2 Propositional Logic

Abstract

If it is raining, the ground is wet.

2.1 Precedence of operators

Operator	Precedence
$\neg$	Most
$\wedge$	
$\vee$	
$\Rightarrow, \Leftarrow, \Leftrightarrow$	Least

Remark ("Backwards conveyor belt"). If operators are of equal precedence, the operator to the right takes precedence.

$$\begin{array}{ll} \neg p \wedge q & ((\neg p) \wedge q) \\ p \wedge \neg q & (p \wedge (\neg q)) \\ p \wedge q \vee r & ((p \wedge q) \vee r) \\ p \vee q \wedge r & (p \vee (q \wedge r) \\ p \Rightarrow q \Rightarrow r & (p \Rightarrow (q \Rightarrow r)) \\ p \Rightarrow q \Leftrightarrow r & (p \Rightarrow (q \Leftrightarrow r)) \end{array}$$

Example (Precedence of Propositional Syntax)**.**

2.2 Semantics of Propositional Logic

2.2.1 Truth assignment and tables

ϕ	ψ	$\phi \iff \gamma$
1	1	1
1	0	0
0	1	0
0	0	1

Example.

Notation. Suppose $p \xrightarrow{i} 1$, $q \xrightarrow{i} 0$, and $r \xrightarrow{i} 1$; we denote:

$$p^i = 1,\ q^i = 0 \text{ and } r^i = 1$$

Remark (Sentential Truth Assignment). Refers to assigning arbitrary sentences a truth value:

$$\begin{aligned} (p \vee q)^i &= 1 \\ (q \vee \neg r)^i &= 0 \\ ((p \vee q) \wedge (q \vee \neg r))^i &= 1 \end{aligned}$$

Note. A false statement implies anything.

2.3 Logic, Algebra and rules.

Consider the statements:

If Mary loves Pat, then Mary loves Quincy. If is is Monday and raining, then Mary loves Pat or Quincy. If is is Monday and raining, does Mary love Quincy?

Proof. Let p :=Mary loves Pat, q :=Mary loves Quincy, m :=Monday and r :=raining.
Then

$$p \Rightarrow q$$
$$m \wedge r \Rightarrow p \vee q$$

since on both sides
$\Longleftrightarrow$

$$\Rightarrow q$$
$$m \wedge r \Rightarrow \vee q$$
$$\overline{m \wedge r \Rightarrow q \vee q}$$
$$m \wedge r \Rightarrow q$$

□