

Equity Spot-Futures Arbitrage Analysis

Andy Andikko
Harrison Zhang

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1 Introduction

This report documents our replication and analysis of equity spot-futures arbitrage as described in Hazelkorn et al. (2021). We follow a reproducible analytical pipeline (RAP) approach to extract, process, and analyze financial data from Bloomberg. Our analysis focuses on measuring arbitrage spreads between the futures-implied forward rate and the OIS benchmark rate for major equity indices.

2 Data Sources and Retrieval

2.1 Overview

The primary objective of our Bloomberg data retrieval script is to extract historical data required for **Equity Spot-Futures Arbitrage Analysis**. The primary data source for this project is the **Bloomberg Terminal**, aligning with the methodology used by the reference paper. The data extraction process is automated using the **Python xbbg package**, allowing seamless interaction with the Bloomberg Terminal API.

To ensure reproducibility, the script supports both **live data extraction** and a fallback to **cached data**. Users can toggle data retrieval behavior using the `USING_XBBG` environment variable in the configuration file. If `USING_XBBG=True`, data is pulled directly from Bloomberg; otherwise, the script defaults to previously saved datasets.

2.2 Data Types and Key Tickers

The script retrieves three key types of financial data:

2.2.1 Spot Prices and Dividend Yields

- Required for computing spot-futures arbitrage.
- Extracted for **S&P 500 (SPX Index)**, **Nasdaq 100 (NDX Index)**, and **Dow Jones Industrial Average (INDU Index)**.
- Bloomberg Fields:
 - `PX_LAST` – Last traded price of the index.
 - `IDX_EST_DVD_YLD` – Estimated annual dividend yield.
 - `INDX_GROSS_DAILY_DIV` – Daily gross dividend.

2.2.2 Equity Index Futures Contracts

- Used to construct **implied forward rates** for arbitrage calculations.
- The script pulls **multiple futures contracts** per index, ordered by contract expiry:
 - **Nearest (Front Month)**: ES1 Index, NQ1 Index, DM1 Index

- **First Deferred:** ES2 Index, NQ2 Index, DM2 Index
- **Second Deferred:** ES3 Index, NQ3 Index, DM3 Index
- **Third Deferred:** ES4 Index, NQ4 Index, DM4 Index
- Bloomberg Fields:
 - PX_LAST – Last traded futures price.
 - PX_VOLUME – Trading volume.
 - OPEN_INT – Open interest.
 - CURRENT_CONTRACT_MONTH_YR – Expiry date of the futures contract.

2.2.3 Overnight Indexed Swap (OIS) Rates

- Used as a benchmark for the risk-free rate in arbitrage calculations.
- The script retrieves various OIS maturities, but **OIS_3M (3-month OIS rate) is the primary reference**, as the paper states that interpolation across OIS rates is unnecessary.
- Bloomberg Fields:
 - PX_LAST – Last recorded interest rate.

2.3 Data Extraction Process

The script is structured into three key functions, each responsible for fetching one category of data:

1. Spot Prices and Dividend Data Retrieval

```
pull_spot_div_data(["SPX Index"], START_DATE, END_DATE)
```

- Fetches index price and dividend-related fields.
- Returns a multi-indexed pandas DataFrame with a timestamped index.

2. Futures Contract Data Retrieval

```
pull_futures_data(["ES1 Index", "ES2 Index"], START_DATE, END_DATE)
```

- Retrieves price, volume, and open interest for futures contracts.
- Data is stored with a date-time index.

3. OIS Rate Retrieval

```
pull_ois_rates(["USSOC CMPN Curncy"], START_DATE, END_DATE)
```

- Pulls OIS rates for different maturities.
- Primarily focuses on 3-month OIS.

Once extracted, all datasets are merged into a **single time-series DataFrame** with a uniform date range defined by `START_DATE` and `END_DATE`.

2.4 Data Storage and Logging

- Extracted data is **saved as a Parquet file** for efficient storage and retrieval.
- The script logs all activities using Python's `logging` module:
 - Logs are stored in `_output/temp/bloomberg_data_extraction.log`.
 - Errors are captured and printed to the console for debugging.

2.5 Challenges and Considerations

During development, several issues were encountered with the `xbbg` package:

- **Import Errors (blpapi not found)**

- Fixed by ensuring installation via:

```
conda install -c conda-forge blpapi
```

- Added to `requirements.txt` for reproducibility:

```
--find-links https://blpapi.bloomberg.com/repository/releases/python/simple/blpapi/index  
blpapi
```

- **Handling Asynchronous Market Closing Times**

- The script avoids spot/futures price mismatch by using **only futures-based implied forward rates** rather than direct spot-futures parity calculations.

3 OIS Data Processing

3.1 Overview

The Overnight Indexed Swap (OIS) rate serves as our benchmark risk-free rate for equity spot-futures arbitrage analysis. This section details our methodology for processing the 3-month OIS rate from raw Bloomberg data into a clean, analysis-ready format.

3.2 Processing Methodology

Our OIS data processing pipeline follows these key steps:

1. **Data Extraction:** We extract the 3-month OIS rate from the Bloomberg multi-index dataset, specifically targeting the ('USSOC CMPN Curncy', 'PX_LAST') column.
2. **Format Conversion:** The extracted OIS rates are converted from percentage format to decimal format (e.g., from 3.25% to 0.0325) to align with standard financial calculations.
3. **Missing Value Handling:** We identify and remove any rows with missing OIS rates to ensure data quality.
4. **Date Range Filtering:** The dataset is filtered to include only data within our analysis period (2010-01-01 to 2024-12-31).
5. **Data Validation:** We implement comprehensive unit tests to ensure the processed data meets all requirements for downstream analysis.
6. **Data Export:** The cleaned dataset is saved as a CSV file for use in subsequent analysis steps.

3.3 Data Quality Assurance

We implemented rigorous unit tests to ensure the OIS data meets all requirements for the equity spot-futures arbitrage calculations:

- **Format Validation:** Ensuring OIS rates are properly converted to decimal format.
- **Completeness Checks:** Verifying no missing values exist in the final dataset.
- **Range Validation:** Confirming OIS rates fall within reasonable bounds (0% to 50%).
- **Time-Series Integrity:** Ensuring the data has a proper `DatetimeIndex` and is sorted chronologically.
- **Data Continuity:** Verifying there are no gaps exceeding 10 trading days, which is critical for reliable as-of merging in downstream calculations.

- **Date Coverage:** Ensuring sufficient coverage across the entire analysis period.

These tests are particularly important because the OIS rate serves as the benchmark for calculating arbitrage spreads. Any issues with the OIS data could significantly distort our arbitrage analysis.

3.4 Integration with Forward Rate Calculations

The processed OIS data integrates with the equity spot-futures arbitrage analysis in several critical ways:

1. **As-Of Merging:** The OIS rates are merged with futures data using a backward-looking as-of merge to ensure we only use rates available at or before each futures observation date.
2. **Dividend Compounding:** The OIS rate is used to compound expected dividends over the futures contract lifetime.
3. **OIS-Implied Forward Rate:** The OIS rate generates a theoretical forward rate as the benchmark for comparison with futures-implied forward rates.
4. **Arbitrage Spread Calculation:** The difference between the futures-implied forward rate and the OIS-implied forward rate forms our key arbitrage spread metric.

4 OIS Rate Analysis

4.1 Historical Context and Patterns

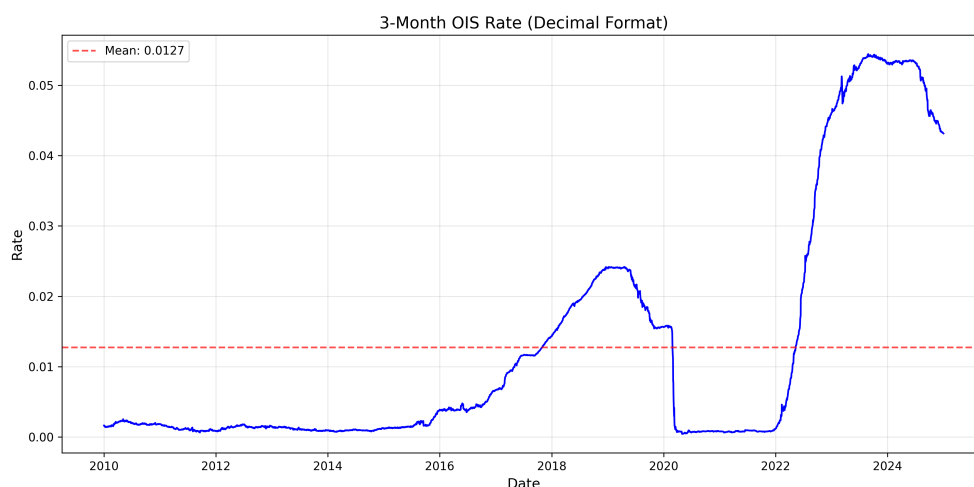


Figure 1: 3-Month OIS Rate (2010-2024). This figure shows the evolution of the 3-month OIS rate over our analysis period. The rate follows distinct regimes corresponding to major monetary policy phases: post-financial crisis low rates (2010-2015), gradual normalization (2016-2019), COVID-19 shock (2020), and inflation-driven tightening (2022-2023). These regime shifts directly impact funding costs for arbitrageurs and help explain variations in the equity spot-futures arbitrage spread.

Examining our visualizations of the 3-month OIS rate data, we observe clear time-varying patterns in both mean and volatility that correspond to significant macroeconomic events:

1. Post-Financial Crisis Period (2010-2015):

- Low and stable OIS rates (near zero) reflecting the Federal Reserve's accommodative monetary policy following the 2008-2009 financial crisis
- Limited volatility as the Fed maintained its [zero interest rate policy \(ZIRP\)](#)

2. Gradual Normalization (2016-2019):

- Steady increase in OIS rates as the Federal Reserve implemented its [policy normalization program](#)
- December 2015 marked the first rate hike after nearly a decade at zero
- Reduced balance sheet and rising federal funds rate target led to a gradual upward trend in OIS rates

3. COVID-19 Pandemic Shock (2020):

- Sharp spike in volatility in March 2020 corresponding to the [global COVID-19 market panic](#)
- Dramatic rate cuts by the Federal Reserve returning to near-zero rates
- Implementation of extensive quantitative easing and emergency liquidity facilities

4. Inflation Surge and Monetary Tightening (2022-2023):

- Pronounced upward trend in OIS rates beginning in early 2022
- Reflects the Federal Reserve's [aggressive rate hiking cycle](#) in response to persistent inflation
- OIS rates reached their highest levels in over a decade

5. Stabilization Period (2024):

- More recent data shows signs of stabilization as markets anticipate the end of the tightening cycle
- Volatility moderates as uncertainty about monetary policy path decreases [CRFB](#)

4.2 OIS Rate Statistical Properties

Table 1: Summary Statistics for 3-Month OIS Rate by Monetary Policy Regime. This table presents key statistical properties of the OIS rate across different monetary policy periods. The stark contrasts in mean levels and volatility across regimes highlight how funding conditions for arbitrageurs have varied dramatically over time. The low volatility during ZIRP (2010-2015) created stable funding conditions, while the high volatility during the COVID-19 shock and subsequent inflation surge created uncertainty that likely widened arbitrage spreads.

| Period | Mean (%) | Std Dev (%) | Min (%) | Max (%) |
|-----------------------------|----------|-------------|---------|---------|
| COVID-19 Shock (2020) | 0.34 | 0.55 | 0.04 | 1.58 |
| Inflation Surge (2021-2023) | 2.47 | 2.28 | 0.06 | 5.44 |
| Normalization (2016-2019) | 1.38 | 0.71 | 0.35 | 2.42 |
| Stabilization (2024) | 5.05 | 0.37 | 4.32 | 5.36 |
| ZIRP (2010-2015) | 0.14 | 0.05 | 0.06 | 0.38 |
| Full Sample | 1.27 | 1.73 | 0.04 | 5.44 |

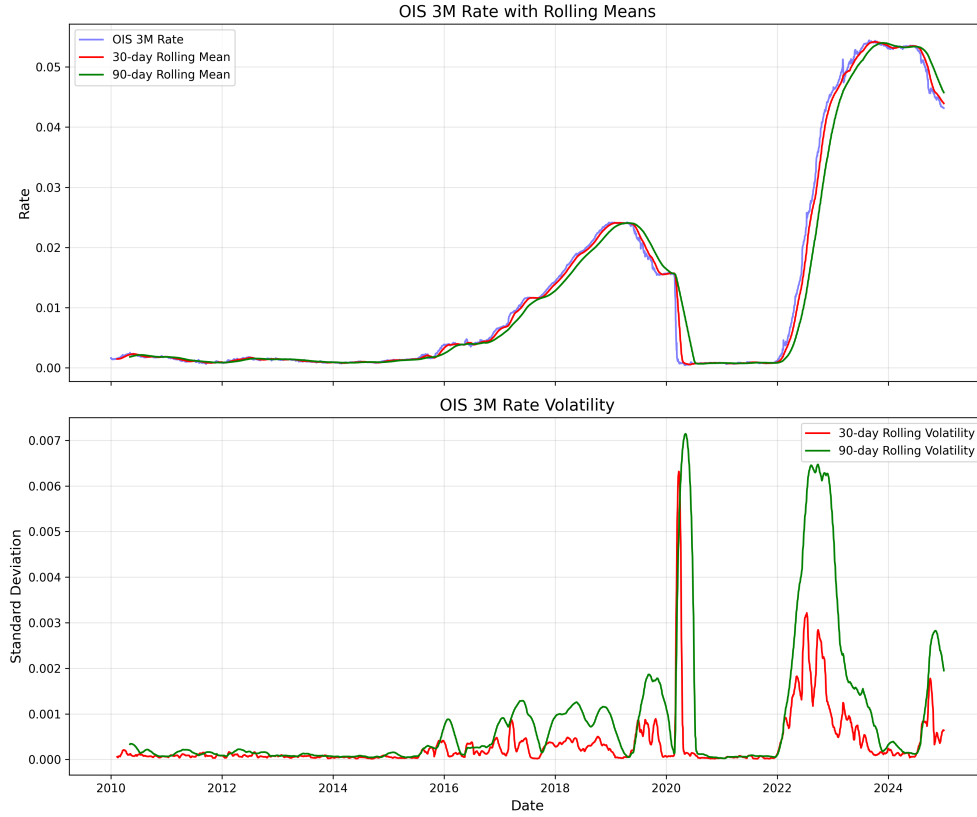


Figure 2: OIS Rate Trends and Volatility (30-day and 90-day rolling windows). This figure illustrates both the evolving trend (top panel) and volatility (bottom panel) in the 3-month OIS rate. The volatility spikes in early 2020 and throughout 2022-2023 coincide with periods of monetary policy uncertainty. These volatility patterns are crucial for understanding when equity spot-futures arbitrage opportunities may have been most pronounced, as uncertainty about funding costs typically widens arbitrage spreads.

These patterns are relevant to our equity spot-futures arbitrage analysis because OIS rates directly impact the theoretical funding costs for arbitrage strategies. Periods of rate volatility often correspond to wider arbitrage spreads as market participants face uncertainty about funding costs.

5 Futures Data Analysis

5.1 Futures Contract Data Processing

The calculation of equity spot-futures arbitrage spreads requires properly processed futures data with accurate settlement dates and time-to-maturity (TTM) calculations. Below we analyze key aspects of the futures contract data for our three indices (S&P 500, Nasdaq 100, and Dow Jones Industrial Average).

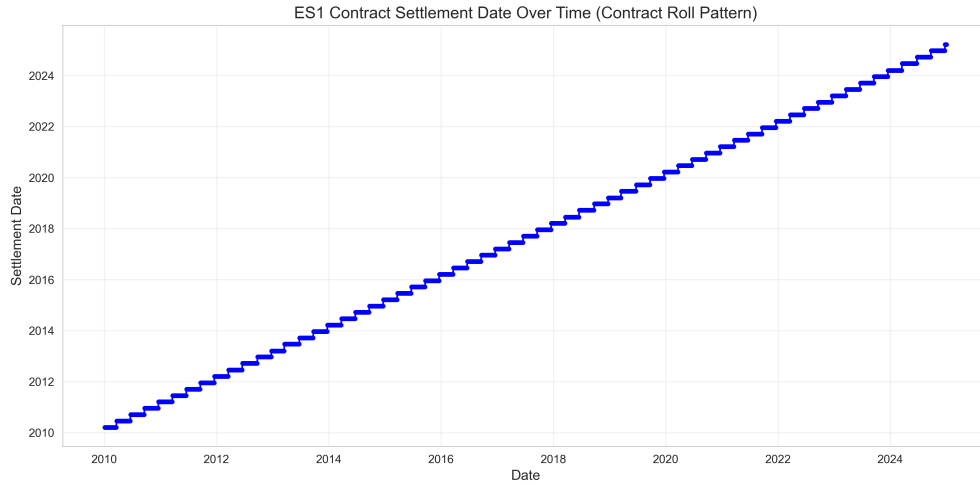


Figure 3: S&P 500 (ES1) Contract Roll Pattern (2010-2024). This figure illustrates how the front-month futures contract rolls over time, with each vertical step representing a roll to a new contract. The pattern shows quarterly rolls (typically March, June, September, December), which correspond to the CME's standard futures expiration cycle. Identifying these roll dates is crucial for our analysis as it helps account for potential price discontinuities in the time series that could distort implied forward rate calculations.

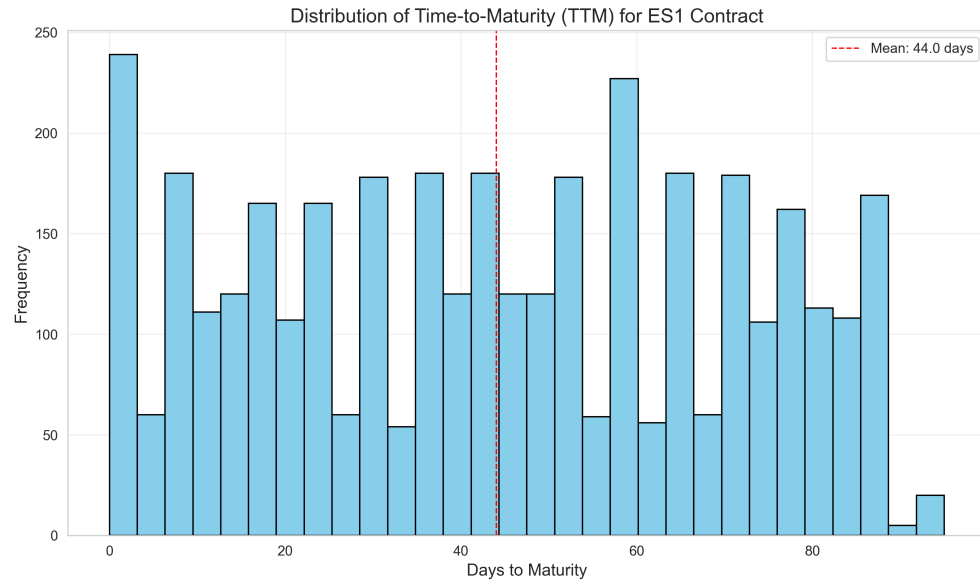


Figure 4: Distribution of Time-to-Maturity for S&P 500 (ES1) Front-Month Contract. This histogram demonstrates that most observations of the ES1 contract occur with 20-80 days to maturity, with a mean of approximately 45 days (red dashed line). The distribution is right-skewed because contracts are typically rolled before expiration to maintain liquidity. Note the small cluster at zero days, representing observations taken on settlement days. This distribution informs our understanding of the typical horizon for the first leg of our arbitrage calculations.

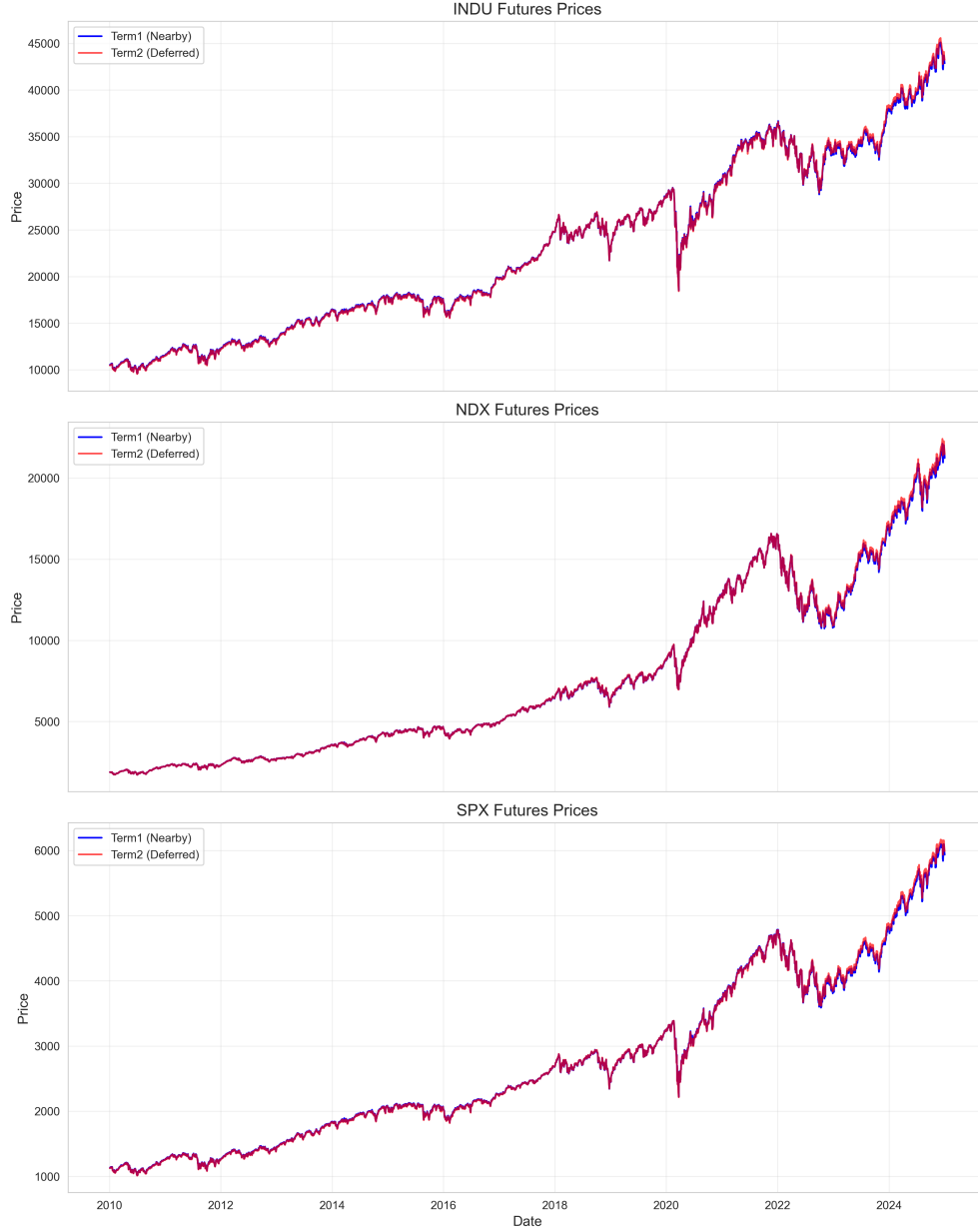


Figure 5: Futures Prices for Term1 (Nearby) and Term2 (Deferred) Contracts by Index (2010-2024). This multi-panel figure shows the evolution of futures prices for all three indices, with the blue line representing the nearby contract and the red line representing the deferred contract. The price trajectories reflect major market movements, including the strong bull market (2010-2019), COVID-19 crash (2020), and subsequent recovery. The consistent premium of deferred contracts over nearby contracts (particularly visible during high interest rate periods like 2022-2023) reflects the cost-of-carry relationship that forms the basis of our arbitrage analysis. Deviations from this normal pattern may signal arbitrage opportunities.

5.2 Forward Rate and Arbitrage Spread Results

After processing the futures data and calculating the implied forward rates, we obtain equity spot-futures arbitrage spreads as shown below.

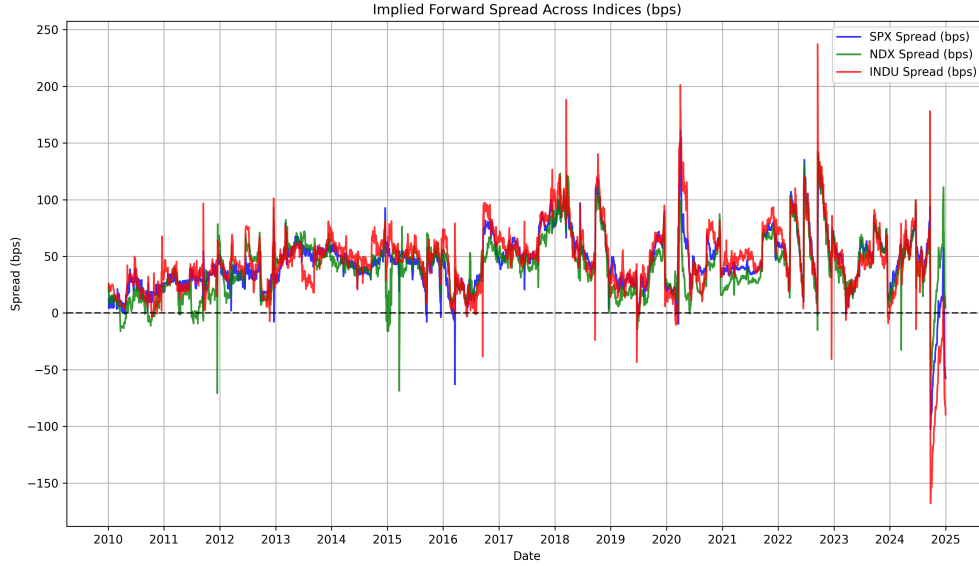


Figure 6: Equity Spot-Futures Arbitrage Spreads Across Indices (2010-2024). This figure displays the arbitrage spread, measured in basis points, between futures-implied forward rates and OIS rates for all three indices. Positive values indicate that the futures-implied funding rate exceeds the OIS rate, suggesting potential funding constraints or risk premiums in the market. Negative values suggest the opposite. The spread exhibits significant time variation and volatility clustering, with notable spikes during periods of market stress (2020) and monetary policy shifts (2022-2023). The co-movement across indices suggests common factors drive these arbitrage opportunities, while the differences in magnitude highlight index-specific dynamics. The Barndorff-Nielsen filter has been applied to remove outliers, ensuring that patterns reflect genuine market phenomena rather than data anomalies.

5.3 Data Quality Considerations

In our analysis of equity futures data, we encountered and addressed several data quality issues. Most notably, we found instances where the Time-to-Maturity (TTM) equals zero, particularly for SPX contracts. These cases represent observations taken exactly on settlement dates. Rather than discarding these data points, our forward rate calculation methodology handles them appropriately by:

1. Using the difference between Term2_TTM and Term1_TTM as the denominator for annualizing rates
2. Applying conditional logic (`np.where(dt > 0, ...)` in the code) to set results to NaN when this difference is not positive
3. Filtering outliers using the Barndorff-Nielsen method to remove any anomalous spreads

This robust approach ensures that our analysis captures genuine arbitrage opportunities while appropriately handling edge cases in the data.

6 Conclusion

Our OIS data processing pipeline ensures reliable and high-quality interest rate data for our equity spot-futures arbitrage analysis. The comprehensive data validation and quality assurance measures implemented guarantee that our arbitrage spread calculations accurately reflect market conditions rather than data artifacts. The historical patterns identified in the OIS rate data provide crucial context for interpreting the equity spot-futures arbitrage spreads. Periods of monetary policy transition or uncertainty typically correspond to increased funding cost volatility, which may create more significant arbitrage opportunities. This analysis forms the foundation for our broader investigation into the

efficiency of equity futures markets and the dynamics of arbitrage spreads across different market conditions.