Project - Installment II

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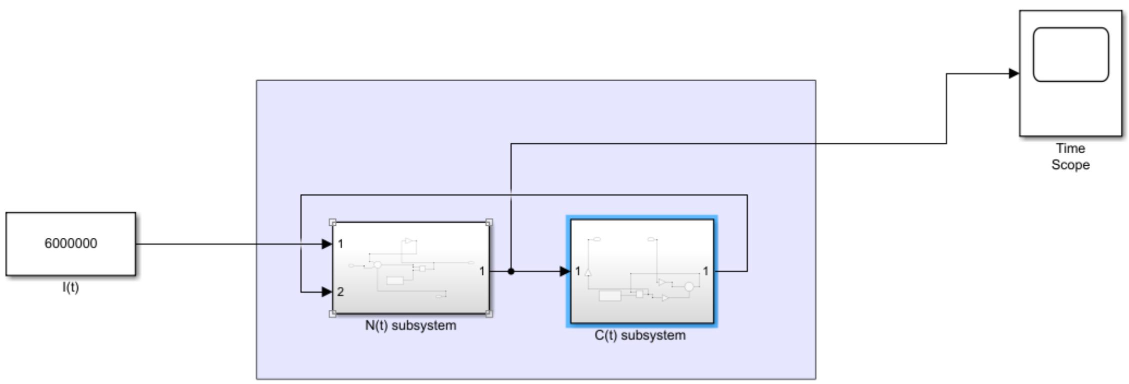
Introduction

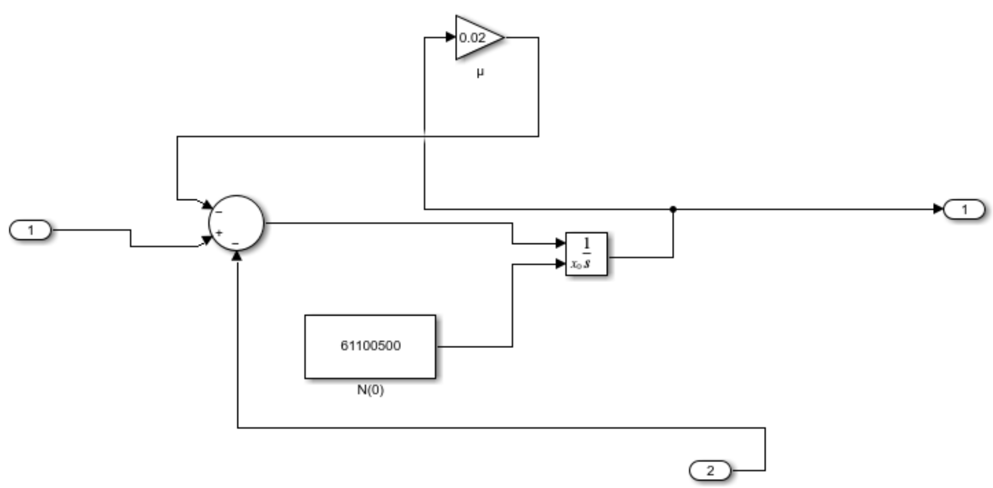
In installment 2 of this control systems project, we will be analyzing the different properties of a diabetes model. We will be using Matlab Simulink to analyze properties of the model such as the order, zero, poles, system response, feedforward gain, etc. We also need to prove that the system is stable and analyze its time response characteristics.

System Response

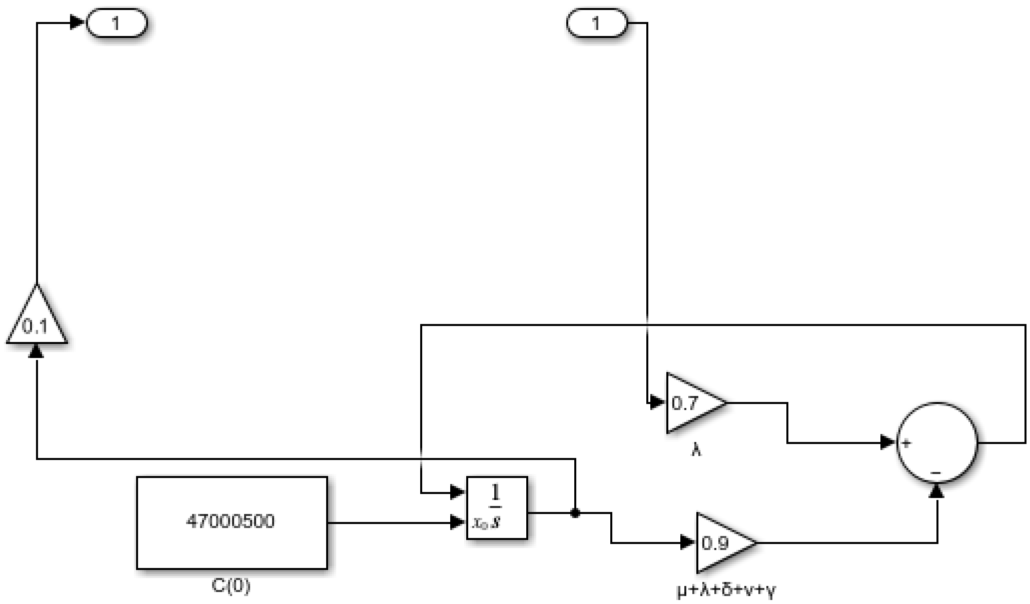
Block Diagram



System Block diagram



N(t) Block diagram



C(t) Block Diagram

Transfer Function Update

Here is the updated system Transfer function for N(s)/I(s)

$$\frac{N(s)}{I(s)} = \left[\frac{s+0.9}{s^2+0.92s+0.088} + \left(\frac{61100500(s+0.9)}{s^2+0.92s+0.088} - \frac{4700050}{s^2+0.92s+0.088} \right) \cdot \frac{s}{6\times10^6} \right]$$

$$\frac{1}{s^2+0.92s+0.088} \left[\frac{(s+0.9)}{1} + \left[\frac{61100500(s+0.9)\cdot s}{6\times10^6} - \frac{4700050\cdot s}{6\times10^6} \right] \right]$$

$$\frac{1}{s^2+0.92s+0.088} \left[\frac{(s+0.9)(6\times10^6) + [61100500s^2 + 54990450s - 4700050s}{6\times10^6} \right]$$

$$\frac{1}{s^2+0.92s+0.088} \left[\frac{6\times10^6 s + 5400000 + 61100500s^2 + 50290400s}{6\times10^6} \right]$$

$$\frac{1}{s^2+0.92s+0.088} \left[\frac{61100500s^2 + 56290400s + 5400000}{6\times10^6} \right]$$

$$\frac{1}{s^2+0.92s+0.088} \left[10.183s^2 + 9.381s + 0.9 \right]$$

$$\frac{10.183\left[s^2 + 0.92s + 0.088\right]}{s\left[s^2+0.92s+0.088\right]}$$

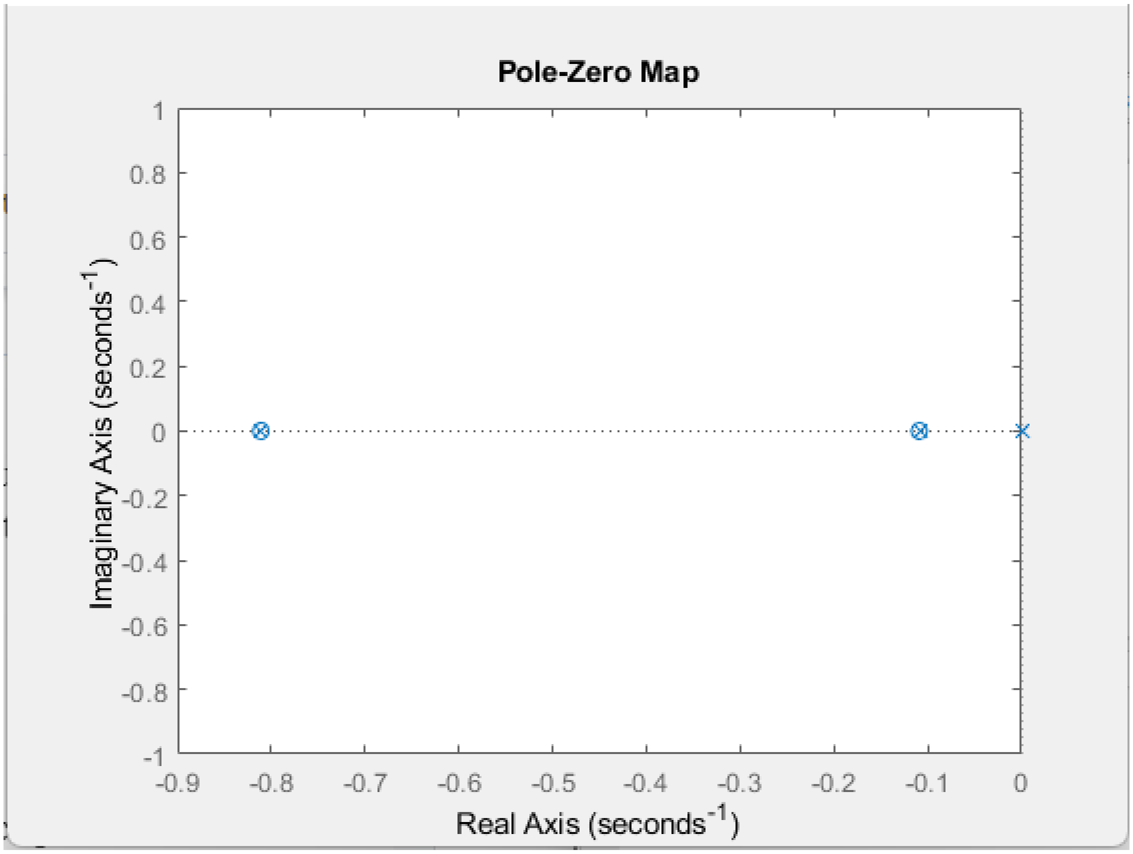
Characteristics of the Transfer function

From the given TF the order is 3.

Pole and Zero code:

```
s = tf([10.183, 9.3686, 0.9], [1, 0.92,0.088 0])
pol = pole(s)
zer = zero(s)
```

Pole-Zero Map:



Pole zero values:

```
>> pol

pol =

         0
   -0.8116
   -0.1084

>> zer

zer =

   -0.8111
   -0.1090
```

****Pole Zero Cancelation and System Parameters**
Since the poles and zeroes cancel the only term remaining is 1/s, there is no wn or zeta values since it would be a first order.

Time Step Response and parameters(on the side rise time, overshoot%)

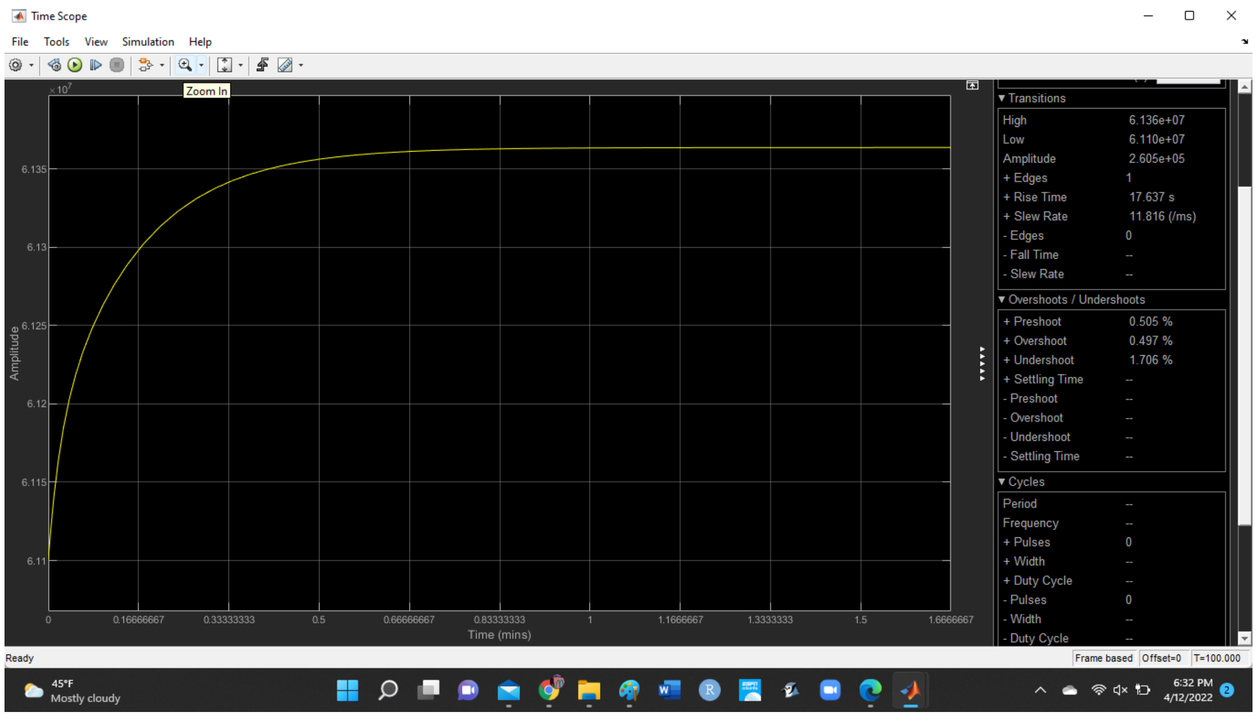
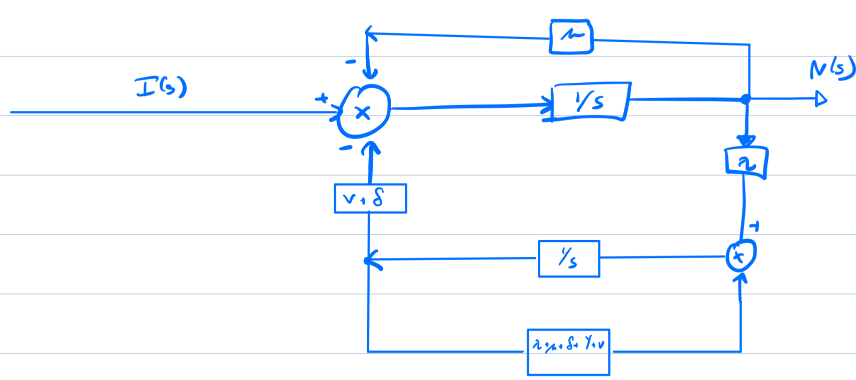
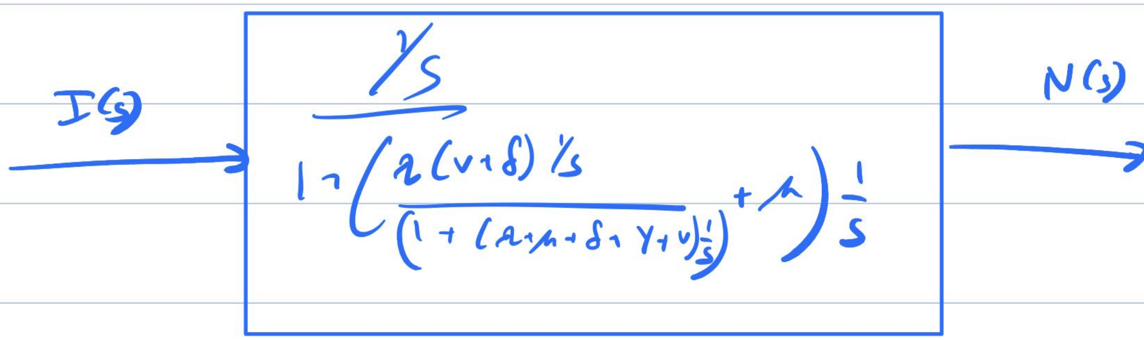


Figure 6. Time step response

Block Diagram Representation



Part 1



Part 2

The Final Transfer Function from the block diagram after simplification

$$\frac{N(s)}{I(s)} = \frac{s + 0.9}{s^2 + 0.92s + 0.088}$$

Stability Analysis

Routh Hurwitz Table and the feedforward gain

This system is stable due to all numbers in the leftmost column being positive.
In order for the feedforward gain RH table to be positive, the K value must be greater than -45 and 0. Since K> 0 overlaps K>-45, the feedforward gain must be greater than 0, essentially being a positive value.

Routh - Hurwitz : $s^2 + 0.92s + 0.088$

s^2	1	0.088
s^1	0.92	0
s^0	0.088	0

$$D(s) \longrightarrow \boxed{\dfrac{K(1/s)}{s + \left[\dfrac{r(v+\delta)(1/s)}{s + (r+\mu+\delta+\gamma+v)(1/s)} + \mu\right]\frac{1}{s}K}} \longrightarrow N(s)$$

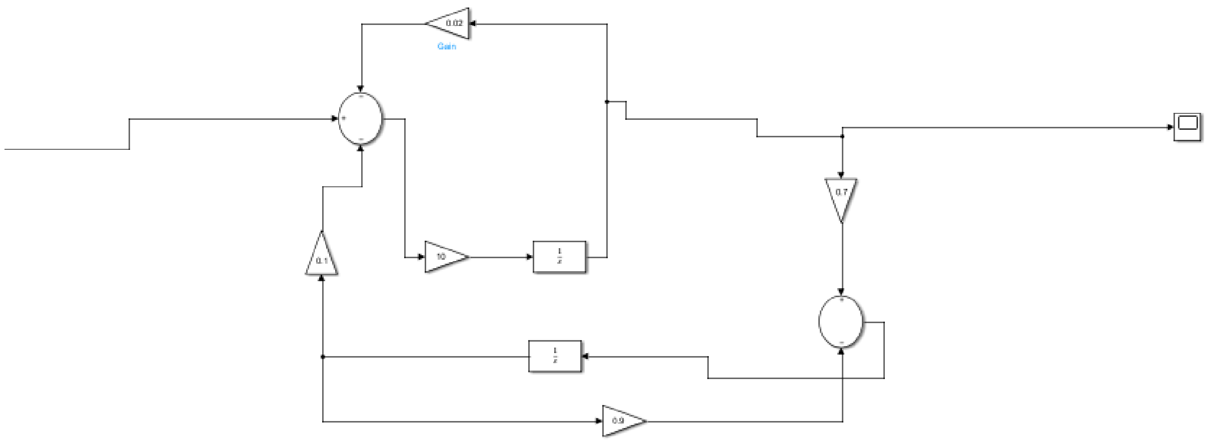
$$\frac{K}{s + \frac{0.072K}{s+0.9} + 0.02K} = \frac{K}{\frac{s^2 + 0.9s + 0.02Ks + 0.015K}{s+0.9}}$$

$$= \frac{K(s+0.9)}{s^2 + (0.9 + 0.02K)s + 0.088K}$$

s^2	1	$0.088K$
s^1	$(0.9 + 0.02K)$	0
s^0	$0.088K$	0

Gain System Characteristics

We are going to use a gain of 15 and below is the system response from the feedforward gain. Below is the block diagram with a gain of 15.

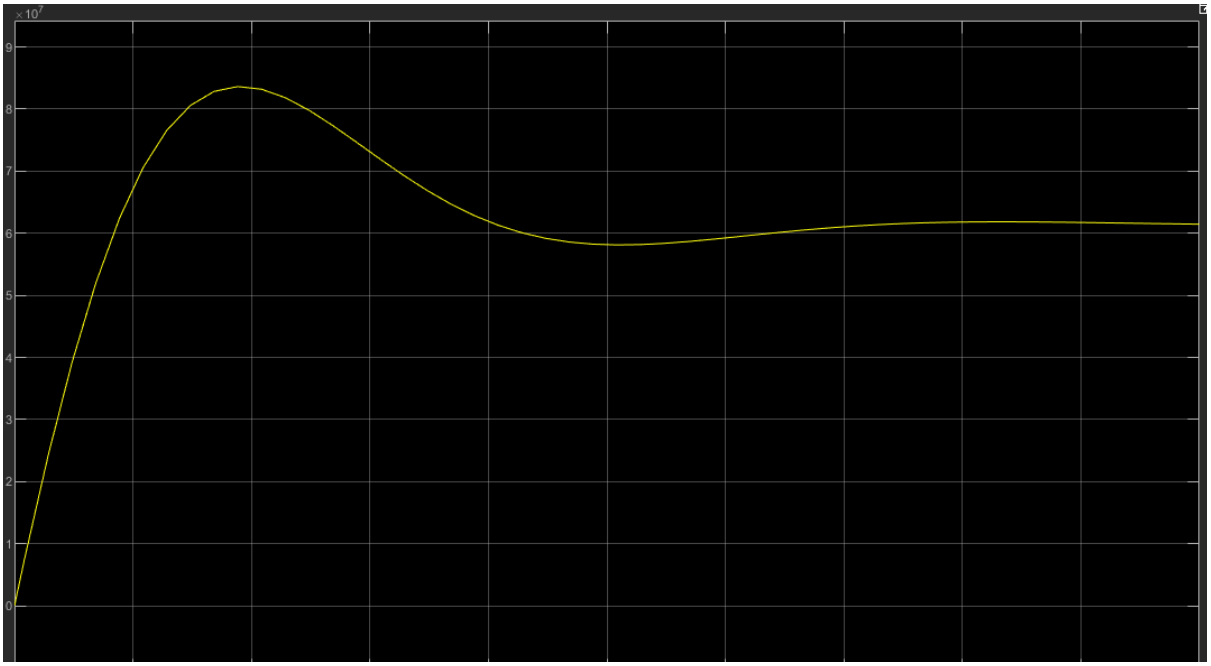


The feedforward TF is $\dfrac{K(s+0.9)}{s^2+(0.9+0.02(15))s+0.088K}$

$$= \frac{15s+13.5}{s^2+1.2s+1.32}$$

Once we reduce the block diagram and substitute 15 for the K value, as we calculated before, it yields this transfer function.

The feedforward gain transfer function yields an underdamped response. The poles will have imaginary components.



Above is the system response for the feed-forward gain.

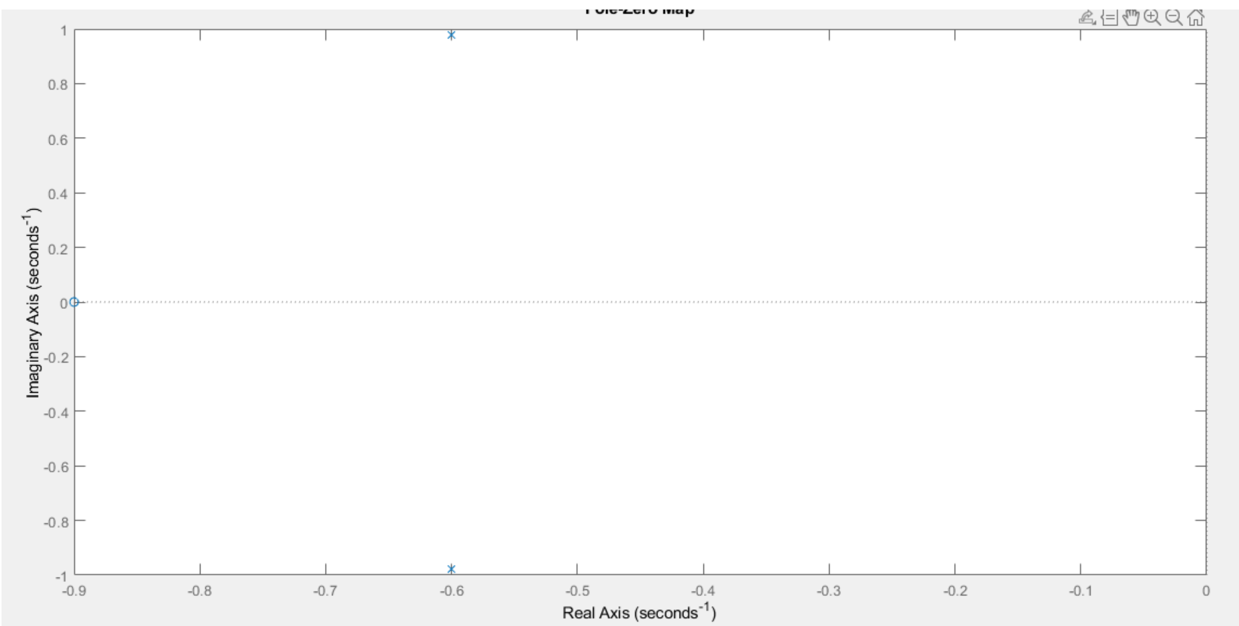
Pole and zero code:

```
s = tf([15, 13.5], [1, 1.2, 1.32])
pol = pole(s)
zer = zero(s)
```

Pole and Zero Value:

```
>> pol = pole(s)

pol =

  -0.6000 + 0.9798i
  -0.6000 - 0.9798i

>> zer = zero(s)

zer =

   -0.9000
```

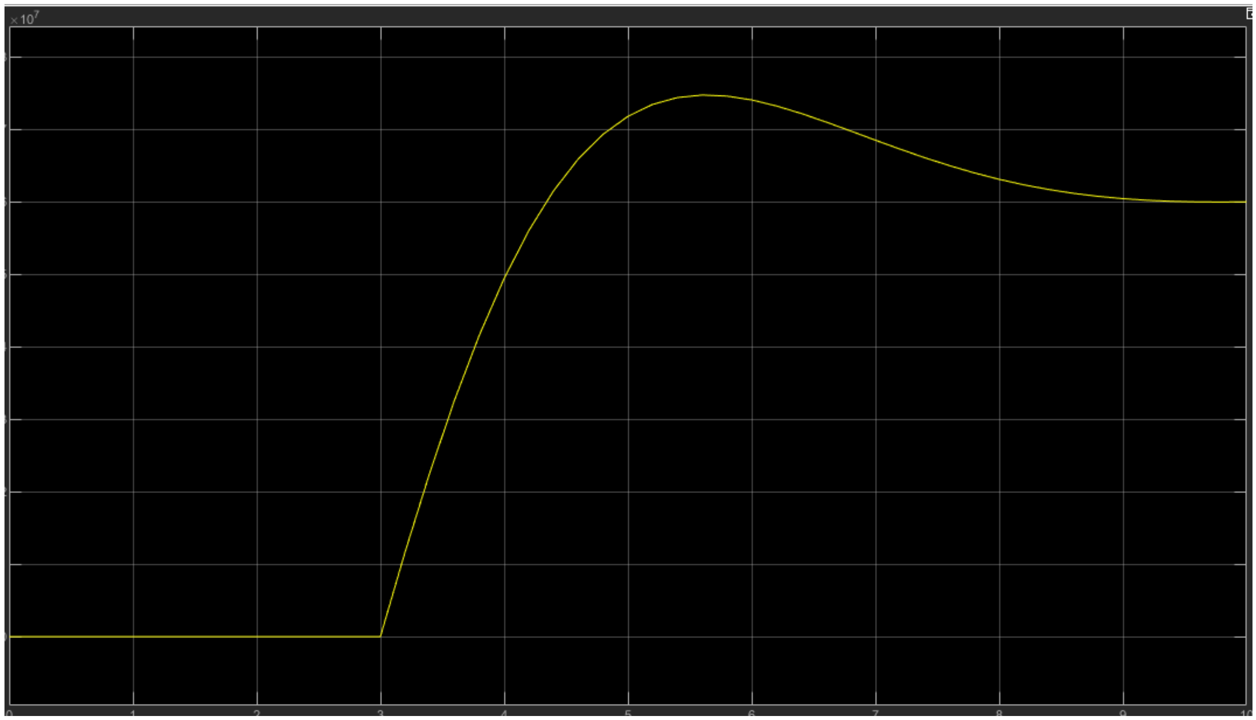
Pole zero map:



As the poles are not on the real axis but on the CLHP, which makes the system underdamped. Since the system is underdamped, it oscillates itself to stability, and its stability is analyzed in the RH section.

The order of the transfer function is second-order as the highest exponent in the denominator is 2.

The feedforward gain makes the system response stable by creating an underdamped response, and due to the tf being underdamped the poles have imaginary components.



This is te step input response for time of 3 seconds of the feed-forward gain transfer function.

Steady State error:

Steady state error

1) Step input : $r_1(t) = u(t)$

$$K_p = \lim_{s \to 0} G(s) = \lim_{s \to 0} \frac{s+0.9}{s(s^2+0.92s+0.088)} = \infty$$

$$e_{ss_1} = \frac{1}{1+K_p} = 0$$

2) ramp input : $r_2(t) = t\,u(t)$

$$\lim_{s \to 0} sG(s) = \lim_{s \to 0} s\left(\frac{s+0.9}{s(s^2+0.92s+0.88)}\right) = 10.27$$

$$e_{ss_2} = \frac{1}{K_v} = \frac{1}{10.27} = 0.0973$$

3) parabolic input $r_3(t)$: $t^2 u(t)$

$$\lim_{s \to 0} s^2 G(s) = \lim_{s \to 0} s^2 \left(\frac{s+0.9}{s(s^2+0.92s+0.088)}\right) = 0$$

$$e_{ss_3} = \frac{1}{K_a} = \infty$$