HW. Longhao Chen

1. Find MLE of the proportion of careal purchased by men.

$$\hat{p} = \frac{12}{70} = \frac{6}{35}$$

2. $X_1, \cdots, X_n$ form a random sample from the Bernoulli distribution with θ, unknown. $0<\theta<1$. Show that MLE of theta does not exist if every observed value is 0 or if every observed value is 1

$$P(n) = p^n (1-p)^{1-n}$$

$$L_n(\theta) = \prod_{i=1}^{n} p^{x_i}(1-p)^{1-x_i} = (1-p)^n \quad \text{if } x_i = 0$$

$$= p^n \quad \text{if } x_i = 1$$

$$\ell(\theta) = n \cdot \log(1-p)$$

$$= -\frac{n}{1-p} = 0 \quad \text{which can not be solved.}$$

3. $X_1, \cdots, X_n$ form a random sample from a Poisson distribution the mean λ is unknown.

a). $$f(x;\lambda) = \frac{\lambda^x e^{-\lambda}}{x!}$$

$$L_n(\lambda) = \prod_{i=1}^{n} \frac{\lambda^{x_i} e^{-\lambda}}{x_i!}$$

$$\ell_n(\lambda) = \sum_{i=1}^{n} (x_i \log \lambda - \lambda - \log x_i!)$$

$$= \log \lambda \sum_{i=1}^{n} x_i - n\lambda - \sum_{i=1}^{n} \log x_i!$$

$$\ell'(\lambda) = \frac{1}{\lambda} \sum_{i=1}^{n} x_i - n = 0$$

so $\hat{\lambda} = \bar{x}$

It can be shown that if $x_i = 0$ for $i = 1$ to n then $\sum_{i=1}^{n} x_i - n < 0$, it is impossible to find one λ that satisfy the equation.

4. $X_1, \dots, X_n$ form a random sample from a normal distribution. for which the mean μ is ~~not~~ known, ~~but~~ the variance σ^2 is unknown. Find the MLE of σ^2.

$$\ell(\mu, \sigma^2; x_1, \dots, x_n) = -\frac{n}{2}\ln(2\pi) - \frac{n}{2}\ln(\sigma^2) - \frac{1}{2\sigma^2}\sum_{j=1}^{n}(x_j - \mu)^2$$

$$\frac{\partial}{\partial \sigma^2}\ell = \frac{\partial}{\partial \sigma^2}\left(-\frac{n}{2}\ln(2\pi) - \frac{n}{2}\ln(\sigma^2) - \frac{1}{2\sigma^2}\sum_{j=1}^{n}(x_j - \mu)^2\right) = 0$$

$$= \frac{1}{2\sigma^2}\left[\frac{1}{\sigma^2}\sum_{j=1}^{n}(x_j - \mu)^2 - n\right] = 0$$

$$\hat{\sigma}_n^2 = \frac{1}{n}\sum_{j=1}^{n}(x_j - \hat{\mu})^2$$