Elephants: Big Birth Control

Final Report MATH 460

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Executive Summary

The project's goal is to develop a mathematical model to fix an elephant population at 11,000 elephants using contraceptives. Based on the data given by park management of a South African National Park, the model will be used to estimate the number of cows (female elephants) to dart with contraceptives every year. To do so, the survival rates and age-sex structure of the population must first be considered. The model should also address how the introduction of contraceptives changes the age-sex structure of the population.

In the last few decades, local communities, environmental agencies, and the South African government have grown concerned about the growing elephant population in South Africa. A contraceptive method of population control was developed to reduce the elephant population growth rate as opposed to the traditional culling and relocation of elephants, methods which were deemed inhumane. Not only will this approach be able to slow down elephant population growth and stabilize the population at low levels, it helps to maintain a natural ecosystem in South Africa, reduce costs compared to relocation, and benefit both surrounding wildlife and the local communities.

To help the park adapt to a modern method of population control, we were given data about elephants which were transported out of a South African National Park. Unfortunately no data was given about elephants which reside in the park so the model we developed does not accurately depict introducing contraceptives into a population. Instead, it gives an illustration of how we would perform modeling this phenomenon. Also due to the lack of accurate data, we were not able to test the model to see its dependability if we were to adapt it to different populations.

The problem is modeled using a system of linear differential equations to find a steady-state solution when the population of the elephants reaches a desired carrying capacity of 11,000. To obtain such a system, we calculated survival rates and modeled the age-sex structure of the population based on values given by the National Park, ideal data values based on a perfect population, and averages of the two. We did so to use a wider range of values, which improved the accuracy of the model. Our system of differential equations uses these survival rates as growth rates and the age-sex structures as initial values. Since different age ranges were found to have different survival rates, we chose to model them separately and use their sum for the total population. Without a known carrying capacity, we decided an exponential model would allow us to avoid making unnecessary assumptions. Then, to maintain a constant population, we introduced a function representing the proportion of un-darted female adult elephants to the birth rate. This allowed us to balance the birth and death rates over time and gave us a model that reached its steady-state within 40 years.

Based on this controlled growth model, we then found a formula for the population of elephants who have received contraceptives, $N(t) = \frac{1}{2}(1 - c(t)P_2$ where $c(t)$ is the proportion of un-darted adult female elephants and P_2 is the adult elephant population. Using the first hundred data points for $N(t)$, we were able to find the following best fit equation, $N(t) = 941.5e^{-02498t} + 1082e^{(-9.654e-05)t}$. Since the contraceptives are only effective for two years, at any time t, the only elephants already in the darted population are the surviving darted elephants from the year before. Thus we found the number of elephants to be darted every year using a recursive sequence based on the number of elephants that were darted the previous year, $a_t = \lceil N(t) - S_2 a_{t-1} \rceil$ with $a_1 = N(1)$.

Introduction

During the 1900s, South African elephant populations experienced a period of irruptive growth. Park management initially controlled the population by culling, killing elephants in herds, and relocation. Both approaches stirred up a controversial debate on the morality of population control methods and so immunocontraceptive vaccines were considered by wildlife management. Dating back to the 1950s and 1960s, there have been numerous studies of the immunocontraceptive dart involving rodents, deer and viral diseases without much success. In 1989, researchers at University of California Davis developed the first immunocontraceptive vaccine for animals, which was initially tested on deer, donkeys, and elephants. The first trials of using immunocontraceptive darts on elephants began in 1996 at the South African Kruger National Park by zoologist Richard Fayrer-Hosken and his colleagues. After a few years of trials, Zhibin Zhang, a researcher at the Chinese Academy of Sciences in Beijing, was interested in developing a practical use for the immunocontraceptive dart and thus developed one of the first mathematical models of wildlife management by contraception.

Traditional population control methods such as culling and relocation aim to increase mortality rate. Consequently, the population leftover has more food and space to grow rapidly, resulting in irruptive growth. On the other hand, population control by contraception intends to control birth rates of animal populations and subsequently maintain the carrying capacity for longer periods. Unlike culling and relocation, administering contraceptives keeps every individual at the park to occupy space and consume food under low stress situations in healthy habitats under the supervision of park management.

A large National Park in South Africa wishes to control the elephant population by contraception so the team developed a mathematical model under certain constraints and data provided by the National Park. Thus, the purpose of this paper is to introduce our mathematical model under such assumptions and explain the results of our solution as well as any general improvements that can be made.

Assumptions

To simplify our model, we assume:

1. There is very little emigration or immigration of elephants.
2. The gender ratio is very close to 1:1 and control measures have endeavored to maintain parity.
3. The gender ratio of newborn calves is also about 1:1. Twins are born about 1.35% of the time.
4. Cows first conceive between the ages of 10 and 12 produce, on average, a calf every 3.5 years until they reach an age of about 60. Gestation is approximately 22 months.
5. The contraceptive dart causes an elephant cow to come into oestrus every month (but not conceiving). Elephants usually have courtship only once in 3.5 years, so the monthly cycle can cause additional stress.
6. A cow can be darted every year without additional detrimental effects. A mature elephant cow will not be able to conceive for 2 years after the last darting.
7. Between 70% and 80% of newborn calves survive to age 1 year. Thereafter, the survival rate is uniform across all ages and is very high (over 95%), until about age 60; it is a good assumption that elephants die before reaching age 70.
8. There is no hunting and negligible poaching in the Park.
9. Elephants are to be studied in age groups: those in their first year (0-1), before mature conceiving age (1-11), conceiving age (11-60), and after conceiving age (60+).
10. Carrying capacity is steadily maintained at 11,000.

These assumptions expect the elephant population in the Park to behave perfectly, which in real life is difficult to achieve. Since our data was derived from the data given, the ideal population, and an average of both, the solution does not give precise results. To obtain a more accurate model, one that gives real world solutions, we would need actual data of the elephants in the park and specifications about the darting process to avoid making any general assumptions. Furthermore, if we received updated data such as survival rate and age-structure from numerous parks, this model can be generalized towards a wider margin of use.

Mathematical Model

Let us define the following variables and terms:

P	Total Population
P_0	Infant Population, Ages 0 - 1
P_1	Juvenile Population, Ages 1 - 11
P_2	Adult Population, Ages 11 - 60
P_3	Elderly Population, Ages 60+
S_n	Survival Rate for population P_n
c(t)	Proportion of undarted adult female population in a year t
N(t)	Number of elephants we need darted in a year t
Ideal	Values from the project description
Data	Actual data sets given to us
Average	Calculated data between data from the project description and the actual data sets

Survival Rates and Ages-Sex Structure

Our project description included survival rates for the different age groups of elephants, but we also derived experimental survival rates from our data. We did this by dividing the number of k year-old elephants by the number of k+1 year-olds for each age in our data sets, getting year-wise survival rates for each age of elephant. Some year-wise rates were above 1, but these aren't as much of a concern since on average they end up less than 1 in their respective age groups. Naturally, we averaged these rates within the P_n groups to get our final experimental survival rates.

We used the year-wise survival rates to simulate a long-term age-sex structure of elephants. Our goal was to discern whether experimental survival rates, ideal rates, or an average produced the most plausible population structure and use that one in our model. To do this, we began with a population of one-year-old elephants and multiplied the population by the one-year-old survival rate to get the two-year-olds. We multiplied the two-year-olds by the two-year-old survival rate to get the speculated three-year olds, and so on and so forth until we had a population of elephants for each age. We repeated this process with the idea rates, data-derived rates, and averages of the two to get three structures. Our end goal was to find which survival-rate scheme produced the closest fit to a pyramidal population structure, and to that end we chose the average of the data-derived rates and the ideal rates for our model.

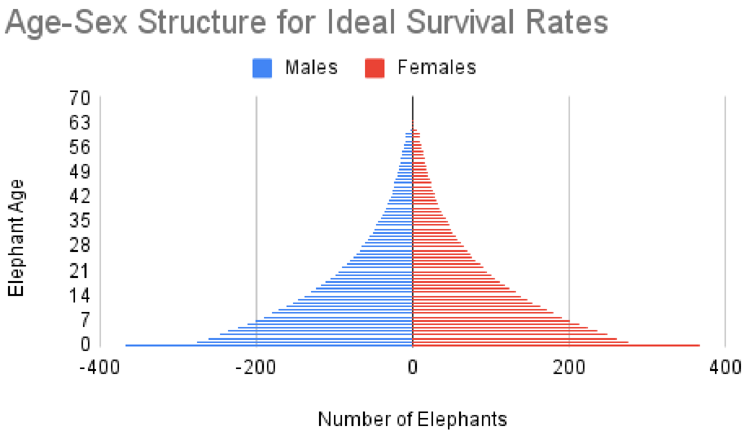

Figure 1: Idealized Age-Sex Structure

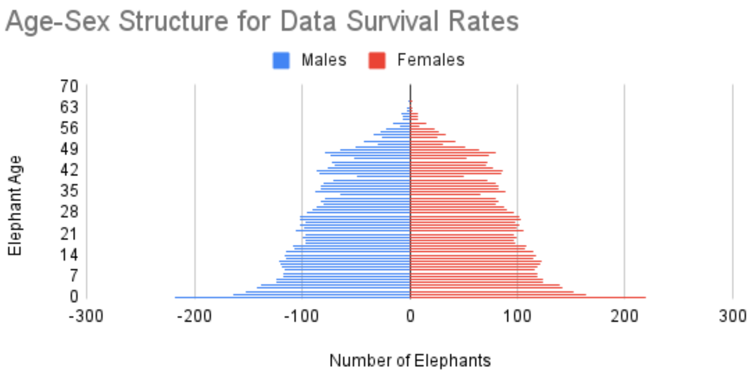

Figure 2: Age-Sex Structure of the Relocated Elephants

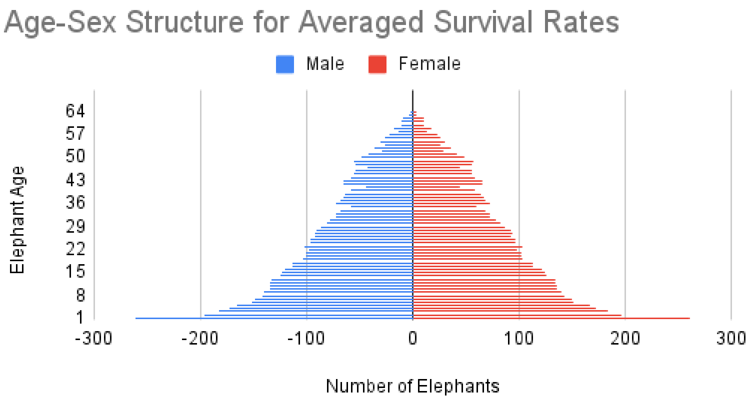

Figure 3: Averaged Age-Sex Structure

Natural Growth Model

Then, we created a system of differential equations to model the natural growth of the elephant population using the survival rates previously mentioned and their age-sex structures as initial values. Since different age ranges of the total population have different survival rates and contribute differently to the overall population growth, we chose to model each age range separately and the total population as a sum of these groups. The rates of change for individual age groups consider the number of elephants entering that age range, dying out, and aging into the next sub-group. The following is this natural growth system:

$$\begin{aligned}
\frac{dP_0}{dt} &= b(t)P_2 - (1-S_0)P_0 - S_0P_0 \qquad (1)\\
\frac{dP_1}{dt} &= S_0P_0 - (1-S_1)P_1 - \frac{S_1}{10}P_1\\
\frac{dP_2}{dt} &= \frac{S_1}{10}P_1 - (1-S_2)P_2 - \frac{S_2}{49}P_2\\
\frac{dP_3}{dt} &= \frac{S_2}{49}P_2 - (1-S_3)P_3\\
\frac{dP}{dt} &= \frac{dP_0}{dt} + \frac{dP_1}{dt} + \frac{dP_2}{dt} + \frac{dP_3}{dt} \qquad (2)\\
P &= P_0 + P_1 + P_2 + P_3
\end{aligned}$$

Using this system we observe the following population graphs:

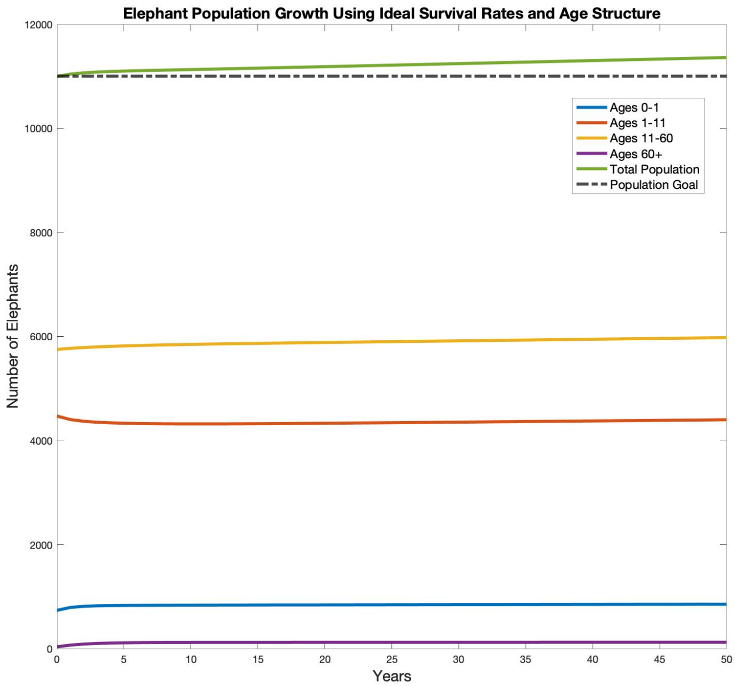

Figure 4: Natural Population Growth Using Ideal Survival Rates and Age Structure

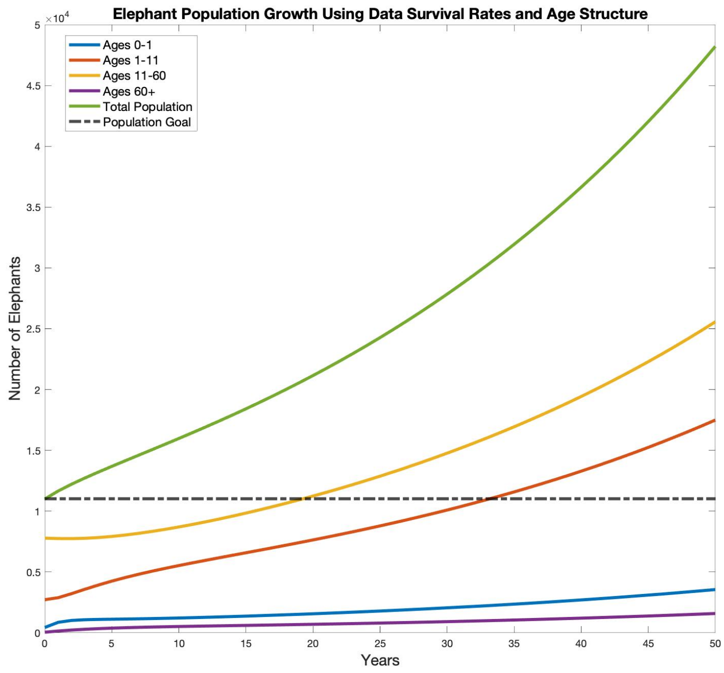

Figure 5: Natural Population Growth Using Data Survival Rates and Age Structure

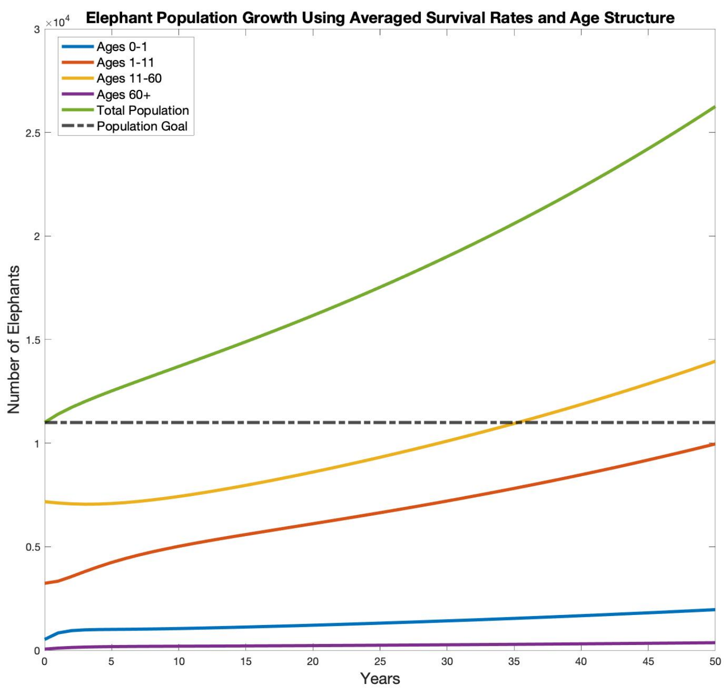

Figure 6: Natural Population Growth Using Averaged Survival Rates and Age Structure

Controlled Growth Model

To prevent the elephant population from growing over 11,000, we introduce a contraceptive term into equation (1) from our natural growth model system. We do this by controlling the birth rate such that

$$b(t)P_2 \rightarrow c(t)b(t)P_2$$

So the system for our controlled growth model becomes:

$$\begin{aligned}
\frac{dP_0}{dt} &= c(t)b(t)P_2 - (1 - S_0)P_0 - S_0P_0 \\
\frac{dP_1}{dt} &= S_0P_0 - (1 - S_1)P_1 - \frac{S_1}{10}P_1 \\
\frac{dP_2}{dt} &= \frac{S_1}{10}P_1 - (1 - S_2)P_2 - \frac{S_2}{49}P_2 \\
\frac{dP_3}{dt} &= \frac{S_2}{49}P_2 - (1 - S_3)P_3 \\
\frac{dP}{dt} &= \frac{dP_0}{dt} + \frac{dP_1}{dt} + \frac{dP_2}{dt} + \frac{dP_3}{dt} \\
P &= P_0 + P_1 + P_2 + P_3
\end{aligned}$$

This $c(t)$ we created represents the proportion of adult female elephants that are still contributing to the birth rate, and thus the proportion of adult female elephants that have not been darted. Since the goal of introducing $c(t)$ is to maintain a constant total population of 11,000 we can solve for it by setting the total population rate of change, equation (2), to zero. Thus we derive::

$$c(t) = \frac{(1 - S_0)P_0}{b(t)P_2} + \frac{(1 - S_1)P_1}{b(t)P_2} + \frac{(1 - S_2)}{b(t)} + \frac{(1 - S_3)P_3}{b(t)P_2} \qquad ((3))$$

By running our adjusted controlled growth system using equation (3) we find the following population graphs for controlled growth:

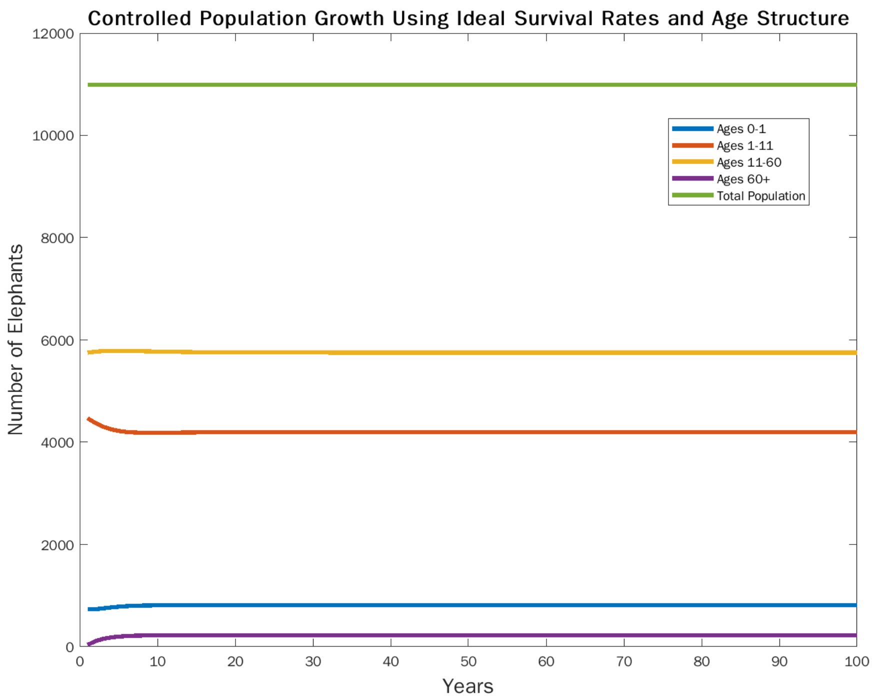

Figure 7: Controlled Population Growth Using Ideal Survival Rates and Age Structure

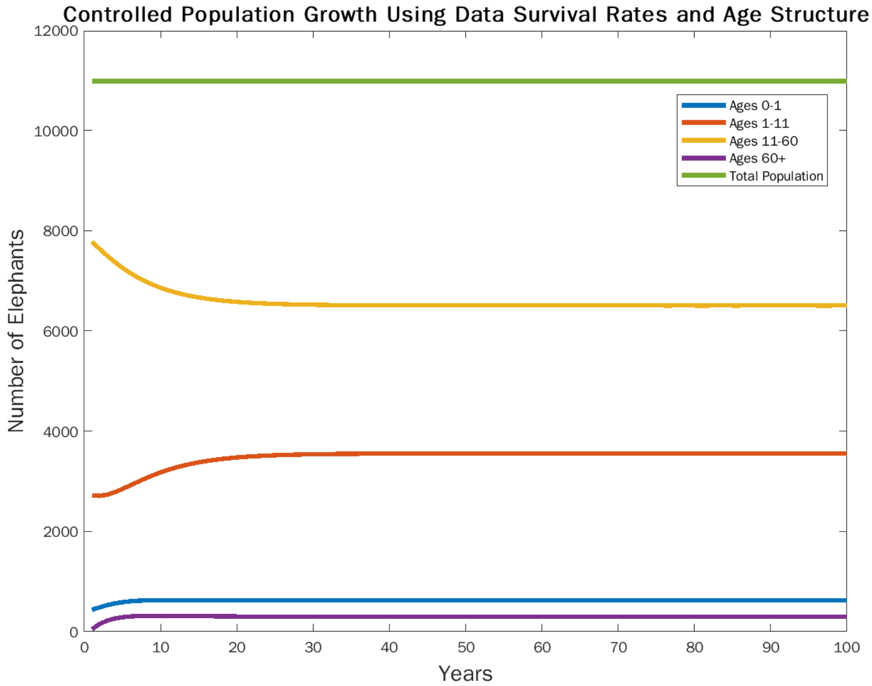

Figure 8: Controlled Population Growth Using Data Survival Rates and Age Structure

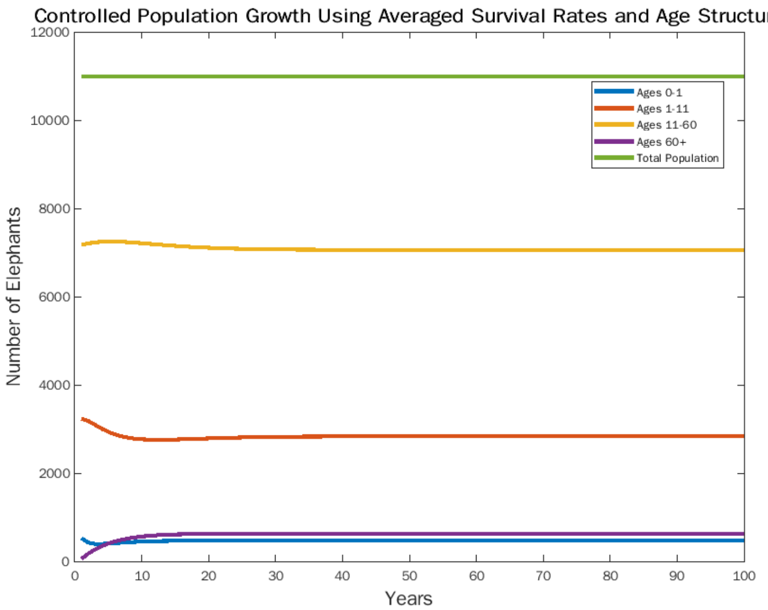

Figure 9: Controlled Population Growth Using Averaged Survival Rates and Age Structure

Since we were given initial conditions and data of elephants that were transported out of the park, we were unable to perform simulations of our model to test its accuracy.

Results

Having found c(t), we can begin finding the number of elephants to dart. Since c(t) is the proportion of adult female elephants that have not been darted, the desired number of darted elephants at any given time can be represented by:

$$N(t) = \frac{1}{2}(1 - c(t))P_2$$

Using our models on MATLAB, we calculated the first hundred data points for N(t) and then found a best fit line for them:

$$N(t) = 941.5e^{-02498t} + 1082e^{(-9.654e-05)t}$$

The best fit line has an R-square value and an adjusted R-square value of 0.9992.

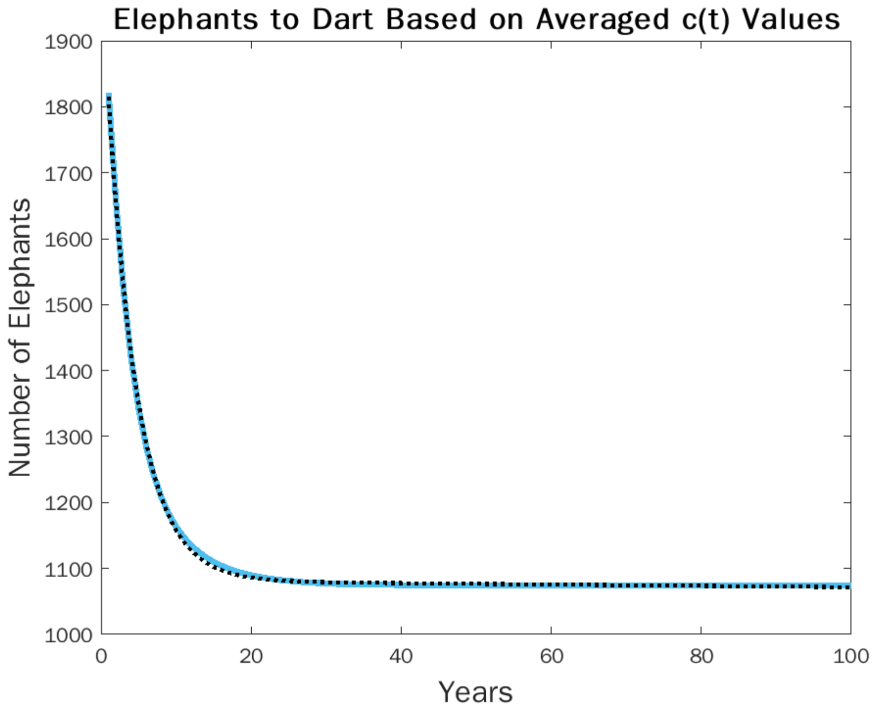

Figure 10: Data Points for N(t) Graphed in Blue with the Best Fit Line Shown Dashed

Since the contraceptives are only effective for two years, at any time t, the number of elephants already in the darted population are the surviving darted elephants from the year before. Thus to have N(t) elephants darted in year t, we need to dart $a(t)$ elephants that year where $a(t)$ is defined by:

$$a_t = \lceil N(t) - S_2 a_{t-1} \rceil$$
$$with \quad a_1 = N(1)$$

Using our formulas for a_t and $N(t)$, we reach our final results for the number of elephants to dart per year to maintain the population at 11,000. The following is a table of the first hundred data points:

Number of Elephants to Dart

Year	Number		Year	Number
1	1822		51	695
2	61		52	403
3	1598		53	686
4	0		54	412
5	1421		55	677
6	0		56	421
7	1286		57	668
8	0		58	430
9	1210		59	660
10	14		60	437
11	1151		61	653
12	35		62	444
13	1103		63	646
14	60		64	451
15	1060		65	639
16	86		66	458
17	1022		67	633
18	112		68	463
19	988		69	628
20	138		70	468
21	957		71	623
22	163		72	473
23	929		73	618
24	186		74	478
25	904		75	613
26	208		76	483
27	880		77	608
28	230		78	488
29	857		79	604
30	251		80	491
31	836		81	601
32	269		82	494
33	817		83	598
34	287		84	497
35	800		85	595
36	303		86	500
37	783		87	592
38	319		88	503
39	768		89	589
40	334		90	506
41	753		91	586
42	347		92	509
43	740		93	583
44	360		94	512
45	727		95	580
46	372		96	515
47	716		97	577
48	383		98	518
49	705		99	575
50	394		100	519

Table 1: Results from a_t

Improvements

The model we developed was based on data that did not reflect the elephant population living within the national park, so if we were to develop a more accurate model, we would need actual data of real elephant populations residing in national parks. This will allow us to base the mathematical model on accurate survival rates and age-sex structures. Additionally, with more information on the darting process we would have a fuller understanding of its effects which would be helpful when developing a definite mathematical model. Once we have a more polished model, we could determine how the contraceptive dart affects population re-growth in response to a dramatic decline in the elephant population. As a higher objective, with a successful model, we could prepare plans to adapt this model for elephant populations with differing sizes, desired capacities, survival rates, and alternative population control methods.

Conclusions

To model the number of female elephants to be darted per year, we first had to analyze our survival rates and age-sex structure. Since the given survival rates did not match the age-sex structure of the relocated elephants, we choose to model them separately then average the two. This averaged age-sex structure was the most suitable for our population model. We then modeled the natural growth of the elephant population. Because each age group of the elephant population had differing survival rates and conditions for growth, we created a system of differential equations to represent each age group. This allowed us to easily observe how total changes in the population affect the age structure. Modeling controlled growth only required inserting a contraceptive term $c(t)$ to birth rate in our system. We solved for this $c(t)$ by setting the sum of our differential equations to 0 since our goal is a fixed population. Once we modeled our controlled growth, we were able to see some changes from the natural growth models. In our age-sex structure for controlled growth, P_1 (population of ages 1-11) grew at a faster rate while P_2 (population of ages 11-60) decayed. Another change was that the elderly population P_3 (ages 60+) outgrew the infant population which may affect tourism in upcoming years with the lack of exposure to an infant elephant population. After noticing these changes through $c(t)$, we then obtained our $N(t)$ term which is the desired number of darted elephants at any given time. Taking into consideration that the contraceptives' effects only last for two years, we had to include the number of darted elephants surviving up to the next year in our final solution. This gave us the completed formula which is used to find how many elephants to dart in a specific year. If we take a closer look at this formula $a_t = \lceil N(t) - S_2 a_{t-1} \rceil$, we can see that as $t \to \infty$, $a_t \to \frac{1}{2}N(\infty)$ which fits the 2 year effect of the contraceptives. Finally, if we were to adapt this model to other populations, we would only require information on age-sex structure, survival rates, and birth rates of the populations therefore, making our model an extremely adaptable one.

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