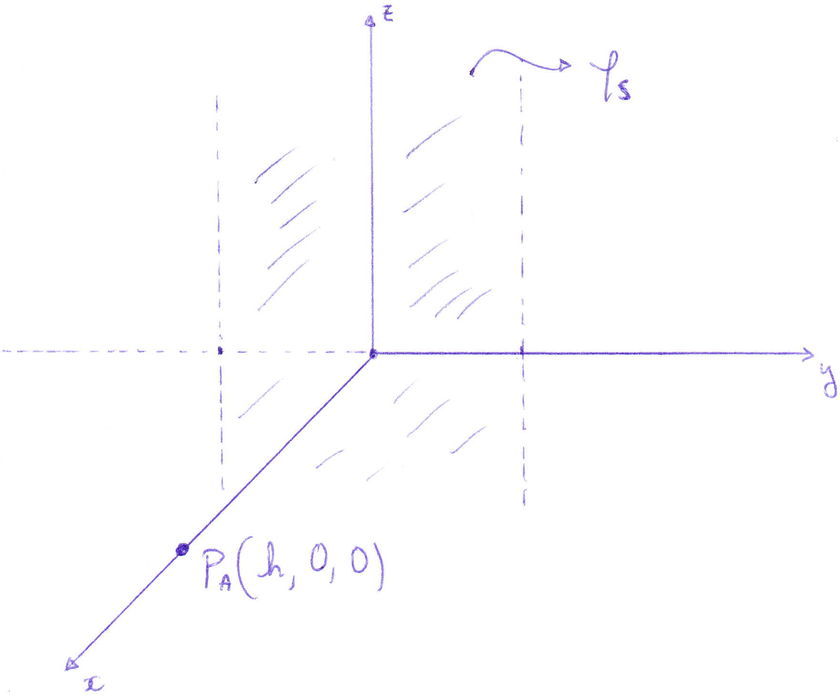
Problema 1:



$$\vec{E} = \iint \frac{\rho_s \, dA \quad (\vec{r} - \vec{r}\,')}{4\pi\varepsilon_0 \, |\vec{r} - \vec{r}\,'|^3}$$

$\vec{r} = h\,\hat{a}_x$

$\vec{r}\,' = y\,\hat{a}_y + z\,\hat{a}_z$

$\vec{r} - \vec{r}\,' = h\hat{a}_x - y\hat{a}_y - z\hat{a}_z \quad , \quad dA = dy\,dz$

$|\vec{r} - \vec{r}\,'|^3 = (h^2 + y^2 + z^2)^{3/2}$

$$\vec{E} = \int_{z=-\infty}^{+\infty} \int_{y=-a}^{y=+a} \frac{\rho_s (h\hat{a}_x - y\hat{a}_y - z\hat{a}_z)\, dy\, dz}{4\pi\varepsilon_0 (h^2 + y^2 + z^2)^{3/2}}$$

Las componentes en $\hat{a}_y$ y $\hat{a}_z$ son cero, funciones impares

Así

$$\vec{E} = \frac{\rho_s h \hat{a}_x}{4\pi\varepsilon_0} \int_{z=-\infty}^{+\infty} \int_{y=-a}^{y=+a} \frac{dy\,dz}{(h^2+y^2+z^2)^{3/2}} \; ;$$

Componente en $\hat{a}_x$;

$$\vec{E} = \frac{\rho_s h}{4\pi\varepsilon_0}\,\hat{a}_x \int_{-a}^{+a} dy \underbrace{\int_{z=-\infty}^{z=+\infty} \frac{dz}{(h^2+y^2+z^2)^{3/2}}}$$

$$= \underbrace{\left.\frac{z}{\sqrt{z^2+y^2+h^2}}\right|_{z=-\infty}^{z=+\infty}}_{2} \cdot \frac{1}{(y^2+h^2)}$$

$$\longrightarrow \quad \vec{E} = \frac{\rho_s \cdot h}{4\pi\varepsilon_0}\,\hat{a}_x \cdot 2 \underbrace{\int_{-a}^{+a} \frac{dy}{y^2+h^2}}_{\left.\frac{\tan^{-1}(y/h)}{h}\right|_{-a}^{+a}}$$

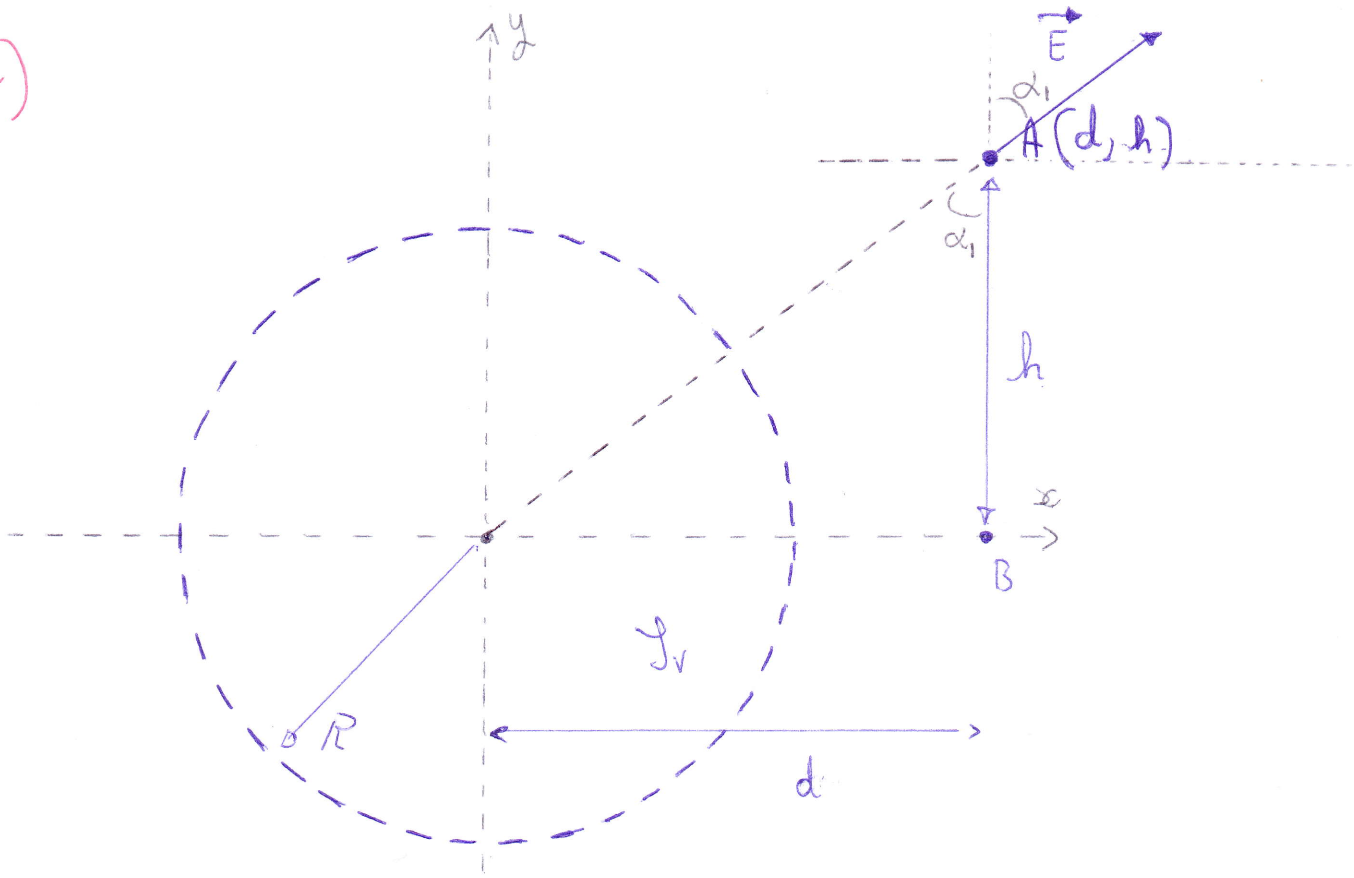
$$\vec{E} = \frac{\rho_s h\, \hat{a}_x}{4\pi\varepsilon_0} \cdot 2 \cdot \frac{1}{h} \cdot \left[\tan^{-1}(a/h) - \tan^{-1}(-a/h)\right]$$

$$= \frac{\rho_s h\, \hat{a}_x}{4\pi\varepsilon_0} \cdot 2 \cdot \frac{1}{h} \cdot 2\tan^{-1}(a/h)$$

$$\vec{E} = \frac{\rho_s\, \hat{a}_x \cdot \tan^{-1}(a/h)}{\pi\varepsilon_0}$$

Problema 2:

a)



$$\oint \vec{D} \cdot d\vec{s} = Q_{enc.}$$

$$D_r(r) \cdot 4\pi r^2 = \frac{4}{3}\pi R^3 \rho_v$$

$$\vec{D} = \frac{R^3}{3r^2}\rho_v \hat{a}_{r1} \; ; \qquad \hat{a}_{r1} = \operatorname{sen}\alpha_1 \hat{a}_x + \cos\alpha_1 \hat{a}_y$$

$$= \frac{d}{(d^2+y^2)^{1/2}}\hat{a}_x + \frac{y}{(d^2+y^2)^{1/2}}\hat{a}_y$$

$$\vec{D} = \frac{R^3}{3r^2}\rho_v \left[\frac{d}{(d^2+y^2)^{1/2}}\hat{a}_x + \frac{y}{(d^2+y^2)^{1/2}}\hat{a}_y\right] \; ; \; r^2 = (d^2+y^2)$$

$$= \frac{R^3\rho_v}{3}\left[\frac{d}{(y^2+d^2)^{3/2}}\hat{a}_x + \frac{y}{(y^2+d^2)^{3/2}}\hat{a}_y\right] \rightarrow \vec{D}\Big|_{y=h} \; ;$$

b)

$$W = -Q\int_{inicio}^{fin} \vec{E}\cdot d\vec{l}$$

$$d\vec{l} = dy\,\hat{a}_y \;;$$

$$\vec{E} = \frac{R^3\rho_v}{3\varepsilon_0}\left[\frac{d}{(y^2+d^2)^{3/2}}\hat{a}_x + \frac{y}{(y^2+d^2)^{3/2}}\hat{a}_y\right] \;;\quad \vec{E}\cdot d\vec{l}$$

$$\vec{E}\cdot d\vec{l} = \frac{R^3\rho_v}{3\varepsilon_0}\,\frac{y}{(y^2+d^2)^{3/2}}\,dy$$

$$\longrightarrow W = -Q\int_{y=h}^{0}\frac{R^3\rho_v}{3\varepsilon_0}\cdot\frac{y}{(y^2+d^2)^{3/2}}\,dy$$

$$= Q\cdot\frac{R^3\rho_v}{3\varepsilon_0}\underbrace{\int_0^h\frac{y}{(y^2+d^2)^{3/2}}\,dy}\;;\quad \begin{aligned} y^2+d^2 &= u \\ 2y\,dy &= du\end{aligned}$$

$$\int_{d^2}^{h^2+d^2}\frac{du}{2\cdot u^{3/2}} = \frac{1}{2}u^{-1/2}\cdot -2$$

$$= -u^{-1/2}\Big|_{d^2}^{h^2+d^2}$$

$$= -\left(\frac{1}{\sqrt{h^2+d^2}} - \frac{1}{\sqrt{d^2}}\right)$$

$$= \frac{1}{d} - \frac{1}{\sqrt{h^2+d^2}}$$

$$W = Q\cdot\frac{R^3\rho_v}{3\varepsilon_0}\left(\frac{1}{d} - \frac{1}{\sqrt{h^2+d^2}}\right)$$

c).

$$\oint \vec{D} \cdot d\vec{S} = Q_{enc.}$$

$\vdots$

$$\Rightarrow D_r(r) \cdot 4\pi r^2 = \frac{4}{3}\pi r^3 \cdot \rho_v$$

$$\left.\begin{aligned} \vec{D} &= \frac{r\rho_v}{3}\,\hat{a}_r \\ \vec{E} &= \frac{r}{3\varepsilon_0}\cdot\rho_v\,\hat{a}_r \end{aligned}\right\} (r < R).$$

Luego.

$$\frac{\Delta W_E}{dV} = \frac{1}{2}\vec{D} \cdot \vec{E}$$

$$= \frac{1}{2} \cdot \frac{r}{3}\rho_v \hat{a}_r \cdot \frac{r}{3\varepsilon_0}\rho_v \hat{a}_r ;$$

$$\frac{\Delta W_E}{\Delta V} = \frac{r^2 \rho_v^2}{18\varepsilon_0}$$