TA Review (Lecture #4)

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Agenda

- Lecture review (CAPM)
- DFA
- Questions

Lecture Review

The CAPM

The most famous Linear Factor Model is the Capital Asset Pricing Model (CAPM).

$$\mathbb{E}\left[\tilde{r}^i\right] = \beta^{i,m}\,\mathbb{E}\left[\tilde{r}^m\right] \tag{1}$$

$$\beta^{i,m} \equiv \frac{\text{cov}\left(\tilde{r}^i, \tilde{r}^m\right)}{\text{var}\left(\tilde{r}^m\right)}$$

where $\tilde{r}^m$ denotes the return on the entire market portfolio, meaning a portfolio that is value-weighted to every asset in the market.

What is CAPM?

- A pricing model that tells us our **expected excess returns** as a function of our market risk *and* the market **risk premium**
  - Each asset still has idiosyncratic risk – we aren't saying that every asset always behaves like the market at all times.
  - In practice, market ⇒ equities
- CAPM is a relative pricing model – what excess return do I expect *relative* to the market risk premium?
  - We are not asserting anything about the market risk premium, which can vary
  - We are asserting something about expected returns *relative* to the market risk premium
- The excess return of an asset should be directly proportional to the risk of the asset (as measured by exposure to the market)

Conceptual Takeaways

- CAPM tells me how much I should be compensated for holding a given level of risk
  - If I can properly estimate the risk of an asset, then I can estimate its mean return – if our model is correct this is far more powerful than estimating the mean return directly.
  - Asset risk characteristics are typically somewhat easy to assess, and they don't often change much.
  - Commonly used in fundamentals-based equity pricing
- CAPM asserts that I can only get more returns by taking more risk
  - This is a common belief among efficient market hypothesis proponents
  - DFA generally believes this, and orients their strategies towards harvesting this risk premia at a low cost to investors
- Beta is the only risk that matters; idiosyncratic risk does not matter.
  - If I can diversify away my idiosyncratic risk (Week #1), then I should not be compensated for it

CAPM and realized returns

The CAPM implies that expected returns for any security are

$$\mathbb{E}\left[\tilde{r}^i\right] = \beta^{i,m}\, \mathbb{E}\left[\tilde{r}^m\right]$$

This implies that realized returns can be written as

$$\tilde{r}_t^i = \beta^{i,m}\, \tilde{r}_t^m + \epsilon_t \tag{4}$$

where ϵ_t is **not** assumed to be normal, but

$$\mathbb{E}\left[\epsilon\right] = 0$$

Of course, taking expectations of both sides we arrive back at the expected-return formulation.

Testing the CAPM on an asset

Using any asset return i, we can test the CAPM.

- Run a time-series regression of excess returns i on the excess market return.
- Regression for asset i, across multiple data points t:

$$\tilde{r}_t^i = \alpha^i + \beta^{i,m}\, \tilde{r}_t^m + \epsilon_t^i$$

  Estimate α and β.

- The CAPM implies $\alpha^i = 0$.

Single Asset CAPM Regression

- Create a linear factor decomposition (LFD) of asset excess returns on the market (e.g. SPY) excess returns

$$\tilde{r}_t^i = \alpha^i + \beta^{i,m}\ \tilde{r}_t^m + \epsilon_t^i$$

- We've estimated the beta to be used in CAPM, but we've also tested the CAPM itself… how?
- How confident are we in our estimate of alpha?
- Should I use an intercept in this regression?
- Do I care about r-squared?

Testing the CAPM

- Our estimate for alpha is poor enough such that we cannot often reject CAPM for any single asset.
- But CAPM is a model for *all* assets. Can we devise a test with more statistical power?
  - Joint alpha test
  - Cross-sectional regression
- Homework #4 is about testing the CAPM

CAPM-implied relation between beta and returns

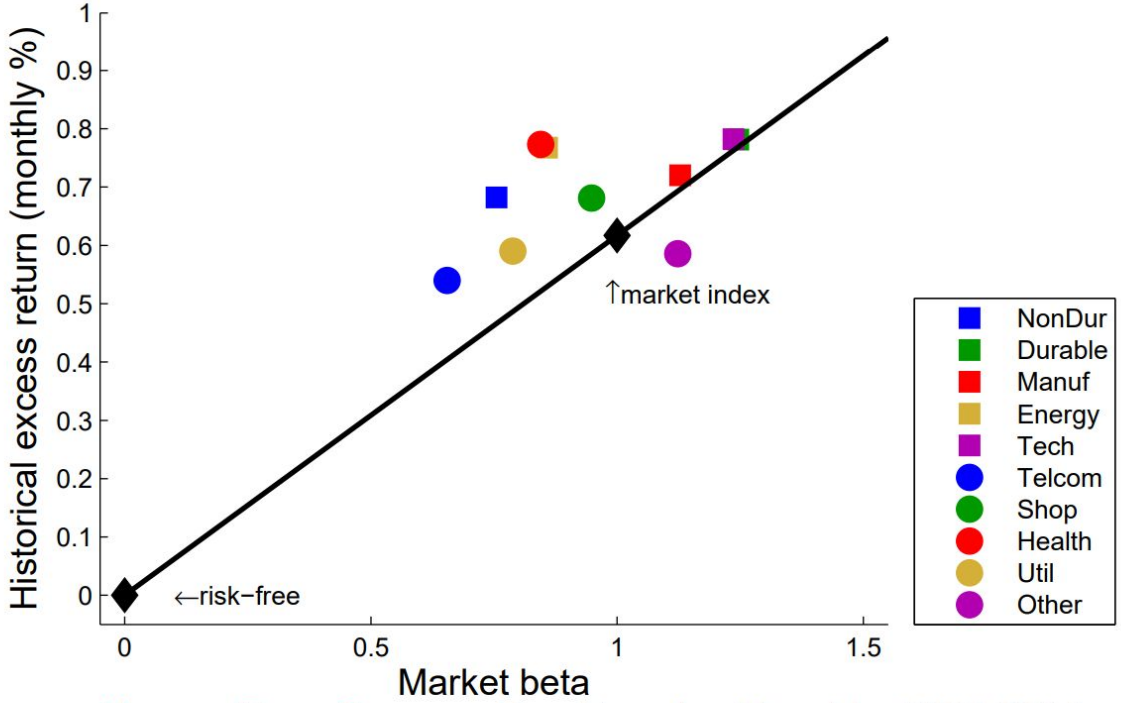

Figure: Data Source: Ken French. Monthly 1926-2011.

Cross Sectional CAPM Regression

- CAPM says all expected excess returns are linear with respect to their market risk
  - We can compute historical excess return means
  - We can compute historical market risk (market beta)
- We can test CAPM by seeing whether this is generally true *across all assets*
  - Cross-sectional regression means this is **across assets**, not over time like our previous regressions

$$\mathbb{E}\left[\tilde{r}^i\right] = \underbrace{\eta}_{\alpha} + \underbrace{\beta^{i,m}}_{x^i}\underbrace{\lambda_m}_{\beta^i} + \underbrace{\upsilon^i}_{\epsilon^i}$$

- What would we expect if CAPM holds?
  - R-squared
  - λ_m
  - η

CAPM implications in the cross-section

$$\mathbb{E}\left[\tilde{r}^i\right] = \eta + \beta^{i,m}\lambda_m + \upsilon^i$$

- CAPM statement (5) implies the R^2 of the cross-sectional regression is 100%.

$$\upsilon^i = 0, \forall i$$

- CAPM statement (6) implies the cross-sectional regression parameters are:

$$\eta = 0, \qquad \lambda_m = \mathbb{E}\left[\tilde{r}^m\right]$$

- That is, the SML goes through zero and the market return. (See slide 24.)

Risk-reward tradeoff is too flat relative to CAPM

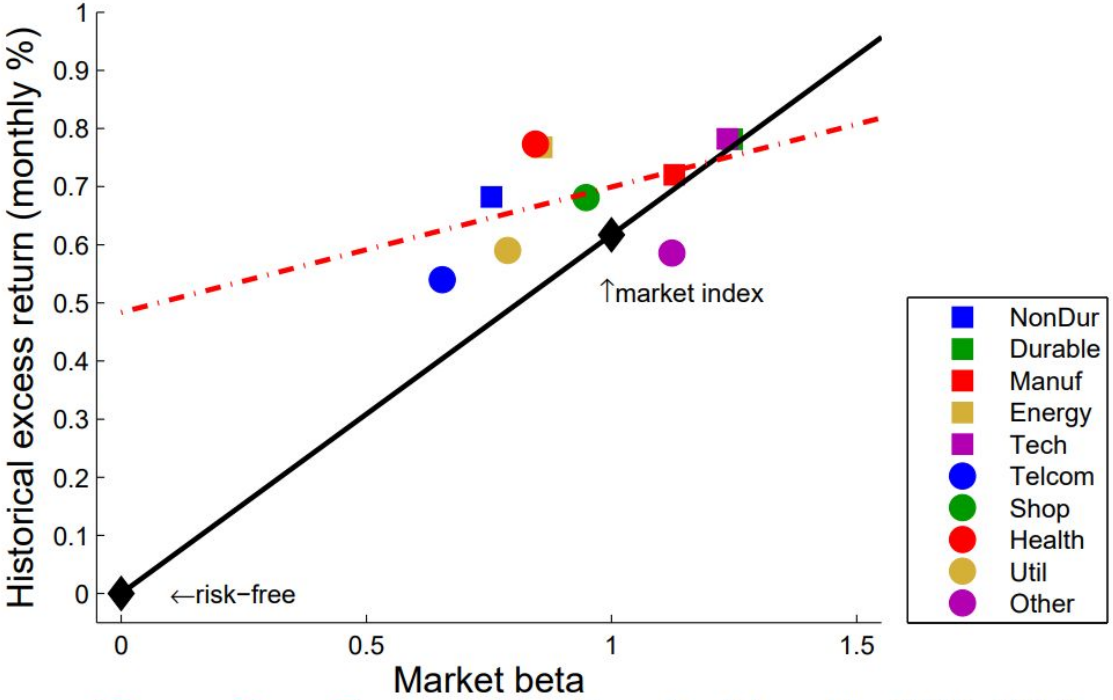

Figure: Data Source: Ken French. Monthly 1926-2011.

Homework #4

Dimensional Fund Advisors

- Founded by David Booth and Rex Sinquefield
- 677 Billion AUM (2023)
- Aligned with academia, even working with Fama and other researchers

DFA's Beliefs

- Markets are generally efficient, and attractive excess returns can be delivered through low-cost exposure to risk factors with attractive risk premiums.
  - No individual stock-picking, macro speculation, etc.
- They (or Fama and French) find that CAPM doesn't seem to hold – it does not properly capture the historical risk and return characteristics of market assets.
- If they cannot capture all excess return characteristics using market beta, this implies that CAPM does not hold, and perhaps stock-picking is necessary.
- However, perhaps these seemingly idiosyncratic returns are actually just a result of some *other* risk factors and their associated risk premiums.
- The Fama-French factors (size, value, and market risk) empirically do a far better job of explaining asset mean returns.

DFA Funds

- Low cost exposure to these risk factors (size, value, etc.) that drive asset returns
  - Passive
  - Highly cost-focused (see pages 6-11)

Questions?