Solución:

$$\rho_v = \begin{cases} K \cdot r & r < a \\ 0 & r > a \end{cases}$$, es esféricas;

b)

b) $\oint \vec{D} \cdot d\vec{s} = Q_{enc}$. $\vec{E}\,(r > a)$

$$\iint D_r(r)\hat{a}_r \cdot r^2 \operatorname{sen}\theta \, d\theta \, d\phi \; \hat{a}_r = Q_e = \int \rho_v \, dV$$

$$D_r(r) \cdot r^2 \cdot 4\pi = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \int_{r=0}^{a} K \cdot r \cdot r^2 dr \cdot \operatorname{sen}\theta \, d\theta \cdot d\phi$$

$$D_r(r) \cdot r^2 \cdot 4\pi = 4\pi \cdot K \cdot \int_0^a r^3 dr$$

$$D_r(r) \cdot r^2 = K \cdot \left.\frac{r^4}{4}\right|_0^a$$

$$\vec{D} = \frac{K \cdot a^4}{4 r^2} \hat{a}_r$$

a)

a) $\vec{E}\,(r < a)$

$$\oint \vec{D} \cdot d\vec{s} = Q_e$$

$$D_r(r) \cdot 4\pi \cdot r^2 = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \int_{\tau=0}^{\tau=r} K \cdot \tau \cdot \tau^2 \cdot d\tau \cdot \operatorname{sen}\theta \, d\theta \, d$$

$$D_r(r) \cdot 4\pi \cdot r^2 = 4\pi \cdot K \underbrace{\int_0^r \tau^3 d\tau}_{\frac{\tau^4}{4}}$$

$$\rightarrow D_r(r) \cdot r^2 = K \cdot \frac{r^4}{4}$$

$$\vec{D} = \frac{K \cdot r^2}{4} \hat{a}_r$$

c)

c) V para $r > a$.

$$V = -\int_{in}^{fin} \vec{E} \cdot \vec{dl}$$

$$= -\int_{+\infty}^{r} \frac{K \cdot a^4}{4 \tau^2} \cdot \frac{1}{\varepsilon_0} \hat{a}_r \cdot d\tau \hat{a}_r$$

$$= \int_{r}^{+\infty} \frac{K \cdot a^4}{4 \varepsilon_0} \cdot \frac{d\tau}{\tau^2} = \frac{K a^4}{4 \varepsilon_0} \cdot \left. \frac{-1}{\tau} \right|_{r}^{+\infty}$$

$$= \underbrace{\frac{K a^4}{4 \varepsilon_0} \cdot \frac{1}{r}}$$

d)

d) V para $r < a$

$$V = \underbrace{-\int_{+\infty}^{r=a} \vec{E}(r>a) \cdot \vec{dl}} + -\int_{a}^{r} \vec{E}(r<a) \cdot \vec{dl}$$

$$= \underbrace{\left. V(r>a) \right|_{r=a}} + -\int_{a}^{r} \frac{K \cdot \tau^2}{4 \varepsilon_0} \hat{a}_r \cdot d\tau \hat{a}_r$$

$$= \frac{K a^4}{4 \varepsilon_0} \cdot \frac{1}{a} + \int_{r}^{a} \frac{K}{4 \varepsilon_0} \cdot \tau^2 d\tau$$

$$= \frac{K a^3}{4 \varepsilon_0} + \frac{K}{4 \varepsilon_0} \cdot \left. \frac{\tau^3}{3} \right|_{r}^{a} = \frac{K a^3}{4 \varepsilon_0} + \frac{K}{12 \varepsilon_0} (a^3 - r^3)$$

$$= \frac{K a^3}{3 \varepsilon_0} - \frac{K \cdot r^3}{12 \varepsilon_0} \Big\}$$

e) e)

$$W_E = \int_V \frac{1}{2} \vec{D} \cdot \vec{E} \, dV$$

Para $r > a$;

$$\vec{D} = \frac{k a^4}{4 r^2} \hat{a}_r \quad ; \quad \vec{E} = \frac{k a^4}{4 r^2} \cdot \frac{1}{\varepsilon_0} \hat{a}_r$$

$$\frac{dW_E}{dV} = \frac{1}{2} \frac{k a^4}{4 r^2} \hat{a}_r \cdot \frac{k a^4}{4 r^2} \cdot \frac{1}{\varepsilon_0} \hat{a}_r$$

$$= \frac{k^2 a^8}{32 \varepsilon_0 r^4}$$

$$W_E = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \int_{r=a}^{+\infty} \frac{k^2 a^8}{32 \varepsilon_0 r^4} \cdot r^2 \, dr \, \operatorname{sen}\theta \, d\theta \, d\phi \, ;$$

$$= \frac{k^2 a^8}{32 \varepsilon_0} \cdot 4\pi \cdot \underbrace{\int_{r=a}^{+\infty} \frac{1}{r^2} \, dr}_{\underbrace{\left. \frac{-1}{r} \right|_a^{+\infty}}_{\frac{1}{a}}}$$

$$\underbrace{W_E = \frac{k^2 a^7 \cdot \pi}{8 \varepsilon_0}}$$ para $r > a$.