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1 We know that $\sum_{i=1}^{N} p_i = 1$ and $\forall i,\ 0 \le p_i \le 1$ - ①

Shannon entropy of X, $H(X) = -\sum_{i=1}^{N} p_i \log p_i$

From ① i.e. $0 \le p_i \le 1\ \forall i \Rightarrow \log p_i \le 0$

$\Rightarrow p_i \log p_i \le 0$ (Note: $\lim_{p_i \to 0} p_i \log p_i = 0$)

Hence $H(X) \ge 0$ ($\because p_i \log p_i \le 0\ \forall i$)

Further we need to prove uniform distribution ~~maximizes $H(X)$~~ maximizes $H(X)$

Under the given constraint $\sum_{\forall i} p_i = 1$, to maximize $H(X)$, we will differentiate $J(X)$ w.r.t. p_i & λ & set it to 0

where $J(X) = H(X) - \lambda(\sum_{i=1}^{N} p_i - 1)$

$$\frac{\partial J(X)}{\partial p_i} = -(\log p_i + 1 + \lambda) = 0$$

$$\Rightarrow \cancel{\frac{\partial J(X)}{\partial p_i}} \log p_i = -(1+\lambda) \quad \forall i \qquad -②$$

$\Rightarrow$ All p_i should be equal to maximize $H(X)$

Find the stationary point of $J(X)$?

$$\frac{\partial J(X)}{\partial \lambda} = 0 \Rightarrow \sum_{i=1}^{N} p_i = 1$$

$\Rightarrow p_i = 1/N$ (Because all p_i are equal) - ③

Using ② and ③

$$\underline{\lambda = \log N - 1}$$