Chapter 0

Part 1 Induction

To prove:

P(n) is true for arbitrary number n

Process:

1. prove the base case(n = 1) is true
2. Assume P(k) is true
3. deduce that P(k+1) is true
4. thus, P(n) for n = 1, 2, 3, 4... 's truth is proved

Part 2 Recursive function

definition:

1. Base case(s) , for example:

$$f(0) = 1$$

2. recusive formula, for example:

$$f(n+1) = 2f(n) + 3,\ \forall n \geq 1$$

Finding closed form(solve):

1. unrolling technique:

$$f(n) = 2f(n-1) + 3 = 2(2f(n-2) + 3) + 3 = \ \ldots$$

2. induction with a claim, for example:

$$f(0) = 2,\ \ f(1) = 3$$
$$\forall n \geq 1,\ \ f(n+1) = 3f(n) - 2f(n-1)$$

we figure out that:

$$f(0) = 2$$
$$f(1) = 3$$
$$f(2) = 5$$
$$f(3) = 9$$

suppose that:

$$f(n) = 2^n + 1$$

then with induction:

$$f(1) = 2^1 + 1,\ f(2) = 2^2 + 1,\ f(2) = 2^3 + 1$$
$$Assume\ f(k) = 2^k + 1,\ \forall n \geq 3$$
$$then,\ f(k+1) = 3(2^k + 1) - 2(2^{k-1} + 1) = 2^{k+1} + 1$$

Part 3 By Contradition

Reason:

1. uneasy to prove a proposition directly
2. easy to prove the contradition of the proposition