Modelling Quasi Birth-and-Death Process

$$\begin{bmatrix} B_0 & B_1 & & & \\ C & A & D & & \\ & C & A & D & \\ & & \ddots & \ddots & \ddots \end{bmatrix} \begin{bmatrix} \mathbf{x}_0 \\ \mathbf{x}_1 \\ \mathbf{x}_2 \\ \vdots \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \end{bmatrix}$$

where

$B_0 = -0.2I_2, B_1 = 0.3I_2, C = 0.2I_2. D = 0.3I_2$ and $A = \begin{bmatrix} -0.6 & 0.2 \\ 0.1 & -0.7 \end{bmatrix}$.

Assuming $\mathbf{x_{i+1}} = R\,\mathbf{x_i}$. Then, we have

$$\begin{cases} (B_0 + B_1 R)\mathbf{x}_0 = 0 & (1) \\ (C + AR + DR^2)\mathbf{x}_0 = 0 & (2) \end{cases}.$$

Defining variables:

```
n = 2;
B0 = -0.2*eye(n)
```

```
B0 =
   -0.2000         0
         0   -0.2000
```

```
B1 = 0.3*eye(n)
```

```
B1 =
    0.3000         0
         0    0.3000
```

```
C = 0.2*eye(n)
```

```
C =
    0.2000         0
         0    0.2000
```

```
D = 0.3*eye(n)
```

```
D =
    0.3000         0
         0    0.3000
```

```
A = [-0.6, 0.2; 0.1, -0.7]
```

```
A =
   -0.6000    0.2000
    0.1000   -0.7000
```

Iterative schemes:

Since $\det(A) \neq 0$,

$$
\begin{aligned}
&C + AR + DR^2 = 0 \\
&-AR = C + DR^2 \\
&R = -A^{-1}(C + DR^2) \\
&R_{k+1} = -A^{-1}\left(C + DR_k^2\right)
\end{aligned}
$$

```
mulA = -1/(-0.6*-0.7 - 0.1*0.2)*[-0.7, -0.2; -0.1, -0.6];
cons = mulA*C;
coef = mulA*D;
R = 0.333*eye(n);
maxIter = 100;
%Solve matrix quadratic eqaution
for i = 1:maxIter
    newR = cons + coef*R^2;
    if i == maxIter
        fprintf('Error matrix:\n')
        abs(newR - R)
        fprintf('Numerical output:\n')
        newR
    end
    R = newR;
end
```

```
Error matrix:
ans =
   1.0e-11 *
    0.4694    0.4694
    0.2347    0.2347
Numerical output:
newR =
    0.5375    0.2583
    0.1291    0.4084
```

```
%Solve for x
precision = 10;
coefMat = round(B0 + B1*R, precision)
```

```
coefMat =
   -0.0387    0.0775
    0.0387   -0.0775
```

```
rref(coefMat) % Numerically unstable without rounding the coefficient matrix
```

```
ans =
    1.0000   -2.0000
         0         0
```

Therefore,

$$
\mathbf{x}_0 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}.
$$