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Pair-wise differential abundance measurements can yield extremely precise estimates of the chemical homogeneity of star clusters

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ABSTRACT

Stars are formed in clusters. Among stars that formed in the same cluster, there is some finite spread of chemical abundances. Knowing how this intrinsic abundance dispersion varies for different star clusters is important for understanding star formation, and places strong limits on how well we can infer the chemodynamic evolution of the Milky Way. Here we use basic statistics to show that the chemical homogeneity of star clusters can be estimated far more precisely from existing data, without the need for any reference (or so called benchmark) stars.

1. INTRODUCTION

Stellar spectroscopy is plagued by systematic effects. When measuring stellar metallicity from spectra, most analyses incorrectly assume the conditions are in local theromdynamic equilibrium, that a stellar photosphere is fully parameterised by one dimension, and an list of electron transitions that often have poorly known properties. These effects conspire to produce both biased and noisy estimates of metallicity.

These effects can be mitigated by measuring the chemical abundance of one star *relative* to another. For two stars of similar stellar parameters, the effects of non-local thermodynamic equilibrium are approximately the same. If a single transition has poorly known parameters that lead to a biased estimate of metallicity, that bias is approximately the same for two very similar stars. This has led to the line-by-line differential abundance analysis technique, where all measurements are made relative to a so called reference star: where the stellar parameters of the reference star are well-measured by non-spectroscopic methods (e.g., asteroseismology, interferometry).

2. METHOD

Here we show that the line-by-line differential abundance technique can be extended to estimate the chemical homogenity of star clusters without the need for reference stars. Let X be a set of observations that are drawn from a normal distribution

$$X \sim \mathcal{N}(\mu_x, \sigma_x^2) \tag{1}$$

with mean μ_x and variance σ_x^2. Here, X represents the metallicity [Fe/H] measured from a star in a cluster, and we are interested in estimating σ_x. Because each estimate of X is biased by systematic effects, we will instead measure the pair-wise metallicity Y

$$Y_{ij} = X_i - X_j \tag{2}$$

in a line-by-line differential manner. We provide a step-by-step example in Section 3. The variance of Y_{ij} is given by

$$\mathrm{Var}(Y_{ij}) = \mathrm{Var}(X_i - X_j) \tag{3}$$

$$\mathrm{Var}(Y_{ij}) = \mathrm{Var}(X_i) + \mathrm{Var}(X_j) - 2\mathrm{Cov}(X_i, X_j) \tag{4}$$

where $\mathrm{Cov}(X_i, X_j)$ is the covariance between metallicity measurements of two (randomly selected) stars in a cluster that have near identical stellar parameters. We assume that the covariance will tend towards zero as the difference in stellar parameters of the two stars approaches zero. We further assume that the X and Y values are each identitically distributed such that

$$\mathrm{Var}(Y) = 2\,\mathrm{Var}(X) \tag{5}$$

By taking many pair-wise abundance differences Y and computing their sample variance $\mathrm{Var}(Y)$ we can estimate the intrinsic chemical homogenity of a cluster, and that estimate is largely unaffected by systematics due to stellar parameters and incorrect line properties:

$$\sigma_x = \frac{\sigma_y}{\sqrt{2}} \quad . \tag{6}$$

The literature on pair-wise differential abundance measurements has shown that we can expect a precision in Y of order 0.005 or 0.010 dex, even for stars that we do not know for sure were born in the same cluster (e.g., just Solar analogues). To first order this would translate to being able to measure cluster homogenity to a precision between $\sigma_x = 0.003$ to 0.007 dex, about two orders of magnitude more precisely than literature estimates of chemical homogenity that are based entirely from sample variance of (systematically limited) classical measurements $\mathrm{Var}(X)$.

3. EXPERIMENTS

Let's say you have some stars in a cluster. Either you have observed them already or you haven't.

1. Pair stars up. Either by their proximity in color-luminosity space, or by ordering a distance matrix of pair-wise things.

2. For each star, estimate the stellar parameters in the way you know how. This could be by a classical excitation/ionization balance, spectrum fitting, etc, or by fixing the temperature from photometry and estimating logg from astrometry or an isochrone.

3. For one atomic transition, compute the abundance in star A and star B. Note the difference $A - B$ and repeat it for all transitions.

4. Take the mean of the line-by-line abundance differences as Y_{ij}. Keep the standard deviation. We will do something with it.

5. Repeat steps 2-4 for all paired stars, then compute the sample variance. Times by $1/\sqrt{2}$ and write the paper.

Set up a simulation where we have: - atomic lines that produce biases that vary smoothly with stellar parameters, and they are all biased in their own ways - each atomic line measurement has some uncertainty - there is some intrinsic cluster dispersion (which we want to measure)

First we take the standard deviation of the observed (biased) abundances, and how that scales if we increase the number of stars. Now we compute the standard deviation from random pair-wise abundances, and show how that estimate scales with the number of stars.

4. CONCLUSIONS

We have shown that freshman statistics is useful.

Software — `numpy` (**?**); `matplotlib` (**?**);