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4. We need to find $\tilde{R} = \text{argmin}_Q \|Q-\hat{R}\|_F^2$ such that $QQ^T = I$

let. $E(Q) = \|Q-\hat{R}\|^2$

$E(Q) = \text{trace}\left((Q-\hat{R})^T(Q-\hat{R})\right)$

$= \text{trace}\left(Q^TQ - Q^T\hat{R} - \hat{R}^TQ + \hat{R}^T\hat{R}\right)$

$= \text{trace}\left(I + \hat{R}^T\hat{R}\right) \underbrace{-2\,\text{trace}(\hat{R}^TQ)}_{\text{maximize.}}$ $\{\because \text{trace}(A) = \text{trace}(A^T)$ and $Q^TQ = I\}$

Now, we need to maximize $\rightarrow \text{trace}(\hat{R}^TQ)$

$\Rightarrow \text{trace}(Q\hat{R}^T)$ $\{\because \text{trace}(AB) = \text{trace}(BA)\}$

$\Rightarrow \text{trace}\left(Q(USV^T)^T\right)$ where, $\hat{R} = USV^T$

$\Rightarrow \text{trace}(QVS^TU^T)$

$\Rightarrow \text{trace}(S^TU^TQV)$ $\because \text{trace}(AB) = \text{trace}(BA)$

$\Rightarrow \text{trace}(S^TX)$

$\Rightarrow \Sigma S_{ii}X_{ii}$ $\hookrightarrow$ orthonormal $\rightarrow \because XX^T = U^TQVV^TQ^TU = I$

Since, all S_{ii} are +ve, therefore all $X_{ii} = 1$. $\because X$ is orthonormal we must have $X = I$.

$\therefore U^TQV = I$

$Q = UV^T$ at which $E(Q)$ is min.

$\therefore \tilde{R} = UV^T$ which is same as given in Ques.

hence, proved.

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