4. We need to find $\tilde{R} = \arg\min_Q \|Q - \hat{R}\|_F^2$ such that $QQ^T = I$

let. $E(Q) = \|Q - \hat{R}\|^2$

$$E(Q) = \mathrm{trace}\left((Q-\hat{R})^T (Q-\hat{R})\right)$$

$$= \mathrm{trace}\left(Q^TQ - Q^T\hat{R} - \hat{R}^TQ + \hat{R}^T\hat{R}\right)$$

$$= \mathrm{trace}(I + \hat{R}^T\hat{R}) \underbrace{-2\,\mathrm{trace}(\hat{R}^TQ)}_{\text{maximize}} \qquad \{\because \mathrm{trace}(A) = \mathrm{trace}(A^T) \text{ and } Q^TQ = I\}$$

Now, we need to maximize $\rightarrow \mathrm{trace}(\hat{R}^TQ)$

$\Rightarrow \mathrm{trace}(Q\hat{R}^T)$ $\{\because \mathrm{trace}(AB) = \mathrm{trace}(BA)\}$

$\Rightarrow \mathrm{trace}(Q(USV^T)^T)$ where, $\hat{R} = USV^T$

$\Rightarrow \mathrm{trace}(QVS^TU^T)$

$\Rightarrow \mathrm{trace}(S^T\underline{U^TQV})$ $\because \mathrm{trace}(AB) = \mathrm{trace}(BA)$

$\Rightarrow \mathrm{trace}(S^TX)$

$\Rightarrow \sum S_{ii}X_{ii}$ $\hookrightarrow$ orthonormal $\rightarrow \because XX^T = \underline{U^TQV}\,\underline{V^TQ^TU} = I$

Since all S_{ii} are +ve, therefore all $X_{ii} = 1$. $\because X$ is orthonormal we must have $X = I$.

$\therefore U^TQV = I$

$Q = UV^T$ at which $E(Q)$ is min.

$\therefore \underline{\tilde{R} = UV^T}$ which is same as given in Ques.

hence, proved.

Limitation which is resolve by this correction is $\rightarrow$ the difraction of light through the pinhole due to which the image formed is not purely the perspective projection of the object.

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