Benford’s Law Graphed in R

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(This article was first published on **triKnowBits**, and kindly contributed to R-bloggers)

Using R to replicate Sara Silverstein’s post at BusinessInsider.com

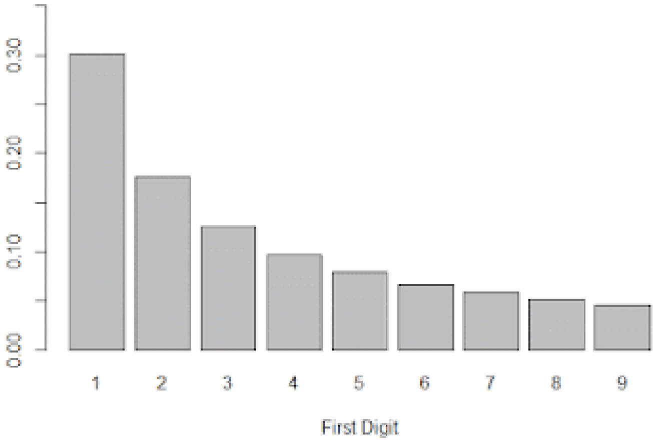
A first-year student near and dear to my heart at the Kellogg School of Management thought I would be interested in this Business Insider story by Sara Silverstein on Benford’s Law. After sitting through the requisite ad, I became engrossed in Ms. Silverstein’s talk about what that law theoretically is and how it can be applied in financial forensics.

I thought I would try duplicating the demonstration in R.[1] This gave me a chance to compare and contrast the generation of combined bar- and line-plots using base R and **ggplot2**. It also gave me an opportunity to learn how to post RMarkdown output to blogger.

Using base R

Define the Benford Law function using log base 10 and plot the predicted values.

```
benlaw <- function(d) log10(1 + 1 / d)
digits <- 1:9
baseBarplot <- barplot(benlaw(digits), names.arg = digits, xlab = "First Digit",
                       ylim = c(0, .35))
```



Remarks:

- That was easy!
- Save the barplot output in an R object so the x-values of the bar centers can be used later in the line plots[2]
- ylim extends the y-axis to 35% to accomodate the highest line value – eventually – below

Define a function that can pick out the first digit of the character respresentation of a number:

```
firstDigit <- function(x) substr(gsub('[0.]', '', x), 1, 1)
```

Define a function that counts the absolute frequencies of each first digit (using “table”), calculates the relative frequencies (by dividing by “length”), and stores the results in a data.frame:

```
pctFirstDigit <- function(x) data.frame(table(firstDigit(x)) / length(x))
```

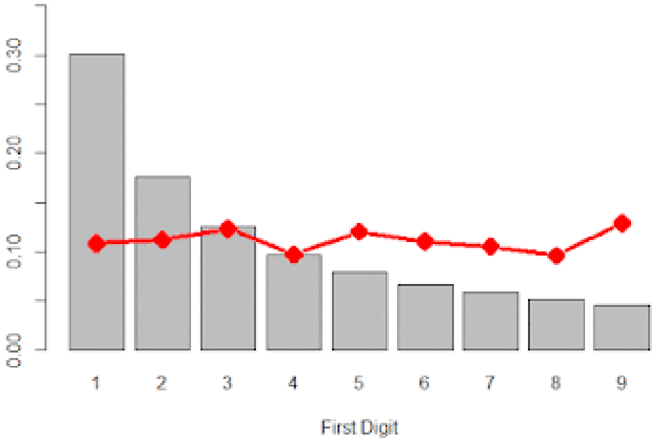
Now generate 1000 random numbers between 0 and 100 (set.seed so your results won’t differ), calculate their first-digit frequencies, …

```
N <- 1000
set.seed(1234)
x1 <- runif(N, 0, 100)
df1 <- pctFirstDigit(x1)
head(df1)
```

```
##   Var1  Freq
## 1    1 0.108
## 2    2 0.112
## 3    3 0.123
## 4    4 0.097
## 5    5 0.120
## 6    6 0.110
```

… and plot the observations relative to Benford’s Law:

```
lines(x = baseBarplot[,1], y = df1$Freq, col = "red", lwd = 4,
      type = "b", pch = 23, cex = 1.5, bg = "red")
```

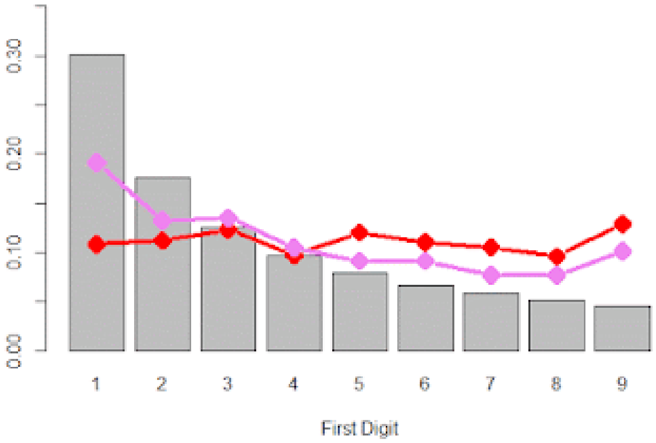


Remarks:

- As Ms. Silverstein notes, not much agreement
- Colors are assigned manually in base R
- Base R’s **lines** function automatically adds lines to the previously generated plot
- Base R allows for both points and lines to be generated in one call to **lines**via the option type = “b”
- **lines** arguments are set (e.g., point symbols are “diamonds” with pch = 23) to approximate Ms. Silverstein’s graphs

Now square each value and add their first-digit frequencies to the plot:

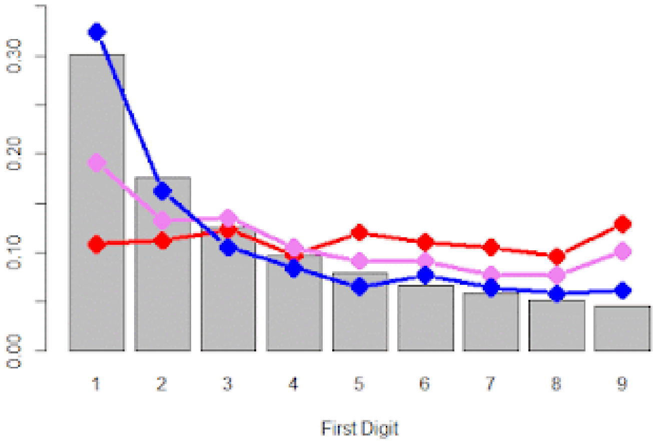
```
df2 <- pctFirstDigit(x1^2)
lines(x = baseBarplot[,1], y = df2$Freq, col = "violet", lwd = 4,
      type = "b", pch = 23, cex = 1.5, bg = "violet")
```



Notice the slight improvement in Benford-conformance.

Next, simulate another 1000 values, divide the square of the first values by these numbers, and add their first-digit frequencies to the plot:

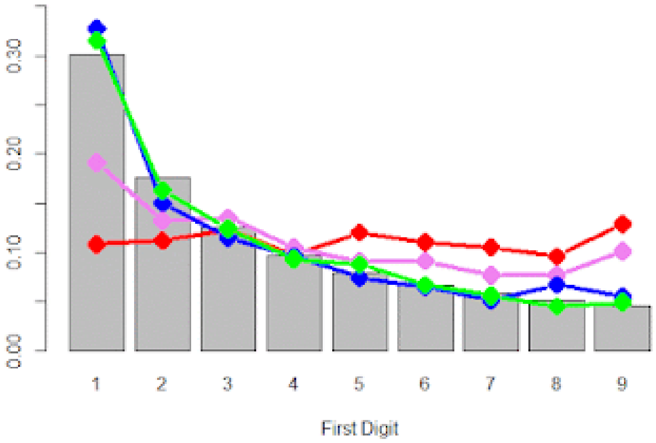
```
x3 <- runif(N, 0, 100)
df3 <- pctFirstDigit(x1^2 / x3)
lines(x = baseBarplot[,1], y = df3$Freq, col = "blue", lwd = 4,
      type = "b", pch = 23, cex = 1.5, bg = "blue")
```



Ms. Silverstein: “We’re getting closer!”

Finally, simulate another set of values, multiply them by the result of the previous operation, and add those frequencies to the plot:

```
x4 <- runif(N, 0, 100)
df4 <- pctFirstDigit(x1^2 / x3 * x4)
lines(x = baseBarplot[,1], y = df4$Freq, col = "green", lwd = 4,
      type = "b", pch = 23, cex = 1.5, bg = "green")
```



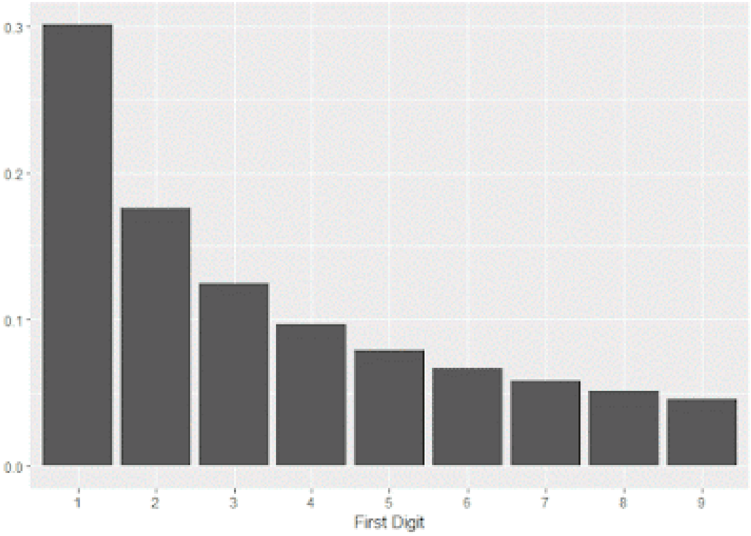
Remarks:

- These plots agree somewhat with those in Ms. Silverstein’s video
- Repeating the exercise with more simulated values (say, N = 100000) gives better results
- A legend would be nice, but legends can be overly tedious in base R!

Using ggplot

In contrast to base R, **ggplot** relies on a data.frame as the source of data to plot, so store the ‘digits’ and Benford’s predictions in a data.frame, then bar-plot the values with “geom_bar”.

```
library(ggplot2)
df <- data.frame(x = digits, y = benlaw(digits))
ggBarplot <- ggplot(df, aes(x = factor(x), y = y)) + geom_bar(stat = "identity") +
  xlab("First Digit") + ylab(NULL)
print(ggBarplot)
```



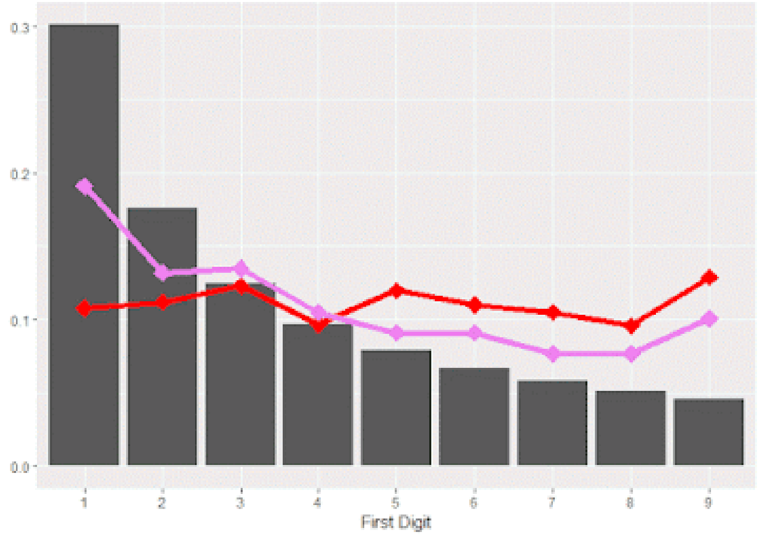
Remarks:

- Per typical **ggplot2** procedure, save the ggplot results in an R object, here named ‘ggBarplot’, for later use
- “print” the saved object to see the plot
- Use “factor(x)” so the bars are labeled on the x-axis with discrete values rather than with continuous values (the default when x is numeric)

- Since there is only one observation per x value, there is nothing for geom_bar's default summary statistic, count, to do, so set 'stat' equal to "identity" to avoid an error

As above, we can add curves corresponding to the actual first-digit frequency resulting from the four operations. One way is to successively add line and point "layers" for each of the four operations, saving each result in turn for display and for subsequent use. We demonstrate that for the first two curves only.

```
p1 <- ggBarplot +
  geom_line(data = df1,
            aes(x = Var1, y = Freq, group = 1),
            colour = "red",
            size = 2) +
  geom_point(data = df1,
            aes(x = Var1, y = Freq, group = 1),
            colour = "red",
            size = 4, pch = 23, bg = "red")
p2 <- p1 +
  geom_line(data = df2,
            aes(x = Var1, y = Freq, group = 1),
            colour = "violet",
            size = 2) +
  geom_point(data = df2,
             aes(x = Var1, y = Freq, group = 1),
             colour = "violet",
             size = 4, pch = 23, bg = "violet")
print(p2)
```



Remarks:

- As with base graphics, manually set the colors of the curves
- Base R's names for the symbol style (**pch**) and symbol background color (**bg**) can be used for **ggplot** as well

However, this manual procedure does not take full advantage of the power of **ggplot2**! The more elegant **ggplot** approach is to store all observations in a single "long" data.frame – i.e., one row per digit and operation. Then let **ggplot**automatically assign colors according to which operation is being plotted. As an added bonus, **ggplot** builds a legend automatically. Here is one way to do that.

First, bind the frequency columns of the four individual data.frames into one "wide" data.frame. Rename the columns so the eventual legend will be more informative.

```
DF <- cbind(df1, df2[2], df3[2], df4[2])
names(DF) <- c("FirstDigit", "X1", "X1^2", "X1^2 / X3", "X1^2 / X3 * X4")
head(DF)
```

```
##   FirstDigit    X1  X1^2 X1^2 / X3 X1^2 / X3 * X4
## 1          1 0.108 0.191     0.327          0.315
## 2          2 0.112 0.132     0.150          0.163
## 3          3 0.123 0.135     0.115          0.124
## 4          4 0.097 0.105     0.096          0.093
## 5          5 0.120 0.091     0.074          0.088
## 6          6 0.110 0.091     0.065          0.067
```

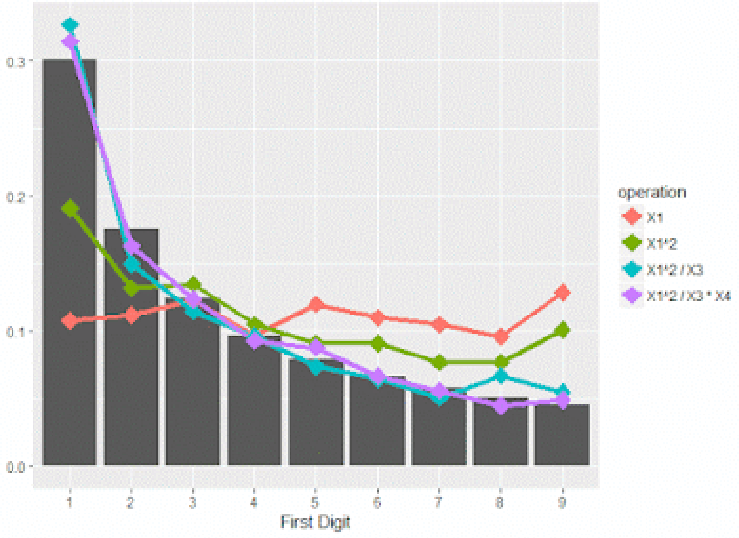
Next, use Hadley's **reshape2** package to melt this "wide" data.frame into the "long" format **ggplot** thrives on.

```
library(reshape2)
mDF <- melt(DF, id = "FirstDigit", variable.name = "operation")
head(mDF)
```

```
##   FirstDigit operation value
## 1          1        X1 0.108
## 2          2        X1 0.112
## 3          3        X1 0.123
## 4          4        X1 0.097
## 5          5        X1 0.120
## 6          6        X1 0.110
```

Now all the data can be plotted all at once.

```
P <- ggBarplot +
  geom_line(data = mDF,
            aes(x = FirstDigit, y = value, colour = operation, group = operation),
            size = 1.5) +
  geom_point(data = mDF,
            aes(x = FirstDigit, y = value, colour = operation,
                group = operation, bg = operation),
            size = 4, pch = 23)
print(P)
```



Remarks:

- **size** and **pch** are outside the **aes** call because they are the same for all curves
- **bg** is inside the **aes** call because the background colors differ by operation

Conclusion

- The Kellogg student was correct

- Base R’s graphics are more useful for quick, exploratory, “one-off” plots because the syntax is relatively straightforward
- **gglot2** is more useful for automated, elegant, presentation-level plots, but the learning curve is more steep

In a subsequent post I will summarize the steps that can turn Rmarkdown output into a blogger post.

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[1] This topic has been previously explored at http://statistic-on-air.blogspot.com/2011/08/benfords-law-or-first-digit-law.html↩

[2] Thanks to https://danganothererror.wordpress.com/2011/01/15/adding-lines-or-points-to-an-existing-barplot/↩