^3 + \left(-\xi + \sqrt{\xi^2 - 1} \right)^2 \left(-\xi - \sqrt{\xi^2 - 1} \right) \text{ and}$$

$$x_2 = \left(-\xi - \sqrt{\xi^2 - 1} \right) \left(-\xi + \sqrt{\xi^2 - 1} \right)^3 - 2 \left(-\xi + \sqrt{\xi^2 - 1} \right)^2 \left(-\xi - \sqrt{\xi^2 - 1} \right)^2.$$

If it is desired to minimize the overshoot (servo response), for $\xi \geq 1$, then $\alpha = 1$.

4.11.7 Non-interacting controller 3

$$U(s) = K_c \left(1 + \frac{1}{T_i s} \right) E(s) - \frac{K_c T_d s}{1 + \frac{s T_d}{N}} Y(s)$$

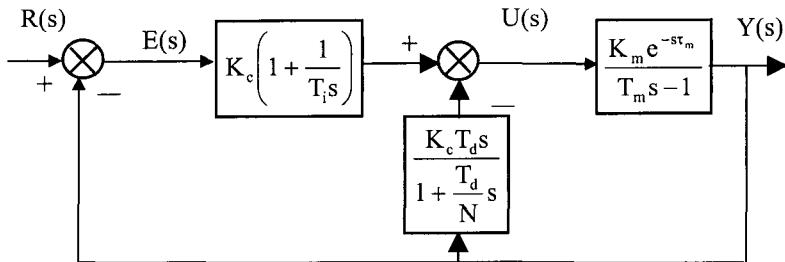


Table 162: PID controller tuning rules – unstable FOLPD model $\frac{K_m e^{-sT_m}}{T_m s - 1}$

Rule	K_c	T_i	T_d	Comment
Minimum performance index				
Huang and Lin (1995) - minimum IAE. Model: Method 2	¹ $K_c^{(617)}$	$T_i^{(617)}$	$T_d^{(617)}$	$0.01 \leq \frac{\tau_m}{ T_m } \leq 0.8 ; N=10$
	² $K_c^{(618)}$	$T_i^{(618)}$	$T_d^{(618)}$	

$$^1 \text{ Servo response: } K_c^{(617)} = -\frac{1}{K_m} \left(-0.433 + 0.2056 \frac{\tau_m}{T_m} + 0.3135 \left[\frac{\tau_m}{T_m} \right]^{-1.0251} \right),$$

$$T_i^{(617)} = T_m \left(-0.0018 + 0.8193 \frac{\tau_m}{T_m} + 7.7853 \left[\frac{\tau_m}{T_m} \right]^{6.6423} \right),$$

$$T_d^{(617)} = T_m \left(-0.0312 + 1.6333 \frac{\tau_m}{T_m} + 0.0399 e^{7.6983 \frac{\tau_m}{T_m}} \right).$$

$$^2 \text{ Regulator response: } K_c^{(618)} = -\frac{1}{K_m} \left(0.2675 + 0.1226 \frac{\tau_m}{T_m} + 0.8781 \left[\frac{\tau_m}{T_m} \right]^{-1.004} \right),$$

$$T_i^{(618)} = T_m \left(0.0005 + 2.4631 \frac{\tau_m}{T_m} + 9.5795 \left[\frac{\tau_m}{T_m} \right]^{2.9123} \right), \quad T_d^{(618)} = 0.0011 T_m + 0.4759 \tau_m.$$

4.11.8 Non-interacting controller 8

$$U(s) = K_c \left(1 + \frac{1}{T_i s} + T_d s \right) E(s) - K_i (1 + T_{di} s) Y(s)$$

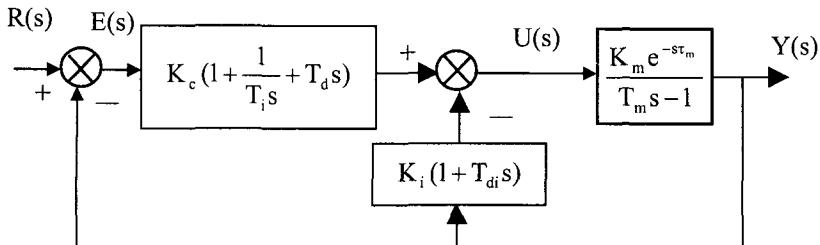


Table 163: PID controller tuning rules – unstable FOLPD model $G_m(s) = \frac{K_m e^{-s\tau_m}}{T_m s - 1}$

Rule	K_c	T_i	T_d	Comment
Robust				
Park <i>et al.</i> . (1998). <i>Model: Method I</i>	³ $K_c^{(619)}$	$T_i^{(619)}$	$T_d^{(619)}$	$K_i = \frac{1}{K_m} \sqrt{\frac{T_m}{\tau_m}}$ $T_{di} = 0$

³ Apply minimum ITAE – regulator tuning rule or minimum ITAE – servo tuning rule defined by Sung *et al.* (1996) for SOSPD model, ideal PID controller (pages 313, 316). For these formulae, K_m , T_{m1} and ξ_m are replaced by the following parameters from the unstable FOLPD model:

$$\frac{K_m}{\sqrt{\frac{T_m}{\tau_m} - 1}} \quad (\text{Unstable FOLPD}) \rightarrow K_m \text{ (SOSPD)}$$

$$\frac{0.71 T_m^{0.25} \tau_m^{0.75}}{\sqrt{\left(\frac{T_m}{\tau_m}\right)^{0.5} - 1}} \quad (\text{Unstable FOLPD}) \rightarrow T_{m1} \text{ (SOSPD)}$$

$$\frac{0.71 |T_m^{0.5} - \tau_m^{0.5}| T_m^{0.25} \tau_m^{-0.75}}{\sqrt{\left(\frac{T_m}{\tau_m}\right)^{0.5} - 1}} \quad (\text{Unstable FOLPD}) \rightarrow \xi_m \text{ (SOSPD)}$$

Rule	K_c	T_i	T_d	Comment
Lee and Edgar (2002). Model: Method 1	$^4 K_c^{(620)}$	$T_i^{(620)}$	$T_d^{(620)}$	$T_{di} = 0$

$$\begin{aligned}
 ^4 K_c^{(620)} &= \frac{K_i^{(620)} K_m - 1}{K_m (\lambda + \tau_m)} \left[\frac{T_m - K_i^{(620)} K_m \tau_m}{K_i^{(620)} K_m - 1} + \frac{\tau_m^2}{2(\lambda + \tau_m)} \right], \\
 T_i^{(620)} &= \frac{T_m - K_i^{(620)} K_m \tau_m}{K_i^{(620)} K_m - 1} + \frac{\tau_m^2}{2(\lambda + \tau_m)}, \\
 T_d^{(620)} &= \frac{K_i^{(620)} K_m - 1}{K_m (\lambda + \tau_m)} \left\{ \frac{K_i^{(620)} K_m \tau_m^2}{2(K_i^{(620)} K_m - 1)} - \frac{\tau_m^3}{6(\lambda + \tau_m)} + \frac{\tau_m^4}{4(\lambda + \tau_m)^2} \right. \\
 &\quad \left. + \frac{(T_m - K_i^{(620)} K_m \tau_m) \tau_m^2}{2(K_i^{(620)} K_m - 1)(\lambda + \tau_m)} \right\}, \\
 K_i^{(620)} &= \frac{1}{K_m} \left[\cos \sqrt{\left(1 - \frac{\tau_m}{T_m}\right) \frac{\tau_m}{T_m}} + \sqrt{\frac{T_m}{\tau_m} \left(1 - \frac{\tau_m}{T_m}\right)} \sin \sqrt{\left(1 - \frac{\tau_m}{T_m}\right) \frac{\tau_m}{T_m}} \right].
 \end{aligned}$$

4.11.9 Non-interacting controller 10

$$U(s) = K_c \left(1 + \frac{1}{T_i s} \right) E(s) - K_f \left(\frac{T_d}{T_m} s + 1 \right) Y(s)$$

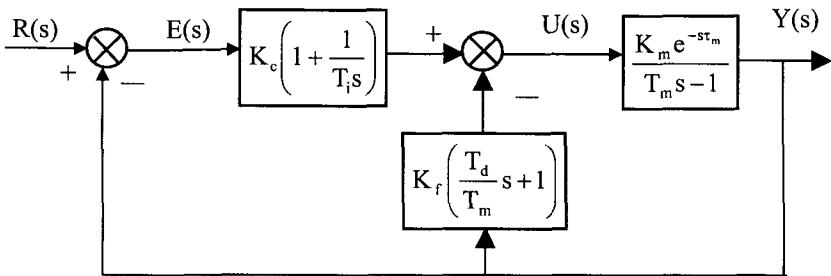


Table 164: PID tuning rules – unstable FOLPD model $G_m(s) = \frac{K_m e^{-st_m}}{T_m s - 1}$

Rule	K_c	T_i	T_d	Comment
Minimum performance index: servo tuning				
Majhi and Atherton (2000) - minimum ITSE	${}^5 K_c^{(621)}$	$T_i^{(621)}$	$0.5\tau_m$	Model: Method I
Majhi and Atherton (2000) - minimum ITSE. Model: Method 3	${}^6 K_c^{(622)}$	$T_i^{(622)}$	$\frac{\ln(1+K)}{4 \tanh^{-1} K} T_m$	$0 < \frac{\tau_m}{T_m} < 0.693$

$${}^5 K_c^{(621)} = \frac{0.8011 T_m \left(1 - 0.9358 \sqrt{\frac{\tau_m}{T_m}} \right)}{K_m \tau_m},$$

$$T_i^{(621)} = T_m \left(0.1227 + 1.4550 \frac{\tau_m}{T_m} - 1.2711 \frac{\tau_m^2}{T_m^2} \right), \quad K_f = \frac{1}{K_m} \sqrt{\frac{2 T_m}{\tau_m}}.$$

$${}^6 K_c^{(622)} = \frac{0.8011 \left(1 - 0.9358 \sqrt{\ln(1+K)} \right)}{K_m \ln(1+K)}, \quad K_f = \frac{1}{K_m} \sqrt{\frac{2}{\ln(1+K)}}, \quad K = A_p / (K_m h),$$

$$T_i^{(622)} = \frac{0.1227 + 1.4550 \ln(1+K) - 1.2711 [\ln(1+K)]^2}{2 \tanh^{-1} K} T_m.$$

Rule	K_c	T_i	T_d	Comment
Majhi and Atherton (2000) – continued.	$^7 K_c^{(623)}$	$T_i^{(623)}$	$T_d^{(623)}$	$0.693 < \frac{\tau_m}{T_m} < 1$

$$^7 K_c^{(623)} = \frac{1}{K_m} \left[\frac{0.8011(K + 0.9946)}{K + 0.0682} - \frac{0.7497\sqrt{K + 0.9946}}{\sqrt{K + 0.0682}} \right],$$

$$T_i^{(623)} = \frac{0.0145K^3 + 0.5773K^2 + 2.6554K + 0.3488}{K^3 + 5.2158K^2 + 4.4816K + 0.2817} T_m,$$

$$T_d^{(623)} = \frac{0.0237(K + 34.5338)}{K + 4.1530} T_m, \quad K_f = \frac{1}{K} \sqrt{\frac{2(K + 0.9946)}{K + 0.0682}}.$$

4.11.10 Non-interacting controller 12

$$U(s) = K_c \left(1 + \frac{1}{T_i s} + T_d s \right) \frac{1}{T_d s + 1} E(s) - K_1 Y(s)$$

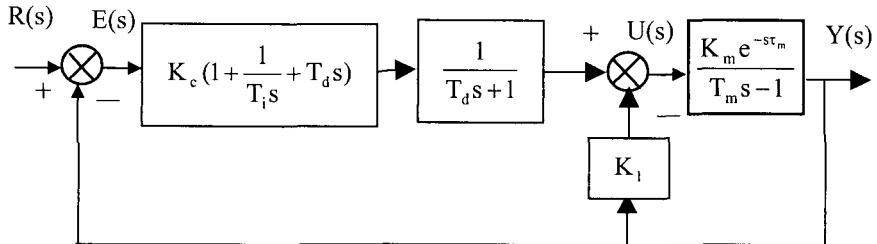


Table 165: PID controller tuning rules – unstable FOLPD model $G_m(s) = \frac{K_m e^{-st_m}}{T_m s - 1}$

Rule	K_c	T_i	T_d	Comment
Robust				
Lee and Edgar (2002). <i>Model: Method 1</i>	${}^8 K_c^{(624)}$	$T_i^{(624)}$	$T_d^{(624)}$	

$$\begin{aligned}
 {}^8 K_c^{(624)} &= \frac{K_1^{(624)} K_m - 1}{K_m (\lambda + \tau_m)} \left\{ \frac{T_m - K_1^{(624)} K_m \tau_m}{K_1^{(624)} K_m - 1} + \frac{\tau_m^2}{2(\lambda + \tau_m)} + \right. \\
 &\quad \left. \tau_m \left[\frac{K_1^{(624)} K_m}{2(K_1^{(624)} K_m - 1)} - \frac{\tau_m}{6(\lambda + \tau_m)} + \frac{\tau_m^2}{4(\lambda + \tau_m)^2} + \frac{(T_m - K_1^{(624)} K_m \tau_m)}{2(K_1^{(624)} K_m - 1)(\lambda + \tau_m)} \right]^{0.5} \right\}; \\
 T_i^{(624)} &= \frac{T_m - K_1^{(624)} K_m \tau_m}{K_1^{(624)} K_m - 1} + \frac{\tau_m^2}{2(\lambda + \tau_m)} + \\
 &\quad \tau_m \left[\frac{K_1^{(624)} K_m}{2(K_1^{(624)} K_m - 1)} - \frac{\tau_m}{6(\lambda + \tau_m)} + \frac{\tau_m^2}{4(\lambda + \tau_m)^2} + \frac{(T_m - K_1^{(624)} K_m \tau_m)}{2(K_1^{(624)} K_m - 1)(\lambda + \tau_m)} \right]^{0.5}; \\
 T_d^{(624)} &= \\
 &\quad \tau_m \left[\frac{K_1^{(624)} K_m}{2(K_1^{(624)} K_m - 1)} - \frac{\tau_m}{6(\lambda + \tau_m)} + \frac{\tau_m^2}{4(\lambda + \tau_m)^2} + \frac{(T_m - K_1^{(624)} K_m \tau_m)}{2(K_1^{(624)} K_m - 1)(\lambda + \tau_m)} \right]^{0.5}; \\
 K_1^{(624)} &= \frac{1}{K_m} \left[\cos \sqrt{\left(1 - \frac{\tau_m}{T_m} \right) \frac{\tau_m}{T_m}} + \sqrt{\frac{T_m}{\tau_m} \left(1 - \frac{\tau_m}{T_m} \right)} \sin \sqrt{\left(1 - \frac{\tau_m}{T_m} \right) \frac{\tau_m}{T_m}} \right].
 \end{aligned}$$

Rule	K_c	T_i	T_d	Comment
Lee and Edgar (2002). Model: Method I	$^9 K_c^{(625)}$	$T_i^{(625)}$	$T_d^{(625)}$	$K_1 = 0.25K_u$

$$\begin{aligned}
 ^9 K_c^{(625)} &= \frac{1 + 0.25K_u K_m}{K_m(\lambda + \tau_m)} \left\{ \frac{T_m - 0.25K_u K_m \tau_m}{1 + 0.25K_u K_m} + \frac{\tau_m^2}{2(\lambda + \tau_m)} + \right. \\
 &\quad \left. \tau_m \left[\frac{0.25K_u K_m}{2(1 + 0.25K_u K_m)} - \frac{\tau_m}{6(\lambda + \tau_m)} + \frac{\tau_m^2}{4(\lambda + \tau_m)^2} + \frac{(T_m - 0.25K_u K_m \tau_m) \tau_m^2}{2(1 + 0.25K_u K_m)(\lambda + \tau_m)} \right]^{0.5} \right\}; \\
 T_i^{(625)} &= \frac{T_m - 0.25K_u K_m \tau_m}{1 + 0.25K_u K_m} + \frac{\tau_m^2}{2(\lambda + \tau_m)} + \\
 &\quad \tau_m \left[\frac{0.25K_u K_m}{2(1 + 0.25K_u K_m)} - \frac{\tau_m}{6(\lambda + \tau_m)} + \frac{\tau_m^2}{4(\lambda + \tau_m)^2} + \frac{(T_m - 0.25K_u K_m \tau_m) \tau_m^2}{2(1 + 0.25K_u K_m)(\lambda + \tau_m)} \right]^{0.5}; \\
 T_d^{(625)} &= \\
 &\quad \tau_m \left[\frac{0.25K_u K_m}{2(1 + 0.25K_u K_m)} - \frac{\tau_m}{6(\lambda + \tau_m)} + \frac{\tau_m^2}{4(\lambda + \tau_m)^2} + \frac{(T_m - 0.25K_u K_m \tau_m) \tau_m^2}{2(1 + 0.25K_u K_m)(\lambda + \tau_m)} \right]^{0.5}.
 \end{aligned}$$

4.12 Unstable SOSPD Model (one unstable pole)

$$G_m(s) = \frac{K_m e^{-s\tau_m}}{(T_{m1}s - 1)(1 + sT_{m2})}$$

4.12.1 Ideal controller $G_c(s) = K_c \left(1 + \frac{1}{T_i s} + T_d s \right)$

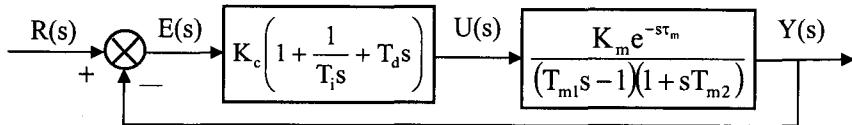


Table 166: PID controller tuning rules – unstable SOSPD model (one unstable pole)

$$G_m(s) = \frac{K_m e^{-s\tau_m}}{(T_{m1}s - 1)(1 + sT_{m2})}$$

Rule	K_c	T_i	T_d	Comment
Direct synthesis				
Wang and Jin (2004).	¹ $K_c^{(626)}$	$T_i^{(626)}$	$T_d^{(626)}$	<i>Model: Method 1</i>

$$^1 K_c^{(626)} = \frac{K_m T_i^{(626)} (T_{m1} - \tau_m)(T_{m2} + \tau_m)}{(T_{CL3} + \tau_m)^3},$$

$$T_i^{(626)} = \frac{T_{CL3}^3 + 3\tau_m T_{CL3}^2 + 3(T_{m1}T_{m2} + \tau_m T_{m1} - \tau_m T_{m2})T_{CL3} + \tau_m T_{m2}(T_{m1} - \tau_m)}{(T_{m1} - \tau_m)(T_{m2} + \tau_m)},$$

$$T_d^{(626)} = \frac{(T_{m2} + \tau_m - T_{m1})T_{CL3}^3 + 3T_{m1}T_{m2}T_{CL3}(T_{CL3} + \tau_m) + T_{m1}T_{m2}\tau_m^2}{T_i^{(626)}(T_{m1} - \tau_m)(T_{m2} + \tau_m)}.$$

Rule	K_c	T_i	T_d	Comment
Robust				
Rotstein and Lewin (1991). <i>Model: Method 1</i>	${}^2 K_c^{(627)}$	$T_i^{(627)}$	$T_d^{(627)}$	
	λ determined graphically – sample values provided			
	$\tau_m/T_m = 0.2, \lambda \in [0.5T_m, 1.9T_m]$			K_m uncertainty = 50%
	$\tau_m/T_m = 0.4, \lambda \in [1.3T_m, 1.9T_m]$			
	$\tau_m/T_m = 0.2, \lambda \in [0.4T_m, 4.3T_m]$			
	$\tau_m/T_m = 0.4, \lambda \in [1.1T_m, 4.3T_m]$			K_m uncertainty = 30%
$\tau_m/T_m = 0.6, \lambda \in [2.2T_m, 4.3T_m]$				
Lee <i>et al.</i> (2000). <i>Model: Method 1</i>	${}^3 K_c^{(628)}$	$T_i^{(628)}$	$T_d^{(628)}$	$0 \leq \frac{\tau_m}{T_m} < 2$

$${}^2 K_c^{(627)} = \frac{T_{m1} \left[\lambda \left(\frac{\lambda}{T_{m1}} + 2 \right) + T_{m2} \right]}{\lambda^2 K_m}, \quad T_i^{(627)} = \lambda \left(\frac{\lambda}{T_{m1}} + 2 \right) + T_{m2},$$

$$T_d^{(627)} = \frac{\lambda \left(\frac{\lambda}{T_{m1}} + 2 \right) T_{m2}}{\lambda \left(\frac{\lambda}{T_{m1}} + 2 \right) + T_{m2}}.$$

$${}^3 K_c^{(628)} = \frac{T_i^{(628)}}{-K_m(2\lambda + \tau_m - \alpha)}, \quad \lambda = \text{desired closed loop dynamic parameter},$$

$$\alpha = T_{m1} \left[\left(\frac{\lambda}{T_{m1}} + 1 \right)^2 e^{\tau_m/T_{m1}} - 1 \right];$$

$$T_i^{(628)} = -T_{m1} + T_{m2} + \alpha - \frac{\lambda^2 + \alpha\tau_m - 0.5\tau_m^2}{2\lambda + \tau_m - \alpha},$$

$$T_d^{(628)} = \frac{-T_{m1}\alpha + T_{m2}\alpha - T_{m1}T_{m2} - \frac{\tau_m^2(0.167\tau_m - 0.5\alpha)}{2\lambda + \tau_m - \alpha}}{T_i^{(628)}} - \frac{\lambda^2 + \alpha\tau_m - 0.5\tau_m^2}{2\lambda + \tau_m - \alpha}.$$

Rule	K_c	T_i	T_d	Comment
Ultimate cycle				
McMillan (1984). Model: Method 1	$^4 K_c^{(629)}$	$T_i^{(629)}$	$T_d^{(629)}$	Tuning rules developed from K_u, T_u

$$\begin{aligned}
 ^4 K_c^{(629)} &= \frac{1.111}{K_m} \frac{T_{m1} T_{m2}}{\tau_m^2} \left\{ \frac{1}{1 + \left[\frac{(T_{m1} + T_{m2}) T_{m1} T_{m2}}{(T_{m1} - T_{m2})(T_{m1} - \tau_m) \tau_m} \right]^{0.65}} \right\}^2, \\
 T_i^{(629)} &= 2\tau_m \left\{ 1 + \left[\frac{(T_{m1} + T_{m2}) T_{m1} T_{m2}}{(T_{m1} - T_{m2})(T_{m1} - \tau_m) \tau_m} \right]^{0.65} \right\}, \\
 T_d^{(629)} &= 0.5\tau_m \left\{ 1 + \left[\frac{(T_{m1} + T_{m2}) T_{m1} T_{m2}}{(T_{m1} - T_{m2})(T_{m1} - \tau_m) \tau_m} \right]^{0.65} \right\}.
 \end{aligned}$$

4.12.2 Ideal controller in series with a first order lag

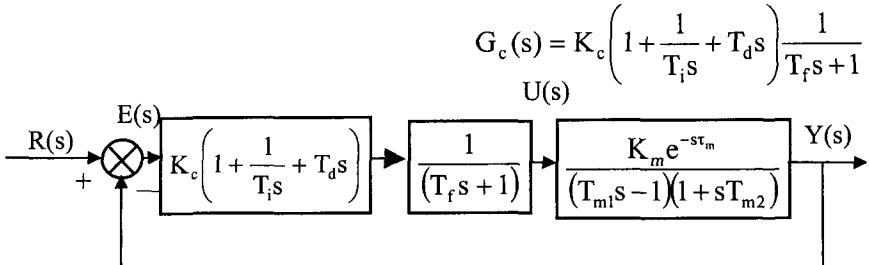


Table 167: PID controller tuning rules – unstable SOSPD model (one unstable pole)

$$G_m(s) = \frac{K_m e^{-s\tau_m}}{(T_{m1}s - 1)(1 + sT_{m2})}$$

Rule	K_c	T_i	T_d	Comment
Robust				
Zhang and Xu (2002). <i>Model: Method 1</i>	${}^5 K_c^{(630)}$	$T_i^{(630)}$	$T_d^{(630)}$	$\tau_m < T_{m1}$

$${}^5 K_c^{(630)} = \frac{T_{m2} T_{m1} (T_{m1} - \tau_m) + \lambda^3 + 3\lambda^2 T_{m1} + 3\lambda T_{m1}^2 + \tau_m T_{m1}^2}{K_m (\lambda^3 + 3\lambda^2 T_{m1} + 3\lambda T_{m1} \tau_m + \tau_m^2 T_{m1})},$$

$$T_i^{(630)} = T_{m2} + \frac{\lambda^3 + 3\lambda^2 T_{m1} + 3\lambda T_{m1}^2 + \tau_m T_{m1}^2}{T_{m1} (T_{m1} - \tau_m)},$$

$$T_d^{(630)} = \frac{T_{m2} (\lambda^3 + 3\lambda^2 T_{m1} + 3\lambda T_{m1}^2 + \tau_m T_{m1}^2)}{T_{m1} T_{m2} (T_{m1} - \tau_m) + \lambda^3 + 3\lambda^2 T_{m1} + 3\lambda T_{m1}^2 + \tau_m T_{m1}^2},$$

$$T_f^{(630)} = \frac{\lambda^3 T_{m1} (T_{m1} - \tau_m)}{T_{m1} (\lambda^3 + 3\lambda^2 T_{m1} + 3\lambda T_{m1} \tau_m + \tau_m^2 T_{m1})};$$

$$\text{for } \frac{\tau_m}{T_{m1}} \leq 0.5, \lambda = 2 - 5(\tau_m + T_{m2}).$$

4.12.3 Ideal controller with set-point weighting 1

$$U(s) = K_c \left(F_p R(s) - Y(s) \right) + \frac{K_c}{T_i s} \left(F_i R(s) - Y(s) \right) + K_c T_d s \left(F_d R(s) - Y(s) \right)$$

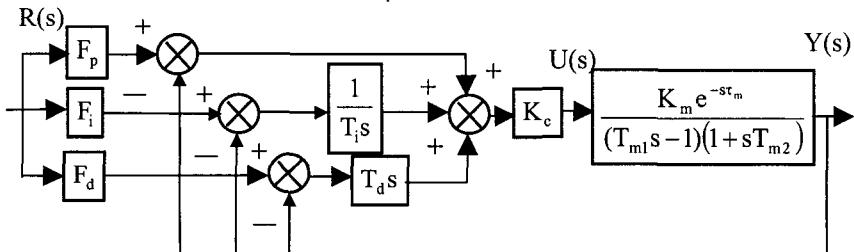


Table 168: PID controller tuning rules – unstable SOSPD model (one unstable pole)

$$G_m(s) = \frac{K_m e^{-s\tau_m}}{(T_{m1}s - 1)(1 + sT_{m2})}$$

Rule	K_c	T_i	T_d	Comment
Robust				
Lee <i>et al.</i> (2000). Model: Method 1	${}^6 K_c^{(631)}$	$T_i^{(631)}$	$T_d^{(631)}$	$0 \leq \frac{\tau_m}{T_m} < 2$

$$F_p = F_i = F_d = \frac{1}{\alpha s + 1}, \quad \alpha = T_{m1} \left[\left(\frac{\lambda}{T_{m1}} + 1 \right)^2 e^{\tau_m/T_{m1}} - 1 \right]$$

$${}^6 K_c^{(631)} = \frac{T_i^{(631)}}{-K_m(2\lambda + \tau_m - \alpha)}, \quad \lambda = \text{desired closed loop dynamic parameter},$$

$$T_i^{(631)} = -T_{m1} + T_{m2} + \alpha - \frac{\lambda^2 + \alpha\tau_m - 0.5\tau_m^2}{2\lambda + \tau_m - \alpha},$$

$$T_d^{(631)} = \frac{-T_{m1}\alpha + T_{m2}\alpha - T_{m1}T_{m2} - \frac{\tau_m^2(0.167\tau_m - 0.5\alpha)}{2\lambda + \tau_m - \alpha}}{T_i^{(631)}} - \frac{\lambda^2 + \alpha\tau_m - 0.5\tau_m^2}{2\lambda + \tau_m - \alpha}.$$

4.12.4 Classical controller 1 $G_c(s) = K_c \left(1 + \frac{1}{T_i s} \right) \left(\frac{1 + T_d s}{1 + \frac{T_d}{N} s} \right)$

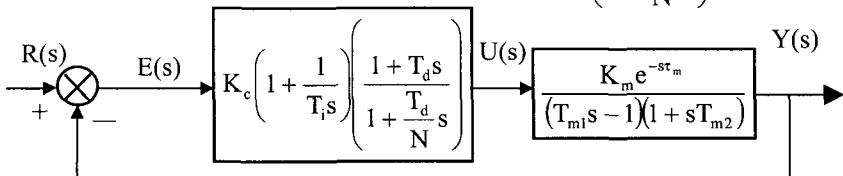


Table 169: PID controller tuning rules – unstable SOSPD model (one unstable pole)

$$G_m(s) = \frac{K_m e^{-s\tau_m}}{(T_{m1}s - 1)(1 + sT_{m2})}$$

Rule	K_c	T_i	T_d	Comment	
Minimum performance index: regulator tuning					
Minimum ITAE – Poulin and Pomerleau (1996), (1997).	${}^7 K_c^{(632)}$	$T_i^{(632)}$	T_{m2}	<i>Model: Method 1</i>	
Coefficient values (deduced from graph)					
	$\frac{\tau_m + T_{m2}}{T_{m1}}$	x_1	x_2	$\frac{\tau_m + T_{m2}}{T_{m1}}$	x_1
Process output step load disturbance	0.05 0.10 0.15 0.20 0.25	0.9479 1.0799 1.2013 1.3485 1.4905	2.3546 2.4111 2.4646 2.5318 2.5992	0.30 0.35 0.40 0.45 0.50	1.6163 1.7650 1.9139 2.0658 2.2080
Process input step load disturbance	0.05 0.10 0.15 0.20 0.25	1.1075 1.2013 1.3132 1.4384 1.5698	2.4230 2.4646 2.5154 2.5742 2.6381	0.30 0.35 0.40 0.45 0.50	1.6943 1.8161 1.9658 2.1022 2.2379
					2.7007 2.7637 2.8445 2.9210 3.0003

$${}^7 K_c^{(632)} = \frac{x_2 T_{m1} \sqrt{1 + \frac{x_1 T_{m2}^2}{4(\tau_m + T_{m2})^2}}}{K_m (x_1 T_{m1} - 4[\tau_m + T_{m2}])}, \quad T_i^{(632)} = \frac{4 T_{m1} (\tau_m + T_{m2})}{x_1 T_{m1} - 4(\tau_m + T_{m2})};$$

$$0 \leq \frac{\tau_m}{(T_d/N)} \leq 2; \quad 0.1 T_{m2} \leq \frac{T_d}{N} \leq 0.33 T_{m2}.$$

4.12.5 Series controller (classical controller 3)

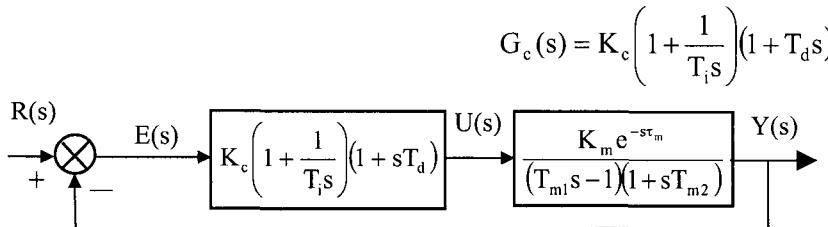


Table 170: PID controller tuning rules – unstable SOSPD model (one unstable pole)

$$G_m(s) = \frac{K_m e^{-sT_m}}{(T_{m1}s - 1)(1 + sT_{m2})}$$

Rule	K_c	T_i	T_d	Comment
Direct synthesis: time domain criteria				
Chidambaram and Kalyan (1996).	⁸ $K_c^{(633)}$	$T_i^{(633)}$	T_{m2}	<i>Model: Method 1</i>
Huang and Chen (1997), (1999). <i>Model: Method 4</i>	⁹ $K_c^{(634)}$	$T_i^{(634)}$	T_{m2}	'Good' servo and regulator response
Direct synthesis: frequency domain criteria				
Ho and Xu (1998). <i>Model: Method 1</i>	$\frac{\omega_p T_{m1}}{A_m K_m}$	¹⁰ $T_i^{(635)}$	T_{m2}	
	$\frac{0.7854 T_{m1}}{K_m \tau_m}$	$-T_{m1}$	T_{m2}	$A_m = 2$, $\phi_m = 45^\circ$

⁸ $K_c^{(633)}$ and $T_i^{(633)}$ are the proportional gain and integral time defined by De Paor and O'Malley (1989), Rotstein and Lewin (1991) or Chidambaram (1995), for the ideal PI control of an unstable FOLPD model [see Table 31].

$${}^9 K_c^{(634)} = \frac{0.5}{K_m} \left[1 + 1.11416 \left(\frac{\tau_m}{T_m} \right)^{-1.13076} \right], \quad T_i^{(634)} = \frac{K_c^{(634)} K_m T_m}{K_c^{(634)} K_m - 1} \left(\frac{2x_1}{T_m} + \frac{\tau_m}{T_m} \right);$$

$$\text{Recommended } x_1 = 0.160367 T_m e^{6.06426 \frac{\tau_m}{T_m}}; \quad 0 < \frac{\tau_m}{T_m} < 0.8.$$

$${}^{10} T_i^{(635)} = \frac{1}{1.57\omega_p - \omega_p^2 \tau_m - \frac{1}{T_m}}.$$

4.12.6 Non-interacting controller 3

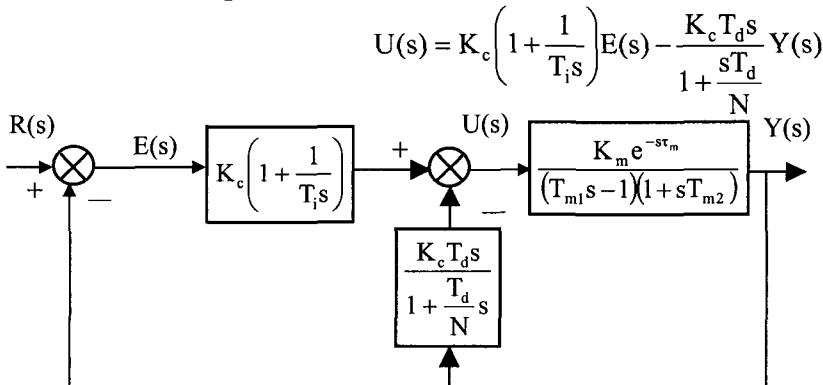


Table 171: PID controller tuning rules – unstable SOSPD model (one unstable pole)

$$G_m(s) = \frac{K_m e^{-s\tau_m}}{(T_{m1}s - 1)(1 + sT_{m2})}$$

Rule	K_c	T_i	T_d	Comment
Minimum performance index: regulator tuning				
Huang and Lin (1995) – minimum IAE.	¹ $K_c^{(636)}$	$T_i^{(636)}$	$T_d^{(636)}$	Model: Method 2; N not specified

¹ Note: equations continued onto page 435. $0.05 \leq \tau_m/T_{m1} \leq 0.4$; $T_{m2} \leq T_{m1}$.

$$K_c^{(636)} = -\frac{1}{K_m} \left[-174.167 - 31.364 \frac{\tau_m}{T_{m1}} + 0.4642 \left(\frac{\tau_m}{T_{m1}} \right)^{-1.164} - 103.069 \frac{T_{m2}}{T_{m1}} \right] \\ - \frac{1}{K_m} \left[-83.916 \left(\frac{T_{m2}}{T_{m1}} \right)^{2.54} - 66.962 \frac{T_{m2} \tau_m}{T_{m1}^2} + 59.496 \left(\frac{T_{m1}}{\tau_m} \right)^{1.065} \left(\frac{T_{m2}}{T_{m1}} \right)^{1.014} \right] \\ - \frac{1}{K_m} \left[-70.79 \frac{T_{m2}}{\tau_m} + 23.121 e^{\frac{\tau_m}{T_{m1}}} + 126.924 e^{\frac{T_{m2}}{T_{m1}}} + 26.944 e^{\frac{T_{m2} \tau_m}{T_{m1}^2}} \right];$$

$$T_i^{(636)} = T_{m1} \left[0.008 + 2.0718 \frac{\tau_m}{T_{m1}} + 6.431 \left(\frac{\tau_m}{T_{m1}} \right)^2 + 0.4556 \frac{T_{m2}}{T_{m1}} + 0.7503 \left(\frac{T_{m2}}{T_{m1}} \right)^2 \right] \\ + T_{m1} \left[2.4484 \frac{T_{m2} \tau_m}{T_{m1}^2} - 18.686 \left(\frac{\tau_m}{T_{m1}} \right)^3 - 2.9978 \left(\frac{T_{m2}}{T_{m1}} \right)^3 - 21.135 \left(\frac{T_{m2} \tau_m}{T_{m1}^3} \right)^2 \right]$$

Rule	K_c	T_i	T_d	Comment
Huang and Lin (1995) – continued.	$^2 K_c^{(637)}$	$T_i^{(637)}$	$T_d^{(637)}$	N not specified; Model: Method 2

$$\begin{aligned}
 & + T_{m1} \left[12.822 \left(\frac{\tau_m T_{m2}^2}{T_{m1}^3} \right) + 39.001 \left(\frac{\tau_m}{T_{m1}} \right)^4 + 22.848 \left(\frac{T_{m2} \tau_m^3}{T_{m1}^4} \right) \right] \\
 & + T_{m1} \left[-4.754 \left(\frac{T_{m2}^3 \tau_m}{T_{m1}^4} \right) - 0.527 \left(\frac{T_{m2}^2 \tau_m^2}{T_{m1}^4} \right) + 1.64 \left(\frac{T_{m2}}{T_{m1}} \right)^4 \right]; \\
 T_d^{(636)} = & T_{m1} \left[1.1766 \frac{\tau_m}{T_{m1}} - 4.4623 \left(\frac{\tau_m}{T_{m1}} \right)^2 + 0.5284 \frac{T_{m2}}{T_{m1}} - 1.4281 \left(\frac{T_{m2}}{T_{m1}} \right)^2 \right] \\
 & + T_{m1} \left[4.6 \frac{T_{m2} \tau_m}{T_{m1}^2} + 11.176 \left(\frac{\tau_m}{T_{m1}} \right)^3 + 1.0886 \left(\frac{T_{m2}}{T_{m1}} \right)^3 - 5.0229 \left(\frac{\tau_m^2 T_{m2}}{T_{m1}^3} \right) \right] \\
 & + T_{m1} \left[-0.0301 - 0.5039 \left(\frac{\tau_m T_{m2}^2}{T_{m1}^3} \right) - 9.8564 \left(\frac{\tau_m}{T_{m1}} \right)^4 - 7.528 \left(\frac{T_{m2} \tau_m^3}{T_{m1}^4} \right) \right] \\
 & + T_{m1} \left[-2.3542 \left(\frac{\tau_m T_{m2}^3}{T_{m1}^4} \right) + 9.3804 \left(\frac{\tau_m^2 T_{m2}^2}{T_{m1}^4} \right) - 0.1457 \left(\frac{T_{m2}}{T_{m1}} \right)^4 \right].
 \end{aligned}$$

² Note: equations continued onto page 436. $0.05 \leq \tau_m / T_{m1} \leq 0.25$, $T_{m1} < T_{m2} \leq 10T_{m1}$;

$$\begin{aligned}
 K_c^{(637)} = & -\frac{1}{K_m} \left[1750.08 + 1637.76 \frac{\tau_m}{T_{m1}} + 1533.91 \left(\frac{\tau_m}{T_{m1}} \right)^{2.1984} - 7.917 \frac{T_{m2}}{T_{m1}} \right] \\
 & - \frac{1}{K_m} \left[6.187 \left(\frac{T_{m2}}{T_{m1}} \right)^{0.791} - 6.451 \frac{T_{m2} \tau_m}{T_{m1}^2} + 0.002452 \left(\frac{T_{m1}}{\tau_m} \right)^{3.2927} \left(\frac{T_{m2}}{T_{m1}} \right)^{1.0757} \right] \\
 & - \frac{1}{K_m} \left[1.3729 \frac{T_{m2}}{\tau_m} - 1739.77 e^{\frac{\tau_m}{T_{m1}}} - 0.000296 e^{\frac{T_{m2}}{T_{m1}}} + 2.311 e^{\frac{T_{m2} \tau_m}{T_{m1}^2}} \right]; \\
 T_i^{(637)} = & T_{m1} \left[51.678 - 57.043 \frac{\tau_m}{T_{m1}} + 1337.29 \left(\frac{\tau_m}{T_{m1}} \right)^2 + 0.1742 \frac{T_{m2}}{T_{m1}} - 0.1524 \left(\frac{T_{m2}}{T_{m1}} \right)^2 \right] \\
 & + T_{m1} \left[7.7266 \frac{T_{m2} \tau_m}{T_{m1}^2} - 6011.57 \left(\frac{\tau_m}{T_{m1}} \right)^3 + 0.0213 \left(\frac{T_{m2}}{T_{m1}} \right)^3 + 9135.52 \left(\frac{\tau_m}{T_{m1}} \right)^4 \right]
 \end{aligned}$$

Rule	K_c	T_i	T_d	Comment
Minimum performance index: servo tuning				
Huang and Lin (1995) – minimum IAE.	³ $K_c^{(638)}$	$T_i^{(638)}$	$T_d^{(638)}$	N not specified; Model: Method 2

$$\begin{aligned}
& + T_{m1} \left[-65.283 \left(\frac{T_{m2} \tau_m^2}{T_{m1}^3} \right) + 0.0645 \left(\frac{\tau_m T_{m2}^2}{T_{m1}^3} \right) + 274.851 \left(\frac{T_{m2} \tau_m^3}{T_{m1}^4} \right) \right] \\
& + T_{m1} \left[0.003926 \left(\frac{T_{m2}^3 \tau_m}{T_{m1}^4} \right) - 2.0997 \left(\frac{T_{m2}^2 \tau_m^2}{T_{m1}^4} \right) - 0.001077 \left(\frac{T_{m2}}{T_{m1}} \right)^4 \right] \\
& + T_{m1} \left[-49.007e^{\frac{\tau_m}{T_{m1}}} + 0.000026e^{\frac{T_{m2}}{T_{m1}}} + 0.2977e^{\frac{T_{m2} \tau_m}{T_{m1}^2}} \right], \\
T_d^{(637)} & = T_{m1} \left[-0.0605 + 4.6998 \frac{\tau_m}{T_{m1}} - 29.478 \left(\frac{\tau_m}{T_{m1}} \right)^2 + 0.0117 \frac{T_{m2}}{T_{m1}} \right] \\
& + T_{m1} \left[-0.0129 \left(\frac{T_{m2}}{T_{m1}} \right)^2 + 0.6874 \frac{T_{m2} \tau_m}{T_{m1}^2} + 140.135 \left(\frac{\tau_m}{T_{m1}} \right)^3 \right] \\
& + T_{m1} \left[0.002455 \left(\frac{T_{m2}}{T_{m1}} \right)^3 + 1.4712 \left(\frac{\tau_m^2 T_{m2}}{T_{m1}^3} \right) - 0.1289 \left(\frac{\tau_m T_{m2}^2}{T_{m1}^3} \right) \right] \\
& + T_{m1} \left[-238.864 \left(\frac{\tau_m}{T_{m1}} \right)^4 + 0.6884 \left(\frac{T_{m2} \tau_m^3}{T_{m1}^4} \right) + 0.007725 \left(\frac{\tau_m T_{m2}^3}{T_{m1}^4} \right) \right] \\
& + T_{m1} \left[-0.1222 \left(\frac{\tau_m^2 T_{m2}^2}{T_{m1}^4} \right) - 0.000135 \left(\frac{T_{m2}}{T_{m1}} \right)^4 \right].
\end{aligned}$$

³ Note: equations continued onto page 437. $0.05 \leq \tau_m/T_{m1} \leq 0.4$, $T_{m2} \leq T_{m1}$;

$$\begin{aligned}
K_c^{(638)} & = -\frac{1}{K_m} \left[10.741 - 13.363 \frac{\tau_m}{T_{m1}} + 0.099 \left(\frac{\tau_m}{T_{m1}} \right)^{-1.344} + 727.914 \frac{T_{m2}}{T_{m1}} \right] \\
& - \frac{1}{K_m} \left[-708.481 \left(\frac{T_{m2}}{T_{m1}} \right)^{0.995} + 9.915 \frac{T_{m2} \tau_m}{T_{m1}^2} + 84.273 \left(\frac{T_{m1}}{\tau_m} \right)^{1.031} \left(\frac{T_{m2}}{T_{m1}} \right)^{0.997} \right] \\
& - \frac{1}{K_m} \left[-90.959 \frac{T_{m2}}{\tau_m} + 9.034e^{\frac{\tau_m}{T_{m1}}} - 2.386e^{\frac{T_{m2}}{T_{m1}}} - 16.304e^{\frac{T_{m2} \tau_m}{T_{m1}^2}} \right]
\end{aligned}$$

$$\begin{aligned}
T_i^{(638)} &= T_{m1} \left[-149.685 - 141.418 \frac{\tau_m}{T_{m1}} - 88.717 \left(\frac{\tau_m}{T_{m1}} \right)^{2.12} - 17.29 \frac{T_{m2}}{T_{m1}} \right] \\
&+ T_{m1} \left[20.518 \left(\frac{T_{m2}}{T_{m1}} \right)^{0.985} - 12.82 \frac{T_{m2}\tau_m}{T_{m1}^2} + 3.611 \left(\frac{\tau_m}{T_{m1}} \right)^{0.286} \left(\frac{T_{m2}}{T_{m1}} \right)^{1.988} \right] \\
&+ T_{m1} \left[0.000805 \frac{T_{m2}}{\tau_m} + 141.702 e^{\frac{\tau_m}{T_{m1}}} - 2.032 e^{\frac{T_{m2}}{T_{m1}}} + 10.006 e^{\frac{T_{m2}\tau_m}{T_{m1}^2}} \right]; \\
T_d^{(638)} &= T_{m1} \left[-0.4144 + 15.805 \frac{\tau_m}{T_{m1}} - 142.327 \left(\frac{\tau_m}{T_{m1}} \right)^2 + 0.7287 \frac{T_{m2}}{T_{m1}} \right] \\
&+ T_{m1} \left[0.1123 \left(\frac{T_{m2}}{T_{m1}} \right)^2 - 18.317 \frac{T_{m2}\tau_m}{T_{m1}^2} + 486.95 \left(\frac{\tau_m}{T_{m1}} \right)^3 - 10.542 \left(\frac{T_{m2}}{T_{m1}} \right)^3 \right] \\
&+ T_{m1} \left[204.009 \left(\frac{\tau_m^2 T_{m2}}{T_{m1}^3} \right) + 47.26 \left(\frac{\tau_m T_{m2}^2}{T_{m1}^3} \right) + 396.349 \left(\frac{\tau_m}{T_{m1}} \right)^4 \right] \\
&+ T_{m1} \left[-138.038 \left(\frac{\tau_m^3 T_{m2}}{T_{m1}^4} \right) + 52.155 \left(\frac{\tau_m T_{m2}^3}{T_{m1}^4} \right) - 646.848 \left(\frac{\tau_m^2 T_{m2}^2}{T_{m1}^4} \right) \right] \\
&+ T_{m1} \left[19.302 \left(\frac{T_{m2}}{T_{m1}} \right)^4 - 4731.72 \left(\frac{\tau_m}{T_{m1}} \right)^5 + 425.789 \left(\frac{\tau_m^4 T_{m2}}{T_{m1}^5} \right) \right] \\
&+ T_{m1} \left[-289.476 \left(\frac{\tau_m T_{m2}^4}{T_{m1}^5} \right) - 841.807 \left(\frac{\tau_m^3 T_{m2}^2}{T_{m1}^5} \right) + 1313.72 \left(\frac{\tau_m^2 T_{m2}^3}{T_{m1}^5} \right) \right] \\
&+ T_{m1} \left[-3.7688 \left(\frac{T_{m2}}{T_{m1}} \right)^5 + 6264.79 \left(\frac{\tau_m}{T_{m1}} \right)^6 - 161.469 \left(\frac{\tau_m^5 T_{m2}}{T_{m1}^6} \right) \right] \\
&+ T_{m1} \left[204.689 \left(\frac{\tau_m T_{m2}^5}{T_{m1}^6} \right) + 25.706 \left(\frac{\tau_m^4 T_{m2}^2}{T_{m1}^6} \right) - 791.857 \left(\frac{\tau_m^2 T_{m2}^4}{T_{m1}^6} \right) \right] \\
&+ T_{m1} \left[648.217 \left(\frac{\tau_m T_{m2}}{T_{m1}^2} \right)^3 - 5.71 \left(\frac{T_{m2}}{T_{m1}} \right)^6 \right].
\end{aligned}$$

Rule	K_c	T_i	T_d	Comment
Huang and Lin (1995) – minimum IAE – continued.	$^4 K_c^{(639)}$	$T_i^{(639)}$	$T_d^{(639)}$	N not specified; Model: Method 2

⁴ Note: equations continued onto page 439. $0.05 \leq \tau_m/T_{m1} \leq 0.25$, $T_{m1} < T_{m2} \leq 10T_{m1}$;

$$K_c^{(639)} = -\frac{1}{K_m} \left[-130.2 + 85.914 \frac{\tau_m}{T_{m1}} + 34.82 \left(\frac{\tau_m}{T_{m1}} \right)^{-0.3055} + 10.442 \frac{T_{m2}}{T_{m1}} \right] \\ - \frac{1}{K_m} \left[-22.547 \left(\frac{T_{m2}}{T_{m1}} \right)^{0.5174} - 14.698 \frac{T_{m2} \tau_m}{T_{m1}^2} + 52.408 \left(\frac{T_{m1}}{\tau_m} \right)^{1.0077} \left(\frac{T_{m2}}{T_{m1}} \right)^{0.9879} \right] \\ - \frac{1}{K_m} \left[-51.47 \left(\frac{T_{m2}}{\tau_m} \right) + 53.378 e^{\frac{\tau_m}{T_{m1}}} - 0.000001 e^{\frac{T_{m2}}{T_{m1}}} + 0.286 e^{\frac{T_{m2} \tau_m}{T_{m1}^2}} \right];$$

$$T_i^{(639)} = T_{m1} \left[72.806 - 268.746 \frac{\tau_m}{T_{m1}} - 4.9221 \left(\frac{T_{m2}}{T_{m1}} \right) + 2468.19 \left(\frac{\tau_m}{T_{m1}} \right)^2 \right] \\ + T_{m1} \left[0.6724 \left(\frac{T_{m2}}{T_{m1}} \right)^2 + 151.351 \frac{T_{m2} \tau_m}{T_{m1}^2} - 6914.46 \left(\frac{\tau_m}{T_{m1}} \right)^3 - 0.0092 \left(\frac{T_{m2}}{T_{m1}} \right)^3 \right] \\ + T_{m1} \left[-795.465 \left(\frac{\tau_m^2 T_{m2}}{T_{m1}^3} \right) - 14.27 \left(\frac{\tau_m T_{m2}^2}{T_{m1}^3} \right) + 5580.17 \left(\frac{\tau_m}{T_{m1}} \right)^4 \right] \\ + T_{m1} \left[1417.65 \left(\frac{\tau_m^3 T_{m2}}{T_{m1}^4} \right) + 0.4057 \left(\frac{\tau_m T_{m2}^3}{T_{m1}^4} \right) + 55.536 \left(\frac{\tau_m^2 T_{m2}^2}{T_{m1}^4} \right) \right] \\ + T_{m1} \left[-0.001119 \left(\frac{T_{m2}}{T_{m1}} \right)^4 - 44.903 e^{\frac{\tau_m}{T_{m1}}} + 0.000034 e^{\frac{T_{m2}}{T_{m1}}} - 15.694 e^{\frac{T_{m2} \tau_m}{T_{m1}^2}} \right] \\ + T_{m1} \left[678778 \left(\frac{\tau_m}{T_{m1}} \right)^{19.056} \left(\frac{T_{m2}}{T_{m1}} \right)^{7.3464} \right];$$

$$T_d^{(639)} = T_{m1} \left[175.515 - 86.2 \frac{\tau_m}{T_{m1}} + 348.727 \left(\frac{\tau_m}{T_{m1}} \right)^{1.1798} - 0.008207 \frac{T_{m2}}{T_{m1}} \right] \\ + T_{m1} \left[-55.619 \left(\frac{T_{m2}}{T_{m1}} \right)^{0.1064} + 0.0418 \frac{T_{m2} \tau_m}{T_{m1}^2} + 78.959 \left(\frac{\tau_m}{T_{m1}} \right)^{0.0355} \left(\frac{T_{m2}}{T_{m1}} \right)^{0.0827} \right]$$

4.12.7 Non-interacting controller 8

$$U(s) = K_c \left(1 + \frac{1}{T_i s} + T_d s \right) E(s) - K_i (1 + T_{di} s) Y(s)$$

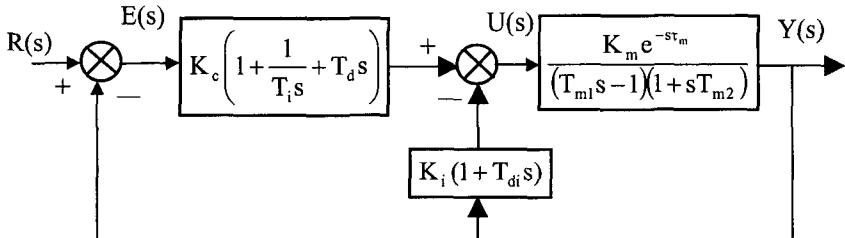


Table 172: PID controller tuning rules – unstable SOSPD model (one unstable pole)

$$G_m(s) = \frac{K_m e^{-s\tau_m}}{(T_{m1}s - 1)(1 + sT_{m2})}$$

Rule	K_c	T_i	T_d	Comment
Direct synthesis				
Kwak <i>et al.</i> (2000). <i>Model: Method 3</i>	$^5 K_c^{(640)}$	$T_i^{(640)}$	$T_d^{(640)}$	Optimal gain margin: regulator response

$$+ T_{m1} \left[0.005048 \left(\frac{T_{m2}}{\tau_m} \right) - 187.01 e^{\frac{\tau_m}{T_{m1}}} + 0.000001 e^{\frac{T_{m2}}{T_{m1}}} - 0.0149 e^{\frac{T_{m2}\tau_m}{T_{m1}^2}} \right].$$

⁵ Note: equations continued onto page 440. $0 \leq \frac{T_{m1}}{\tau_m} \leq 1 - \frac{T_{m2}}{T_{m1}}$.

$$K_c^{(640)} = \frac{1}{K_m} \left[-0.67 + 0.297 \left(\frac{\tau_m}{T_{m1}} \right)^{-2.001} + 2.189 \left(\frac{\tau_m}{T_{m1}} \right)^{-0.766} \xi_m \right], \quad \frac{\tau_m}{T_{m1}} < 0.9 \text{ or}$$

$$K_c^{(640)} = \frac{1}{K_m} \left[-0.365 + 0.260 \left(\frac{\tau_m}{T_{m1}} - 1.4 \right)^2 + 2.189 \left(\frac{\tau_m}{T_{m1}} \right)^{-0.766} \xi_m \right], \quad \frac{\tau_m}{T_{m1}} \geq 0.9;$$

$$T_i^{(640)} = T_{m1} \left[2.212 \left(\frac{\tau_m}{T_{m1}} \right)^{0.520} - 0.3 \right], \quad \frac{\tau_m}{T_{m1}} < 0.4 \text{ or}$$

Rule	K_c	T_i	T_d	Comment
Kwak <i>et al.</i> (2000) - continued.	$^6 K_c^{(641)}$	$T_i^{(641)}$	$T_d^{(641)}$	Optimal gain margin: servo response

$$T_i^{(640)} = T_{m1} \left\{ -0.975 + 0.910 \left(\frac{\tau_m}{T_{m1}} - 1.845 \right)^2 + \alpha \right\}, \quad \frac{\tau_m}{T_{m1}} \geq 0.4 \text{ with}$$

$$\alpha = \left[1 - e^{-\frac{\xi_m}{0.15+0.33\frac{\tau_m}{T_{m1}}}} \right] \left[5.25 - 0.88 \left(\frac{\tau_m}{T_{m1}} - 2.8 \right)^2 \right];$$

$$T_d^{(640)} = \frac{T_{m1}}{\left[1 - e^{-\frac{\xi_m}{-0.15+0.939\left(\frac{\tau_m}{T_{m1}}\right)^{-1.121}}} \right] \left[1.45 + 0.969 \left(\frac{T_{m1}}{\tau_m} \right)^{1.171} \right] - 1.9 + 1.576 \left(\frac{\tau_m}{T_{m1}} \right)^{-0.530}};$$

$$T_{di} = \left(X_1 + X_2 \frac{\tau_m}{T_{m1}} + X_3 \left[\frac{\tau_m}{T_{m1}} \right]^2 \right) T_{m1} \text{ with}$$

$$X_1 = -0.0030 + 0.6482 \frac{T_{m2}}{T_{m1}} - 2.2841 \left(\frac{T_{m2}}{T_{m1}} \right)^2 + 2.6221 \left(\frac{T_{m2}}{T_{m1}} \right)^3 - 0.9611 \left(\frac{T_{m2}}{T_{m1}} \right)^4,$$

$$X_2 = 0.2446 - 1.0410 \frac{T_{m2}}{T_{m1}} - 13.6723 \left(\frac{T_{m2}}{T_{m1}} \right)^2 - 16.7622 \left(\frac{T_{m2}}{T_{m1}} \right)^3 + 5.1471 \left(\frac{T_{m2}}{T_{m1}} \right)^4,$$

$$X_3 = 0.1685 + 0.8289 \frac{T_{m2}}{T_{m1}} - 9.3630 \left(\frac{T_{m2}}{T_{m1}} \right)^2 + 2.9855 \left(\frac{T_{m2}}{T_{m1}} \right)^3 + 7.3803 \left(\frac{T_{m2}}{T_{m1}} \right)^4,$$

$$K_i = \frac{1}{\sqrt{|G_m(j\omega_u)(1+jT_{di}\omega_u)|K_m}}.$$

⁶ Note: equations continued onto page 441. $0 \leq \frac{T_{m1}}{\tau_m} \leq 1 - \frac{T_{m2}}{T_{m1}}$.

$$K_c^{(641)} = \frac{1}{K_m} \left[-0.04 + \left[0.333 + 0.949 \left(\frac{\tau_m}{T_{m1}} \right)^{-0.983} \right] \xi_m \right], \quad \xi_m \leq 0.9 \text{ or}$$

$$K_c^{(641)} = \frac{1}{K_m} \left[-0.544 + 0.308 \frac{\tau_m}{T_{m1}} + 1.408 \left(\frac{\tau_m}{T_{m1}} \right)^{-0.832} \xi_m \right], \quad \xi_m > 0.9.$$

$$T_i^{(641)} = T_{m1} [2.055 + 0.072(\tau_m/T_{m1})] \xi_m, \quad \tau_m/T_{m1} \leq 1 \text{ or}$$

$$T_i^{(641)} = T_{m1} [1.768 + 0.329(\tau_m/T_{m1})] \xi_m, \quad \tau_m/T_{m1} > 1;$$

$$T_d^{(641)} = \frac{T_{m1}}{\left[1 - e^{-\frac{\left(\frac{\tau_m}{T_{m1}} \right)^{1.060} \xi_m}{0.870}} \right] \left[0.55 + 1.683 \left(\frac{T_{m1}}{\tau_m} \right)^{1.090} \right]};$$

$$T_{di} = \left(X_1 + X_2 \frac{\tau_m}{T_{m1}} + X_3 \left[\frac{\tau_m}{T_{m1}} \right]^2 \right) T_{m1} \text{ with}$$

$$X_1 = -0.0030 + 0.6482 \frac{T_{m2}}{T_{m1}} - 2.2841 \left(\frac{T_{m2}}{T_{m1}} \right)^2 + 2.6221 \left(\frac{T_{m2}}{T_{m1}} \right)^3 - 0.9611 \left(\frac{T_{m2}}{T_{m1}} \right)^4,$$

$$X_2 = 0.2446 - 1.0410 \frac{T_{m2}}{T_{m1}} - 13.6723 \left(\frac{T_{m2}}{T_{m1}} \right)^2 - 16.7622 \left(\frac{T_{m2}}{T_{m1}} \right)^3 + 5.1471 \left(\frac{T_{m2}}{T_{m1}} \right)^4,$$

$$X_3 = 0.1685 + 0.8289 \frac{T_{m2}}{T_{m1}} - 9.3630 \left(\frac{T_{m2}}{T_{m1}} \right)^2 + 2.9855 \left(\frac{T_{m2}}{T_{m1}} \right)^3 + 7.3803 \left(\frac{T_{m2}}{T_{m1}} \right)^4;$$

$$K_i = \frac{1}{\sqrt{|G_m(j\omega_u)(1+jT_{di}\omega_u)|K_m}}.$$

4.13 Unstable SOSPD Model (two unstable poles)

$$G_m(s) = \frac{K_m e^{-s\tau_m}}{(T_{m1}s - 1)(T_{m2}s - 1)}$$

4.13.1 Ideal controller $G_c(s) = K_c \left(1 + \frac{1}{T_i s} + T_d s \right)$

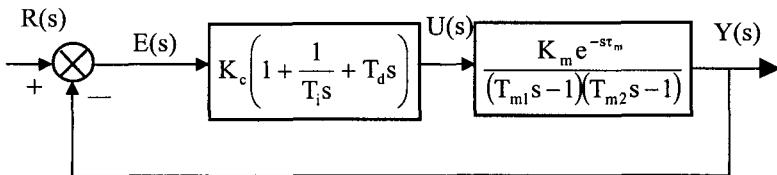


Table 173: PID controller tuning rules – unstable SOSPD model (two unstable poles)

$$G_m(s) = \frac{K_m e^{-s\tau_m}}{(T_{m1}s - 1)(T_{m2}s - 1)}$$

Rule	K_c	T_i	T_d	Comment
Direct synthesis				
Wang and Jin (2004).	${}^7 K_c^{(642)}$	$T_i^{(642)}$	$T_d^{(642)}$	Model: Method I

$${}^7 K_c^{(642)} = \frac{K_m T_i^{(642)} (T_{m1} - \tau_m)(T_{m2} - \tau_m)}{(T_{CL3} + \tau_m)^3},$$

$$T_i^{(642)} = \frac{T_{CL3}^3 + 3\tau_m T_{CL3}^2 + 3(T_{m1}T_{m2} + \tau_m T_{m1} - \tau_m T_{m2})T_{CL3} + \tau_m T_{m2}(T_{m1} - \tau_m)}{(T_{m1} - \tau_m)(\tau_m - T_{m2})},$$

$$T_d^{(642)} = \frac{(T_{m2} + \tau_m - T_{m1})T_{CL3}^3 + 3T_{m1}T_{m2}T_{CL3}(T_{CL3} + \tau_m) + T_{m1}T_{m2}\tau_m^2}{T_i^{(642)}(T_{m1} - \tau_m)(T_{m2} - \tau_m)}.$$

Rule	K_c	T_i	T_d	Comment
Robust				
Lee <i>et al.</i> (2000). Model: Method I	$^8 K_c^{(643)}$	$T_i^{(643)}$	$T_d^{(643)}$	$0 \leq \frac{\tau_m}{T_m} < 2$

$$^8 K_c^{(643)} = \frac{T_i^{(643)}}{K_m(4\lambda + \tau_m - \alpha_1)}, \quad \lambda = \text{desired closed loop dynamic parameter.}$$

$$T_i^{(643)} = -T_{m1} - T_{m2} + \alpha_1 - \frac{6\lambda^2 - \alpha_2 + \alpha_1\tau_m - 0.5\tau_m^2}{4\lambda + \tau_m - \alpha_1};$$

$$T_d^{(643)} = \frac{\alpha_2 - (T_{m1} + T_{m2})\alpha_1 + T_{m1}T_{m2} - \frac{4\lambda^3 + \alpha_2\tau_m + 0.167\tau_m^3 - 0.5\alpha_1\tau_m^2}{4\lambda + \tau_m - \alpha_1}}{T_i^{(643)}} - \frac{6\lambda^2 - \alpha_2 + \alpha_1\tau_m - 0.5\tau_m^2}{4\lambda + \tau_m - \alpha_1};$$

with α_1, α_2 values determined by solving $1 - \frac{(\alpha_2 s^2 + \alpha_1 s + 1)e^{-\tau_m s}}{(\lambda s + 1)^4} \Big|_{s=\frac{1}{T_{m1}}, \frac{1}{T_{m2}}} = 0$.

4.13.2 Ideal controller with set-point weighting 1

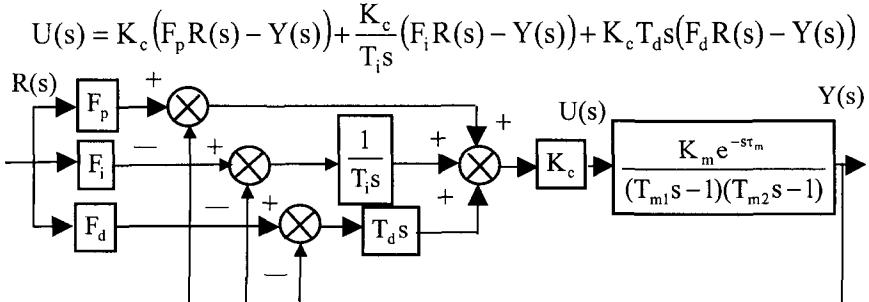


Table 174: PID controller tuning rules – unstable SOSPD model (two unstable poles)

$$G_m(s) = \frac{K_m e^{-st_m}}{(T_{m1}s - 1)(T_{m2}s - 1)}$$

Rule	K_c	T_i	T_d	Comment
Robust				
Lee <i>et al.</i> (2000). Model: Method 1	${}^9 K_c^{(644)}$	$T_i^{(644)}$	$T_d^{(644)}$	$0 \leq \frac{\tau_m}{T_m} < 2$

$F_p = F_i = F_d = \frac{1}{\alpha_2 s^2 + \alpha_1 s + 1}, \alpha_1, \alpha_2 \text{ determined by}$

$$1 - \frac{(\alpha_2 s^2 + \alpha_1 s + 1) e^{-\tau_m s}}{(\lambda s + 1)^4} \Big|_{s=\frac{1}{T_{m1}}, \frac{1}{T_{m2}}} = 0$$

$${}^9 K_c^{(644)} = \frac{T_i^{(644)}}{K_m (4\lambda + \tau_m - \alpha_1)}, \lambda = \text{desired closed loop dynamic parameter.}$$

$$T_i^{(644)} = -T_{m1} - T_{m2} + \alpha_1 - \frac{6\lambda^2 - \alpha_2 + \alpha_1 \tau_m - 0.5\tau_m^2}{4\lambda + \tau_m - \alpha_1},$$

$$T_d^{(644)} = \frac{\alpha_2 - (T_{m1} + T_{m2})\alpha_1 + T_{m1}T_{m2} - \frac{4\lambda^3 + \alpha_2 \tau_m + 0.167\tau_m^3 - 0.5\alpha_1 \tau_m^2}{4\lambda + \tau_m - \alpha_1}}{T_i^{(644)}}$$

$$- \frac{6\lambda^2 - \alpha_2 + \alpha_1 \tau_m - 0.5\tau_m^2}{4\lambda + \tau_m - \alpha_1}.$$

4.14 Unstable SOSPD Model with a Positive Zero

$$G_m(s) = \frac{K_m(1 - T_{m3}s)e^{-sT_m}}{T_{m1}^2 s^2 - 2\xi_m T_{m1}s + 1}$$

4.14.1 Ideal controller in series with a first order lag

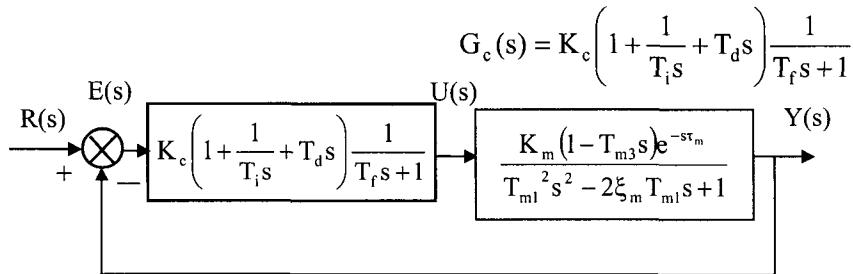


Table 175: PID controller tuning rules -- unstable SOSPD model with a positive zero

$$G_m(s) = \frac{K_m(1 - T_{m3}s)e^{-sT_m}}{T_{m1}^2 s^2 - 2\xi_m T_{m1}s + 1}$$

Rule	K_c	T_i	T_d	Comment
Direct synthesis: time domain criteria				
Sree and Chidambaram (2004).	¹⁰ $K_c^{(645)}$	¹¹ $T_i^{(645)}$	$T_d^{(645)}$	<i>Model: Method 1</i>

$$^{11} \text{Desired closed loop transfer function} = \frac{(1 - sT_{m3})(1 + T_i^{(645)}s)e^{-sT_m}}{(1 + sT_{CL}^{(1)})(1 + sT_{CL}^{(2)})(1 + sx_1)}.$$

$$\begin{aligned} ^{10} K_c^{(645)} &= \frac{T_i^{(645)}}{K_m(T_{CL}^{(1)} + T_{CL}^{(2)} + x_2 + T_{m3} + 0.5\tau_m - T_i^{(645)})}, \\ T_d^{(645)} &= \frac{0.5(T_i^{(645)} - 0.5\tau_m)\tau_m}{T_i^{(645)}}, \quad T_f = \frac{0.5\tau_m T_{CL}^{(1)} T_{CL}^{(2)} x_2}{T_{m1}^2 (T_{CL}^{(1)} + T_{CL}^{(2)} + x_2 + T_{m3} + 0.5\tau_m - T_i^{(645)})}. \end{aligned}$$

¹¹ Equations continued into the footnote on page 446.

$$T_i^{(645)} = \frac{x_1 - T_{m1}^2 (T_{CL}^{(1)} + T_{CL}^{(2)} + x_2 + T_{m3} + \tau_m)}{0.5T_{m1}^2 T_{m3} \tau_m - T_{m1}^4} \text{ with}$$

$$x_1 = T_{CL}^{(1)} T_{CL}^{(2)} x_2 + 0.5\tau_m T_{m1}^2 (T_{CL}^{(1)} T_{CL}^{(2)} + T_{CL}^{(2)} T_{CL}^{(3)} + T_{CL}^{(3)} T_{CL}^{(1)}) + \xi_m T_{m1} \tau_m T_{CL}^{(1)} T_{CL}^{(2)} x_2,$$

$$\begin{aligned}
 x_2 &= \frac{T_{m1}^4 (0.5T_{m3}\tau_m - T_{m1}^2) (T_{CL}^{(1)}T_{CL}^{(2)} + 0.5\tau_m [T_{CL}^{(1)} + T_{CL}^{(2)} - T_{m3}]T_{CL}^{(2)} + 2\xi_m T_{m3}x_3)}{x_4} \\
 &\quad + \frac{T_{m1}^4 (-2\xi_m T_{m1} + T_{m3} + 0.5\tau_m) (0.5\tau_m T_{CL}^{(1)}T_{CL}^{(2)} - T_{m3}^2 x_3)}{x_4}, \\
 x_3 &= T_{CL}^{(1)} + T_{CL}^{(2)} + T_{m3} + \tau_m, \\
 x_4 &= -T_{m1}^2 (0.5T_{m3}\tau_m - T_{m1}^2) (T_{m1}^2 [T_{CL}^{(1)} + T_{CL}^{(2)} + 0.5\tau_m + 2\xi_m T_{m1}] - 0.5\tau_m T_{CL}^{(1)}T_{CL}^{(2)}) \\
 &\quad - (-2\xi T_{m1} + T_{m3} + 0.5\tau_m) \\
 &\quad (T_{m1}^2 [T_{CL}^{(1)}T_{CL}^{(2)} + 0.5\tau_m (T_{CL}^{(1)} + T_{CL}^{(2)})] + \xi_m T_{m1} \tau_m T_{CL}^{(1)}T_{CL}^{(2)} - T_{m1}^4).
 \end{aligned}$$

4.14.2 Non-interacting controller based on the two degree of freedom structure 1

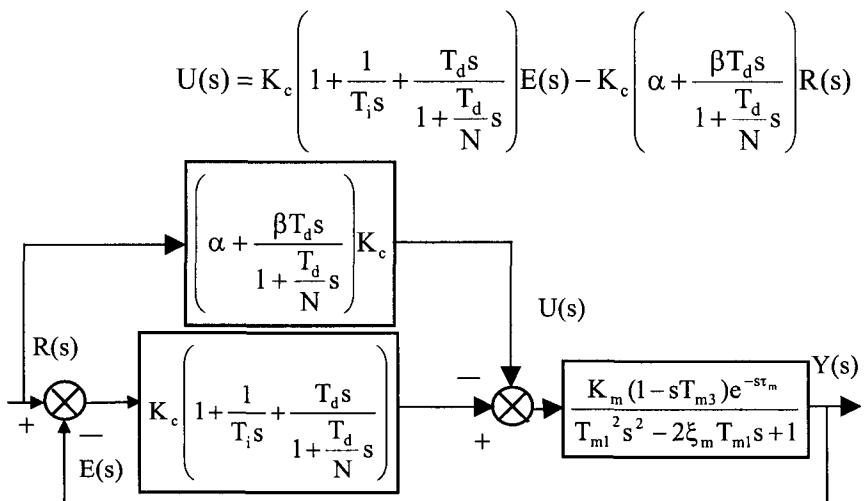


Table 176: PID controller tuning rules — unstable SOSPD model with a positive zero

$$G_m(s) = \frac{K_m (1 - T_{m3}s) e^{-s\tau_m}}{T_{m1}^2 s^2 - 2\xi_m T_{m1} s + 1}$$

Rule	K_c	T_i	T_d	Comment
Direct synthesis: time domain criteria				
Sree and Chidambaram (2004). Model: Method I	¹² $K_c^{(646)}$	¹³ $T_i^{(646)}$	$T_d^{(646)}$	$\beta = 0$; $N = \infty$

$$\text{Desired closed loop transfer function} = \frac{(1 - s T_{m3})(1 + T_i^{(646)} s) e^{-s\tau_m}}{(1 + s T_{CL}^{(1)})(1 + s T_{CL}^{(2)})(1 + s x_1)}.$$

$$\begin{aligned} {}^{12} K_c^{(646)} &= \frac{T_i^{(646)}}{K_m (T_{CL}^{(1)} + T_{CL}^{(2)} + x_2 + T_{m3} + 0.5\tau_m - T_i^{(646)})}, \\ {}^{13} T_d^{(646)} &= \frac{0.5(T_i^{(646)} - 0.5\tau_m)\tau_m}{T_i^{(646)}} , \quad \alpha = 1 - \frac{T_{m3} + 0.5(T_{CL}^{(1)} + T_{CL}^{(2)})}{T_i^{(646)}}. \end{aligned}$$

¹³ Equations continued into the footnote on page 448.

$$\begin{aligned}
T_i^{(646)} &= \frac{x_1 - T_{m1}^2 (T_{CL}^{(1)} + T_{CL}^{(2)} + x_2 + T_{m3} + \tau_m)}{0.5 T_{m1}^2 T_{m3} \tau_m - T_{m1}^4} \text{ with} \\
x_1 &= T_{CL}^{(1)} T_{CL}^{(2)} x_2 + 0.5 \tau_m T_{m1}^2 (T_{CL}^{(1)} T_{CL}^{(2)} + T_{CL}^{(2)} T_{CL}^{(3)} + T_{CL}^{(3)} T_{CL}^{(1)}) + \xi_m T_{m1} \tau_m T_{CL}^{(1)} T_{CL}^{(2)} x_2, \\
x_2 &= \frac{T_{m1}^4 (0.5 T_{m3} \tau_m - T_{m1}^2) (T_{CL}^{(1)} T_{CL}^{(2)} + 0.5 \tau_m [T_{CL}^{(1)} + T_{CL}^{(2)} - T_{m3}] T_{CL}^{(2)} + 2 \xi_m T_{m3} x_3)}{x_4} \\
&\quad + \frac{T_{m1}^4 (-2 \xi_m T_{m1} + T_{m3} + 0.5 \tau_m) (0.5 \tau_m T_{CL}^{(1)} T_{CL}^{(2)} - T_{m3}^2 x_3)}{x_4}, \\
x_3 &= T_{CL}^{(1)} + T_{CL}^{(2)} + T_{m3} + \tau_m, \\
x_4 &= -T_{m1}^2 (0.5 T_{m3} \tau_m - T_{m1}^2) (T_{m1}^2 [T_{CL}^{(1)} + T_{CL}^{(2)} + 0.5 \tau_m + 2 \xi_m T_{m1}] - 0.5 \tau_m T_{CL}^{(1)} T_{CL}^{(2)}) \\
&\quad - (-2 \xi_m T_{m1} + T_{m3} + 0.5 \tau_m) \\
&\quad (T_{m1}^2 [T_{CL}^{(1)} T_{CL}^{(2)} + 0.5 \tau_m (T_{CL}^{(1)} + T_{CL}^{(2)})] + \xi_m T_{m1} \tau_m T_{CL}^{(1)} T_{CL}^{(2)} - T_{m1}^4).
\end{aligned}$$

4.15 Delay Model $G_m(s) = K_m e^{-s\tau_m}$

4.15.1 Ideal controller $G_c(s) = K_c \left(1 + \frac{1}{T_i s} + T_d s \right)$

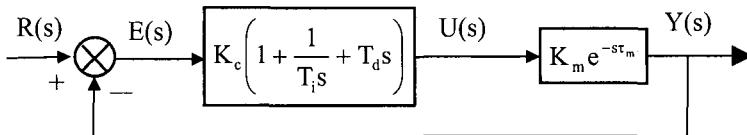


Table 177: PID controller tuning rules – Delay model $G_m(s) = K_m e^{-s\tau_m}$

Rule	K_c	T_i	T_d	Comment
Process reaction				
Callender <i>et al.</i> (1935/6). <i>Model: Method 1</i>	¹ $\frac{0.749}{K_m \tau_m}$	$2.734\tau_m$	$0.303\tau_m$	<i>Representative results deduced from graphs</i>
	² $\frac{1.24}{K_m \tau_m}$	$1.31\tau_m$	$0.303\tau_m$	

¹ Decay ratio = 0.015; Period of decaying oscillation = $10.5\tau_m$

² Decay ratio = 0.12; Period of decaying oscillation = $6.28\tau_m$

4.15.2 Ideal controller in series with a first order lag

$$G_c(s) = K_c \left(1 + \frac{1}{T_i s} + T_d s \right) \frac{1}{T_f s + 1}$$

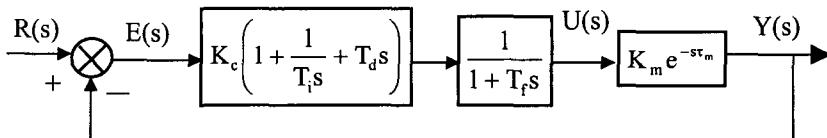


Table 178: PID controller tuning rules – Delay model $G_m(s) = K_m e^{-s\tau_m}$

Rule	K_c	T_i	T_d	Comment
Robust				
Bequette (2003) – page 300. <i>Model: Method 1</i>	$\frac{\tau_m}{K_m(4\lambda + \tau_m)}$	$0.5\tau_m$	$0.167\tau_m$	$T_f = \frac{2\lambda^2 - 0.167\tau_m^2}{4\lambda + \tau_m}$

4.15.3 Classical controller 1 $G_c(s) = K_c \left(1 + \frac{1}{T_i s}\right) \begin{pmatrix} 1 + T_d s \\ 1 + \frac{T_d s}{N} \end{pmatrix}$

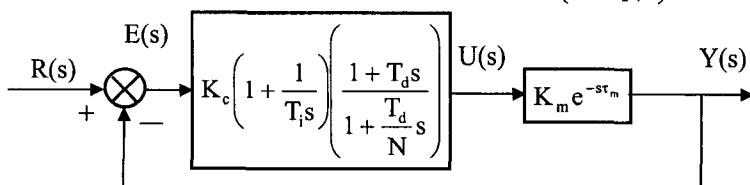


Table 179: PID controller tuning rules – Delay model $G_m(s) = K_m e^{-st_m}$

Rule	K_c	T_i	T_d	Comment
Process reaction				
Hartree <i>et al.</i> (1937). <i>Model: Method 1</i>	³ $\frac{0.70}{K_m \tau_m}$	$2.66\tau_m$	τ_m	<i>Representative results</i>
	⁴ $\frac{0.78}{K_m \tau_m}$	$2.97\tau_m$	$0.5\tau_m$	

³ N = 2. Decay ratio = 0.15; Period of decaying oscillation = $4.49\tau_m$

⁴ N = 2. Decay ratio = 0.042; Period of decaying oscillation = $5.03\tau_m$

4.16 General Model with a Repeated Pole $G_m(s) = \frac{K_m e^{-sT_m}}{(1+sT_m)^n}$

4.16.1 Ideal controller $G_c(s) = K_c \left(1 + \frac{1}{T_i s} + T_d s \right)$

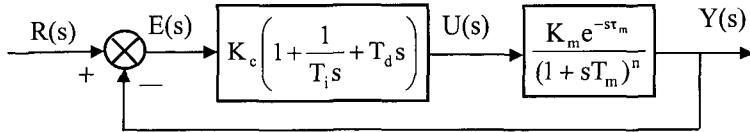


Table 180: PID controller tuning rules – general model with a repeated pole

$$G_m(s) = \frac{K_m e^{-sT_m}}{(1+sT_m)^n}$$

Rule	K_c	T_i	T_d	Comment
Direct synthesis				
Skoczowski and Tarasiejski (1996).	${}^5 K_c^{(647)}$	T_m	$T_d^{(647)}$	<i>Model: Method 1</i>

$${}^5 K_c^{(647)} \leq \frac{\omega_g T_m}{K_m} \sqrt{\frac{\left(1 + \omega_g^2 T_m^2\right)^{n-1}}{1 + \omega_g^2 [T_d^{(647)}]^2}}, \quad T_d^{(647)} = \frac{n-1}{n+2} T_m, \quad n \geq 2 \text{ with}$$

$$\omega_g = \frac{-T_m \left[n^2 - 2n - 2 + \frac{2n+4}{\pi} \frac{\tau_m}{T_m} + \frac{4n+2}{\pi} \phi_m \right] \pm b}{2T_m^2 \left[n^2 - 4n + 3 + \frac{4n+2}{\pi} \frac{\tau_m}{T_m} + \frac{2n-2}{\pi} \phi_m \right]}, \quad \text{and with}$$

$$b = T_m \sqrt{\left[n^2 - 2n - 2 + \frac{2n+4}{\pi} \frac{\tau_m}{T_m} + \frac{4n+2}{\pi} \phi_m \right]^2} + c, \quad \text{with}$$

$$c = 4(n+2) \left(1 + \frac{2}{\pi} \phi_m \right) \left[n^2 - 4n + 3 + \frac{4n+2}{\pi} \left(\frac{\tau_m}{T_m} \right) + \frac{2n-2}{\pi} \phi_m \right],$$

ϕ_m = specified phase margin.

4.16.2 Ideal controller in series with a first order lag

$$G_c(s) = K_c \left(1 + \frac{1}{T_i s} + T_d s \right) \frac{1}{T_f s + 1}$$

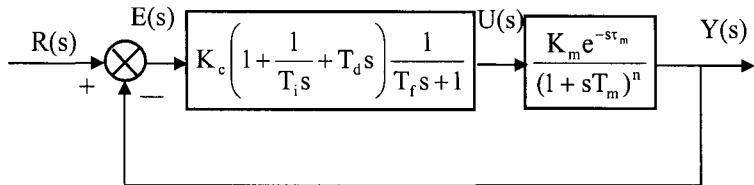


Table 181: PID controller tuning rules – general model with a repeated pole

$$G_m(s) = \frac{K_m e^{-s\tau_m}}{(1 + sT_m)^n}$$

Rule	K_c	T_i	T_d	Comment
Direct synthesis: frequency domain criteria				
Schaedel (1997). Model: Method 2	$\frac{0.563}{K_m} \left[\frac{n+1}{n-2} \right]$	${}^6 T_i^{(648)}$	${}^6 T_d^{(648)}$	$T_f = T_d^{(648)} / N$, $5 \leq N \leq 20$

⁶ $T_i^{(648)} = \frac{3(n+1)}{5n-1} (\tau_m + T_{ar})$, $T_d^{(648)} = \frac{n+1}{6n} (\tau_m + T_{ar})$.

4.17 General Stable Non-Oscillating Model with a Time Delay

4.17.1 Ideal controller $G_c(s) = K_c \left(1 + \frac{1}{T_i s} + T_d s \right)$

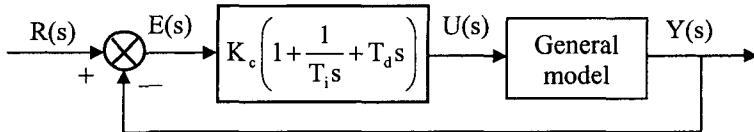


Table 182: PID controller tuning rules – general stable non-oscillating model with a time delay

Rule	K_c	T_i	T_d	Comment
Direct synthesis				
Gorez and Klàn (2000). <i>Model: Method I</i>	$^7 K_c^{(649)}$	$T_i^{(649)}$	$T_d^{(649)}$	
			$T_d^{(650)}$	alternative tuning
			$0.25T_i^{(649)}$	alternative tuning

$$\begin{aligned}
 ^7 K_c^{(649)} &= \frac{T_i^{(649)}}{T_i^{(649)} + \tau_m}, \quad T_i^{(649)} = T_{ar} \frac{1 + \sqrt{1 + 2\left(\frac{\tau_m}{T_{ar}}\right)^2} - 2\frac{\tau_m}{T_{ar}}}{2}, \\
 T_d^{(649)} &= \frac{T_{ar}}{T_i^{(649)}} \left[\left(T_i^{(649)} + T_{cr} \right) \frac{T_{cr}}{T_{ar}} + T_i^{(649)} - T_{aa} - \frac{\tau_m^2}{2T_{ar}} \left(1 - K_c^{(649)} \left\{ 1 + \frac{2\tau_m}{3T_i^{(649)}} \right\} \right) \right], \\
 T_d^{(650)} &= T_i^{(649)} \frac{\tau_m}{T_{ar}} \left(1 - \frac{\tau_m}{T_{ar}} \right), \quad T_{ar} = \text{average residence time of the process (which equals } T_m + \tau_m \text{ for a FOLPD process, for example); } T_{aa} = \text{additional apparent time constant; } T_{cr} = \left[1 - K_c^{(649)} \left(1 + \frac{\tau_m}{2T_i^{(649)}} \right) \right] \tau_m.
 \end{aligned}$$

4.18 Fifth Order System plus Delay Model

$$G_m(s) = \frac{K_m(1+b_1s+b_2s^2+b_3s^3+b_4s^4+b_5s^5)e^{-s\tau_m}}{(1+a_1s+a_2s^2+a_3s^3+a_4s^4+a_5s^5)}$$

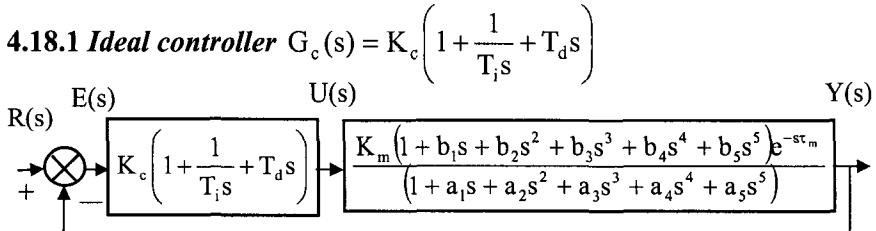


Table 183: PID controller tuning rules – fifth order model with delay

$$G_m(s) = \frac{K_m(1+b_1s+b_2s^2+b_3s^3+b_4s^4+b_5s^5)e^{-s\tau_m}}{(1+a_1s+a_2s^2+a_3s^3+a_4s^4+a_5s^5)}$$

Rule	K_c	T_i	T_d	Comment
Direct synthesis				
Vrančić <i>et al.</i> (1999).	⁸ $K_c^{(651)}$	$T_i^{(651)}$	$T_d^{(651)}$	<i>Model: Method I</i>

⁸ Note: equations continued into the footnote on page 456.

$$K_c^{(651)} = \frac{a_1^3 - a_1^2 b_1 + a_1 b_2 - 2a_1 a_2 + a_2 b_1 + a_3 - b_3 + y_1}{2K_m [-a_1^2 b_1 + a_1 a_2 + a_1 b_2^2 - a_3 - b_1 b_2 + b_3 + y_2]}, \text{ with}$$

$$y_1 = \tau_m (a_1^2 - a_1 b_1 - a_2 + b_2) + 0.5(a_1 - b_1) \tau_m^2 + 0.167 \tau_m^3 \text{ and}$$

$$y_2 = (a_1 - b_1)^2 \tau_m + (a_1 - b_1) \tau_m^2 + 0.333 \tau_m^3 - (a_1 - b_1 + \tau_m)^2 T_d^{(651)};$$

$$T_i^{(651)} = \frac{a_1^3 - a_1^2 b_1 + a_1 b_2 - 2a_1 a_2 + a_2 b_1 + a_3 - b_3 + y_1}{[a_1^2 - a_1 b_1 - a_2 + b_2 + (a_1 - b_1) \tau_m + 0.5 \tau_m^2 - (a_1 - b_1 + \tau_m) T_d^{(651)}]};$$

$T_d^{(651)}$ is found by solving the following equation:

$$T_d^{(651)} x_1 + 360x_2 - 360\tau_m x_3 + 180\tau_m^2 x_4 - 60\tau_m^3 x_5 - 15\tau_m^4 x_6$$

$$- 3\tau_m^5 x_7 + 7\tau_m^6 (b_1 - a_1) - \tau_m^7 = 0, \text{ with}$$

$$x_1 = 360[a_1^4 b_2 - a_1^3 (a_2 b_1 + a_3 + b_1 b_2 + b_3)]$$

$$+ a_1^2 (a_2^2 + a_2 (b_1^2 - 2b_2) + 3a_3 b_1 + 2a_4 + b_1 b_3 + b_2^2 - b_4)$$

$$- a_1 (a_2 (2a_3 - 3b_3) + a_3 (2b_1^2 - b_2) + 3a_4 b_1 + a_5 - b_1 b_4 + 2b_2 b_3 - b_5)$$

$$\begin{aligned}
& -a_2 b_1 b_3 + a_3^2 + a_3(b_1 b_2 - 2b_3) + a_4 b_1^2 + a_5 b_1 - b_1 b_5 + b_3^2] \\
& - 360\tau_m[a_1^4 b_1 - a_1^3(a_2 + b_1^2 + 2b_2) + 3a_1^2(a_3 + b_1 b_2) \\
& + a_1(3a_2 b_2 - 3a_3 b_1 - 3a_4 - b_1 b_3 - 2b_2^2 + 2b_4) \\
& - a_2(b_1 b_2 + b_3) + a_3(b_1^2 - b_2) + 2a_4 b_1 + a_5 - b_1 b_4 + 2b_2 b_3 - b_5] \\
& + 180\tau_m^2[a_1^4 - 4a_1^3 b_1 + 3a_1^2(b_1^2 + b_2) + a_1(3a_2 b_1 - 3a_3 - 5b_1 b_2 + b_3) \\
& - a_2(b_1^2 + 2b_2) + a_3 b_1 + 2a_4 + b_1 b_3 + 2(b_2^2 - b_4)] \\
& + 60\tau_m^3[4a_1^3 - 9a_1^2 b_1 - a_1(3a_2 - 5b_1^2 - 4b_2) + 4a_2 b_1 - a_3 - 5b_1 b_2 + b_3] \\
& + 15\tau_m^4[9a_1^2 - 14a_1 b_1 - 4a_2 + 5b_1^2 + 4b_2] - 42\tau_m^5(b_1 - a_1) + 7\tau_m^6,
\end{aligned}$$

$$x_2 = a_1^4 b_3 - a_1^3(a_2 b_2 + a_1 b_3 + a_4 + b_1 b_3)$$

$$+ a_1^2(a_2^2 b_1 + a_2(2a_3 + b_1 b_2 - 3b_3) + a_3(b_1^2 + b_2) + 2a_4 b_1 + a_5 + b_2 b_3 - b_5)$$

$$- a_1(a_2^3 + a_2^2(b_1^2 - 2b_2) + a_2(2a_3 b_1 - 2b_1 b_3 + b_2^2 - b_4)$$

$$+ 2a_3^2 + a_3(2b_1 b_2 - 3b_3) + a_4(b_1^2 + b_2) + a_5 b_1 - b_1 b_5 + b_3^2)$$

$$+ a_2^2 a_3 + a_2[a_3(b_1^2 - 2b_2) - a_5 - b_1 b_4 + b_5] + a_3^2 b_1$$

$$+ a_3(a_4 - b_1 b_3 + b_2^2 - b_4) + a_4(b_1 b_2 - b_3) + a_5 b_2 - b_2 b_5 + b_3 b_4,$$

$$x_3 = a_1^4 b_2 - a_1^3(a_2 b_1 + a_3 + b_1 b_2 + b_3)$$

$$+ a_1^2(a_2^2 + a_2(b_1^2 - 2b_2) + 3a_3 b_1 + 2a_4 + b_1 b_3 + b_2^2 - b_4)$$

$$- a_1(a_2(2a_3 - 3b_3) + a_3(2b_1^2 - b_2) + 3a_4 b_1 + a_5 - b_1 b_4 + 2b_2 b_3 - b_5)$$

$$- a_2 b_1 b_3 + a_3^2 + a_3(b_1 b_2 - 2b_3) + a_4 b_1^2 + a_5 b_1 - b_1 b_5 + b_3^2,$$

$$x_4 = a_1^4 b_1 - a_1^3(a_2 + b_1^2 + 2b_2)$$

$$+ 3a_1^2(a_3 + b_1 b_2) + a_1(3a_2 b_2 - 3a_3 b_1 - 3a_4 - b_1 b_3 - 2b_2^2 + 2b_4)$$

$$- a_2(b_1 b_2 + b_3) + a_3(b_1^2 - b_2) + 2a_4 b_1 + a_5 - b_1 b_4 + 2b_2 b_3 - b_5),$$

$$x_5 = a_1^4 - 4a_1^3 b_1 + 3a_1^2(b_1^2 + b_2) + a_1(3a_2 b_1 - 3a_3 - 5b_1 b_2 + b_3)$$

$$- a_2(b_1^2 + 2b_2) + a_3 b_1 + 2a_4 + b_1 b_3 + 2(b_2^2 - b_4),$$

$$x_6 = 4a_1^3 - 9a_1^2 b_1 - a_1(3a_2 - 5b_1^2 - 4b_2) + 4a_2 b_1 - a_3 - 5b_1 b_2 + b_3,$$

$$x_7 = 9a_1^2 - 14a_1 b_1 - 4a_2 + 5b_1^2 + 4b_2.$$

4.18.2 Controller with filtered derivative

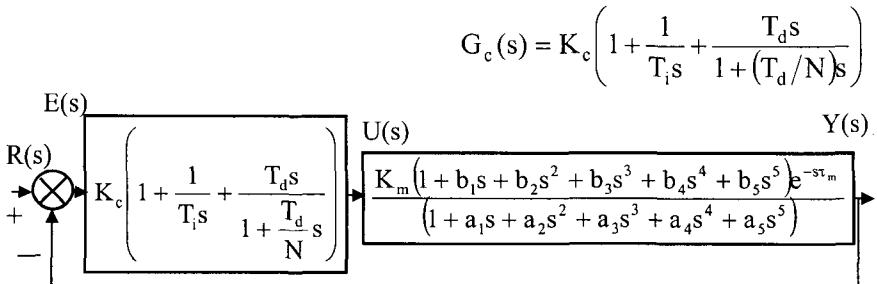


Table 184: PID controller tuning rules – fifth order model with delay

$$G_m(s) = \frac{K_m(1+b_1s+b_2s^2+b_3s^3+b_4s^4+b_5s^5)e^{-s\tau_m}}{(1+a_1s+a_2s^2+a_3s^3+a_4s^4+a_5s^5)}$$

Rule	K_c	T_i	T_d	Comment
Direct synthesis				
Vrančić (1996) – pages 118-119. Model: Method 1	$\frac{T_i^{(652)}}{2(A_1 - T_i^{(652)})}$	${}^9T_i^{(652)}$	$T_d^{(652)}$	$N < 10$

$${}^9T_i^{(652)} = \frac{A_3}{A_2 - T_d^{(652)}A_1 - \frac{[T_d^{(652)}]^2}{N}},$$

$$T_d^{(652)} = \frac{- (A_3^2 - A_5 A_1) + \sqrt{(A_3^2 - A_5 A_1)^2 - \frac{4}{N}(A_3 A_2 - A_5)(A_5 A_2 - A_4 A_3)}}{\frac{2}{N}(A_3 A_2 - A_5)},$$

$$A_1 = K_m(a_1 - b_1 + \tau_m), \quad A_2 = K_m(b_2 - a_2 + A_1 a_1 - b_1 \tau_m + 0.5 \tau_m^2),$$

$$A_3 = K_m(a_3 - b_3 + A_2 a_1 - A_1 a_2 + b_2 \tau_m - 0.5 b_1 \tau_m^2 + 0.167 \tau_m^3),$$

$$A_4 = K_m(b_4 - a_4 + A_3 a_1 - A_2 a_2 + A_1 a_3 - b_3 \tau_m + 0.5 b_2 \tau_m^2 + 0.167 b_1 \tau_m^3 + 0.042 \tau_m^4)$$

$$A_5 = K_m(a_5 - b_5 + A_4 a_1 - A_3 a_2 + A_2 a_3 - A_1 a_4 + b_4 \tau_m - 0.5 b_3 \tau_m^2) \\ + K_m(0.167 b_2 \tau_m^3 - 0.042 b_1 \tau_m^4 + 0.008 \tau_m^5).$$

Rule	K_c	T_i	T_d	Comment
Vrančić <i>et al.</i> (1999). Model: Method 1	$^{10}K_c^{(653)}$	$T_i^{(653)}$	$T_d^{(653)}$	$8 \leq N \leq 20$

$$^{10}K_c^{(653)} = \frac{a_1^3 - a_1^2 b_1 + a_1 b_2 - 2a_1 a_2 + a_2 b_1 + a_3 - b_3 + y_1}{2K_m [a_1^2 b_1 + a_1 a_2 + a_1 b_1^2 - a_3 - b_1 b_2 + b_3 + (a_1 - b_1)^2 \tau_m + y_2]}, \text{ with}$$

$$y_1 = \tau_m (a_1^2 - a_1 b_1 - a_2 + b_2) + 0.5(a_1 - b_1) \tau_m^2 + 0.167 \tau_m^3 \text{ and}$$

$$y_2 = (a_1 - b_1) \tau_m^2 + 0.333 \tau_m^3 - (a_1 - b_1 + \tau_m)^2 T_d^{(653)} - \frac{[T_d^{(653)}]^2}{N} (a_1 - b_1 + \tau_m);$$

$$T_i^{(653)} = \frac{a_1^3 - a_1^2 b_1 + a_1 b_2 - 2a_1 a_2 + a_2 b_1 + a_3 - b_3 + y_1}{\left[a_1^2 - a_1 b_1 - a_2 + b_2 + (a_1 - b_1) \tau_m + 0.5 \tau_m^2 - (a_1 - b_1 + \tau_m) T_d^{(653)} - \frac{[T_d^{(653)}]^2}{N} \right]}$$

$T_d^{(653)}$ is found by solving the following equation:

$$[T_d^{(653)}]^4 N^{-3} z_1 + [T_d^{(653)}]^3 N^{-2} z_2 + [T_d^{(653)}]^2 N^{-1} z_3 +$$

$$T_d^{(653)} x_1 + 360 x_2 - 360 \tau_m x_3 + 180 \tau_m^2 x_4 - 60 \tau_m^2 x_5 - 15 \tau_m^4 x_6$$

$$- 3 \tau_m^5 x_7 + 7 \tau_m^6 (b_1 - a_1) - \tau_m^7 = 0$$

with $x_1, x_2, x_3, x_4, x_5, x_6$ and x_7 as defined in Table 183; z_1, z_2 and z_3 are defined as follows:

$$z_1 = 360[a_1^3 - a_1^2 b_1 + a_1(b_2 - 2a_2) + a_2 b_1 + a_3 - b_3]$$

$$+ 360 \tau_m [a_1^2 - a_1 b_1 - a_2 + b_2] + 180 \tau_m^2 (a_1 - b_1) + 60 \tau_m^3,$$

$$z_2 = 360(a_1 - b_1)[a_1^3 - a_1^2 b_1 + a_1(b_2 - 2a_2) + a_2 b_1 + a_3 - b_3]$$

$$+ 360 \tau_m [2a_1^3 - 3a_1^2 b_1 - a_1(3a_2 - b_1^2 - 2b_2) + 2a_2 b_1 + a_3 - b_1 b_2 - b_3]$$

$$+ 180 \tau_m^2 (3a_1^2 - 4a_1 b_1 - 2a_2 + b_1^2 + 2b_2) + 240 \tau_m^3 (a_1 - b_1) + 60 \tau_m^4,$$

$$z_3 = z_4 + z_5 \tau_m + z_6 \tau_m^2 + z_7 \tau_m^3 + 135(a_1 - b_1) \tau_m^4 + 27 \tau_m^5, \text{ with}$$

$$z_4 = -360[a_1^4 b_1 - a_1^3 (a_2 + b_1^2 + b_2) + a_1^2 (2(a_3 + b_1 b_2) - a_2 b_1)]$$

$$+ a_1 (a_2^2 + a_2 (b_1^2 + b_2) - a_3 b_1 - 2a_4 - b_1 b_3 - b_2^2 + b_4)$$

$$- a_2 (a_3 + b_1 b_2) + a_4 b_1 + a_5 + b_2 b_3 - b_5],$$

$$z_5 = 360[a_1^4 - 3a_1^3 b_1 + a_1^2 (2(b_1^2 + b_2) - a_2) + a_1 (3a_2 b_1 - a_3 - 3b_1 b_2)]$$

$$- a_2 (b_1^2 + b_2) + a_4 + b_1 b_3 + b_2^2 - b_4],$$

$$z_6 = 540[a_1^3 - 2a_1^2 b_1 - a_1(a_2 - b_1^2 - b_2) + b_1(a_2 - b_2)] \text{ and}$$

$$z_7 = 180[2a_1^2 - 3a_1 b_1 - a_2 + b_1^2 + b_2].$$

Rule	K_c	T_i	T_d	Comment
Vrančić <i>et al.</i> (2004a), Vrančić and Lumbar (2004). Model: Method I	$^{11} K_c^{(654)}$	$T_i^{(654)}$	$T_d^{(654)}$	$8 \leq N \leq 20$

$^{11} K_c^{(654)}$, $T_i^{(654)}$ and $T_d^{(654)}$ are determined by solving the following equations:

$$K_c^{(654)} \cdot T_d^{(654)} = \frac{0.5 \frac{A_2}{A_1} (A_4 A_1 - A_5 K_m) - 0.5 \frac{A_4}{K_m} (A_1 A_2 - A_3 K_m)}{A_5 A_1 K_m + A_1 A_2 A_3 - A_4 A_1^2 - A_3^2 K_m},$$

$$\frac{K_c^{(654)}}{T_i^{(654)}} = \frac{1 + 2 K_m K_c^{(654)} + K_m^2 [K_c^{(654)}]^2}{2(A_1 + K_m^2 K_c^{(654)} T_d^{(654)})} \text{ and}$$

$$K_c^{(654)} = \frac{A_3 K_m - A_1^2 K_m K_c^{(654)} T_d^{(654)} - A_2 A_1 + A_2 K_m^2 K_c^{(654)} T_d^{(654)} + x}{2 A_1 A_2 K_m - A_1^3 - A_3 K_m^2}, \text{ with}$$

$$x^2 = \left[(2 A_1 K_m K_c^{(654)} T_d^{(654)} + A_2) A_2 - A_1 A_3 - A_3 (K_m^2 + A_1^2) K_c^{(654)} T_d^{(654)} \right] \\ \left[A_1 + K_m^2 K_c^{(654)} T_d^{(654)} \right],$$

and with

$$A_1 = K_m (a_1 - b_1 + \tau_m), \quad A_2 = K_m (b_2 - a_2 + A_1 a_1 - b_1 \tau_m + 0.5 \tau_m^2),$$

$$A_3 = K_m (a_3 - b_3 + A_2 a_1 - A_1 a_2 + b_2 \tau_m - 0.5 b_1 \tau_m^2 + 0.167 \tau_m^3),$$

$$A_4 = K_m (b_4 - a_4 + A_3 a_1 - A_2 a_2 + A_1 a_3 - b_3 \tau_m + 0.5 b_2 \tau_m^2 + 0.167 b_1 \tau_m^3 + 0.042 \tau_m^4)$$

$$A_5 = K_m (a_5 - b_5 + A_4 a_1 - A_3 a_2 + A_2 a_3 - A_1 a_4 + b_4 \tau_m - 0.5 b_3 \tau_m^2) \\ + K_m (0.167 b_2 \tau_m^3 - 0.042 b_1 \tau_m^4 + 0.008 \tau_m^5).$$

4.18.3 Non-interacting controller 7

$$U(s) = \frac{K_c}{T_i s} E(s) - K_c \left(1 + \frac{T_d s}{1 + \frac{T_d}{N} s} \right) Y(s)$$

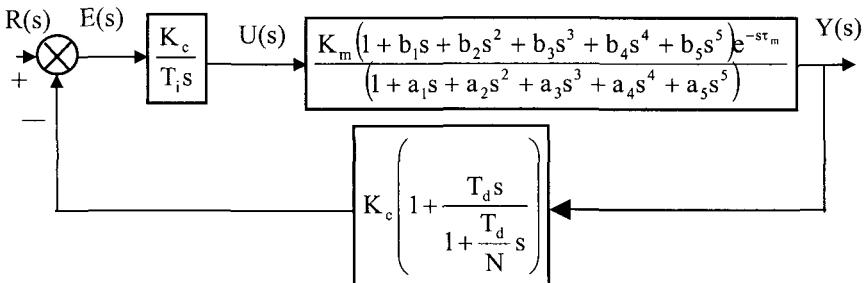


Table 185: PID controller tuning rules – fifth order model with delay

$$G_m(s) = \frac{K_m (1 + b_1 s + b_2 s^2 + b_3 s^3 + b_4 s^4 + b_5 s^5) e^{-s\tau_m}}{(1 + a_1 s + a_2 s^2 + a_3 s^3 + a_4 s^4 + a_5 s^5)}$$

Rule	K_c	T_i	T_d	Comment
Direct synthesis				
Vrančić <i>et al.</i> (2004a), Vrančić and Lumbar (2004).	$^{12} K_c^{(655)}$	$T_i^{(655)}$	$T_d^{(655)}$	<i>Model: Method 1;</i> $8 \leq N \leq 20$

¹² Note: equations continued into the footnote on page 461. $K_c^{(655)}$, $T_i^{(655)}$ and $T_d^{(655)}$ are determined by solving the following equations:

$$K_c^{(655)} \cdot T_d^{(655)} = \frac{0.5 \frac{A_2}{A_1} (A_4 A_1 - A_5 K_m) - 0.5 \frac{A_4}{K_m} (A_1 A_2 - A_3 K_m)}{A_5 A_1 K_m + A_1 A_2 A_3 - A_4 A_1^2 - A_3^2 K_m},$$

$$\frac{K_c^{(655)}}{T_i^{(655)}} = \frac{1 + 2K_m K_c^{(655)} + K_m^2 [K_c^{(655)}]^2}{2(A_1 + K_m^2 K_c^{(655)} T_d^{(655)})} \text{ and}$$

$$K_c^{(655)} = \frac{A_3 K_m - A_1^2 K_m K_c^{(655)} T_d^{(655)} - A_2 A_1 + A_2 K_m^2 K_c^{(655)} T_d^{(655)} + x}{2 A_1 A_2 K_m - A_1^3 - A_3 K_m^2}, \text{ with}$$

$$x^2 = \left[(2 A_1 K_m K_c^{(655)} T_d^{(655)} + A_2) A_2 - A_1 A_3 - A_3 (K_m^2 + A_1^2) K_c^{(655)} T_d^{(655)} \right] \left[A_1 + K_m^2 K_c^{(655)} T_d^{(655)} \right],$$

and with

$$A_1 = K_m(a_1 - b_1 + \tau_m), \quad A_2 = K_m(b_2 - a_2 + A_1 a_1 - b_1 \tau_m + 0.5 \tau_m^2),$$

$$A_3 = K_m(a_3 - b_3 + A_2 a_1 - A_1 a_2 + b_2 \tau_m - 0.5 b_1 \tau_m^2 + 0.167 \tau_m^3),$$

$$A_4 = K_m(b_4 - a_4 + A_3 a_1 - A_2 a_2 + A_1 a_3 - b_3 \tau_m + 0.5 b_2 \tau_m^2 + 0.167 b_1 \tau_m^3 + 0.042 \tau_m^4)$$

$$A_5 = K_m(a_5 - b_5 + A_4 a_1 - A_3 a_2 + A_2 a_3 - A_1 a_4 + b_4 \tau_m - 0.5 b_3 \tau_m^2)$$

$$+ K_m(0.167 b_2 \tau_m^3 - 0.042 b_1 \tau_m^4 + 0.008 \tau_m^5).$$

Chapter 5

Performance and Robustness Issues in the Compensation of FOLPD Processes with PI and PID Controllers

5.1 Introduction

This chapter will discuss the compensation of FOLPD processes using PI and PID controllers whose parameters are specified using appropriate tuning rules. The gain margin, phase margin and maximum sensitivity of the compensated system as the ratio of time delay to time constant of the process varies, are used as ways of judging the performance and robustness of the system. This work follows the method of Ho *et al.* (1995b), (1996), in which good approximations for the gain and phase margins of the PI or PID controlled system are analytically calculated. The method will be extended to determine analytically the maximum sensitivity of the compensated system. Insight will be obtained into the range of time delay to time constant ratios over which it is sensible to apply various tuning rules, to compensate a FOLPD process.

The chapter is organised as follows. Formulae for calculating analytically the gain margin, phase margin and maximum sensitivity are outlined in Sections 5.2 and 5.3. The performance and robustness of FOLPD processes compensated with a variety of PI and PID tuning rules are evaluated in Section 5.4. In Section 5.5, tuning rules are designed to achieve constant gain and phase margins for all values of delay, for a number of process models and controller structures. Conclusions of the

work are drawn in Section 5.6. This work was previously published by O'Dwyer (1998), (2001a).

5.2 The Analytical Determination of Gain and Phase Margin

5.2.1 PI tuning formulae

The controller and process model are respectively given by

$$G_c(s) = K_c \left(1 + \frac{1}{T_i s} \right) \quad (5.1)$$

$$G_m(s) = \frac{K_m e^{-s\tau_m}}{1 + sT_m} \quad (5.2)$$

Then

$$G_m(j\omega)G_c(j\omega) = \frac{K_m e^{-j\omega\tau_m}}{1 + j\omega T_m} \frac{K_c(j\omega T_i + 1)}{j\omega T_i} \quad (5.3)$$

From the definition of gain and phase margin, the following sets of equations are obtained:

$$\phi_m = \arg[G_c(j\omega_g)G_m(j\omega_g)] + \pi \quad (5.4)$$

$$A_m = \frac{1}{|G_c(j\omega_p)G_m(j\omega_p)|} \quad (5.5)$$

where ω_g and ω_p are given by

$$|G_c(j\omega_g)G_m(j\omega_g)| = 1 \quad (5.6)$$

$$\arg[G_c(j\omega_p)G_m(j\omega_p)] = -\pi \quad (5.7)$$

From Equation (5.3),

$$G_m(j\omega)G_c(j\omega) = \frac{K_m K_c \sqrt{1 + \omega^2 T_i^2}}{\omega T_i \sqrt{1 + \omega^2 T_m^2}} \angle -0.5\pi + \tan^{-1} \omega T_i - \tan^{-1} \omega T_m - \omega \tau_m \quad (5.8)$$

Therefore, from Equation (5.4)

$$\phi_m = \pi - 0.5\pi + \tan^{-1} \omega_g T_i - \tan^{-1} \omega_g T_m - \omega_g \tau_m \quad (5.9)$$

with ω_g given by the solution of Equation (5.6) i.e.

$$\frac{K_m K_c \sqrt{1 + \omega_g^2 T_i^2}}{\omega_g T_i \sqrt{1 + \omega_g^2 T_m^2}} = 1 \quad (5.10)$$

Also, from Equations (5.5) and (5.8),

$$A_m = \frac{1}{|G_m(j\omega_p)G_c(j\omega_p)|} = \frac{\omega_p T_i}{K_m K_c} \sqrt{\frac{1 + \omega_p^2 T_m^2}{1 + \omega_p^2 T_i^2}} \quad (5.11)$$

with ω_p given by the solution of Equation (5.7) i.e.

$$-0.5\pi + \tan^{-1} \omega_p T_i - \tan^{-1} \omega_p T_m - \omega_p \tau_m = -\pi \quad (5.12)$$

From Equation (5.10), ω_g may be determined analytically to be

$$\omega_g = \sqrt{\frac{T_i(K_c^2 K_m^2 - 1) + \sqrt{(K_c^2 K_m^2 - 1)^2 T_i^2 + 4K_c^2 K_m^2 T_m^2}}{2T_i T_m^2}} \quad (5.13)$$

An analytical solution of Equation (5.12) (to determine ω_p) is not possible. An approximate analytical solution may be obtained if the following approximation for the arctan function is made:

$$\tan^{-1} x \approx \frac{\pi}{4}x, \quad |x| < 1 \quad \text{and} \quad \tan^{-1} x \approx \frac{\pi}{2} - \frac{\pi}{4x}, \quad |x| > 1 \quad (5.14)$$

This is quite an accurate approximation, as is shown in Figures 1a and 1b (page 465). Considering Equation (5.12), four possibilities present themselves if the approximation in Equation (5.14) is to be used. These possibilities, together with the formula for ω_p that may be determined analytically for each of these cases, are

$$(i) \quad \omega_p T_i > 1, \quad \omega_p T_m > 1 : \quad \omega_p = \frac{\pi \pm \sqrt{\pi^2 - 4\pi\tau_m \left(\frac{1}{T_i} - \frac{1}{T_m} \right)}}{4\tau_m} \quad (5.15)$$

$$(ii) \quad \omega_p T_i > 1, \quad \omega_p T_m < 1 : \quad \omega_p = \frac{\pi \pm \sqrt{\pi^2 - \frac{\pi}{T_i} (0.25\pi T_m + \tau_m)}}{2(0.25\pi T_m + \tau_m)} \quad (5.16)$$

$$(iii) \quad \omega_p T_i < 1, \quad \omega_p T_m > 1 : \quad \omega_p = \sqrt{\frac{\pi/T_m}{4\tau_m - \pi T_i}} \quad (5.17)$$

$$(iv) \quad \omega_p T_i < 1, \quad \omega_p T_m < 1 : \quad \omega_p = \frac{2\pi}{4\tau_m + \pi(T_m - T_i)} \quad (5.18)$$

The gain and phase margin of the compensated system, for each of the tuning rules, as a function of τ_m/T_m , may be calculated by applying Equations (5.9), (5.11), (5.13) and the relevant approximation for ω_p from Equations (5.15) to (5.18).

Figure 1a: Arctan x and its approximation (Equation 5.14)

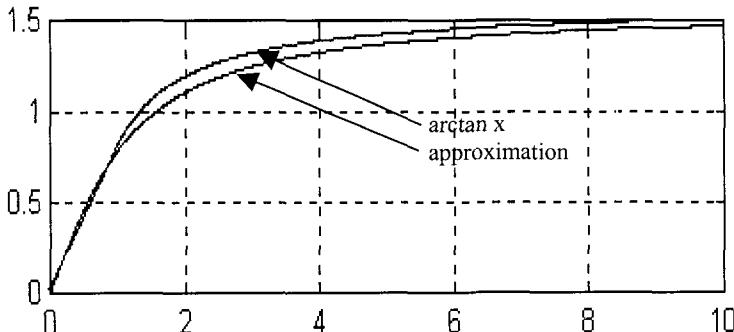
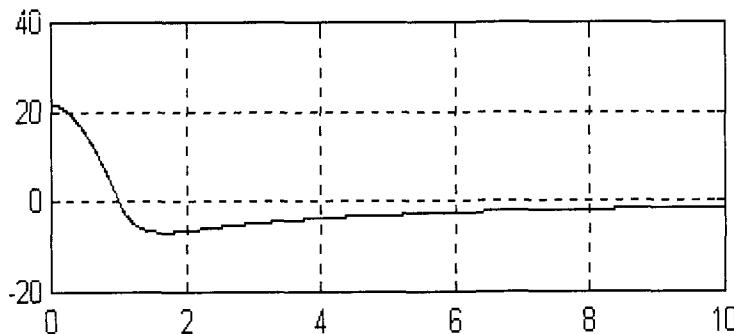


Figure 1b: % error in taking approximation (Equation 5.14) to arctan(x)



5.2.2 PID tuning formulae

The classical PID controller structure is considered, whose transfer function is given as

$$G_c(s) = K_c \left(1 + \frac{1}{T_i s} \right) \left(\frac{1 + s T_d}{1 + s \alpha T_d} \right) \quad (5.19)$$

and the process model is given by Equation (5.2). Substituting Equations (5.2) and (5.19) into Equations (5.4) and (5.5) gives

$$\phi_m = 0.5\pi + \tan^{-1} \omega_g T_i + \tan^{-1} \omega_g T_d - \tan^{-1} \omega_g T_m - \tan^{-1} \omega_g \alpha T_d - \omega_g \tau_m \quad (5.20)$$

and

$$A_m = \frac{\omega_p T_i}{K_c K_m} \sqrt{\frac{(1 + \omega_p^2 T_m^2)(1 + \omega_p^2 \alpha^2 T_d^2)}{(1 + \omega_p^2 T_i^2)(1 + \omega_p^2 T_d^2)}} \quad (5.21)$$

From Equation (5.6), ω_g is given by the solution of

$$\frac{K_m K_c \sqrt{1 + \omega_g^2 T_i^2} \sqrt{1 + \omega_g^2 T_d^2}}{\omega_g T_i \sqrt{1 + \omega_g^2 T_m^2} \sqrt{1 + \alpha^2 \omega_g^2 T_d^2}} = 1 \quad (5.22)$$

The equation for ω_g may be determined to be

$$\omega_g^6 + a_1 \omega_g^4 + a_2 \omega_g^2 + a_3 = 0 \quad (5.23)$$

$$\text{with } a_1 = \frac{T_i^2 (T_m^2 + \alpha^2 T_d^2) - K_c^2 K_m^2 T_i^2 T_d^2}{T_i^2 T_m^2 \alpha^2 T_d^2},$$

$$a_2 = \frac{T_i^2 - K_c^2 K_m^2 (T_i^2 + T_d^2)}{T_i^2 T_m^2 \alpha^2 T_d^2} \text{ and } a_3 = -\frac{K_c^2 K_m^2}{T_i^2 T_m^2 \alpha^2 T_d^2}.$$

Following the procedure outlined by Ho *et al.* (1996), the analytical solution of Equation (5.23) is given as

$$\omega_g = \sqrt[3]{R + \sqrt{Q^3 + R^2}} + \sqrt[3]{R - \sqrt{Q^3 + R^2}} - \frac{a_1}{3} \quad (5.24)$$

$$\text{with } Q = \frac{3a_2 - a_1^2}{9} \text{ and } R = \frac{9a_1 a_2 - 27a_3 - 2a_1^3}{54}.$$

From Equation (5.7), ω_p is given by the solution of

$$0.5\pi + \tan^{-1} \omega_p T_d + \tan^{-1} \omega_p T_i - \tan^{-1} \omega_p T_m - \tan^{-1} \omega_p \alpha T_d - \omega_p \tau_m = 0 \quad (5.25)$$

As with Equation (5.12), an analytical solution of this equation is not possible. An approximate analytical solution may be obtained if the approximation detailed in Equation (5.14) is made. Looking at Equation (5.25), twelve possibilities present themselves if the approximation in Equation (5.14) is to be used; these possibilities are

$$(1) \underline{\omega_p T_i > 1, \omega_p T_m > 1}$$

$$(a) \underline{\omega_p T_d \leq 1, \omega_p \alpha T_d < 1}$$

$$(b) \underline{\omega_p T_d > 1, \omega_p \alpha T_d < 1}$$

$$(c) \underline{\omega_p T_d > 1, \omega_p \alpha T_d > 1}$$

$$(2) \underline{\omega_p T_i > 1, \omega_p T_m < 1}$$

$$(a) \underline{\omega_p T_d \leq 1, \omega_p \alpha T_d < 1}$$

$$(b) \underline{\omega_p T_d > 1, \omega_p \alpha T_d < 1}$$

$$(c) \underline{\omega_p T_d > 1, \omega_p \alpha T_d > 1}$$

$$(3) \underline{\omega_p T_i < 1, \omega_p T_m > 1}$$

$$(a) \underline{\omega_p T_d \leq 1, \omega_p \alpha T_d < 1}$$

$$(b) \underline{\omega_p T_d > 1, \omega_p \alpha T_d < 1}$$

$$(c) \underline{\omega_p T_d > 1, \omega_p \alpha T_d > 1}$$

$$(4) \underline{\omega_p T_i < 1, \omega_p T_m < 1}$$

$$(a) \underline{\omega_p T_d \leq 1, \omega_p \alpha T_d < 1}$$

$$(b) \underline{\omega_p T_d > 1, \omega_p \alpha T_d < 1}$$

$$(c) \underline{\omega_p T_d > 1, \omega_p \alpha T_d > 1}$$

The formulae for ω_p that may be determined for each of these cases are as follows:

$$(1) \underline{\omega_p T_i > 1, \omega_p T_m > 1}$$

$$(a) \underline{\omega_p T_d \leq 1, \omega_p \alpha T_d < 1}:$$

$$\omega_p = \frac{\pi \pm \sqrt{\pi^2 - \pi(4\tau_m - \pi T_d(1-\alpha))\left(\frac{1}{T_i} - \frac{1}{T_m}\right)}}{4\tau_m - \pi T_d(1-\alpha)} \quad (5.26)$$

$$(b) \underline{\omega_p T_d > 1, \omega_p \alpha T_d < 1}:$$

$$\omega_p = \frac{2\pi \pm \sqrt{4\pi^2 - \pi(4\tau_m + \pi T_d\alpha)\left(\frac{1}{T_d} + \frac{1}{T_i} - \frac{1}{T_m}\right)}}{4\tau_m + \pi T_d\alpha} \quad (5.27)$$

(c) $\omega_p T_d > 1, \omega_p \alpha T_d > 1$:

$$\omega_p = \frac{\pi \pm \sqrt{\pi^2 - 4\pi\tau_m \left(\frac{1}{T_d} + \frac{1}{T_i} - \frac{1}{\alpha T_d} - \frac{1}{T_m} \right)}}{4\tau_m} \quad (5.28)$$

(2) $\omega_p T_i > 1, \omega_p T_m < 1$:

(a) $\omega_p T_d \leq 1, \omega_p \alpha T_d < 1$:

$$\omega_p = \frac{2\pi \pm \sqrt{4\pi^2 + \frac{\pi}{T_i} (\pi T_d - \pi T_m - \pi \alpha T_d - 4\tau_m)}}{\pi T_m + 4\tau_m + \pi \alpha T_d - \pi T_d} \quad (5.29)$$

(b) $\omega_p T_d > 1, \omega_p \alpha T_d < 1$:

$$\omega_p = \frac{3\pi \pm \sqrt{9\pi^2 - \pi \left(\frac{1}{T_d} + \frac{1}{T_i} \right) (\pi T_m + \pi \alpha T_d + 4\tau_m)}}{\pi T_m + 4\tau_m + \pi \alpha T_d} \quad (5.30)$$

(c) $\omega_p T_d > 1, \omega_p \alpha T_d > 1$:

$$\omega_p = \frac{2\pi \pm \sqrt{4\pi^2 + \pi \left(\frac{1}{\alpha T_d} - \frac{1}{T_d} - \frac{1}{T_i} \right) (\pi T_m + 4\tau_m)}}{\pi T_m + 4\tau_m} \quad (5.31)$$

(3) $\omega_p T_i < 1, \omega_p T_m > 1$:

$$(a) \omega_p T_d \leq 1, \omega_p \alpha T_d < 1: \omega_p = \sqrt{\frac{\pi}{T_m (4\tau_m + \pi [\alpha T_d - T_d - T_i])}} \quad (5.32)$$

(b) $\omega_p T_d > 1, \omega_p \alpha T_d < 1$:

$$\omega_p = \frac{-\pi \pm \sqrt{\pi^2 - \pi \left(\frac{1}{T_m} - \frac{1}{T_d} \right) (\pi T_i - \pi \alpha T_d - 4\tau_m)}}{\pi T_i - 4\tau_m - \pi \alpha T_d} \quad (5.33)$$

$$(c) \omega_p T_d > 1, \omega_p \alpha T_d > 1: \omega_p = \sqrt{\frac{\frac{\pi}{T_d} - \frac{\pi}{T_m} - \frac{\pi}{\alpha T_d}}{\pi T_i - 4\tau_m}} \quad (5.34)$$

(4) $\omega_p T_i < 1, \omega_p T_m < 1 :$

$$(a) \omega_p T_d \leq 1, \omega_p \alpha T_d < 1 : \omega_p = \frac{2\pi}{\pi(\alpha T_d - T_d) + \pi(T_m - T_i) + 4\tau_m} \quad (5.35)$$

(b) $\omega_p T_d > 1, \omega_p \alpha T_d < 1 :$

$$\omega_p = \frac{2\pi \pm \sqrt{4\pi^2 - \frac{\pi}{T_d} (\pi T_m - \pi T_i + \pi \alpha T_d + 4\tau_m)}}{\pi T_m - \pi T_i + 4\tau_m - \pi \alpha T_d} \quad (5.36)$$

(c) $\omega_p T_d > 1, \omega_p \alpha T_d > 1 :$

$$\omega_p = \frac{\pi \pm \sqrt{\pi^2 - \pi \left(\frac{1}{\alpha T_d} - \frac{1}{T_d} \right) (\pi T_i - \pi T_m - 4\tau_m)}}{4\tau_m - \pi T_m - \pi T_i} \quad (5.37)$$

The gain and phase margin of the compensated system, for each of the tuning rules, as a function of τ_m/T_m may now be calculated by applying Equations (5.20), (5.21), (5.24) and the relevant approximation for ω_p from Equations (5.26) to (5.37). As Equation (5.19) shows, the procedure applies to tuning rules defined for the classical PID controller structure only; however, Ho *et al.* (1996) suggest that the method may be applied to tuning rules defined for the ideal PID controller structure, by using equations that approximately convert these tuning rules into their equivalent in the classical controller structure.

5.3 The Analytical Determination of Maximum Sensitivity

The maximum sensitivity is the reciprocal of the shortest distance from the Nyquist curve to the (-1,0) point on the Re-Im axis. It is defined as follows:

$$M_{\max} = \text{Max}_{\text{all } \omega} \left| \frac{1}{1 + G_m(j\omega)G_c(j\omega)} \right| \quad (5.38)$$

For a FOLPD process model controlled by a PI controller,

$$\left| G_m(j\omega)G_c(j\omega) \right| = \frac{\sqrt{1 + \omega^2 T_i^2}}{\sqrt{1 + \omega^2 T_m^2}} \frac{K_c K_m}{\omega T_i} \quad (5.39)$$

and

$$\arg[G_m(j\omega)G_c(j\omega)] = -0.5\pi - \tan^{-1}\omega T_m + \tan^{-1}\omega T_i - \omega\tau_m \quad (5.40)$$

For a FOLPD process model controlled by a PID controller,

$$|G_m(j\omega)G_c(j\omega)| = \frac{\sqrt{(1+\omega^2T_i^2)(1+\omega^2T_d^2)}}{\sqrt{(1+\omega^2T_m^2)(1+\alpha^2\omega^2T_d^2)}} \frac{K_c K_m}{\omega T_i} \quad (5.41)$$

and $\arg[G_m(j\omega)G_c(j\omega)] =$

$$-0.5\pi - \tan^{-1}\omega T_m - \tan^{-1}\omega\alpha T_d + \tan^{-1}\omega T_i + \tan^{-1}\omega T_d - \omega\tau_m \quad (5.42)$$

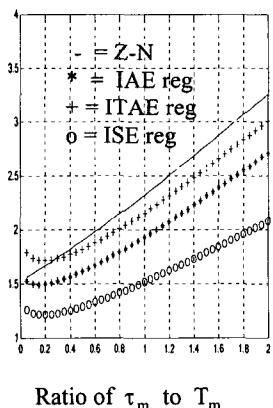
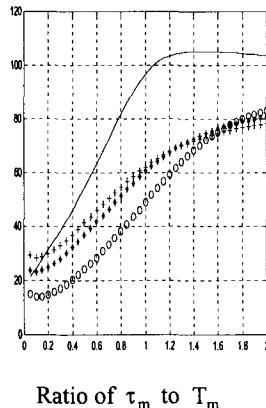
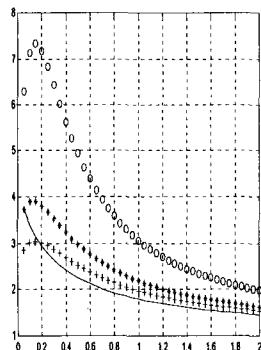
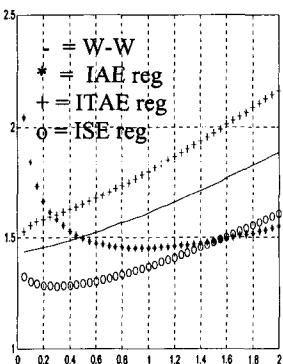
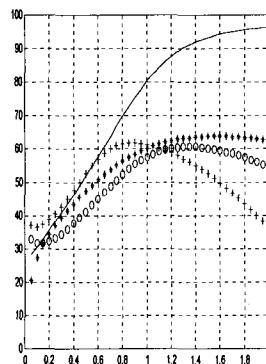
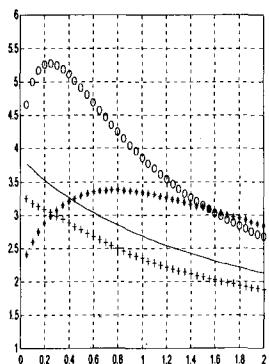
The maximum sensitivity may be calculated over an appropriate range of frequencies corresponding to phase lags of 100° to 260° .

5.4 Simulation Results

Space considerations dictate that only representative simulation results may be provided; an extensive set of simulation results covering 103 PI controller tuning rules and 125 PID controller tuning rules (for the ideal and classical controller structures) are available (O'Dwyer, 2000). The MATLAB package has been used in the simulations. Figures 2 to 7 show how gain margin, phase margin and maximum sensitivity vary as the ratio of time delay to time constant varies, if some PI tuning rules are used (Figures 2 to 4) and corresponding PID tuning rules for the classical controller structure (with $\alpha = 0.1$) are used (Figures 5 to 7). In these results, Z-N refers to the process reaction curve method of Ziegler and Nichols (1942); W-W refers to the process reaction curve method of Witt and Waggoner (1990); IAE reg, ISE reg and ITAE reg refer to the tuning rules for regulator applications that minimise the integral of absolute error criterion, the integral of squared error criterion and the integral of time multiplied by absolute error criterion, respectively, as defined by Murrill (1967) for PI tuning rules and Kaya and Scheib (1988) for PID tuning rules based on the classical controller structure. Figures 8 to 15 show gain and phase margin comparisons between corresponding PI and PID controller tuning rules.

It is clear that the gain margin is generally less when the PID rather than the PI tuning rules are considered, over the ratios of time delay to

time constant taken; the difference between the phase margins is less clear cut. This suggests that these PID tuning rules should provide a greater degree of performance than the corresponding PI tuning rules, but may be less robust. Comparing the individual tuning rules, it is striking that the ISE based tuning rules have generally the smallest gain margin and have also a small phase margin, suggesting that this is a less robust tuning strategy. The results in Figures 4 and 7 confirm these comments.

Figure 2: Gain marginFigure 3: Phase marginFigure 4: Max. sensitivityRatio of τ_m to T_m Ratio of τ_m to T_m Ratio of τ_m to T_m Figure 5: Gain marginFigure 6: Phase marginFigure 7: Max. sensitivityRatio of τ_m to T_m Ratio of τ_m to T_m Ratio of τ_m to T_m

No general conclusion can be reached as to the best tuning rule (as expected); it is interesting, though, that a full panorama of simulation results (O'Dwyer, 2000) show that many tuning rules may be applied at ratios of time delay to time constant greater than that normally recommended. One example may be seen in Figures 5 to 7, where the gain margin, phase margin and maximum sensitivity (associated with the use of the PID tuning rule for obtaining minimum IAE in the regulator mode) tends to level out when the ratio of time delay to time constant is greater than 1; normally, the tuning rule is used when the ratio is less than 1 (Murrill, 1967). On the other hand, it is clear from Figures 8 and 9 that there is a significant degradation of performance when using the PID tuning rule of Witt and Waggoner (1990) and the PI tuning rule of Ziegler and Nichols (1942) for large ratios of time delay to time constant, which is compatible with application experience.

The decision between the use of a PI and PID controller to compensate the process, depends on the ratio of time delay to time constant in the FOLPD model, together with the desired trade-off between performance and robustness, as expected. It turns out, however, that the analytical method explored allows the calculation of a far wider range of gain and phase margins for PI controllers; it is also true that stability tends to be assured when a PI controller is used (O'Dwyer, 2000). Thus, a cautious design approach is to use a PI controller, particularly at larger ratios of time delay to time constant.

Finally, the volume of tuning rules and data generated means that the use of an expert system to recommend a tuning rule based on user defined requirements is indicated; work is ongoing on such an implementation (Feeney and O'Dwyer, 2002).

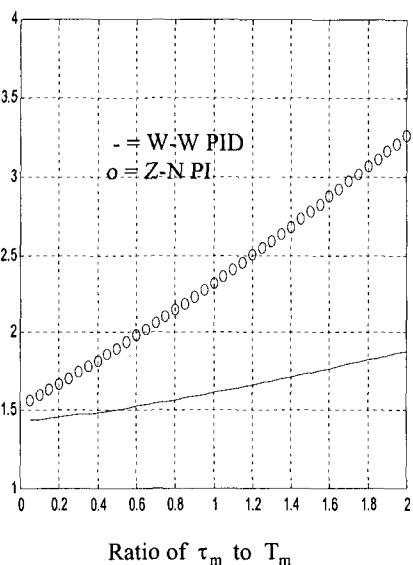
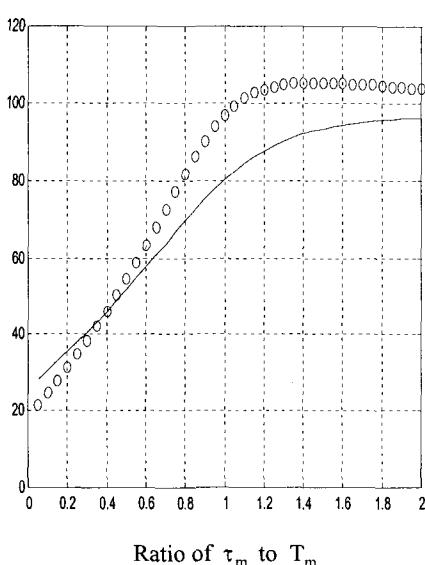
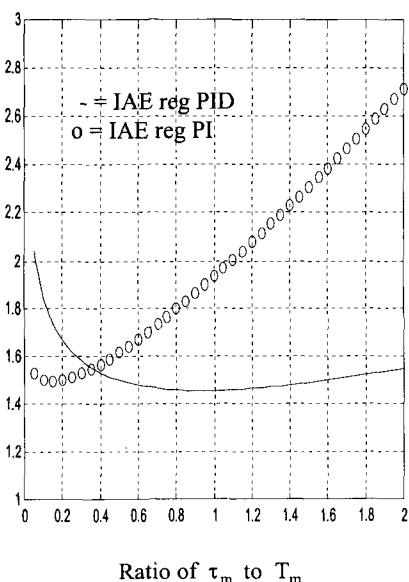
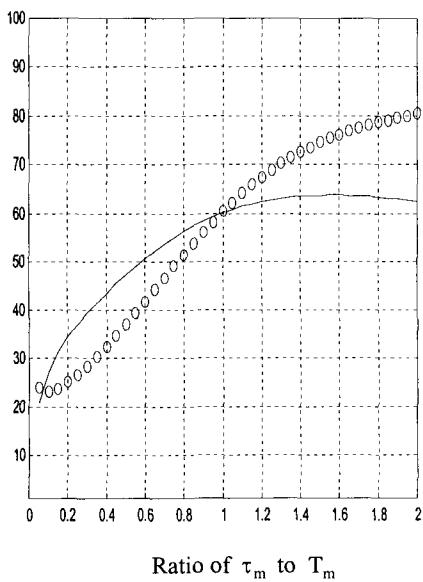
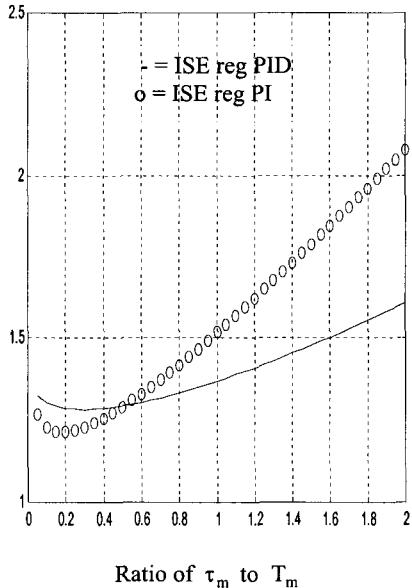
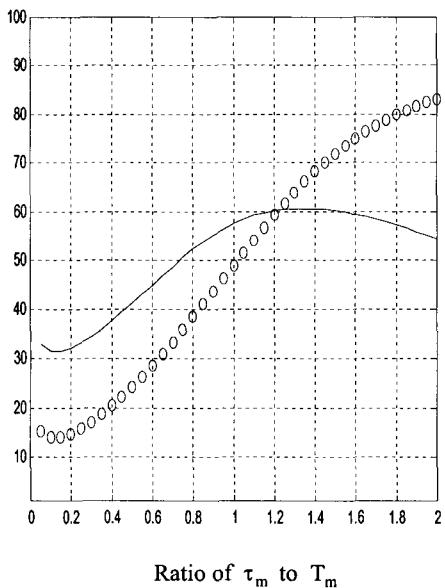
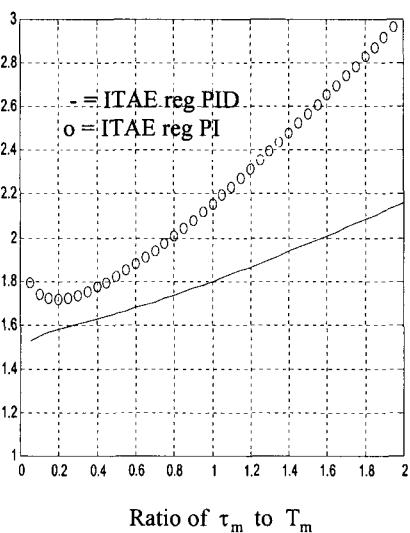
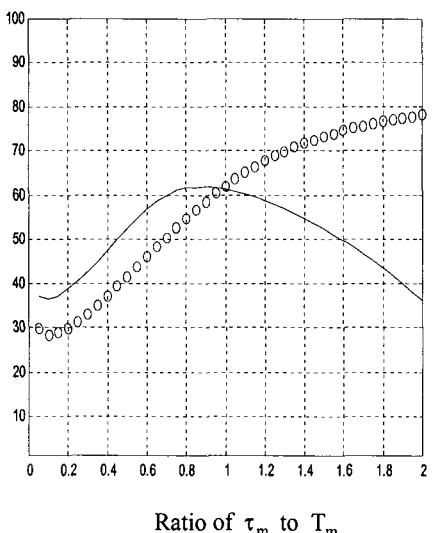
Figure 8: Gain margin comparisonFigure 9: Phase margin comparisonFigure 10: Gain margin comparisonFigure 11: Phase margin comparison

Figure 12: Gain margin comparisonFigure 13: Phase margin comparisonFigure 14: Gain margin comparisonFigure 15: Phase margin comparison

5.5 Design of Tuning Rules to Achieve Constant Gain and Phase Margins, for all Values of Delay

Normally, the gain and phase margins of the compensated systems tend to increase as the time delay increases, supporting the common view that PI and PID controllers are less suitable for the control of dominant time delay processes. However, for the PI control of a FOLPD process model, O'Dwyer (2000) shows that the tuning rules proposed by Chien *et al.* (1952), Haalman (1965) and Pemberton (1972a), among others, facilitate the achievement of a constant gain and phase margin as the time delay of the process model varies. All of these tuning rules have the following structure: $K_c = aT_m/K_m \tau_m$, $T_i = T_m$. Following this observation, original approaches to the design of tuning rules for PI, PD and PID controllers are proposed, for a wide variety of process models, which allow constant gain and phase margins for the compensated system.

This work organised as follows. In Section 5.5.1, PI controller tuning rules are specified for processes modelled in FOLPD form and IPD form. In Section 5.5.2, PID controller tuning rules are described for processes modelled in FOLPD form, SOSPD form and SOSPD form with a negative zero. Section 5.5.3 deals with the design of PD controller tuning rules for the control of processes modelled in FOLIPD form.

5.5.1 PI controller design

5.5.1.1 Processes modelled in FOLPD form

For such processes and controllers, Equations (5.1) and (5.2) apply i.e.,

$$G_m(s) = \frac{K_m e^{-s\tau_m}}{1 + sT_m}$$

with $G_c(s) = K_c \left(1 + \frac{1}{T_i s} \right)$

From Section 5.2.1,

$$\phi_m = \pi - 0.5\pi + \tan^{-1} \omega_g T_i - \tan^{-1} \omega_g T_m - \omega_g \tau_m \quad (\text{Equation 5.9})$$

with ω_g given by the solution of

$$\frac{K_m K_c \sqrt{1 + \omega_g^2 T_i^2}}{\omega_g T_i \sqrt{1 + \omega_g^2 T_m^2}} = 1 \quad (\text{Equation 5.10})$$

and

$$A_m = \frac{\omega_p T_i}{K_m K_c} \sqrt{\frac{1 + \omega_p^2 T_m^2}{1 + \omega_p^2 T_i^2}} \quad (\text{Equation 5.11})$$

with ω_p given by the solution of

$$-0.5\pi + \tan^{-1} \omega_p T_i - \tan^{-1} \omega_p T_m - \omega_p \tau_m = -\pi \quad (\text{Equation 5.12})$$

If K_c and T_i are designed as follows:

$$K_c = \frac{a T_m}{K_m \tau_m} \quad (5.43)$$

and

$$T_i = T_m \quad (5.44)$$

Then Equation (5.12) becomes

$$-0.5\pi - \omega_p \tau_m = -\pi \quad (5.45)$$

i.e.

$$\omega_p = \pi/2\tau_m \quad (5.46)$$

Substituting Equation (5.46) into Equation (5.11) gives

$$A_m = \pi T_m / 2 K_m K_c \tau_m \quad (5.47)$$

Substituting Equation (5.44) into Equation (5.10) gives

$$K_m K_c / \omega_g T_m = 1 \quad (5.48)$$

i.e.

$$\omega_g = K_m K_c / T_m \quad (5.49)$$

Substituting Equations (5.44) and (5.49) into Equation (5.9) gives

$$\phi_m = 0.5\pi - K_m K_c \tau_m / T_m \quad (5.50)$$

Finally, substituting Equation (5.43) into Equation (5.47) gives

$$A_m = \pi/2a \quad (5.51)$$

and substituting Equation (5.43) into Equation (5.50) gives

$$\phi_m = 0.5\pi - a \quad (5.52)$$

Some typical tuning rules are shown in Table 186.

Table 186: Typical PI controller tuning rules – FOLPD process model

a	K_c	T_i	A_m	ϕ_m
$\pi/3$	$1.047T_m/K_m\tau_m$	T_m	1.5	$\pi/6$
$\pi/4$	$0.785T_m/K_m\tau_m$	T_m	2.0	$\pi/4$
$\pi/6$	$0.524T_m/K_m\tau_m$	T_m	3.0	$\pi/3$

These rules are also provided in Table 3, Chapter 3. It may also be demonstrated that, if K_c and T_i are designed as follows:

$$K_c = \frac{aT_m}{\tau_m} \frac{T_u K_u}{\sqrt{T_u^2 + 4\pi^2 T_m^2}} \quad (5.53)$$

and

$$T_i = T_m \quad (5.54)$$

where K_u and T_u are the ultimate gain and ultimate period, respectively, then the constant gain and phase margins provided in Equations (5.51) and (5.52) are obtained.

5.5.1.2 Processes modelled in IPD form

A similar analysis to that of Section 5.5.1.1 may be done for the design of PI controllers for processes modelled in IPD form. The process is modelled as follows:

$$G_m(s) = K_m e^{-s\tau_m} / s \quad (5.55)$$

Corresponding to Equations (5.4) and (5.9), the phase margin is

$$\phi_m = \pi - 0.5\pi + \tan^{-1} \omega_g T_i - 0.5\pi - \omega_g \tau_m \quad (5.56)$$

with ω_g given by the solution of

$$\frac{K_m K_c \sqrt{1 + \omega_g^2 T_i^2}}{\omega_g^2 T_i} = 1 \quad (5.57)$$

If K_c and T_i are designed as follows:

$$K_c = \frac{a}{K_m \tau_m} \quad (5.58)$$

and

$$T_i = b\tau_m \quad (5.59)$$

then it may be shown that

$$\omega_g = \left[\frac{a^2}{2\tau_m^2} \pm \frac{a}{2b\tau_m} \sqrt{a^2b^2 + 4} \right]^{0.5} \quad (5.60)$$

and, substituting Equations (5.59) and (5.60) into Equation (5.56),

$$\phi_m = \tan^{-1} \left[0.5a^2b^2 + 0.5ab\sqrt{a^2b^2 + 4} \right]^{0.5} - \sqrt{0.5a^2 + 0.5 \frac{a}{b} \sqrt{a^2b^2 + 4}} \quad (5.61)$$

Corresponding to Equations (5.5) and (5.11), the gain margin,

$$A_m = \frac{\omega_p^2 T_i}{K_m K_c \sqrt{1 + \omega_p^2 T_i^2}} \quad (5.62)$$

with ω_p given by the solution of

$$-0.5\pi + \tan^{-1} \omega_p T_i - 0.5\pi - \omega_p \tau_m = -\pi \quad (5.63)$$

i.e. ω_p is given by the solution of

$$\tan^{-1} \omega_p T_i = \omega_p \tau_m \quad (5.64)$$

An analytical solution to this equation is not possible, though if the approximation $\tan^{-1} \omega_p T_i \approx 0.5\pi - \frac{\pi}{4\omega_p T_i}$ (when $|\omega_p T_i| > 1$) is used, simple calculations show that the following analytical solution for ω_p may be obtained:

$$\omega_p = \frac{0.5}{\tau_m} \left[0.5\pi + \sqrt{0.25\pi^2 - \frac{\pi}{b}} \right] \quad (5.65)$$

The inequality $|\omega_p T_i| > 1$ may be shown to be equivalent to $b > 1.273$.

Substituting Equations (5.58), (5.59) and (5.65) into Equation (5.62), calculations show that:

$$A_m = \frac{\frac{b}{4a} \left[\frac{\pi}{2} + \sqrt{\frac{\pi^2}{4} - \frac{\pi}{b}} \right]^2}{\sqrt{1 + \frac{b^2}{4} \left[\frac{\pi}{2} + \sqrt{\frac{\pi^2}{4} - \frac{\pi}{b}} \right]^2}} \quad (5.66)$$

Some typical tuning rules are shown in Table 187.

Table 187: Typical PI controller tuning rules – IPD process model

K_c	T_i	A_m	ϕ_m
$0.558/K_m \tau_m$	$1.4\tau_m$	1.5	46.2°
$0.484/K_m \tau_m$	$1.55\tau_m$	2.0	45.5°
$0.458/K_m \tau_m$	$3.35\tau_m$	3.0	59.9°
$0.357/K_m \tau_m$	$4.3\tau_m$	4.0	60.0°
$0.305/K_m \tau_m$	$12.15\tau_m$	5.0	75.0°

It may also be shown that the maximum sensitivity is a constant if K_c and T_i are specified according to Equations (5.58) and (5.59). The maximum sensitivity may be calculated to be:

$$M_{max} = \left[1 - 2a \cos(\omega_r \tau_m) - \frac{a^2}{\omega_r^2 \tau_m^2} \right]^{-0.5} \quad (5.67)$$

with $\omega_r \tau_m$ being a constant obtained numerically from the solution of the equation $(a^2/\omega_r^2 \tau_m^2) + \cos \omega_r \tau_m = \sin(\omega_r \tau_m)/\omega_r \tau_m$.

It may also be shown that, if K_c and T_i are designed as follows:

$$K_c = a K_u \quad (5.68)$$

and

$$T_i = b T_u \quad (5.69)$$

then the following constant gain and phase margins are determined:

$$A_m = \frac{\frac{2b}{\pi a} \left[\frac{\pi}{2} + \sqrt{\frac{\pi^2}{4} - \frac{\pi}{4b}} \right]^2}{\sqrt{1 + \frac{1}{a^2 \pi^2} \left[\frac{\pi}{2} + \sqrt{\frac{\pi^2}{4} - \frac{\pi}{4b}} \right]^2}} \quad (5.70)$$

and $\phi_m = \tan^{-1} \left[2\pi^2 a^2 b^2 + 2\pi a b \sqrt{4\pi^2 a^2 b^2 + 4} \right]^{0.5}$

$$- \sqrt{0.125\pi^2 a^2 + \frac{\pi a}{16b} \sqrt{4\pi^2 a^2 b^2 + 4}} \quad (5.71)$$

5.5.2 PID controller design

5.5.2.1 Processes modelled in FOLPD form - classical controller 1

The classical PID controller is given by

$$G_c(s) = K_c \left(1 + \frac{1}{T_i s} \right) \left(\frac{1 + s T_d}{1 + s \alpha T_d} \right) \quad (\text{Equation 5.19})$$

and the process model is given by

$$G_m(s) = \frac{K_m e^{-sT_m}}{1 + s T_m} \quad (\text{Equation 5.2})$$

From Section 5.2.2,

$$\phi_m = 0.5\pi + \tan^{-1} \omega_g T_i + \tan^{-1} \omega_g T_d - \tan^{-1} \omega_g T_m - \tan^{-1} \omega_g \alpha T_d - \omega_g \tau_m \quad (\text{Equation 5.20})$$

with ω_g given by the solution of

$$\frac{K_m K_c \sqrt{1 + \omega_g^2 T_i^2} \sqrt{1 + \omega_g^2 T_d^2}}{\omega_g T_i \sqrt{1 + \omega_g^2 T_m^2} \sqrt{1 + \alpha^2 \omega_g^2 T_d^2}} = 1 \quad (\text{Equation 5.22})$$

Also

$$A_m = \frac{\omega_p T_i}{K_c K_m} \sqrt{\frac{(1 + \omega_p^2 T_m^2)(1 + \omega_p^2 \alpha^2 T_d^2)}{(1 + \omega_p^2 T_i^2)(1 + \omega_p^2 T_d^2)}} \quad (\text{Equation 5.21})$$

with ω_p given by

$$0.5\pi + \tan^{-1} \omega_p T_d + \tan^{-1} \omega_p T_i - \tan^{-1} \omega_p T_m - \tan^{-1} \omega_p \alpha T_d - \omega_p \tau_m = 0 \quad (\text{Equation 5.25})$$

If K_c , T_i and T_d are designed as follows:

$$K_c = \frac{aT_m}{K_m \tau_m} \quad (5.72)$$

$$T_i = \alpha T_m \quad (5.73)$$

and

$$T_d = T_m \quad (5.74)$$

then, Equation (5.25) becomes

$$0.5\pi - \omega_p \tau_m = 0 \quad (5.75)$$

i.e.

$$\omega_p = \pi/2\tau_m \quad (5.76)$$

and Equation (5.21) becomes

$$A_m = \alpha \pi T_m / 2K_m K_c \tau_m \quad (5.77)$$

Equation (5.22) becomes

$$K_m K_c / \omega_g T_m = 1 \quad (5.78)$$

i.e.

$$\omega_g = K_m K_c / T_m \quad (5.79)$$

Finally, substituting Equations (5.73), (5.74) and (5.79) into Equation (5.20) gives

$$\phi_m = 0.5\pi - K_m K_c \tau_m / \alpha T_m \quad (5.80)$$

Substituting Equation (5.72), (5.73), (5.74) and (5.76) into Equation (5.21) gives

$$A_m = \pi \alpha / 2a \quad (5.81)$$

and substituting Equation (5.72) into Equation (5.80) gives

$$\phi_m = 0.5\pi - a/\alpha \quad (5.82)$$

This design reduces to the PI controller design when $\alpha = 1$.

5.5.2.2 Processes modelled in SOSPD form – series controller

For such processes and controllers,

$$G_m(s) = \frac{K_m e^{-s\tau_m}}{(1+sT_{m1})(1+sT_{m2})} \quad (5.83)$$

with

$$G_c(s) = K_c \left(1 + \frac{1}{T_i s} \right) (1+sT_d) \quad (5.84)$$

Therefore,

$$G_m(s)G_c(s) = \frac{K_m K_c e^{-s\tau_m} (1+sT_i)(1+sT_d)}{T_i s (1+sT_{m1})(1+sT_{m2})} \quad (5.85)$$

If T_i and T_d are designed as follows:

$$T_i = T_{m1} \quad (5.86)$$

and

$$T_d = T_{m2} \quad (5.87)$$

then, from Equation (5.85)

$$G_m(s)G_c(s) = \frac{K_m K_c e^{-s\tau_m}}{T_i s} \quad (5.88)$$

which is equal to $G_m(s)G_c(s)$ in Section 5.5.1.1 when $T_i = T_m$. Therefore, designing

$$K_c = \frac{a T_m}{K_m \tau_m} \quad (5.89)$$

will allow $A_m = \pi/2a$ and $\phi_m = 0.5\pi - a$, as before.

5.5.2.3 Processes modelled in SOSPD form with a negative zero - classical controller 1

For such processes,

$$G_m(s) = \frac{K_m e^{-s\tau_m} (1+sT_{m3})}{(1+sT_{m1})(1+sT_{m2})} \quad (5.90)$$

with $G_c(s)$ given by Equation (5.19). Therefore,

$$G_m(s)G_c(s) = \frac{K_m K_c e^{-s\tau_m} (1+sT_{m3})(1+sT_i)(1+sT_d)}{T_i s(1+sT_{m1})(1+sT_{m2})(1+\alpha sT_d)} \quad (5.91)$$

If T_i , T_d and α are designed as follows:

$$T_i = T_{m1} \quad (5.92)$$

$$T_d = T_{m2} \quad (5.93)$$

and

$$\alpha = T_{m3}/T_{m2} \quad (5.94)$$

therefore, designing

$$K_c = \frac{aT_m}{K_m \tau_m} \quad (5.95)$$

will allow $A_m = \pi/2a$ and $\phi_m = 0.5\pi - a$, as in Sections 5.5.1.1 and 5.5.2.2.

5.5.3 PD controller design

In this case, the process is modelled in FOLIPD form, i.e.

$$G_m(s) = \frac{K_m e^{-s\tau_m}}{s(1+sT_m)} \quad (5.96)$$

with

$$G_c(s) = K_c (1 + T_d s) \quad (5.97)$$

Therefore,

$$G_m(s)G_c(s) = \frac{K_m K_c e^{-s\tau_m} (1+sT_d)}{s(1+sT_m)} \quad (5.98)$$

Therefore, designing

$$K_c = \frac{aT_m}{K_m \tau_m} \quad (5.99)$$

and

$$T_d = T_m \quad (5.100)$$

will allow $A_m = \pi/2a$ and $\phi_m = 0.5\pi - a$, as in Sections 5.5.1.1, 5.5.2.2 and 5.5.2.3.

5.6 Conclusions

This chapter has considered the performance and robustness of a PI and PID controlled FOLPD process, with the parameters of the controllers determined by a variety of tuning rules. The original contributions of this work are as follows: (a) an expansion of the analytical approach of Ho *et al.* (1995b; 1996) to determine the approximate gain and phase margin analytically under all operating conditions (b) the analytical determination of the approximate maximum sensitivity of the compensated system and (c) the application of the algorithms using a wide variety of PI and PID tuning rules. The implementation of autotuning algorithms in commercial controllers means that choice of a suitable tuning rule is an important issue; the techniques discussed allow an analytical evaluation to be performed of candidate tuning rules.

Finally, the chapter discusses an original approach to design tuning rules for both PI and PID controllers, for a variety of delayed process models, with the objective of achieving constant gain and phase margins for all values of delay. In one of the cases discussed (PI control of an IPD process model), an analytical approximation is used in the development; this approximation may also be used to determine further tuning rules for other process models with delay, and for other PID controller structures.

Appendix 1

Glossary of Symbols Used in the Book

a_{f1}, a_{f2}, b_{f1} = Parameters of a filter in series with some PID controllers

$a_1, a_2, a_3, b_1, b_2, b_3$ = Parameters of a third order process model

$a_1, a_2, a_3, a_4, a_5, b_1, b_2, b_3, b_4, b_5$ = Parameters of a fifth order process model

$A_1 = y_1(\infty), A_2 = y_2(\infty), A_3 = y_3(\infty), A_4 = y_4(\infty), A_5 = y_5(\infty)$

A_m = Gain margin

A_p = Peak output amplitude of limit cycle determined from relay autotuning

b, c = Weighting factors in some PID controller structures

D_R = Desired closed loop damping ratio

$d(t)$ = Disturbance variable (time domain)

du/dt = Time derivative of the manipulated variable (time domain)

$e(t) = \text{Desired variable, } r(t), \text{ minus controlled variable, } y(t)$ (time domain)

$E(s) = \text{Desired variable, } R(s), \text{ minus controlled variable, } Y(s)$

FOLPD model = First Order Lag Plus time Delay model

FOLIPD model = First Order Lag plus Integral Plus time Delay model

F_p, F_i, F_d = Weights on the desired variable for one PID controller structure

$G_c(s)$ = PID controller transfer function

$G_{CL}(s)$ = Closed loop transfer function

$G_{CL}(j\omega)$ = Desired closed loop frequency response

$G_{cp}(s)$ = Ideal PID controller transfer function

$G_{cs}(s)$ = Series PID controller transfer function

$G_m(s)$ = Process model transfer function

$G_p(s)$ = Process transfer function

$G_p(j\omega)$ = Process transfer function at frequency ω

$|G_p(j\omega)|$ = Magnitude of $G_p(j\omega)$

$|G_p(j\omega_\phi)|$ = Magnitude of $G_p(j\omega)$, at frequency ω , corresponding to phase lag of ϕ

$\angle G_p(j\omega)$ = Phase of $G_p(j\omega)$

$\pm h$ = Relay amplitude (relay autotuning)

IE = Integral of Error = $\int_0^\infty |e(t)|dt$

IAE = Integral of Absolute Error = $\int_0^\infty |e(t)|dt$

IMC = Internal Model Controller

IPD model = Integral Plus time Delay model

I²PD model = Integral Squared Plus time Delay model

ISE = Integral of Squared Error = $\int_0^\infty e^2(t)dt$

ISTES = Integral of Squared Time multiplied by Error, all to be Squared
 $= \int_0^\infty [t^2 e(t)]^2 dt$

ISTSE = Integral of Squared Time multiplied by Squared Error =
 $\int_0^\infty t^2 e^2(t)dt$

ITSE = Integral of Time multiplied by Squared Error = $\int_0^\infty t e^2(t)dt$

ITAE = Integral of Time multiplied by Absolute Error = $\int_0^\infty t |e(t)|dt$

K_c = Proportional gain of the controller

K_{cp} = Proportional gain of the ideal PID controller

K_{cs} = Proportional gain of the series PID controller

K_f = Feedback gain used in non-interacting PID controller 10

$$K_H = \frac{9}{2\tau_m^2 K_m} \left[\frac{\tau_m^2}{18} - T_{ml}^2 - \frac{\xi_m T_{ml} \tau_m}{9} \right] \quad (\text{Hwang, 1995})$$

$$K_{H1} = \frac{9}{2K_m \tau_m^2} \left[\frac{\tau_m^2}{18} - \frac{\tau_m T_m}{18} + \frac{1.884 K_c K_u \tau_m}{9\omega_u} \right] \\ + \frac{9}{2K_m \tau_m} \left[\sqrt{\frac{\tau_m^2}{324} + \frac{49T_m^2}{324} + \frac{7T_m \tau_m}{162}} - \frac{4.71 K_u K_c (\tau_m + 1.6T_m)}{81\omega_u} - \frac{1.775 K_u^2 K_c^2}{81\omega_u^2} \right] \quad (\text{Hwang, 1995})$$

$$K_{H2} = \frac{9}{2K_m \tau_m^2} \left[\frac{\tau_m^2}{18} - T_{ml}^2 - \frac{\xi_m \tau_m T_{ml}}{9} + \frac{1.884 K_u K_c \tau_m}{9\omega_u} \right] + \frac{9}{2K_m \tau_m^2} \alpha$$

with

$$\alpha = \left[\sqrt{T_{ml}^4 + \frac{\tau_m^4}{324} + \frac{49\tau_m^2 \xi_m^2 T_{ml}^2}{81}} - \frac{\tau_m^2 T_{ml}^2}{9} + \frac{7T_{ml} \xi_m \tau_m^3}{81} + \frac{10\tau_m T_{ml}^3 \xi_m}{9} - a \right]$$

and

$$a = \frac{0.471 K_u K_c}{\omega_u} \left[\frac{10\tau_m^3}{81} + \frac{4T_{ml} \tau_m (8\xi_m \tau_m + 9T_{ml})}{81} \right] - \frac{1.775 K_u^2 K_c^2 \tau_m^2}{81\omega_u^2}$$

(Hwang, 1995)

K_i = Feedback gain term used in non-interacting PID controller 8, 11

K_I = Integral-only controller gain (Pinnella *et al.*, 1986)

K_L = Gain of a load disturbance process model

K_m = Gain of the process model

K_o = Weighting parameter used in one PID controller structure

K_p = Gain of the process

K_u = Ultimate proportional gain

K_ϕ = Proportional gain when $G_c(j\omega)G_p(j\omega)$ has a phase lag of ϕ

\hat{K}_u = Ultimate proportional gain estimate determined from relay autotuning

K_1 = Feedback gain term used in one PI controller structure (labeled controller with proportional term acting on the output 2) and the PID non-interacting controller 12 structure

$K_{25\%}$ = Proportional gain required to achieve a quarter decay ratio

$K_x\%$ = Proportional gain required to achieve a decay ratio of $0.01x$

K_{135° = Controller gain when $G_c(j\omega)G_p(j\omega)$ has a phase lag of 135°

K_{150° = Controller gain when $G_c(j\omega)G_p(j\omega)$ has a phase lag of 150°

M_s = Closed loop sensitivity

M_{max} = Maximum value of closed loop sensitivity, M_s

n = Order of a process model with a repeated pole

N = Parameter that determines the amount of filtering on the derivative term on some PID controller structures

OS = Closed loop response overshoot

PI controller = Proportional Integral controller

PID controller = Proportional Integral Derivative controller

r = Desired variable (time domain)

$$r_1 = \frac{0.7071M_{max}^2 - M_{max}\sqrt{1 - 0.5M_{max}^2}}{M_{max}^2 - 1} \quad (\text{Chen et al., 1999a})$$

$R(s)$ = Desired variable (Laplace domain)

s = Laplace variable

SOSPD model = Second Order System Plus time Delay model

SOSIPD model = Second Order System plus Integral Plus time Delay model

T_{ar} = Average residence time = time taken for the open loop process step response to reach 63% of its final value

T_d = Derivative time of the controller

T_{di} = Derivative feedback term used in non-interacting PID controller 8

T_{dp} = Derivative time of the ideal PID controller

T_{ds} = Derivative time of the series PID controller

T_{CL} = Desired closed loop system time constant

T_{CL2} = Desired parameter of second order closed loop system response

T_{CL3} = Desired parameter of third order closed loop system response,
with a repeated pole

T_f = Time constant of the filter in series with some PI or PID controllers

T_i = Integral time of the controller

T_{ip} = Integral time of the ideal PID controller

T_{is} = Integral time of the series PID controller

T_L = Time constant of a load disturbance FOLPD process model

T_m = Time constant of the process model

T_{m1}, T_{m2}, T_{m3} = Time constants of second or third order process models, as appropriate.

$T_{m1i}, T_{m2i}, T_{m3i}, T_{m4i}$ = Time constants of a general process model

T_p = Time constant of the process

T_r = Parameter of the filter on the set-point in some PID controller structures

T_R = Closed loop rise time

T_S = Closed loop settling time

T_u = Ultimate period

\hat{T}_u = Ultimate period estimate determined from relay autotuning

$T_{25\%}$ = Period of the quarter decay ratio waveform, when the closed loop system is under proportional control

$T_{x\%}$ = Period of the waveform with a decay ratio of $0.01x$, when the closed loop system is under proportional control

TOLPD model = Third Order Lag Plus time Delay model

$u(t)$ = Manipulated variable (time domain).

u_∞ = Final value of the manipulated variable (time domain)

$U(s)$ = Manipulated variable (Laplace domain)

V = Closed loop response overshoot (as a fraction of the controlled variable final value)

x_1, x_2, x_3, x_4, x_5 = Coefficient values.

y = Controlled variable (time domain)

y_∞ = Final value of the controlled variable (time domain)

$Y(s)$ = Controlled variable (Laplace domain)

$$y_1(t) = \int_0^t \left(K_m - \frac{y(\tau)}{\Delta u} \right) d\tau, \quad y_2(t) = \int_0^t (A_1 - y_1(\tau)) d\tau,$$

$$y_3(t) = \int_0^t (A_2 - y_2(\tau)) d\tau, \quad y_4(t) = \int_0^t [A_3 - y_3(\tau)] d\tau,$$

$$y_5(t) = \int_0^t [A_4 - y_4(\tau)] d\tau$$

α, β, γ = Weighting factors in some PI or PID controller structures

$$\varepsilon = \frac{6T_{ml}^2 + 4\xi_m T_{ml} \tau_m + K_H K_m \tau_m^2}{2T_{ml}^2 \tau_m \omega_H} \quad (\text{Hwang, 1995})$$

$$\varepsilon_1 = \frac{2T_m \omega_u + K_{H1} K_m \tau_m \omega_u - 1.884 K_u K_c}{0.471 K_c K_u \omega_{H1} \tau_m} \quad (\text{Hwang, 1995})$$

$$\varepsilon_0 = \frac{6T_{ml}^2 + 4T_{ml} \xi_m \tau_m + K_u K_m \tau_m^2}{2\tau_m T_{ml}^2 \omega_u} \quad (\text{Hwang, 1995})$$

$$\varepsilon_2 = \frac{6T_{ml}^2 \omega_u + 4T_{ml} \xi_m \tau_m \omega_u + K_{H2} K_m \tau_m^2 \omega_u - 1.884 K_u K_c \tau_m}{(0.471 K_u K_c \tau_m^2 + 2\tau_m T_{ml}^2 \omega_u) \omega_{H2}}$$

(Hwang, 1995)

λ = Parameter that determines robustness of compensated system.

ξ = Damping factor of the compensated system

ξ_m = Damping factor of an underdamped process model

ξ_m^{des} = Desired damping factor of an underdamped process model

$\xi_{m_{n,i}}, \xi_{m_{o,i}}$ = Damping factors of a general underdamped process model

$\kappa = 1/K_m K_u$

ϕ = Phase lag

ϕ_c = Phase of the plant at the crossover frequency of the compensated system, with a ‘conservative’ PI controller (Pecharroman and Pagola, 2000).

ϕ_m = Phase margin

ϕ_ω = Phase lag at an angular frequency of ω

τ_L = Time delay of a load disturbance process model

τ_m = Time delay of the process model

$$\tau = \tau_m / (\tau_m + T_m)$$

ω = Angular frequency

ω_{bw} = -3dB closed loop system bandwidth (Shi and Lee, 2002)

ω_{-6dB} = Frequency where closed loop system magnitude is -6dB (Shi and Lee, 2004)

ω_c = Maximum cut-off frequency

ω_d = Bandwidth of stochastic disturbance signal (Van der Grinten, 1963)

$$\omega_{CL} = 2\pi/T_{CL}$$

ω_g = Specified gain crossover frequency (Shi and Lee, 2002)

$$\omega_H = \sqrt{\frac{1 + K_H K_m}{T_{ml}^2 + \frac{2T_{ml}\tau_m\xi_m}{3} + \frac{K_H K_m \tau_m^2}{6}}} \quad (\text{Hwang, 1995})$$

$$\omega_{H1} = \sqrt{\frac{1 + K_{H1} K_m}{\frac{\tau_m T_m}{3} + \frac{K_{H1} K_m \tau_m^2}{6} - \frac{0.942 K_c K_u \tau_m}{3\omega_u}}} \quad (\text{Hwang, 1995})$$

$$\omega_{H2} = \sqrt{\frac{1 + K_{H2} K_m}{T_{ml}^2 + \frac{2\xi_m \tau_m T_{ml}}{3} + \frac{K_{H2} K_m \tau_m^2}{6} - \frac{0.942 K_c K_u \tau_m}{3\omega_u}}} \quad (\text{Hwang, 1995})$$

$\omega_{M_{max}}$ = Frequency where the sensitivity function is maximized (Rotach, 1994)

ω_n = Undamped natural frequency of the compensated system

$$\omega_p = \frac{A_m \phi_m + 0.5\pi A_m (A_m - 1)}{(A_m^2 - 1)\tau_m} \quad (\text{Hang et al. 1993a, 1993b})$$

ω_r = Resonant frequency (Rotach, 1994)

ω_u = Ultimate frequency

ω_ϕ = Angular frequency at a phase lag of ϕ

ω_{90° = Angular frequency at a phase lag of 90°

ω_{135° = Angular frequency at a phase lag of 135°

ω_{150° = Angular frequency at a phase lag of 150°

Appendix 2

Some Further Details on Process Modelling

Processes with time delay may be modelled in a variety of ways. The modelling strategy used will influence the value of the model parameters, which will in turn affect the controller values determined from the tuning rules. The modelling strategy used in association with each tuning rule, as described in the original papers, is indicated in the tables in Chapters 3 and 4. Some outline details of these modelling strategies are provided, together with the references that describe the modelling method in detail. The references are given in the bibliography. For all models, the label “Model: Method 1” indicates that the model method has not been defined or that the model parameters are assumed known. Of the 1134 tuning rules specified (442 PI controller tuning rules, 692 PID controller tuning rules), it is startling to relate that only 386 (or 34%) of tuning rules have been specified based on a defined process modelling method.

$$\text{A2.1 FOLPD Model } G_m(s) = \frac{K_m e^{-s\tau_m}}{1 + sT_m}$$

A2.1.1 *Parameters estimated from the open loop process step or impulse response*

Method 2: Parameters estimated using a tangent and point method (Ziegler and Nichols, 1942).

Method 3: K_m, τ_m determined from the tangent and point method of Ziegler and Nichols (1942); T_m determined at 60% of the

total process variable change (Fertik and Sharpe, 1979).

- Method 4: K_m, τ_m assumed known; T_m estimated using a tangent method (Wolfe, 1951).
- Method 5: Parameters estimated using a second tangent and point method (Murrill, 1967).
- Method 6: Parameters estimated using a third tangent and point method (Davydov *et al.*, 1995).
- Method 7: τ_m and T_m estimated using the two-point method; K_m estimated from the open loop step response (ABB, 2001).
- Method 8: τ_m and T_m estimated from the process step response:

$$T_m = 1.4(t_{67\%} - t_{33\%}), \quad \tau_m = t_{67\%} - 1.1T_m; \quad K_m \text{ assumed known (Chen and Yang, 2000).}$$
¹
- Method 9: τ_m and T_m estimated from the process step response:

$$T_m = 1.245(t_{70\%} - t_{33\%}), \quad \tau_m = 1.498t_{33\%} - 0.498t_{70\%}; \quad K_m \text{ assumed known (Vítečková } et al., 2000b).$$
²
- Method 10: K_m estimated from the process step response; τ_m = time for which the process variable does not change; T_m determined at 63% of the total process variable change (Gerry, 1999).
- Method 11: K_m estimated from the process step response; τ_m = time for which the process variable has to change by 5% of its total value; T_m determined at 63% of the total process variable change (Kristiansson, 2003).
- Method 12: Parameters estimated using a least squares method in the time domain (Cheng and Hung, 1985).
- Method 13: Parameters estimated from linear regression equations in the time domain (Bi *et al.*, 1999).
- Method 14: Parameters estimated using the method of moments (Åström and Hägglund, 1995).
- Method 15: Parameters estimated from the process step response and its first time derivative (Tsang and Rad, 1995).
- Method 16: Parameters estimated from the process step response using numerical integration procedures (Nishikawa *et al.*, 1984).

¹ $t_{67\%}, t_{33\%}$ are the times taken by the process variable to change by 67% and 33%, respectively, of its total value.

² $t_{70\%}$ is the time taken by the process variable to change by 70% of its total value.

- Method 17: Parameters estimated from the process impulse response (Peng and Wu, 2000).
- Method 18: Parameters estimated using a “process setter” block (Saito *et al.*, 1990).
- Method 19: Parameters estimated from a number of process step response data values (Kraus, 1986).
- Method 20: Parameters estimated in the sampled data domain using two process step tests (Pinnella *et al.*, 1986).
- Method 21: A model is obtained using the tangent and point method of Ziegler and Nichols (1942); label the parameters K_m' , T_m' and τ_m' . Subsequently, τ_m is estimated from the process step response; then, a parameter labelled $\tau^a_m = \tau'_m - \tau_m$ (Schaedel, 1997).
- Method 22: A model is obtained using the tangent and point method of Ziegler and Nichols (1942); label the parameters K_m' , T_m' and τ_m' . Subsequently, τ_m is estimated from the process step response (Henry and Schaedel, 2005).

A2.1.2 Parameters estimated from the closed loop step response

- Method 23: K_m estimated from the open loop step response. $T_{90\%}$ and τ_m estimated from the closed loop step response under proportional control (Åström and Hägglund, 1988).
- Method 24: Parameters estimated using a method based on the closed loop transient response to a step input under proportional control (Sain and Özgen, 1992).
- Method 25: Parameters estimated using a second method based on the closed loop transient response to a step input under proportional control (Hwang, 1993).
- Method 26: Parameters estimated using a third method based on the closed loop transient response to a step input under proportional control (Chen, 1989; Taiwo, 1993).

- Method 27: Parameters estimated from a step response autotuning experiment – Honeywell UDC 6000 controller (Åström and Hägglund, 1995).
- Method 28: Parameters estimated from the closed loop step response when process is in series with a PID controller (Morilla *et al.*, 2000).
- Method 29: Parameters estimated by modelling the closed loop response as a second order system (Ettaleb and Roche, 2000).

A2.1.3 *Parameters estimated from frequency domain closed loop information*

- Method 30: Parameters estimated from two points, determined on process frequency response, using a relay and a relay in series with a delay (Tan *et al.*, 1996).
- Method 31: T_m and τ_m estimated from the ultimate gain and period, determined using a relay in series with the process in closed loop; K_m assumed known (Hang and Cao, 1996).
- Method 32: T_m and τ_m estimated from the ultimate gain and period, determined using a relay in series with the process in closed loop; K_m estimated from the process step response (Hang *et al.*, 1993b).
- Method 33: T_m and τ_m estimated from the ultimate gain and period, determined using the ultimate cycle method of Ziegler and Nichols (1942); K_m estimated from the process step response (Hang *et al.*, 1993b).
- Method 34: T_m and τ_m estimated from relay autotuning method (Lee and Sung, 1993); K_m estimated from the closed loop process step response under proportional control (Chun *et al.*, 1999).
- Method 35: Parameters estimated in the frequency domain from the data determined using a relay in series with the closed loop system in a master feedback loop (Hwang, 1995).

- Method 36: K_u and T_u estimated from relay autotuning method; τ_m estimated from the open loop process step response, using a tangent and point method (Wojsznis *et al.*, 1999).
- Method 37: Parameters estimated by including a dynamic compensator outside or inside an (ideal) relay feedback loop (Huang and Jeng, 2003).
- Method 38: Parameters estimated from measurements performed on the manipulated and controlled variables when a relay with hysteresis is introduced in place of the controller (Zhang *et al.*, 1996).
- Method 39: Non-gain parameters estimated using a relay, with hysteresis, in series with the process in closed loop; K_m is estimated with the aid of a small step signal added to the reference (Potočnik *et al.*, 2001).
- Method 40: Parameters estimated using a relay in series with the process in closed loop (Perić *et al.*, 1997).
- Method 41: Parameters estimated using an iterative method, based on data from a relay experiment (Leva and Colombo, 2004).
- Method 42: K_m estimated from relay autotuning method; τ_m , T_m estimated using a least squares algorithm based on the data recorded from the relay autotuning method, with the aid of neural networks (Huang *et al.*, 2005).

A2.1.4 Other methods

- Method 43: Parameter estimates back-calculated from a discrete time identification method (Ferretti *et al.*, 1991).
- Method 44: Parameter estimates determined graphically from a known higher order process (McMillan, 1984).
- Method 45: Parameter estimates determined from a known higher order process.
- Method 46: The parameters of a second order model plus time delay are estimated using a system identification approach in discrete time; the parameters of a FOLPD model are subsequently determined using standard equations (Ou and Chen, 1995).

Method 47: Parameter estimates back-calculated from a discrete time identification non-linear regression method (Gallier and Otto, 1968).

Table 188 summaries the most common FOLPD process modelling methods.

Table 188: The most common FOLPD process modelling methods

Model Method	PI	PID	Total
Method 1	112	147	259
Method 2	18	36	54
Method 5	12	26	38
Method 31	6	6	12

It is interesting that in 57% of tuning rules based on the FOLPD process model (252 out of 443), the modelling method has not been specified or the model parameters are known *a priori*.

A2.2 FOLPD Model with a Negative Zero $G_m(s) = \frac{K_m(1+sT_{m2})e^{-sT_m}}{1+sT_{m1}}$

Method 2: Non-delay parameters are estimated in the discrete time domain using a least squares approach; time delay assumed known (Chang *et al.*, 1997).

A2.3 Non-Model Specific

Modelling methods have not been specified in the tables in most cases, as typically the tuning rules are based on K_u and T_u . Modelling methods have been specified whenever relevant data have been estimated using a relay method.

Method 2: K_u and T_u estimated from an experiment using a relay in series with the process in closed loop (Jones *et al.*, 1997).

Method 3: K_u and T_u estimated with the assistance of a relay (Lloyd, 1994).

Method 4: ω_{90° and A_p estimated using a relay in series with an integrator (Tang *et al.*, 2002).

$$\text{A2.4 IPD Model } G_m(s) = \frac{K_m e^{-s\tau_m}}{s}$$

A2.4.1 Parameters estimated from the open loop process step response

- Method 2: τ_m assumed known; K_m estimated from the slope at start of the process step response (Ziegler and Nichols, 1942).
- Method 3: K_m, τ_m estimated from the process step response (Hay, 1998).
- Method 4: K_m, τ_m estimated from the process step response using a tangent and point method (Bunzemeier, 1998).

A2.4.2 Parameters estimated from the closed loop step response

- Method 5: Parameters estimated from the servo or regulator closed loop transient response, under PI control (Rotach, 1995).
- Method 6: Parameters estimated from the servo closed loop transient response under proportional control (Srividya and Chidambaram, 1997).
- Method 7: Parameters estimated from the closed loop response, under the control of an on-off controller (Zou *et al.*, 1997).
- Method 8: Parameters estimated from the closed loop response, under the control of an on-off controller (Zou and Brigham, 1998).

A2.4.3 Parameters estimated from frequency domain closed loop information

- Method 9: Parameters estimated from K_u and T_u values, determined from an experiment using a relay in series with the process in closed loop (Tyreus and Luyben, 1992).
- Method 10: Parameters estimated from values determined from an

experiment using an amplitude dependent gain in series with the process in closed loop (Pecharromán and Pagola, 1999).

The most common IPD modeling method is the one where the model parameters are known *a-priori* or the modeling method is unspecified (i.e. Model: Method 1); 46 PI controller tuning rules, and 47 PID controller tuning rules have been specified using the method. Altogether 76% of tuning rules based on the IPD process model (93 out of 122) have been specified using the method.

$$\text{A2.5 FOLIPD Model } G_m(s) = \frac{K_m e^{-s\tau_m}}{s(1+sT_m)}$$

- Method 2: Parameters estimated from the open loop process step response and its first and second time derivatives (Tsang and Rad, 1995).
- Method 3: Parameters estimated using the method of moments (Åström and Hägglund, 1995).
- Method 4: Parameters estimated from values determined from an experiment using an amplitude dependent gain in series with the process in closed loop (Pecharromán and Pagola, 1999).
- Method 5: Parameters estimated from the open loop response of the process to a pulse signal (Tachibana, 1984).
- Method 6: Parameters estimated from the waveform obtained by introducing a single symmetrical relay in series with the process in closed loop (Majhi and Mahanta, 2001).
- Method 7: K_m estimated from the response of the process to a square wave pulse; other process parameters estimated from the waveform obtained by introducing a relay in series with the process in closed loop (Wang *et al.*, 2001a).
- Method 8: Parameters estimated using a relay in series with the process in closed loop (Perić *et al.*, 1997).

The most common FOLIPD modeling method is the one where the model parameters are known *a-priori* or the modeling method is

unspecified (i.e. Model: Method 1); 25 PI controller tuning rules, and 49 PID controller tuning rules have been specified using the method. Altogether 85% of tuning rules based on the FOLIPD process model (74 out of 87) have been specified using the method.

$$\text{A2.6 SOSPD Model } G_m(s) = \frac{K_m e^{-s\tau_m}}{T_{m1}^2 s^2 + 2\xi_m T_{m1}s + 1} \text{ or}$$

$$\frac{K_m e^{-s\tau_m}}{(1 + T_{m1}s)(1 + T_{m2}s)}$$

A2.6.1 Parameters estimated from the open loop process step response

- Method 2: K_m , T_{m1} and τ_m are determined from the tangent and point method of Ziegler and Nichols (1942) (Shinskey, 1988, page 151); T_{m2} assumed known.
- Method 3: FOLPD model parameters estimated using a tangent and point method (Ziegler and Nichols, 1942); corresponding SOSPD parameters subsequently deduced (Auslander *et al.*, 1975).
- Method 4: $T_{m1} = T_{m2}$. τ_m , T_{m1} estimated from process step response: $T_{m1} = 0.794(t_{70\%} - t_{33\%})$, $\tau_m = 1.937t_{33\%} - 0.937t_{70\%}$. K_m assumed known (Vítečková *et al.*, 2000b).
- Method 5: $T_{m1} = T_{m2}$. K_m , τ_m and T_{m1} estimated from the process step response; τ_m = time for which the process variable does not change; T_{m1} determined at 73% of the total process variable change (Pomerleau and Poulin, 2004).
- Method 6: Parameters estimated from the underdamped or overdamped transient response in open loop to a step input (Jahanmiri and Fallahi, 1997).

A2.6.2 Parameters estimated from the closed loop step response

Method 7: Parameters estimated from a step response autotuning experiment – Honeywell UDC 6000 controller (Åström and Hägglund, 1995).

A2.6.3 Parameters estimated from frequency domain closed loop information

Method 8: In this method, $T_{m1} = T_{m2} = T_m$. T_m and τ_m estimated from K_u , T_u determined using a relay autotuning method; K_m estimated from the process step response (Hang *et al.*, 1993a).

Method 9: Parameters estimated using a two-stage identification procedure involving (a) placing a relay and (b) placing a proportional controller, in series with the process in closed loop (Sung *et al.*, 1996).

Method 10: Parameters estimated in the frequency domain from the data determined using a relay in series with the closed loop system in a master feedback loop (Hwang, 1995).

Method 11: Parameters estimated from values determined from an experiment using an amplitude dependent gain in series with the process in closed loop (Pecharromán and Pagola, 1999).

Method 12: Parameters estimated from data obtained when the process phase lag is -90° and -180° , respectively (Wang *et al.*, 1999).

Method 13: Model parameters estimated by including a dynamic compensator outside or inside an (ideal) relay feedback loop (Huang and Jeng, 2003).

Method 14: K_m estimated from a relay autotuning method; τ_m , T_{m1} and ξ_m estimated using a least squares algorithm based on the data recorded from the relay autotuning method, with the aid of neural networks (Huang *et al.*, 2005).

A2.6.4 Other methods

- Method 15: Parameter estimates back-calculated from a discrete time identification method (Ferretti *et al.*, 1991).
- Method 16: Parameter estimates back-calculated from a second discrete time identification method (Wang and Clements, 1995).
- Method 17: Parameter estimates back-calculated from a third discrete time identification method (Lopez *et al.*, 1969).
- Method 18: Model parameters estimated assuming higher order process parameters are known.
- Method 19: Parameter estimates back-calculated from a discrete time identification non-linear regression method (Gallier and Otto, 1968).

The most common SOSPD modeling method is the one where the model parameters are known *a-priori* or the modeling method is unspecified (i.e. Model: Method 1); 15 PI controller tuning rules, and 93 PID controller tuning rules have been specified using the method. Altogether 68% of tuning rules based on the SOSPD process model (108 out of 160) have been specified using the method.

$$\text{A2.7 SOSIPD model – Repeated Pole } G_m(s) = \frac{K_m e^{-sT_m}}{s(1+sT_m)^2}$$

- Method 2: Parameters estimated from values determined from an experiment using an amplitude dependent gain in series with the process in closed loop (Pecharromán and Pagola, 1999).

A2.8 SOSPD Model with a Positive Zero $G_m(s) = \frac{K_m(1-sT_{m3})e^{-s\tau_m}}{(1+sT_{m1})(1+sT_{m2})}$

- Method 2: Parameters estimated from a closed loop step response test using a least squares approach (Wang *et al.*, 2001a).
- Method 3: $T_{m1} = T_{m2}$. K_m , τ_m , T_{m1} and T_{m3} estimated from the process step response; the latter three parameters are estimated using a tabular approach (Pomerleau and Poulin, 2004).

A2.9 SOSPD model with a Negative Zero $G_m(s) = \frac{K_m(1+sT_{m3})e^{-s\tau_m}}{(1+sT_{m1})(1+sT_{m2})}$

- Method 2: Parameters estimated from a closed loop step response test using a least squares approach (Wang *et al.*, 2001a).
- Method 3: $T_{m1} = T_{m2}$. K_m , τ_m , T_{m1} and T_{m3} estimated from the process step response; the latter three parameters are estimated using a tabular approach (Pomerleau and Poulin, 2004).

A2.10 Unstable FOLPD Model $G_m(s) = \frac{K_m e^{-s\tau_m}}{T_m s - 1}$

- Method 2: Parameters estimated by least squares fitting from the open loop frequency response of the unstable process; this is done by determining the closed loop magnitude and phase values of the (stable) closed loop system and using the Nichols chart to determine the open loop response (Huang and Lin, 1995; Deshpande, 1980).
- Method 3: Parameters estimated using a relay feedback approach (Majhi and Atherton, 2000).

- Method 4: Parameters estimated using a biased relay feedback test (Huang and Chen, 1999).

A2.11 Unstable SOSPD Model $G_m(s) = \frac{K_m e^{-s\tau_m}}{(T_{m1}s - 1)(1 + sT_{m2})}$ or
 $\frac{K_m e^{-s\tau_m}}{(T_{m1}s - 1)(T_{m2}s - 1)}$

- Method 2: Parameters estimated by least squares fitting from the open loop frequency response of the unstable process; this is done by determining the closed loop magnitude and phase values of the (stable) closed loop system and using the Nichols chart to determine the open loop response (Huang and Lin, 1995; Deshpande, 1980).
- Method 3: Parameters estimated by minimising the difference in the frequency responses between the high order process and the model, up to the ultimate frequency. The gain and delay are estimated from analytical equations, with the other parameters estimated using a least squares method (Kwak *et al.*, 2000).
- Method 4: Parameters estimated using a biased relay feedback test (Huang and Chen, 1999).

A2.12 General Model with a Repeated Pole $G_m(s) = \frac{K_m e^{-s\tau_m}}{(1 + sT_m)^n}$

- Method 2: A FOLPD model is obtained using the tangent and point method of Ziegler and Nichols (1942); label the parameters K'_m , T'_m and τ'_m . Then, $n = 10 \frac{\tau'_m}{T'_m} + 1$. Subsequently, τ_m is estimated from the open loop step response, and T_{ar} is estimated using “known methods” (Schaedel (1997)).

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