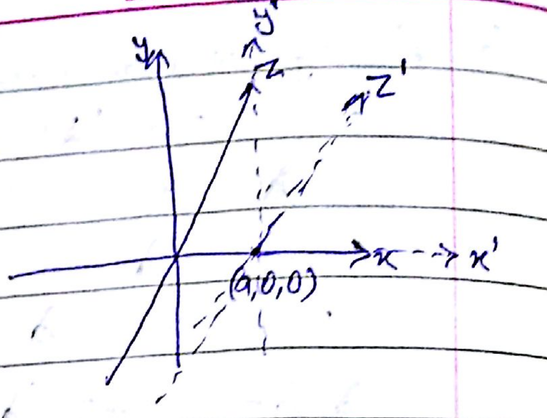
1.(a) Since, the optical axis are parallel
we will have R as ~~Inden~~ Identity matrix.
Now, as given in the question,
$T = [a \ 0 \ 0]$ (T_x T_y T_z) where, $P' = R(P-T)$
P': point w.r.t $x'y'z'$; P: point w.r.t xyz.



Now,

$$S = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -a \\ 0 & a & 0 \end{bmatrix}$$

conventions taken from slides.

and $E = RS$

$$\boxed{E = S = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -a \\ 0 & a & 0 \end{bmatrix}}$$

(b) We know that for any point x in the first camera's coordinate frame, x' (in other camera's coordinate frame) will lie on a line given by $l' = Fx$ (called as epipolar line)
Since, e' is ~~the~~ a epipole
$\therefore$ for any x, e' must lie on the line l'
$\therefore$ $e'l' = 0$

$$e'Fx = 0 \quad \forall x \Rightarrow e'F = 0 \Rightarrow F^T e'^T = 0.$$

$\therefore$ e' lies in the ~~is the left~~ null space of F^T.
Similiarly, $l = F^T x'$

$$e^T F^T x' = 0 \quad \forall x \Rightarrow e^T F^T = 0 \Rightarrow Fe = 0$$

Hence, e lies in the ~~is the right~~ null space of F.
$\therefore$ left epipole will lie in the null space of F
and right " " " " " " F^T.

Note: Using the convention of part (a).

(c) Finding singular value of matrix S.

~~$|S-\lambda I|=0$~~ ~~$|\lambda I - S| = 0$~~ $|\lambda I - S| = 0$

$$\begin{vmatrix} \lambda & T_z & -T_y \\ -T_z & \lambda & T_x \\ T_y & -T_x & \lambda \end{vmatrix} = 0$$

$$\lambda(\lambda^2 + T_x^2) + T_z(\lambda T_z + T_x T_y) - T_y(T_z T_x - \lambda T_y) = 0$$

$$\lambda(\lambda^2 + T_x^2 + T_z^2 + T_y^2) = 0$$

$\lambda = 0$ and $\lambda = \pm i\sqrt{T_x^2 + T_y^2 + T_z^2}$

we know that singular value is matrix A is the magnitude of its eigenvalue.

$\therefore \Delta = |\lambda|$

$$\Delta = \{0, \sqrt{T_x^2 + T_y^2 + T_z^2}, \sqrt{T_x^2 + T_y^2 + T_z^2}\} \longleftarrow (i)$$

Now, these are the singular value of S and we know that multiplying any matrix with orthonormal matrix will not change its singular value $\xrightarrow{\text{proof.}}$

$A = USV^T$

$RA = \underbrace{RU}_{U'}SV^T$

$\because U'^T U' = U^T R^T R U = I$.

$\therefore$ S remain same)

$\therefore$ Multiplying matrix S with the orthonormal rotation matrix will not change its singular value.

$\Rightarrow$ RS will have same singular values as S

$\Rightarrow$ Singular value of $E = RS$ is the set Δ. {using (i)}

clearly, one of them is zero and other two are identical..

As derived above the singular value (identical) = magnitude of translation vector.

It does not depend on the rotational matrix.

$$E^T E = (RS)^T RS = S^T R^T RS = S^T S$$

$$= \begin{bmatrix} 0 & T_z & -T_y \\ -T_z & 0 & T_x \\ T_y & -T_x & 0 \end{bmatrix} \begin{bmatrix} 0 & -T_z & T_y \\ T_z & 0 & -T_x \\ -T_y & T_x & 0 \end{bmatrix}$$

$$= \begin{bmatrix} T_z^2 + T_y^2 & -T_x T_y & -T_x T_z \\ -T_x T_y & T_z^2 + T_x^2 & -T_z T_y \\ -T_x T_z & -T_z T_y & T_x^2 + T_y^2 \end{bmatrix}$$

$$= \begin{bmatrix} T_x^2+T_y^2+T_z^2 & 0 & 0 \\ 0 & T_x^2+T_y^2+T_z^2 & 0 \\ 0 & 0 & T_x^2+T_y^2+T_z^2 \end{bmatrix} - \begin{bmatrix} T_x^2 & T_xT_y & T_xT_z \\ T_yT_x & T_y^2 & T_yT_z \\ T_xT_z & T_yT_z & T_z^2 \end{bmatrix}$$

$$= \alpha\, I_{3\times3} - \begin{bmatrix} T_x \\ T_y \\ T_z \end{bmatrix} \begin{bmatrix} T_x & T_y & T_z \end{bmatrix}$$ { where, $\alpha = T_x^2+T_y^2+T_z^2$

$E^T E = \alpha\, I_{3\times3} - \alpha\, t_u t_u^T$ where, $t_u = \frac{1}{\sqrt{\alpha}} \begin{bmatrix} T_x \\ T_y \\ T_z \end{bmatrix}$

$= \alpha \left(I_{3\times3} - t_u t_u^T \right)$

~~comparing~~ this eq^n^ matches with the given eq^n^

$\therefore\ \alpha = T_x^2 + T_y^2 + T_z^2$

(α = square of magnitude of translational vector.)

does not depend on R.

(d) The essential matrix has five degrees of freedom.
Since, rotation matrix R and translation vector t have 3 degree of freedom each, so total is 6.
But we need to substract one degree of freedom related to scaler multiplication.
If E satisfies $P_r^T E P_l = 0$
then αE will also satisfy the same.
Hence, it has total 5 degrees of freedom only.

(e) Since, the fundamental matrix has rank = 2 only,
the 8-point algorithm does not ~~gaur~~ guarantee this criteria.
To require this
& We need to further do the rank-2 approximation using SVD.
But this is not in the case of 7-point algorithm.
This algorithm takes care of the rank = 2 requirement.
It gives rank-2 fundamental matrix and only require 7 correspondences.