An Implementation of Halfedge Data Structure in Catmull-Clark Subdivision for 2-Manifold Single-sided Surface

Yu Wang

August 2015

1 Introduction

Catmull-Clark subdivision is widely applied to construct a smooth surface from an initial mesh of polygons. It is independent of the topology of initial mesh.

2 Halfedge Data Structure

An object in the 3D Euclid space can be modeled as several meshes of polygons. For a single mesh, it comprises three types of geometry elements: vertex, edge, and face. Adjacency data structure is need to store the topological information (adjacency and connectivity) between these elements.
Several adjacency structures have been fully developed, including simple data structure, winged edge data structure (Baumgart, 1975), halfedge data structure (Eastman, 1982), QuadEdge Data structure (Guibas and Stolfi), and FaceEdge Data Structure (Dobkin and Laszlo, 1987).
Among all these data structures, the author chose halfedge data structure in this project to realize Catmull-Clark subdivision, because 1) the storage is independent of the mesh topology, and 2) simple implementation. The author also extends its definition to add the ability in dealing with single-sided surfaces (or non-orientable object).

2.1 Vertex, Halfedge, and Face

The author assumes The definitions and assumptions of vertex, halfedge and face are shown in Table 1. A quadrilateral face made with four halfedges is shown in Figure 1.

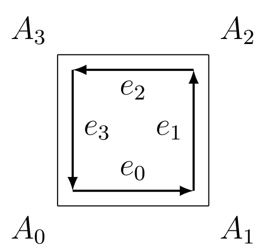

Figure 1: A quadrilateral face made with four halfedges

	Definition	Assumption
Vertex	A 3-dimensional point.	No overlapping vertices exits in a mesh. But overlapping vertices can exist in different meshes.
Halfedge	An edge that starts from one vertex and end at another vertex.	A halfedge connects exactly two non-overlapping vertices and it has a direction. Less than two halfedges start from the same vertex and end at the same vertex in a single mesh.
Face	A polygon that contains a loop of vertices and halfedges.	A face has at least three non-overlapping vertices so it makes a polygon. The face has to be constructed with a complete loop of halfedges with on openings.

Table 1: Definitions and assumptions of vertex, halfedge, and face

Elements store two types of data: self-information and adjacency information, where the adjacency information include links in a face and links between faces. As shown in Table 2.

2.2 Face Connections

There are two types of face connections, the normal connection and the mobius connection, as shown in Figure 2. In a typical halfedge data structure, the pair of halfedges between two faces are defined with opposite direction. The author extend this idea and add another type of connection In a normal connection, the pair of halfedges between two faces are in opposite direction. In the mobius connection, the pair are in same direction.

2.3 Traversals

Three basic traversals are necessary to implement Catmull-Clark subdivision: 1) traversal around a face, 2) The traversal around an edge, and 3) traversal

Element	Self-Information	Adjacency Information
Vertex	1. vertex ID 2. vertex position 3. vertex normal	1. one outgoing halfedge 2. on mobius connection?
Halfedge	1. edge ID	1. start and end vertex 2. link to parent face 3. predecessor and successor in the parent face 4. sibling links to adjacent face 5. boundary links to adjacent face
Face	1. face ID 2. face normal	1. one side halfedge

Table 2: Definitions and assumptions of vertex, halfedge, and face

around a vertex.

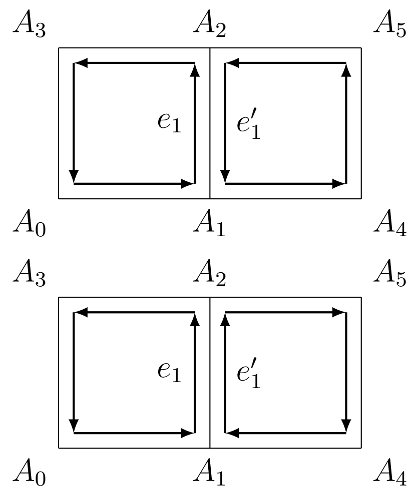

Figure 2: Normal junction (left) and mobius junction (right) between two faces

2.3.1 Traverse around a face

2.3.2 Traverse around an edge

2.3.3 Traverse around a vertex

2.4 Mesh Construction

2.4.1 Build from Elements

2.4.2 Instantiation and Rotation

2.4.3 Build by Merging Meshes

3 Catumll-Clark Subdivision

3.1 General Approach of Catmull-Clark Subdivision

3.1.1 Compute Vertex Positions of New Mesh

3.1.2 Make Connections of New Mesh

3.2 Sharp Crease and Boundary Feature

3.3 Mobius Connection

4 Offset Surface

4.1 Compute Vertex Normals

4.2 Positive and Negative Offsets

4.3 Mobius Connection Issue