

Let $A, B \in \mathbb{R}^{n \times n}$ be upper triangular matrices.

To show that $C = AB \in \mathbb{R}^{n \times n}$ is also an upper triangular matrix.

Consider the entries of C which is given as

$$C_{ij} = \sum_{k=0}^{n-1} A_{ik} B_{kj} \quad \text{for } i, j = 0, \dots, n-1$$

$$C_{ij} = \sum_{k=0}^{i-1} A_{ik} B_{kj} + \sum_{k=i}^{n-1} A_{ik} B_{kj}$$

Consider the case where $i > j$ then $A_{ik} = 0$
and $B_{kj} = 0$ for $k < i$ and $k > j$ respectively.

\therefore For $i > j$

$$C_{ij} = \sum_{k=0}^{i-1} A_{ik} B_{kj} + \sum_{k=i}^{n-1} A_{ik} B_{kj}$$

$$= \cancel{0} + 0$$

$$C_{ij} = 0, \text{ hence } C_{ij} = 0 \text{ for } i > j$$

Therefore C is also an upper triangular matrix.