# **Mathematica Cheat Sheet**

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# **Getting Started**

### **Mathematica Document**

The Mathematica document acts exactly like a Jupyter Notebook. When you set a variable, it sets it for the entire document environment. While it's more complicated than this, what it means for you is if you set

```
In[98]:= X = 5;
```

on one line, it will save that value until you clear it or change it to a different value or you even delete the line where you defined it! The first step in debugging is always to check that all your variables are what you think they are.

Also be careful if you have multiple Mathematica files open. It likes to bring variables from one document to another!

## **Defining Basic Variables**

You can define basic variables using "=". You can suppress the output by ending the line with ";", but it is not necessary.

```
ln[2]:= x = 1

y = 2;

z = x + y
```

Out[2]= 1

Out[4]= 3

You can use this to also define strings as well, but I have rarely done this.

```
In[5]:= school = "Princeton"
```

Out[5]= Princeton

## Special Characters + Useful Key-binds

Mathematica makes it incredibly easy to use Greek letters and various other symbols. You can write  $\alpha$  by typing: 'Esc' + alpha + 'Esc'

Hitting the 'Esc' key gives you this symbol - 'E'. You actually don't need to type out the full word either! You can make  $\alpha$  by just typing 'Esc' + a + 'Esc'. When you type in a letter, look to the autocomplete line to see what symbol Mathematica thinks you want.

Below I've made a table of the special characters I end up using the most.

Symbol	Syntax (w/ spaces)	Used For
Greek Letters	: letter :	All Greek letters, $\pi$ , $\gamma$ , $\mu$ , etc
ħ	፤ hbar ፤	Quantum Stuff!
€	elem	Sets, you'll use this with complicated funcitons.
t	į ct į	Conjugate Transpose, more Quantum Mechanics
0	i deg i	Degrees, switching between radians and degrees
$\int \Box \times d\Box$	intt:	Special note below
∞	inf:	Infinite sums / integrals

Note on | ■ × d □ - To set the bounds, use Ctrl + - to set lower bound. Then use Ctrl + 5 to 'jump' to upper bound. Using Ctrl + then Ctrl + 6 causes problems. I would highly recommend against doing integrals like this. Use t

There are also a lot of useful ways you can format your equations to look very nice. This allows you to make fractions that look like a numerator over a denominator and a square root with the actual square root. I find are much easier to use and find typos when writing massive equations.

Format	Keybind for Windows	Notes
$\frac{1}{2}$	Ctrl + /	Maybe use Cmd for Mac?
$\sqrt{2}$	Ctrl + 2	
x <sup>2</sup>	Ctrl + 6	
<b>x</b> <sub>1</sub>	Ctrl + -	Be careful when using this to distinguish variables. Sometimes Mathematica can mess up variables when you have x, $x_1$ , and $x_2$
$\int_1^2$	Ctrl + 5	Jumps from subscript to superscript or reverse. Needed for $\int\!\!\!\!\!\!\!\Box\times d\!\!\!\!\!\!\!\!\!\!\!\!\Box$

Clear[x, y, z]

### **Calling Functions**

A basic function is Simplify[]. All Mathematica functions start with a capital letter and use brackets [] to call the function. Let's simplify x+x into 2x.

```
Simplify[x + x]
In[7]:=
```

Out[7]= 2 x

Hovering over a function, you'll see



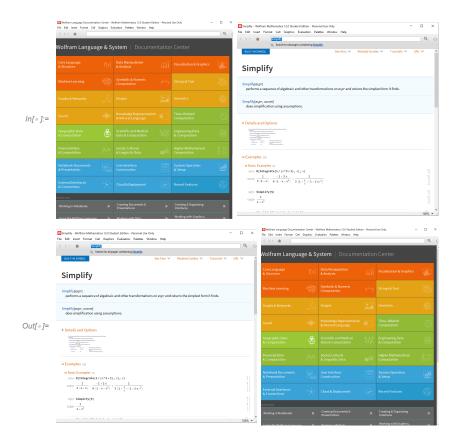
Clicking on the down arrows gives you a brief overview of how to use the function. Clicking the i will open the help window.

### Dropdown help:



### Using the Help Window

The help window should look like this. You can also access it by clicking on 'Help' in the top right corner. If you need a common sounding function but aren't sure of the exact name or syntax, the search does a very good job at getting something very close to what you want. This is also a very easy way to explore other functions in Mathematica by category if you're not sure exactly what you need.



**PRO TIP:** Mathematica has functions for nearly everything, so if you're doing something kind of complicated the function probably already exists.

In[8]:= ClearAll[x, y, z]

## **Useful Functions**

### Writing Equations (Sums, Integrals, Derivative, Taylor Series etc.)

Mathematica will let you write equations symbolically. Let's write out the force of gravity:

$$\ln[9] = \frac{G \, m1 \, m2}{r^2}$$

$$Out[9] = \frac{G \, m1 \, m2}{r^2}$$

Since we haven't defined any of these variables, Mathematica treats them as symbols that we can manipulate. Let's combine this with some other forces we may have and see what happens:

$$\ln[10] = \frac{G \text{ m1 m2}}{r^2} + a + b x + c x^2 - \frac{2G \text{ m1 m2}}{r^2}$$

Out[10]= 
$$a - \frac{G \text{ m1 m2}}{r^2} + b x + c x^2$$

Note on Equation Syntax: Spaces are interpreted as multiplication (or you can use \*), but make sure you don't forget them! ax is not the same as a x.

Now let's get more complicated equations going! For these, it is far easier to use the function form than using all the symbols. I'll show the first couple in both forms so you can get the idea, but I'll switch to just functions later. I personally find it much easier to type/read them as functions, but do whatever you find easiest!

Taking Sums:

$$\ln[11] = \sum_{k=1}^{\infty} \frac{1}{x^2}$$

$$Sum \left[ \frac{1}{x^2}, \{x, 1, \infty\} \right]$$

Out[11]= 
$$\frac{\pi^2}{6}$$

Out[12]= 
$$\frac{\pi^2}{6}$$

Integrals:

Integrate [a+b x) dlx

Integrate [a+b x, x]
$$\int_{-\infty}^{\infty} E^{-x^2} dx$$
Integrate [Exp[-x<sup>2</sup>], {x,-\infty,\infty}]

Out[13]= 
$$a x + \frac{b x^2}{2}$$

Out[14]= 
$$a x + \frac{b x^2}{2}$$

Out[15]= 
$$\sqrt{\pi}$$

Out[16]= 
$$\sqrt{\pi}$$

**Derivatives:** 

$$D[x^2, x]$$
 $D[x^2, \{x, 2\}]$ 
 $D[x^2, \{x, 3\}]$ 
 $D[x^2, \{x, 3\}]$ 

Out[17]= 2 x

Out[18]= 2

Out[19]= **0** 

Out[20]= -Sin[x]

Note that you can also take derivatives with the 'when used before a function bracket.

**Taylor Series:** 

Out[21]= 
$$1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + 0[x]^5$$

Out[22]= 
$$1 - \frac{x^2}{2} + \frac{x^4}{24} + 0[x]^5$$

Out[23]= 
$$x - \frac{x^3}{6} + 0[x]^5$$

Out[24]= 
$$x + x^2 + \frac{x^3}{3} + 0[x]^5$$

Something important to note here: we see the  $O[x]^5$  represents our error term from cutting off the Taylor series. This is useful in the case of the last term as Mathematica knows when to drop higher order terms.

### **Defining Your Own Basic Function**

If you have a simple function repeated many times, it is often far simpler to define it yourself. You can do this with the following code:

$$ln[25]:= f[x_] = \frac{Cos[x]}{x^2 + 4x + 1} Exp[-\frac{x^2}{5}]$$

Out[25]= 
$$\frac{e^{-\frac{x^2}{5}} \cos [x]}{1 + 4 x + x^2}$$

Keep in mind, you NEED the syntax f[x], not just f[x]. We can now manipulate this using what we learned before.

$$\text{Out}[26] = -\frac{e^{-\frac{4\,x^2}{5}}\,\left(4+4\,x\right)\,\text{Cos}\left[\,2\,x\,\right]}{\left(\,1+8\,x+4\,x^2\,\right)^{\,2}} \,-\, \frac{4\,e^{-\frac{4\,x^2}{5}}\,x\,\text{Cos}\left[\,2\,x\,\right]}{5\,\left(\,1+8\,x+4\,x^2\,\right)} \,-\, \frac{e^{-\frac{4\,x^2}{5}}\,\text{Sin}\left[\,2\,x\,\right]}{1+8\,x+4\,x^2}$$

$$\begin{aligned} & \text{Out}[27] = & -\frac{\text{e}^{-\frac{x^2}{5}} \; \left(4 + 2 \; x\right) \; \text{Cos} \left[x\right]}{\left(1 + 4 \; x + x^2\right)^2} - \frac{2 \; \text{e}^{-\frac{x^2}{5}} \; x \; \text{Cos} \left[x\right]}{5 \; \left(1 + 4 \; x + x^2\right)} - \\ & -\frac{\text{e}^{-\frac{x^4}{5}} \; \left(4 + 2 \; x^2\right) \; \text{Cos} \left[x^2\right]}{\left(1 + 4 \; x^2 + x^4\right)^2} - \frac{2 \; \text{e}^{-\frac{x^4}{5}} \; x^2 \; \text{Cos} \left[x^2\right]}{5 \; \left(1 + 4 \; x^2 + x^4\right)} - \frac{\text{e}^{-\frac{x^2}{5}} \; \text{Sin} \left[x\right]}{1 + 4 \; x + x^2} - \frac{\text{e}^{-\frac{x^4}{5}} \; \text{Sin} \left[x^2\right]}{1 + 4 \; x^2 + x^4} \end{aligned}$$

### Simplifying Real Equations

Let's start with an obvious simplification:

$$\frac{(x+1) (x-1)}{x^2-1}$$
FullSimplify  $\left[\frac{(x+1) (x-1)}{x^2-1}\right]$ 

Out[28]= 
$$\frac{(-1+x)(1+x)}{-1+x^2}$$

Out[29]= **1** 

FullSimplify[] will solve 90% of all your problems! Some of the easiest debugging steps if it's not working out, try using Expand[] and it will separate as many terms as it can. Don't bother with this everytime, but occasionally it may help.

$$\text{Out} [\text{30}] = -\frac{4 \, \text{e}^{-\frac{x^4}{5}} \, \text{Cos} \left[ x^2 \right]}{\left( 1 + 4 \, x^2 + x^4 \right)^2} \, - \, \frac{2 \, \text{e}^{-\frac{x^4}{5}} \, x^2 \, \text{Cos} \left[ x^2 \right]}{\left( 1 + 4 \, x^2 + x^4 \right)^2} \, - \, \frac{2 \, \text{e}^{-\frac{x^4}{5}} \, x^2 \, \text{Cos} \left[ x^2 \right]}{5 \, \left( 1 + 4 \, x^2 + x^4 \right)} \, - \, \frac{\text{e}^{-\frac{x^4}{5}} \, \text{Sin} \left[ x^2 \right]}{1 + 4 \, x^2 + x^4}$$

$$\begin{array}{c} \text{Out[31]=} & \frac{ e^{-\frac{x^4}{5}} \, \left(-2 \, \left(10+6 \, x^2+4 \, x^4+x^6\right) \, \text{Cos} \left[\, x^2\,\right]\, -\, 5 \, \left(1+4 \, x^2+x^4\right) \, \text{Sin} \left[\, x^2\,\right]\, \right)}{5 \, \left(1+4 \, x^2+x^4\right)^2} \end{array}$$

### **Simplifying Complex Equations**

Lets try to simplify this complex exponential function:

In[32]:= FullSimplify 
$$\left[ \text{Exp} \left[ \frac{\text{I k y d}}{2\text{z}} \right] + \text{Exp} \left[ -\frac{\text{I k y d}}{2\text{z}} \right] \star \text{Exp} \left[ \text{I T k (1-n)} \right] \right]$$

$$\text{Out} \text{[32]=} \quad \text{@}^{-\frac{1}{2}} \text{ is k } \left(2 \text{ (-1+n) } \text{ T} + \frac{\text{d} \, y}{\text{z}}\right) \\ + \text{ @}^{\frac{\text{i} \, \text{d} \, \text{k} \, y}{2 \, \text{z}}}$$

Notice how that didn't do anything. Let's try using ComplexExpand[] first and see if we can get something better:

In[33]:= FullSimplify [ComplexExpand 
$$\left[ Exp \left[ \frac{I k y d}{2z} \right] + Exp \left[ -\frac{I k y d}{2z} \right] * Exp[I T k(1-n)] \right] \right]$$

$$\text{Out} \text{[33]= } 2 \, \text{@}^{-\frac{1}{2} \, \text{i} \, k \, \, (-1+n) \, \, T} \, \, \text{Cos} \left[ \frac{k \, \, (d \, y \, + \, (-1+n) \, \, T \, z)}{2 \, z} \, \right]$$

That looks quite a bit better! When dealing with complex functions (will happen a lot with waves), this almost always improves the simplification!

### Solving Equations

The next thing you'll want to do is solve equations to avoid lots of algebra. Let's start off with a basic quadratic:

$$ln[34]:=$$
 equation = a  $x^2$  + b  $x$  + c;

Note; can be used to suppress the output.

Now we want to solve for x when this equation equals 0. We can do this with

$$\text{Out[35]= } \left\{ \left\{ x \to \frac{-\,b\,-\,\sqrt{b^2\,-\,4\,a\,c}}{2\,a} \right\} \text{, } \left\{ x \to \frac{-\,b\,+\,\sqrt{b^2\,-\,4\,a\,c}}{2\,a} \right\} \right\}$$

Now we want to use this solution to do other things. You can just copy paste the solution and set some variable to be that, or do this fancy step: (those are double brackets [[1]])

Out[36]= 
$$\frac{-b - \sqrt{b^2 - 4 a c}}{2 a}$$

You can change the number in the double brackets to select the solution that you want.

Out[37]= 
$$\frac{-b + \sqrt{b^2 - 4 a c}}{2 a}$$

### **Solving Systems of Equations**

You can define systems of equations a few ways. I usually write it all out in one line, but for the sake of demonstration I'll separate everything out.

```
eq1 = a+b+c==0;
In[38]:=
      eq2 = a-b-c=5;
      eq3 = a+b-c=2;
      eqs = {eq1, eq2, eq3};
```

Here we use {} to show that we want to solve a set of equations. Also note the double equals == instead of the single = we use when defining variables. If you accidentally use =, it will try to assign a value and run into a lot of errors. You will also need to clear your variables before trying again.

$$\label{eq:outsign} \begin{split} & \textit{Info} \ \textit{j:=} \ \ \textbf{Solve[eqs, \{a,b,c\}]} \\ & \textit{Out[s]=} \ \left\{ \left\{ a \to \frac{5}{2} \text{, } b \to -\frac{3}{2} \text{, } c \to -1 \right\} \right\} \end{split}$$

So that's it, we solved it! It's important to say which variables you want to solve for, because if we try solving something like the equation below, it won't automatically give us {a, b, c}.

In[110]:= Clear[x] FullSimplify[Solve[{a x+b y+c z ==0, a 
$$x^2+b+c$$
  $z^2==1$ , a + b + c ==1}]] Solve[{a x+b y+c z ==0, a  $x^2+b+c$   $z^2==1$ , a + b + c ==1},{a,b,c}]

$$\text{Out[111]= } \left\{ \left\{ b \to \frac{-1 + z^2 + a \; (x - z) \; (x + z)}{-1 + z^2} \;,\; c \to \frac{a - a \; x^2}{-1 + z^2} \;,\; y \to \frac{a \; (x - z) \; (1 + x \; z)}{-1 + z^2 + a \; (x - z) \; (x + z)} \right\} , \\ \left\{ a \to \frac{z^2}{1 + z^2} \;,\; b \to 0 \;,\; c \to \frac{1}{1 + z^2} \;,\; x \to -\frac{1}{z} \right\} \;,\; \left\{ a \to 0 \;,\; b \to 1 - c \;,\; y \to -\frac{c}{-1 + c} \;,\; z \to -1 \right\} , \\ \left\{ b \to 1 - a - c \;,\; x \to -1 \;,\; y \to -\frac{a + c}{-1 + a + c} \;,\; z \to -1 \right\} \;,\; \left\{ a \to 0 \;,\; b \to 1 - c \;,\; y \to \frac{c}{-1 + c} \;,\; z \to 1 \right\} , \\ \left\{ b \to 1 - a - c \;,\; x \to 1 \;,\; y \to \frac{a - c}{-1 + a + c} \;,\; z \to 1 \right\} \;,\; \left\{ a \to 0 \;,\; b \to 1 - c \;,\; y \to \frac{c}{-1 + c} \;,\; z \to 1 \right\} , \\ \left\{ b \to 1 - a - c \;,\; x \to -1 \;,\; y \to \frac{-a + c}{-1 + a + c} \;,\; z \to 1 \right\} \;,\; \left\{ b \to 1 - a - c \;,\; x \to 1 \;,\; y \to 1 + \frac{1}{-1 + a + c} \;,\; z \to 1 \right\} , \\ \left\{ a \to \frac{1}{2} \;,\; b \to 0 \;,\; c \to \frac{1}{2} \;,\; x \to 1 \;,\; z \to -1 \right\} \;,\; \left\{ a \to \frac{1}{2} \;,\; b \to 0 \;,\; c \to \frac{1}{2} \;,\; x \to -1 \;,\; z \to 1 \right\} \right\}$$

### **Solving Differential Equations**

The syntax for this is very similar, there are just a few extra things to include. You can solve for a general solution for a differential equation, but almost all of the time you'll have some information about the system. Lets solve for  $y''(t) + 2y'(t) = -\cos(t)$  with the initial conditions that y'(0) = 0.

$$\text{Out}[45] = \left\{ \left\{ y \left[ t \right] \right. \right. \rightarrow \left. -\frac{1}{5} \right. e^{-2t} \left( -1 + e^{2t} \, \mathsf{Cos} \left[ t \right] \right. - 2 \cdot e^{2t} \, \mathsf{Sin} \left[ t \right] \right) \right\} \right\}$$

Note you can define any number of other initial conditions this way, just make sure you include the == separate them appropriately.

### **Numeric Solving**

Depending on the class, you may be using actual numbers instead of variables (AH disgusting I know!). If this is the case, you may want to integrate things numerically because Mathematica cannot solve them symbolically. The most common ones are NSolve[] and NIntegrate[].

```
NSolve [y^{3.42}-2y^2=0,y]
In[46]:=
Out[46]= \{\,\{\,y\rightarrow\textbf{1.62927}\,\}\,,\,\,\{\,y\rightarrow\textbf{0.}\,\}\,,\,\,\{\,y\rightarrow\textbf{0.}\,\}\,\}
              NIntegrate [Exp[x^2]Tan[x], \{x, 0.1, 1\}]
In[114]:=
```

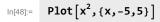
Out[114]= 1.09167

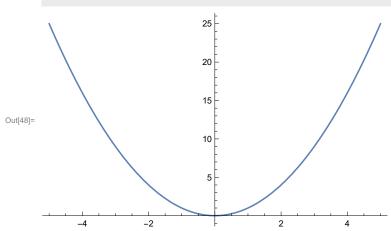
These don't really necessitate NSolve[] and NIntegrate[] since they already have decimals, but hopefully you get the idea. You'll need this when you get red errors telling you that some solutions were lost.

# **Plotting**

There's not too much going on here, so I'm just going to speed through a bunch of syntax that you can look at/copy if you need it later.

### **Basic Plot**

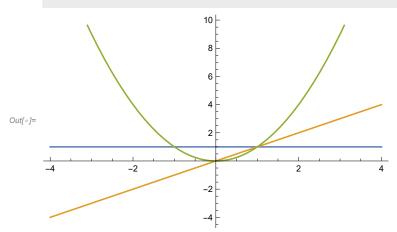




## **Plotting Multiple Functions**

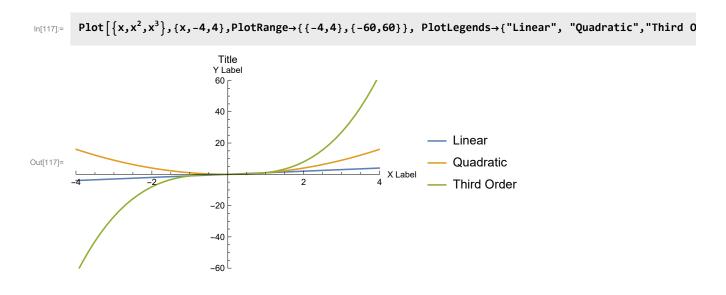
Note the use of the brackets to make a list of functions.

Plot[
$$\{1,x,x^2\},\{x,-4,4\}$$
]



## Prettying and Tidying Up Your Plot

This next part is likely overkill, but should have all the major syntax you'll need to make a nice looking plot.



# Linear Algebra Basics

### **Defining Matrices and Vectors**

The syntax for vectors and matrices can be a bit confusing, so be careful when writing things out!

```
vec={{a},{b},{c}};
In[51]:=
      vec//MatrixForm
      matrix={{a,b},{c,d}};
      matrix//MatrixForm
```

Out[52]//MatrixForm=

b

Out[54]//MatrixForm=

Make sure you don't include //MatrixForm when defining the matrix because I have run into some errors that way. If you are using a vector for anything other than a basic dot product, make sure to use double brackets! Here are some issues:

```
In[55]:=
      badvec={a,b,c};
      badvec//MatrixForm
      Transpose[badvec]//MatrixForm
      Transpose[vec]//MatrixForm
```

Out[56]//MatrixForm=

C

Out[57]//MatrixForm=

b

Out[58]//MatrixForm=

(a b c)

Notice how the last line gives the desired row vector while we can't take the transpose of **badvec**.

There's also a neat way to define large arrays that only have a few elements:

```
MatrixForm[SparseArray[\{\{3,3\}\rightarrow 1, \{2,1\}\rightarrow 4, \{5,1\}\rightarrow 5, \{1,5\}\rightarrow 5\}]]
In[59]:=
```

Out[59]//MatrixForm=

0 0 0 0 5 4 0 0 0 0 0 0 1 0 0 00000 5 0 0 0 0

If you don't like the bracket form of inputting a matrix, check out this site https://reference.wolfram.com/language/howto/InputAMatrix.html and how you can input a matrix in MatrixForm.

### **Matrix Multiplication**

Now that we have our matrices, we can start to do fun things! (Note I define the other matrix like this by right clicking and then going to "Add Table/Matrix")

```
matrix2 = \left(\frac{e|f|}{g|h|}\right);
In[60]:=
        matrix matrix2 //MatrixForm
        matrix.matrix2 //MatrixForm
```

Out[61]//MatrixForm=

ae bf cg dh/

Out[62]//MatrixForm=

(ae+bg af+bh) ce+dg cf+dh,

Note the difference between the two products here. Without any symbols, each term is multiplied. With the ".", Mathematica performs matrix multiplication.

Out[133]//MatrixForm=

1 1 0 1

### **Matrix Functions**

Besides regular matrix multiplication, here's a bunch of useful functions you may need:

```
MatrixPower[matrix,2]//MatrixForm
  In[63]:=
         MatrixPower[matrix,3]//MatrixForm
Out[63]//MatrixForm=
          a^2 + bc \quad ab + bd
          ac+cd bc+d^2
Out[64]//MatrixForm=
          a(a^2 + bc) + b(ac + cd) a(ab + bd) + b(bc + d^2)
         c(a^2 + bc) + d(ac + cd) c(ab + bd) + d(bc + d^2)
         Transpose[matrix]
  In[65]:=
 Out[65]= \{ \{a, c\}, \{b, d\} \}
         Det[matrix]
  In[66]:=
 Out[66]= -bc+ad
  In[67]:=
         Conjugate[matrix]//MatrixForm
         ConjugateTranspose[matrix]//MatrixForm
Out[67]//MatrixForm=
          Conjugate[a] Conjugate[b] \)
          Conjugate[c] Conjugate[d] /
Out[68]//MatrixForm=
          Conjugate[a] Conjugate[c] \rangle
         Conjugate[b] Conjugate[d]
         Tr[matrix]
  Out[\ \ \ \ ]=\ \ a\ +\ d
         mat = \{\{0,1\},\{0,0\}\};
 In[131]:=
         mat //MatrixForm
         MatrixExp[mat]//MatrixForm
Out[132]//MatrixForm=
          0 1
         0 0
```

Here are some other useful functions to make standard matrices you'll run into. This will save you some time if you don't want to write out the full matrices yourself.

```
IdentityMatrix[2]//MatrixForm
In[69]:=
      IdentityMatrix[3]//MatrixForm
      PauliMatrix[1]//MatrixForm
      PauliMatrix[2]//MatrixForm
      PauliMatrix[3]//MatrixForm
```

Out[69]//MatrixForm=

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Out[70]//MatrixForm=

$$\begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{pmatrix}$$

Out[71]//MatrixForm=

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Out[72]//MatrixForm=

$$\left(\begin{array}{cc}
\mathbf{0} & -\mathbf{i} \\
\mathbf{i} & \mathbf{0}
\end{array}\right)$$

Out[73]//MatrixForm=

$$\left(\begin{array}{cc} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & -\mathbf{1} \end{array}\right)$$

### Eigenvalues, Eigenvectors, etc

```
m = \{\{4,2\},\{-1,2\}\};
Eigenvalues[m]
Eigenvectors[m]
Eigensystem[m]//MatrixForm
```

```
Out[75]= \{3 + i, 3 - i\}
  Out[76]= \{\{-1-\dot{\mathbb{1}}, 1\}, \{-1+\dot{\mathbb{1}}, 1\}\}
Out[77]//MatrixForm=
                \begin{pmatrix} 3 + i & 3 - i \\ \{-1 - i, 1\} & \{-1 + i, 1\} \end{pmatrix}
```

#### **Tensors**

I personally dislike trying to use tensors in Mathematica and prefer the regular matrix representation. However there are some functions you can try out and see if you like:

TensorProduct[vec,vec]//MatrixForm TensorSymmetry[TensorProduct[vec,vec]]

```
Out[136]//MatrixForm=
Out[137]= \{\{Cycles[\{\{1,3\}\}], 1\}, \{Cycles[\{\{2,4\}\}], 1\}\}
```

Clear[x]

# Miscellaneous Useful Functions and Simplification Tricks

### Simplifying with Assumptions

A lot of the time, you'll know more about the variables than Mathematica does. For example, the standard deviation in a Gaussian will be real and positive or you may only be looking at x>0.

$$Integrate \left[ Exp \left[ -\frac{x^2}{2\sigma^2} \right], \{x, -\infty, \infty\} \right]$$
 
$$Refine \left[ Integrate \left[ Exp \left[ -\frac{x^2}{2\sigma^2} \right], \{x, -\infty, \infty\} \right], Assumptions \rightarrow \{\sigma \in Reals, \sigma > \emptyset\} \right]$$

Out[138]= 
$$\sqrt{\frac{\sqrt{2 \pi}}{\sqrt{\frac{1}{\sigma^2}}}}$$
 if  $Re\left[\sigma^2\right] > 0$ 

Out[139]= 
$$\sqrt{2 \pi} \sigma$$

FullSimplify 
$$\left[ \sqrt{x^2} \right]$$
FullSimplify  $\left[ \sqrt{x^2} \right]$ , Assumptions  $\rightarrow \{x > 0\}$ 

Out[83]= 
$$\sqrt{x^2}$$

Out[84]= X

This is especially useful for when you want to take the conjugate of things.

```
FullSimplify[(a+b I)Conjugate[a+b I]]
        FullSimplify[(a+b I)Conjugate[a+b I],Assumptions\rightarrow{{a,b} \in Reals}]
Out[85]= (a + ib) (Conjugate [a] - i Conjugate [b])
Out[86]= a^2 + b^2
```

#### **TeXForm**

This is incredibly useful if you LATEX your sets. You can take some massive expression and then get LATEX code that you can copy paste over.

```
TeXForm \left[ \frac{a + b + x^2 + c + x^3}{Exp[5x^2]} \right]
```

Out[87]//TeXForm=

e^{-5 x^2} \left(a x+b x^2+c x^3\right)

```
TeXForm[matrix]
In[88]:=
```

Out[88]//TeXForm=

```
\left(
\begin{array}{cc}
a & b \\
c & d \\
\end{array}
\right)
```

This works with //TeXForm just like //MatrixForm

$$ln[140] = \frac{a x + b x^2 + c x^3}{Exp[5 x^2]}$$
 // TeXForm

Out[140]//TeXForm=

```
e^{-5 x^2} \left(a x+b x^2+c x^3\right)
```

### **Units and Unit Simplifications**

This is sometimes incredibly useful, but really only worth it if you want to solve a lot of the problem this way. Sometimes, it's easier to just go to WolframAlpha instead. But here's how you can do it in Mathematica.

```
Quantity[1,"Meters"]
In[89]:=
Out[89]= 1 m
        Quantity[1,"Joules"]
In[90]:=
Out[90]= 1 J
```

```
Quantity[1,"Joules"]
In[161]:=
            Quantity[1,"Newtons"]
           \label{eq:unitSimplify} \textbf{UnitSimplify} \Big[ \frac{\textbf{Quantity} \texttt{[1,"Joules"]}}{\textbf{Quantity} \texttt{[1,"Newtons"]}} \Big]
           UnitConvert[Quantity[760,"MillimetersOfMercury0C"]]
           UnitConvert[Quantity[760, "MillimetersOfMercuryOC"], "Atmospheres"]
```

```
Out[161]= 1 J / N
Out[162]= 1 m
Out[163]= 101325 \cdot \text{kg}/(\text{ms}^2)
Out[164]= 0.999998 atm
```

Note that **UnitConvert** [] converts to base SI and is very useful.

Some other useful units to have: (note these all have the correct values and units if you use Unit

```
Quantity[1,"ReducedPlanckConstant"]
Quantity[1,"PlanckConstant"]
Quantity[1,"BoltzmannConstant"]
UnitConvert[Quantity[1, "BoltzmannConstant"]]
```

```
Out[94]= ħ
Out[95]= h
Out[96]= k
              1380649
                              - kg m^2 / (s^2 K)
```

### **Delayed Evaluation**

You can use the operator := (also known as the Walrus operator) to assign a value without actually evaluating it. Let's see what this looks like.

```
c = 1;
In[197]:=
         f[x_{-}] = c x^{2};
         g[x_{-}] := c x^{2};
         f[x]
         g[x]
```

```
Out[200]= x^2
Out[201]= x^2
```

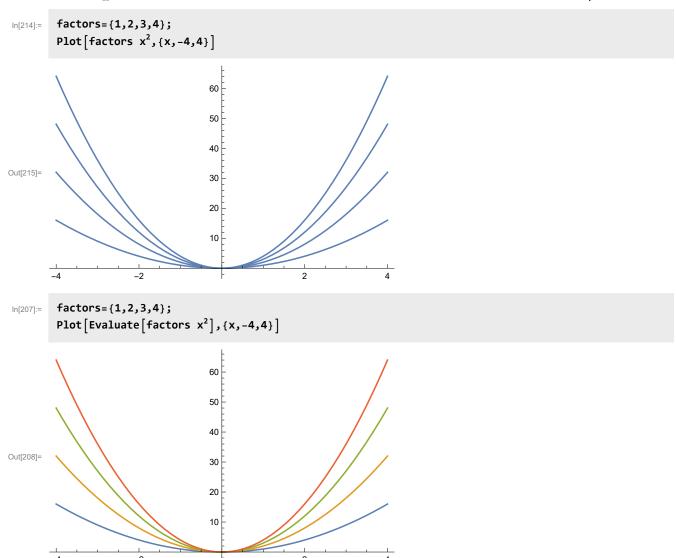
Now let's change the value of c.

```
c = 2;
In[202]:=
         f[x]
         g[x]
```

Out[203]= x<sup>2</sup>

Out[204]=  $2 x^2$ 

Why did this happen? Well by delaying our evaluating with :=, we don't know what g[x] is until we call it. However, f[x] is set to the function  $1x^2$ . So when we change c, g(x) doesn't care since it doesn't know c is even in the function while f(x) no longer has a c since it already let c=1. You should also look at the Evaluate[] function and how it forces immediate evaluation. Look at the differences in the plots:



Notice the difference here . When we call Evaluate[], we make sure to get our list of values first, then plot . When we don't evaluate, Mathematica thinks we have a single object and gives them all the same color. Calling Evaluate[] makes sure we have our list first that we can plot. You can also call functions

using the @ symbol, so it may also be written as (this makes more sense if you want to do something like Evaluate[Table[]])

Plot[Evaluate@(factors  $x^2$ ), $\{x,-4,4\}$ ]

