METHODS OF TEACHING LIMIT

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Goal

In our project we wanted to compare two different methods of teaching the limit: the 'Rule Of Convenience' and the 'Traditional Teaching of Limit'.

Introduction

Multiple studies have shown that students have difficulty understanding limits. The reason is that the difference between 'approaching' an answer and 'equaling' an answer is taught as being static. To solve this, we have come up with a teaching method called the 'Rule Of Convenience'. This method appears to be effective when *tutoring* students. The big question: can this be used to *teach* the limit? How does it stack up to the traditional ways of teaching the limit?

Explanations

Suppose our limit was $\lim_{x\to1}\left[\frac{x^3-1}{x-1}\right]$. We define the *Traditional* method of teaching the limit as the *"Rule Of Approaching"* (left figure), and the *"Rule Of Convenience"* involves a 'convenient choice' (right figure).

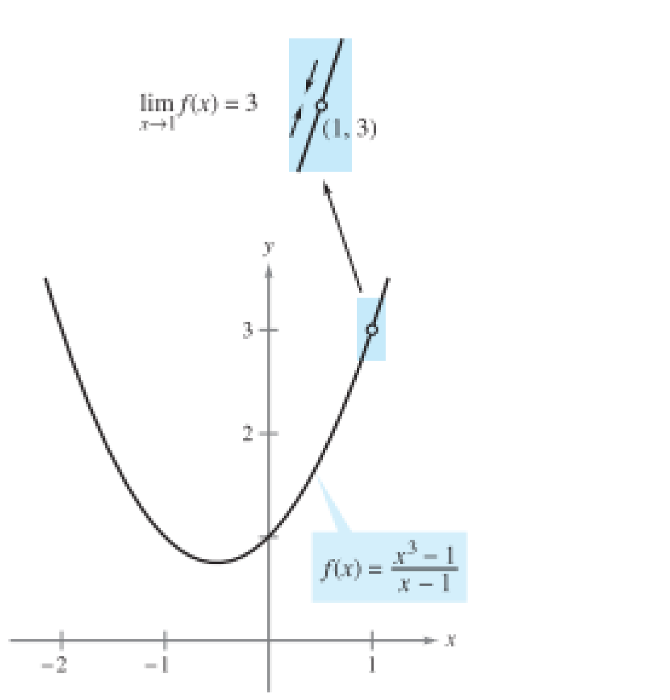


Figure: The "Rule Of Approaching" as described by Larson Edwards's *Calculus 9th Edition* textbook

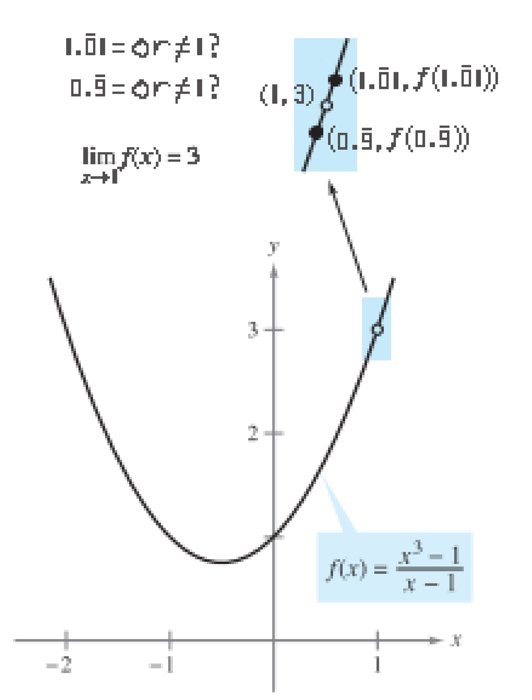


Figure: The "convenient choice" that guides the method we call the "Rule Of Convenience"

The Experiment

We split our group of participants based on whether they would be learning the Rule Of Convenience or the Traditional Method. To teach the participants, we had them watch a 10-minute video that would explain limits with the method we assigned. Each video had the same person doing the voice-over, person drawing, production value, problems, answers, and even the same color scheme.

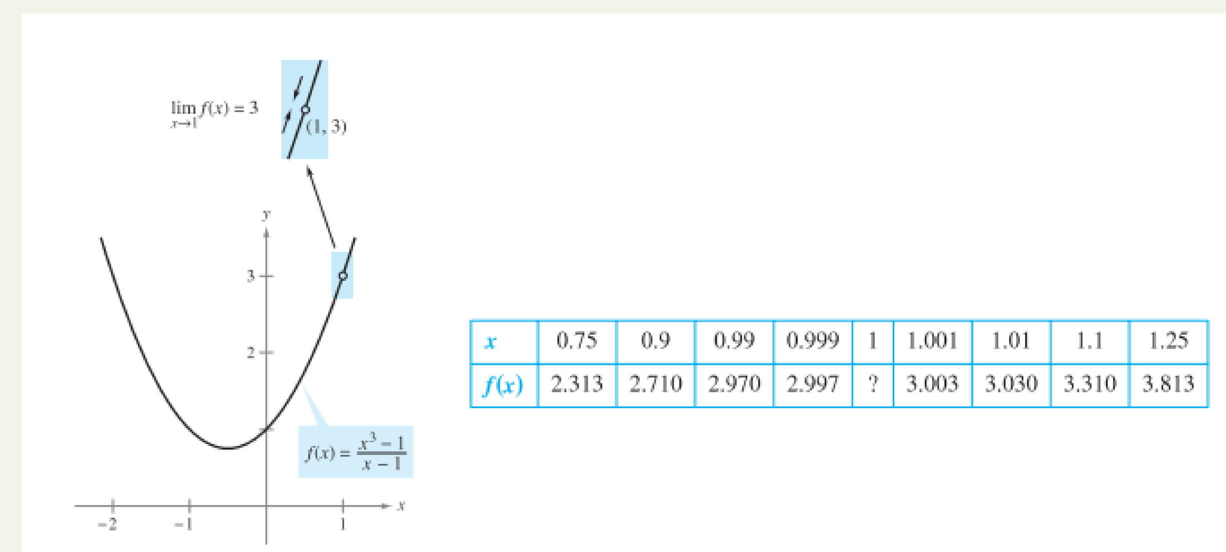
We then interviewed each participant one-at-a-time over Zoom as they filled out a worksheet. The worksheet was split into Section A, which focused on definitions and understanding, and Section B, which focused on procedural knowledge. Any time a function showed up on the worksheet, the graph of that function immediately followed. Our hope was that if the function was unfamiliar, the student would use the graph to solve the problem.

Results

We only got 3 students to participate, so our findings are not statistically significant. However, there are a few interesting emerging trends.

Preference to Approach

$\frac{2}{3}$ students said that the 'plug numbers in that get closer to what you're approaching' concept was the most helpful understanding of how the limit behaved.



x	0.75	0.9	0.99	0.999	1	1.001	1.01	1.1	1.25
$f(x)$	2.313	2.710	2.970	2.997	?	3.003	3.030	3.310	3.813

Figure: $\frac{2}{3}$ students preferred this concept of the limit. However, their thought processes never used this definition when solving problems, which led them to the wrong answer.

The new question this raises: What major drawbacks appear from relying on this concept of the limit?

Helpful or Unhelpful?

All of the students said that graphs were helpful, but the way the students were solving the problems indicated that the graphs didn't help at all or led the students to the wrong answer.

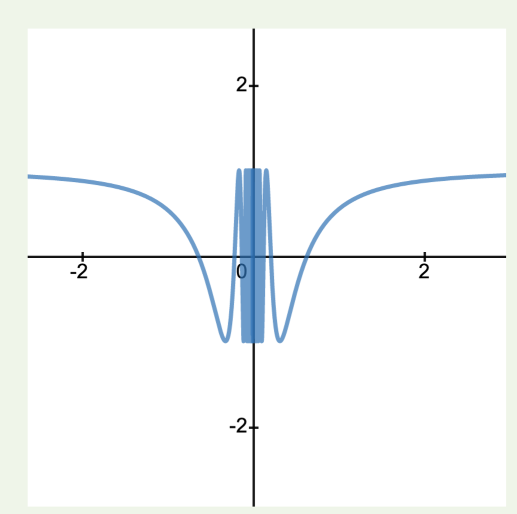


Figure: $\frac{2}{3}$ of the students were confused by this graph, and one even gave up on the problem due to lack of trig knowledge.

The new question this raises: Do students simply not understand how to use the graphs, or are the graphs themselves misleading?

In general, students shown the "Rule Of Convenience" did better on the worksheet and had better understanding of how to apply the different limit-solving methods.

Worksheet Excerpt

8) $\lim_{x\to1}[k(x)]$, $k(x) = \begin{cases} x^3+1 & , x \neq 1 \\ 1 & , x = 1 \end{cases}$

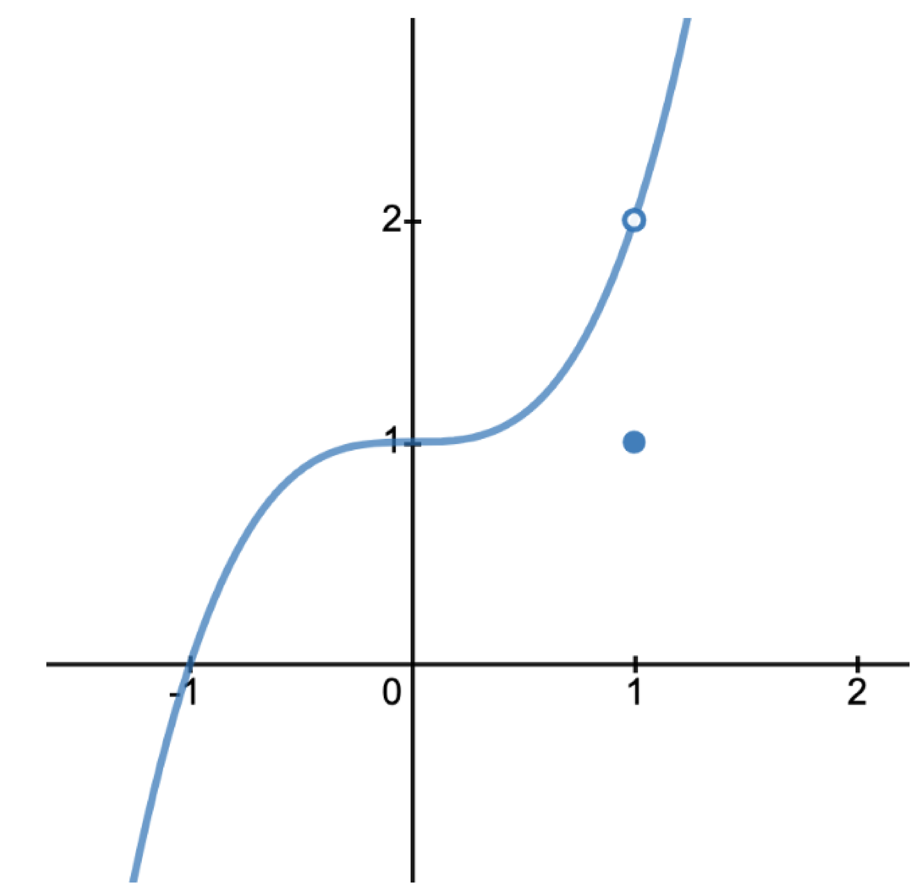


Figure: Problem 8 of Section B

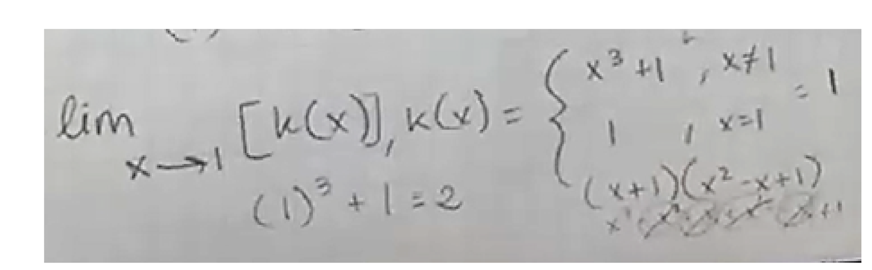


Figure: One student's response

Future Research

The hard work has already been done. We have all of the materials created, and the process worked beautifully and ethically. We just did not get enough participants to come to any significant conclusion.

In the future, we would also like to study understanding of infinity, since this often went hand-in-hand with understanding of the limit in the literature. We would like to continue this research in the future with (hopefully) a larger group of participants.

Acknowledgements

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