HW3_LonghaoChen

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1. Suppose that one letter is to be selected at random from the 42 letters in the sentence, "The shortest distance between two points is a taxi." If Y denotes the number of letters in the word in which the selected letter appears, what is the value of E(Y)?

Y can be c(1, 2, 3, 4, 6, 7, 8) with corresponding probability of (1/9, 1/9,2/9,1/9,1/9,1/9,2/9) So

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E=1*1/9+2*1/9+3*2/9+4*1/9+6*1/9+7*1/9+8*2/9
E
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## [1] 4.666667
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2. Suppose that X and Y have a continuous joint distribution for which the joint ppf is: f(x, y) = 12y^2 for 0 y x 1 Find the value of E(XY).

$$E(XY) = \int_0^1 \int_0^x xyf(x,y)dydx = \int_0^1 \int_0^x 12xy3dydx = \int_0^1 4xy4|x0dx = \int_0^1 4x5dx = (4x6/6)|10 = 2/3 = 0.6667$$

3. Suppose that three random variables X1, X2, X3 form a random sample from the uniform distribution on the interval [0, 1]. Find E[(X1 − 2X2 + X3)^2].

The expected value of the uniform distribution on interval is (1-0)/2=0.5 Expanding E[(X1-2*X2+X3)^2] We can get $E(X_1^2) + 4E(X_2^2) + E(X_3^2) - 4E(X_1X_2) + 2E(X_1X_2) - 4E(X_2X_3)$ We can further calculate $E(X_1^2) = \int_0^1 x^2dx = 1/3$ Since X1 X2, X3 are independent of each other $E(X1X2) = E(X1) * E(X2) = 1/2 * 1/2$ So, we can calculate the final result as 1/2

4. X has pdf f(x)=e^−x, x>0. Y=e^(3x/4) Find E(Y) Using the lazy statistician theorem. We can easily calculate the E(Y)

$$E(Y) = \int_0^\infty e^{-x}e^{3x/4}dx = -4\mathrm{e}^{-\frac{x}{4}} = 4$$

5. X is the outcome of rolling a fair die. Y = g(X) = 2X^2 + 1 Find E(Y)

$$\mathrm{E}[g(X)] = \sum_x g(x)f_X(x) = \sum_x (2x^2+1) * 1/6 = 31.3$$

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y = 0
for (i in 1:6) {
  y= y+(2*i^2+1)*1/6
}
y
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## [1] 31.33333
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6. X has pdf f(x)=2(1−x), 0<x<1 Y =(2X+1) Find E(Y^2).

Using rule of lazy statistician

$$E(Y^2) = \int_x g(x)f_X(x) = \int_x (2*(1-x))(2x+1)(1x+1)dx = 3$$

7. Remember the binomial theorem: (x + y)n = n Show that E[(ax + b)n] = n n an−ibiE(Xn−i) i=0

When we are interested in the expectation of a ramdom variable, we can take a constant out of the expectation. So

$$E[(ax+b)^n] = \sum_{i=0}^{n} \binom{n}{i} (ax)^{n-i} b^i$$

$$E[(ax+b)^n] = E[\sum_{i=0}^{n} \binom{n}{i} a^{n-i} b^i x^{n-i}]$$

$$E[(ax+b)^n] = \sum_{i=0}^{n} \binom{n}{i} a^{n-i} b^i x^{n-i} E[x^{n-i}]$$

8. The proportion of defective parts in a large shipment is p. A random sample of n parts is selected from the shipment. Let X denote the number of defective parts in the sample, and Y denote the number of good parts in the sample. Find E(X − Y). If the sample size is 20 and p is 5%, what is E(X −Y)? Write out your answer as a complete sentence that expresses the meaning of your result.

Since the defective parts comes from a large shipment. We will consider this as a binomial distribition. E(X-Y)=np-n(1-p)=2np-n

The defective parts are expected to be 18 units less than good parts.