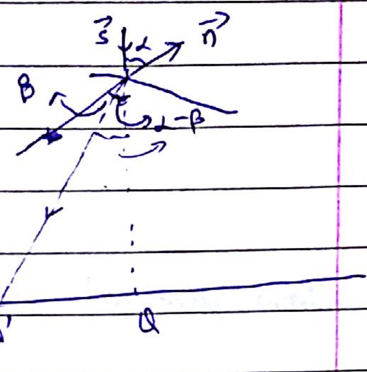
DATE | | PAGE

(a) $$\vec{n}(x,y) = \frac{(-h_x, -h_y, 1)}{\sqrt{h_x^2 + h_y^2 + 1}} \quad -①$$ where $h_x = \dfrac{\partial h(x,y)}{\partial x}$

$h_y = \dfrac{\partial h(x,y)}{\partial y}$

(b) $\vec{s}$, $\vec{n}$ & $\vec{r}$



The law of refraction states that ~~the~~ at the point of incidence, the incident ray, reflected ray & normal lie in the same plane. Hence $\vec{r}$ can be written in the following way. (linear combination of $\vec{s}$ & $\vec{n}$)

$$\vec{r} = A\vec{s} + B\vec{n} \qquad A \,\&\, B \text{ are constant}$$

$$(r_x, r_y, r_z) = A(0,0,-1) + B(n_x, n_y, n_z)$$

where r_x, r_y, r_z are x,y,z component of $\vec{r}$

n_x, n_y, n_z are x,y,z component of $\vec{n}$

Compare only x & y components.

$r_x = B n_x$ & $r_y = B n_y$

$n_x = \gamma h_x$ & $n_y = \gamma h_y$ $\quad \gamma$ is constant

From ① ~~$A_x = \beta h_x \quad n_y = \beta h_y \cdot \beta$~~ ~~$\beta$ is constant~~

$$\gamma = \frac{-1}{\sqrt{h_x^2 + h_y^2 + 1}} \qquad h_x = \frac{\partial h(x,y)}{\partial x} \quad h_y = \frac{\partial (x,y)}{\partial y}$$

$r_x = B n_x = \gamma B h_x$

$r_y = B n_y = \gamma B h_y$

$\Rightarrow (r_x, r_y) = \gamma B (h_x, h_y)$

Hence, we conclude that (r_x, r_y) is parallel to (h_x, h_y)

(c) $\overrightarrow{QQ'}$ is the projection of $\vec{r}$ in x-y plane

From the above figure

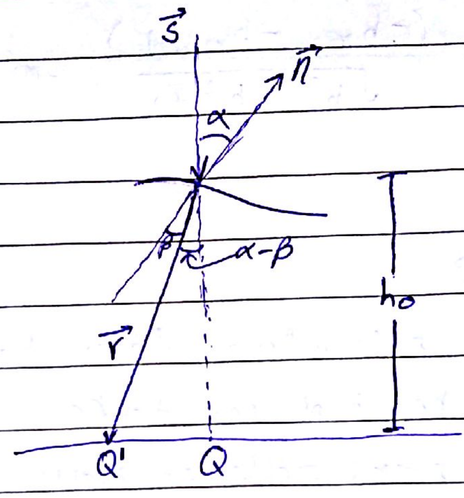
$$\tan(\alpha - \beta) = \frac{|QQ'|}{h_0}$$

$$\Rightarrow |QQ'| = h_0 \tan(\alpha - \beta)$$

$\overrightarrow{QQ'} = (r_x, r_y)$. where r_x is the x-component of r

r_y is the y-component of r

(d)



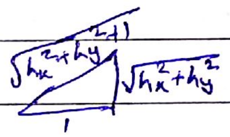
from part (a), $\vec{n} = \dfrac{\left(-\frac{\partial h}{\partial x}, -\frac{\partial h}{\partial y}, 1\right)}{\sqrt{h_x^2+h_y^2+1}}$ where, $h_x = \dfrac{\partial h}{\partial x}$

$h_y = -\dfrac{\partial h}{\partial y}$

and $\vec{s} = (0, 0, -1)$

$\vec{n}\cdot\vec{s} = \cos(\pi-\alpha)$ (angle bet^{n} both unit vectors)

$\dfrac{-1}{\sqrt{h_x^2+h_y^2+1}} = -\cos\alpha$



$\Rightarrow \tan\alpha = \sqrt{h_x^2+h_y^2}$ —— (i)

Now, from part (c) QQ' is the projection of $\vec{r}$ in x-y plane

$\vec{QQ'} = (r_x, r_y)$

and $|\vec{QQ'}| = h_0\tan(\alpha-\beta)$

we know that unit vecter along (r_x, r_y) is $\dfrac{(h_x, h_y)}{\sqrt{h_x^2+h_y^2}}$ {from (b)}

Now, $(r_x, r_y) = h_0\tan(\alpha-\beta)\dfrac{(h_x, h_y)}{\sqrt{h_x^2+h_y^2}}$

$= h_0\dfrac{\tan(\alpha-\beta)}{\tan\alpha}(h_x, h_y)$ (using (i))

$= \dfrac{h_0}{\tan\alpha}\dfrac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}(h_x, h_y)$

$= h_0\dfrac{\left(1-\frac{\tan\beta}{\tan\alpha}\right)}{1+0}(h_x, h_y)$ neglecting $\tan\alpha\tan\beta$ w.r.t 1. since α, β small.

$= h_0\left(1-\dfrac{\sin\beta}{\sin\alpha}\right)(h_x, h_y)$ {for small α & β putting $\tan\alpha = \sin\alpha$ & $\tan\beta = \sin\beta$

$$\boxed{(r_x, r_y) = h_0\left(1-\frac{1}{K}\right)(h_x, h_y)}$$

(e) Given $\overline{h(x,y,t)} = h_0$ (av. over time)

∴ $\overline{\frac{\partial h(x,y,t)}{\partial x}} = 0$ and $\overline{\frac{\partial h(x,y,t)}{\partial y}} = 0$

Using the eq^n derived in part (d), we will have

$\overline{r_x} = \overline{r_y} = 0$ (averaged over time).

Now, ~~the~~ let the still image is $C(x,y)$ without any deformation. and let $I(x,y,t)$ is the video frame at time t.
Using brightness constantcy, we can write,

$$I(x+r_x, y+r_y, t) = C(x,y)$$

$$\underbrace{I(x,y,t) + r_x(x,y,t)\frac{\partial}{\partial x}I(x,y,t) + r_y(x,y,t)\frac{\partial}{\partial y}I(x,y,t)}_{R(r_x,r_y)} = C(x,y) \quad \text{—(i)}$$

(using tayler series)

taking time average, we get

$$\overline{I(x,y,t)} + 0 + 0 = C(x,y)$$

$(\because \overline{r_x(x,y,t)} = \overline{r_y(x,y,t)} = 0)$

$\therefore C(x,y) = \overline{I(x,y,t)}$ (time average)

Now, for any time $t=t$, ~~eq (i) can be solved~~

put $r_x(x,y,t) = h_0\left(1-\frac{1}{K}\right) h_x(x,y,t)$

and $r_y(x,y,t) = h_0\left(1-\frac{1}{K}\right) h_y(x,y,t)$

in eq^n (i)

then the problem will ~~come~~ become similiar to shape from shading

where, $h_x(x,y,t)$ is analogous to $p(x,y)$ for a given time t.

$h_y(x,y,t)$ " " " $q(x,y)$ " , "

and $h(x,y,t)$ is " $z(x,y)$.

function $R(p(x,y), q(x,y))$ here will be left side of eq^n (i) ~~when~~ ~~r_x and r_y are replaced with~~.

and $C(x,y)$ will act as $I(x,y)$

So, we can solve for h_x and h_y iteratively by minimizing the cost function

$$\min \iint \left\{ \left(C(x,y) - R\left(h_x(x,y,t), h_y(x,y,t) \right) \right)^2 + \lambda \left(h_{xx}^2 + h_{xy}^2 + h_{yx}^2 + h_{yy}^2 \right) \right\} dx\,dy$$

and then estimate the $h(x,y,t)$ by solving the Poisson equation. at every time t.