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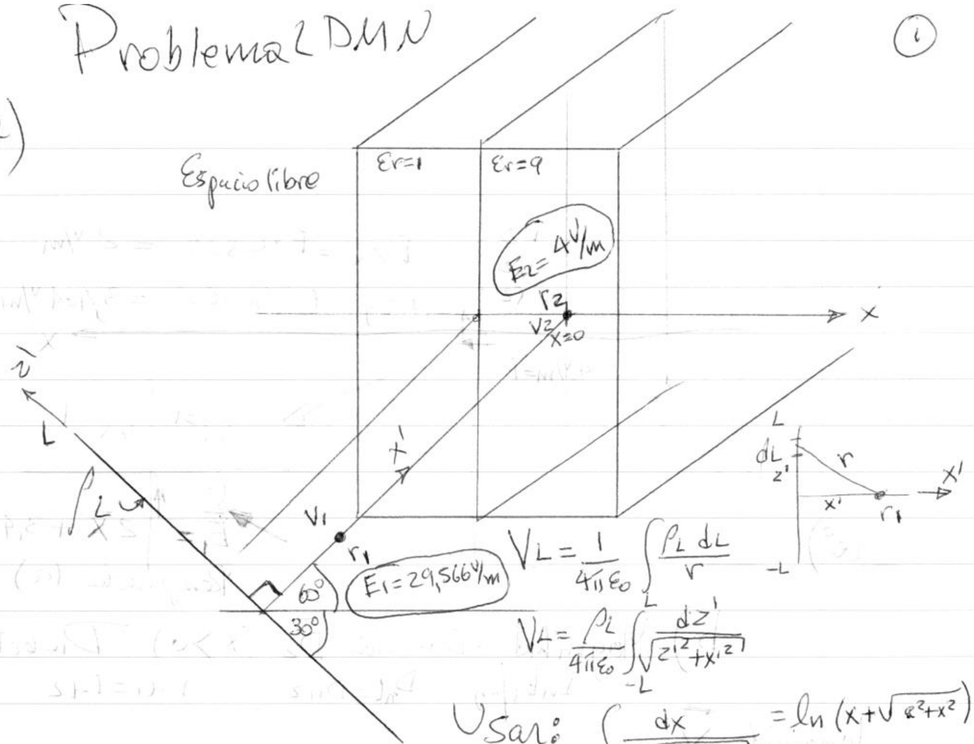
Solución:

Problema 2 DMN (1)

a)



$$V_L = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho_L\, dL}{r}$$

$$V_L = \frac{\rho_L}{4\pi\varepsilon_0}\int_{-L}^{L} \frac{dz'}{\sqrt{z'^2+x'^2}}$$

Usar: $\displaystyle\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left(x+\sqrt{a^2+x^2}\right)$

En r_1

$$\Rightarrow V_1 = 2\int_0^L \frac{\rho_L}{4\pi\varepsilon_0}\frac{dz'}{\sqrt{z'^2+r_1^2}} = \frac{2\rho_L}{4\pi\varepsilon_0}\ln\left(z'+\sqrt{z'^2+r_1^2}\right)\Big|_0^L$$

$$= \frac{\rho_L}{2\pi\varepsilon_0}\left[\ln\left(L+\sqrt{L^2+r_1^2}\right) - \ln r_1\right]$$

En r_2

$$\Rightarrow V_2 \Rightarrow V_2 = \frac{\rho_L}{2\pi\varepsilon_0}\left[\ln\left(L+\sqrt{L^2+r_2^2}\right) - \ln r_2\right]$$

Si $L \gg r_2$, $L \gg r_1$

$$V_1 \cong \frac{\rho_L}{2\pi\varepsilon_0}\left(\ln 2L - \ln r_1\right) ; \quad V_2 \cong \left(\ln 2L - \ln r_2\right)$$

respecto a 2

$$V_{12} = V_1 - V_2 \approx \frac{\rho_L}{2\pi\varepsilon_0}\ln\frac{r_2}{r_1}$$

Para línea de carga (Libro): $E_r = \dfrac{\rho_L}{2\pi\varepsilon_0}\dfrac{1}{r}$

Datos: $29{,}566\frac{V}{m}\cdot(0{,}406\,m) = \dfrac{\rho_L}{2\pi\varepsilon_0}$

$$\Rightarrow \frac{\rho_L}{2\pi\varepsilon_0} \cong 12 \; ; \quad V_1 - V_2 = 24V = \frac{\rho_L}{2\pi\varepsilon_0}\ln\frac{r_2}{r_1} \Rightarrow r_2 \cong 3m$$

Entonces $E_r = 12\,\frac{1}{3} = 4\,V/m$ ← Usar este valor en r_2 → Interfaz!!

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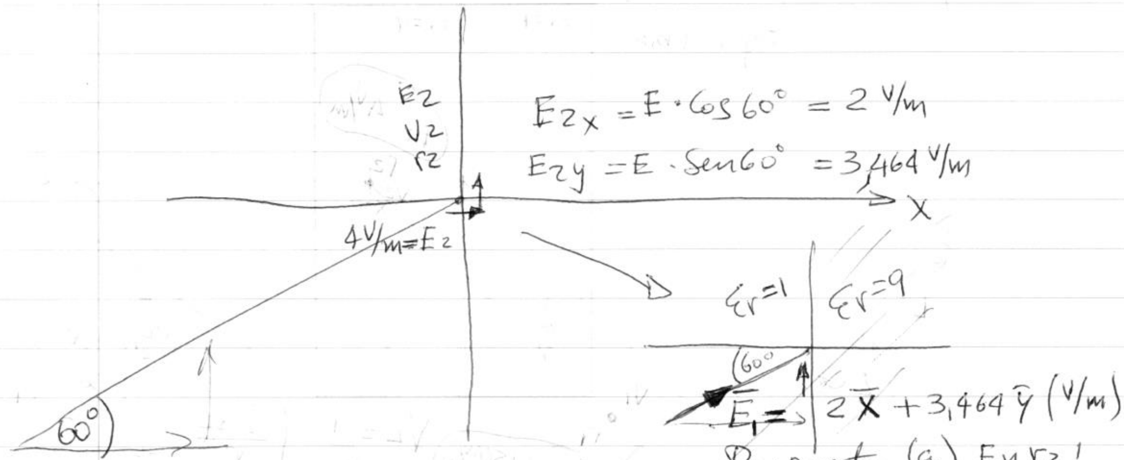
IE-0307 – II ciclo 2016

Primer examen parcial - 10/09/2016 Duración 3 horas 1.2

(2)

$E_{2x} = E \cdot \cos 60° = 2\ V/m$

$E_{2y} = E \cdot \operatorname{Sen} 60° = 3{,}464\ V/m$



$\bar{E}_1 = 2\bar{X} + 3{,}464\bar{Y}\ (V/m)$

Respuesta (a) En r2!

b) Magnitud ∧ Dirección $\bar{D}_2$ $(x > 0)$ Dieléctricos.
Interfaz $D_{n1} = D_{n2}$ $E_{t1} = E_{t2}$

Tangencial $\bar{Y}$ $\Longrightarrow E_{t1} = E_{t2} = 3{,}464\ V/m$

Normal $\bar{X}$

$D_{n1} = D_{n2}$ $\varepsilon_0 \varepsilon_{r1} E_{n1} = \varepsilon_0 \varepsilon_{r2} E_{n2}$

$\varepsilon_0 \cdot 2 = \varepsilon_0\, 9\, E_{n2}$

$\frac{1}{9} \cdot 2 = E_{n2} \Longrightarrow E_{n2} = 0{,}222\ V/m$

$$\bar{E}_2 = 0{,}222\,\bar{X} + 3{,}464\,\bar{Y}\ (V/m)$$

$$\bar{D}_2 = 9 \cdot \varepsilon_0 \cdot 0{,}222\,\bar{X} + 9 \cdot \varepsilon_0 \cdot 3{,}464\,\bar{Y}\ (C/m^2)$$

$$\bar{D}_2 = 2\varepsilon_0 \bar{X} + 31{,}176\,\varepsilon_0 \bar{Y}\ (C/m^2)$$

etc!!

PROBLEM 2.20 Vacuum diode. (a) We apply the one-dimensional Poisson's equation in the x-coordinate, Eq.(2.98), from which the volume charge density in the vacuum ($\varepsilon = \varepsilon_0$) diode in Fig.2.43 amounts to

$$\rho(x) = -\varepsilon_0 \frac{\mathrm{d}^2 V(x)}{\mathrm{d}x^2} = -\frac{4\varepsilon_0 V_0 x^{-2/3}}{9d^{4/3}} \quad (0 < x < d) . \qquad \text{(P2.40)}$$

(b)-(c) As in Eq.(2.101), the electric field intensity in the diode is given by

$$\mathbf{E}(x) = -\nabla V = -\frac{\mathrm{d}V(x)}{\mathrm{d}x}\,\hat{\mathbf{x}} = -\frac{4V_0 x^{1/3}}{3d^{4/3}}\,\hat{\mathbf{x}} \quad (0 < x < d) . \qquad \text{(P2.41)}$$

Eq.(1.190) then tells us that the surface charge densities on the cathode and anode (Fig.2.43) are

$$\rho_{s1} = \varepsilon_0\,\hat{\mathbf{x}} \cdot \mathbf{E}(0^+) = 0 \quad \text{and} \quad \rho_{s2} = \varepsilon_0(-\hat{\mathbf{x}}) \cdot \mathbf{E}(d^-) = \frac{4\varepsilon_0 V_0}{3d} , \qquad \text{(P2.42)}$$

respectively.

(d) Performing a similar integration as in Eq.(1.149) and using the second expression in Eqs.(1.30), the total charge of the diode turns out to be

$$Q = \int_v \rho(x) \underbrace{S\,\mathrm{d}x}_{\mathrm{d}v} + \rho_{s1} S + \rho_{s2} S = -\frac{4\varepsilon_0 V_0 S}{9d^{4/3}} \int_{x=0}^{d} x^{-2/3}\,\mathrm{d}x + \frac{4\varepsilon_0 V_0 S}{3d} = 0 , \qquad \text{(P2.43)}$$

El valor de la capacitancia es cero porque la carga total es cero.

Hay muchas formas de resolverlo

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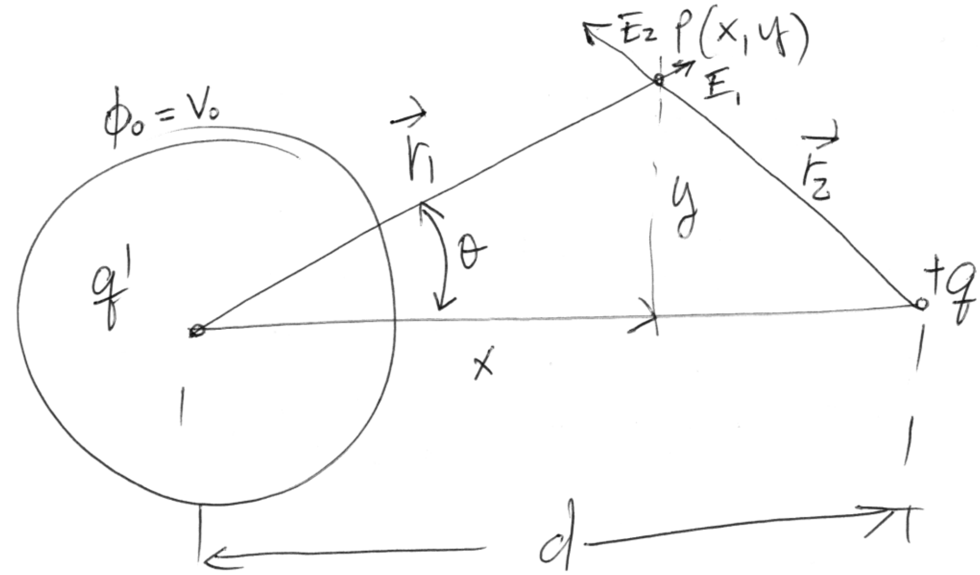
#1



$q' = 4\pi\varepsilon_0 V_0 a$ (Se considera como carga puntual)

$$\vec{E}_1 = \frac{q'}{4\pi\varepsilon_0 |r_1|^2}\vec{a}_{r_1} = \frac{\cancel{4\pi\varepsilon_0} V_0 a(\cos\theta\hat{a}_x + \sin\theta\hat{a}_y)}{\cancel{4\pi\varepsilon_0}(x^2+y^2)}$$

$$\cos\theta = \frac{x}{\sqrt{x^2+y^2}} \quad \sin\theta = \frac{y}{\sqrt{x^2+y^2}} \quad \text{entonces:}$$

$$\vec{E}_1 = \frac{V_0 a}{(x^2+y^2)^{3/2}}\left[x\hat{a}_x + y\hat{a}_y\right]$$

Mientras tanto $\vec{E}_2$

$$\vec{E}_2 = \frac{q\,\vec{r}_2'}{4\pi\varepsilon_0 |\vec{r}_2|^{3/2}}$$

donde $\vec{r}_2 = (x-d)\hat{a}_x + y\vec{a}_y$

$$\vec{E}_2 = \frac{q}{4\pi\varepsilon_0} \frac{(x-d)\hat{a}_x + y\vec{a}_y}{\left[(x-d)^2 + y^2\right]^{3/2}}$$

$$E_T = \vec{E}_1 + \vec{E}_2 = \frac{V_0 a}{(x^2+y^2)^{3/2}}\left[x\hat{a}_x + y\hat{a}_y\right] +$$

$$\frac{q(x-d)\hat{a}_x}{4\pi\varepsilon_0} + \frac{q y \vec{a}_y}{4\pi\varepsilon_0}$$