Partial Derivative Working:

Let $K \in \{1, 2, \dots, N\}$

Then $$\frac{\partial L(\underline{w})}{\partial w_K} = \frac{\partial}{\partial w_n}\left[-\frac{1}{2m}\sum_{i=1}^{m}\left[t^{(i)}\log(y(\underline{x}^{(i)},\underline{w})) + (1-t^{(i)})\log(1-y(\underline{x}^{(i)},\underline{w}))\right] + \phi\sum_{j=1}^{N} w_j^2\right]$$

$$= -\frac{1}{2m}\sum_{i=1}^{m}\left[\frac{t^{(i)}\frac{\partial}{\partial w_n}(y(\underline{x}^{(i)},\underline{w}))}{y(\underline{x}^{(i)},\underline{w})} + \frac{(1-t^{(i)})\left(-\frac{\partial}{\partial w_n}(y(\underline{x}^{(i)},\underline{w}))\right)}{1-y(\underline{x}^{(i)},\underline{w})}\right] \quad \boxdot$$

$$+ 2\phi w_n$$

where $$y(\underline{x}^{(i)},\underline{w}) = \frac{1}{1+e^{-\sum_{j=1}^{N} x_j^{(i)} w_j}} = \left(1+e^{-\sum_{j=1}^{N} x_j^{(i)} w_j}\right)^{-1}$$

$$\Rightarrow \frac{\partial}{\partial w_n}(y(\underline{x}^{(i)},\underline{w})) = -\left(1+e^{-\sum_{j=1}^{N} x_j^{(i)} w_j}\right)^{-2}\left(-x_n^{(i)} e^{-\sum_{j=1}^{N} x_j^{(i)} w_j}\right)$$

$$= \left(-y(\underline{x}^{(i)},\underline{w})^2\right)\left(-\left(\frac{1}{y(\underline{x}^{(i)},\underline{w})} - 1\right) x_n^{(i)}\right)$$

$$= \frac{x_n^{(i)} y(\underline{x}^{(i)},\underline{w})^2 (1-y(\underline{x}^{(i)},\underline{w}))}{y(\underline{x}^{(i)},\underline{w})}$$

$$= x_n^{(i)} y(\underline{x}^{(i)},\underline{w})(1-y(\underline{x}^{(i)},\underline{w})) \quad - \circledast$$

Substituting $\circledast$ into $\boxdot$:

$$\frac{\partial L(\underline{w})}{\partial w_n} = -\frac{1}{2m}\sum_{i=1}^{m}\left[t^{(i)} x_n^{(i)}(1-y(\underline{x}^{(i)},\underline{w})) + (1-t^{(i)})(-x_n^{(i)})(y(\underline{x}^{(i)},\underline{w}))\right]$$

$$+ 2\phi w_n.$$

$$= \frac{1}{2m}\sum_{i=1}^{m}\left[(1-t^{(i)}) x_n^{(i)}(y(\underline{x}^{(i)},\underline{w})) - t^{(i)} x_n^{(i)}(1-y(\underline{x}^{(i)},\underline{w}))\right] + 2\phi w_n.$$

$$= \frac{1}{2m} \sum_{i=1}^{m} [x_n^{(i)} y(\underline{x}^{(i)}, \underline{w}) - t^{(i)} x_n^{(i)} y(\underline{x}^{(i)}, \underline{w}) - t^{(i)} x_n^{(i)} + t^{(i)} x_n^{(i)} y(\underline{x}^{(i)}, \underline{w})] + 2\phi w_n$$

$$= \frac{1}{2m} \sum_{i=1}^{m} [x_n^{(i)} (y(\underline{x}^{(i)}, \underline{w}) - t^{(i)})] + 2\phi w_n$$

$$= -\frac{1}{2m} \sum_{i=1}^{m} [x_n^{(i)} (t^{(i)} - y(\underline{x}^{(i)}, \underline{w}))] + 2\phi w_n$$