3. (a) $I = \tilde{N}\tilde{D}$

$\tilde{N} \in R^{M \times 3}$ $\quad$ $\tilde{D} \in R^{3 \times K}$

If we consider that $M \geq 3$ and $K \geq 3$

$\Rightarrow$ Rank of $\tilde{N} = 3$

Rank of $\tilde{D} = 3$

Hence, rank of $I = 3$ in the absence of noise

(b) Given the matrix I, we compute its SVD

$$\underset{M \times K}{I} = \underset{M \times M}{U}\ \underset{M \times K}{S}\ \underset{K \times K}{V^T}$$

Since rank of $I = 3$, we can write the reduced form of SVD as follows:

$$\underset{M \times K}{I} = \underset{M \times 3}{U}\ \underset{3 \times 3}{S}\ \underset{K \times 3}{V^T}$$

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$$\Rightarrow \underset{m\times k}{I} = \underset{m\times 3}{U}\ \underset{3\times 3}{S^{1/2}}\ \underset{3\times 3}{S^{1/2}}\ \underset{k\times 3}{V^T} \quad -①$$

$$I = \tilde{N}\tilde{D} \quad -②$$

On comparing ① & ②

$$\tilde{N} = U S^{1/2} \qquad \tilde{D} = S^{1/2} V^T$$

But this $\tilde{N}$ & $\tilde{D}$ are not unique because for any invertible 3x3 matrix A, we can write I as

$$I = \tilde{N} A A^{-1} \tilde{D}$$

The final $\tilde{N}$ and $\tilde{D}$ will be

$$\tilde{N} \leftarrow \tilde{N} A$$

$$\tilde{D} \leftarrow A^{-1} \tilde{D}$$

But even then, these solutions are unique only up to some unknown orthonormal transformation R,

$$I = \tilde{N}\tilde{D} = \tilde{N} R R^T \tilde{D} \quad (\because RR^T = \text{identity matrix})$$

This R cannot be uniquely obtained by exploiting some property (This is explanation for this part & next part since solving these eqn require some Minimum m needed ? information

A → 3x3 matrix (invertible) → Total 9 variables

Hence, we require atleast 9 independent equations to find A. ↑ Rows of $\tilde{N}$ = magnitude of "albedo" at that point

Note: Albedo at a point will result in 1 equation

Therefore we should know albedo at atleast 9 different point to find A.

$$m \geq 9$$

When we don't know the actual albedo at these $m(9)$ points but given that they are all equal.

In this case we will require 1 more point i.e. 10 different points to find A.

Why ? Since in all the eqn there will be ρ, so you can simply divide all the 1 to 9 eqn with the 10th eqn. Hence,

$$m \geq 10$$

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(c) The information about intensity of light source is in $\tilde{D}$. So now we will take columns of $\tilde{D}$. i^{th} column of $\tilde{D}$ contains $L_i d_i$

d_i → unit vector

Therefore magnitude of columns of $\tilde{D}$ are L_i

(Explaination about R done in part b)

As in the previous part, to find A (9 unknown) we require 9 idependent equation. L_i for a particular lighting condition gives one eq^n.

9 eq^n are required.

$\therefore$ Requires $m \geq 9$

When lighting intensity is equal in m image then we require 1 more eq^n i.e 10 equation (since now we don't know about intensity, hence one more equation)

$m \geq 10$