Crosstalk on Transmission Lines

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1 Introduction

When two or more transmission lines are in the vicinity of one another, a wave propagating along one line, the primary line, can induce a wave on another line, the secondary line, due to capacitive (electric field) and inductive (magnetic field) coupling between the two lines, resulting in the undesirable phenomenon of crosstalk between the lines.

Here, we analyze a pair of coupled transmission lines for the determination of induced waves on the secondary line for a given wave on the primary line. To keep the analysis simple, we shall consider both lines to be like figure 1. It is also convenient to assume the coupling to be weak, so that we shall be concerned only with the crosstalk from the primary line to the secondary line and not vice versa.

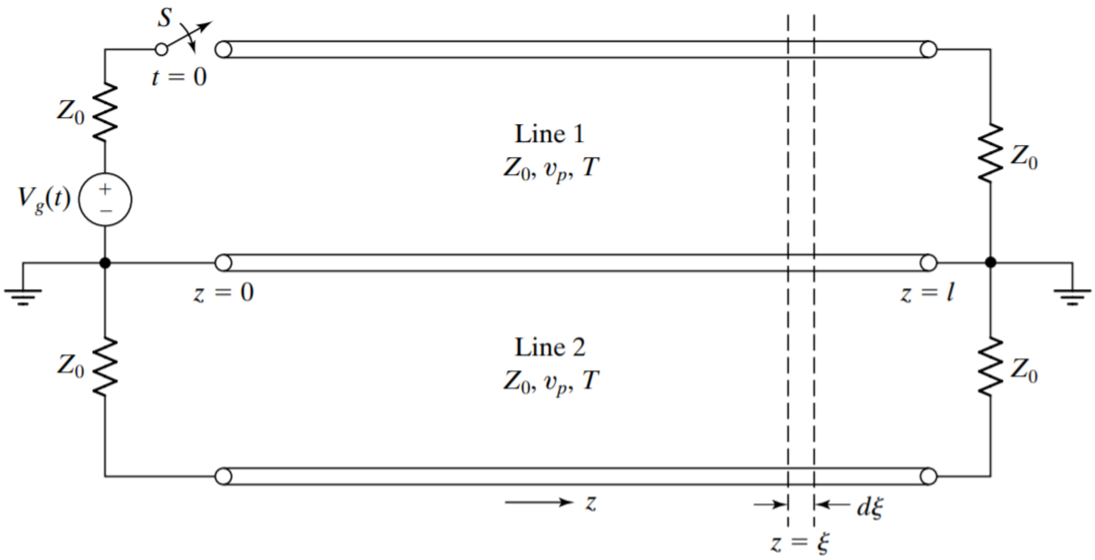

Figure 1: Coupled transmission-line pair for analysis of crosstalk.

2 Derivation

First, we talk about the capacitive coupling, which induces a differential crosstalk current ΔI_{c2}, flowing into the nongrounded conductor of line 2, given by

$$\Delta I_{c2}(\xi, t) = \mathscr{C}_m \Delta\xi \frac{\partial V_1(\xi, t)}{\partial t} \tag{1}$$

where $V_1(\xi, t)$ is the line-1 voltage. This induced current is modeled as shown in figure 2(a), and the equivalent circuit is shown as in figure 2(b).

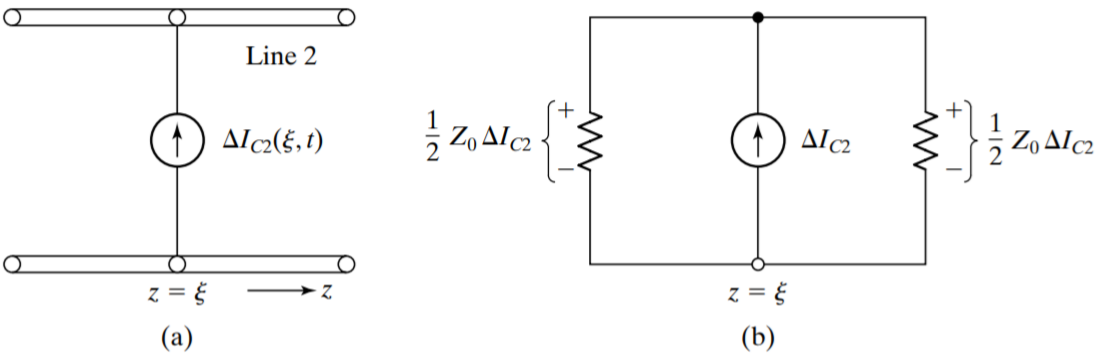

Figure 2: (a) Modeling for capacitive coupling (b) Equivalent circuit for (a)

Then, we talk about the inductive coupling. It induces a differential crosstalk voltage, ΔI_{c2}, which is given by

$$\Delta V_{c2}(\xi, t) = \mathscr{L}_m \Delta\xi \frac{\partial I_1(\xi, t)}{\partial t} \tag{2}$$

This induced voltage is modeled as shown in figure 3(a), and the equivalent circuit is as shown in figure 3(b).

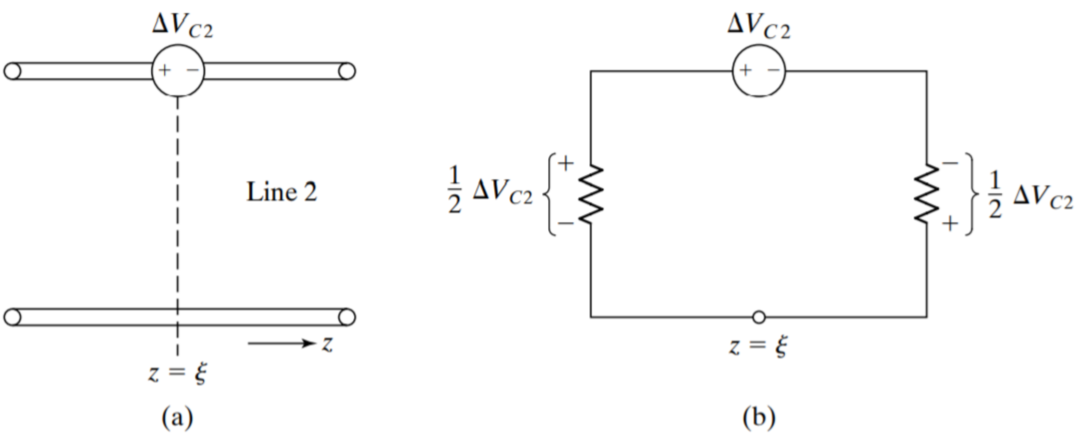

Figure 3: (a) Modeling for inductive coupling (b) Equivalent circuit for (a)

Combining the contributions due to capacitive coupling and inductive coupling, we obtain the total differential voltages produced to the right and left of $z = \xi$ to be

$$\Delta V_2^+ = \frac{1}{2} Z_0 \Delta I_{c2} - \frac{1}{2} \Delta V_{c2} \tag{3}$$

$$\Delta V_2^- = \frac{1}{2} Z_0 \Delta I_{c2} + \frac{1}{2} \Delta V_{c2} \tag{4}$$

Substituting (1) and (2) into (3) and (4), we obtain

$$\begin{aligned} \Delta V_2^+(\xi, t) &= \left[\frac{1}{2} \mathscr{C}_m Z_0 \frac{\partial V_1(\xi, t)}{\partial t} - \frac{1}{2} \mathscr{L}_m \frac{\partial I_1(\xi, t)}{\partial t} \right] \Delta \xi \\ &= \frac{1}{2} \left(\mathscr{C}_m Z_0 - \frac{\mathscr{L}_m}{Z_0} \right) \frac{\partial V_1(\xi, t)}{\partial t} \Delta \xi \end{aligned} \tag{5}$$

$$\Delta V_2^-(\xi, t) = \frac{1}{2} \left(\mathscr{C}_m Z_0 + \frac{\mathscr{L}_m}{Z_0} \right) \frac{\partial V_1(\xi, t)}{\partial t} \Delta \xi \tag{6}$$

To obtain the (+) wave voltages at any location on line 2, we integrate (5).

$$\begin{aligned} V_2^+(z, t) &= \int_0^z \frac{1}{2} \left(\mathscr{C}_m Z_0 - \frac{\mathscr{L}_m}{Z_0} \right) \frac{\partial}{\partial t} \left[V_1 \left(t - \frac{\xi}{v_p} - \frac{z - \xi}{v_p} \right) \right] d\xi \\ &= \frac{1}{2} \left(\mathscr{C}_m Z_0 - \frac{\mathscr{L}_m}{Z_0} \right) \int_0^2 \frac{\partial V_1 \left(t - z/v_p \right)}{\partial t} d\xi \end{aligned} \tag{7}$$

or

$$\boxed{V_2^+(z, t) = z K_f V_1' \left(t - z/v_p \right)} \tag{8}$$

where we have defined

$$\boxed{K_f = \frac{1}{2} \left(\mathscr{C}_m Z_0 - \frac{\mathscr{L}_m}{Z_0} \right)} \tag{9}$$

and the prime associated with V_1 denotes differentiation with time. The quantity K_f is called the *forward-crosstalk coefficient*.

To obtain $V_2^-(z, t)$, we integrate (6).

$$\begin{aligned} V_2^-(z, t) &= \int_z^l \frac{1}{2} \left(\mathscr{C}_m Z_0 + \frac{\mathscr{L}_m}{Z_0} \right) \frac{\partial}{\partial t} \left[V_1 \left(t - \frac{\xi}{v_p} - \frac{\xi - z}{v_p} \right) \right] d\xi \\ &= \frac{1}{2} \left(\mathscr{C}_m Z_0 + \frac{\mathscr{L}_m}{Z_0} \right) \int_z^l \frac{\partial}{\partial t} \left[V_1 \left(t + \frac{z}{v_p} - \frac{2\xi}{v_p} \right) \right] d\xi \\ &= -\frac{1}{4} v_p \left(\mathscr{C}_m Z_0 + \frac{\mathscr{L}_m}{Z_0} \right) \int_z^l \frac{\partial}{\partial \xi} \left[V_1 \left(t + \frac{z}{v_p} - \frac{2\xi}{v_p} \right) \right] d\xi \\ &= -\frac{1}{4} v_p \left(\mathscr{C}_m Z_0 + \frac{\mathscr{L}_m}{Z_0} \right) \left[V_1 \left(t + \frac{z}{v_p} - \frac{2\xi}{v_p} \right) \right]_{\xi = z}^l \end{aligned} \tag{10}$$

or

$$\boxed{V_2^-(z, t) = K_b \left[V_1 \left(t - \frac{z}{v_p} \right) - V_1 \left(t - \frac{2l}{v_p} + \frac{z}{v_p} \right) \right]} \tag{11}$$

where we have defined the *backward-crosstalk coefficient*

$$\boxed{K_b = \frac{1}{4} v_p \left(\mathscr{C}_m Z_0 + \frac{\mathscr{L}_m}{Z_0} \right)} \tag{12}$$

3 Example

Let $V_g(t)$ in figure 1 be the function shown in figure 4, where $T_0 < T(= l/v_p)$. We wish to determine the $(+)$ and $(-)$ wave voltages on line 2.

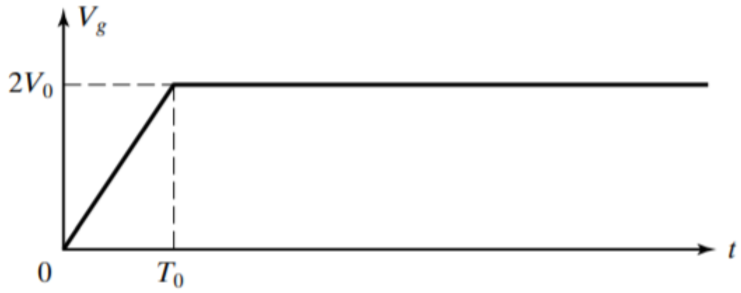

Figure 4: Source Voltage in Example

We can find that

$$V_1(t) = \frac{1}{2}V_g(t) = \begin{cases} (V_0/T_0)\,t & \text{for} \quad 0 < t < T_0 \\ V_0 & \text{for} \quad t > T_0 \end{cases}$$

and hence

$$V_1'(t) = \begin{cases} V_0/T_0 & \text{for} \quad 0 < t < T_0 \\ 0 & \text{for} \quad t > T_0 \end{cases}$$

Using (8), we can write the $(+)$ wave voltage on line 2 as

$$\begin{aligned} V_2^+(z,t) &= zK_f V_1'\left(t - z/v_p\right) \\ &= \begin{cases} zK_f V_0/T_0 & \text{for } (z/l)T < t < \lfloor (z/l)t + T_0 \rfloor \\ 0 & \text{otherwise} \end{cases} \end{aligned}$$

Using (11), the $(-)$ wave voltage can be written as

$$V_2^-(z,t) = K_b\left[V_1\left(t - z/v_p\right) - V_1\left(t - 2l/v_p + z/v_p\right)\right]$$

where

$$V_1\left(t - \frac{z}{v_p}\right) = \begin{cases} \frac{V_0}{T_0}\left(t - \frac{z}{l}T\right) & \text{for } \frac{z}{l}T < t < \left(\frac{z}{l}T + T_0\right) \\ V_0 & \text{for } t > \left(\frac{z}{l}T + T_0\right) \end{cases}$$

$$V_1\left(t - \frac{2l}{v_p} + \frac{z}{v_p}\right) = \begin{cases} \frac{V_0}{T_0}\left(t - 2T + \frac{z}{l}T\right) & \text{for } \left(2T - \frac{z}{l}T\right) < t < \left(2T - \frac{z}{l}T + T_0\right) \\ V_0 & \text{for } t > \left(2T - \frac{z}{l}T + T_0\right) \end{cases}$$