An Implementation of Halfedge Data Structure in Catmull-Clark Subdivision for 2-Manifold Single-sided Surface

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1 Introduction

Catmull-Clark Subdivision is a

2 Halfedge Data Structure

An object in 3D Euclid space can be represented by multiple meshes of polygons. A mesh comprises three types of geometry element: vertex, edge, and face. The adjacency structure stores the topological information (adjacency and connectivity) of the mesh. The author chose halfedge data structure as the adjacency structure in this project to realize Catmull-Clark subdivision.

2.1 Vertex, Halfedge, and Face

The definitions and assumptions of vertex, halfedge and face are shown in Table 1. A quadrilateral face made with four halfedges is shown in Figure 1 .

	Definition	Assumption
Vertex	A 3-dimensional point.	No overlapping vertices exits in a mesh. But overlapping vertices can exist in different meshes.
Halfedge	An edge that starts from one vertex and end at another vertex.	A halfedge connects exactly two non-overlapping vertices and it has a direction. Less than two halfedges start from the same vertex and end at the same vertex in a single mesh.
Face	A polygon that contains a loop of vertices and halfedges.	A face has at least three non-overlapping vertices so it makes a polygon. The face has to be constructed with a complete loop of halfedges with on openings.

Table 1: Definitions and assumptions of vertex, halfedge, and face

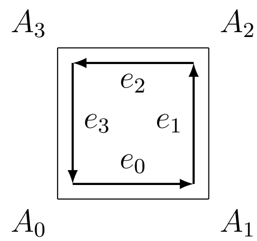

Figure 1: A quadrilateral face made with four halfedges

Elements store two types of data: self-information and adjacency information, where the adjacency includes within a face and between faces. As shown in Table 2.

Element	Self-Information	Adjacency Information
Vertex	1. vertex ID 2. vertex position 3. vertex normal	1. one outgoing halfedge 2. on mobius connection?
Halfedge	1. edge ID	1. start and end vertex 2. link to parent face 3. predecessor and successor in the parent face 4. sibling links to adjacent face 5. boundary links to adjacent face
Face	1. face ID 2. face normal	1. one side halfedge

Table 2: Definitions and assumptions of vertex, halfedge, and face

2.2 Face Connections

There are two types of face connections, the normal connection and the mobius connection, as shown in Figure 2. In a typical halfedge data structure, the pair of halfedges between two faces are defined with opposite direction. The author extend this idea and add another type of connection In a normal connection, the pair of halfedges between two faces are in opposite direction. In the mobius connection, the pair are in same direction.

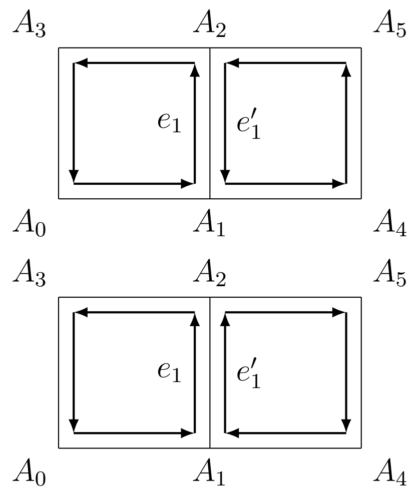

Figure 2: Normal junction (left) and mobius junction (right) between two faces

2.3 Traversals

Three basic traversals are necessary to implement Catmull-Clark subdivision: 1) traversal around a face, 2) The traversal around an edge, and 3) traversal around a vertex.

2.3.1 Traverse around a face

2.3.2 Traverse around an edge

2.3.3 Traverse around a vertex

2.4 Mesh Construction

2.4.1 Build from Elements

2.4.2 Instantiation and Rotation

2.4.3 Build by Merging Meshes

3 Catumll-Clark Subdivision

3.1 General Approach of Catmull-Clark Subdivision

3.1.1 Compute Vertex Positions of New Mesh

3.1.2 Make Connections of New Mesh

3.2 Sharp Crease and Boundary Feature

3.3 Mobius Connection

4 Offset Surface

4.1 Compute Vertex Normals

4.2 Positive and Negative Offsets

4.3 Mobius Connection Issue

5 Input and Output

6 Test Cases and Discussions

7 Future Researches