TA Review (Lecture #2)

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Agenda

- Lecture review (measuring and evaluating performance)
- Hedging and tracking
- Homework / Questions

Lecture Review

Log returns

Let r denote the log-return:

$$\mathrm{r}_t \equiv \log(1 + r_t)$$

Log returns are particularly useful when dealing with compounding returns across different time horizons.

$$\mathrm{r}_{t,t+h} \equiv \log(1 + r_{t,t+h}) = \sum_{i=1}^{h} \mathrm{r}_{t+i}$$

Thus, the cumulative return from t to $t + h$ is just the sum of one-period returns.

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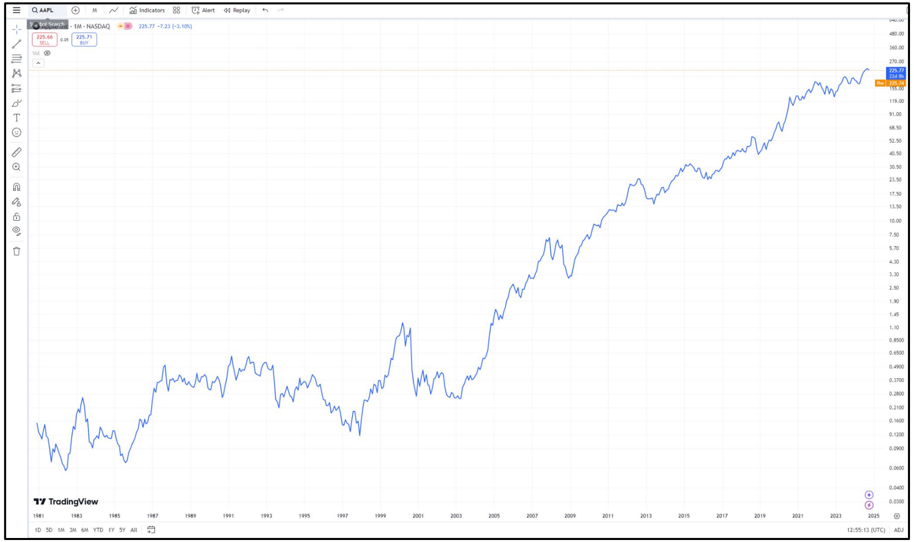
Why log returns?

- Log returns represent the continuously-compounded return
- Log returns *may* have more friendly distributions when modeled (note that a common assumption is log-normal prices; we live in a GBM world)
- Often times we may look at log prices
- Log returns are additive



Annualizing Returns and Volatility

- Assuming i.i.d log returns:

$$\mathbb{E}\left[\mathrm{r}_{t,t+h}\right] = h\left(\mathbb{E}\left[\mathrm{r}_t\right]\right)$$
$$\mathrm{var}\left[\mathrm{r}_{t,t+h}\right] = h\left(\mathrm{var}\left[\mathrm{r}_t\right]\right)$$

- Are these assumptions reasonable?
- Quoting convention
  - Consider interest rates quoting convention

Skewness is defined as the (scaled) third centralized moment of the distribution:

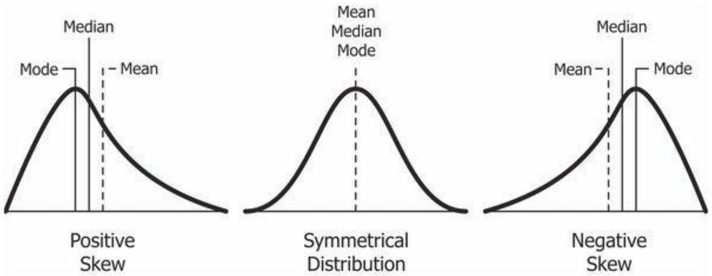
$$\varsigma = \frac{\mathbb{E}\left[(x-\mu)^3\right]}{\sigma^3}$$

Kurtosis is defined as

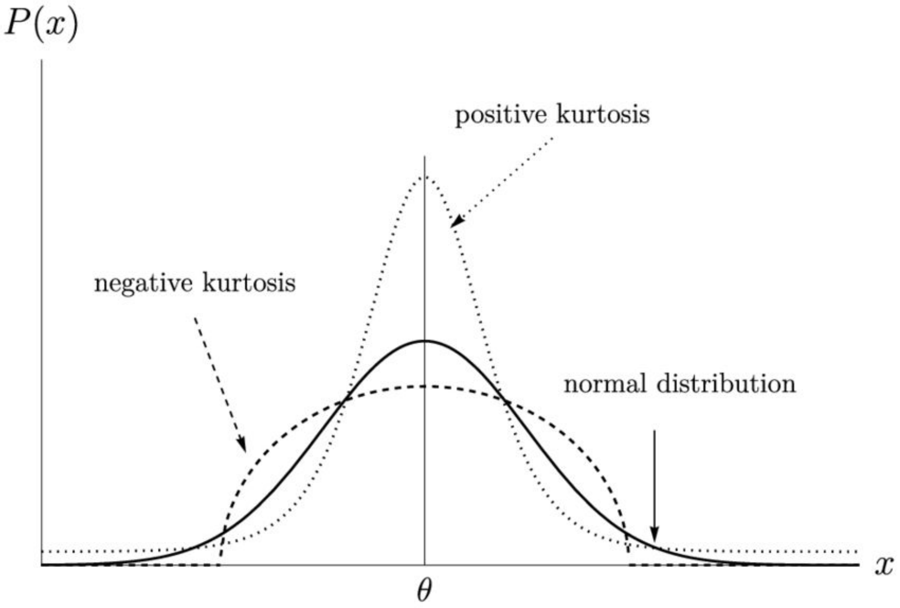
$$\kappa = \frac{\mathbb{E}\left[(x-\mu)^4\right]}{\sigma^4}$$

A normal distribution has kurtosis equal to 3, so *excess kurtosis* refers to $\kappa - 3$.

Higher Moments Visualized



Source: Diva Dugar
(https://codeburst.io/2-important-statistics-terms-you-need-to-know-in-data-science-skewness-and-kurtosis-388fef94eeaa)



Source: Research Gate
(https://www.researchgate.net/figure/Examples-of-positive-negative-and-zero-excess-kurtosis_fig4_373105776)

Other Distribution Quantifications

- VaR
- CVaR (expected shortfall)
- Maximum drawdown

The VaR expressed as a return rate is then,

$$r_{\pi,\tau}^{\mathrm{VaR}} = F_{\tau}^{r(-1)}(\pi)$$

where F_{τ}^{r} is the cdf of the return distribution.

The VaR in terms of returns is simply the π quantile of the observed returns.

Note: Think about how the values here relate to our distribution properties (moments, etc.) – what are we trying to measure here?

VaR Estimation

"an airbag that works all the time, except when you have a car accident" – Einhorn

- VaR requires a deep understanding of distribution tails, which are rarely realized
- Estimation considerations:
  - Historical quantile?
  - Fit parameters of an imposed distribution?
  - Monte Carlo?

Factor decomposition of return variation

A Linear Factor Decomposition (LFD) of $\tilde{r}^i$ onto the factor $\mathbf{x}_t$ is given by the regression,

$$\tilde{r}^i_t = \alpha + \boldsymbol{\beta}^{i,\mathbf{x}}\mathbf{x}_t + \epsilon_t$$

- The variation in returns is decomposed into the variation explained by the benchmark, x_t and by the residual, ϵ_t.
- These factors, $\mathbf{x}$, in the LFD should give a high R-squared in the regression if they really explain the variation of returns well.

Treynor's measure is an alternative measure of the risk-reward tradeoff. For the return of asset, i,

$$\text{Treynor Ratio} = \frac{\mathbb{E}\left[\tilde{r}^i\right]}{\beta^{i,m}}$$

The **information ratio** refers to the Sharpe Ratio of the non-factor component of the return: $\alpha + \epsilon_t$.

$$\text{IR} = \frac{\alpha}{\sigma_\epsilon}$$

where σ_ϵ and α come from

$$\tilde{r}^i_t = \alpha + \beta^{i,j}\tilde{r}^j_t + \epsilon_t$$

Hedging & Tracking

Net exposure

Investor is long \$1 of i and hedges by selling h\$ of j.

- Time t net exposure is,

$$\epsilon_t = r_t^i - h\, r_t^j$$

- This net exposure after hedging is known as **basis**.
- A position in i is perfectly hedged over horizon t if $\epsilon_t = 0$ with probability one.

What is a “basis” in industry?

- Broadly speaking, the difference between two (or more) instruments
- The basis is a more meaningful concept when the instruments are correlated
    - Sure, I can talk about the BTC - 3mo. TBill basis, but this is a very broad idea
- Examples:
    - Spot - Future
    - Brent - WTI
- More specifically, it is the risk held after hedging instrument A with instrument B

Basis risk

$$\epsilon_t = r_t^i - h\, r_t^j$$

Basis risk refers to volatility in ϵ_t, denoted as σ_ϵ.

$$\sigma_\epsilon^2 = \sigma_i^2 + h^2\sigma_j^2 - 2h\,\sigma_i\sigma_j\,\rho_{i,j}$$

- Denoted as σ_ϵ^2 because basis is the error in the hedge.
- For $\rho_{i,j} = \pm 1$, the basis risk can be eliminated.

Optimal hedge ratio

The **optimal hedge ratio**, h^*, minimizes basis risk.

$$\begin{aligned} h^* &= \arg\min_h \sigma_\epsilon^2 \\ &= \arg\min_h \left\{\sigma_i^2 + h^2\sigma_j^2 - 2h\,\sigma_i\sigma_j\,\rho_{i,j}\right\} \end{aligned}$$

Solve by taking derivative,

$$h^* = \frac{\sigma_i}{\sigma_j}\,\rho_{i,j}$$

- Higher correlation implies larger hedge ratio, h.
- High relative volatility of i implies larger hedge ratio.
- With negative correlation, must go long the hedging security.

Basis as a regression residual

From the previous slide, we can write

$$r_t^i = \beta^{i,j} r_t^j + \epsilon_t$$

where

$$\beta^{i,j} = h^*$$

- The optimal hedge ratio, h^*, is simply a regression beta!
- Optimized basis risk is simply the regression residual variance.
- (Thus the notation of using ϵ to denote basis.)

Include an intercept?

In regression for optimal hedge ratio, should we include a constant, (alpha?) Depends on our purpose...

- Do we want to explain the total return (including the mean) or simply the excess-mean return?
- In short samples, mean returns may be estimated inaccurately, (whether in r^i or $\tilde{r}^i$,) so we may want to include α (eliminate means) to focus on explaining variation.

Investment with hedging a factor

- Suppose a hedge fund wants to trade on information regarding a certain asset return, r^i.
- But does not want the trade to be subject to the overall market factor, r^m.
- More generally, imagine anyone that wants to trade on the performance of return r^i **relative** to another factor r^j.

This idea of trading on specific information while hedging out broader market movements is the origination of the term, hedge funds.

Building the market-hedged position

A hedge fund would first run the regression

$$\tilde{r}^i_t = \alpha + \beta^{i,m}\tilde{r}^m_t + \epsilon_t$$

- Then the hedge-fund can go long $\tilde{r}^i$, while shorting $\beta^{i,m}$ times the overall market.
- The fund is then holding

$$\tilde{r}^i_t - \beta^{i,m}\tilde{r}^m_t = \alpha + \epsilon_t$$

Properties of the market-hedged position

$$\tilde{r}_t^i - \beta^{i,m}\tilde{r}_t^m = \alpha + \epsilon_t$$

- This hedged position has mean excess return α; volatility σ_ϵ.
- Compared to simply going long $\tilde{r}^i$, the strategy is no longer subject to the volatility coming from $\beta^{i,m}\tilde{r}_t^m$.
- This allows the hedge fund to minimize the variance of the hedged position.

Tracking portfolios

Regress

$$\tilde{r}^i_t = \alpha + \beta \tilde{r}^j_t + \epsilon_t$$

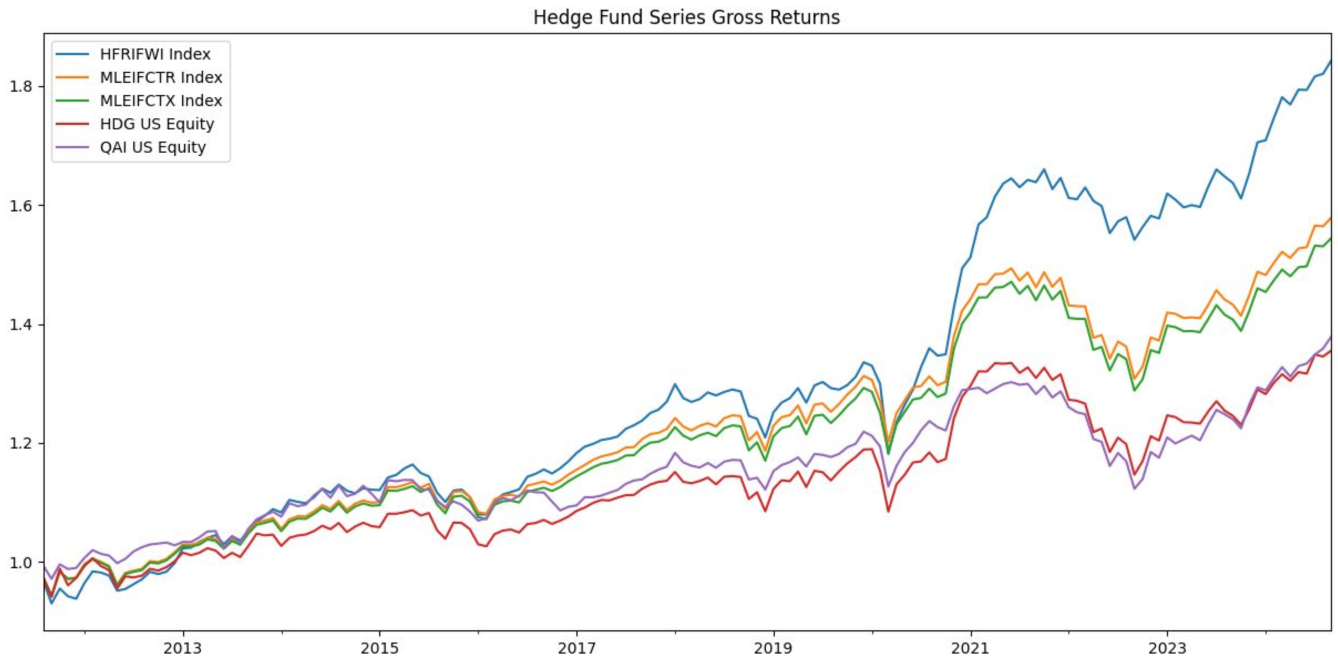
- ϵ is known as the **tracking error** of $\tilde{r}^i$ relative to $\tilde{r}^j$.
- R-squared measures how well j tracks i.
- The Information Ratio, α/σ_ϵ, measures the tradeoff between obtaining extra mean return α at the cost of taking on tracking error ϵ from the target portfolio.

Of course, this is just another way of looking at the hedging problem.

Homework #2

What is HDG?

What is the proposed benefit to HDG?



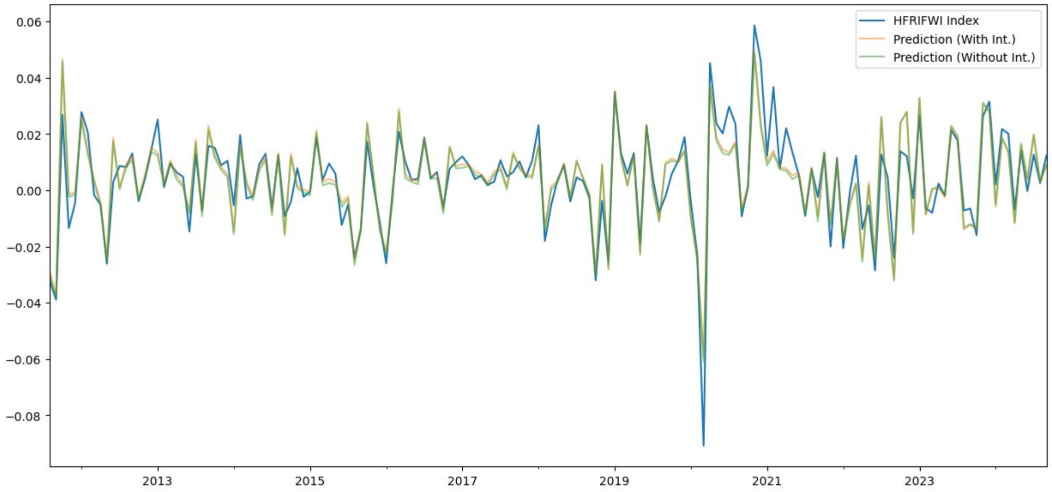
Annualizing Statistics

- Treynor
- Information Ratio

The intercept term

- Should we include an intercept term for replication?

$$\tilde{r}_t^i = \alpha + \beta \tilde{r}_t^j + \epsilon_t$$


HFRIFWI Index
Prediction (With Int.)
Prediction (Without Int.)
0.06
0.04
0.02
0.00
−0.02
−0.04
−0.06
−0.08
2013
2015
2017
2019
2021
2023


HFRIFWI Index
Prediction (With Int.)
Prediction (Without Int.)
1.8
1.6
1.4
1.2
1.0
2013
2015
2017
2019
2021
2023

HFRIFWI Index
Prediction (With Int.)
Prediction (With Int., actual portfolio)
Prediction (Without Int.)
1.0
1.2
1.4
1.6
1.8
2013
2015
2017
2019
2021
2023

Questions?