Benford Group assignment

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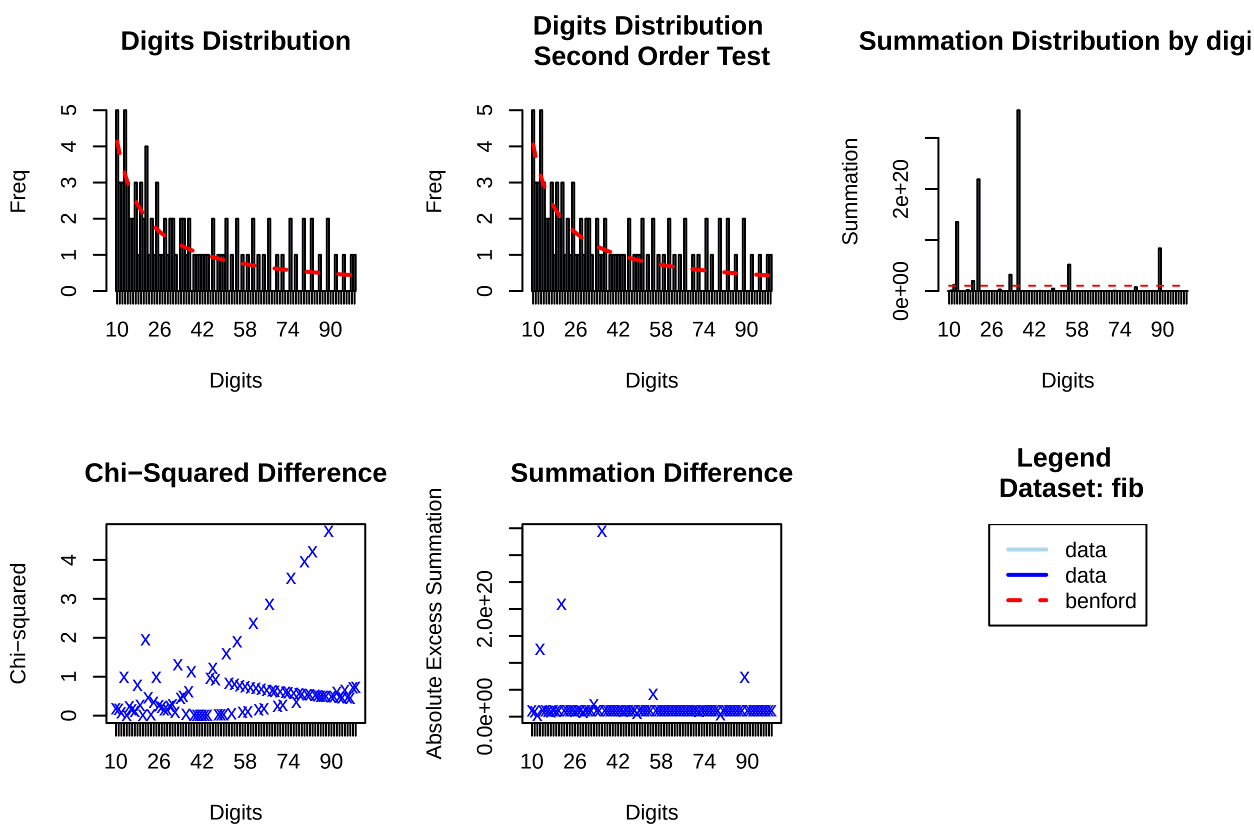
Fibonacci Numbers

To investigate the effect of increasing sample size on relation to Benford's Law, I chose to use Fibonacci numbers. I started with the first 100 numbers of the sequence and performed Benford's analysis on the first two leading digits.

100 Numbers

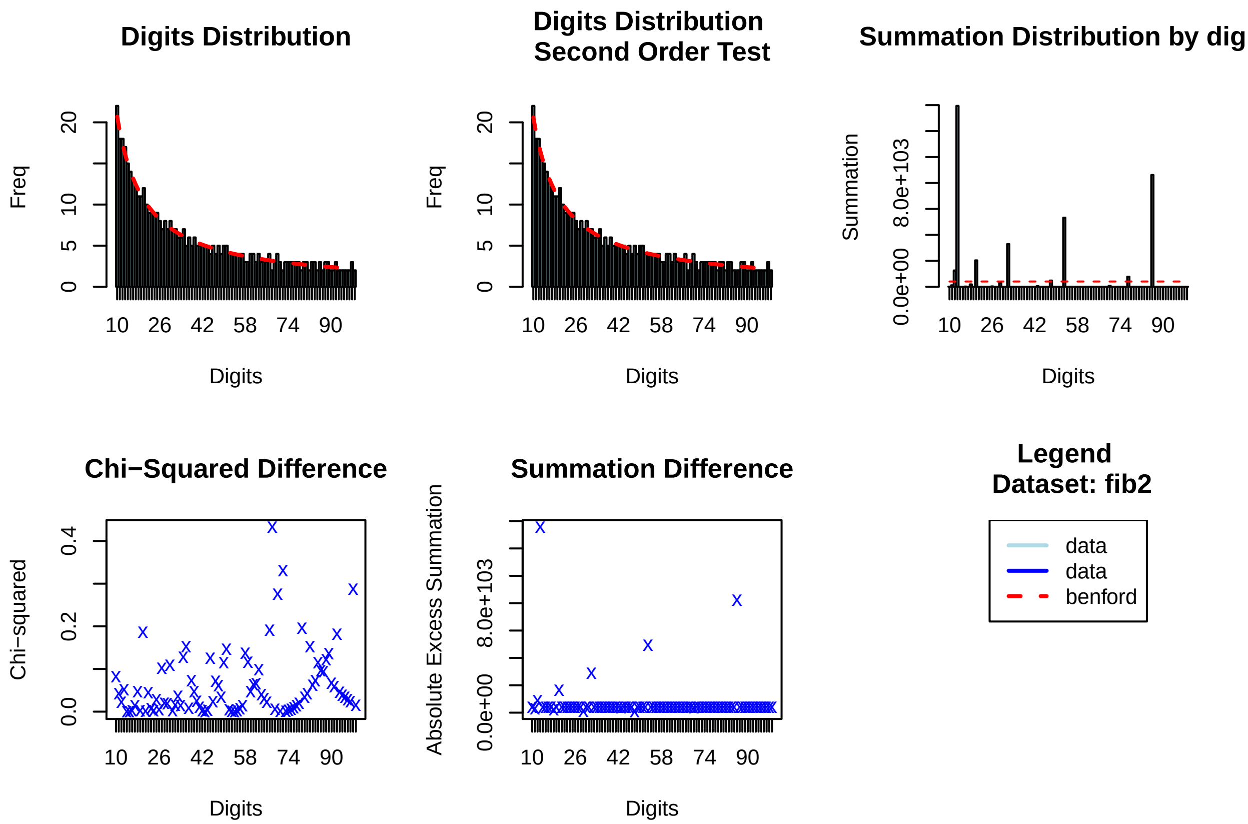



```
##     digits absolute.diff
##  1:     21      1.979661
##  2:     13      1.781532
##  3:     89      1.514750
##  4:     83      1.479881
##  5:     80      1.460497
##  6:     75      1.424767
##  7:     67      1.356589
##  8:     18      1.348110
##  9:     25      1.296666
```

```
## 10:      33       1.296498
```

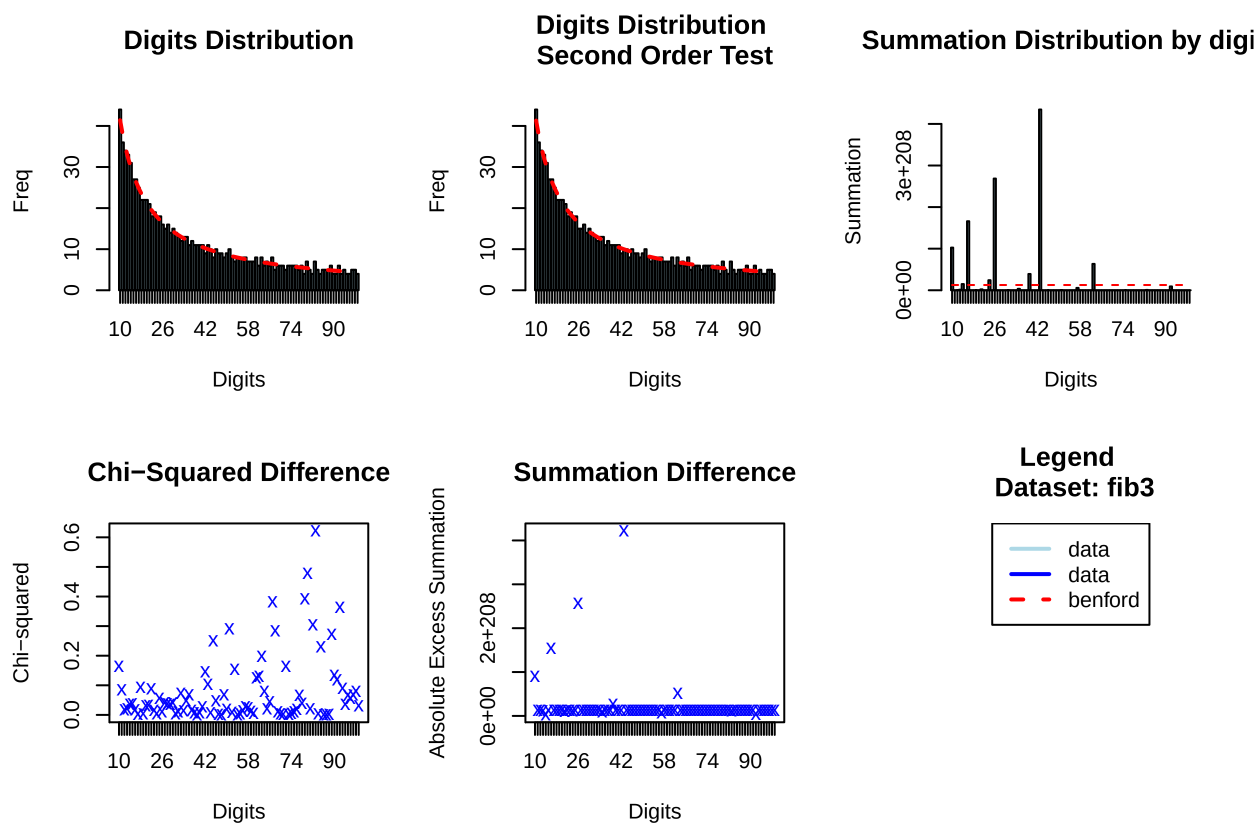
There is clearly a staggered pattern in the numbers, which creates a very interesting pattern on the chi-squared difference plot.

500 Numbers



Increasing our numbers from 100 to 500 has made a big difference in our analysis. The distribution of digits seems to conform to Benford very well. The pattern in the chi-squared difference is still evident, but the differences have decreased dramtically.

1,000 numbers

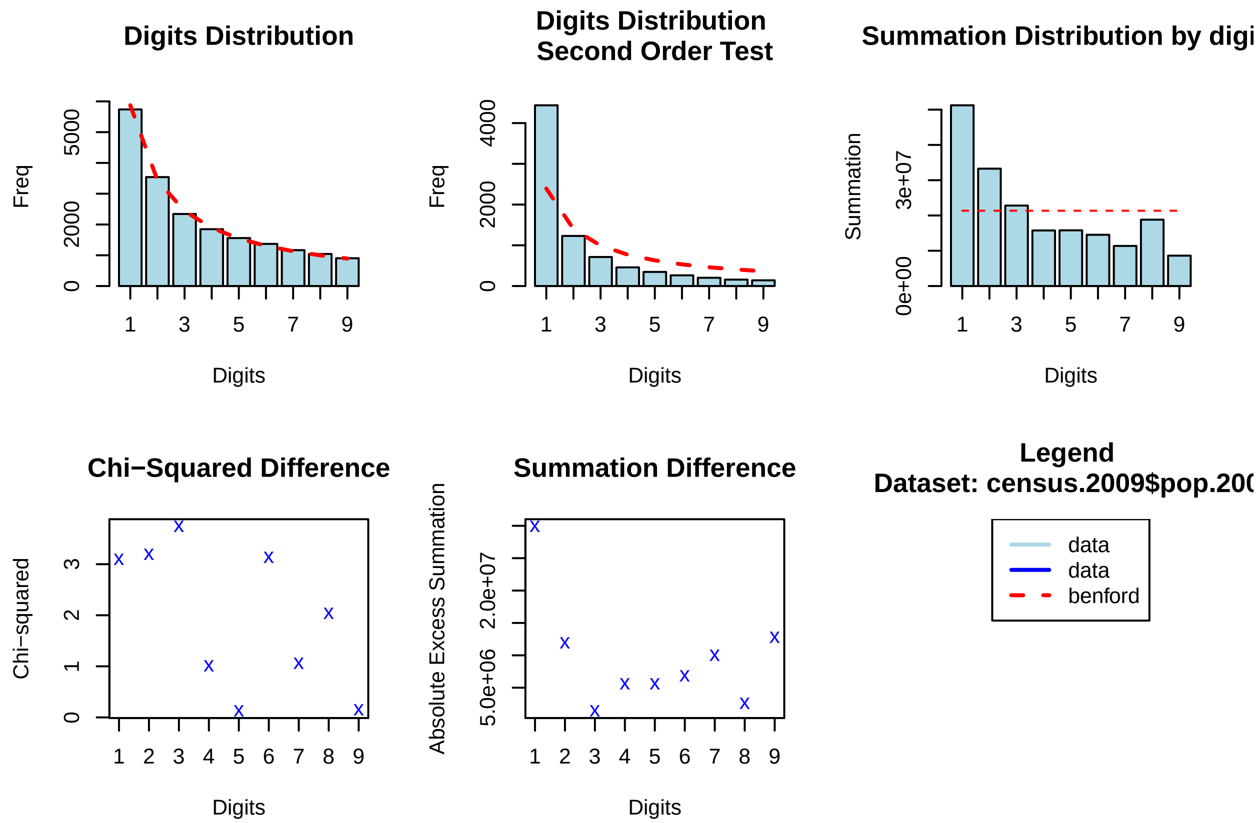



```
##     digits absolute.diff
##  1:     10      2.607315
##  2:     83      1.798806
##  3:     11      1.788561
##  4:     80      1.604968
##  5:     51      1.566832
##  6:     67      1.565890
##  7:     45      1.545318
##  8:     18      1.481096
##  9:     79      1.462896
## 10:     68      1.340178
```

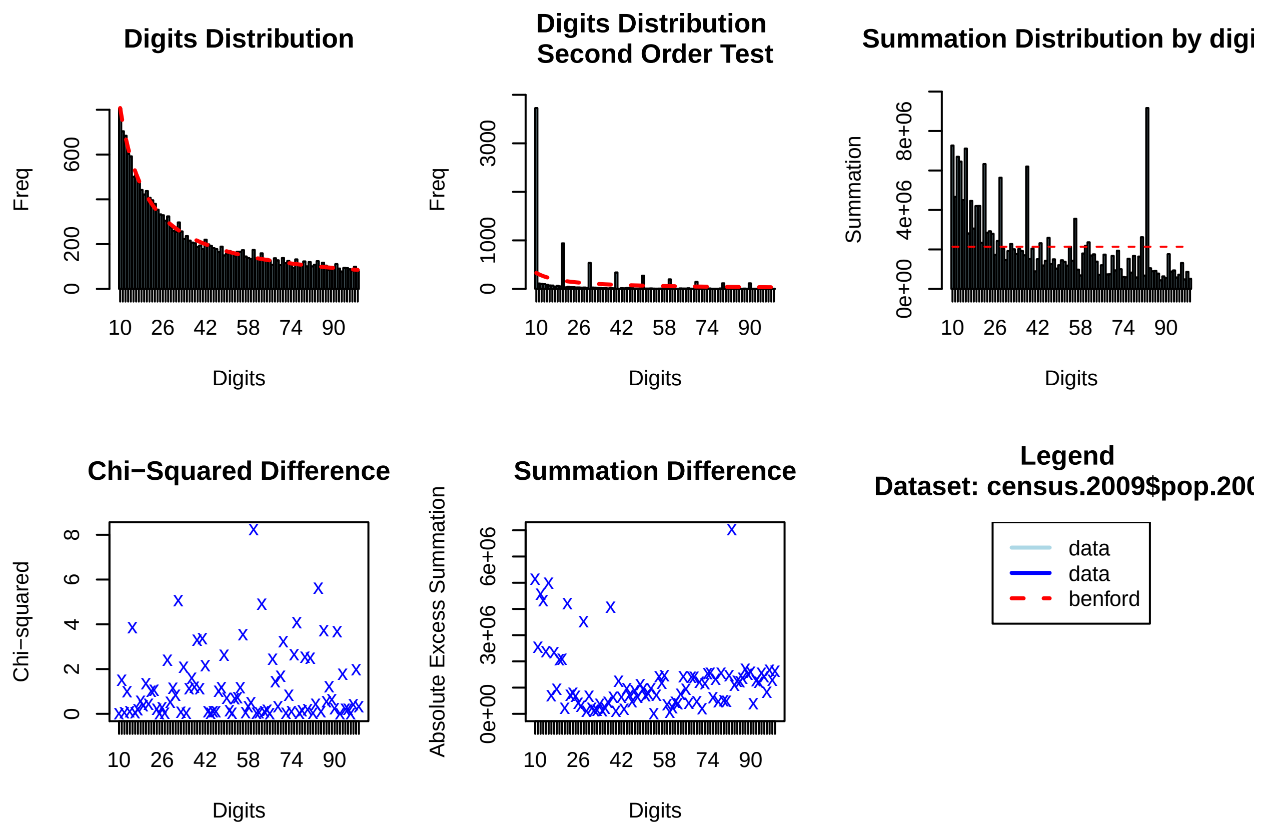
Interestingly, it seems as if increasing numbers to 1,000 is actually further from Benford's distribution in some ways. Overall, the pattern from the chi-squared difference has decreased, and most digits are closer to the law, but some digits have strayed further away. It is interesting that many multiples of tens are seen in the suspects list.

Census Data

```
d <- data(package = "benford.analysis")
## names of data sets in the package
#d$results[, "Item"]
data(census.2009)

#The first code is to look at the high level test of reasonableness.
census1 <- benford(census.2009$pop.2009, number.of.digits = 1, sign = "positive", discrete = TRUE, rounc
#We can take a closer look at the first two digits, which is designed to select audit targets.
census2 <- benford(census.2009$pop.2009, number.of.digits = 2, sign = "positive", discrete = TRUE, rounc

plot(census1)
```




```
#From this plot we can see that rea dotted line, which is the benford line, generally matches good with

plot(census2)
```




```
#This plot reveals a closer look at first two digits data.
```

By looking at the general information of the dataset. We can see that the values of mean, variance, Ex.Kurtosis and skewness well match with the expected values of 0.5, 0.0833, -1.2, and 0.

```
census1
```

```
##
## Benford object:
##
## Data: census.2009$pop.2009
## Number of observations used = 19509
## Number of obs. for second order = 7950
## First digits analysed = 1
##
## Mantissa:
##
##    Statistic  Value
##         Mean  0.503
##          Var  0.084
##  Ex.Kurtosis -1.207
##     Skewness -0.013
##
##
## The 5 largest deviations:
##
##   digits absolute.diff
```

```
## 1      1        134.79
## 2      2        104.64
## 3      3         95.43
## 4      6         63.94
## 5      8         45.07
##
## Stats:
##
##  Pearson's Chi-squared test
##
## data:  census.2009$pop.2009
## X-squared = 17.524, df = 8, p-value = 0.0251
##
##
##  Mantissa Arc Test
##
## data:  census.2009$pop.2009
## L2 = 4.198e-05, df = 2, p-value = 0.4409
##
## Mean Absolute Deviation: 0.003119261
## Distortion Factor: 0.7404623
##
## Remember: Real data will never conform perfectly to Benford's Law. You should not focus on p-values!
```

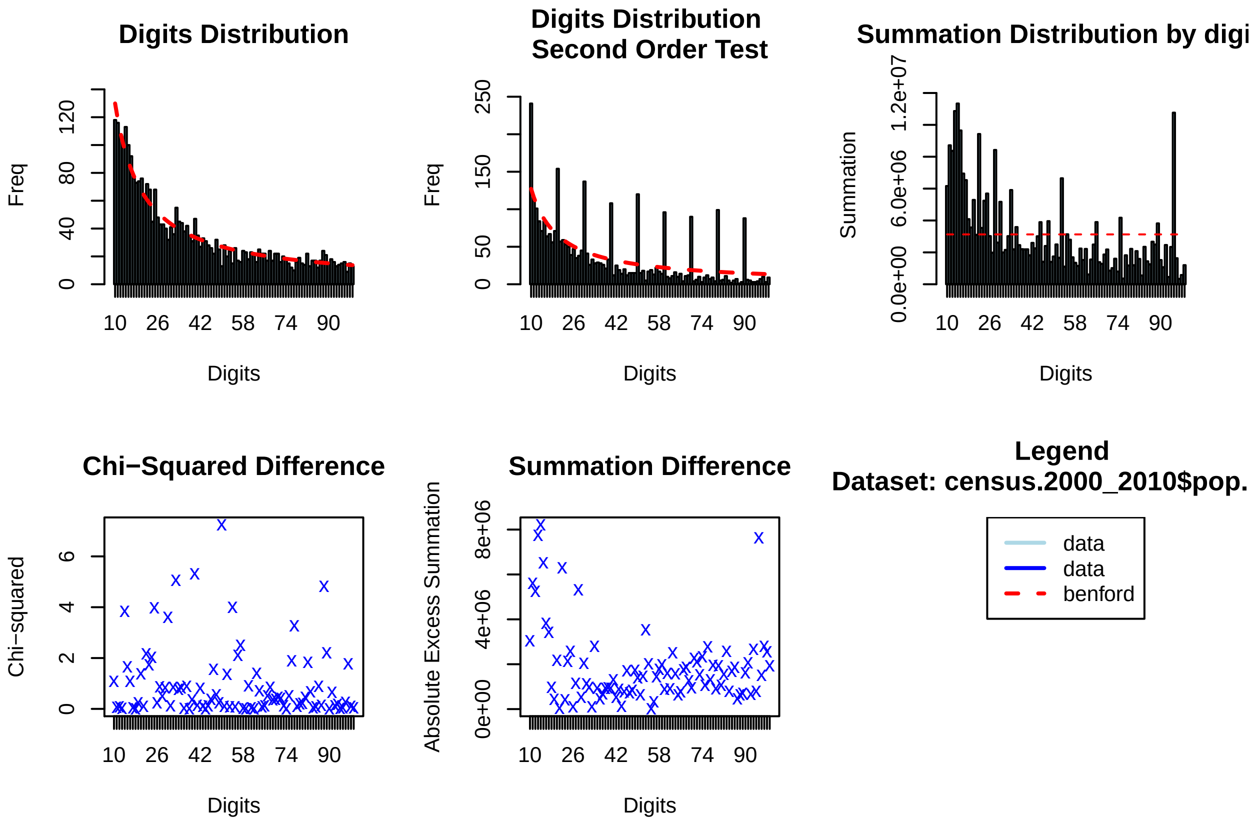
```r
#This step is to find the suspicious targets using the first 2 digits Benford model.
suspects <- getSuspects(census2, census.2009)
suspects
```

```
##          state                town pop.2009
##   1:   Alabama Alexander City city    15114
##   2:   Alabama      Bakerhill town      322
##   3:   Alabama   Center Point city    15519
##   4:   Alabama     Crossville town     1513
##   5:   Alabama        Cullman city    15302
##  ---
## 794: Wisconsin      Whitehall city     1582
## 795: Wisconsin  White Lake village      321
## 796:   Wyoming      Ten Sleep town      328
## 797:   Wyoming      Wheatland town     3236
## 798:   Wyoming         Wright town     1550
```

From the suspect function, we can see that Alabama state has a handful of data that are suspicious.

Census 2000

```r
data("census.2000_2010")
census2000_bfd <- benford(census.2000_2010$pop.2000)
plot(census2000_bfd)
```




```r
MAD(census2000_bfd)
```

```
## [1] 0.001419661
```

```r
chisq(census2000_bfd)
```

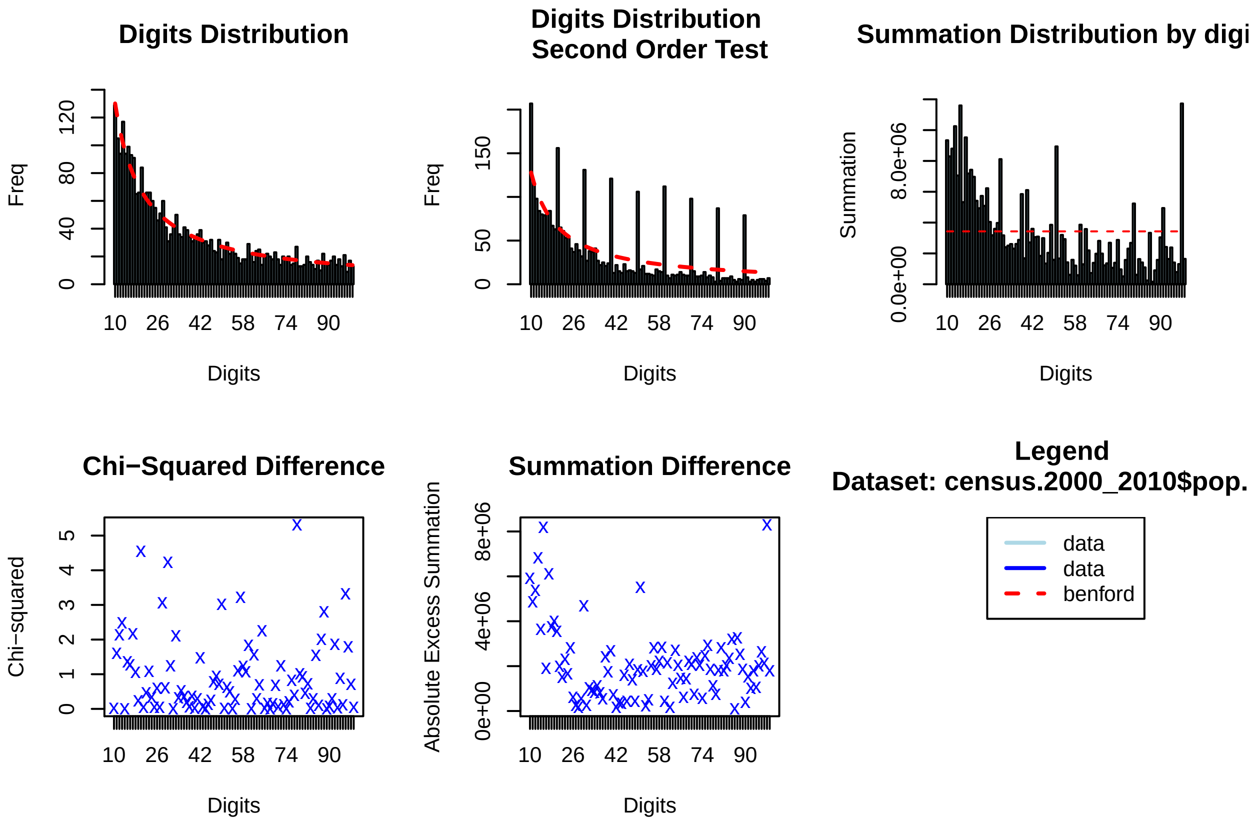
```
##
##  Pearson's Chi-squared test
##
## data:  census.2000_2010$pop.2000
## X-squared = 87.647, df = 89, p-value = 0.5207
```

```r
# Get suspecious value from the data
suspects_2000 <- getSuspects(bfd = census2000_bfd, data = census.2000_2010)
```

For the Year of 2000 census data, generally we have a pretty good result from the benford analysis. However, we have a few abnormal test results for the sequence starts with 2 and 3. From the Chi-squared test, we have a p-value of 0.5207 and it would indicate fail to reject the null hypothesis, which indicates that the distribution of the data is very close to the distribution of Benford Law.

Census 2010

```r
census2010_bfd <- benford(census.2000_2010$pop.2010)
plot(census2010_bfd)
```




```r
MAD(census2010_bfd)
```

```
## [1] 0.001428017
```

```r
chisq(census2010_bfd)
```

```
##
##  Pearson's Chi-squared test
##
## data:  census.2000_2010$pop.2010
## X-squared = 82.809, df = 89, p-value = 0.6646
```

```r
# Get suspecious value from the data
suspects_2010 <- getSuspects(bfd = census2010_bfd, data = census.2000_2010)
```

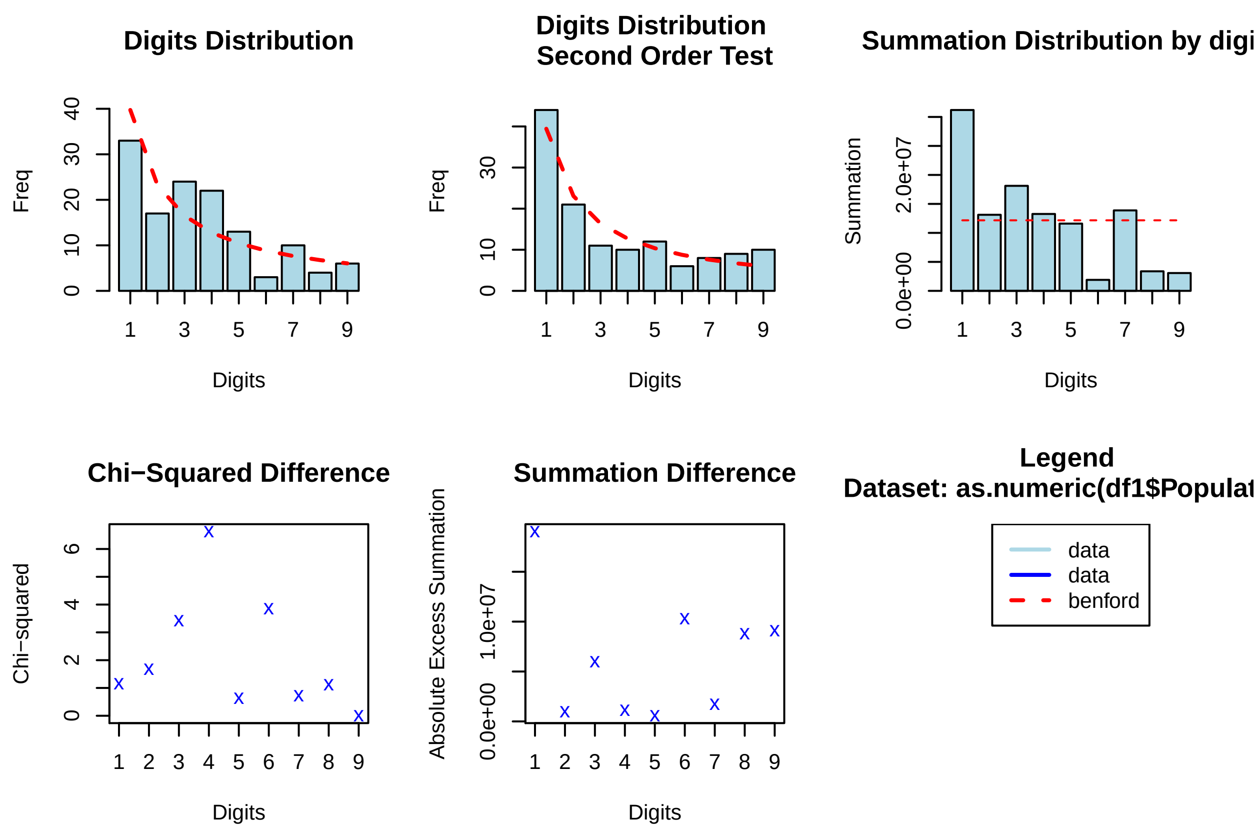
For the Year of 2010 census data, generally we have a pretty good result from the benford analysis. However, we have a few abnormal test results for the sequence starts with 2, 3 and 7. From the Chi-squared test, we have a p-value of 0.6646 and it would indicate fail to reject the null hypothesis, which indcates that the distribution of the data is very close to the distribution of Benford Law.

Chinese city population data

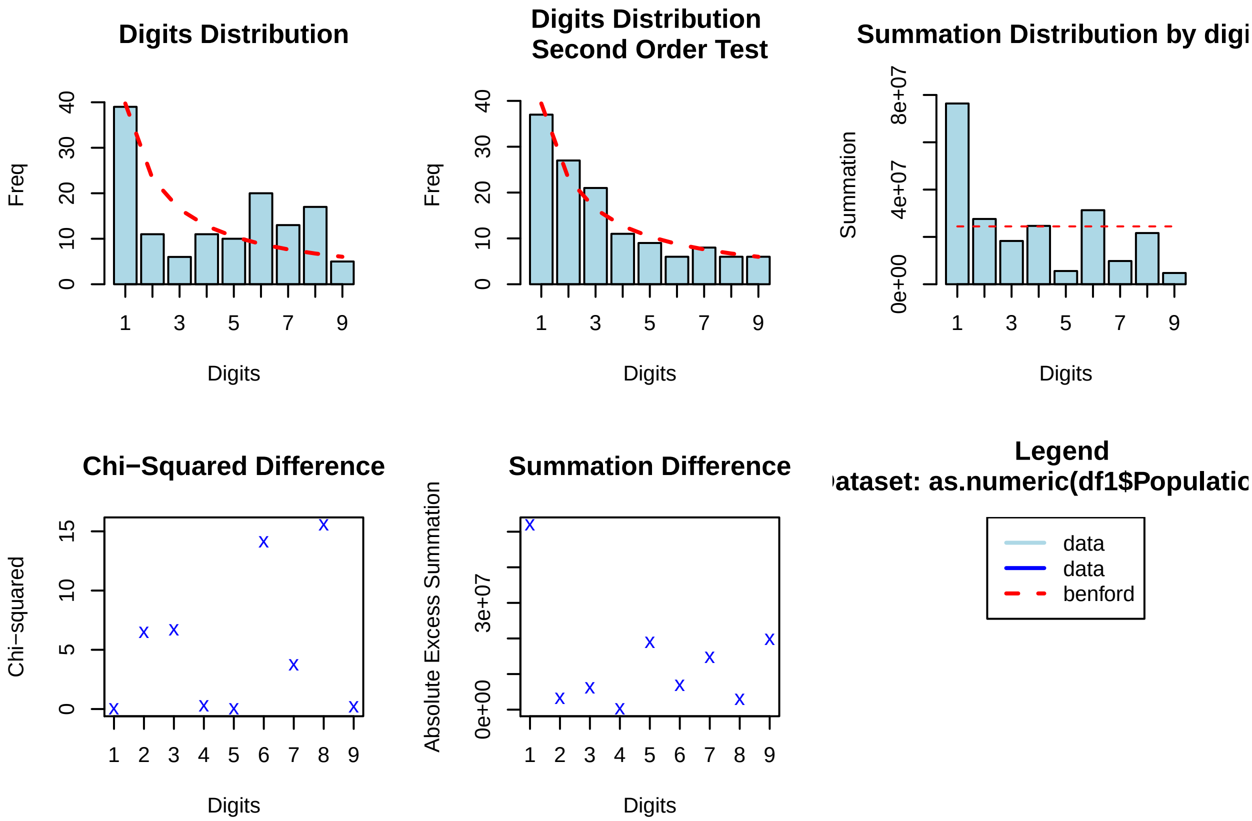
data source: http://www.citypopulation.de/China-UA.html

```r
library(tidyverse)
library(benford.analysis) # loads package data(corporate.payment) # loads data
```

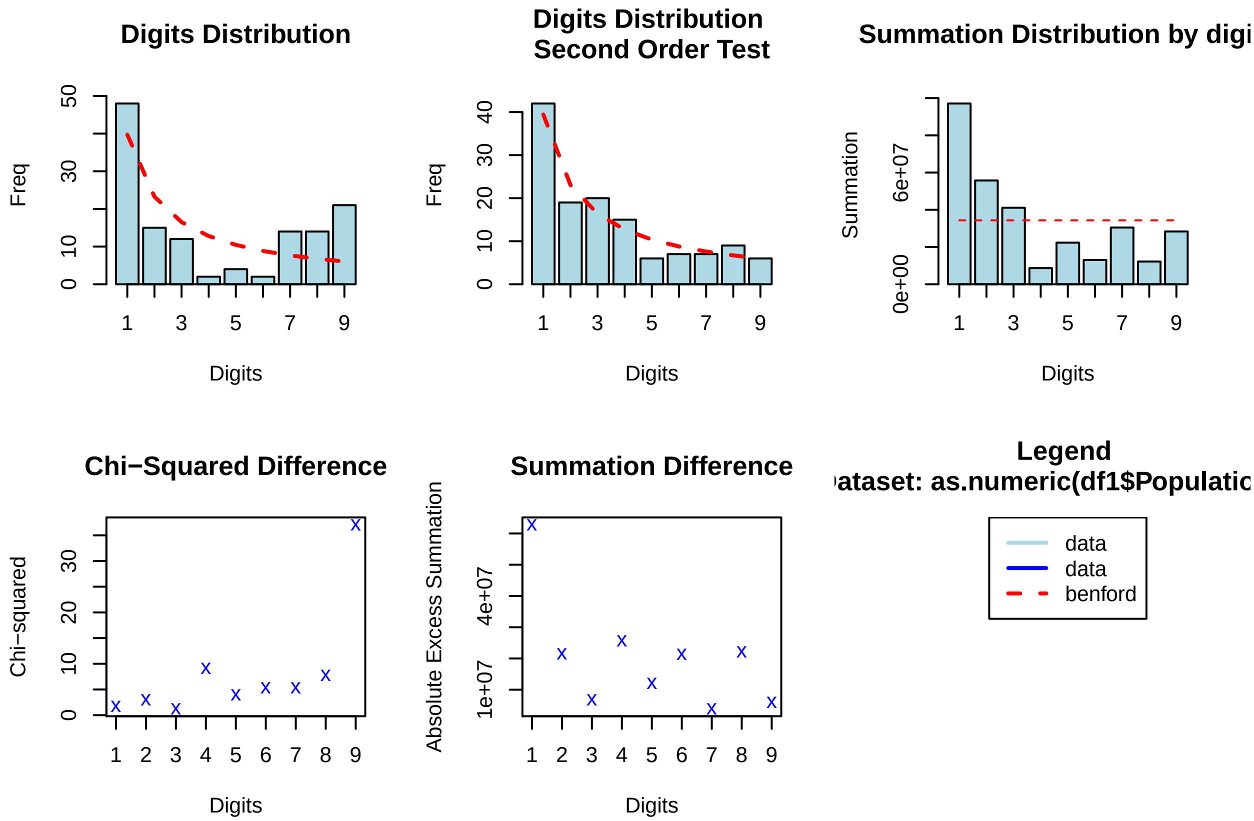
```r
library(readxl)
chinese_cencus <- read_excel("chinese cencus.xlsx")
df <- na.omit(chinese_cencus)
df1 <- select(df, Population, Population__1, Population__2)



bfd.pop <- benford(as.numeric(df1$Population),
                   number.of.digits = 1)
plot(bfd.pop) #Census 1990
```




```r
bfd.pop1 <- benford(as.numeric(df1$Population__1),
                    number.of.digits = 1)
plot(bfd.pop1) #Census 2000
```




```r
bfd.pop2 <- benford(as.numeric(df1$Population__2),
                    number.of.digits = 1)
plot(bfd.pop2) #Census 2010
```



The results do not well fit benford law. The reason could be: 1. Dataset is relatively small (132 obs only) 2. Data is not complete. This dataset only contains population of those cities over 750,000, and they are urban populations.